Compensation for Dynamic Errors
of
Coordinate Measuring Machines

Proefschrift

ter verkrijging van de graad van doctor
aan de Technische Universiteit Eindhoven,
op gezag van de Rector Magnificus, prof.dr. J.H. van Lint,
voor een commissie aangewezen door het College van Dekanen
in het openbaar te verdedigen op
donderdag 27 juni 1996 om 16.00 uur

door

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geboren te Weert
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ISBN 90-386-0178-6

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Summary

This thesis presents a method for compensating the dynamic measurement errors of coordinate measuring machines (CMMs). Dynamic errors degrade the accuracy of the measurement result and they occur when the CMM is still subjected to accelerations when taking measurements. Therefore accelerations at the time of probing are often minimised by performing measurements with low probing speed and long settling times. However, in order to increase CMM productivity higher probing speeds of CMMs are desired. The main goal of the research described in this thesis is the estimation of the dynamic errors that occur at the CMM's probe position in case of higher probing speeds. Using the estimated probe error, compensation of the measuring result can be achieved. The assessment of dynamic errors of CMMs has received little attention until now and therefore this thesis can be an useful contribution to the research concerning dynamic error modelling and accuracy improvement of CMMs.

Mainly the dynamic errors of CMMs due to axis motion are studied in this thesis. During acceleration and deceleration of the machine's axes, the various components of the CMM's structural loop are deformed. Analysis of the most commonly used CMM types, makes clear that for most conventional CMMs dynamic errors can be expected in case of fast probing. This is shown in practice by conducting measurements on an existing CMM. Quasi-static as well as vibration errors were found to be quite large in relation to the static errors of the CMMs.

The method developed here for estimating the dynamic errors consists of the identification of the so-called parametric errors and calculation of their effects on the measuring error at probe position, using a kinematic model. The same kinematic model that has been used for other type of errors, is used here for calculating the effects of the dynamic errors. This is advantageous, since in this way a
modular compensation system can be obtained. In the kinematic model the CMM structure is considered rigid and the parametric errors can be considered as the errors in the degrees of freedom of the kinematic model. However, the components of a CMM are actually flexible elements, introducing quasi-static deformations and vibrations due to the accelerations. These deformations result into dynamic parametric errors. In order to estimate these errors of a CMM, a combined analytical and empirical approach has been followed. With additional sensors attached to the carriages of a CMM, rotations and translations of the carriages with respect to their guideways have been measured on-line. In this way the sensors measure part of the deformations. Simple relationships have been formulated between the deformations measured by the sensors and the other deformations. These relations have been used to express the parametric errors (i.e. the combined effect of the deformations) into the sensor measurements. With the kinematic model the effects of the estimated parametric errors on the probe position have been calculated.

The developed method has been applied to an existing CMM. For this CMM a limited number of parametric errors were found significant. These are mainly rotation errors of the CMM's joints. Deformation of the guideways is negligible, but support motion must be accounted for. Inductive position sensors have been mounted on the CMM's x- and y-carriage for on-line measurement of the errors. Test have shown that the sensors can accurately measure the deformation during axis motion of the CMM. Based on the sensor readings and a kinematic model of the CMM, the time history of the dynamic error at probe position can be calculated. For verification, experiments have been conducted, comparing the estimated error at the probe position with the actual error measured during linear motion using laser interferometry. The compensation method proved to be very successful. When operating the CMM at maximum traverse speed, 90% of the dynamic error could be compensated for, leaving a maximum residue of 2.5 μm. For motion under joy-stick control, causing higher accelerations, also 90% of the error could be compensated for with a maximum residue of 5 μm. The residues have the same order of magnitude as the CMM's static inaccuracy. These results show that, by compensating for dynamic errors of CMMs, still a high accuracy can be obtained when the CMM is subjected to accelerations. This means that fast probing is possible without degradation of measurement accuracy.
Samenvatting

In dit proefschrift wordt een methode gepresenteerd voor de compensatie van dynamische afwijkingen bij coördinaten meetmachines (CMMs). Dergelijke afwijkingen beïnvloeden het meetresultaat en worden veroorzaakt door versnellingen op de CMM tijdens het meten. Deze versnellingen worden vaak geminimaliseerd door met lage snelheid aan te tasten en door lange wachttijden in acht te nemen. Om echter de produktiviteit van CMMs te verhogen zijn hogere aantastsnelheden en kortere wachttijden gewenst. Doel van het onderzoek is de schatting van de dynamische afwijkingen op de taster positie in geval van snel aantasten. Met het geschatte verloop van de afwijkingen, kan het meetresultaat gecompenseerd worden. Tot nu toe is weinig aandacht besteed aan dynamische afwijkingen bij CMMs. Dit proefschrift is daarom een nuttige bijdrage aan het onderzoek betreffende afwijkingen modellering en nauwkeurigheidsverbetering van CMMs.

Met name de dynamische afwijkingen ten gevolge van asbewegingen van de CMM, worden in het onderzoek beschouwd. Tijdens versnellen en vertragen van de assen worden de verschillende componenten van de mechanische CMM structuur gedeformeerd. Uit analyse van veel toegepaste typen CMMs, blijkt dat voor de meeste machines dynamische afwijkingen te verwachten zijn bij snel aantasten. Metingen aan een bestaande CMM bevestigen dit en laten zien dat zowel quasi-statische afwijkingen als trillingen optreden die relatief groot zijn vergeleken met statische afwijkingen.

De hier ontwikkelde methode voor het schatten van dynamische afwijkingen bestaat uit de bepaling van de zogenaamde parametrische afwijkingen en de doorrekening van hun effecten op de tasterpositie met behulp van een kinematisch model. Hiervoor wordt hetzelfde kinematisch model toegepast dat ook wordt gebruikt voor de doorrekening van andere typen afwijkingen, waarmee een modu-
lair compensatie systeem verkregen wordt. In de kinematische modellering wordt de CMM star verondersteld en zijn de parametrische afwijkingen te beschouwen als afwijkingen in de vrijheidsgraden van het kinematisch model. In werkelijkheid zijn de CMM componenten flexibel, zodat optredende versnellingen deformaties veroorzaken. Deze deformaties veroorzaken de dynamisch, parametrische afwijkingen. Voor de schatting van deze afwijkingen is een gecombineerde analytische- en empirische aanpak gevolgd. Door toevoeging van sensoren aan de sledes van een CMM kunnen translaties en rotaties van de sledes ten opzichte van de geleidingen gemeten worden. Hiermee meten de sensoren een deel van de optredende deformaties. Eenvoudige relaties zijn opgesteld om andere optredende deformaties uit te drukken in de gemeten deformaties. Met behulp van deze relaties zijn de parametrische afwijkingen (het gecombineerde effect van alle deformaties) uitgedrukt in de sensor metingen. Met het kinematisch model is het effect van de geschatte parametrische afwijkingen op de tasterpositie bepaald.

De ontwikkelde methode is toegepast op een bestaande CMM. Bij deze CMM zijn een beperkt aantal parametrische afwijkingen significant. Het betreft hier met name rotatieafwijkingen bij de sledes. Buiging van de geleidingen is niet significant, maar afwijkingen van de y-as ondersteuning moeten in acht genomen worden. Op de x- en y-sledes van de machine zijn inductieve positie sensoren geplaatst voor het on-line meten van de afwijkingen. Uit experimenten blijkt dat met deze sensoren de deformaties tijdens asbewegingen nauwkeurig bepaald kunnen worden. Met behulp van de sensorsignalen en het kinematisch model van de meetmachine, kan het verloop van de afwijking in de tijd op tasterpositie berekend worden. Ter verificatie zijn experimenten uitgevoerd, waarbij de berekende afwijking vergeleken is met de afwijking op tasterpositie tijdens lineaire beweging, gemeten door een laserinterferometer. De compensatiemethode blijkt zeer succesvol te zijn. In geval van automatische aansturing van de CMM met maximale snelheid, kan voor 90% van de afwijkingen gecompenseerd worden, waarbij het maximale residu 2.5 μm bedraagt. Tijdens joystick besturing van de machine kan eveneens voor 90% van de afwijkingen gecompenseerd worden. In dit geval bedraagt het maximale residu 5 μm, hetgeen vergelijkbaar is met statische onnauwkeurigheid van de CMM. Uit de resultaten blijkt dat door compensatie van de dynamische afwijkingen toch een hoge nauwkeurigheid bereikt kan worden indien er nog versnellingen op de machine werken. Dit houdt in dat sneller meten zeer wel mogelijk is zonder groot verlies aan nauwkeurigheid.
## Notation

The list presented here gives an overview of the notation with respect to the most frequently used symbols.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i$</td>
<td>-</td>
<td>CMM axis ($i = x, y, z$)</td>
</tr>
<tr>
<td>$j$</td>
<td>-</td>
<td>direction of translation or axis of rotation ($j = x, y, z$)</td>
</tr>
<tr>
<td>$ir_j$</td>
<td>arcsec</td>
<td>parametric rotation error of the $i$-axis carriage about the $j$-axis</td>
</tr>
<tr>
<td>$\varepsilon_j$</td>
<td>arcsec</td>
<td>rotation error about the $j$-axis at some part of the machine's structural loop</td>
</tr>
<tr>
<td>$i\varepsilon_j$</td>
<td>arcsec</td>
<td>rotation error about the $j$-axis of some part of the machine's structural loop belonging to the $i$-axis</td>
</tr>
<tr>
<td>$i\varepsilon_{j,c}$</td>
<td>arcsec</td>
<td>contribution to the rotation error about the $j$-axis of a particular component $c$ belonging to the $i$-axis</td>
</tr>
<tr>
<td>$it_j$</td>
<td>$m$</td>
<td>parametric translation error of the $i$-axis carriage in the $j$-direction</td>
</tr>
<tr>
<td>$\delta_j$</td>
<td>$m$</td>
<td>translation error in $j$-direction at some part of the machine's structural loop</td>
</tr>
<tr>
<td>$i\delta_j$</td>
<td>$m$</td>
<td>translation error in $j$-direction of some part of the machine's structural loop belonging to the $i$-axis</td>
</tr>
<tr>
<td>$i\delta_{j,c}$</td>
<td>$m$</td>
<td>contribution to the translation error in $j$-direction of a particular component $c$ belonging to the $i$-axis</td>
</tr>
<tr>
<td>$iS_j$</td>
<td>$m$</td>
<td>measured displacement by a position sensor. The subscript $i$ indicates the corresponding CMM axis and $j$ the axis along which the sensor is measuring</td>
</tr>
<tr>
<td>$p$</td>
<td>$m$</td>
<td>vector indicating the actual position of the CMM probe</td>
</tr>
</tbody>
</table>
vector indicating the nominal position of the CMM probe, based on the readings of the machine's scales

vector indicating the total error at probe position

vector containing the parametric rotation errors belonging to the i-axis

vector containing the parametric translation errors belonging to the i-axis

vector containing the effective arm between the scale at the i-axis and the probe position.

vector indicating the position of the probe tip relative to the position of the probe head

mass matrix of a dynamic system

damping matrix of a dynamic system

stiffness matrix of a dynamic system

vector containing generalised coordinates of the system

vector representing the dynamic load acting on the system

distributed load

concentrated force

distributed load in the direction i

concentrated force in the direction i

concentrated moment

displacement of a segment of a beam

rotation of a segment of a rod

cross-sectional area of a beam element

the Young's modulus

the second moment of inertia of a component c

the shear modulus of elasticity

mass moment of inertia of a component c

mass of a component c

stiffness of a component c belonging to the j-axis with respect to the i-axis or in i-direction

length of a component c

effective arm of rotation in direction i

carriage width
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With quality control being an important issue in industry, there is a great need for dimensional measurements in manufacturing. Coordinate measuring machines (CMMs) are nowadays widely used for a large range of such measurement tasks. They are used to check if a product has been manufactured within the tolerances, as well as for identifying trends in the manufacturing process. By monitoring a manufacturing process in this way, adjustment to the process can be made and the manufacturing of products that don't meet the specifications can be avoided. With manufacturing processes becoming increasingly more flexible, there is also a demand for increasing flexibility and integration of measuring machines. Due to their structure with at least three degrees of freedom, and their high level of automation CMMs are especially suitable for flexible measurement of complex products.

1.1 CMM description and operation

In practice the Cartesian, orthogonal coordinate system, necessary to reach and measure any position in 3-dimensional space within the measuring range, is most commonly achieved by an arrangement of three perpendicular translation axes with linear scales. Figure 1.1 is showing an example of a common CMM structure. For efficiency reasons some machines are equipped with an extra, rotary axis.
Figure 1.1: Example of a common CMM structure.

Figure 1.2: Schematic overview of the major CMM components.
In the schematic overview of Figure 1.2 the most important CMM components are shown. The base of the CMM is formed by a table on which the workpiece to be measured is placed. The CMM axes are arranged around this table. Each CMM axis consists of a guideway, a carriage that can move along the guideway, and a measurement system. For accurate motion along the guideways, most modern CMM carriages have air bearings. The carriage position of a certain axis is accurately indicated by the linear scale attached to the respective guideway. The readings of all three scales together indicate the 3D-position of the probe connected to the last axis. The probe is used to establish measuring points on the workpiece. Depending on the type of control, the CMM can be equipped with drive systems consisting of motors and transmissions. The CMM can be either manual controlled, joystick controlled or Computer Numerical Controlled (CNC). With manual control no drives are available. The CMM carriages are so-called free-floating and they are moved to the various measuring positions on the workpiece by an operator. With joystick control the CMM axes are servo-controlled and motion commands for each axis are given by the operator using joysticks. In case of CNC the CMM axes are moved automatically using servo-control and a computer that provides the motion commands. The latter method is the most efficient, since similar measurements can be repeated automatically. Furthermore a high accuracy can be reached, because measuring points can be taken in a well controlled manner keeping the level of the accelerations low. However for economic reasons manual controlled CMMs are still very popular and widely used. Joystick control is often used in order to generate the position data for the CNC measurement programs.

Measurement points are generated by establishing a defined contact (i.e. with known measurement force) between workpiece and a probing device connected to end of the last CMM axis. Usually the probe is a mechanical device, consisting of a housing (the probe head) that is supporting a stylus with at the end a sphere, the stylus tip. Displacement of the stylus in the support, due to a mechanical load at the stylus tip, is electronically detected and a trigger signal is provided to the controller. This signal is used to read the values of the scales of all axes by a computer. Measurement software is used to transform the measured points into a local workpiece coordinate system. Based on these coordinate values dimensions and shapes of the workpiece can be calculated.
Chapter 1

Considering the performance of CMMs, the most important criteria are accuracy, speed and flexibility (see e.g. Neumann 1993). Measurement tasks are expected to be carried out with ever increasing performance in these terms: higher accuracy and speed are demanded as well as the ability to operate under worse environmental conditions. Research is necessary to meet these demands. Until now research effort regarding CMM accuracy is mainly spent on quasi-static mechanical errors, not considering dynamic errors. However there are some trends concerning the use of CMMs that make an assessment of the dynamic errors of CMMs increasingly more important. These trends are:

- **Increase in variety** of measurement tasks. Due to their structure and high level of automation CMMs are universal measurement machines, and very flexible with respect to the often complex measurement tasks they can handle. This ability is being recognised more and more, resulting in the replacement of measurement devices for specific measurement tasks by CMMs. Roundness measurements for example are usually carried out on a dedicated instrument, but depending on the accuracy demanded, a CMM might be used. The flexibility of CMMs makes it also possible to measure complex surfaces and profiles. Compared to simple dimensional measurements, these more complex measurement tasks often involve more complex motion, making such tasks more prone to errors (e.g. dynamic errors).

- **Changes in the location** of CMMs. In principle, (accurate) measurement tasks are best carried out in the controlled environment of a measurement laboratory. However there is an increasing trend to locate CMMs near the manufacturing process or even integrate them with production lines (e.g. Weule 1987, Fix 1989, Weckenmann 1990, Neumann 1993). This trend is mainly driven by the demand for faster inspection of produced parts, reinforced by an increasing frequency of inspections. Of course environmental conditions at the shop floor (vibrations, thermal effects) are far worse than laboratory conditions, resulting into errors and a degradation of measurement accuracy. In order to maintain a high level of accuracy the influence of these errors on the CMM has to be reduced.
• **Shorter cycle times** of measurement tasks. In order to obtain accurate measurement results, probing speeds are often kept very low. In laboratories this is generally acceptable. However for inspections tasks on (semi)manufactured products, short cycle times are demanded for economic reasons. As a consequence CMMs are expected to operate with higher speed (e.g. McMurty 1980, Sutherland 1987, Lu 1992, Jones 1993, Lotze 1993, Neumann 1993). This trend is reinforced by the trend mentioned before: the increasing integration of CMMs in manufacturing processes. With increasing operation speeds and thus accelerations the effects of dynamic errors will also be increased.

• **Certification of measurement results.** The increasing awareness of customers for quality is pushing manufacturers to proof the quality of their products by certified measurements, especially in the case of semi-manufactured products. This trend results in more attention for traceability of the accuracy of measurement results and the level of confidence of these results. In order to calculate the (traceable) accuracy and the uncertainty of a measurement task, sufficient knowledge about the systematic and random errors affecting measurement accuracy and methods to calculate their effects are necessary (see e.g. Kunzmann 1993, Phillips 1993, Soons 1993).

From the trends mentioned here, it is obvious that extensive knowledge about the errors affecting the accuracy of CMMs is necessary. Among these errors the dynamic errors are becoming more important, mainly due to the demand for better CMM performance in terms of speed. Before stating the research goals with respect to these errors, an overview of the most important error sources will be given.

### 1.2 CMM error sources

Being complex machines with multiple axes, generally servo-controlled and used for complex measuring tasks with high accuracy specifications, CMMs are prone to many error sources. Based on the functional components of a CMM, an overview will be given of the most important error sources affecting the accuracy of a CMM:
• **Mechanical system.** The main components of a CMM structure are the table for supporting the measurement objects, the guideways, and the carriages with the bearings. These components are causing errors due to inaccuracies related to manufacturing, adjustment and component properties such as stiffness and thermal expansion. The nature of these errors can be static or quasi-static, as well as dynamic. These errors will be discussed later in more detail.

• **Drive systems.** For CNC operated CMMs the axes are equipped with drives, transmissions and a servo-control unit. Errors that can be related to the drive system and may affect the measuring accuracy are: an incorrect, non-constant measuring speed, mechanical load on the carriage causing unwanted carriage motion, and introduction of vibrations to the mechanical structure. Positioning errors are in general not important, since the coordinates of measurement points are derived from the *measured* positions (by the scales) and not by the *commanded* positions.

• **Measurement system.** The actual coordinates of the measuring points are derived from the values indicated by the linear scales of the CMM. The main errors introduced by the scales are inaccuracy of the scale pitch, misalignment and adjustment of the reading device, interpolation errors, and digitisation errors. For the detection of a measuring point at the surface of the workpiece the probe system is used. Several error sources can be distinguished with the probe system, such as hysteresis in the stylus support, stylus bending, and errors in the measuring system. Also the electrical (trigger) signals from the probe system are error sources, mainly due to (variation of) time delays. A more detailed discussion on probe errors can be found in Butler 1990 and Vliet 1996.

• **Computer system.** The computer system, including the control unit, involves both the hardware and the software. Errors in the hardware are not very common, therefore they will not be discussed here. An important task of the software is performing the calculations to transform the measuring points into workpiece coordinates and to derive the demanded workpiece dimensions and shapes. Errors in these calculation algorithms do occur and they can seriously affect and thus degrade the accuracy of the measuring result.
Introduction

Besides the above mentioned CMM related error sources, CMM measuring accuracy is also influenced by external influences that can be related to the operator or the environment of the CMM. Major operator influences and error sources are product handling, measurement strategy, and actual CMM operation. Product handling refers to preparatory tasks before measuring, like climatisation, cleaning and clamping of the workpiece. If not done correctly, such tasks introduce errors due to a dirty workpiece or due to its temperature- or mechanical deformation. Measurement strategy involves the selection of probing points on the workpiece. Their position on the workpiece has great influence on the accuracy of the calculated measuring result. CMM operation includes correct probing with constant measuring speed perpendicular to the workpiece surface in order to establish a defined contact. Especially when operating a manual CMM, probing is prone to errors, because it is difficult to control the measurement force.

Very important with respect to the measuring accuracy is the environment in which the CMM is placed. Temperature disturbances of the environment in general seriously affect the geometry of the mechanical structure and thus the measuring accuracy. Similar, vibrations, mainly from other machines located near the CMM, can degrade measuring accuracy. Most often these vibrations are transmitted via the ground and through the CMM support and they cause relative motion of the CMM probe with respect to the workpiece on the table. These errors are discussed later in more detail. Another environmental error source is the variation of the humidity of the air, causing component deformation. Especially granite is prone to humidity variations.

With respect to the research described in this thesis, especially the errors affecting the mechanical structure are important. The nature of these errors is either quasi-static or dynamic. Quasi-static mechanical errors are defined as errors related to the structural loop of the machine that are slowly varying in time (see Hocken 1980). Whether or not an error can be considered varying slowly, depends on the time scale of the relevant process (i.e. measuring). The structural loop of a CMM comprises the mechanical elements that together define the relative position and orientation of the measuring probe towards the measuring object. The accuracy of a measuring task is primarily determined by the accuracy of the structural loop and thus by the errors affecting it. Most of the research concerning CMM accuracy has been focused on the quasi-static errors. When considering
the mechanical accuracy of multi-axis machines such as CMMs three main sources of quasi-static mechanical errors can be distinguished (see e.g. Schellekens 1993, Soons 1993):

• **Geometric errors.** These are errors due to the limited accuracy of the components, like guideways and measurement systems and depend on the manufacturing accuracy of these components and the adjustment accuracy during installation or maintenance. Geometric errors of the guideways are straightness and rotation errors and their relative orientation is subjected to squareness errors. The measuring scales cause errors in the measured position along the axes (the linearity errors).

• **Errors due to mechanical loads.** These are errors related to static- or slowly varying forces on the CMMs components in combination with the compliance of the components. This variation of the mechanical load is mainly caused by the weight of moving parts. As a result components will deform from their nominal shape and cause geometric errors as described above. These errors depend on the stiffness and weight of the components and their configuration.

• **Thermally induced errors.** These are errors due to the temperature field in the machine and workpiece. Two types of thermally induced errors can be distinguished. First, a uniform difference between the temperature of the measuring standard (i.e. the measuring scales of the CMM) and the workpiece will cause measuring errors. Second, temperature gradients introduced in the machines components will cause deformations like bending of the guideways and thus geometric errors. The errors depend on the machine structure, material properties and the temperature distribution of the CMM, influenced by external sources such as the environmental temperature and by internal heat sources such as the drives.

Besides these quasi-static errors, which behaviour is in general well known, CMMs are also influenced by **dynamic errors.** These are errors that vary relatively fast in time, like acceleration dependent deformation of CMM components due to part movements and vibrations, both self-induced and forced. Similar as to the quasi-static errors, dynamic errors affect the geometry of the CMM mechani-
Introduction

This results in time-depending measuring errors. These dynamic errors depend on the CMMs structural properties, like mass distribution, component stiffness and damping characteristics, as well as on control- and disturbing forces.

For high measurement accuracy the effects of error sources on the CMM accuracy, mentioned in the previous paragraph, have to be small. So a lot of effort is spent to eliminate these error sources or to keep them small. With respect to the errors affecting the mechanical structure this yields in principle the following conditions for CMM design and operating conditions:

- high manufacturing and adjusting accuracy.
- high component stiffness, low mass, and good temperature properties.
- temperature conditioned environment and small internal heat sources.
- vibration isolation and well defined motion during probing.

The research described in this thesis deals with the measuring accuracy of CMMs. More particularly the influence of dynamic errors on the mechanical structure of CMMs and by this on the CMM measuring accuracy is studied. In the next paragraph the objectives with respect to this research will be stated and discussed in more detail.

1.3 Research objectives

Some of the design and operating conditions for high measuring accuracy, mentioned in the previous paragraph, are either difficult to combine, like high stiffness and low mass or they are putting restrictions on some of the trends concerning the use of CMMs, like locating CMMs at the shop floor or the demands for shorter cycle times. An important restriction for cycle time reduction is the probing procedure. During probing, CMM speed is limited in order to avoid dynamic errors. An alternative for restricting operating conditions is to obtain sufficient knowledge of all the errors and to apply software error compensation for these errors. This method has been applied successfully by several researchers for geometric, and thermal errors and errors due to mechanical loads, since these errors are highly systematic and can be well described. Due to their more com-
plex nature, dynamic errors of CMMs received little attention until now. Since it is difficult to obtain an accurate description of dynamic errors at the probe position, they are in general regarded as random and not suitable for software error compensation. Therefore only design measures and improved CMM control to minimise accelerations are used.

However, in order to avoid restrictions with respect to shorter cycle times, accelerations (thus dynamic errors) during probing have to be accepted. Research effort is needed to find descriptions of dynamic errors during fast probing. The research described in this thesis is focused on this subject. The main goal of this research is to estimate the dynamic errors on the probe position at the time of probing in case the CMM is still subjected to accelerations. Based on the estimated errors, compensation of the measuring result is possible. If successful compensation for dynamic errors can be achieved, faster probing without degradation of the measuring accuracy will be possible. With these objectives, the research described in thesis is a continuation of the important research on error modelling and accuracy improvement of CMMs. This research proved successful with respect to the compensation of geometric errors, errors due to static and slowly varying forces (mechanical loads), and thermally induced errors. By assessing the dynamic errors of CMMs in this thesis a new subject regarding CMM accuracy is being studied.

In this thesis mainly dynamic errors due to axis accelerations are studied. Before the dynamic errors at the probe position can be estimated a better understanding of the dynamic errors of CMMs has to be obtained. Therefore measurement and analysis of the dynamic behaviour of CMMs is necessary in order to identify the CMM components that introduce significant dynamic errors. A strategy has to be developed to capture these dynamic errors during probing, either based on modelling or on the use of additional sensors for on-line measurement of the errors. A method for calculating the effect of all the identified errors on the probe position has to be derived. Thus an estimation of the dynamic errors at the probe position can be achieved. Based on the developed method for error estimation, a compensation method for the measurement errors has to be developed and tested on an existing CMM. Especially, attention should be paid to finding possibilities to avoid the use of many sensors, since this constrains the economic usefulness of the developed method.
1.4 Outline of the thesis

The assessment of dynamic errors of CMMs, described in this thesis, consists of four main parts: the analysis of the dynamic errors, the modelling and measurement of the errors and a compensation strategy for reducing the effect of dynamic errors on the measurement result.

In Chapter 2 first an overview of the literature concerning the accuracy of CMMs is given. Next the concept of fast probing is discussed. A clear definition of the resulting dynamic errors is given and a distinction between types of dynamic errors is made. In order to indicate the significance of the dynamic errors, both a theoretical and an experimental analysis are performed. The analysis is followed by a brief discussion of different type of methods for reducing the dynamic errors of CMMs. Finally the strategy adopted here is presented.

Chapter 3 deals with the modelling of the dynamic errors. Similar to the way quasi-static errors are dealt with, the assessment of the dynamic errors consists of two parts: identification of the individual dynamic, parametric errors and prediction of their effects on the probe position, using a kinematic model. A general kinematic model for CMMs is presented. The components of a CMM structure are considered as flexible elements. Thus dynamic errors are introduced in case of accelerations. In order to calculate their effects on the probe position, these errors have to be identified. A general approach, for estimating these errors is adopted. This approach is based on the use of additional position sensors. Mathematical expressions relating the dynamic errors to the sensor readings are given. Based on the estimated error values, the error on probe position can be calculated using the kinematic model.

In Chapter 4 the significant dynamic errors of an existing CMM are identified. The various measurements conducted in order to identify these errors, are described. The most important measurement results are presented and an overview of the most significant errors is given. Based on the measurement results, suitable displacement sensors are selected for measuring dynamic errors of the CMM on-line. Several test are conducted to verify their performance.
Chapter five covers the actual compensation of the investigated CMM for dynamic errors during fast probing. A kinematic error model for the CMM is given. The significant errors are measured on-line by the implemented sensors. Error models are given that relate the dynamic errors to the sensor readings. Based on these models and the sensor readings the measurement error at probe position is calculated using the kinematic model. The calculated errors are used as compensation values for the measurement result. The compensation method is verified for the CMM by comparing calculated error values with measured values, using laser interferometry. The results of the error compensation are largely depending on the accuracy of the modelling and the sensors, as well as the number of sensors used. To a certain extend this is a balance between the costs and benefits. The possibility of reducing the number of sensors are discussed. This thesis will be completed by conclusions and recommendations given in Chapter 6.
2

Analysing dynamic errors

In this chapter dynamic errors of CMMs are discussed in more detail. First a brief overview of CMM accuracy research is given. Measuring concepts and probe types for different measurement tasks are described and the influence of fast probing is considered. The resulting dynamic errors are discussed and the sensitivity of the most common types of CMMs for these errors is established. In order to indicate the significance of the dynamic errors, examples of these errors for existing CMMs are presented. The examples are followed by an overview of the relevant literature with respect to dynamic errors of CMMs and a brief discussion on different methods for reducing the dynamic errors of CMMs, like design-, control-, and error compensation. At the end of this chapter the strategy that has been adopted here is presented.

