Estimation of Transmission Line Parameters Single-Core XLPE Cables Considering Semiconducting layer

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Abstract—A power cable model for partial discharge (PD) pulse propagation requires knowledge of all factors influencing the transmission line parameters of the cable (characteristic impedance, attenuation coefficient and propagation velocity). These frequency dependent quantities are related, amongst others, to the dielectric properties of the semiconducting layers. This paper discusses methods to estimate the characteristic impedance, attenuation coefficient and propagation velocity using the cable geometry and the material properties of conductor, conductor shield, XLPE and semiconducting layers. Measurements on cable samples show that the cable characteristics can be approximated with sufficient accuracy for PD signal propagation modeling, e.g. estimating the PD magnitude at their origins.

Keywords—modeling, parameter estimation, partial discharges, power cable insulation.

I. NOMENCLATURE

\( \alpha \) attenuation coefficient (Np/km)
\( \gamma \) propagation coefficient
\( \varepsilon' \) relative permittivity of semiconducting material
\( \varepsilon_i \) relative permittivity of XLPE
\( \rho_1 \) conductor resistivity (Ωm)
\( \rho_2 \) semi-conducting screen resistivity (Ωm)
\( \rho_3 \) conductor shield resistivity (Ωm)
\( v_p \) propagation velocity (m/μs)
\( Y \) distributed shunt admittance (S/m)
\( Z \) distributed series impedance (Ω/m)
\( Z_c \) characteristic cable impedance (Ω)

II. INTRODUCTION

In Partial Discharge (PD) based diagnosis of power cables the measured signals must be traced back to the PD magnitudes at the defect site for proper interpretation. A cable model that describes high frequency signal propagation along the cable is required [1]. A transmission line model is normally used for a coaxial structure, such as a power cable [2]. Reference [3] proposed a way to approximate attenuation, and pointed out that detailed knowledge of the dielectric properties of the semiconducting screens is demanded, which in reality is not easy. Reference [4] incorporated the thickness of semiconducting layer into insulation layer to estimate the characteristic impedance and propagation velocity. The attenuation was approximated with measured semiconducting material’s permittivity in [3].

This paper proposes a way to predict the characteristic impedance, propagation velocity and attenuation with the cable geometry and the material properties of conductor, conductor shield, XLPE and semi-conducting layers. This paper differs from [4] in the aspect that the affects to both the distributed series impedance and the distributed shunt admittance from the semiconducting layer are considered.

III. TRANSMISSION LINE MODEL AND SINGLE CORE XLPE CABLE

A typical single core XLPE cable is shown in Figure 1. The conductor is made of aluminum or copper. The conductor screen is a semi-conducting layer extruded around conductor. Around the XLPE insulation, there is insulation screen which is also a semi-conducting layer. In many modern cables semiconducting swelling tapes are wrapped around the insulation screen. Because the electrical properties of this layer are similar to the insulation screen [5], we consider this layer to be part of the insulation screen. The conductor shield is a metallic layer. An often-used construction is copper wires wrapped helically around the cable. These wires are held into place by a counter-wound copper tape. An aluminum foil may be wrapped over the wires and tapes. The outer sheath is usually polyethylene (PE).

Figure 1. Configuration of a single-core XLPE cable.
A cable modeled as transmission line can be characterized by two parameters: the characteristic impedance $Z_c$ and the propagation coefficient $\gamma$. They can be expressed by distributed shunt admittance and distributed series impedance

$$Z_c = \sqrt{Z/Y}$$  \hspace{1cm} (1)$$

$$\gamma = \sqrt{ZY} = \alpha + j\beta$$  \hspace{1cm} (2)

The real part of $\gamma$ is the attenuation coefficient $\alpha$. This frequency dependent parameter describes how waves attenuate due to losses as they propagate through the transmission line. The propagation velocity $v_p$ can be derived from the imaginary part of $\gamma$:

$$v_p = \frac{\omega}{\beta}$$  \hspace{1cm} (3)

