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Pena Ramirez, J.; Fey, R.H.B.; Nijmeijer, H.

Published: 01/01/2012

Document Version
Accepted manuscript including changes made at the peer-review stage

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Download date: 29. Nov. 2018
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Introduction

In this work, the occurrence of synchronization in weakly nonlinear self-sustained oscillators that interact via Huygens’ coupling, i.e., a suspended rigid bar, is investigated. To this end, the mechanical system depicted in Figure 1 is considered. This system is a generalized version of the original Huygens setup of pendulum clocks. Note that in this model, the pendulum clocks have been replaced by two (actuated) mass-spring-damper (MSD) oscillators.

![Figure 1: Two MSD oscillators with Huygens coupling.](image)

Model description

It is assumed that the oscillators are identical and that the dynamic behaviour of the coupled system is described by

\[
\ddot{x}_i = -\frac{K}{m}(x_i - x_3) + \mu F(x_i, \dot{x}_i, x_3, \dot{x}_3), \quad i = 1, 2, \tag{1}
\]

\[
\ddot{x}_3 = -\mu \sum_{i=1}^{2} \dot{x}_i - \frac{Kc}{M} x_3 - \frac{\beta c}{M} \dot{x}_3, \tag{2}
\]

where \(\mu = \frac{m_i}{M}\) is the coupling strength, and \(F(x_i, \dot{x}_i, x_3, \dot{x}_3) = -\frac{\beta}{M} (\dot{x}_i - \dot{x}_3) + U_i(x_i, \dot{x}_i), i = 1, 2\) is assumed to be an analytic function. This function describes the damping characteristic and the internal source of energy in the two subsystems (1). The need of having \(U_i\) in the oscillators can be linked to Huygens’ pendulum clocks where the energy loss, due to friction, is compensated by an escapement mechanism.

How to determine the existence and stability of synchronous solutions?

By using the Poincaré method of perturbations, in combination with well-known stability criteria for periodic motion, it is possible to derive conditions for the existence and stability of synchronous motion in the oscillators. In fact, it is possible to show that the mass of the coupling bar, which is directly associated with the coupling strength, determines the limit synchronized behaviour in the oscillators, namely in-phase or anti-phase synchronization [1].

Example

Consider system (1-2) and assume that the resupply of energy into the oscillators is provided by the Hamiltonian escapement

\[
U_i = -\lambda (H_i - H^*) \dot{x}_i, \quad i = 1, 2, \tag{3}
\]

where \(H_i = \frac{1}{2} m_i \dot{x}_i^2 + \frac{1}{2} \kappa x_i^2\), \(i = 1, 2\), is the Hamiltonian for the uncoupled and unforced oscillator \(i = 1, 2\), and \(H^* = \frac{1}{2} \kappa x_{ref}^2\) is a reference energy level with \(x_{ref}\) being a reference amplitude. A theoretical analysis has revealed that for a large coupling strength \(\mu\) (a coupling bar with small mass), the anti-phase motion is globally asymptotically stable, whereas for a small coupling strength (a coupling bar with large mass), the in-phase and anti-phase synchronized regimes co-exist and are locally stable. This has been experimentally validated as depicted in Figures 2.

![Figure 2: Experimental results. Left: for large coupling strength the oscillators practically synchronizes in anti-phase. Right: the experiment is repeated by using the same parameters except that now the mass of the coupling bar has been increased. As a result the oscillators synchronize in-phase.](image)

Discussion

The occurrence of synchronization is not influenced by the type of ‘escapement’ used to compensate the energy loss in the oscillators. The key element is the coupling strength, i.e., the ratio between the oscillators’ mass and the mass of the coupling bar.

References