2.1 Research on CMM accuracy

Considering CMMs a lot of research effort has been paid to enhance their performance, mainly their measuring accuracy. First research was aimed at the assessment of CMM accuracy, developing measuring methods and procedures for testing and calibrating CMMs. Originally CMMs were mainly used in laboratories, often having a controlled environment with respect to temperature, and also with measures against the influence of vibrations. Therefore most early research was focused at the geometric errors of CMMs. Since CMMs have a high degree of
automation, improvement of CMM accuracy by means of software error compensation turned out to be an effective as well as an economically efficient alternative for design measures. Because CMMs are used for measuring and not positioning, off-line compensation by correction of the measurement result is sufficient. Software error compensation methods have been studied by many researchers (e.g. Busch 1984, Zhang 1985, Teeuwen 1989, Kruth 1992, Soons 1993). A recent summary of this research has been published by Sartori 1995. Nowadays most CMM manufacturers have implemented software error compensation algorithms on their machines for at least part of the geometric errors. Although effective design measures can be taken to make CMMs less sensitive for thermal errors, software error compensation for this type of errors proved to be effective as well (e.g. Trapet 1989, Balsamo 1990, Breyer 1991, Theuws 1991, Schellekens 1993, Soons, 1993, Spaan 1995). Besides geometric and thermal errors, also varying mechanical loads (moving weights) are an important source of quasi-static errors. Similar to dynamic errors, they are depending on the machine stiffness. They are mainly due to the compliance of CMM components in combination with the weight of moving machine components. Often these errors are already included in the geometric errors, since it is not useful to separate them from the measurements made to identify the geometric errors. However, care should be paid to the dependency of errors due to mechanical loads on the positions of more than one axis (e.g. Soons 1993). Like thermal errors, errors due to mechanical loads are especially important in the case of machine tools. Workpiece weight and process forces can have significant influence on the position accuracy. Schellekens 1993, reports a software compensation technique, applied to a five axis milling machine, taking into account geometric, and thermal errors as well as errors due to mechanical loads.

In CMM research as well as machine tool research much attention has been paid to the improvement of the accuracy. Besides design measures software error compensation has proved to be an effective tool for improving the machine accuracy, affected by several, quasi-static error sources. As stated before also dynamic errors are becoming more important for the accuracy of CMMs. This is related to several trends, mentioned in the first chapter: a demand for shorter cycle times of measurement tasks, and thus higher measurement speeds, more complex measuring tasks involving more complex motion, placement of CMMs closer to the manufacturing process and an increasing need for knowledge about measure-
ment uncertainty. The last five years there has been an increasing awareness of these trends among many researchers, especially with respect to the need for higher measurement speeds (see e.g. McMurty 1980, Sutherland 1987, Weckemann 1990, Lu 1992, Jones 1993, Katebi 1993/1, Kunzmann 1993, Lotze 1993, Neumann 1993, Phillips 1993).

2.2 Fast probing

The main subject of this research is the CMM measuring accuracy that is limited by dynamic errors during fast probing. When referring to fast probing opposed to normal probing, it does not only mean a higher CMM speed, but more general a reduction of the total cycle time of a measuring task. Several factors can be identified that influence the cycle time of a measuring task (see Neumann 1993):

- traverse and measuring speed, acceleration/deceleration, approach distance.
- probe changing time, angular speed of a rotation table
- calculation, data storage times, output measuring results
- operation and measuring strategy

The first group of factors has to be seen in relation to the measuring accuracy. The relation between these speed influencing factors and the (dynamic) accuracy is very much depending on the measuring procedure used. When we consider the collection of measuring points (i.e. probing) in more detail, three aspects are important with respect to the accuracy of the measuring result of a certain task:

- The measuring task itself: Basically two types of measuring tasks can be distinguished: measuring dimensions and profiles. A dimension is a geometrical parameter indicating the size of some part of the measured object. Typical dimensions are length, diameter, distance, angle etc. In case of profile measurements the shape of a certain part of the object is identified. Examples are roundness- and gear wheel measurements.

- The measurement concept. Either a limited amount of single points are measured and, assuming that the geometry of the element is ideal, parameters (dimensions) are calculated that define this geometry, or many points are
measured to identify the real geometry of the element (i.e. scanning). Using filtering techniques, profiles as well as dimensions can be calculated from the collected data points. With the first concept only dimensions can be calculated.

- **The probe type.** The probe is used to define the contact (usually mechanical) between workpiece and CMM. At the time of contact a trigger signal is provided and values of the scales of all axes are read by a computer. The way probes operate depends on the probe type.

Mechanical probes can be divided in two main groups: touch-trigger and measuring probes. Both types have a similar mechanical structure, consisting of a probe head that is carrying a stylus with at the end a sphere, the stylus tip. In the case of a touch-trigger probe the stylus carrier and its support form an electronic circuit. A displacement of the stylus in the support, due to a mechanical load at the stylus tip, is electronically detected and a trigger signal for the scale readout is provided. An example of a touch-trigger probe is depicted in Figure 2.1.

In order to ensure proper probing the CMM has to move with a well defined constant measuring speed at the time of contact. In this way errors in the measured position, due probing errors are mainly systematic and can be calibrated. Since a touch-trigger probe can only accurately detect a measuring point at the instant of contact, and when moving with a constant measuring speed, each data point has to be collected, following the same pattern of motion.

![Figure 2.1: Touch-trigger probe.](image-url)
This particular pattern of motion greatly affects the cycle time of the measurement task as well as the accuracy. In the scheme of Figure 2.2 the motion is described, indicating the acceleration, speed and position error of the probe versus time. Moving from one measurement point to another one, the CMM will first accelerate to the maximum traverse speed. When reaching a point at a predefined approach distance from the measuring point, the machine has to decelerate to probing speed. During the speed changes, the inertial forces will cause dy-

![Figure 2.2: Pattern of motion during a measuring task.](image)

Top: axis acceleration.
Middle: axis velocity.
Bottom: probing error.
namic position errors. In case of relative position errors between the actual probe position and the measured position, measurement errors are introduced. In order to avoid unacceptable dynamic errors, some time between decelerating and probing is necessary to allow the vibrations to settle (i.e. settling time). However, it is not always possible to achieve a well-defined constant probing speed in practice. In the case of short approach distances the CMM will still be in the course of acceleration when contacting the measuring object (Breyer 1994). Especially in the case of small measuring elements, approach distances can be often very short and thus the CMM is likely to be subjected to accelerations during the time of probing. In general touch-trigger probes are suitable for measuring dimensions based on a limited amount of single points. The measurement accuracy for each point is relative high, however the accuracy of the measured dimension strongly depends on the selection of the measuring points and on possible form errors. Touch-trigger probes can also be used for scanning, but due to the large number of points needed, measurement times will be long if the necessary settling time is taken into account.

The other main class of probes are measuring probes. These probes have their own measuring system that measures the relative 3D-position of the probe tip to the probe head (see e.g. Vliet 1996). Compared to touch-trigger probes, a measuring probe can sample multiple measuring points without renewed contact. The CMM only has to keep the tracking error within the range of the probe measuring system. This makes measuring probes very suitable for scanning and thus profile measurements. Still important is a defined contact in the sense of known measuring force perpendicular to the surface of the object. Because of the friction force between probe tip and workpiece during scanning, it is difficult to realise a well defined measuring force. Obviously during high speed scanning of a profile the CMM will be subjected to dynamic errors due to axes accelerations and drive induced vibrations. Measuring probes can be used for measuring dimensions as well as profiles. Especially in the case of profile measurements they can measure relative fast. However the measurement accuracy of the individual points is worse compared to a single point measuring strategy, due to the probing itself as well as the dynamic behaviour of the CMM (see Lotze 1993, Phillips 1995). But because of the many data points filtering techniques can be used and still accurate results can be obtained when calculating dimensions and profiles.
With respect to dynamic errors during probing there is a significant difference between CNC measuring machines and manual CMMs. Dynamic effects are one of the principal reasons why manual CMMs are less accurate than their computer controlled counterparts. The variability of acceleration, velocity, and probe approach distance that are inherent in manual operation often limit the level of accuracy which can be achieved with manual CMMs (see also Phillips 1995). So for accurate measurements CNC machines are preferred.

Regardless which type of probe or control is being used, the cycle time of a measuring task is limited by the dynamic behaviour of the CMM’s mechanical structure. For reduction of the cycle time faster probing is needed and thus higher accelerations and decelerations are necessary. As a consequence the CMM will be affected by an increase of dynamic errors. Without proper measures this may lead to an unacceptable degradation of the measurement accuracy.

2.3 Dynamic errors of CMMs

2.3.1 Nature and causes of dynamic errors of CMMs

In fact dynamic errors are only indirectly related to probing speed, but directly by acceleration (Sutherland 1987). The relationship between measurement error and acceleration is quite obvious. Acceleration of CMM components constituting the structural loop of a CMM and having certain mass, results in forces acting on these components. Due to component compliance these forces cause deflections of the components, leading to a relative position error of the probe to the measuring scales and thus to a measuring error. From this relationship it is clear that whenever the CMM is subjected to accelerations, deflections will exist, since the structural loop of a CMMs will always have compliance to a some degree. Especially CMMs used for fast probing will experience large accelerations and as a consequence large deflections. So, if such accelerations are applied to the CMM at the time of probing, significant dynamic (measurement) errors will result. In contrast to quasi-static errors that are constant or only varying slowly in time, dynamic errors are varying relatively fast in time. Due to their time varying nature accurate modelling of dynamic errors is difficult and therefore they are generally considered as random errors. With respect to their behaviour in time two types of
Chapter 2

dynamic error causes external:
- environment
- forced components:
- controller
- bearings
- spindels
free - axis accelerations
- environment

Table 2.1: Overview of dynamic errors of a CMM.

dynamic errors can be distinguished (see also Table 2.1): vibrations and inertial effects.

Vibrations

If a statically loaded elastic system, such as a CMM, is disturbed in some manner from its position of equilibrium, the internal forces and moments in the deformed configuration will no longer be in balance with the external forces; and vibrations may occur. If the disturbing force is applied only initially to the structure, the resulting vibration is maintained by the elastic forces in the structure alone. Such a vibration is called a free or natural vibration. If, however, the structure is subjected to time varying disturbing forces, the dynamic response of the system is referred to as forced vibrations. The characteristic symptom of these forced vibrations is that the machine system vibrates with the frequency of the excitation force. This can involve particularly high amplitudes if the excitation frequency is close to one of the natural frequencies of the CMM. The forced vibrations generally come from outside sources through the foundation (externally forced vibrations) or from internal sources (component forced vibrations) like the controller, bearing defects, spindels, drives etc. (see e.g. Hocken 1980, Week 1981).
External (environmental) vibrations are coming from the ground, the air and utilities which serve the machine. In a manufacturing environment especially ground vibrations are often difficult to avoid. These vibrations can shake a CMM and so cause an undesired change in relative position between probe and workpiece. The way accuracy is affected by vibrations depends on the machine's construction, mounting, and the direction and amplitude of the acceleration experienced by the machine. Effective measures against the distorting effects of vibrations are either designing measures making the machine robust or isolating the machine from the vibrations. A machine is robust if the distortion is minimised for a given acceleration. Isolation is aimed at attenuating the ground motions or the motions from the other error sources so that the machine experiences a sufficiently low level of acceleration so that the relative vibrations are acceptable.

A complicating factor with respect to vibration isolation is the difference between the responsibilities that both manufacturer and customer of a CMM have. The manufacturer is providing the measuring machine and the customer 'provides' the environment. According to standards with respect to the evaluation of the performance of measuring machines the user is responsible for a correct site selection (see e.g. ANSI/ASME B89 1990, VDI/VDE 2617). The B89 standard states: "The user shall be responsible for site selection, environmental shock and vibration analysis, and additional special isolators required to ensure compliance with the maximum permissible vibration levels specified by the supplier." This means that vibration isolation often is not an integrated part of a purchased CMM. Thus in order to avoid degradation of measuring accuracy by vibrations special care should be spent on CMM isolation. Not to be overlooked with respect to machine's performance is the fact that isolation not only effects vibration amplitude but also such matters as the settling time after deceleration from traverse to probing speed. Preferably settling should not be degraded by isolation measures. Vibration isolation of machines is a general problem and a lot of literature is available about this subject (see e.g. Rao 1990, DeBra 1992, Stühler 1992). Many references can also be found in a keynote paper by DeBra 1992, who discusses the problem for precision engineering applications. Although serious attention should be paid to the problem of environmental vibrations, it can be and should be accounted for by adequate isolation measures and therefore vibrations due external sources will not be considered here.
Disturbing sources from inside the CMM should be minimised by adequate design measures since this is the best way to avoid unacceptable position errors at the probe during measurement. Acceleration of the axes of the CMM will also disturb the mechanical structure, generally causing the structure to vibrate in one or more of its natural frequencies. Since fast probing is the main subject of this study, research is focused on vibrations induced by axes accelerations. The degree to which the resulting vibrations affect the accuracy at probe position, depends on the vibration force or probing concept and the dynamic behaviour of the CMM. This behaviour is characterised by properties like the natural frequencies, mode shapes, damping and stiffness of the components of the CMM. The effects of these vibrations on the accuracy of the CMM at the probe position are difficult to predict. Especially relationships describing the exact probe position, which are sufficiently accurate are difficult to obtain. The major problem being the fact that, in general, an elastic structure like a CMM can perform vibrations of different patterns, or modes.

**Inertial effects**

Inertial effects refer to acceleration dependent joint and link deflections. Due to accelerations of the CMM axes the machine parts, like the joints and links, are subjected to inertial forces. Because of part compliance these forces cause deflections of the machine parts, affecting the accuracy at probe position. Modelling of inertial effects is equivalent to modelling of errors due to mechanical loads. In fact this type of dynamic errors can also be regarded as quasi-static (see e.g. Hocken 1980, Weck 1981, Slocum 1992). Since CMM measurement accuracy during fast probing is the subject of this research and vibrations as well as inertia both affect the measuring accuracy due to fast probing, they are both treated here. Because the two types of error have common base (i.e. the dynamic behaviour of the CMM) they are also both considered as dynamic errors. The term inertial is used for the type of dynamic error with a quasi-static behaviour. This can be somewhat confusing, since inertial forces are responsible for both types of errors mentioned here.
2.3.2 CMM sensitivity for dynamic errors

The way a CMM is affected by dynamic errors, strongly depends on its structural loop. The structural loop is the part of the mechanical structure that comprises all the components that together define the position of the probe relative to the workpiece. The main components of the loop are the frame of the CMM, the table on which the workpiece is mounted, possible mounting aids, the workpiece itself, and three mutually orthogonal axes. Each axis generally consists of connection elements, one guideway and a carriage. At the end of the last axis the probe system is attached. In some cases a rotary table can be part of the loop. There are many different configurations of the axes possible, that form an orthogonal mechanical structure. In Figure 2.3 the most common types of CMM structures are depicted (see e.g. ANSI/ASME B89 1990). Each CMM configuration has its advantages and disadvantages with respect to properties like accessibility, measuring volume, load capacity, rigidity, and attainable accuracy and speed (e.g. Warnecke 1984). In this case especially the dynamic accuracy is important.

The CMM’s mechanical structure is being used for two tasks: positioning of the probe and as part of the coordinate system, because it is acting as a frame for the measuring systems. Thus deformations of the structural loop e.g. due to driving forces and moving loads that cause (dynamic) errors with respect to the probe position, will inevitably affect the measuring accuracy. There are several factors that influence the sensitivity of the machines components for dynamic errors and the effects of these errors on the probe position. They can be categorised as factors related to the CMM configuration, the components properties and the dynamic load on the CMM.

CMM configuration

The configuration of a machine refers to the arrangement of the carriages and guideways. In the first place, the location of each of these components is important with respect to the influence of the dynamic errors of a component on the probe position (error propagation). A rotation error of a carriage yields an error at probe position that is proportional with the effective arm between the measuring scale and the probe tip, the Abbe offset.
a. Column CMM.

b. Horizontal arm CMM.

c. Cantilever CMM.

d. Fixed Bridge CMM.

e. Moving Bridge CMM.

f. Gantry CMM.

Figure 2.3: The most common types of CMM structures.
The location of the components is also important with respect to the way they are affected by the dynamic loads induced by the acceleration of the axes. A moving carriage lower in the structure (i.e. closer to the base of the machine) will directly affect elements higher in the structure because these will also be accelerated. This will yield dynamic errors at more elements. Carriages higher in the structure will also influence the lower elements (reaction forces), but the total accelerated mass will be lower as will be the dynamic errors. Thus the influence of the lowest carriage is the most important. Depending on the configuration the dynamic load can be more symmetric or more eccentric, and in the latter case this will yield larger errors. E.g. gantry and bridge type CMMs have a higher degree of symmetry than cantilever and horizontal arm types and thus will suffer less from errors in the vertical direction. The same is valid for an individual carriage. E.g. the vertical moving pinole of a bridge type CMM will yield a rotation error of the supporting carriage about the x-axis if mounted outside the carriage. But if, at the same time, it is mounted symmetrically with respect to the bearings in x-direction of this carriage, it won't cause a rotation error about the y-axis. In the latter case there is no effective arm between the load and the rotation point between the bearings.

Component properties

Besides the configuration of the CMM, the dynamic behaviour of a CMM is influenced by the mass, stiffness and damping properties of the several components. Obviously the ratio between stiffness and mass should be as high as possible in order to minimise dynamic errors. Thus high stiffness and low mass of all these components is necessary. Especially the rotation stiffness of the bearing systems can cause problems. E.g. in the case of a moving bridge CMM, the bridge itself is forming a large eccentric mass relative to the carriage moving the bridge. Therefore the (bearing) stiffness against pitch and yaw movements of this carriage has to be very high. However traditional CMM design has been based on demands with respect to static accuracy, rather than dynamic accuracy, mainly focusing on gravity forces that cause (quasi-static) errors related to the finite stiffness of the CMM. This makes the CMM configurations, such as the gantry and bridge types, less sensitive for additional dynamic loads in the vertical direction. But in the horizontal directions they will suffer from more severe dynamic errors.
Dynamic load

The structural loop of a CMM is subject to several forces: gravity forces, forces applied to the CMM by the drives, and the resulting acceleration forces. The gravity forces cause errors due to the weight of moving parts in relation to the finite stiffness of the components. The resulting errors are considered quasi-static (see also Paragraph 1.2). The drive forces induce a dynamic load, that causes deformations. Ideally drive forces should only act in the in direction of motion of the carriages and they should be introduced in the centre of mass of the moving parts. The first condition is aimed at avoiding the coupling of unwanted degrees of freedom between carriage and drive mechanism. This can be fulfilled by proper design of so-called kinematic drives. The second condition is difficult to fulfil for all axes and often at the expense of accessibility of the CMM. E.g. moving bridge type CMMs can have a central driven bridge, but an extra column in the middle of the bridge is needed, blocking one side of the CMM. The magnitude of the dynamic load is, besides the masses, determined by the accelerations induced by the drives. Traditionally, accelerations during probing have to be minimised. Higher accelerations will directly affect dynamic errors.

Based on the previously made considerations, an overview of error sources of the most common CMM types (see Figure 2.3) is given in Table 2.2. This overview comprises the main rotation errors, since these generally yield the largest errors at probe position. The indicated error type $ir_j$ is a rotation error around the j-axis while moving in i-direction. The error includes both a rigid body rotation of the mentioned elements due to bearing- and carriage compliance as well as bending of these elements.

The table gives only a global indication of the sensitivity of the several types for dynamic errors. But there are significant differences between the types, depending on their structural loops. It is interesting to notice that also the traditionally more rigid designed CMM types, such as the bridge and gantry types, will still suffer severe dynamic errors when induced with higher accelerations. Especially, large rotation errors of the carriage of the lowest axis can be expected, causing dynamic errors in the horizontal plane. On the other hand, dynamically advantageous in terms of high stiffness is the use of a moving table, either in one or two directions, instead of a series of three linked axes.
### Table 2.2: Overview of the main rotation errors of the most common CMM types.

<table>
<thead>
<tr>
<th>CMM type</th>
<th>moving axis</th>
<th>type of error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column</td>
<td>y</td>
<td>yrz column</td>
</tr>
<tr>
<td></td>
<td></td>
<td>yrz pinole</td>
</tr>
<tr>
<td>Horizontal arm</td>
<td>x</td>
<td>xrz arm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>xry column</td>
</tr>
<tr>
<td></td>
<td>z</td>
<td>zrx arm</td>
</tr>
<tr>
<td>Cantilever</td>
<td>x</td>
<td>xry pinole</td>
</tr>
<tr>
<td></td>
<td>y</td>
<td>yrz arm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>yrz arm</td>
</tr>
<tr>
<td></td>
<td>z</td>
<td>zry arm</td>
</tr>
<tr>
<td>Fixed bridge</td>
<td>x</td>
<td>xry pinole</td>
</tr>
<tr>
<td>Moving bridge</td>
<td>x</td>
<td>xry pinole</td>
</tr>
<tr>
<td></td>
<td>y</td>
<td>yrz bridge</td>
</tr>
<tr>
<td></td>
<td></td>
<td>yrz bridge</td>
</tr>
<tr>
<td></td>
<td></td>
<td>yrz pinole</td>
</tr>
<tr>
<td>Gantry</td>
<td>x</td>
<td>xry pinole</td>
</tr>
<tr>
<td></td>
<td>y</td>
<td>yrz traverse</td>
</tr>
<tr>
<td></td>
<td></td>
<td>yrz traverse</td>
</tr>
<tr>
<td></td>
<td></td>
<td>yrz pinole</td>
</tr>
</tbody>
</table>

2.3.3 Examples of dynamic errors of CMMs

To illustrate the dynamic errors mentioned in the previous paragraphs some examples of these errors will be given in this paragraph. Using experiments on two different type of machines, examples of inertial effects, axes induced vibrations, and environmental vibrations will be given.
Inertial effects

Inertial effects arise if a CMM is still in the course of acceleration or deceleration. Proper use of the CMM during probing implies that the machine is moving at a relative slow rate, avoiding high accelerations and thus dynamic errors. However if a machine is being speed up in order to reduce cycle times, significant deformations will be the result. To illustrate this some experiments were performed on an existing CMM. During the experiments this gantry type CMM was programmed to move to a certain position in y-direction with high traverse speed. Actual probing was not possible since the probe system only allowed slow speeds for measuring. During deceleration before reaching the position, the machine’s components are deformed due to inertial forces. In this period the rotation \( yrz \) of the traverse about the vertical axis was measured. Because of the Abbe offset such a rotation will directly yield a measuring error when actually probing. In Figure 2.4 the CMM used for the experiments is depicted, together with the measured rotation.

![Diagram of CMM and graph showing rotation error](image)

Figure 2.4: Rotation error caused by inertial effects on a gantry type CMM.

The rotation error was measured at the probe position of the CMM using a laser interferometer with rotational optics. From the graph it is clear that the structural loop of the CMM is subject to large deformations during deceleration from traverse speed (70 mm/s) to rest. The maximum rotational error is over 4 arcsec, which yields translation errors at probe position of 20 \( \mu \)m for an Abbe offset of
1 m. As will be shown later, the main cause for this error is the limited stiffness of the air-bearings of the y-carriage and the y-carriage itself.

Axes induced vibrations

Depending on the CMM motion control, also vibrations can occur. In the case that a CMM is accelerating or decelerating very fast the structural loop will be subjected to vibrations. Most likely the CMM structure will start to vibrate in a mode shape related to one of the lower natural frequencies. As an example measurements from the same set of experiments on the gantry type CMM from the previous example, are shown in Figure 2.5. In this example the CMM is first moving at constant traverse speed (70 mm/s) and then suddenly stopped. As a result the CMM starts to vibrate.

![Figure 2.5: Rotational vibrations yrxi and yrz after sudden deceleration.](image)

The graph is showing the similar rotation error yrz of the previous example, as well as a the rotation xry from another set of measurements. In this case even larger deformations occur. After some time the vibrations are damped out. It is clear that in this situation sufficient time must be allowed after deceleration before an actual measurement can be made. Another example of a vibration is given in Figure 2.6. In this case a measurement on a moving bridge type CMM was made. A schematic picture of this existing CMM is depicted, indicating also
the measured rotation error. As in the previous situation, again the rotation \( yrz \) was measured. The CMM was first accelerated from rest to traverse speed and then decelerated to rest again. The graph clearly shows the rotations during acceleration and deceleration. Again they are damped out after some time. Errors of over 15 \( \mu \text{m} \) are yielded by a maximum rotation of almost 5 arcsec.

\[\begin{array}{c}
\text{Figure 2.6: Rotational vibrations after acceleration and deceleration of bridge type CMM.}
\end{array}\]

**Environmental vibrations**

The last example given here shows the effects of environmental vibrations on the gantry CMM from the first two examples. This CMM is located in a measuring laboratory with a foundation separated from the main building. The machine itself is mounted on simple rubber pads. Nevertheless, due to shock waves from a ram used on a construction site located 300 m from the laboratory, the machine was subjected to severe environmental vibrations. In Figure 2.7 the resulting rotation \( yrz \) is depicted that was measured during the vibrations. In this case the CMM vibrates with the frequencies of the forced vibration. In general these vibrations should be accounted for by adequate isolation measures.
In this paragraph several CMM configurations commonly used have been analysed with respect to their sensitivity for dynamic errors. It is clear that for most conventional CMMs dynamic errors can be expected in case of fast probing. Measurements conducted at existing CMMs also show that during motion of CMM axes such errors occur. For the CMMs quasi-static as well as vibration errors were found. It has been shown that they can be quite large in relation to the static errors of CMMs.

2.4 Dynamic error reduction

2.4.1 Literature overview

Most research concerning dynamic behaviour of CMMs has been focused on theoretical and experimental methods for identifying the vibration modes and estimations of error amplitudes of CMMs in order to improve CMM design and/or control. In this paragraph the relevant literature with respect to the assessment and improvement of dynamic errors of CMMs will be presented.
Assessment of dynamic errors

Ricciardi 1985 recognises the accuracy problem of CMMs caused by dynamic errors. Inertial forces, due to acceleration of moving masses, always excite lower natural frequencies. Modal analysis and finite element techniques are used to identify the dynamic behaviour of structures like CMMs. They are considered valuable tools for improved machine design. Okuba 1989 uses laser interferometry as well as acceleration pick ups for measuring the relative vibration between the probe and the base of a CMM in steady state. From the measurements the dynamic behaviour is analysed. Nijs 1988 developed a model, based on the Lagrange energy method, for estimating the lower natural frequencies of a CMM. The model yields good results that are verified by performing modal analysis on an existing CMM. The method can be used to optimise the design of a mechanical structure before actually realising a prototype. Grimbergen 1990, uses the same modelling technique for the development of a new concept for a CMM. Terken 1986, gives an estimation of the maximum amplitudes that can be expected for a given CMM structure during acceleration of its axes. This method is used to calculate the necessary stiffness of the components. In Phillips 1993 an overview of factors affecting measurement accuracy of a CMM is given. With respect to dynamic errors, probing speed, probing direction, probe approach rate, and acceleration/deceleration are identified as relevant factors. Deceleration to probing speed causes structural oscillations. At large distances these oscillations are damped out prior to probe triggering, but in case of shorter approach rates dynamic errors result. According to the authors these errors are highly dependent on the probe approach distance and thus can be accounted for by calibration. Measurements, indicating the relation between measurement error and approach rate, are also used by manufacturers to identify an appropriate settling time. Jones 1993, reports an analysis of variance study, showing how the parameters that affect measuring speed affect measurement quality. Modal analysis and other experimental methods are used to develop an optimisation technique, with respect to the selection of time optimal measuring speeds.
Error reduction

A number of authors give suggestions and methods for the reduction of dynamic errors. In Ni 1992, Ni 1993, and Huang 1995, a laser measurement system for identifying geometric error components of CMMs is described. According to the authors the system is also suitable for time-variant errors and might be used for real-time compensation of CMM errors. Ax 1983 also presented an optical measurement system for real-time measurement of geometric errors. However the system has relative low accuracy. In McMurry 1980 the need for faster movements and the resulting effects of inertia forces are recognised. Assuming uniform acceleration, a system is proposed and patented for measuring the acceleration at probe position and to calculate probe deflections. Breyer 1994 describes a patented method for measuring oscillatory motion of a CMM by using two parallel linear scales. The signals of the scales are used for compensation of dynamic errors. The method is primarily aimed at the identification of one rotatory error component, but it is claimed in the invention that, by using signal analysis, another error component can be identified.

One way of enhancing the dynamic behaviour of CMMs is to improve controller performance. Efforts for improving CMM control are aimed at the reduction of transient structural vibrations induced by deceleration from traverse to probing speed (i.e. reducing settling time) and at the suppression of steady state vibrations caused by motors, servo control, air supply etc. With respect to tracking errors only errors in the measured position are important, not errors in the commanded position. When probing single points at constant speed, in general no tracking errors will exist. However in the case of scanning, motion is more complex and tracking errors are more likely. Literature on (theoretical) research with respect to control of mechanical structures in general is extensively available. Practical research is mainly focusing on experimental laboratory set-ups or industrial robots. Robots often need to perform positioning tasks. Due to their structure (most commonly a series of revolute joints), the mass and stiffness of the components, the operating speed and the weight of products and tools to be handled, industrial robots are prone to large dynamic positioning errors. However, compared with CMMs, the demanded accuracy is generally at least one order of magnitude lower. Proposed models and control algorithms are taking into account rigid manipulators as well as manipulators with either flexibility of the
links or the joints or even with flexibility of both. Good introductions to the problem are given by Asada 1986 and Spong 1989. An extended overview of literature regarding flexible manipulators (flexible with respect to joints and links) is e.g. given by Lammerts 1993. In Park 1994 the authors recognise the fact that residual vibration after stopping prevents a robot from quick positioning and prolongs cycle times. Minimising settling time is considered the main issue in position control. Their conclusion is that an overall optimal performance is only attainable if structure and control are designed concurrently.

A number of researchers report practical applications for CMMs. In Sutherland 1987 the authors identify increased acceleration as the principle factor for cycle time reduction as well as accuracy degradation. They describe the development of a CMM servo system, taking into account both requirements. Similar objectives are formulated by Katebi 1993/1 and Katebi 1993/2, and the design of an optimal CMM position controller with respect to the conflicting design requirements speed and accuracy has been presented. Jones 1993 describes a method for vibration reduction by filtering the controller input commands. Having identified acceleration, approach distance and approach rate as the factors affecting measurement quality, he also describes a time optimisation method for CMM path control. For specific measurement tasks reductions in time up to 25% are achieved. Lu 1992 uses similar methods. He reports improved steady state control and a reduction of the settling time for the transient vibrations. He also proposed an increase of probing speed in order to decrease the speed gap and thus decelerations and structural vibrations.