IV. PARAMETER MODELING

Reference [6] and [7] provide a general formulation of impedance and admittance of cables without considering semiconducting layer based on the telegraph equation:

$$\begin{cases}
-\frac{dU}{dx} = Z\mathbf{I} \\
-\frac{d\mathbf{I}}{dx} = Y\mathbf{U}
\end{cases}$$  \hspace{1cm} (4)

Here

$$\mathbf{U} = \begin{bmatrix} U_c \\ U_g \end{bmatrix}, \quad \mathbf{I} = \begin{bmatrix} I_c \\ I_g \end{bmatrix},$$

$$\mathbf{Z} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}, \quad \mathbf{Y} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$  \hspace{1cm} (5)

where the subscript $c$ refers to the core conductor to ground and g conductor shield to ground.

A. Distributed series impedance

In [8], a general expression for the impedance of a two-layered conductor is given, which is applicable to a semiconducting layer on the outer surface of a core, on the inner surface of a conductor shield and on the outer surface of the conductor shield of a cable.

Figure 2 Equivalent circuit diagram for matrix $\mathbf{Z}$. The impedances of a cable consisting of conductor, conductor screen, insulation layer, insulation screen and outer sheath are given in the following form [6,8]:

$$Z = \begin{bmatrix}
    z_{10s} + z_{12} + z_{2Is} + z_6 + z_7 - 2z_{2ms} & z_{20s} + z_6 + z_7 - z_{2ms} \\
    z_{20s} + z_6 + z_7 - z_{2ms} & z_{20s} + z_6 + z_7
\end{bmatrix}$$  \hspace{1cm} (6)

where $z_{10s}$ is the internal impedance of core outer surface considering the conductor screen, $z_{12}$ is the XLPE insulation impedance due to the time-varying magnetic field, which is not affected by the semiconducting layers, $z_{2Is}$, $z_{2ms}$ and $z_{2ms}$ are the internal impedance of the conductor shield inner surface, internal impedance of conductor shield outer surface and conductor shield mutual impedance incorporating semiconducting layer, $z_{6}$ is the impedance due to the time-varying magnetic field in outer sheath and $z_{7}$ the self-impedance of the earth-return path. All these impedances are per-unit-length. The equivalent circuit is shown in Figure 2. It should be noted that the circuit in Figure 2 is only to help understand the expression in (6). The detailed deduction of (6) can be found in [6].

For partial discharge detection, a general method is to look into the coaxial mode, which is determined by the coaxial mode impedance of the cable and the admittance between the conductor and the conductor shield. To get the coaxial mode impedance, it is assumed that

$$I_c = -I_g$$  \hspace{1cm} (7)

Thus the current flows only through conductor and earth sheath. Substitution into (5) results in the coaxial mode impedance

$$\frac{d}{dx}(U_c - U_g) = \frac{d}{dx}(U_c + U_g) = (Z_{11} + Z_{22} - 2Z_{12})I_c$$

$$= (z_{10s} + z_{12} + z_{2Is})I_c$$

$$= Z_{coax}I_c$$  \hspace{1cm} (8)

The equations to get these impedances are shown as following:

$$z_{10s} = z_{2Is} - \frac{\pi}{2} \frac{\mu_0}{\omega} \ln\left(\frac{r_1}{r_2}\right)$$

$$z_{12} = j \omega \frac{\mu_0}{2\pi} \ln\left(\frac{r_1}{r_2}\right)$$

$$z_{2Is} = \frac{m_2 \rho_2 N}{2\pi \eta_3 M + z_{22I}}$$

where $z_{2Is}$ is the internal impedance of outer conductor screen surface; $z_{22I}$ is the mutual impedance of conductor screen; $z_{10s}$ is the internal impedance of core outer surface; $z_{2Is}$ is the internal impedance of inner conductor screen surface; $z_{22I}$ is the internal impedance of inner insulation screen surface; $z_{21s}$ is the internal impedance of outer insulation screen surface; $z_{22I}$ is the internal impedance of inner conductor shield surface; $m_2$, $\eta_3$, $M$, $N$ are computation factor. Formulas to derive these variables are listed in the appendix.