With respect to the dynamic behaviour of CMMs several aspects have been considered by researchers. The need for faster CMMs and the significance of dynamic errors is being recognised. Experimental and theoretical methods for assessing the dynamic behaviour of CMMs are used such as modal analysis, laser interferometry, and finite element techniques. Isolation measures of CMMs in order to reduce the effect of external vibrations are available. Settling time of transient vibrations and steady state vibrations can be reduced by improved CMM control. Some on-line measurement methods are proposed for error compensation of CMM errors. However these methods are either limited or inaccurate with respect to the error components taken into account, or they need an extensive measurement set-up.
2.4.2 Methods for reduction of dynamic errors

In order to obtain sufficient measuring accuracy in the case of fast probing, the effect of the dynamic errors has to be minimised. Three different approaches can be adopted. These approaches are based on CMM design, control or error compensation.

Design

The design approach is aimed at the improvement of the machine's structural loop. First of all the CMM configuration should be such that Abbe offsets are small, minimising the propagation of rotation errors to the probe position. In general the structural loop should form a path between probe and workpiece that is as small as possible. In this way the sensitivity of the CMM for dynamic errors is minimised. Large dynamic errors should be avoided by using components with low mass and high stiffness. This will reduce the deflections resulting from the acceleration forces acting on the components. The use of other materials having a higher specific stiffness can be helpful, but can affect other properties such as thermal conductivity and expansion. Extra damping can reduce vibration amplitudes and thus settling times. Deformations of the structural loop can be further minimised by incorporating a high degree of symmetry in all components and the use of kinematic drives in order to avoid drive forces in unwanted directions (see Teague 1989). Environmental disturbances can also seriously degrade measuring accuracy. Isolation measures should be used to shield the CMM from these environmental influences.

A more fundamental solution of the problem of structural loop deformations and resulting measuring errors, is the use of a separate so-called metrology frame (Teague 1989). Such a frame supports only the measuring system and is separated from the structural loop containing the drive systems. In this way the measuring loop is not affected by the forces of the structural loop. However, for economic reasons most CMMs only have one mechanical structure constituting the positioning loop as well as the measuring loop.
Control

Since it is the task of CMMs to take accurate measurements, not accurate positioning of the carriages, it is sensible to distinguish between the control loop and the measuring loop of the CMM. The control loop is the part of the structure involved in the control task. The so-called closed loop control system, comprises the drive system, guideway, carriage and measuring scale, whereas the open loop control system is extended by the part of the structure connecting the probe to the carriage. The measuring loop is the part of the structural loop between the measuring scale and the probe. For accurate measurements the relative position of the probe to the CMM’s reference coordinate system must be known. In practice this means that the coordinates of the probe in three directions relative to each of the three scales must be known, and also possible displacements of the scales relative to the machines reference coordinate system. The control task here is to ensure that measuring points can be reached fast and that at probing time deflections within the measuring loop are minimal. Therefore the control system must be capable of controlling acceleration, besides position and velocity. At the time of probing the acceleration parameter must be kept within limits. This has two drawbacks: minimum acceleration is in conflict with fast probing, and control of the closed loop system is not sufficient, since it doesn’t comprise the measuring loop. Thus an accurate model of the open loop system is necessary in order to guarantee minimal deflections. In general accurate control of the CMM probe position during fast probing will be time consuming, especially when during a complex measurement task a large number of points have to be measured. At each of these probing points the dynamic position error has to be kept small.

Error compensation

In case the CMM’s structural loop and its control provide insufficient measurement accuracy, the measurement result can also be compensated for errors in the position of the probe, relative to the reference coordinate system of the CMM. As mentioned in the literature overview of this paragraph this approach has been applied successfully for quasi-static errors. Software error compensation is also considered a serious possibility for dealing with the dynamic errors of a CMM.
Analysing dynamic errors

(see e.g. Weckenmann 1990, Breyer 1994, Huang 1995, Sartori 1995, Weekers 1995). In order to make software error compensation possible, the significant dynamic errors of the CMM have to be known, either by measurement or modelling. The deviation at probe position has to be obtained very accurately, but only at discrete times, when probing, and not as a function of time.

2.4.3 Adopted strategy

For reduction of dynamic errors design, control as well as compensation methods can be useful. Thus an integrated approach should be adopted, taking into account all these methods, especially for the development of new CMMs. Important considerations with respect to dynamic errors are the measuring tasks that have to be fulfilled, the demanded accuracy and the allowable measuring time that is economically acceptable. Obviously these considerations will be different for a CMM used for precision measurements in a laboratory and a CMM at the shop floor used for part inspection. In the first case the strategy can be aimed at totally avoiding dynamic errors, and design and control measures are preferred. In the latter case this might be economically unacceptable and compensation methods can be an alternative.

The dynamic errors described in Paragraph 2.3 are summarised again in Table 2.3. Included are the measures proposed in order to deal with dynamic errors of CMMs in general. First of all, avoiding the problem of dynamic errors by eliminating possible error sources and minimising the sensitivity of the structural loop for these sources, is the best way to achieve acceptable measurement accuracy. So external vibrations should be dealt with by adequate isolation of the CMM. Component vibrations should be minimised by appropriate design measures and proper control of the CMM motion. However, also compensation of the dynamic errors can be advantageous. When shorter cycle times of measurements are demanded, eventually higher speeds and thus accelerations during probing time cannot be avoided. This is especially true in the case of scanning, were generally non-linear movements are required. This means that vibrations and inertial effects due to axis accelerations have to be accepted to some degree. In order to maintain an acceptable accuracy at probe position, estimation of these dynamic errors at the time of probing is necessary. For CMMs exact knowledge of
the (probe) position is sufficient in contrast to machine tools, where the programmed position has to be reached exactly. By applying compensation of the measuring results for position errors of the probe, in principle time consuming position control is unnecessary. Furthermore advantageous is that in principle compensation for dynamic errors can also be applied to manual CMMs. These are very prone to dynamic errors since probing on a manual CMM is often performed in a rather uncontrolled way.

Considering these advantages, the object of the research described in this thesis is the description of dynamic errors CMMs due to axes accelerations and decelerations, in order to achieve software error compensation for these errors. By developing such a software compensation method for dynamic errors, this research is aiming at the improvement of the efficiency of conventional CMMs. With the compensation method faster probing on such CMMs will be possible without degradation of measurement accuracy. The approach adopted here is of a combined analytical and empirical nature and contains the following steps:

1. identifying the significant dynamic errors of CMM components.
2. using (additional) sensors for measurement of these dynamic errors.
3. using models to relate the dynamic errors to the input signals of the sensors.
4. calculating the effect of all the identified errors on the probe position.
5. compensating the measurement results for the calculated errors.

### Table 2.3: Measures for minimising dynamic errors of a CMM.

<table>
<thead>
<tr>
<th>dynamic error</th>
<th>causes</th>
<th>measures</th>
</tr>
</thead>
<tbody>
<tr>
<td>vibrations forced</td>
<td>external:</td>
<td>isolation</td>
</tr>
<tr>
<td></td>
<td>- environment</td>
<td></td>
</tr>
<tr>
<td></td>
<td>components:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- controller</td>
<td>design/control</td>
</tr>
<tr>
<td></td>
<td>- bearings</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- spindels</td>
<td></td>
</tr>
<tr>
<td>free</td>
<td>- axis accelerations</td>
<td>control/compensation</td>
</tr>
<tr>
<td></td>
<td>- environment</td>
<td>isolation</td>
</tr>
<tr>
<td>inertial effects</td>
<td>axis accelerations</td>
<td>control/compensation</td>
</tr>
</tbody>
</table>


First an understanding of the dynamic behaviour of the CMM has to be obtained in order to identify the components yielding significant dynamic errors. These errors and their effects on the position of the probe at the time of probing, have to be estimated. For these estimations, input signals (e.g. accelerations or measured carriage rotations) and models relating the dynamic errors to the input signals are necessary. Depending on the input signals used, these models can be either relative simple, or more complex. For instance extra sensors can be used, measuring directly the deformations due to axes accelerations (such as carriage rotations) and thus little modelling is necessary to estimate the correct deformations. The input signals can also be derived from the already available scales readings (e.g. accelerations), in which case more complex modelling is needed to relate the input signals to the resulting deformations. Ideally (with respect to economical reasons) no extra sensors are needed for deriving the input signals. However direct measurement of the resulting deformations generally will yield more accurate estimations of the errors. Contrary the use of more complex models will introduce larger modelling errors. Therefore the number of sensors will be a pay-off between accuracy and economical reasons. Our approach is aimed at the use of a minimum number of extra sensors, but with sufficient accuracy.

After estimating the errors, their effects on the probe position have to be calculated, using a kinematic model of the CMM. Last step is to compensate the measuring result for the errors at probe position. The next chapter of this thesis, Chapter three, will treat the modelling aspects mentioned here. The chapters four and five will deal with respectively the measurement part and the compensation part.
Modelling of dynamic errors

This chapter deals with the modelling of the dynamic errors. Assessment of the dynamic errors consists of two parts: identification of the individual dynamic, parametric errors and calculation of their effects on the measuring error at the probe position, using a kinematic model. The parametric errors can be considered as the errors in the degrees of freedom of the kinematic model. In the kinematic modelling the CMM structure is considered rigid. However, the components of a CMM are actually flexible elements, introducing quasi-static deformations and vibrations due to the accelerations. These deformations have to be expressed in the chosen degrees of freedom, i.e. the parametric errors. In order to estimate these errors for a CMM, a combined analytical and empirical approach is followed. With additional sensors carriage deformations are measured directly. Based on these measurements the parametric errors can be derived, using relative simple relationships between the measured- and other deformations.

3.1 Modelling CMM errors

The main task here is the estimation of the exact probe position of a CMM each time a measurement is taken. In general, when taking a measurement by probing an object, the error sources affecting the CMM will cause differences between the actual probe position and the nominal probe position, indicated by the scale. As mentioned before in Paragraph 1.2, the main errors that affect the structural
loop of a CMM are geometric- and thermal errors, errors due to mechanical loads and dynamic errors. For assessment of all these errors the same modelling approach can be used (see also *Soons 1993*). This is important, since in this way a modular compensation system is obtained. Depending on the circumstances, the various error sources will have more or less influence on the measuring accuracy. Taking into account the significance of the error sources and economical considerations, only compensation for some of the errors will be desirable. With a modular structure of the machine's error model this is more easy to achieve. The method is also systematic and 'transparent' with respect to the actual deformations of the machine's structure. Thus there is less danger of 'overlapping' of error compensation (i.e. compensating more than once for the same error due to inadequate separation of the different error types).

In the parametric modelling approach, the machine's errors are described as an analytical synthesis of errors introduced in the structural loop components. The basis of this approach is the kinematic error model. This model relates the errors in the relative location of the probe position to errors in the geometry of consecutive structural loop segments. The latter so-called parametric errors describe the combined effect of the various error sources on the geometry of the structural loop components that constitute such a segment, including the joints. The propagation of the parametric errors to the errors at the probe position, indicated by the machine's scales, is a geometric problem, completely defined by the nominal geometry of the structural loop. Therefore a complete mathematical description of the kinematic model can be given. In general, the parametric errors of a CMM are small to such an extend that the parametric errors of the different segments do not affect each other. Thus the kinematic model enables the separation of the structural loop into different segments whose errors can be modelled and measured individually.

So for instance a CMM guideway can have a limited manufacturing accuracy, with respect to its geometry, and can be loaded simultaneously by a temperature gradient, the weight of the moving mass of a carriage, and by an acceleration due to the movement of the carriage carrying the particular guideway (see also Figure 3.1). The combined effect of all these error sources will result in translation as well as rotation errors of the carriage supported by the guideway. In case of the CMM depicted in Figure 3.1, the error sources mentioned above cause for in-
stance rotation errors $z_{ry}$ of the $z$-carriage about the $y$-axis. Superposition of these errors yields the parametric rotation error $z_{ry}$ of the $z$-carriage:

$$z_{ry} = z_{ry_{geom}} + z_{ry_{weight}} + z_{ry_{temp}} + z_{ry_{dyn}}$$  \hspace{1cm} (3.1)$$

Within the scope of this thesis only the dynamic part of the parametric errors is of interest. Using a kinematic model the effect of this parametric error on the probe position can be described:

$$\Delta z_{probe} = F(z_{ry})$$  \hspace{1cm} (3.2)$$
The operation \( F \) is defined by the kinematic model of the CMM. In the next paragraph the kinematic model used will be presented. The Paragraphs 3.3 and 3.4 will deal with the dynamic part of the parametric errors.

### 3.2 Kinematic error modelling

The kinematic error model of a CMM defines the spatial relationship between the machine components and the probe position. The purpose of the modelling is the estimation of the actual probe position. The probe position of a CMM is nominally described by the position of each carriage, relative to the machine's coordinate system and is indicated by its scales. The actual probe position, however, is influenced by errors in the location of each of these carriages, due to deformations of the structural loop by the several error sources. These errors in the location of a carriage can be described by three translation and three rotations of a reference point of the carriage, corresponding to the degrees of freedom of that carriage. In Figure 3.2 these errors are depicted for the carriage of a prismatic joint. The notation with respect to the errors is according to the VDI-2617 guideline on the performance evaluation of CMMs (see VDI/VDE 2617, 1991). The first character indicates the axis of motion, the second character the type of error, and the third character the direction of the error (either the axis of rotation or the direction of translation). The errors represent the differences between the nominal and actual geometry of the part of the structural loop, enclosed by consecutive carriages and they are described by the parametric errors. When more axes are combined,

---

**Figure 3.2**: The errors in the location of a carriage.
extra parameters have to be included to specify the relative location of the axes. For three nominally perpendicular axes three parameters, representing squareness errors, are necessary to describe the actual angles between the axes. Thus the kinematic error model for a CMM with three perpendicular axes, incorporates 21 so-called parametric errors. These errors have to be related to the actual probe position. With respect to the error notation it is important to note the difference between e.g. $irj$, $\varepsilon_j$, $\varepsilon_j$, and $\varepsilon_{j,c}$. The notation $irj$ is used explicitly to indicate a parametric error, i.e. an error in the a chosen degree of freedom at a given location of the CMM. These errors are the input of the kinematic model. In our case the parametric errors represent rotations and translations of the CMM's carriages. The notation $\varepsilon_j$ is used more generally to indicate any rotation error in the structural loop of the CMM about the $j$-axis with respect to the machine's reference coordinate system. The symbol $i\varepsilon_j$ denotes a rotation error that can be related to part of the structural loop belonging to the $i$-axis. The contribution of a particular component $c$ to such an error is denoted as $i\varepsilon_{j,c}$. The parametric error $irj$ at its defined location is in general a combination of several errors $i\varepsilon_{j,c}$.

In literature several different kinematic models are described. Most models are based on the use of coordinate frames that are attached to the various components of the structural loop. The parametric errors in the degrees of freedom of the components are described relative to these frames. Main differences between the models are the mathematical representation chosen, the position of the reference points on the elements (i.e. the location of the coordinate frames), and the flexibility with respect to the type of joints that can be used. Many of the models are based on the use of (homogeneous) transformations of the errors between the coordinate frames. This yields a general model for multi-axis machines, having prismatic as well as revolute joints, such as industrial robots and machine tools (see e.g. Soons 1993). For machines consisting of only prismatic joints in a cartesian configuration, like most CMMs, another model using a more convenient vectorial notation can be used. Here this vector model is chosen for the propagation of the parametric errors. In order to obtain a useful and efficient kinematic model the following assumptions and limitations are made with respect to the behaviour of the parametric errors and the machine's structure:
• A first order approximation is being used for the angular errors (i.e. \( \cos \varepsilon = 1, \sin \varepsilon = \varepsilon \)).

• The difference between the nominal and actual geometry of the CMM doesn't affect the active arm of the angular errors and the direction of the errors significantly. Thus the effects of the various parametric errors on the probe position can be calculated individually.

• Only CMMs with mutually perpendicular translation axes are considered here, allowing the use of a convenient and compact vectorial notation.

To achieve an unambiguous description of the kinematic model, some properties of the model have to be well defined. For combining the different errors and for allowing a correct interpretation of the results a clear sign convention is necessary. Naturally translation errors are positive when acting in the positive direction of the respective axis of the coordinate frame. Rotation errors are positive according to the right hand rule. All coordinate frames have their axes parallel and in the same directions as the corresponding axes of the machine's reference frame. Furthermore an error at probe position is defined as the actual realised position minus the nominal position, indicated by the scales. In this way the actual probe position can be found by adding the error to the scale readings.

For squareness errors no additional parameters are introduced. For most CMM configurations they can be considered as an offset for certain angular errors, and thus no extra parameters are needed. However, this is for completeness only, since squareness errors are fully described by the parametric errors belonging to the geometric error modelling. Any deformation due to dynamic errors can be described by the 6 parametric errors for each axis.

The kinematic model describes the relations between the errors in the relative location of the coordinate frames attached to the machine's elements and the errors on probe position, so a well defined choice of the location of the coordinate frame is important. There are several possible choices for these locations (Soons 1993, Slocum 1992). An important consideration is the fact that angular errors are unaffected by other errors and therefore they can be defined with respect to any set of axes. Translation errors, on the other hand, are caused by direct linear
motion of an element as well as linear motion resulting from an Abbe offset. Only if the coordinate frames are located in the origin of the angular errors, the translation errors are not 'contaminated' with the effect of angular errors. Another consideration is whether or not the errors should have a close relationship with the actual geometry of the machine's errors. For instance the coordinate frames can be attached to the machine's scales, so the translations errors *directly* reflect the linearity errors (i.e. errors in the measurement systems). With the frames located in the centroids of the joints, straightness errors are directly related to the straightness of the guideways. The frames can also be conveniently located in the workspace in such a way that allows measurements of the errors to be transferred directly to the model. This yields great advantages with the implementation of compensation methods (Spaan 1995). Here the coordinate frames are located at the scales of the axes, because for the dynamic errors it is assumed that in most cases they don't affect linearity (in contrast to, for instance, thermal errors that cause scale errors). Thus in case of dynamic rotations the scale readings generally don't have to be corrected for translation errors. This doesn't apply when dynamic errors affect the position of the entire guideway in the scale direction, leading to a variation in the scale zero point that cannot be taken into account by a straightness error of the previous element. This is the case if the lowest joint in the structural loop is weakly supported.

For the kinematic model of a CMM, having three perpendicular axes, we can now derive a relationship, describing the effect of the parametric errors on the probe position, using a vectorial notation (see also the C-type CMM depicted in Figure 3.3). The actual position of the probe is defined by the vector:

\[
\mathbf{p} = \begin{pmatrix}
x \\
y \\
z
\end{pmatrix}
\]

(3.3)

The nominal probe position is defined by the displacements *x*, *y*, and *z*, indicated by the scales:

\[
\mathbf{d} = \begin{pmatrix}
x \\
y \\
z
\end{pmatrix}
\]

(3.4)
Figure 3.3: Definition of the various vectors used in the kinematic model of a CMM.

By definition the error at the probe position can be written:

$$ e = p - d $$

where the error vector is defined as:

$$ e = \begin{pmatrix} e_x \\ e_y \\ e_z \end{pmatrix} $$

The parametric errors related to an axis $i$ of the CMM are also described by vectors. The translation errors are defined by the vector:

$$ t_i = \begin{pmatrix} i_{tx} \\ i_{ty} \\ i_{tz} \end{pmatrix} $$
The angular errors are defined as rotations about mutually perpendicular axes. In case the axes are not exactly perpendicular, certain angular errors contain an offset representing a squareness error. However, as stated before this is only important for geometric errors, not for dynamic errors. The rotations can be combined in a vector. Geometrically the direction of this vector represents the axis of rotation and its length the magnitude of the rotation. The rotation vector is defined as:

\[ r_i = \begin{pmatrix} irx \\ iry \\ irz \end{pmatrix} \]  \hspace{1cm} (3.8)

The propagation to the probe position of the errors described by these vectors, is according to the following rules:

- Translation errors affect the errors at probe position on a one to one basis. Thus they can be added directly to the nominal probe position, taking into account the correct sign convention.

- Rotational errors have to be multiplied by their effective arm between the probe position and the respective scale, also taking into account the correct sign convention.

The effect of the parametric errors of the segment belonging to the i-th axis, can now be described as:

\[ e_i = t_i + r_i \times a_i \]  \hspace{1cm} (3.9)

Where \( \times \) denotes the cross product and \( a_i \) is the vector containing the effective arm between the scale at axis i and the probe position. This vector is defined as:
Where the parameters $b_{ij}$ define the machine's configuration: $b_{ij}=1$ if the axis contributes to the effective arm, else $b_{ij}=0$. The values $x$, $y$, and $z$ are the scale readings also defined by Formula 3.4. The parameters $o_{ij}$ are the elements of a vector $O_i$, containing the offsets between the probe head and the coordinate frame of the $i$-th axis, with all axes in zero position. The parameters $s_i$ are the elements of the probe vector $S_i$, containing the distances between the probe tip and the probe head. In general a probe can have several different styli. For each of these styli a separate probe vector has to be defined. This implies that the model has to be adapted during a measuring task, each time another stylus is used. Combining the contributions of all axes yields the total position error at the probe position:

$$\varepsilon = \sum_i \varepsilon_i$$

### 3.3 Modelling dynamic parametric errors

Although a rigid-body kinematic model is used for the error propagation of the parametric errors, the components of a CMM structure are here considered as flexible elements. Dynamic loading of the machine will therefore introduce deformations of these elements. These deformations have to be described and expressed in the dynamic, parametric errors, which will be used later as the inputs of the kinematic model.

#### 3.3.1 Modelling approach

The parametric error models have to include quasi-static errors due to inertia effects as well as vibrations. As already mentioned with the kinematic model, the CMM axes are considered to be mutually perpendicular. Based on this, the as-
assumption is made that there is no dynamic coupling between the CMM axes that affects the errors significantly. This is a reasonable assumption for orthogonal CMM types (see also Jones 1993). The errors are related to each other because they can be induced by the same CMM motion, but they don’t affect each other directly. This implies that the effect of the motion of a certain axis on a parametric error is affected by the motion of this axis only and by the properties and geometry of the structural loop. The latter being dependent on the position of the various axes. In this way the description of the errors can be separated for each axis. The contributions of the various axes to one parametric error can be simply added together in order to get the total error.

Let us consider the physical behaviour that causes position errors at the probe position due to the motion of a single axis. The motion is generated by a motor current, based on a control scheme, that generates a torque and by a transmission provides a driving force to a carriage (see also Figure 3.4). This results in an acceleration, a velocity, and a certain position of the carriage. The acceleration of the carriage induces vibrations and quasi-static deformations of the axis components. In the modelling approach the parametric errors have to be described, i.e. the errors in the location of a element of the structural loop, such as a carriage, relative to the previous element. As input of the model of a axis, we can take the motor current, measured spindle rotation, carriage acceleration, or measured carriage deflection(s), such as rotation(s). The choice of the input effects the modelling complexity, the sensors needed and the accuracy of the estimated errors. Modelling will be more complex if a greater part of the CMM structure is involved, and the accuracy will decrease. On the other hand the input signals might be obtained more easy. Since here only the measurement accuracy is important, only the part of the structural loop that affects this accuracy will be

\[
\begin{align*}
I & \rightarrow \text{Motor} & T & \rightarrow \text{Transmission} & x, \dot{x}, \ddot{x} & \rightarrow \text{Mechanics} & \Delta x_{\text{probe}} \\
\end{align*}
\]

Figure 3.4: Schematic representation of a CMM axis’ control, causing an error at probe position.
modelled (i.e. the carriage and the connecting parts to the other axes). In order to avoid unnecessary complexity, and modelling inaccuracies, the drive system (motor and transmission part) will not be included in the modelling. As inputs for the model, either the carriage accelerations or measured deflections can be chosen. Both possibilities will be discussed in Paragraph 3.4.

3.3.2 Single axis model

In general a single axis of a typical CMM can be described by a carriage with a (air) bearing system, moving along a guideway, and a connecting element with the succeeding axis. Usually the connection is formed by the guideway for the carriage of this axis. The last axis often exists of a pinole guided by a bearing system directly connected to the previous carriage. Due to the dynamic load on the axis, induced by its own motion or the other axes, the components are subjected to deformations. Depending on the load situation and the type of element, these deformations can be characterised by rotations, translations, bending and torsion. For each relevant deformation we have to derive a relationship between this deformation and possible input signals.

Based on the Newton-Euler or Lagrange methods, the equations of motion describing the physical relations between the deformations and the dynamic load can be derived. Using matrix notation we can express these equations in the well known form:

$$M \ddot{q}(t) + B \dot{q}(t) + Kq(t) = f(t)$$  \hspace{1cm} (3.12)

Where $M$ denotes the mass matrix, $B$ the damping matrix, and $K$ the stiffness matrix. The vector $q(t)$ contains the generalised coordinates of the system, and the vector $f(t)$ the dynamic load acting on the system. The generalised coordinates of the system are the 'degrees of freedom' of the system at the position of the deformations. Note that especially the mass matrix is not constant but depends on the position of the various axes. The several influence factors such as mass-, stiffness-, and damping parameters, as well as the load situation are very type dependent with respect to the CMM considered. Thus for each CMM type
individual descriptions have to be made of all deformations that will affect the parametric errors belonging to the different axes. Next the deformations have to be expressed in the various parametric errors, which allows the use of the general kinematic model. In the flow chart of Figure 3.5 an overview is given of the mentioned modelling steps that relate the errors in the measured values to the dynamic load on the CMM. In the remainder of this paragraph some common situations are presented, showing how rotation as well as translation errors can be described. Paragraph 3.3.3. will deal with expressing the deformations in the parametric errors.

**Rotations**

In Figure 3.6, as an example, parts of the structural loop of the gantry type CMM studied are depicted. The y-axis shown actually exists of two guideways, to the right- and left side of the machine. The drive force is introduced on the carriage of the right y-guideway. Both y-carriages are connected to each other by the x-guideway, which is guiding the x-carriage. The y-guideways are supported by two columns, that are attached to the machine's frame. The x-carriage is also carrying the support and the bearing system for the z-pinole.

A dynamic load due to acceleration of the y-carriage along the y-axis will cause torsion of the carriage with its bearings and its support (i.e. the joint), resulting in a rotation around the z-axis. Furthermore the x-guideway (i.e. the link) can be subjected to bending. In Figure 3.7 these relevant deformations, in the x-y-plane due to the y-axis motion, are indicated by their rotation angles \( \varepsilon \) (omitting the subscripts that indicate the axis of rotation). Because motion takes place in the horizontal x-y-plane, the gravity can be ignored. It is assumed that damping can be neglected. For the generalised coordinates of these system the rotation angles \( \varepsilon_i \) are chosen:

\[
\varepsilon = \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_c \\ \varepsilon_g \end{pmatrix}
\]  

(3.13)
Figure 3.5: Overview of the modelling steps for relating measurement values to the various deformations of the CMM
Figure 3.6: Parts of the structural loop of a gantry type CMM.

Figure 3.7: The rotational deformations of the y-axis of a CMM due to y-motion.
These angles denote the absolute rotations of the considered components relative to a fixed coordinate system. The rotations $\varepsilon_s$ and $\varepsilon_c - \varepsilon_b$ represent the torsion of the support and carriage respectively. The rotation $\varepsilon_b - \varepsilon_s$ represents the rotation of the carriage due to deflection of the bearings. The angle $\varepsilon_g - \varepsilon_c$ is the so-called virtual rotation representing the transverse displacement of a point of the x-guideway due to the bending of the guideway. The virtual rotation of the point is expressed relative to the same fixed coordinate system, and is defined by the beam deformation:

$$\varepsilon_g - \varepsilon_c = \frac{\delta_x}{l_x}$$  (3.14)

Where $\delta_x$ is the transverse displacement of the guideway at the position $x = l_x$ along the guideway. The deformations of all the components of the axis considered, have to be expressed in the chosen generalised coordinates. Hence, the behaviour of each continuous element has to be described in terms of these coordinates. In general the components of a CMM axis are elements of complex geometry, that are difficult to model in detail. Therefore more simple elements are taken in order to describe the behaviour of these components. The bearings are modelled as springs that have only a certain stiffness in the direction perpendicular to the guideway. The individual bearings have zero stiffness with respect to rotations. The stiffness of the carriage against rotations due to bearing deflections is obtained from the configuration of several bearings. In general the stiffness of air bearings is frequency dependent.

The guideway is modelled as a simple Euler beam, thus assuming no shear deformation and rotary inertia of the beam parts. The latter condition is satisfied if the guideway is sufficiently slender. Applying also Bernoulli’s hypotheses that cross sections don’t deform and remain perpendicular to the beams axis, the motion $w(x,t)$ of a segment of the beam is governed by the well known partial differential equation (Timoshenko 1974, Thomson 1988):

$$\frac{\partial^2}{\partial x^2}(EI \frac{\partial^2 w(x,t)}{\partial x^2}) + pA \frac{\partial^2 w(x,t)}{\partial t^2} = q(x,t)$$  (3.15)
With the parameters $\rho$ as the density, $A$ the cross-sectional area, $E$ the Young's modulus and $I$ the second moment of inertia of the beams cross-section, $q(x,t)$ is the load per unit of length. When $E$ and $I$ are considered constant and the load per unit of length is due to an acceleration $\dot{y}(t)$ of the base of the beam (i.e. the carriage) Formula 3.15 can be written as:

$$EI \frac{\partial^4 w(x,t)}{\partial x^4} + pA \frac{\partial^2 w(x,t)}{\partial t^2} = \rho A \ddot{y}(t)$$  \hspace{1cm} (3.16)

The load due to the mass of the $x$-carriage and the mass of the remaining part of the beam beyond the position of the $x$-carriage, acts as an external force on the beam. The effect of this force has to be taken into account by defining proper boundary conditions. In order to express the deformations $w(x,t)$ of the beam in the discrete, generalised coordinates that can be associated with the beam (in this case $\epsilon_g$ and $\epsilon_c$), a displacement field that describes the deformation between the discrete coordinates, has to be assumed. For the bending of the $x$-guideway it is assumed that the displacement field of the beam element is similar as in the static situation. This field can be described using a third order polynomial in the position coordinate along the beam's axis (Lammerts 1993). A displacement field that satisfies the boundary conditions $w(0,t) = 0$ and $w(l_x,t) = \delta_x$ for all $t$ is:

$$w(x,t) = \frac{1}{2} \frac{\delta_x}{l^3} (3x^2 - x^3)$$  \hspace{1cm} (3.17)

Thus only one bending mode of the beam element is considered here. With this mode the largest deflections at the point of interest can be described. However, depending on the frequencies of the dynamic load, higher order modes can be induced. When these higher modes have significant contributions, the beam element has to be modelled as a continuous element, taking into account more modes. By substituting the found displacement function (3.17) and relation (3.14) into Equation 3.18 a differential equation in the generalised coordinates can be obtained.
Similar to the bending of a beam, a relation can be derived for the torsion of a component, such as the support column. Assuming this element can be modelled as a rod, we can write down an expression for the torsion of a segment of the rod:

\[ G \frac{\partial^2 \theta(x,t)}{\partial x^2} - \rho \frac{\partial^2 \theta(x,t)}{\partial t^2} = 0 \]  

(3.18)

With \( G \) the shear modulus of elasticity, \( \theta(x,t) \) is the angular deformation of the element. Here a linear displacement function is taken, that has the following form:

\[ \theta(x,t) = \theta(0,t) + \frac{\theta(l,t) - \theta(0,t)}{l} \cdot x \]  

(3.19)

The angular rotations for \( x = 0 \) and \( x = l \) represent the corresponding generalised coordinates located at the end points of the regarding component. Thus differential equation 3.18 can also be expressed in the generalised coordinates.

Based on the differential equations expressed in the chosen generalised coordinates, the inertia and stiffness terms with respect to these generalised coordinates can be determined. A convenient approach is to use Lagrange's method. The mass- and stiffness matrices for each individual component can be calculated using the kinetic and potential energy terms of the differential equations. These matrices can then be assembled to form the global mass- and stiffness matrices (see e.g. Thomson 1988). In Appendix A, expressions for these matrices are given for the system depicted in Figure 3.17. The mass- and stiffness matrices both depend on the position of the x-carriage along the x-guideway.

The driving force that is accelerating the y-axis is applying a force as well as a moment to the y-carriage. The extra moments on the elements due to the eccentric drive force and the resulting internal forces on the elements, have to be taken into account as external moments. Also the moments due to the induced accelerations have to be accounted for. For the y-axis this yields the following load vector containing the moments:
Modelling of dynamic errors

Here $F(t)$ is the force that is applied to the y-carriage by the drive system, $\ddot{y}(t)$ the resulting acceleration and $m_c$, $m_b$, $m_g$, $m_x$, and $m_z$ the masses of the y-carriage, the bearing system, the guideway, the x-carriage, and the z-axis respectively. The dimension $d$ is the width of the carriage and bearing system, acting as the effective arm for the moment due to the driving force and internal forces. Applying equilibrium of forces, the driving force $F(t)$ can be expressed into the acceleration $\ddot{y}(t)$ of the total mass. This allows the expression of Equation 3.20 in a more compact form:

$$\ddot{f}(t) = \text{diag}(\ddot{y}(t))m$$  \hspace{1cm} (3.21)$$

Here $\text{diag}(\ddot{y}(t))$ is a square matrix with only zeros except for its diagonal. The elements on the diagonal all have value $\ddot{y}(t)$. $m$ is a vector holding the masses and the effective arms of the forces. Thus the equations of motion of the y-axis can be expressed as:

$$M\ddot{\xi}(t) + K\xi(t) = \text{diag}(\ddot{y}(t))m$$ \hspace{1cm} (3.22)$$

This set of second order differential equations of the undamped system describes the relationship between the acceleration load on the y-axis and the resulting deformations, expressed into the rotations of elements belonging to the y-joint and the bending of the x-guideway.
Translations

Dynamic translation errors can be modelled relatively simple. This is due to the fact that the translation errors are defined in coordinate frames located near the centroids of the carriages. The structural loop is not very sensitive in the direction of the relevant translation errors, i.e. the errors perpendicular to the guide- ways. The stiffness of the various elements of the CMM’s axes with respect to these directions is relatively high, and in general only the bearing compliance will significantly contribute to the translation errors. The stiffness in the direction of movement of an axis can be much lower, in contrast to the stiffness in the other two directions. This will only cause linearity errors that are directly measured by the scales. Thus these translation errors will not affect the measuring accuracy. However if an axis is weakly supported, deformations of this support, e.g. bending, can contribute to the translation errors.

In Figure 3.8 again the y-axis of a gantry type CMM is depicted, now as an example of translational deformations. The translation errors of the y-carriage are here caused by accelerations of the x-axis. Due to this acceleration there will be a reaction force on the bearings of the y-carriage, causing it to displace perpendicular to its guideway. Furthermore bending of the support column yields a translation error of the y-carriage relative to the reference coordinate frame. Such an error can be accounted for by including the translation in the errors that are described relative to the coordinate frame of the y-carriage. Besides bending of the support column, also torsion of this column will cause translation errors, especially when the y-carriage is in one of its end-positions. In this case we can chose the translational errors due to the bearing system ($\delta_b$), due to the support bending about the y-axis ($\delta_y$), and the support torsion about the z-axis ($\delta_z$) respectively as the generalised coordinates of the system:

$$\hat{\delta} = \begin{bmatrix} \delta_b \\ \delta_y \\ \delta_z \end{bmatrix}$$ (3.23)

For these translation errors we can derive in a similar way as for the rotation errors a mass matrix and a stiffness matrix. We can use these matrices to define
the differential equations that describe the relationship between the load due to x-axis motion and the translation errors of the y-carriage:

$$M \ddot{\delta}(t) + K \delta(t) = \text{diag}(\ddot{x}(t))m$$  \hspace{1cm} (3.24)

### 3.3.3 Deformations expressed in parametric errors

In the previous paragraph it has been shown how expressions for translational as well as rotational deformations of a CMM can be derived. In this way all translational and rotational deformations of the CMM, due to the dynamic loading by motion of the various axes, can be described in principle. These deformations are expressed as errors in the chosen generalised coordinates. In order to be able to use the kinematic model for calculating the effects of the errors on the probe position, the parametric errors have to be known. Therefore the deformations have to be related to the parametric errors that are described relative to coordinate frames attached to the carriages at the scale positions.
The way in which the deformations can be expressed in the parametric errors is not quite straightforward. Since the CMM's components are in fact continuous elements, the deformations are also continuously distributed between the various coordinate frames. Therefore it is an arbitrary choice, relative to which coordinate frame a certain deformation is described. Let's consider e.g. the bending of a beam that is enclosed by two coordinate frames (see Figure 3.9). Frame 1 is connected to the carriage that is moving along the y-guideway and located near the position where the beam is attached to the y-carriage. Frame 2 is connected with the x-carriage that is moving along the beam. Note that frame 2 is not rigidly attached to the carriage but that it maintains in its nominal position. This is according the assumption that the parametric errors are small enough not to affect the kinematic model (see Paragraph 3.2). The displacement of the beam, at the position of the x-carriage, can be expressed either as a parametric rotation error in frame 1, or as a parametric translation error in frame 2. Thus, in case the bending of the beam is the only deformation contributing to the parametric errors, we can write:

\begin{align*}
yrz &= \gamma \varepsilon_z \\
\text{or:} \\
xty &= \gamma \delta_y
\end{align*}

Figure 3.9: Possibilities for describing the bending of a beam as a parametric error.
In the first case the position error can be obtained directly from the parametric error, in the second case the effective arm of rotation has to be used to obtain the error. Like the bending deformations, also the deformation due to torsion of a component can be related to either one of the frames. In order to derive the parametric errors unambiguously, some rules are followed for relating the deformations to the parametric errors.

- Deformations of a carriage and components closely related and located to a carriage, such as the bearing system, are naturally related to parametric errors of the carriage concerned. In most cases the main error source will be the finite stiffness of the bearing system, with respect to rotations as well as translations. In principle this can cause errors in all degrees of freedom, except for linearity errors. The rotation errors are generally the most significant ones. Another important error source is torsion of the carriage, since an partly open structure is needed for the guideway along which the carriage is moving. This is especially the case if the carriage is not forming a closed structure around the guideway but rather a U-profile (see e.g. y-carriage in Figure 3.8).

- As mentioned in the example of Figure 3.9 beam deformations can be related to the frames of either one of the connected carriages. For other types of parametric errors, such as geometric errors, guideway (beam) deformations are usually expressed in the frame belonging to the carriage that is moving along the guideway (i.e. in frame 2 of the example). This is sensible since errors are only introduced during motion of this carriage along the deformed beam. However in the case of dynamic errors the physical background of the deformations is mainly the motion of the carriage carrying the beam (see the example of Figure 3.7). For this reason beam deformations are here entirely expressed as a rotation with respect to the coordinate frame belonging to the carriage carrying the beam. This choice is also more advantageous in the case that various errors can be lumped together, as will be seen later.

- Care should be taken for the rotation at the position of the carriage moving along the beam. The actual rotation of the x-carriage due to the beam deformation is larger than the rotation $\gamma \varepsilon_z$ used for expressing the displacement error of the carriage due to beam deformation in y-direction (see Figure 3.10). The difference $\gamma \varepsilon_z'$ between the actual and used rotations can result in an
additional error $\Delta x'$ in the estimated position of the probe in $x$-direction, if there is an effective arm in the plane of bending. In case of an arm $l_y$ and for small angles this error can be expressed as:

$$
\Delta x' = y \varepsilon_z' l_y
$$  \hspace{1cm} (3.27)

In most cases this error is not important for orthogonal system, since besides the beam itself there will be no large effective arm in the plane of bending. Thus with the difference between the angles being small and a small effective arm, the error usually can be neglected. Only for very long probe extensions the effect might be relevant. In this case the difference between the actual and used rotations has to be taken into account as an parametric rotation error of the $x$-carriage.
A special case arises if the lowest axis of the CMM is weakly supported. Due to the dynamic load on the machine, the support will also be subjected to deformations. The resulting support motion has to be included in the parametric error modelling. A possibility to account for support motion is to introduce an extra coordinate frame at the base of the support, relative to which the errors can be described. A more convenient method, used here, is to include these errors in the parametric errors belonging to the carriage of the lowest axis. Displacements of the support and thus the axis are now expressed as translation errors of the carriage. This implies that, due to the support motion, in this case also parametric errors in the direction of axis motion, i.e. linearity errors, are introduced. Thus, in contrast with most situations, even when the coordinate frames are chosen to be located at the scales, still dynamic linearity errors can occur (see also the choice of the location of coordinate frames in Paragraph 3.2).

Using these considerations, all deformations of the various components can be expressed as part of a certain parametric error, belonging to one of the three CMM axes. Thus e.g. the parametric rotation error $irj$ can be written as a summation of the deformations of several components $c$:

$$irj = \sum_{c=1}^{n} i \epsilon_{j,c}$$

(3.28)

In this way the kinematic error model, based on 18 parametric errors, can be used to calculate the effect of the dynamic errors on the probe position. For the y-axis of the gantry type CMM depicted in Figure 3.6, we can define the six parametric errors, taking into account the dynamic load of the three axes and the deformation of the carriage, bearing system, guideways and support (the deformations are also depicted in Figure 3.11):

- The rotation error $yrx$ is induced by motion of the y- and z-axis. The y-motion results in a moment around the x-axis, causing torsion of the x-guideway, rotation of the carriage and bearings, and bending of the support components. The force due to z-motion causes mainly bending of the support and y-guideway.
• The rotation error \( y_{ry} \) can be neglected, since the structure with two parallel y-guideways will prevent any significant rotation around the y-axis.

• The rotation error \( y_{rz} \) is induced by motion of the x- and y-axis. The force due to x-motion causes mainly torsion and bending of the support and y-guideway. The y-motion results in a moment around the z-axis, causing bending of the x-guideway, torsion of the carriage and bearings, and torsion and bending of the support components.

• The translation error \( y_{tx} \) is induced by a force due to x-motion, causing bearing deflections of the y-carriage and bending and torsion of the support and y-guideway. The moment due to motion of the y-axis that causes the rotation error \( y_{rz} \), will also cause support displacement in x-direction.

• The linearity error \( y_{ty} \) is caused by support motion, mainly due to a force induced by y-motion, that causes bending of the support. The z-motion can also cause a moment around the x-axis, resulting in bending of the support. This yields a support displacement in y-direction.

• The translation error \( y_{tz} \) is induced by a force due to z-motion, causing bearing deflections of the y-carriage and bending of the support and y-guideway. The moment due to motion of the y-axis that causes the rotation error \( y_{rx} \), will also cause support displacement in z-direction.

The amplitudes of the deformations depicted in Figure 3.11 are strongly exaggerated, and in practice there will be considerable differences between the influences of the load due to the different axes. The last axis (here the z-axis) will in general have little influence compared to the first axis (here the y-axis). For completeness here all effects are mentioned. The errors, including the parametric errors of the x- and z-axis, are summarised in Table 3.1.
Modelling of dynamic errors

Figure 3.11: The dynamic parametric errors, belonging to the y-carriage, due to the deformations of the various components.
<table>
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Table 3.1: Overview of the parametric errors of a gantry type CMM due to the deformation of the various components.
3.4 Estimation of dynamic parametric errors

In the previous paragraphs of this chapter the way in which CMM components are deformed due to axis motion has been described. Also rules for expressing the various deformations into the parametric errors were given. These parametric errors are used as the input for the kinematic model that relates them to the measuring error at probe position. In order to estimate actual values for a certain parametric error at the time of probing, accurate values of all relevant deformations that contribute to this parametric error have to be found.

3.4.1 Methods for estimating the dynamic errors

There are several different methods for obtaining these deformations. These methods can be classified according to the complexity of the modelling that is necessary to derive the values for the deformations. Here we distinguish three different methods, that are either analytical or combined analytical/empirical, with or without the use of additional sensors.

The analytical method is aimed at finding solutions for the mathematical relations that describe all deformations of the CMM structure. Therefore differential equations that relate all deformations to the dynamic load on the CMM (like the examples of the previous paragraphs) have to be derived. Next the differential equations have to be solved for each measuring situation. This means solving sets of differential equations of the type, that are expressed by Equation 3.12, or more specific for rotation and translation errors of a single axis (the Equations 3.22 and 3.24). However, the parameters of these equations, expressed in the mass- and stiffness matrices, are dependent on the machines axes positions. So, these matrices have to be calculated for each individual measuring position. Finally, for finding correct solutions of the equations also the initial conditions and the input signal have to be known. Hence, the time history of the excitation force, i.e. the acceleration, has to be recorded from the time that the whole system is at rest (the initial conditions are then known: \( \dot{x}(0) = 0 \) and \( \ddot{x}(0) = 0 \)) until the time of probing. The acceleration signal can be estimated using the scale readings. Solving the equations, and using the dynamic load as input, yields the advantage that in principle no extra sensors are needed. However, there are
many factors that can influence the accuracy of the estimated errors. Hence, the analytical approach will have a limited level of confidence with respect to the estimated errors. The most important factors affecting the accuracy of the analytical modelling of the dynamic deformations are:

- **Discretisation.** Because it is difficult to find precise analytical solutions for the differential equations that describe the complex geometry of the continuous machine components, an approximation is necessary. Therefore discretisation is used to describe the mathematical relation between structural loop deformations and the mechanical load on the CMM. Thus the machine is divided in discrete parts that can be described by more simple equations, which can be solved by a computer.

- **Assumptions and simplifications.** Several assumptions and simplifications are being made in order to obtain more workable sets of equations. The components of the machine are modelled as ideal elements, such as prismatic beams, and assumptions are made with respect to the behaviour of these elements; e.g. the way a beam deforms. In practice the components will not be ideal, and thus uncertainty in their described behaviour will be introduced. For the sake of simplicity, the influence of damping is neglected. This greatly simplifies solving the differential equations, since the differential equations can be uncoupled easily. The eigenvalues and the eigenvectors of the system are also real. This means that there is no phase lag between the components. Thus all considered masses move in phase, i.e. they reach there positions of maximum amplitude at the same. If damping cannot be neglected, the amplitudes of the components will be smaller since they will be damped out, and their combined effect will be different due to phase lags between the components. As already mentioned before the inertia and stiffness coefficients are not constant but position dependent. In the example of the rotations described in Paragraph 3.3.2, the derived mass and stiffness matrices are both depending on the position of the x-carriage (see Appendix A). This greatly increases the complexity of the differential equations and there solution methods. Furthermore terms in the differential equations are neglected that are related to the moving carriage, such as inertial forces in the direction of the beam axis, and the coriolis and centrifugal forces. Only in the case of small movements, where the CMM configuration can be considered constant for a certain meas-
uring position, parameters can be calculated, that are constant for that particular position. In practice this will be often the case for measuring with touch-trigger probes, especially when measuring points that are located close together. However, for scanning using a measuring probe this is generally not true.

- **Parameter uncertainty.** Besides the above mentioned simplifications with respect to the components, there can also be a considerable uncertainty in the values of the various parameters, such as: masses, mass moments of inertia, moduli and shear moduli of elasticity, second moments of inertia, and damping coefficients. The exact values of these parameters are not always known. Thus often these parameters have to be identified by experiments. A problem remains the fact that in general the parameters are not constant, not only with respect to the carriage positions, but also for a single component such as a beam. In the latter case the parameters may vary for different cross-sections of the beam. Obviously these parameter uncertainties affect the accuracy of the estimated deformations.

- **Uncertainty of the input signal.** Besides uncertainties in the derived model, the accuracy of the solutions for the deformations are also influenced by the uncertainty of the input signal. In order to solve the differential equations, the input signal has to be known. This means that the input signal must be available as a function of time from the moment that initial conditions for the differential equations are known, i.e. when the whole system is at rest. Thus with the dynamic load as input to the model, the time history of the acceleration of the moving carriage has to be obtained. This signal can be derived by differentiating the carriage position twice with respect to the time. The position information can be read obtained from the scales of the CMM axis. In order to avoid strong discontinuities in the acceleration signal, the scale signal has to be low-pass filtered with zero phase shift.

Taking into account all these factors, the reliability of the calculated values for the dynamic errors at probe position should be judged with caution. A quite different way of finding values for the deformations is to follow a combined analytical/empirical approach using additional sensors. Such an approach involves the measurement of the actual deformations by suitable sensors, implemented on
the CMM. Obviously this requires less modelling effort with respect to the dynamic, parametric errors. In this way a great deal of possible modelling inaccuracy can be avoided. Furthermore measuring the deformations is advantageous since no time history of an input signal is needed. Insight in the physical behaviour of the CMM is used in the investigation of the 'weak' spots of the CMM, that can cause significant deformations. Knowledge about the physics of the CMM is also necessary in order to derive the parametric errors from the deformations. In general the sensors that are placed on the CMM will not measure a certain parametric error directly. Thus modelling is required to relate the sensor readings to the corresponding parametric errors. Measurement of the deformations is advantageous, because reliable estimations of the parametric errors can be obtained. Thus there is less risk of an unacceptable inaccuracy. The obvious disadvantage is the fact that extra sensors are needed.

Another alternative is to follow a analytical/empirical approach without the use of additional sensors. In this approach a model is used relating the probe position to the time histories of the input signals such as the scale readings and the derived velocity and acceleration. The parameters of the differential equations that describe the relationships are non-constant during axis motion. These parameters are also estimated using the input signals. By experimentally comparing the estimated probe position with the actual probe position, the parameters of the model are adjusted in such a way that the model yields good estimations of the measuring error at the probe position. This method has the advantages over the previous two methods that there is less effect of modelling inaccuracies due to the adaptation of the parameters and that no extra sensors are necessary. However a lot of effort is needed to find adequate models that give a good description of the errors and to calculate all the model parameters each time after motion. Furthermore it is difficult to predict the precision in the results that can be expected by using this method.

The main purpose of our research project on dynamic errors of CMMs is to develop a method for improving the accuracy of a CMM subjected to dynamic errors by error compensation. It is also demanded that actual compensation is achieved for a certain CMM. Therefore the method using additional sensors for measuring the deformations was adopted first. This approach is quite straightforward and empirical methods for assessment of errors from other error sources have proved
their successful application in several projects executed at the Precision Engineering section of the Eindhoven University of Technology (Theuws 1991, Schellekens 1993, Soons 1993, Spaan 1995). Furthermore the sensor measurements give more direct information about CMM’s dynamic behaviour. Based on the knowledge about the CMM’s behaviour, assessment of the other methods can be more effective. At this moment another project is being carried out to investigate the possibilities of the analytical/empirical approach using only the scale readings as input signals. However, this thesis will concentrate on the method using additional sensors. With these sensors the deformation of a component, that can be related to a certain parametric error, is measured directly. The contributions of other component deformations to the same parametric error are estimated, using relative simple relations between these deformations and the measured deformations.

3.4.2 Measuring the dynamic deformations

For the direct measurement of CMM structural deformations, suitable sensors have to be mounted on several locations at the CMM. The type of sensors and their locations depend on the actual occurrence of relevant deformations at the CMM. In the error descriptions of Paragraph 3.3 a CMM is modelled as a series of flexible joints and links. The carriages with their bearing systems are forming the joints, and the guideways are forming the links of the CMM. Possible joint deformations that can occur are translations and rotations of the carriages, and possible link deformations are bending and torsion of the guideways (see e.g. Figure 3.11 and table 3.1).

An obvious possibility for measuring link deformations of a machine is the use of strain gauges (Spaan 1995). These gauges can be attached to the beams that are subjected to bending or torsion. When a load is being applied to the beam, strains are induced that can be measured. However, a problem of this method is that in general the load induced strains are small compared to the deformations. Thus if the parametric errors due to beam deformations are significant, very sensitive strain gauges are needed to measure these errors. The significance of bending deformations will be discussed for an actual CMM in Chapter 4.
Chapter 3

In practice the deformations of the joints will often give the largest contributions to the error at probe position (see e.g. Chapter 4 and Spaan 1995). A good possibility for measuring these deformations of a particular joint is to attach displacement sensors to the respective carriage. From the sensor measurements the dynamic translation- and rotation errors of the carriage relative to the guideway can be derived (see also the schematic drawing of Figure 3.12). The sensors measure the relative displacements of the carriage sides perpendicular to the guideway. From the combined measurements of the two sensors on both sides of the carriage, one of the carriage’s rotations and one of the translations can be found. Let’s define the measured displacement by one sensor as \( i S_j \). Here the subscript \( i \) indicates the corresponding CMM axis and \( j \) the axis along which the sensor is measuring. A positive sensor value corresponds to a displacement in the positive direction of the \( j \)-axis. The superscript \(+/-\) indicates whether the sensor is attached to the side in the positive or negative axis direction. Thus the value \( yS_j^+ \) is the vertical displacement of the back-side of the \( y \)-carriage. Hence, we can write for the carriage rotation \( y \varepsilon_y \) of the \( y \)-carriage about the \( x \)-axis:

\[
y\varepsilon_y = \frac{yS_z^- - yS_z^+}{l_y}
\]  

(3.29)

With \( l_y \), the distance between the two sensors. Note that the measured rotation error \( y \varepsilon_y \) is generally not the same as the parametric error \( yrx \). The latter describes the whole rotation about the \( x \)-axis for the respective carriage, while the
former describes only the relative rotation between the carriage and the guideway. For the translation error $\delta_z$, a similar expression is obtained:

$$\delta_z = \frac{y S_z + y S_z^t}{2}$$  \hspace{1cm} (3.30)

With four sensors on a carriage the two translations perpendicular to the guideway and the two rotations about axes perpendicular to the guideway can be measured. The third translation, in the direction of the guideway, is measured by the carriage's scale and is not of interest. The third rotation is the rotation about the guideway axis (i.e. the roll). Accurate measurement of this rotation is generally more difficult, due to the relative small height and width of the guideways. Thus the distance between the sensors measuring this rotation is small. This puts higher demands on the sensor accuracy. In most cases dynamic rotation errors of a carriage about the guideway axis will be relative small and of no interest. For all CMM types (see the overview of Figure 2.3) sensor arrangements can be made, that can measure carriage errors.

However, many sensors are needed for measuring all possible deformations. Of course it is often not desirable, from an economical point of view, to use a large number of sensors. This would also increase the complexity of the hardware, making the CMM more sensitive for failures, and increasing maintenance costs. Therefore the number of sensors has to be limited. As already mentioned at the end of Chapter 2, the number of sensors will be a pay-off between accuracy and economical reasons. Our approach is aimed at the use of a minimum number of extra sensors, but with sufficient accuracy. This can be achieved by relating the measured deformations to the other relevant deformations.

### 3.4.3 Relating parametric errors to measured deformations

Let's consider again the rotation errors about the $z$-axis that can be related to the $y$-axis (see also Figure 3.7). The components that can contribute to the parametric rotation error $yrz$ are (according to the rules presented in Paragraph 3.3.3): the $y$-carriage, its bearing system, its support, and the $x$-guideway. The rotations
Figure 3.13: The component deformations belonging to the y-carriage joint that contribute to the parametric rotation error \( y_{rz} \).

of these elements are denoted as \( y_\text{carriage}, y_\text{bearings}, y_\text{support}, \) and \( y_\text{z-x-guideway} \) respectively. Using Equation 3.28 the parametric error \( y_{rz} \) can be expressed as:

\[
    y_{rz} = \sum_{\epsilon=1}^{n} y_\epsilon = y_\text{carriage} + y_\text{bearings} + y_\text{support} + y_\text{z-x-guideway}
\]

First we consider only the first three components that together form the y-carriage joint. In Figure 3.13 the y-carriage joint with these components is depicted. For convenience the subscripts \( y \) and \( z \) are further omitted in this paragraph. In contrary to the definitions of Paragraph 3.3, in this case the rotations are not absolute with respect to a reference coordinate frame, but they are defined relative to the previous component. The rotation of the support is defined relative to the CMM’s reference coordinate system. In this way the values for these rotations correspond directly with their respective deformations and Equation 3.31 can be used to obtain the parametric rotation error. The y-carriage joint can be represented schematically by a mass-spring system, as depicted in Figure 3.14a.
In this representation the support, the carriage, and the part of the structural loop that is carried by the carriage (i.e. the traverse) all have a certain mass moment of inertia with respect to the joint’s z-axis, which is the axis of rotation. The mass of the bearings is negligible. The indicated rotation error of the carriage is the rotation between the top and bottom of the carriage, due to torsion of the carriage. Similar the support rotation is the rotation between the CMM frame and the top of the y-guideway, also caused by torsion. The bearing rotation between the guideway and the bottom of the y-carriage, is due to deflections of the individual bearing pads. All these deformations are a result of the (finite) stiffnesses, that are represented as springs between the elements, and the (internal) moments acting on the various components. In general the relationship between the various deformations can be described by sets of differential equations (see Paragraph 3.3). However for the joint the mass moment of inertia of the traverse will be much larger than the moments of inertia of the other components, that are located closely to the axis of rotation. In this case a simplified model can be obtained by neglecting the moments of inertia of the carriage and the support. Furthermore it is assumed that the stiffness of the bearing system is constant over the frequency range of interest and can be represented by its static stiffness.
The frequencies of interest are the lower frequencies of the CMM induced by motion of the axis. In this model the system is represented by a series of springs and a single mass moment of inertia (see Figure 3.14b). Due to the absence of the inertia moments between the springs, the same moment \( M_{\text{traverse}} \) is acting on each of the springs. Thus for \( J_{\text{carriage}} \) and \( J_{\text{support}} \approx M_{\text{traverse}} \), we can write:

\[
M_{\text{support}} = M_{\text{bearings}} = M_{\text{carriage}} = M_{\text{traverse}}
\]

or:

\[
k_{\text{support}} \varepsilon_{\text{support}} = k_{\text{bearings}} \varepsilon_{\text{bearings}} = k_{\text{carriage}} \varepsilon_{\text{carriage}}
\]

Where the stiffness parameters \( k_i \) are assumed constant. Using this relationship we can express the support- and carriage rotations in terms of bearing rotations:

\[
\varepsilon_{\text{support}} = \frac{k_{\text{bearings}}}{k_{\text{support}}} \varepsilon_{\text{bearings}}
\]

\[
\varepsilon_{\text{carriage}} = \frac{k_{\text{bearings}}}{k_{\text{carriage}}} \varepsilon_{\text{bearings}}
\]

Substituting these expressions in Equation 3.31 and considering only the joint rotations, we can write for the parametric error of the joint:

\[
y_{\text{rz}} = \varepsilon_{\text{bearings}} + \frac{k_{\text{bearings}}}{k_{\text{carriage}}} \varepsilon_{\text{bearings}} + \frac{k_{\text{bearings}}}{k_{\text{support}}} \varepsilon_{\text{bearings}}
\]

or:

\[
y_{\text{rz}} = \left(1 + \frac{k_{\text{bearings}}}{k_{\text{carriage}}} + \frac{k_{\text{bearings}}}{k_{\text{support}}} \right) \varepsilon_{\text{bearings}}
\]

If the bearing rotations (in fact the rotations between the bottom of the carriage and the guideway) are measured and the stiffness ratio's are known the parametric error for the joint can be estimated using this relationship.
Expressing Equation 3.37 in a more general form yields:

\[ irj = \left( \sum_{c=1}^{n} \frac{k_m}{k_c} \right) \varepsilon_{j,m} \]  

(3.38)

where \( \varepsilon_{j,m} \) denotes the measured rotation error of component \( m \) about the \( j \)-axis during motion in \( i \)-direction, and \( k_m / k_c \) the stiffness ratio between the measured component \( m \) and component \( c \). This relationship gives an estimation of the parametric rotation error \( irj \) based on measurements, such as proposed in the previous paragraph.

Relating link deformations, such as the bending of a guideway, to the measured carriage rotations, is less straightforward than dealing with the joint rotations. In order to estimate the deformation of a link, some assumptions are made regarding its dynamical behaviour. It is assumed that the displacement field of the link in the dynamic situation is similar to the static deformation and that the bending rotation is in phase with the carriage rotation. In general link deformations can be more complex, but the deformation assumed here will cause the largest errors. Furthermore, the link deformations of CMMs and other (precision) machines, are often small compared to joint deformations (which will be shown in Chapter 4). Therefore it is reasonable to expect that the contributions of the more complex deformations of the links are negligible compared to the joint deformations and the assumed bending deformation of the link.