B. Distributed shunt admittance

The admittance between the conductor and the conductor shield is given by [8]...
where
\[ y_{s1} = \frac{j\omega \mu \sigma}{\ln \left( \frac{r_2}{r_1} \right)} \]
\[ y_{s2} = \frac{j\omega \mu \sigma}{\ln \left( \frac{r_3}{r_1} \right)} \]
\[ y_{s1} = \frac{j\omega \mu \sigma}{\ln \left( \frac{r_4}{r_3} \right)} \]
\[ \varepsilon' = \varepsilon' + j/\omega \rho \]

(10)
(11)

V. PARAMETER APPROXIMATION

Based on the section IV, the characteristic impedance, propagation velocity and attenuation can be predicted from the cable geometry and the material properties of conductor, semi-conducting screen, XLPE and shielding screen. In order to verify the model, the approximation is compared with measurement result from [4], in which the cable sample is a piece of 82 m, single-core 220 kV XLPE cable with stranded aluminum conductor; a wire screen of 60 wires with a diameter of 1.8 mm helically wound around the cable with a lay length of 1.0 m. The parameters are listed in Table 1. It should be noted that the parameters for semi-conducting material \( (\rho_2, \varepsilon'_s) \) are taken as typical values from [8,9], since the actual values were not specified.

<table>
<thead>
<tr>
<th>Table 1 Parameters to Model Cable Characters</th>
</tr>
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<tbody>
<tr>
<td>( \rho_1 (\Omega \cdot m) )</td>
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<tr>
<td>( \rho_2 (\Omega \cdot m) )</td>
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<tr>
<td>( r_1 (\text{mm}) )</td>
</tr>
<tr>
<td>( r_2 (\text{mm}) )</td>
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<td>( r_5 (\text{mm}) )</td>
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<td>( \varepsilon'_s )</td>
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<td>( \varepsilon )</td>
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A. Characteristic impedance

According to (1), the characteristic impedance is calculated with \( Z_{\text{calc}} \) and \( Y \). The calculated result is compared with the measured result as shown in Figure 3.

B. Propagation velocity

The propagation velocity is calculated by (3). The comparison between the measurement result and calculated result is shown in Figure 4. It needs to be noted here, that the velocity is compensated for the helical wire screen according to [4,10].

\[ v_{p} = \frac{1}{\sqrt{1 + \frac{\left( \frac{r_2}{r_1} \right)^2}{2 \ln \left( \frac{r_3}{r_2} \right)}}} \]

(12)

C. Attenuation

Figure 5 shows the comparison of the attenuation between the calculated result and measured result. It can be observed that the attenuation is within the accuracy of tens of percent. Compared with [4], the semiconducting layers affect to both the distributed series impedance and distributed shunt admittance are considered. The remaining deviation may be caused by the cable dimension tolerance and the choice of resistivity for the semi-conducting material.

VI. CONCLUSION

A model is proposed to predict the characteristic impedance, propagation velocity and attenuation from the cable geometry and the material properties of conductor shield,
XLPE and semi-conducting layers. The model matches well the test data up to 30 MHz for the impedance and propagation velocity. For the attenuation the model has tens of percent deviation between prediction and measurement. Since for PD magnitudes, especially in on-line power cable monitoring, usually the order of magnitude of the apparent charge is relevant, the models’ accuracy is acceptable. Better predictions would require more precise knowledge of the semi-conductor parameters, which in practice is hardly available.