In Figure 3.15 the \( y \)-carriage with the connected link (x-guideway) is depicted. During \( y \)-axis motion the beam is bent due to a distributed inertia force \( q_y \) acting on the x-guideway in \( y \)-direction, and a concentrated inertia force \( F_y \) due to the mass of the x-carriage and z-axis, also in \( y \)-direction. The forces cause a deformation \( \delta_y \) of the guideway at the x-carriage position. The virtual rotation related to this deformation is \( \varepsilon_y \), and the measured carriage rotation about the z-axis is \( \varepsilon_m \). The guideway has length \( l \), the \( y \)-carriage width \( d \), and the x-carriage is positioned at a distance \( l_x \) from the y-carriage, \( k_m \) is the related stiffness and \( M \) the moment causing this carriage rotation. In case of small angular acceleration compared with linear acceleration (\( \varepsilon_m \ll \ddot{y} \)) we can write for the distributed- and concentrated force on the beam:
Figure 3.15: Deformation of the link (x-guideway) connected to the y-carriage.

\[ q_y = \frac{m_x \cdot \ddot{y}}{l} \]  \hspace{1cm} (3.39)

and:

\[ F_x = (m_x + m_c) \cdot \ddot{y} \]  \hspace{1cm} (3.40)

Where \( m_x \), \( m_y \), and \( m_c \) denote the masses of the x-guideway, the x-carriage and the z-axis respectively. On basis of force equilibrium we can express the drive force applied to the carriage as:

\[ F_u = q_y (l + d) + F_x \]  \hspace{1cm} (3.41)

Applying moment equilibrium with respect to the z-axis of the y-joint yields:

\[ M = \frac{1}{2} q_y l^2 + \frac{1}{2} F_u d + F_x (l_y + \frac{1}{2} d) \]  \hspace{1cm} (3.42)
Substitution of (3.41) into (3.42) yields:

\[ M = \frac{1}{2} g_y (l + d)^2 + F_y (l_x + d) \]  

(3.43)

Substituting (3.39) and (3.40) into this relation gives:

\[ M = \left( \frac{1}{2} \frac{m_g}{l} (l + d)^2 + (m_x + m_z) \cdot (l_x + d) \right) \ddot{y} \]  

(3.44)

This moment can be expressed as:

\[ M = k_m \cdot \varepsilon_m \]  

(3.45)

From the Equations (3.45) and (3.44) we can derive the relationship between the measured rotation and the acceleration:

\[ \ddot{y} = \frac{k_m}{\frac{1}{2} \frac{m_g}{l} (l + d)^2 + (m_x + m_z) \cdot (l_x + d)} \cdot \varepsilon_m \]  

(3.46)

Substituting this acceleration into the force equations (3.39) and (3.40) yields:

\[ q_y = \frac{k_m}{\frac{1}{2} \frac{m_g}{l} (l + d)^2 + (m_x + m_z) \cdot (l_x + d)} \cdot \frac{m_g}{l} \cdot \varepsilon_m \]  

(3.47)

and:

\[ F_y = \frac{k_m \cdot \varepsilon_m (m_x + m_z)}{\frac{1}{2} \frac{m_g}{l} (l + d)^2 + (m_x + m_z) \cdot (l_x + d)} \]  

(3.48)

For the deformation of the link we assume a displacement \( \delta_g \) that can be calculated on basis of the known deformations for a statically loaded beam. The distributed force will cause a displacement:
The concentrated force due to the x-carriage and z-axis, and the 'free' part of the x-guideway (i.e. the part beyond the x-carriage seen from the y-carriage) causes a displacement:

\[
\delta_{g,q} = \frac{q_x \cdot l_x^4}{8EI} \tag{3.49}
\]

\[
\delta_{g,F} = \frac{(F_x + q_x(l - l_x)) \cdot l_x^3}{3EI} \tag{3.50}
\]

The total beam displacement at the x-carriage position is found by applying the superposition principle for both load cases:

\[
\delta_g = \delta_{g,q} + \delta_{g,F} \tag{3.51}
\]

For the virtual rotation that describes this displacement relative to the y-carriage, we can write:

\[
\varepsilon_x = \frac{\delta_g}{l_x} = \frac{\delta_{g,q} + \delta_{g,F}}{l_x} \tag{3.52}
\]

Using the relations (3.47) - (3.51), we can express the rotation error \(\varepsilon_g\) of the x-guideway in terms of the measured carriage rotation \(\varepsilon_m\):

\[
\varepsilon_g = \frac{k_m (m_x + m_z) + m_x^g (l - \frac{5}{8} l_x))}{\frac{1}{2} m_x (l + d)^2 + (m_x + m_z) \cdot (l_x + d)} \cdot \frac{l_x^2}{3EI} \cdot \varepsilon_m \tag{3.53}
\]

or written in a more compact form:

\[
\varepsilon_g = \frac{k_m}{k_g} \varepsilon_m \tag{3.54}
\]
with $k_g$:

$$k_g = \frac{\frac{m_k}{l^2}}{\frac{1}{l^2} + \frac{m_k}{l} \left(1 + \frac{m_k}{l^2} \right)} \cdot \frac{EI}{l_x} \tag{3.55}$$

In case of bending of the x-guideway the above derived term describing the beam deformation has to be taken into account when calculating the relevant parametric error. Thus for the error $yrz$ Equation 3.37 is expanded with the rotation term expressed by Equation 3.54

$$yrz = \left(1 + \frac{k_{\text{bearings}}}{k_{\text{carriage}}} + \frac{k_{\text{bearings}}}{k_{\text{support}}} + \frac{k_{\text{bearings}}}{k_{\text{guideway}}} \right) \epsilon_{\text{bearings}} \tag{3.56}$$

Where the measured rotation $\epsilon_{m}$ is represented by $\epsilon_{\text{bearings}}$. In a similar way all relevant deformations of a CMM (see e.g. Table 3.1) can be related to the respective parametric errors and their total contribution to this parametric error can be estimated on basis of the measured value of a certain carriage deformation.

Summarising, we can describe our approach for improving the accuracy of a CMM subjected to dynamic errors, by the following steps:

- Describing the CMM structure with a kinematic model. With this model the degrees of freedom of the CMM are defined. Dynamic errors in the structural loop of the CMM have to be expressed in these degrees of freedom, which allows the calculation of their effects on the probe position.

- Analysing the dynamic behaviour of the CMM in order to identify the significant deformations. Based on the results suitable sensors can be implemented on the CMM for measuring these significant errors on-line.

- Measuring of deformations with the sensors implemented on the CMM. Based on the measured values and machine parameters such as the stiffness of components, the dynamic load on the CMM that caused a certain deformation can be estimated. This allows the estimation of other relevant errors that are not measured by the sensors.
• Expressing the combined effect of measured- and estimated errors into the parametric errors. These are the errors in the degrees of freedom of the kinematic model. The model is used for calculation of the error propagation to the probe position. The calculated error values at probe position during a certain measurement task are used for compensation of the measurement result.

It is important to realise, with respect to the efficiency of the proposed method, that the modelling and analysis are not dependent on a particular CMM, but only on the type of CMM. This means that the error modelling and analysis of the dynamic behaviour has to be carried out only once for a certain type of CMM. The results can be used for all CMMs of the same type. In general differences between the actual machine parameters (e.g. stiffness values) that are used for estimating deformations based on the sensor measurements, will be small for different CMMs of the same type. However in order to obtain a high accuracy and reliability of the method it is sensible to identified these parameters for each individual CMM.

In Chapter 4 the use of additional sensors for these measurements will be researched in detail for an existing CMM. Chapter 5 will deal with the actual compensation for dynamic errors. In this chapter it will be shown that the simple relationships derived in this paragraph are sufficient for estimating the parametric errors based on the sensor measurements.
4

Measuring dynamic errors of a CMM

The dynamic behaviour of an existing CMM is studied in detail in this chapter. The goal of this investigation is to identify the significant dynamic errors of this CMM during axis motion. The obtained measurement results are important with respect to the implementation of sensors on the CMM, which can measure the dynamic errors on-line. First, a description of the investigated CMM is given. Taking into account the CMM configuration and possible locations for implementing sensors, a measurement strategy is presented. The measurements are carried out using laser interferometry and displacement sensors. From all measurements the most important dynamic errors of this CMM are identified. Different sensor types are discussed for on-line measurement of dynamic carriage errors. Inductive position sensors are chosen and the accuracy and calibration of these sensors is discussed. After implementation of the sensors on the CMM, test measurements are conducted, both static and dynamic, in order to test the usefulness of the sensors.

4.1 Description of the investigated CMM

The CMM under investigation is a Mitutoyo FN 905 measuring machine. This is a gantry type, CNC coordinate measuring machine (see Figure 2.3 for the different types of CMMs). In Figure 4.1 a schematic drawing is given of the CMM's most important components.
Figure 4.1: Schematic drawing of the Mitutoyo FN 905 CMM, used for the experiments.

The base of the machine is formed by a frame with a granite table, on which the objects to be measured are clamped. The frame itself is mounted on rubber pads isolating external vibrations. On the left side as well as the right side of the frame, two columns are supporting the two y-guideways of the machine. Each y-guideway has one carriage with air-bearings for accurate motion. The carriage on the right side contains six air-bearings, two located on top of the y-guideway and acting in vertical direction, and two on both sides of the guideway, acting in horizontal direction. The latter four bearings actually form two sets of two bearings preloaded against each other. The preload for the bearings on top of the guideway is provided by the weight of the machine components they are carrying. The bearing system of this carriage constrains 4 degrees of freedom: the two translations in the directions perpendicular to the guideway and the rotations around axes in the same directions. Both y-carriages are connected by beam, forming the x-guideway. The y-carriage to the left contains two more air-bearings, located on top of the guideway. These bearings constrain the fifth degree of freedom for the y-axis: the rotation around the y-axis itself. Actually, by using two bearings for the left y-carriage instead of just one, the system is over constrained. However,
due to bearing flexibility this is acceptable. The sixth degree of freedom along the y-axis is controlled by the drive system. This drive system consists of a motor and a spindle, connected to the y-carriage on the right side. The position of the carriage along the y-guideway is measured by a linear scale with $1 \mu m$ resolution. The x-axis contains similar components as the y-axis. In this case one carriage, having nine air-bearings, is used. Three bearings act in vertical direction: two on top of the guideway and one at the bottom, providing a preload. The other six bearings form two sets of each three bearings acting on either side of the guideway, and preloaded against each other. Except for the motion in x-direction, these bearings constrain all degrees of freedom. The drive- and measuring system are identical to the those of the y-axis. The z-axis consists of a pinole, guided by a bearing system with eight bearings: four sets of two against each other preloaded bearings. Two bearings are acting at each of the long side surfaces of the pinole. Four degrees of freedom are restricted in this way. Torsion about the pinole axis is only restricted due to the bearing pads sizes. However, moments about the z-axis will be very small. The bearing system of the z-axis is directly attached to the x-carriage. Again the drive- and measuring system are similar to those of the other axes. Because the z-pinole is moving in vertical direction it's subjected to gravity forces in the direction of motion. Therefore its weight is balanced by a counter weight. The support for the counter weight is located on top of the x-carriage.

The three perpendicular axes form the orthogonal coordinate system of a three dimensional measuring space with a range of 550x900x450 mm. At the end of the z-pinole a Renishaw touch-trigger probe is connected. As explained in Paragraph 2.2 this probe is used for establishing measuring points on the object to be measured (see also Figure 2.1). The 1D-inaccuracy for each axis of this CMM is specified as follows:

$$|dL| = 4 + \frac{5L}{1000}$$

With $L$ the measuring range in mm and $dL$ the allowable error in $\mu m$. The 3D-inaccuracy is not specified by the manufacturer, but measurements show a 3D-inaccuracy of approximately 12 $\mu m$. These values specify the machine's inaccuracy without software compensation for geometric errors. In case software compensation for geometric errors is applied, a 3D-inaccuracy of lower than 4 $\mu m$ can
be obtained (see Theeuwse 1989). The repeatability for each axis of the CMM is specified by the standard deviation: $\sigma < 1 \mu m$.

### 4.2 Identifying the dynamic errors of the CMM

#### 4.2.1 Measuring strategy

The goal of the experiments, that will be described in this chapter, is to identify the dynamic behaviour of the CMM under investigation. From the measurement examples presented in Chapter 2 it is already known that quasi-static deformations and vibrations of the CMM’s probe with respect to the table occur, when the CMM is subjected to acceleration. The magnitude of these dynamic errors will depend on the level of acceleration. Furthermore, it is clear from the analysis in Chapter 3 that the deformations generally will depend on the position of the carriages. One of the purposes of the measurements to be carried out, is to get insight in the magnitude of the dynamic errors, what levels of accelerations cause significant errors, and if there is a dependency on the position of the carriages.

In Chapter 3 an approach for achieving error compensation has been adopted that is based on the on-line measurement of certain dynamic errors. For the implementation of this method on the CMM under investigation, it is necessary to identify the components that cause significant dynamic errors of the CMM. Based on the measurement results, the actual locations at the CMM can be chosen, where the dynamic errors have to be measured. In paragraph 3.4.2 it was shown how the rotation- and translation errors of a CMM carriage can be measured using position sensors. Based on these sensor measurements the related parametric errors have to be estimated. The main goal of the measurements conducted here, is to find out which parametric errors contribute significantly to the error at probe position. In Figure 4.2 a schematic overview of the three axes of the investigated CMM is depicted. In this overview all parametric errors and possible sensor locations for the measurement of carriage errors are indicated. For clarity the z-axis has been drawn separately from the x-axis. The y-carriage on the left side has been omitted.
Figure 4.2: Possible dynamic errors and locations for displacement sensors that can measure the carriage rotation- and translation errors of the respective axes of the investigated CMM. The spots indicate the sensor locations.

Each spot indicates a sensor location. At this location a displacement sensor can be attached to the side of the respective carriage. Most carriage errors can be measured using the displacement sensors. As explained in Paragraph 3.4.2, with four sensors on a carriage the two translations perpendicular to the guideway and the two rotations about axes perpendicular to the guideway can be measured. The third translation, in the direction of the guideway, is measured by the carriage's scale and is not of interest. Due to the relative small height and width of the guideways, accurate measurement of the rotation about the guideway axis is more difficult. In most cases dynamic rotation errors of a carriage about the guideway axis will be relative small and of no interest. For the CMM under investigation the rotation error \( \xi_x \) can be important. The x-carriage is carrying the z-axis. In the case of y-axis motion this can result in a large moment with respect to the x-axis due to inertia effects of the x-carriage and z-axis. Combined with a low stiffness of the carriage's bearing system with respect to the x-rotation
(due to the small bearing distance), significant rotation errors can be expected. Therefore a fifth sensor might be necessary for the x-carriage. Suitable measurements on the CMM have to show which errors are important.

Figure 4.2 is taken as a guideline for systematically carrying out the necessary measurements. In principle all the carriage translation and rotations with respect to the guideways, as well as all deformations of the other components, such as the guideways, have to be identified individually. In order to measure all these errors many, often complicated measurements, have to be carried out. Therefore a different strategy was adopted to limit the number of measurements. It is expected that the translation errors are small compared to the rotation errors (see also Paragraph 3.3.2). This is checked first for this CMM. Next rotation measurements are conducted, measuring the rotation error at probe position relative to the CMM table using a laser interferometer. The advantage of rotation measurements over translation measurements in the direction of motion, is that the measured rotations can be directly interpreted as errors about a single axis. Translation measurements also include the actual carriage position and the effects of translation and rotation about difference axes. Hence, the results are more difficult to interpret. The measured rotations are in general also the combined effect of several parametric errors (e.g. the measured rotation $\varepsilon_z$ at probe position about the z-axis for y-axis motion is a combination of the parametric errors $y_{rz}$ belonging to the y-carriage, $x_{rz}$ belonging to the x-carriage, and $z_{rz}$ of the z-pinole. More detailed measurements are necessary to distinguish these parametric errors and the contributions of the various components of a certain axis to the regarding parametric error (e.g. bending of the x-guideway and deflections of the y-carriage bearings can both contribute to the parametric error $y_{rz}$). However only for the rotation errors at the probe position that were found significant, the various CMM component were investigated in more detail. By performing stiffness measurements on the regarding components as well as by making calculations of the deformations that can be expected, it was found which link and joint deformations are relevant. Summarising, the following measurements were carried out for identifying the CMM components with relevant contributions to the dynamic errors of the CMM:
• laser interferometry measurements at the probe position for identifying the magnitude of the dynamic errors, the influence of the acceleration level, and the influence of the carriage positions.

• measurements using position sensors to test the significance of translation errors.

• rotation measurements, using a laser interferometer, in order to identify the major rotation errors at probe position and additional rotation measurements in order to distinguish the several parametric rotation errors contributing to the error at probe position.

• more detailed sensor measurements on the CMM components in order to distinguish between their contributions to the significant parametric errors.

4.2.2 Results of the measurements on the CMM

The first set of measurements that was carried out, was aimed at identifying the magnitude of the errors in relation to acceleration on the CMM and their dependency on the position of the carriages. These measurements were conducted using a laser interferometer with linear and angular optics. With this equipment accurate time histories can be obtained of rotation- and translation errors along straight lines in the measuring volume. In this way a good estimation of errors at the probe position, induced by (single) axis motion, can be measured.

In Figure 4.3 a typical measurement set-up for measuring rotation errors at the probe position of the investigated CMM is shown. The laser head and angular interferometer are placed on the granite table. The angular retro-reflector is mounted at the end of the z-pinole of the CMM. The laser beam from the laser head is split into two beams both parallel to the axis of motion. After reflection on the angular retro-reflector at the CMM, both beams pass the interferometer where they interfere. Rotations of the retro-reflector relative to the interferometer cause length differences between both beams. These differences in length are measured by the interferometer. In this way rotations can be measured, during linear motion of the CMM. In the depicted set-up of Figure 4.3 rotations \( \varepsilon_z \) about
the vertical axis can be measured during motion in \( y \)-direction. The system used for most measurements is a Renishaw ML 10 laser interferometer. It can perform measurements with a sample frequency of maximum 5 kHz. The estimated inaccuracy of angular laser measurements on the CMM is approximately 0.2 arcsec. Using different optics instead of angular optics, different errors can be measured.

With linear optics translations in the direction of motion and with straightness optics translations perpendicular to the direction of motion can be measured. For the first set of measurements rotation optics were used to measure rotation errors. As explained in the previous paragraph rotation errors generally have the largest contribution to the probe error. In Paragraph 2.3.3 already some examples of typical measurement results for the investigated CMM have been given (Figures 2.4 and 2.5). In Paragraph 2.2 the typical pattern of motion during a measuring task, using a touch trigger probe, has been shown. In order to illustrate the effects of such motion on the error at probe position two different measurements were performed. The rotation error at the probe position was measured. With linear optics instead of angular optics, the laser interferometer was used for measuring the position of the pinole as a function of time. From the
measured signal the time histories of the velocity as well as the acceleration during motion of one axis were calculated. Although the signals were obtained from different measurements, comparison of the load (acceleration) and deformation (rotation) was possible. The Figures 4.4 and 4.5 show the velocity- and the acceleration signals of the z-pinole respectively, for motion of the y-axis. Figure 4.6 shows the rotation error $e_z$ measured at probe position. During this experiment the CMM was commanded to move to a certain position with a traverse speed of 70 mm/s. Before reaching the commanded position the machine decelerates with a maximum deceleration of $160 \text{ mm/s}^2$. During deceleration the traverse of the machine is rotating about the z-axis. A maximum rotation error of 4.5 arcsec has been measured. This rotation error is mainly due to quasi-static deformation, but there are also vibration components. During travelling with traverse speed, the vibrations are drive induced. Part of the error depicted in Figure 4.6 is due to a geometric error source. Because the y-guideway is not perfectly flat but bent due to manufacturing inaccuracies, there is a linear dependency between the rotation error $y e_z$ and the position along the y-guideway. This effect is shown in the first part of the graph, when the CMM is moving with traverse speed along the y-guideway. This geometric error is approximately 1 arcsec.

![Figure 4.4: Time history of the velocity, measured at probe position of the investigated CMM during change of motion of the y-axis from 70 mm/s to rest.](image-url)
Figure 4.5: Time history of the acceleration, measured at probe position of the investigated CMM during change of motion of the y-axis from 70 mm/s to rest.

Figure 4.6: Time history of the rotation error $\varepsilon_z$, measured at probe position of the investigated CMM during change of motion of the y-axis from 70 mm/s to rest.
Comparing the rotation error with the acceleration, there is a high degree of correlation between both signals for the deceleration part of the motion. From the data it is quite clear that for this CMM moderate accelerations already yield significant dynamic errors. As already mentioned before in Chapter 2, dynamic errors with CMMs are avoided by minimising the accelerations during probing. For touch-trigger probes this is achieved by probing at a constant, generally low, probing speed. The specified maximum probing speed for the described CMM is 8 mm/s. At this speed level dynamic errors due to axis motion are negligible compared with errors from other error sources. In our experiments the CMM's behaviour is being studied when subjected to much higher speeds, involving higher accelerations. Thus dynamic errors will have a much larger influence.

The dependency of the deformations on the dynamic load can be illustrated by a set of experiments with varying accelerations. In these experiments the rotation error $\varepsilon_z$ has been measured at probe position during motion of the y-axis for different commanded velocities. In this way also different levels of acceleration have been obtained. Some results can be found in Appendix B. As already mentioned before, when travelling with traverse speed the vibrations are drive induced. In this case the level of vibration is at maximum $\pm 0.6$ arcsec. With an effective arm of 1 m, rotation errors of that magnitude will yield displacement errors at probe position of approximately 3 $\mu$m. This is comparable to the geometric accuracy of the CMM after software compensation (see Paragraph 4.1). Of more interest are the errors induced during deceleration of the CMM. With CNC-control quasi-static as well as vibrational deformations are found. For a traverse speed of 10 mm/s the errors are fairly small (less than 1 arcsec). However, for speeds of 40 mm/s and 70 mm/s the errors become rather significant (2.5 and 4.5 arcsec respectively). In case of joystick control the accelerations can be much higher than with CNC-control, resulting in relative large rotational vibration errors (over 12 arcsec). This vibration as well as the vibration components during deceleration in CNC-mode are due to the low stiffness of the y-drive. This causes the y-carriage of the CMM to vibrate in y-direction with a relative low frequency of approximately 5 Hz (i.e. the lowest eigenfrequency). This translation motion induces at the same time a rotation about the z-axis due to inertia effects of the x- and z-axis.
Figures 4.7: The effect of a variation in x-position on the rotation error $\varepsilon_z$, measured at the probe position of the investigated CMM. The rotation errors are the maximum values due to deceleration of the y-carriage from a traverse speed of 70 mm/s to rest. The error bars are $2\sigma$-values that indicate the variation in the measured error.

Besides acceleration, the positions of the carriages can also affect the deformations. The acceleration forces on the CMM result into moments about the axes of rotation of the CMM's carriages. Due to the changing position of a certain carriage, these moments can vary.

For the rotation error $\varepsilon_z$ from the previous examples, the position of the x-carriage is of interest. The effect of a varying x-carriage position is shown in the graph of Figure 4.7. At several different x-positions the rotation error $\varepsilon_z$ as measured under similar motion conditions. In the graph the maximum measured rotation error is depicted as a function of the carriage position. Variations in the induced motion cause relative large variations in the measured deformations. The ranges of these variations for each position are indicated in the graph by error bars. Because the machine's coordinate system is located at the left side (see Figure 4.1), an increase in the x-position means a smaller distance between the x-carriage and the relevant rotation point in the centroid of the y-carriage. The
moment about the z-axis consists of contributions due to the acceleration forces in y-direction, acting on the x-guideway and on the x-carriage. If the angular acceleration of the x-guideway about the z-axis is small compared to the acceleration of the y-carriage in y-direction, the acceleration in y-direction will be approximately constant along the x-axis. Considering the moving mass of the x-carriage, the moment acting on the y-carriage is linear dependent on the x-carriage position. Hence, the rotation error will decrease linearly with increasing x-position. The dependency of an error at probe position on, for instance, the position of the x-carriage is important with respect to the sensor measurements. Obviously the method of estimating the probe error based on the sensor measurements must be valid for any position of the carriages. As will be seen later with the implementation of the sensors, the sensor measurements can be used to estimate the error for any position of the various carriages. In the next sections the relevant translation- and rotation errors of the CMM will be identified.

**Translations**

It is expected that dynamic errors, that are pure translational (thus those that are not caused by rotations of components) are small compared to rotation errors. Bending or torsion of guideways or support elements will also results in translations, but these effects will be considered when measuring the rotation errors. As explained in Paragraph 3.3.2 the structural loop of the CMM is not very sensitive in the direction of the relevant translation errors, i.e. the errors perpendicular to the guideways. The stiffness of the various elements of the CMM's axes with respect to these directions is relatively high, and in general only the bearing compliance will significantly contribute to the translation errors. The largest influence of pure translations errors can be expected during acceleration of the x-carriage. In this situation the largest dynamic load is applied to a set of bearings. The acceleration forces in x-direction will cause deflections of the y-carriage bearings due to the reaction force on the y-carriage. The maximum deflection can be estimated on basis of the bearing stiffness. The y-carriage has two sets of bearings acting in the x-direction. For an acceleration \( \ddot{x} \) of the x-carriage, we can write for the deflection \( \delta_x \):
\[ \delta_x = \frac{(m_x + m_z) \cdot \ddot{x}}{2k_x} \] (4.2)

Where \( m_x \) and \( m_z \) denote the masses of the x-carriages and the z-axis respectively. \( k_x \) is the stiffness of one set of bearings. With \( m_x = 34 \text{ kg} \), \( m_z = 10 \text{ kg} \), \( k_x = 70 \text{ N/\mu m} \), and \( \ddot{x} = 0.5 \text{ m/s}^2 \), the deflection in x-direction is approximately \( 0.2 \text{ \mu m} \). A similar result was found experimentally. In the conducted experiment the x-carriage was accelerated (in CNC-mode from rest to \( 70 \text{ mm/s} \)) and the translation of the y-carriage was measured, using position sensors. The measurement showed typical values for the deflection \( \delta_x \) of \( 0.3 \text{ \mu m} \). The maximum value found was \( 0.6 \text{ \mu m} \) during joy-stick controlled motion. Similar measurements were performed for other translation errors. For the error \( \delta_y \) caused by accelerations in y-direction maximum values of \( 0.3 \text{ \mu m} \) for joy-stick mode and \( 0.15 \text{ \mu m} \) for CNC mode were found. The translations due to deflections of the other bearings are even smaller. The errors \( \delta_z \) and \( \delta_y \) induced by z-axis motion are negligible since the acceleration forces of the z-pinole are very small. Likewise, the errors \( \delta_x \) and \( \delta_y \) of the z-axis due to accelerations of the x-axis and y-axis respectively are also negligible due to the small mass of the z-pinole. Thus the pure translation errors due bearing deflections are not significant compared to the rotation errors.

**Rotations**

In order to find out which rotation errors are possibly relevant for the dynamic error at probe position, several measurements were conducted, measuring the rotation error at probe position relative to the CMM table using laser interferometer (see Hazenberg 1993). The measurements were carried out for motion in all three axis directions. For each carriage the two rotations about the axes perpendicular to the guideway were measured for motion of the regarding axis. The rotations about the own axes of the carriages during their motion couldn’t be measured with the used measuring set-up. However during motion of a certain axis there will be no acceleration force that causes a moment about its own axis. Thus no significant rotations were expected about the own axes of the carriages due to their acceleration. The measurement results show that only a limited
number of rotation errors are significant. As expected significant errors were found for motion of the x-axis and the y-axis (see also sensitivity analysis in Chapter 2). Y-axis motion yields significant errors about the x-axis as well as the z-axis. X-axis motion yields rotations about the y-axis. Errors due to z-axis motion are negligible. Figure 4.6 shows an example of the z-rotation due to deceleration from 70 mm/s to rest. Another example is given in Figure 4.8, showing the rotation about the x-axis for similar y-axis motion. Again quasi-static deformations as well as vibrations are found. Compared to the z-axis rotations the errors due to the x-rotation are smaller.

As explained before the rotation errors measured at probe position are generally a combination of parametric rotation errors belonging to different axes. Additional measurements were carried out in order to distinguish between the contributions of the different parametric errors. For instance the rotation about the z-axis of the pinole relative to the x-guideway was measured during y-motion. From this measurement it could be concluded that the contribution \( z_{rz} \) of the z-pinole and \( x_{rz} \) of the x-carriage to the z-rotation are insignificant. In the contrary, the rotation \( y_{rz} \) was found quite significant. From measurements about the

![Figure 4.8: Time history of the rotation error \( \varepsilon_x \), measured at probe position of the investigated CMM during change of motion of the y-axis from 70 mm/s to rest.](image-url)
x-axis for y-axis motion it could be concluded that the contribution \( zrx \) of the z-pinole is insignificant. However both the contributions \( xrx \) of the x-carriage and \( yrx \) of the y-carriage are significant for y-axis motion. In case of x-axis motion \( xry \) was found significant, whereas \( zry \) and \( yry \) were insignificant.

In order to find out which components of a certain axis contribute to the rotation errors that were identified in the previous section, additional measurements had to be carried out. For this reason several static stiffness measurements on the various components have been performed (see also Hazenberg 1993, Haanen 1995). In general the rotation errors are influenced by the finite stiffness of the joints and links of the CMM (see Chapter 3). For measuring the stiffness of these components a measuring set-up was created. With this set-up static forces were applied to the CMM's probe position. The resulting displacements at the various components were measured. From these measured displacements and the applied forces the static stiffnesses of the components were calculated. The results of these measurements show that rotations due to bending of the guideways (i.e. links) are relative small compared with the rotations of the joints (e.g. due to the bearings). For the CMM under investigation bending effects of the guideways were found to be one order of magnitude smaller. Thus also the translations that result from these deformations will be small. However rotations and translations due to bending and torsion of the support of the y-guideway were found significant. In Appendix C the most important results of the static stiffness measurements as well results of calculation of the stiffnesses can be found. From these results conclusions can be drawn with respect to the contribution of the various components to the rotation errors that were found significant. The deformations of all joint components are significant, but the guideway deformations are negligible. From the joint deformations not only the bearing deflections are important, but also the deformations of e.g. the carriage itself. Furthermore, the y-axis is also affected by the finite stiffness of the support of this axis. The support contributes to rotation errors as well as a translation error in y-direction. This latter error can be regarded as a linearity error \( yty \). Thus in general several components belonging to an axis contribute to the parametric errors of that axis. Hence, relationships such as derived in Paragraph 3.4.2 are necessary to estimate the total parametric error from the deformations measured by the sensors to be implemented on the carriages. In Chapter 5, that deals with the actual compensation of the CMM, those relations will be derived for this CMM.
Based on the measurement results, the dynamic parametric errors and the contributions of the various components to these errors have been identified. An overview of all parametric errors of the investigated CMM is given in Table 4.1. In the table it is indicated which errors are significant for motion of what axis (x, y, or z) or which errors are insignificant (-). For the significant parametric errors also the components that contribute to the errors are mentioned. In summary the dynamic behaviour of the investigated CMM can be characterised as follows:

- Significant dynamic errors are obtained by rotations induced by motion of the x- and y-axis. The most significant parametric errors are: $yrz$, $yrx$, $xrx$, $xry$. Most translation errors were found to be small compared to the effects of rotations. This applies to pure translations, resulting from bearing deflections as well as from guideway deformation. However translations $yty$ caused by bending of the support of the y-guideway cannot be neglected.

- The behaviour of the y-axis is rather dominant. The low stiffness of its drive causes translational vibrations, and this also induces rotational vibrations.

- The error levels are affected by the accelerations that are set by the commanded velocities or type of control. In case of higher (commanded) velocities the accelerations are higher and thus the errors. Especially during joystick control large errors are found.

- Errors can be dependent on the position of the various carriages. A change in the position of a certain carriage, changes the effective arm of the acceleration forces on the respective carriage. This causes changes in the moment applied to another carriage and thus variations in the errors.

- Bending of the guideways and pinole (i.e. link deformations) are negligible compared to joint deformations. However support deformations (bending and torsion) cannot be neglected.

- The stiffness of the joints is only partly due to the finite stiffness of the air bearings, other components such as the carriages may also have significant contributions.
Table 4.1: Overview the significant parametric errors of the investigated CMM and the CMM components contributing to these errors.

The relevant errors of this CMM are limited to several joint rotations. Especially the rotations of the carriages relative to their guideways are significant. Therefore the measurement of these carriage rotations is considered a good method for obtaining accurate estimations of the errors. Based on the readings of such sensors, the parametric errors of the CMM during a measuring task can be estimated, and the effect on the probe position can be calculated. In the next paragraph sensors that can be used for such measurements will be discussed.
4.3 Selecting additional sensors

4.3.1 Displacement sensors for measuring carriage errors

In Chapter 3 it already has been explained that rotation- as well as translation errors of carriages can be measured by displacement sensors (see paragraph 3.4). For the CMM under investigation only the dynamic rotation errors are important. Using two sensors on either side of a carriage, the respective rotation is measured during motion of the carriage. In this configuration (see Figure 3.12) the guideway is acting as a reference for the sensor measurements. Therefore it is important that this guideway is not subjected to large changes in geometry. The geometry of the guideway between the sensor positions should at least be stable. Dynamic changes of the guideway geometry will influence the measurement of the dynamic carriage rotations. In Figure 4.9 a carriage has been depicted, moving along a guideway that is subjected to deformations. Due to these deformations the distance between the sensors attached to the carriage and the guideway varies. Obviously the sensors will give readings that are not indicating the correct carriage rotation. However, from the measurement results of the previous paragraph it is clear that in case of the investigated CMM, the guideway deformations are small. They do not affect the sensor measurements. Variations are only allowed as long as these are constant with respect to time. In case they

![Figure 4.9: Influence of guideway deformations on the measurement of carriage rotations, using two displacement sensors.](image-url)
only depend on the position of the axes, geometry corrections are possible by static calibration of the sensors for different positions. There are several sensor types that can be used for measuring the displacements. They can be classified according to their physical principle. The main principles that were considered here, are:

- optical
- inductive
- pneumatic
- capacitance

Preferably non-contact sensors are used, because guideway damage should be avoided. From all principles mentioned here, non-contact sensors are available. Important aspects with respect to a correct operation of the sensors, are the guideway properties. For optical measurements the guideway surface should have good reflective properties. Little is known about the reflective properties of the guideways of the investigated CMM and possible disturbances from environment light. In the case of inductive- and capacitance sensors appropriate magnetic and electric properties are needed. On the steel guideways of the CMM both sensor types can be used, but they have to be calibrated for the material they are used at. For capacitance sensors also the dielectric medium between sensor and guideway surface is important. In order to avoid variations in the dielectric constant a clean surface is needed. In principle pneumatic sensors can be used on any type of guideway, without a very dedicated calibration. A study has been conducted to the usefulness of such sensors for other applications (see Bruls 1995). They might be good candidates for future implementation on commercially available CMMs. With respect to the dynamic measuring possibilities of the pneumatic sensors, until now little is known, whereas inductive-, capacitance-, and optical sensors have good dynamic properties. Taking into account all considerations, the inductive sensors were chosen for our experiments, mainly because they proved successful in other projects that were carried out at the Precision Engineering Section (Theuws 1991, Spaan 1995). Furthermore, knowledge with respect to their interfacing was also available as well as software.
4.3.2 Inductive displacement sensors

The measuring principle of the is selected inductive displacement sensors based on variations in the inductance of a coil caused by differences in the air gap between the sensor and the object surface, i.e. the guideway (for the physical principle of the sensor see Appendix D.1). The actual sensor consists of two coils: one measures over a reference gap $l_r$ against a reference plate, the other measures the gap $l_M$ with the object surface (see Figure 4.10). Both coils are part of a bridge circuit. The AC supply and signal conditioning are realised by a carrier amplifier system, consisting of a carrier generator, a resistor half-bridge, an AC-amplifier, and a demodulator (see also Boll 1989). The carrier generator produces a stable AC voltage $U_G$ which feeds the precision resistor/inductor network. The carrier frequency is 5 kHz. By adjusting the position of the reference plate the reference gap is changed, and by this the measured bridge voltage $U_M$ is balanced to zero if the sensor is in zero position. If the bridge is now detuned by displacing the sensor relative to the guideway over a distance $\Delta L$, a voltage $U_M$ will appear. The ratio $U_M / U_G$ at the input of the amplifier represents the output of the variable inductance sensor. This signal is now amplified and demodulated to produce a DC output signal.

![Diagram of TR 102 inductive sensor and readout equipment](image)

Figure 4.10: Layout of the TR 102 inductive sensor and readout equipment, used for measuring displacements.
The amplifier system used here contains 6 half bridges for reading six sensors at the same time. The amplifier is IEEE interfaced by a PC (see also Figure 4.10). Using dedicated software the amplifier can be programmed for a certain measurement configuration. During sampling of the sensor readout, the data is stored in the amplifier. After finishing the measuring sequence it is passed to the PC for further processing. The sensors can be sampled with a frequency of up to 10 kHz. The 3 dB point of the measured signals is approximately 200 Hz. Thus the important lower frequencies of the CMM can be measured by these sensors.

Other important properties of the selected inductive sensors are the measuring range and the accuracy that can be obtained. Based on the CMM accuracy that can be obtained after software compensation for geometric errors (better than 4 μm), a measuring accuracy of 1 μm at probe position is desired. Taking into account the ratio’s between the sensor distances and the active arms between the carriages and the probe, the sensors should have an accuracy of 0.25 μm or better. Since rotation errors at probe position are below 20 arcsec, a measuring range of the sensors of ± 25 μm will be sufficient. Furthermore (temperature) drift should be well below 0.25 μm during a measuring sequence. Off-line calibration of the sensors, using a laser interferometer as a reference, shows that these accuracy can be achieved. However, of more importance is the measuring accuracy of the sensors on the CMM. Therefore they have to be carefully calibrated after mounting on the CMM.

4.4 Sensor implementation

For the measurement of carriage rotations the inductive sensors selected in the previous paragraph have been implemented on the CMM under investigation. Before actual mounting of the sensors, some test were conducted in order to identify parameters that affect the sensor readings (Haanen 1995). The results of these tests show that the CMM sensors are influenced by the guideway material as well as the material of the holders, that are used for attachment of the sensors to the carriages. Also the nominal distance to the guideway is important, since this distance affects the sensitivity of the sensor measurements (see Appendix D.1). With respect to (temperature) drift, only short term variations are of importance. Long term drift effects can be eliminated by resetting the sensor
readings to zero, each time the machine is at rest. During a period of 10 minutes
the drift is less than 0.2 μm. The parameters mentioned here, have to be taken
into account when mounting the sensors at the CMM. The sensors holders that
are needed for attachment of the sensors to the CMM carriages, must also pro­
vide the possibilities for calibrating the sensors against the guideway surface.

4.4.1 Static sensor calibration

Calibration of the sensors can be achieved relatively simple by using a holder as
depicted in Figure 4.11. This holder, used at the y-carriage, can carry the sensor
to be mounted on the CMM (further referred to as CMM sensors), as well as a
reference sensor. This reference sensor is an inductive contact sensor, which
measurements are not influenced by the guideway material. It was calibrated off­
line against a laser interferometer with a traceable accuracy. In this way the

Figure 4.11: Experimental set-up for calibrating the CMM-sensors against reference sen­
sors after mounting on the carriage. To the left, the sensor holder attached
to the y-carriage holding a CMM-sensor and a reference sensor is shown. To
the right, the calibration method of providing displacements by applying a
force to the CMM is depicted.
traceability of the accuracy of the CMM sensors is guaranteed. The reference sensor is placed opposite the CMM sensor. During calibration a varying force $F$ is applied to the CMM. This causes rotations of the respective carriage and displacements of the sensor holder relative to the guideway. These displacements $\Delta L$ are measured by both sensors. For small displacements the dependency between the CMM-sensor output and the displacement is approximately linear (see Appendix D.1). From the calibration data of both sensors the parameters that describe this linear dependency, can be estimated. This relationship can be described as:

$$R = \hat{c}_1 \hat{S} + \hat{c}_2 + \gamma$$

(4.3)

Where $\hat{c}_1$ and $\hat{c}_2$ denote the coefficients of the linear calibration function and the vectors $R$, $\hat{S}$, and $\gamma$ contain the reference sensor data, the CMM-sensor data, and the residuals of the estimation respectively. The residuals found during the calibration measurements, are not randomly distributed, so the actual relationship is not quite linear (see Appendix D.2). At the end of the measuring range, the estimated measuring error is almost 0.40 $\mu$m, which is more than the desired accuracy of 0.25 $\mu$m but acceptable. The coefficients $\hat{c}_1$ for all CMM-sensors are implemented in the software that is being used for interfacing the amplifier system. The amplifier outputs are multiplied with these coefficients. The coefficients $\hat{c}_2$ are not important since these represent only the offset between the CMM-sensors and the reference sensor at the time of calibration. Before measuring with a sensor its output is reset to zero. In this way the situation when the CMM is at rest, is taken as a reference for the dynamic measurements.

Important for accurate measurement of dynamic carriage errors by the CMM-sensors is the geometry of the respective guideways. At least the geometry of a guideway has to be stable (see Paragraph 4.3.1). For the CMM under investigation this is guaranteed, since dynamic guideway deformations were found to be insignificant. Furthermore, static geometry errors (i.e. flatness errors) will generally result into different sensor readings for different carriage positions. These readings can be considered as a position dependent offset for the CMM-sensors, and have to be used to correct the sensor readings of the dynamic measurements.
In Figure 4.12 an example is given of a set of static measurements at several positions along the y-axis. The graph indicates the offset dependency on the y-carriage position for the two sensors, mounted according to the configuration depicted in Figure 4.11. Both show significant offset variations, and one of the two is showing even very large variations. But the variations are repeatable (their range is approximately 0.1 µm) and rather smooth. Hence, correction of the dynamic measurements for the static sensor offsets is relatively easy to achieve. However, during implementation of the sensors on the CMM, serious problems were encountered for other sensor positions. The graph of Figure 4.13 indicates the readings of the two y-carriage sensors measuring in z-direction for different y-carriage positions (measured statically). The values of the readings of these sensors are very large and they show considerable fluctuations. Considering the magnitude of these fluctuations, it is very unlikely that they are caused by the guideway geometry. To test this, one of the sensors was replaced by a contact sensor (the reference sensor used for the calibration). The readings measured by this sensor are also displayed in the graph of Figure 4.13. As expected, the offset variations are much smaller. Thus the variations measured by the CMM-sensors must be caused by the sensors themselves. Most likely they are caused by the fact that the guideway material is not homogeneous with respect to its magnetic properties. Doebeлин 1990 mentions the effect of “electrical runout”. This refers to a variation in magnetic permeability along the guideway surface (resulting from inhomogeneities of heat treatment, hardness, etc.). In the case of the investigated CMM, the variations are probably due to thickness variations in the protective coating layer on the guideway. The variation in permeability changes the sensor inductance, that causes an electrical output even for a constant distance between sensor and surface, thus giving false motion readings. In principle correction for the found offset variations is possible, but a more reliable alternative is to use contact sensors instead. The (reference) contact sensor shows a much more gradual offset variation and is more repeatable. Therefore contact sensors are used at positions were non-contact sensors don't measure properly. Clearly, non-contact sensors are preferred over contact sensors, since there is no risk of damaging the guideway surface. But for the limited number of experiments during our research the contact sensors are acceptable. For later use on CMMs in practice more appropriate sensors have to be selected (e.g. capacitance- or pneumatic sensors can be considered). With respect to the readout by the amplifier system and its interfacing, the contact and non-contact sensors are identical.
Figure 4.12: The readings of the two y-carriage sensors measuring in x-direction for static measurements at different y-carriage positions. The readings indicate the offset dependency on the y-carriage position.

Figure 4.13: The readings of the two y-carriage sensors measuring in z-direction for static measurements at different y-carriage positions. The readings of a (contact) reference sensor at the same position as one of the non contact sensors are also indicated.
The photo of Figure 4.14 shows an example of implemented position sensors on the CMM under investigation. The machine's y-carriage that is also drawn schematically in Figure 4.11, is displayed. Two contact sensors can be seen. The one measuring in horizontal direction was used for calibration of the non-contact sensor, located at the opposite side of the guideway (not to be seen at the photo). This sensor is being used for measuring $\gamma_{Ez}$ rotations. The sensor measuring in vertical direction is being used for measuring $\gamma_{Ex}$ rotations.

### 4.4.2 Dynamic sensor measurements

In order to verify the capability of the displacement sensors to measure dynamic rotations of the CMM-carriages, some tests were conducted using the CMM-sensors and the laser interferometer, that was also used for the measurements described in Paragraph 4.2. Both static and dynamic tests were performed. A similar set-up as for the sensor calibration (depicted in Figure 4.11) was used in order to measure the static performance. Again a static force was applied to the machine resulting in a carriage rotation. From the sensor readings of the two
sensors attached to the carriage, the rotation was calculated using Equation 3.29. In this case the laser interferometer was also used for verification. The optics were arranged in such a way that the carriage rotations with respect to the guideway could be measured. Figure 4.15 is showing the measurement set-up. The sensors are connected to the amplifier system for their readout. Both the laser- and the amplifier system are interfaced by a personal computer. The collected data is stored in data files and processed later using a standard mathematical software package (Matlab).

In the graph of Figure 4.16 the results of a static test are shown. Two curves are plotted: one shows the rotation measured by the sensors as a function of the load, the other shows the rotation measured by the laser interferometer. The curves are both approximately linear. However there is a (constant) factor between the rotation indicated by the sensors and the 'actual' rotation. This is due to the deformation of the y-carriage itself. The moment that acts on the carriage causes torsion of the carriage itself.

Figure 4.15: Experimental set-up for testing the static performance of the CMM-sensors. The carriage rotation due to a static load was measured by the sensors as well as the laser interferometer.
Figure 4.16: The static rotation error $\gamma \varepsilon_2$ for different loads, measured by both, the sensors and the laser interferometer.

Figure 4.17: The dynamic rotation error $\gamma \varepsilon_2$, during y-axis motion. The error is measured by both, the sensors and the laser interferometer.
As can be seen from the difference between both curves this deformation is quite significant. Nevertheless, if this effect is accounted for, the sensors give a good indication of the actual rotation in the static situation. In order to test the dynamic performance of the sensors, measurements were conducted during carriage motion. The resulting (dynamic) carriage rotations were again measured by both the sensors and the laser interferometer, taking samples simultaneously. Figure 4.17 shows the results of a measurement run. In the graph, curves are plotted for the measured (sensors) and ‘actual’ rotation (laser). Both curves show good similarity, which indicates that the sensors can be also used as a measure for the dynamic rotations. But again there is a difference in magnitude between the sensor- and laser measurements, that can be related to the carriage torsion.

In the next chapter, that deals with the actual compensation of the investigated CMM, it will be shown that a good estimation of the total (joint) rotation can be obtained from the sensor measurements, if the stiffness of all joint components are accounted for. Based on such error estimations of all relevant components, and by using a kinematic model of the CMM, actual compensation for dynamic errors of this CMM can be achieved.
5

Compensation for dynamic errors of a CMM

In this chapter the compensation for dynamic errors of an existing CMM is described. First the kinematic model of the CMM is presented. With the description of this model the parametric errors are also defined. The actual values of these errors at the time of probing have to be estimated. Estimation of the errors is based on measurement of the significant deformations, using the sensors selected in Chapter 4. For the investigated CMM relations between the parametric errors and the measured deformations are given. Experiments are conducted in order to verify the estimations of the parametric errors. Next, the kinematic model is used to calculate the effect of the parametric errors on the probe position. The measuring result, indicated by the scale readings, can be compensated for this error. Verification measurements are conducted to verify the estimated probe error. At the end of the chapter, the number of sensors that is necessary for the error estimation, is being discussed. The possibilities of sensor reduction and other methods using no additional sensors are considered.

5.1 Kinematic model of the investigated CMM

The CMM for which compensation will be applied has been described in Paragraph 4.1. The first step in the compensation strategy is to obtain a kinematic model of the respective CMM. The kinematic model relates the errors between
the various elements of the structural loop to the error at probe position, where measurements are taken. In Paragraph 3.2 a general kinematic model for CMMs has been derived. The basis of the kinematic modelling is the definition of the location of the coordinate frames. Their location is important with respect to the parametric errors (see Chapter 3). Different locations of the coordinate frames will yield different values for the parametric errors. By choosing the locations of the coordinate frames, the parametric errors are also defined. Here each of the three carriages has one coordinate frame. Each frame is located at the respective scale, symmetrically with respect to the carriage. The frames’ coordinate axes are all parallel to the axes of the machine’s reference frame. Furthermore, the frames are not attached to the scales, but they are moving with the carriages along the scales. Their orientation with respect to the fixed reference coordinate frame remains the same during carriage motion. For the investigated CMM the kinematic model depicted in Figure 5.1 was derived. The figure shows the three coordinate frames and the dimensions that are necessary for calculating the length of the effective arms of rotation. These arms, used to calculate the effect of the parametric rotation errors, are represented by the position vectors expressed by the general Equation (3.10). For the position vectors of the investigated CMM, we can write:

\[
\begin{align*}
    a_x &= \begin{pmatrix} a_{xx} \\ a_{yx} \\ a_{zx} \end{pmatrix} = \begin{pmatrix} 0 \\ -l_{xz} \\ z-l_z-s_z \end{pmatrix} \\
    a_y &= \begin{pmatrix} a_{xy} \\ a_{yy} \\ a_{zy} \end{pmatrix} = \begin{pmatrix} x-l_x \\ l_{xz} \\ z+l_y-z-s_z \end{pmatrix} \\
    a_z &= \begin{pmatrix} a_{xz} \\ a_{yz} \\ a_{zz} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ z-l_z-s_z \end{pmatrix}
\end{align*}
\]

Where \( x, y, \) and \( z \) denote the scale readings, \( l_x \) the length of the \( x \)-guideway between its zero point and the \( y \)-frame, \( l_z \) the length of the \( z \)-pinole between its zero...
point and the z-frame, $l_{yx}$ the vertical distance between the y- and x-frame, $l_{xz}$ the horizontal distance between the x- and z-frame, and $s_z$ the length of the probe between the end of the pinole and the stylus tip. The kinematic model is given for a probe with only one stylus, pointing in the z-direction. In general a probe can have more styli, pointing in different directions. In that case different position vectors must be defined for each stylus taking into account the different relative positions of the styli tips with respect to the end of the pinole.

With the presented kinematic model the propagation of the errors in the structural loop of the CMM to the probe position can be calculated. An important advantage of this kinematic model is its simplicity due to the orthogonal structure of the CMM. As a result the geometry of the machine is completely described by the position vectors expressed by the Equations (5.1)-(5.3) and the location of the coordinate frames. Using these position vectors and the Equations (3.7)-(3.9) the contributions of the individual axes to the error at the probe position can be expressed as follows:
In Paragraph 5.3 this model will be used to calculate the measuring errors at probe position, caused by dynamic errors. The next paragraph will deal with the dynamic parametric errors of the investigated CMM.

### 5.2 Deriving the parametric errors

Thus in order to find the measuring error at probe position due to dynamic errors, all these individual dynamic errors must be identified and related to one of the parametric errors (see Paragraph 3.3.3). In Chapter 4 it has been shown that the deviations of the CMM's carriages with respect to their guideways (mainly the carriage rotations), can be measured using position sensors attached to the carriages. These carriage deviations generally form a substantial part of the total dynamic parametric errors. By establishing relationships between the measured carriage deformations and the deformation of other components, an estimation of the total parametric error can be obtained (see Paragraph 3.3.4). It will be shown
for the CMM under investigation, that the simple relationships derived in Paragraph 3.3.4, can be used for the estimation of parametric errors on basis of the measured carriage errors.

In Chapter 4 it was found that for the CMM under investigation the deformations of the link elements (i.e. the guideways) are small compared to the deformations of the joints. Therefore these deformations are ignored here. However the deformation of the support of the y-guideway was also found to be significant, thus this deformation must be accounted for. Furthermore the parametric error \( yrz \) is the most significant dynamic error for this CMM. Let's consider the rotation errors about the \( z \)-axis that can be related to this parametric error. In Figure 5.2 the \( y \)-axis of the CMM is depicted, showing the \( y \)-carriage, the \( y \)-guideway, and the column that is supporting the guideway. To the sides of the carriage, position sensors are attached. Using these sensors the part of the carriage rotation, that is mainly due to the finite stiffness of the bearing system, is measured with respect to the guideway. From the measuring results of Paragraph 4.4.2, regarding the capabilities of the position sensors to measure dynamic rotation errors, it is clear that also the other components contribute to the parametric error. In Paragraph 3.4 a similar carriage joint has been modelled. In this paragraph an expression has been derived that relates the parametric rotation error of the joint to the measured bearings rotation, taking into account the deformations of the other components. The dynamic rotations of these elements during motion, are denoted simply as \( \varepsilon_{\text{carriage}} \), \( \varepsilon_{\text{bearings}} \), and \( \varepsilon_{\text{support}} \) respectively (without the subscripts \( y \) and \( z \) that indicate the carriage considered and the axis of rotation). The rotation error \( \varepsilon_{\text{bearings}} \) is measured using the position sensors. Actually the rotation of the bottom of the carriage with respect to the guideway is measured. Since this rotation is mainly due to the deflections of the bearings, the measured rotation is denoted by \( \varepsilon_{\text{bearings}} \). Equation (3.43) gives the expression for the parametric error:

\[
yrz = \left( 1 + \frac{k_{\text{bearings}}}{k_{\text{carriage}}} + \frac{k_{\text{bearings}}}{k_{\text{support}}} \right) \varepsilon_{\text{bearings}}
\]
The actual stiffness ratios are experimentally determined by identifying the ratio between the rotation measured by the implemented sensors and the rotation of a certain component. For these experiments the measuring set-up described in Paragraph 4.2.2. was used. With this set-up the y-carriage of the CMM was loaded with a static force. Both the rotation of a certain component and the bearing rotation were measured at the same time using the laser interferometer and the sensors respectively. The ratio between these rotations is the reciprocal value of the respective stiffness ratio. In the graph of Figure 5.3 the rotations of the different components for the same static load are displayed. Clearly the air bearings have the greatest contribution to the parametric error, but also the support and the carriage itself show considerable deformation (see also Appendix C on static stiffness measurements). The difference between the joint error, calculated on basis of the individually measured errors, and measured directly is not very significant (7.6 arcsec vs. 8.1 arcsec) considering the measuring accuracy of the laser interferometer (approximately 0.2 arcsec). Using the values for the

Figure 5.2: The rotation deformations related to the y-carriage of the investigated CMM. Bending of the x-guideway is ignored.
Compensation for dynamic errors of a CMM

Figure 5.3: Contributions to the parametric error of the y-joint due to the deformations of the various components. The last two columns indicate the total joint error obtained by calculation on basis of the individual errors and by measurement respectively.

In order to verify the ratio found in the case of dynamic loading of the CMM, dynamic measurements were conducted. During y-axis motion of the CMM the dynamic rotation of the carriage with respect to the guideway was measured by the sensors. At the same time a laser measurement was conducted to identify the dynamic rotation of the support or the carriage. From these measurement results the ratios between the sensor- and laser measurements were calculated: 

\[ \frac{\epsilon_{\text{carriage}}}{\epsilon_{\text{bearings}}} = \frac{2.4}{3.6} \text{ and } \frac{\epsilon_{\text{support}}}{\epsilon_{\text{bearings}}} = \frac{2.0}{3.6}. \]

For the support the value found is higher compared to the value found for a static load. Partly, this can be due to measuring inaccuracies. But the difference can also be caused by the influence of damping. Compared to the support, the air bearings have more damping, resulting into smaller deformations and thus a higher ratio \( \frac{\epsilon_{\text{support}}}{\epsilon_{\text{bearings}}} \).
Using the values for the ratios under dynamic conditions, we can express the parametric error $yrz$ as:

$$yrz = \left(1 + \frac{2.4}{3.6} + \frac{2.0}{3.6}\right) \epsilon_{\text{bearings}} = 2.2 \cdot \epsilon_{\text{bearings}} \quad (5.10)$$

The factor between the measured rotation and the parametric error that is depending on the stiffness ratios is further referred to as stiffness factor. This factor was applied to the sensor readings of a measurement conducted during motion. Thus an estimation of the total parametric joint error $yrz$ was obtained, which was verified using the laser interferometer. In the graph of Figure 5.4 the results of this measurement are plotted. The graph shows good similarity between the estimated values and the measured values.

The difference between the estimated and measured values is plotted in the graph of Figure 5.5. The absolute difference varies between $\pm 0.5$ arcsec for most of the signal and is at maximum $1$ arcsec. Part of this residue is caused by the inaccuracies of both the sensor- and laser measurements as well as the effects of geometric errors of the guideway. In the latter case the dynamic measurements at the probe position are contaminated with static errors of the guideway geometry. In order to verify the sensor measurements, these static effects have to be removed from the laser measurements. Part of the residue is also due to the inaccuracy of the simple joint model. Relating the residue to the absolute rotation error, gives an indication of the accuracy improvement that can be achieved. For the relative low level of vibration during motion with traverse speed, the improvement is approximately $50\%$. For the high vibration amplitudes at the time of deceleration improvement up to $90\%$ can be achieved for the parametric error $yrz$.

Important for the correct estimation of the dynamic parametric error is the validity of the stiffness factor. For negligible link deformations, the factor is only dependent on joint deformations. In that case the stiffness factor will be constant for different carriage positions. In order to verify the stiffness factor several sets of measurements were conducted for different carriage positions. During each measurement run sensor readings as well as laser readings were taken.
Figure 5.4: Comparison of the total parametric joint error $y_{rz}$, measured by the laser interferometer and estimated on basis of the sensor measurements.

Figure 5.5: Difference between the estimated- and measured values of the parametric joint error $y_{rz}$ during motion.
In order to check the x-position dependency, dynamic measurements during y-axis motion were conducted at different x-positions. In all cases the laser interferometer was measuring the rotation at the probe position (Figure 4.2. is showing the set-up used for the laser measurements). From other measurements it is known that the dynamic rotation errors of the x-carriage and the z-pinole about their z-axis are negligible for y-axis motion. This is understandable regarding their small moment of inertia with respect to the z-axis. Thus from the laser measurements at the probe position the total parametric rotation error \( y_{rz} \) is obtained. The photo of Figure 5.6 shows the practical realisation of a measuring set-up. The configuration displayed is used for identifying the parametric error \( y_{rx} \) and has a different arrangement of the optics. On the granite table a laser interferometer is placed. At the top right corner the y-carriage is shown with attached position sensors.

Figure 5.7 shows the results of repeated measurements at several x-positions. Both the sensor- and the laser measurements show an approximately linear dependency of the rotation on the x-carriage position. As explained in Paragraph 4.2.2. this is caused by the dependency on the x-position of the moment about the z-axis due to acceleration forces. However their ratio is constant as can be seen in Figure 5.8. The average dynamic stiffness ratio (indicated by the horizontal line) is approximately 2.3, which is somewhat higher than the ratio from Equation (5.10). This difference is due to the effect of the connection between the x-guideway and the y-carriage.

For testing the dependency of the stiffness ratio on the y-carriage position, similar measurements were conducted. From these measurements no trend can be identified that indicates bending of the y-guideway. The stiffness ratio of the y-joint is also constant for different y-carriage positions. Also measurements with respect to the dependency on the z-pinole position were performed. As to be expected the position of the z-pinole does not affect the parametric error \( y_{rz} \).