VII. APPENDIX

As described in [8], \( z_{1a} \) in (9) can be derived. While for \( z_{2a} \), it is derived as the following: for a two-layered coaxial structure as in Figure 6, the inner surface impedance \( Z_{a} \) is:

\[
Z_a = \frac{(m_a \rho_a') F}{2 \pi i D} \left( m_a \rho_a' F + m_b \rho_b' E \right)
\] (A.1)

Parameters \( D \) to \( S \) are as follows:

\[
D = m_a \rho_a' F + m_b \rho_b' E
\]

\[
E = I_s(x_a) \cdot K_n(x_a) + I_f(x_a) \cdot K_n(x_a)
\]

\[
F = I_s(x_a) \cdot K_n(x_a) - I_f(x_a) \cdot K_n(x_a)
\]

\[
G = I_s(x_a) \cdot K_n(x_a) + I_f(x_a) \cdot K_n(x_a)
\]

\[
H = I_s(x_a) \cdot K_n(x_a) - I_f(x_a) \cdot K_n(x_a)
\]

\[
P = I_s(x_a) \cdot K_n(x_a) + I_f(x_a) \cdot K_n(x_a)
\]

\[
Q = I_s(x_a) \cdot K_n(x_a) - I_f(x_a) \cdot K_n(x_a)
\]

where \( m_a = \frac{j \omega \mu_a}{\rho_a} \), \( m_b = \frac{j \omega \mu_b}{\rho_b} \) and

\[
x_1 = m_a a, x_2 = m_b b, x_3 = m_b b, x_4 = m_b c.
\]

Thus,

\[
Z_a = \frac{(m_a \rho_a') F}{2 \pi i D} \left( m_a \rho_a' F + m_b \rho_b' E \right)
\] (A.2)

\[
Z_{2a} = \frac{(m_a \rho_a') P}{2 \pi i H} \left( m_a \rho_a' P + m_b \rho_b' E \right)
\] (A.3)

\[
Z_{2a} = \frac{m_a \rho_a' P}{2 \pi i H} \left( m_a \rho_a' P + m_b \rho_b' E \right)
\]

It is the same format as \( z_{2a} \) in (9). To derive all equations in (9),

\[
z_{2a} = \frac{m_a \rho_a' (I_s(m_a) \cdot K_n(m_a) + I_f(m_a) \cdot K_n(m_a))}{2 \pi i} \left( m_a \rho_a' (I_s(m_a) \cdot K_n(m_a) + I_f(m_a) \cdot K_n(m_a)) \right)
\]

\[
z_{2a} = \frac{m_a \rho_a' (I_s(m_a) \cdot K_n(m_a) - I_f(m_a) \cdot K_n(m_a))}{2 \pi i} \left( m_a \rho_a' (I_s(m_a) \cdot K_n(m_a) - I_f(m_a) \cdot K_n(m_a)) \right)
\]

\[
z_{2a} = \frac{m_a \rho_a' I_s(m_a)}{2 \pi i} \left( m_a \rho_a' I_s(m_a) \right)
\]

\[
z_{2a} = \frac{m_a \rho_a' I_f(m_a)}{2 \pi i} \left( m_a \rho_a' I_f(m_a) \right)
\]

\[
z_{2a} = \frac{(m_a \rho_a') P}{2 \pi i H} \left( m_a \rho_a' P + m_b \rho_b' E \right)
\]

\[
z_{2a} = \frac{(m_a \rho_a') P}{2 \pi i H} \left( m_a \rho_a' P + m_b \rho_b' E \right)
\]

Where \( m_k = \frac{\sqrt{j \omega \mu_k}}{\rho_k} \) \((k = 1, 2, 3)\), \( \mu_k \) is the permeability for conductor, semi-conducting screen and conductor respectively. For all, the permeability of vacuum is taken.

VIII. ACKNOWLEDGMENT

The authors gratefully acknowledge the helpful discussion of Professor Akihiro Ametani via Email.

IX. REFERENCES