From the various measurement results presented here, it can be concluded that the parametric rotation \( y_{rz} \) related to the y-joint can be estimated using the sensor measurements and the stiffness ratio. The simple y-joint model yields good results for different positions of the axes with this CMM. Other parametric errors are even less dependent on the various axes positions. These errors can be esti-
mated in a similarly using sensor measurements and simple joint models. For the other significant parametric errors the following relations were derived (for clarity the subscripts indicating the carriages and rotation axes are used):

\[
y_{rx} = \left(1 + \frac{y_{k_x,bearings}}{y_{k_x,carriage}} + \frac{y_{k_x,bearings}}{y_{k_x,support}}\right) \cdot y\varepsilon_{x,bearings} = 2.0 \cdot y\varepsilon_{x,bearings} \tag{5.11}
\]

\[
x_{ry} = \left(1 + \frac{x_{k_x,bearings}}{x_{k_x,carriage}}\right) \cdot x\varepsilon_{y,bearings} = 1.9 \cdot x\varepsilon_{y,bearings} \tag{5.12}
\]

\[
x_{rx} = \left(1 + \frac{x_{k_y,bearings}}{x_{k_y,carriage}}\right) \cdot x\varepsilon_{x,bearings} = 1.7 \cdot x\varepsilon_{y,bearings} \tag{5.13}
\]
Figure 5.7: The parametric error $yrz$ measured by the laser interferometer and the carriage rotation measured by the sensors for different x-positions.

Figure 5.8: Dependency on the x-carriage position of the ratio between the parametric rotation error $yrz$ and the rotation measured by the sensors.
The parametric translation $y_{ty}$, caused by support motion, cannot be measured by the sensors. Therefore this error is accounted for in a similar way as described in Paragraph 3.4.3 for the bending of a guideway. The acceleration $\ddot{y}$ of the y-carriage can be related to the moment on the y-carriage and the measured carriage rotation $\dot{y}\varepsilon_z$ (Equation (3.46)). Using equilibrium of forces, the reaction force that acts on the y-guideway can also be expressed into the acceleration of the y-carriage. With the known translation stiffness of the support at the position of the y-guideway, the support motion $y_{ty}$ can be expressed into the measured deformation of the y-carriage.

$$y_{ty} = f_{yy}(x) \cdot \frac{y_{k_z \text{bearings}}}{y_{k_y \text{support}}} \cdot \varepsilon_{z \text{bearings}} = 0.5 \cdot f_{yy}(x) \cdot \dot{y}\varepsilon_{z \text{bearings}}$$  \hspace{1cm} (5.14)

Where $y_{k_y \text{support}}$ and $y_{k_z \text{bearings}}$ denote the stiffness of the support in y-direction and of the carriage's bearing system with respect to the z-axis respectively. The factor $f_{yy}(x)$ denotes the relationship between the moment applied to the y-carriage and the reaction force of the carriage on the support. Note that this factor is dependent on the x-carriage position. The maximum value for this factor is obtained when the x-carriage is in home position ($f_{yy}(x = 0) = 1.1$).

5.3 Dynamic error compensation

From the measurement results presented in Chapter 4 it is clear that a limited number of parametric errors are significant, these are: $yrz$, $yrx$, $xxr$, $xxy$ and $yty$ (see Paragraph 4.2.2). In order to obtain the error at the probe position during CMM motion, all these parametric errors have to be estimated. In the previous paragraph it has been shown that these errors can be estimated accurately, based on on-line sensor measurements in combination with simple models that describe the relations between the various deformations. Using the estimations of the parametric errors, the error at the probe position is calculated with the kinematic model. This error can then be used as a compensation value for measurements. In paragraph 5.3.1 an estimation for the error at probe position is given in case of y-axis motion. The results of the estimation are verified with di-
rect measurement of the error at probe position. These results are presented in paragraph 5.3.2.

5.3.1 Estimation of the probe error

Using the kinematic model of Paragraph 5.1, the probe error of the investigated CMM can be expressed into the above mentioned significant parametric errors. Taking into account these relevant parametric errors and the expressions for the position vectors of the CMM (given by the Equations (5.1)-(5.3)), we can write for the components of the error $\mathbf{e}$ at probe position (where $\mathbf{e}=(e_x, e_y, e_z)$):

$$
e_x = yrz \cdot l_x + xry \cdot (z-l_z-s_z)$$  \hspace{1cm} (5.15)

$$
e_y = -xrx \cdot (z-l_z-s_z) + yty + yrz \cdot (x-l_x) - yrz \cdot (z+l_x-l_z-s_z)$$  \hspace{1cm} (5.16)

$$
e_z = 0$$  \hspace{1cm} (5.17)

The latter error in z-direction is zero, since none of the relevant parametric error causes an error in this direction. In order to estimate these probe errors, on-line measurement of the parametric errors involved is necessary. Therefore in total eight sensors (four sets of each two sensors) had to be implemented on the investigated CMM. Each set measures one of the parametric rotations. Although not necessary for this CMM, each set can also measure one translation perpendicular to the guideway.

In Figure 5.9 the investigated CMM is depicted together with the implemented eight position sensors. Two sets of sensors are attached to the y-carriage and also two sets to the x-carriage. In the realised measuring set-up actually only six sensors could actually be used simultaneously, since only six channels at the amplifier system were available for reading the sensor values. As an example of the error that can be expected at probe position during motion, an estimation of the error in y-direction will be given here. This error is the most significant error for this CMM, and there are significant contributions of four parametric errors.
Three of these errors can be measured by the sensors. The support motion, expressed by the parametric error $yty$, is estimated on basis of the sensor readings for the error $yruz$. In this example the CMM was programmed to move with traverse speed (70 mm/s) along the y-axis to a certain position. The x-carriage and the z-pinole were both in their zero positions, resulting in maximum effective arms for the respective parametric rotation errors. During motion the sensor readings were recorded and buffered in the amplifier system. At the end of the experiment the values were read from the buffer by the connected computer and stored on disk for further processing. Based on the sensor data the time histories of the parametric errors that occurred were calculated, taking into account the stiffness ratios for each parametric error. The Figures 5.10-5.12 show the parametric errors $xrx$, $yrx$, and $yruz$ respectively. As expected from the analysis of this CMM, as reported in Chapter 4, all these parametric errors are significant. They are clearly correlated with respect to each other. This is due to the fact that they are all caused by the accelerations and decelerations of the y-carriage.
Figure 5.10: The parametric error $x_{\text{rx}}$ calculated on the basis of the sensor readings that were recorded during motion in the y-direction.

Figure 5.11: The parametric error $y_{\text{rx}}$ calculated on the basis of the sensor readings that were recorded during motion in the y-direction.
Compensation for dynamic errors of a CMM

Figure 5.12: The parametric error $y_{rz}$ calculated on the basis of the sensor readings that were recorded during motion in y-direction.

Figure 5.13: The dynamic error in y-direction at the probe position during y-axis motion, calculated on the basis of the parametric errors.
Using Equation (5.16) the time history of the error at probe position was calculated on basis of the recorded values for the parametric errors. This error signal is depicted in Figure 5.13. The graph shows clearly that the dynamic error at the probe position is quite large compared to the static inaccuracy (maximum 25 \( \mu \text{m} \) dynamically vs. less than 4 \( \mu \text{m} \) statically). This means that if measurements are made when the CMM is still subjected to accelerations, compensation of the measuring result is necessary in order to obtain a sufficient accuracy (i.e. comparable to the static inaccuracy). In the next paragraph the accuracy of the method will be verified by comparing the estimated error with the actual error at probe position.

5.3.2 Verification

In order to verify the performance of the compensation method, a measurement set-up was created for measuring the actual error at probe position during motion. Using the laser interferometer with linear optics the displacement at probe position was measured. In order to obtain the error at probe position (i.e. the difference with respect to the scale readings) the CMM's y-scale readings had to be read simultaneously. Because the CMM's controller couldn't be interfaced fast enough, the CMM readings had to be derived directly from the scale sensors. Therefore the pulses from these scale sensors were sent to a counter board, that was implemented in the same computer that was used for interfacing the laser system (see also Figure 5.9). From the counter readings the actual carriage position could be calculated. By simultaneously interfacing the laser and the counter board the error at probe position was obtained. Several sets of experiments were made, measuring the actual error at the probe position as well as the various rotation errors by the sensors.

In Figure 5.14 the probe error is depicted that was measured simultaneously with the sensor readings belonging to the example of the previous paragraph. In Figure 5.15 the difference between the estimated- and the measured error is given. This residue is at maximum 2.5 \( \mu \text{m} \), which is acceptable considering the static accuracy of approximately equal magnitude (better than 4 \( \mu \text{m} \)).
Figure 5.14: The measured dynamic error in y-direction at probe position during y-axis motion. The CMM is accelerating to- and decelerating from traverse speed.

Figure 5.15: The difference between the measured- and the estimated dynamic error in y-direction at probe position during y-axis motion. The CMM is accelerating to- and decelerating from traverse speed.
Figure 5.16: Overview of the contributions of the various parametric errors to the error at the probe position in y-direction, during y-axis motion. Also the total calculated error and the actual, measured error are given.

Compared to the maximum error found, the residue is approximately 10%. Thus in this case the dynamic error that occurs can be compensated for 90%, using the compensation method based on position sensors. Summarising, the graph of Figure 5.16 gives an overview of the contributions of the various parametric errors to the total error at probe position. The calculated- and measured probe error are also depicted.

In the previous example an estimation of a dynamic error was given that showed mainly quasi-static effects. In Figure 5.17 the estimated error for a vibration, that occurs after suddenly decelerating, is shown. The difference with the actual error is shown in Figure 5.18. In this case the CMM was joy-stick controlled. Due to the larger accelerations, the error at probe position is higher than in the previous case. Also the residue is higher in this case, approximately 5 μm, but the relative error is again about 10%.
Figure 5.17: The estimated dynamic error in y-direction at probe position during y-axis motion. The CMM is decelerating suddenly.

Figure 5.18: The difference between the measured- and the estimated dynamic error in y-direction at probe position during y-axis motion.
Other sets of measurements were performed at different locations, for instance with the pinole in upward position. The results were similar except for the fact that the contribution of the parametric rotation errors about the x-axis were now negligible due to the small arm in z-direction. From these examples it is clear that for one-axis motion of this CMM, the dynamic errors can be compensated for very well. This means that if compensation is used, higher acceleration levels during probing are acceptable. In this way faster CMM motion, without serious degradation of the measuring accuracy, is possible.

Until now the method has only been tested for single y-axis motion. Further experiments are necessary to test the performance for multi-axes motion. Because the errors that can be related to y-axis motion are dominant and accurate compensation for these errors proved to be possible, it is expected that for 2D- and 3D-motion also a high accuracy can be obtained. In order to verify this, different tests are necessary, such as linear motion in 3D-measurement space or the measurement of 3D-objects. Unfortunately the CMM under investigation is not vector controlled. This means that the CMM cannot traverse accurately along a straight line in an arbitrary direction. Because the probe motion is not very linear, especially with high traverse speed, the probe cannot be tracked by the laser interferometer and verification of the probe error is not possible. A good alternative method would be measuring a test object with known dimensions (e.g. a ring gauge or a sphere), either by scanning or by single point measurements. However the CMM under investigation is not equipped with a measuring probe that allows scanning measurements (see Chapter 2). The CMM control software also doesn't allow actual high speed probing with its touch-trigger probe (for normal probing operations a low measurement speed must be selected). For further evaluation of the proposed method it is therefore recommended to realise a measuring set-up, suitable for performing high speed measurements with touch-trigger probes.

5.4 Efficiency improvement of the error compensation

From the results presented in the previous paragraph it is clear that compensation for dynamic errors is possible with the adopted method. By applying dynamic error compensation to CMMs, higher accelerations during probing can be accepted. For scanning measurements this implies that higher scanning speeds
can be allowed. In case of probing with touch-trigger probes, long settling times in order to reduce the vibration level before probing, can be avoided, and also higher traverse speeds are acceptable. This means that the cycle time of measurement tasks can be reduced without serious degradation of the accuracy of the measuring results.

An important aspect of the proposed method is the use of additional sensors that measure the actual deformation of several CMM components on-line. In this way an accurate and reliable estimation of the various parametric errors of the CMM is achieved. However the implementation of a large number of sensors also has some disadvantageous. From a economical point of view, many sensors can be expensive. Not only due to the costs of the sensors themselves, but also due to the costs of the actual implementation. More sensors will also increase the complexity of the hardware, making the CMM more sensitive for failures, and increasing maintenance costs. Therefore it is important that only a limited number of sensors will be used in order to obtain a more efficient error compensation. In the test set-up that was created for the CMM used in our investigations, in total eight position sensors were implemented. Although all the errors that are measured by these sensors are significant, there are possibilities for reducing the number of sensors. These possibilities will be discussed briefly here.

In general it is assumed that the dynamic error at probe position of a certain CMM is affected by rotation errors as well as translation errors. With the adopted method in principle both type of errors can be measured on-line at the CMM's carriages. A major reduction of the number of sensors can be obtained if translation errors are small compared to rotation errors. In this case the rotations between the carriages and the guideways can be measured using only one sensor instead of two sensors. In order to calculate the rotation based on the measured displacement of one sensor, it is important that the effective arm of the rotation is known. Thus the actual location of the axis of rotation at the carriage must be known. In the case of the investigated CMM, it was found that translation errors of the carriages are small. Hence, it should be possible to reduce the number of sensors for this CMM. This has been tried for the experiments described in Paragraph 5.3.1. The parametric errors that were calculated using the readings of two sensors, were calculated again, but now using the readings of just one sensor.
Figure 5.19: The difference between the measured- and the estimated dynamic error in y-direction at probe position during y-axis motion. The probe error has been estimated using only one sensor for each parametric rotation error.

Based on these values for the parametric errors the probe error was calculated again. Comparing this with the measured probe error, the residue for the situation with the reduced number of sensors was obtained. In Figure 5.19 the time history of this residue is depicted. This time the residue is about twice as high as in the case where 6 sensors were used (approximately 5 μm). In reality the residue will be somewhat higher, due to x-axis motion. In Paragraph 4.2.2 it has been mentioned that for x-axis motion bearing deflections of the y-carriage in x-direction of maximum 0.6 μm were found (for joy-stick controlled motion). Typical values for CNC motion with traverse speed are 0.3 μm. This error is actually a carriage translation. However, if only one sensor is used for measuring the y-carriage rotation about the z-axis, this carriage translation will be interpreted as a rotation error. The magnitude of this 'rotation error' is 0.2 arcsec. This causes a maximum error at probe position of 1 μm. Thus in the worst case situation the residue for the estimation of the probe error will be 6 μm. Regarding the total dynamic error, compensation could be achieved for about 75% of the error. If this
inaccuracy is acceptable, the number of sensors can be reduced with a factor 2. In case of the investigated CMM 4 sensors would be sufficient.

Even a further reduction can be achieved, if different parametric errors of a certain axis can be related to each other. From the readings of just one sensor, the dynamic load (i.e. the moment on the carriage) that caused the respective deformation can be estimated. Based on this dynamic load and knowledge about the CMM's axes positions, an estimation of the acceleration acting on the axis can be made. This acceleration can then be used to estimated all other parametric errors that are affected by the same acceleration. For the investigated CMM this method has been used for estimating the translation error $y_t$ of the $y$-guideway, since this translation could not be measured with other sensors. In this case good results were achieved. However it is dependent on the dynamic behaviour of the respective CMM if this method can be used with sufficient accuracy. In order to relate the different parametric error to each other, the significant vibrations that occur due to axis motion must be identical for the errors, with respect to their frequency and their phase. It can not be assumed that this will be generally the case with CMMs, but the results from the measurements conducted at the CMM used, show good comparison between the various parametric errors (see Figures 5.10-5.12). Therefore a further reduction of the number of sensors might be possible for this CMM. This is also an interesting topic for future research. If the dynamic parametric errors can be estimated accurately enough on basis of the acceleration acting on a certain axis, it is also worthwhile to investigate the possibility of only using the readings from the scales. From these scale readings the actual accelerations of the carriages can be derived. Although this will be less accurate than direct measurement of the dynamic errors, good results can be expected, considering the deterministic behaviour of the investigated CMM with respect to accelerations. At the moment a research project at the Precision Engineering section of the Eindhoven University of Technology is being carried out for investigating this possibility.

During the experiments on the CMM, at first non-contact inductive position sensors have been used for measuring the dynamic deformations. With these sensors the deformations could be measured with sufficient accuracy. But at some positions also problems occurred, due to inhomogeneities of the guideway surfaces. Thus if non-contact inductive sensors are considered for implementation on a cer-
tain CMM, it is important to take into account the guideway properties. The problem could be solved for the investigated CMM, by using inductive contact sensors at the positions where inhomogeneities were found. A disadvantage of these sensors is the possibility of guideway and sensor wear. Especially, in manufacturing where CMMs are used for considerable longer times than in research, this can be a problem. However, the major advantage of contact sensors is the fact that they can be used on any guideway material. In contrary inductive sensors can not be used on for instance granite guideways, without extra means such as a metal strip attached on the granite. Therefore it is worthwhile to consider also other type of sensors.

In Paragraph 4.3.1 it has been suggested that capacitance and pneumatic displacement sensors might be good candidates for future implementation on commercially available CMMs. Like inductive sensors, capacitance sensors are restricted to the use on conductive materials. In principle pneumatic sensors can be used on any type of guideway, without a very dedicated calibration. A study has been conducted to the use of such sensors for other applications (see Bruls 1995). This study shows that accurate displacement measurement are possible with pneumatic sensors The possibility of using capacitance and pneumatic sensors on a CMM will also be subjects of future research.

In this chapter the proposed method for error compensation of the dynamic errors of CMMs has been tested for single axis motion. For the tests conducted here the estimated dynamic error at probe position showed good resemblance with the actual error. When using sufficient sensors for measuring all the significant parametric errors, compensation can be achieved for 90% of the dynamic errors. The main conclusion with respect to these results is that by compensating for dynamic errors of CMMs, still a high accuracy can be obtained when the CMM is subjected to accelerations. This means that fast probing is possible without significant degradation of measurement accuracy. Thus a powerful method has been developed for compensating dynamic errors of CMMs. Application of this method enables the improvement of CMM productivity, because faster probing can be allowed.
Conclusions and Recommendations

In order to increase CMM productivity higher probing speeds of CMMs are desired. The main goal of the research described in this thesis is the estimation of the dynamic errors that occur at the CMM's probe position in case of higher probing speeds. Based on the estimated probe error the measuring result can be compensated. This means that faster probing speeds will be possible without serious degradation of measuring accuracy. In this chapter conclusions with respect to the obtained results will be drawn and recommendations for future research will be given.

6.1 Conclusions

The way a CMM is affected by dynamic errors, depends on its structural loop. The structural loop is the part of the mechanical structure that comprises all the components that together define the position of the probe relative to the workpiece. In general the structural loop is being used for two tasks: positioning of the probe and for constituting the coordinate system, because it is acting as a frame for the measuring systems. Thus deformations of the structural loop e.g. due to driving forces and moving loads will affect the measuring accuracy. There are different configurations of the structural loop components possible. Each configuration has its advantages and disadvantages with respect to properties like accessibility, measuring volume, load capacity, rigidity, and attainable accuracy and speed. Analysing the CMM configurations commonly used, it is clear that for most conventional CMMs dynamic errors can be expected in case of fast probing.
Measurements conducted at existing CMMs show that during motion of CMM axes such errors occur. For the CMMs quasi-static as well as vibration errors were found. It turned out that they can be quite large in relation to the static errors of CMMs, as demonstrated in Chapter 2.

By improved CMM design aimed at high stiffness and low mass of the CMM's components or CMM control aimed at minimising accelerations, dynamic errors can be reduced. However, a different philosophy is to accept the accelerations and the resulting errors and to compensate for their effects afterwards. In this way there are no restrictions put on CMM motion, and fast probing is possible. This approach can be especially advantageous for scanning measurements or manually operated CMMs, where accelerations are difficult to minimise. In this research this approach has been followed. In order to make compensation of a measurement possible, the dynamic error at probe position must be known at the time of probing. Therefore a general method has been developed for accurate estimation of the dynamic error at probe position during CMM operation.

This method for estimating dynamic errors with CMMs is similar to error modelling methods used for estimating other types of errors with CMMs. The developed error modelling consists of two parts: identification of the individual dynamic, parametric errors and calculation of their effects on the measuring error at probe position, using a kinematic model. The same kinematic model that is also used for other errors, is used here for calculating the effect of the dynamic errors. This is advantageous, since in this way a modular compensation system is obtained. In the kinematic modelling the CMM structure is considered rigid and the parametric errors are considered as the errors in the degrees of freedom of the kinematic model. However, the components of a CMM are actually flexible elements, introducing quasi-static deformations and vibrations due to the accelerations. These deformations result into dynamic parametric errors. In order to estimate these errors of a CMM, a combined analytical and empirical approach is followed. With additional sensors attached to the carriages of a CMM, rotations and translations of the carriages with respect to their guideways are measured on-line. In this way the sensors measure part of the deformations. Simple relationships are formulated between the deformations measured by the sensors and the other deformations. These relations are used to express the parametric errors (i.e. the combined effect of the deformations) into the sensor measurements. With
the kinematic model the effect of the estimated parametric errors on the probe position is calculated. Summarising the developed method for achieving actual error compensation for dynamic errors of a certain CMM, contains the following steps:

- Describing the CMM structure in a kinematic model. With this model the degrees of freedom of the CMM are defined and the effect of the parametric errors in these degrees of freedoms on the probe position can be calculated.

- Analysing the dynamic behaviour of the CMM in order to identify the significant deformations. Based on the results suitable sensors can be implementation on the CMM's carriages for measuring these significant errors on-line. Using simple models, other deformations that are not measured are related to these errors.

- Measuring of deformations with the sensors implemented on the CMM and expressing the combined effect of measured- and estimated errors into the parametric errors.

- Using the kinematic model for calculation of the error propagation of all estimated parametric errors to the probe position. The calculated error values at the probe position during a certain measurement task are used for compensation of the measurement result.

The modelling and analysis of the dynamic behaviour are not dependent on a particular CMM, but only on the type of CMM. This means that the error modelling and analysis of the dynamic behaviour has to be carried out only once for a certain type of CMM. The results can be used for all CMMs of the same type. This is important with respect to the efficiency of the proposed method. In general differences between the actual machine parameters (e.g. stiffness values) that are used for estimating deformations based on the sensor measurements, will be small for different CMMs of the same type. However in order to obtain a high accuracy and reliability of the method it is sensible to identified these parameters for each individual CMM.
The developed method has been applied to an existing CMM in order to verify its performance in practice. The CMM’s dynamic parametric errors that contribute to the probe error, were identified. A limited number of parametric errors were found significant. These are mainly rotation errors of the CMM’s joints. Deformation of the guideways were found negligible, but support motion must be accounted for. Inductive position sensors were mounted on the CMM’s x- and y-carriage for on-line measurement of rotations and translations. Tests showed that the sensors can accurately measure the deformations during axis motion of the CMM. Based on the sensor readings and a kinematic model of the CMM, the time history of the dynamic error at probe position can be calculated.

For verification of a single axis experiments were conducted, comparing the estimated error with the actual error. In these experiments the position of the probe of the investigated CMM was measured during linear motion using laser interferometry. The actual probe error is the difference between the position of the probe and the CMM’s scale readings that are interfaced simultaneously. The estimated probe error showed good resemblance with the actual error. When operating the CMM at maximum traverse speed, the maximum residue was 2.5 μm, which is approximately 10% of the error at the probe tip position. For motion under joy-stick control a larger residue of 5 μm was found, which is also 10% of the dynamic error. Because translation errors of the investigated CMM are small compared to rotation errors, the rotations between the carriages and the guideways could also be measured using only one sensor instead of two sensors. In this way the number of sensors was reduced with 50%. With the reduced number of sensors compensation could be achieved for about 75% of the error. Thus it turned out that with little degradation of accuracy sensor reduction was possible.

The results of the verification measurements show that, at least for single axis motion, by compensating for dynamic errors of CMMs, still a high accuracy can be obtained when the CMM is subjected to accelerations. In this way fast probing is possible without significant degradation of measurement accuracy. Thus a powerful method has been developed for compensating dynamic errors of CMMs. Application of this method enables the improvement of CMM productivity, because faster probing can be allowed.
6.2 Recommendations

The developed method for compensating of dynamic errors shows good performance for single axis motion. However further experiments are necessary in order to determine the accuracy improvement that can be obtained for multi-axes motion. Therefore the following experiments are recommended:

- The measurement of 2D- and 3D test objects with known dimensions (e.g. a ring gauge or a sphere) on the investigated CMM, using a touch-trigger probe, which signal is also used to trigger the reading of the position sensors. Using the sensor readings the measurements can be compensated for dynamic errors and the dimensions of the test objects can be calculated. These dimensions can be compared with the known dimensions of the workpiece.

- Implementation of the method on a CMM that can operate with a scanning probe. If test objects, such as mentioned with the previous test method, are scanned, their compensated measuring results can also be compared with the actual dimensions. In this way the performance of the method for scanning can be tested.

- Testing of the compensation method on a manual CMM. Probing at a manually controlled CMM is often under bad dynamic conditions. Therefore these CMMs are very prone to dynamic errors. Thus a relative high accuracy improvement can be expected if compensation for dynamic errors can be achieved on manual CMMs. For these CMMs this is only interesting if low cost implementation of sensors is possible.

In order to obtain a higher efficiency of the compensation method (with respect to the costs) it is also important to reduce the number of sensors used for estimating the parametric errors. For the CMM investigated it turned out that with little degradation of accuracy the number of sensors could be reduced with 50%. The following steps are recommended to achieve a further reduction of the number of sensors:
• A further reduction can be achieved, if different parametric errors of a certain axis can be related to each other. From the readings of just one sensor, the dynamic load (i.e. the moment on the carriage) that caused the respective deformation can be estimated. Based on this dynamic load and knowledge about the CMM’s axes positions, an estimation of the acceleration acting on the axis can be made. This acceleration can then be used to estimate all other parametric errors that are affected by the same acceleration. This approach can be applied to the investigated CMM.

• Another possibility is to use only the readings from the scales to estimate the parametric errors. From these scale readings the accelerations of the carriages can be derived, which can be used to estimate the various parametric errors. At the moment a research project is being carried out in which this possibility is investigated.

For implementing the compensation on other CMMs, other sensors might be necessary. Furthermore, if the method will be used on commercial CMMs low cost sensors are desired for economical reasons. Therefore it is worthwhile to consider also other type of sensors than the sensors used during our investigations. Capacitance- and pneumatic displacement sensors are interesting options. Research for their implementation on a CMM is therefore recommended.
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Appendix A

Expressions for the mass- and stiffness matrices of a single axis

In this appendix examples of the mass- and stiffness matrices, that can be derived for the CMM axis depicted in Figure 3.7, will be given. The set of differential equations describing the undamped system of Figure 3.7 is given by:

\[ M\ddot{\xi}(t) + K\dot{\xi}(t) = diag(\ddot{y}(t))m \]  \hspace{1cm} (A.1)

The generalised coordinates \( \xi \) are defined as:

\[ \xi = \begin{pmatrix} \xi_s \\ \xi_b \\ \xi_c \\ \xi_g \end{pmatrix} \]  \hspace{1cm} (A.2)

The guideway is modelled as an Euler beam, which behaviour can be described by the partial differential equation of Equation 3.16. For the bending of this beam due to axis accelerations, a displacement field is assumed that is similar as the deformation under static load (see Equation 3.17). The components of the y-axis that are subjected to torsion are modelled as a rod, which dynamic behaviour is expressed by Equation 3.18. For the angular deformation a linear displacement function is assumed (expressed by Equation 3.19).
Based on the differential equations expressed in the chosen generalised coordinates, the inertia and stiffness terms with respect to these generalised coordinates can be determined. A convenient approach is to use Lagrange's method. The mass- and stiffness matrices for each individual component can be calculated using the kinetic and potential energy terms of the differential equations. These matrices can then be assembled to form the global mass- and stiffness matrices (see e.g. Thomson 1988). For the defined generalised coordinates and the assumptions made, we can now calculate the kinetic energy and derive the mass matrix of the system depicted in Figure 3.7:

\[
M = \begin{pmatrix}
    J_y + \frac{1}{2}(J_z + J_b) & \frac{1}{6} J_b & 0 & 0 \\
    \frac{1}{6} J_b & \frac{1}{2}(J_b + J_c) & \frac{1}{6} J_c & 0 \\
    0 & \frac{1}{6} J_c & J_c + \frac{2}{105} \frac{l_x^3}{l} m_g & \frac{11}{280} \frac{l_x^3}{l} m_g \\
    0 & 0 & \frac{2}{280} \frac{l_x^3}{l} m_g & (\frac{23}{140} \frac{l_x}{l} + \frac{1}{3}(l - l_x)^3) \frac{m_g}{l_x} + (m_x + m_z) l_x^2
\end{pmatrix} \quad (A.3)
\]

Here \( J_y, J_z, J_b, J_c \) denote the mass moments of inertia around the z-axis of the column of the support, the y-guideway fixed to the column, the bearing system and the carriage respectively. \( m_g, m_x, m_z \), denote the masses of the x-guideway, the x-carriage and the z-axis. The x-guideway has a total length \( l \) and the distance between the y-carriage and the x-carriage is \( l_x \). Note that the mass matrix is dependent on the position of the x-carriage along the x-guideway. The extra load on the beam due to the motion along the beam axis of a moving mass is not considered here as are coriolis- and centrifugal forces. Taking into account the effects of a moving mass on the beams deformation leads to rather complex modelling (see e.g. Timoshenko 1974, Khalily 1994). For the sake of simplicity the direct influence of the motion of other axes on the beam deformation is not considered in this analysis.
Calculating the potential energy of the system yields the stiffness matrix:

$$K = \begin{pmatrix} k_s + k_b & -k_b & 0 & 0 \\ -k_b & k_b + k_c & -k_c & 0 \\ 0 & -k_c & k_c + k_g & -k_g \\ 0 & 0 & -k_g & k_g \end{pmatrix}$$  \hspace{1cm} (A.4)

Where \( k_s, k_b, k_c \) denote the stiffnesses of the support, the bearing system and the carriage respectively. The rotation stiffness of the bearing system can be derived from the stiffnesses of the individual bearings (see Appendix C.1, Equation C.10). The carriage and support elements are subjected to torsion and their stiffness can be expressed as:

$$k_i = \frac{G_i l_i}{l_i}$$  \hspace{1cm} (A.5)

With \( l_i \) the second moment of inertia, \( G_i \) the shear modulus of elasticity, and \( l_i \) the length of the i-th element. The bending stiffness \( k_g \) of the guideway is related to the chosen displacement field and can be expressed as:

$$k_g = \frac{EI}{3l_x}$$  \hspace{1cm} (A.6)

With \( E \) the Young's modulus, \( I \) the moment of inertia and \( l_x \) the length of the x-guideway that is subjected to bending, i.e. the part from the x-carriage position to the effective rotation point, located in the centroid of the y-carriage. Depending on the position of the x-carriage the length \( l_x \) will vary. Hence, also the stiffness matrix is position dependent.
Appendix B

Load dependency of the dynamic errors
of a CMM

The dependency of the deformations on the dynamic load can be illustrated by a set of experiments with varying accelerations. In these experiments the rotation error $\varepsilon_z$ was measured at the probe position of the CMM, during motion of the $y$-axis for different commanded velocities. During each measurement run the CMM is at first moving with constant traverse speed. After a approximately 2 seconds the CMM decelerates to rest. Depending on the traverse speed, different levels of acceleration were obtained and thus different deformation amplitudes. The Figures B.1 to B.3 show the rotation errors for the situations where the CMM is CNC controlled with traverse speeds of $10 \, mm/s$, $40 \, mm/s$, and $70 \, mm/s$. The maximum accelerations that occur during deceleration are $20 \, mm/s^2$, $90 \, mm/s^2$, and $160 \, mm/s^2$ respectively. In Figure B.4 the error is shown when operating the machine with joystick control instead of CNC-control. In this case the maximum acceleration was $440 \, mm/s^2$. 
Figures B.1: The time history of the rotation error $\varepsilon_\varphi$, measured at the probe position of the investigated CMM. The CMM is CNC controlled and decelerating from a constant traverse speed ($10 \text{ mm/s}$) to rest.

Figures B.2: The time history of the rotation error $\varepsilon_\varphi$, measured at the probe position of the investigated CMM. The CMM is CNC controlled and decelerating from a constant traverse speed ($40 \text{ mm/s}$) to rest.
Figures B.3: The time history of the rotation error $\varepsilon_z$, measured at the probe position of the investigated CMM. The CMM is CNC controlled and decelerating from a constant traverse speed (70 mm/s) to rest.

Figures B.4: The time history of the rotation error $\varepsilon_z$, measured at the probe position of the investigated CMM. The CMM is joystick controlled and decelerating very suddenly from maximum traverse speed to rest.
During travelling with traverse speed the vibrations measured at the probe position are caused by the CMM drive system. In this case the level of vibration is at maximum ± 0.6 \text{arcsec}. This is comparable to the geometric accuracy of the CMM after software compensation (see Paragraph 4.1). Of more interest are the errors induced during deceleration of the CMM. Due to the inertia forces on the CMM, deformations of the machine's components occur. With CNC-control quasi-static as well as vibrational deformations are found. For a traverse speed of 10 \text{mm/s} the errors are still fairly small (less than 1 \text{arcsec}). However, for speeds of 40 \text{mm/s} and 70 \text{mm/s} the errors become rather significant (2.5 and 4.5 \text{arcsec} respectively). In case of joystick control the accelerations can be much higher than with CNC-control, resulting in relative large vibration errors (over 12 \text{arcsec}).
Appendix C

Joint- and link deformations of a CMM

In order to identify the contribution of joint- and link components to a certain parametric error, calculations and measurements on the various components of the investigated CMM were performed (see also Hazenberg 1993, Haanen 1995). Components contributing to the finite stiffness of the CMM's joints (i.e. the carriages) are the air bearing systems of the carriages and the carriages themselves. The machine's links are formed by the guideways, which have in principle finite bending- and torsion stiffness. Also torsion and bending of the support of the first guideway (the y-guideway) can contribute to the dynamic errors. The deformations of the joints and links that can be expected during accelerations of the CMM axes, were calculated first. The results were verified by measurements of the deformations under static load, using several position sensors. From the calculations and measurements it can be concluded which components will contribute to the parametric errors.

C.1 Calculation of the deformations

The deformations of the various joint- and link components that can be expected when they are subjected to accelerations, are estimated by calculations on these components. Here the calculations of the following, most important, deformations will be presented: the bending of the z-pinole and the x-guideway, the z-rotation due to the bearing system of the y-carriage and the torsion of the column support-
ing the y-axis. For the components relative simple elements are taken, and an acceleration force in y-direction is assumed. Due to the complex shape of the carriages it is difficult to calculate values for their deformations with a reasonable level of confidence.

**Bending of the z-pinole**

The z-pinole can be considered as a simple beam element with constant cross section. The acceleration due to y-axis motion can be considered as a distributed force on the pinole. This causes bending of the pinole. For the resulting displacement at probe position in y-direction, we can write:

\[
\delta_{y,q} = \frac{q_y \cdot l_z^4}{8EI} \quad (C.1)
\]

With \( q_y \) the distributed load per unit of length due to an acceleration, \( l_z \) the pinole length between the bearing system and the its free end, \( E \) the Young’s modulus, and \( I \) the second moment of inertia. The load can be expressed as:

\[
q_y = \frac{m_g \cdot \ddot{y}}{l_z} \quad (C.2)
\]

Here \( m_g \) denotes the mass of the pinole and \( \ddot{y} \) the acceleration. For the second moment of inertia with respect to the cross-sectional area, we can write:

\[
l = \frac{W^4 - w^4}{12} \quad (C.3)
\]

The dimensions \( W \) and \( w \) of the pinole are the outside and inside width respectively. With \( W = 0.055 \text{ m} \), \( w = 0.035 \text{ m} \), \( l_z = 0.550 \text{ m} \), \( m_g = 10 \text{ kg} \), \( E = 2.1 \times 10^{11} \text{ N/m}^2 \), and \( \ddot{y} = 0.5 \text{ m/s}^2 \) we find for the displacement of the pinole end \( \delta_{y,q} = 0.8 \mu \text{m} \). This is very small considering the rotation errors that were found for a similar acceleration level. The measurement results presented in Appendix B show that for an
acceleration of 0.44 m/s² rotations of over 10 arcsec are found, resulting in displacements of 50 µm. Compared to these results the bending of the pinole is negligible.

**Bending of the x-guideway**

When accelerating in y-direction a distributed force will be applied to the x-guideway. Furthermore, the accelerated mass of the x-carriage and z-axis will also act as a concentrated force on the guideway (see the example of Figure 3.15 in Chapter 3). The maximum displacement resulting from bending about the z-axis due to these forces, will be found if the x-carriage is at the end of the guideway. In this case we can write for the displacement $\delta_{y,q}$ in y-direction, due to the distributed force:

$$\delta_{y,q} = \frac{q_y \cdot l_x^4}{8EI}$$  \hspace{1cm} (C.4)

with:

$$q_y = \frac{m_y \cdot \ddot{y}}{l_x}$$  \hspace{1cm} (C.5)

The concentrated force causes a displacement $\delta_{y,F}$:

$$\delta_{y,F} = \frac{(m_x + m_z) \cdot \ddot{y} \cdot l_x^3}{3EI}$$  \hspace{1cm} (C.6)

The x-guideway is considered as a simple, hollow beam element with constant cross section and length $l_x$. In reality the guideway structure is more complex. Plates are welded inside the hollow guideway in a diagonal pattern. It can be assumed that the actual stiffness against bending will be higher. The second moment of inertia with respect to the z-axis can be expressed as:

$$l_z = \frac{1}{2} b h^2 t$$  \hspace{1cm} (C.7)
Where $b$, $h$, and $t$ denote the width, the height, and the thickness of the beam respectively. The total beam displacement at the end of the x-guideway is found by applying the super position principle for both load cases:

$$\delta_y = \delta_{y,d} + \delta_{y,l}$$  \hspace{1cm} (C.8)

Substituting (C.4) and (C.5) into (C.8) yields:

$$\delta_y = \frac{(m_x + m_z + \frac{3}{8} m_g) \cdot \ddot{y} \cdot l_x^3}{3EI}$$  \hspace{1cm} (C.9)

With $l_x = 1 \text{ m}$, $b = 0.25 \text{ m}$, $h = 0.15 \text{ m}$, $t = 0.01 \text{ m}$, $m_x = 34 \text{ kg}$, $m_z = 10 \text{ kg}$, $m_g = 134 \text{ kg}$, $E = 2.1 \cdot 10^1 \text{ N/m}^2$, and $\ddot{y} = 0.5 \text{ m/s}^2$ we find for the displacement of the end of the x-guideway $\delta_{y,g} = 2.5 \mu\text{m}$. Again the displacement is small compared to the displacements resulting from the rotations measured. The actual displacement will be even lower than calculated, since the second moment of inertia is estimated conservatively.

From the calculations on link components such as the z-pinole and the x-guideway, it is clear that the guideways contribute only little to the displacement error at probe position, if subjected to accelerations that are maximum values for the CMM investigated.

**Rotation due to bearing compliance**

Deformations of the joints of the CMM are caused by forces as well as moments acting at the joints. The forces result into translation errors, the moments into rotation errors. In Paragraph 4.2 it has been shown that pure translation errors can be neglected. However the rotation errors are expected to be quite significant. The moment causing these errors results from the accelerations acting on the connected beam elements. The largest moment for the investigated CMM is found for y-direction motion, causing rotation of the x-guideway about the z-axis at the y-carriage position. With the following calculations the effects of this moment on the bearing system and the support of the y-axis will be shown. In Fig-
Figure 3.15 shows the load on the x-guideway for this situation. Using Equation 3.44, we can write for the maximum moment (x-carriage at the end of the beam):

\[ M_z = -\left( \frac{1}{2} \frac{m_y}{l_x} (l_x + d)^2 + (m_x + m_z) (l_x + d) \right) \ddot{y} \]  \hspace{1cm} (C.9)

Here \( d \) denotes the width of the y-carriage. With \( l_x = 1.2 \, m \), \( d = 0.2 \, m \), \( m_x = 34 \, kg \), \( m_z = 10 \, kg \), \( m_y = 134 \, kg \), and \( \ddot{y} = 0.5 \, m/s^2 \) the maximum moment about the z-axis of the y-carriage is \( M_z = 85 \, Nm \).

The joints of the CMM contain air bearings. These are often the major cause of the limited stiffness of machine components. Due to their mounting, the individual bearings of a joint have zero rotation stiffness. But together they form a system that has a stiffness against rotations. Based on the stiffness of the individual bearings and their configuration the stiffness of the bearing system can be calculated. The y-carriage has two sets of preloaded air bearings that contribute to the stiffness about the z-axis. If \( k_{sb} \) is the stiffness of one set of bearings, the rotation stiffness can be expressed as:

\[ k_{z,b} = \frac{1}{2} k_{sb} \cdot a^2 \] \hspace{1cm} (C.10)

Where \( a \) denotes the distance between the sets of bearings. With \( k_{sb} = 70 \cdot 10^6 \, N/m \) and \( a = 0.4 \, m \) we find for the rotation stiffness \( k_{z,b} = 5.6 \cdot 10^6 \, Nm/\text{rad} \). For the rotation \( \varepsilon_{z,b} \) of the bearings caused by a moment about the z-axis, we can write:

\[ \varepsilon_{z,b} = \frac{M_z}{k_{z,b}} \] \hspace{1cm} (C.11)

For \( M_z = 85 \, Nm \) and \( k_{z,b} = 5.6 \cdot 10^6 \, Nm/\text{rad} \) this yields \( \varepsilon_{z,b} = 3 \text{arcsec} \). As expected this is a significant contribution.
Torsion of the support column

Another contribution is expected from the support column of the y-axis. For the deformation $\varepsilon_{z,c}$ due to torsion of the column, we can write:

$$\varepsilon_{z,c} = \frac{M_z \cdot l_c}{G \cdot I_z}$$  \hspace{1cm} (C.12)

Where $l_c$ denotes the length of the column, $G$ is the shear modulus of elasticity, and $I_z$ is the second moment of inertia, which can be expressed as:

$$I_z = \frac{2t(a-t)^2(b-t)^2}{a+b}$$  \hspace{1cm} (C.13)

Where $a$, $b$, and $t$ denote the side dimensions and the thickness of the column respectively. With $l_c = 0.5 m$, $a = 0.3 m$, $b = 0.15 m$, $t = 0.01 m$, $G = 8 \cdot 10^{10} N/m^2$, and $M_z = 85 Nm$ we find for the rotation $\varepsilon_{z,c} = 1.4 \, \text{arcsec}$. This is smaller than the rotation due to the bearings, but still significant.

C.2 Measurement of the deformations

The calculations of the previous paragraph give an estimation of the deformations that can be expected in case of acceleration due to y-axis motion. The results of the calculations are verified by a set of experiments. In these experiments the rotations of the joints as well as the bending of the links for a given load were measured. From these data the stiffness of the relevant components could be calculated. A set-up was created by which a static force could be applied to the CMM's probe position (see Figure C.1). This static force at the pinole causes internal forces and moments that act on the components. The resulting displacements at these various components were measured by displacement sensors or by laser interferometric rotation measurements.
Joint- and link deformations

Figure C.1: Experimental set-up for measuring the static stiffness of some CMM components. At probe position a static force is applied. The deformations are measured with displacement sensors or laser interferometry.

Guideway deformations

In Figure C.2 the measurement configuration for identifying the z-pinole bending is shown. At several different heights above the CMM table inductive sensors measure the relative y-displacement of the pinole with respect to the CMM table. In the graph of Figure C.3 the displacements at the four z-heights are shown for a unit load of 1 N. Because the curve through the four data points is a straight line, it is clear that the influence of pinole bending is not significant. The results of other experiments on different components, show similar results with respect to guideway deformations. The rotations due to bending of the guideways are relative small compared with the rotations of the joints. For the CMM under investigation bending effects of the guideways were found to be one order of magnitude smaller. Here static measurements were conducted using a concentrated force. The load in the dynamic situation will be distributed. However, the measurements give a good indication of the significant errors in case of dynamic loads. The found results on insignificant guideway deformations are also checked for dynamic loads. These results are reported together with the measurements on the displacement sensors, that were implemented on the CMM (see Paragraph 5.2).
Figure C.2: The configuration of the used experimental set-up for identifying bending of the z-pinole. A static force is applied at the end of the pinole. The displacements at different z-heights are measured with inductive position sensors.

Figure C.3: The measured displacements at four heights above the CMM table for a static force at the end of the pinole of 1 N.
Joint and support deformations

Based on several experiments with the described measuring set-up, the rotation stiffness values of the joints with respect to the different degrees of freedom, could be calculated, using the relation between the moment due to the load and the measured rotation, e.g.:

\[ k_{z,y\text{-joint}} = \frac{M_z}{y\varepsilon_z} \]  \hspace{1cm} \text{(C.14)}

With \( M_z \) the moment about the z-axis due to the applied force, and \( y\varepsilon_z \) the resulting rotation that is measured. With the depicted set-up of Figure C.1, stiffness values could be identified that are related to a load in y-direction. In Table C.1 the stiffness results are displayed for the components that yield significant rotations (see Paragraph 4.2.2). From the calculations in the previous paragraph it was clear that especially the bearings have a significant contribution to the joint compliance. Therefore the calculated values for the bearing stiffnesses are also displayed in Table C.1. Comparing these calculated values with the measured values, yields rather large differences. The total stiffness is approximately half of the bearing stiffness. Thus besides the bearings, other components like the carriage contribute to the joint compliance. In the case of the y-axis, a rotation measured at the y-carriage is partly due to torsion of the support. For the torsion stiffness of the support it was found that \( k_{z,\text{support}} = 1.3 \times 10^7 \text{ Nm/rad} \). It was also found that the support causes translation errors of the y-carriage in y-

<table>
<thead>
<tr>
<th>rotation</th>
<th>total joint stiffness</th>
<th>bearing stiffness</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x\varepsilon_x )</td>
<td>( 0.7 \times 10^6 )</td>
<td>( 1.5 \times 10^6 )</td>
</tr>
<tr>
<td>( y\varepsilon_y )</td>
<td>( 4.0 \times 10^6 )</td>
<td>( 8.0 \times 10^6 )</td>
</tr>
<tr>
<td>( y\varepsilon_z )</td>
<td>( 2.9 \times 10^6 )</td>
<td>( 5.6 \times 10^6 )</td>
</tr>
</tbody>
</table>

Table C.1: The stiffness values belonging to the relevant rotations. The values for the total joint stiffness are measured values, the values for the respective bearing stiffness are calculated.
direction. In general such a linearity error of a carriage is not of interest, because the carriage's position is measured by the scales attached to the guideway. However, in this case the guideway itself is translating due to support motion. Thus a translation error is introduced. For the relevant stiffness of the support in the y-direction it was found $k_{y,\text{support}} = 1 \cdot 10^7 \text{ N/m}$.

From the results of the calculations and measurements presented here, some conclusions can be drawn with respect to the contribution of the various components to the rotation errors that were found significant. The deformations of all joint components are significant, but the guideway deformations are negligible. From the joint deformations not only the bearing deflections important, but also the deformations of e.g. the carriage itself. Furthermore, the y-axis is also affected by the finite stiffness of the support of this axis. The support contributes to rotation as well as translation errors.
Appendix D

Inductive displacement sensors

D.1 Physical principle of inductive non-contact sensors

The physical principle of the inductive non-contact sensors is based on variations of the inductance, caused by changes of the air gap within a magnetic circuit. Such a magnetic circuit is realised by winding a coil on a c-shaped ferromagnetic core (see Figure D.1).

Figure D.1: Principle of an inductive sensor with variable air gap between sensor and object surface.
The inductance $L$ of the magnetic circuit can be expressed as (Rohrbach 1967):

$$L = \frac{\mu_0 \cdot N^2 \cdot A}{\left(\frac{l_{Fe}}{\mu_{Fe}} + \frac{l_L}{\mu_L}\right)}$$  \hspace{1cm} (D.1)

where:

- $N$ : number of coil windings
- $A$ : cross section of the core
- $\mu_0$ : vacuum permeability
- $\mu_{Fe}$ : iron permeability
- $\mu_L$ : air gap permeability ($\mu_L = 1$)
- $l_{Fe}$ : length of the magnetic path
- $l_L$ : length of the air gap

The maximum inductance is given at $l_L = 0$:

$$L_{max} = \mu_0 \cdot \mu_{Fe} \cdot N^2 \cdot \frac{A}{l_{Fe}}$$  \hspace{1cm} (D.2)

There is hyperbolic dependence of the normalised inductance $L / L_{max}$ to the ratio $l_L \cdot \mu_{Fe} / l_{Fe} \cdot \mu_I$, which is shown in Figure D.2.

![Figure D.2: Hyperbolic dependence of the inductance on the normalised displacement of the sensor relative to the object surface.](image-url)
For small displacements the sensitivity is almost linear. With the initial gap as small as possible, the highest sensitivity is obtained. To achieve a better performance with respect to linearity and temperature dependence, the used sensor is based on two coils. One coil is the measuring coil, the other coil is the reference coil that is placed opposite a reference plate (see Figure 4.8). Both coils are part of a bridge circuit. In Figure D.3 a schema of the circuit is depicted. The bridge is supplied with a stable AC voltage $U_G$. The carrier frequency is 5 kHz. If the bridge is detuned by displacing the sensor relative to the guideway over a distance $\Delta L$, a voltage $U_M$ will appear. For this voltage we can write:

$$U_M = U_G \cdot \left( \frac{L_M}{L_R + L_M} - \frac{R_2}{R_1 + R_2} \right)$$

(D.3)

Where $L_M$ and $L_R$ denote the inductance of the measuring coil and the reference coil respectively. $R_1$ and $R_2$ are bridge resistors. Using Formula D.1 we can derive the dependency between the measured voltage and the gap length:

$$U_M = U_G \cdot \left( \frac{C_1}{C_2 + l_L} - \frac{R_2}{R_1 + R_2} \right)$$

(D.4)

Figure D.3: Scheme of the bridge circuit that measures variations in the inductance of the sensor, using a measuring coil and a reference coil.
Where the constants are:

\[ C_1 = \frac{L_{Fe} \cdot i_{Fe}}{L_R} \quad (D.5) \]

\[ C_2 = \left( \frac{L_{Fe}}{L_R} \right) \cdot i_{Fe} \quad (D.6) \]

For small variations \( \Delta L \) of the air gap \( l_L \) we can write:

\[ \Delta U_M = \left( \frac{\partial U_M}{\partial l_L} \right)_{l_L=L_0} \cdot \Delta L = -U_G \frac{C_1}{(C_2 + l_{L0})^2} \cdot \Delta L \quad (D.7) \]

or for the relative voltage:

\[ \frac{\Delta U_M}{U_G} = -\frac{C_1}{(C_2 + l_{L0})^2} \cdot \Delta L \quad (D.8) \]

By adjusting the position of the reference plate the reference gap is changed, and by this the measured bridge voltage \( U_M \) is balanced to zero if the sensor is in zero position, thus for \( l_L = l_{L0} \). Thus the ratio \( \Delta U_M / U_G \), that forms the output of the amplifier system, is a measure for small displacements around zero.

**D.2 Calibration of the inductive sensors**

The inductive CMM-sensors are calibrated against a reference sensor. The measuring principle of the used reference sensor is also based on inductance. However in contrast to the CMM-sensors it's a contact sensor using a stylus. The stylus is attached to a core that can move inside two coils. Displacement of the core causes variations in the inductance of both coils. This variation is measured, using the same equipment as for the CMM-sensors. From measurements with a CMM-sensor and the reference sensor, a calibration curve can be estimated between the readout of the CMM-sensor and the actual displacement measured by the refer-
ence sensor. Based on the theoretical relation (D.8), a linear dependency is assumed. Hence, the estimation can be expressed by the relationship:

\[ R = \hat{c}_1 S + \hat{c}_2 + \gamma \]  

(D.9)

Where \( \hat{c}_1 \) and \( \hat{c}_2 \) denote the coefficients of the linear calibration function and the vectors \( R \), \( S \), and \( \gamma \) contain the reference sensor data, the CMM-sensor data, and the residuals of the estimation respectively. In the graph of Figure D.4 the results of a typical sensor calibration are shown. The measuring range over which the respective sensor is calibrated is only \( \pm 10 \mu m \), because this turned out to be the actual range of the displacements. The sensor readings are the output of the digital amplifier and are given in counts. In Figure D.5 the residues of the estimated values are shown. The residues are not randomly distributed, so the actual relationship is not quite linear. However the measuring error, using the linear calibration function, is at maximum \( 0.1 \mu m \) over the calibrated range. Based on several sets of data the mean value for the linearity coefficient and standard

![Figure D.4: Results of a the calibration of a CMM-sensor vs. the reference sensor over small measuring range.](image-url)
Figure D.5: Residues after application of a linear calibration curve to CMM-sensor values.

declaration of the coefficient were calculated. The calculated values are $c_1 = -0.0303$ \( \mu \text{m}/\text{count} \) for the linearity coefficient and $\sigma_{c_1} < 5 \times 10^{-5}$ \( \mu \text{m}/\text{counts} \) for its standard deviation. The maximum measuring error will occur at the end of the measuring range. We can estimate the error $\delta_{\text{max}}$ for the worst case situation:

$$\delta_{\text{max}} = \left| \delta_{\text{fit}} + 2\sigma_{c_1} \cdot S_{\text{max}} \right|$$  \hspace{1cm} (D.10)

Where $\delta_{\text{fit}}$ is the fitting error and $S_{\text{max}}$ the maximum sensor reading. For a maximum reading of -300 counts (10 \( \mu \text{m} \)), $\delta_{\text{fit}} = -0.1 \mu \text{m}$, and $\sigma_{c_1} = 5 \times 10^{-5}$ \( \mu \text{m}/\text{counts} \) this yields $\delta_{\text{max}} = 0.4 \mu \text{m}$. This is higher than the demanded specifications (0.25 \( \mu \text{m} \)), but for smaller displacements the sensor accuracy will be considerably better.

Using the linear calibration curve yields the advantage that the absolute sensor readings are of no interest. Thus when the sensor readings are reset to zero by the amplifier system, the displacements can still be calculated by multiplying the amplifier output with the calibration coefficient. In case of a non-linear calibration curve the absolute sensor readings must be known.
Acknowledgements

The author wishes to thank all people who contributed to the research described in this thesis. Special thanks go to my first promotor Piet Schellekens, professor of the Precision Engineering group of the Eindhoven University of Technology. He introduced me to the field of precision engineering and gave me the opportunity to start this research project. Throughout the project he has supported me with great enthusiasm and must have spent many hours critically reading several versions of this thesis. I am very grateful for his contribution to my work and our pleasant cooperation.

During my research I greatly benefited from the research conducted by former PhD-students, so many thanks to them. Many thanks also go to Ryoshu Furutani and Leon Levasier for their help with the measuring equipment, to John Hazenberg and Rik Haanen for performing many experiments for me, and to Renishaw Transducer Systems Ltd. for supplying a fast laser interferometer system, that has proved very useful for my measurements. Furthermore I am much obliged to the members of the main promotion committee, prof. dr. ir. M.J.W. Schouten, prof. dr. ir. J.E. Rooda and prof. ir. R.J.G.A. van der Hoorn for the time spent reviewing this document. I also would like to thank in particular Frits Verhoeven for giving the document a final review.

Furthermore I wish to express my thanks to the present and former staff members, PhD-students, and students of the Precision Engineering group for their cooperation and the pleasant working climate. I very much enjoyed being their colleague. A special word of thanks goes to my fellow PhD-student and friend Wim van Vliet for his support by the many fruitful, scientific- and less scientific discussions we have had. Finally, I would like to thank my family, who have always encouraged and supported me throughout my education and work.
Curriculum Vitae

Wim Weekers was born on June 14th, 1967 in Weert, The Netherlands. He attended the Scholengemeenschap Bisschoppelijk College in Weert, where he obtained his Atheneum B diploma in 1985. The same year he commenced his study of Mechanical Engineering at the Eindhoven University of Technology. The last year of this study he worked on a project concerning the manufacturing accuracy of automobiles assembled by industrial robots. During this period he was employed by Volvo Car B.V. in Born. He received his Master’s degree in November 1990.

From January 1991 until April 1991 he worked at the Mitutoyo Corporation in Kawasaki, Japan, having been offered a trainee job. In October 1991 he was engaged as a research assistant by the Faculty of Mechanical Engineering at the Eindhoven University of Technology, where he started his doctoral studies in the field of precision engineering. During his studies he worked on the compensation for dynamic errors of coordinate measuring machines. Besides his research work, he has been a co-author of the new precision engineering syllabus.
Compensation for Dynamic Errors of Coordinate Measuring Machines

Wim Weekers
1. De bouwvormen van conventionele meetmachines, vaak bepaald op grond van meetvolume en toegankelijkheid, leiden dikwijls tot een grote gevoeligheid voor dynamische afwijkingen.

*Bij dit proefschrift, hoofdstuk 2.*

2. Nauwkeurige bepaling van dynamische afwijkingen van een machineslede met behulp van on-line sensormetingen, stelt hoge eisen aan zowel de sensoren als aan de geleiding van de slede.

*Bij dit proefschrift, hoofdstuk 4.*

3. Ten aanzien van de rotatiestijfrheid van een luchtgelagerde slede bij meetmachines kan naast het lagersysteem ook de slede zelf een belangrijke bijdrage leveren.

*Bij dit proefschrift, hoofdstuk 4.*

4. Compensatie voor dynamische afwijkingen bij coördinaten meetmachines is een uitstekend middel om sneller meten zonder groot verlies aan nauwkeurigheid mogelijk te maken.

*Bij dit proefschrift, hoofdstuk 6.*

5. In geval van snelle aantasting op een coördinaten meetmachine met een mechanische taster, is met betrekking tot de meetnauwkeurigheid niet alleen het dynamische gedrag van de meetmachine van belang, maar ook het botsingsgedrag tussen taster en werkstuk.

6. Een nauwkeurig meetresultaat impliceert dat het meetmiddel herleidbaar gekalibreerd is naar de standaard en dat het betrouwbaarheidsinterval van een meetresultaat verkregen met het meetmiddel bekend is. Bij complexe meetmiddelen zoals coördinaten meetmachines is het daarom lastig om nauwkeurige meetresultaten te verkrijgen.

7. “The known is finite, the unknown infinite; intellectually we stand on an islet in the midst of an illimitable ocean of inexplicability. Our business in every generation is to reclaim a little more land, add something to the extent and solidity of our possessions.”

*Thomas Henry Huxley (1887).*
8. “When you cannot measure what you are speaking about, when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind; it may be the beginning of knowledge, but you have scarcely in your thoughts advanced to the stage of science, whatever the matter may be.”

Lord Kelvin.

9. In het vergoedingensysteem van de TUE wordt een eenzame autorit naar een ver conferentieoord positief en een vruchtbaar diner met mede-congresgangers negatief gewaardeerd. Dit in tegenstelling tot de feitelijke bedoeling van een congres.

10. Het opbouwen van een staatsschuld kan voor een land een legitiem middel zijn om noodzakelijke investeringen in bijvoorbeeld de infrastructuur te spreiden over meerdere generaties. Het getuigt echter van weinig solidariteit met toekomstige generaties als een land een staatsschuld opbouwt voor consumptieve uitgaven.

11. De weigering van Groot-Brittannië om iets van haar soevereiniteit in te leveren ten behoeve van “Brussel” kan niet zonder meer als dwarsliggerij betiteld worden, gezien de stabiliserende factor die ze door de eeuwen heen vaak geweest is tegen overheersende machten in Europa.

12. Kritiek op historische beslissingen zegt vaak meer over het gebrek aan inzicht van de critici omtrent de toenmalige politieke en sociale context, dan over de kwaliteiten van de toenmalige leaders.

13. De toekomst is niet wat gaat gebeuren, maar wat wij gaan doen.

_Uit: Kwartaalblad Amnesty International Nederland, 1995._

14. Reizen geeft niet alleen een kijk op andere culturen, maar ook een andere kijk op de eigen cultuur.

Eindhoven, 15 mei 1996