Droplet collection in a scaled-up rotating separator

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Droplet collection in a scaled-up rotating separator

PROEFSCHRIFT

ter verkrijging van de graad van doctor aan de Technische Universiteit Eindhoven, op gezag van de rector magnificus, prof.dr.ir. C.J. van Duijn, voor een commissie aangewezen door het College voor Promoties in het openbaar te verdedigen op dinsdag 20 maart 2012 om 16.00 uur

door

Johannes Pieter Kroes

geboren te Amsterdam
Abstract

Separation of droplets from a gas stream is a frequent operation in natural gas processing. The trend towards remote, contaminated fields demands compact, efficient and reliable demisters. A specific challenge is the removal of condensates. In a new process called condensed rotational separation (CRS), natural gas contaminants (CO$_2$ and H$_2$S) are removed by condensation. The low surface tension of the condensing species leads to a fine mist of 1–10 micron droplets. In CRS, the condensed droplets are separated from the clean gas in a rotating phase separator (RPS).

The core of the RPS is a rotating element: a bundle of channels (tubes), contained in a cylinder which rotates around its axis. When a particle laden gas is led through the rotating channels, the centrifugal force drives the particles (droplets) towards the walls. The radial traveling distance of droplets is small as compared to the channel length. This effects the efficient separation of particles as small as 1 micron.

To verify scaling laws and liquid removal, a full scale prototype was built at the Eindhoven University of Technology, based on a previous small scale CH$_4$/CO$_2$ test unit at Shell Global solutions in Amsterdam. The prototype was tested at atmospheric conditions, using air and water. The test rig models an 80 MMSCFD (24 m$^3$/s) natural gas installation, which, in terms of volume flow, is equivalent to an entire gas well.

In previous RPS applications (e.g. air filtration, flue gas filtering), the flow inside the channels was laminar. However, high volume flow/high fluid density applications in the oil and gas industry are characterized by unstable or even turbulent channel flow. To quantify the effect of flow instabilities and turbulent mixing on the separation efficiency, a good measurement accuracy is needed.

In order to determine the separation efficiency as a function of droplet size, a mist injection system was built and droplet size distributions were measured by means of laser diffraction particle sizing. The accuracy was improved by paying specific attention to channel entrance effects and vignetting in the laser diffraction system, plus reducing side leakage along the rotating element. By varying the gas flow rate and element rotation speed, the efficiency curve was measured in a large operating range. Compared to previous measurements in literature obtained with laminar RPS units, the accuracy was improved and better correspondence to theory was obtained. We further completed the theory of laminar efficiency for rectangular channels.
In high pressure natural gas installations like CRS, a large gas density and high flowrates induce turbulent conditions within the rotating channels. To simulate high Reynolds numbers in the atmospheric test setup, an element with enlarged channels was built. To maintain the same separation efficiency, the element and prototype also had to be elongated. The subsequently obtained results are the first with unstable/turbulent flow till date. In the turbulent regime, measurements showed good correspondence to direct numerical simulations (DNS) of particle laden rotating pipe flow. Further investigation of the DNS results yielded a new model which characterizes the effect of mixing on the separation efficiency.

For bulk Reynolds numbers below 2000, poor separation efficiencies were found due to flow instabilities in otherwise laminar flow. It is well known that nonrotating pipe flow becomes turbulent at bulk Reynolds numbers $Re > 2000$. However, sufficient rotation causes pipe flow to become unstable against infinitesimal disturbances already at $Re = 83$. The instabilities induce traveling spiral waves inside the rotating channels, which tend to trap particles and undo their separation. This is specifically relevant in RPS applications for oil/water separation. The negative effect of the spiral waves was captured in a new empirical correction factor.

The measurement method was further applied to cyclones and vane packs. For axial cyclones an accompanying model was introduced, based on realistic vortex profiles. A benchmark was made for the efficiency of three types of demisters for natural gas processing: vane packs, cyclone decks and rotating elements.
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Introduction

1.1 RPS technology

Many processes require the separation of fine particles from a gas stream. Techniques employed are: scrubbers, fabric filters, electrostatic separators and (multi)cyclones. However, there is still a drive to develop new technologies: scrubbers are sizeable and fail to remove micron sized particles, fabric filters/electrostatic precipitators are limited to dry/chargeable particulate matter and involve large installations, and cyclones in industrial applications involving large volumetric flows fail to collect micron sized particles as well. A new development [8] which overcomes the above limitations is the rotational particle separator (RPS).

The core of the RPS is an axially rotating element, consisting of a many small channels contained in a cylinder (Figure 1.1a). Particles or droplets entrained in the fluid flowing through the channels are centrifuged towards the walls of the channels, where they form a layer or film of particles material (Figure 1.1b). The film is removed by applying pressure pulses or by breakup of the liquid itself (Figure 1.1c). The channels provide the means to collect micron sized particles at limited rotation speed, pressure drop and short residence time (small building volume).

(a) Rotating element and closeup  (b) Film formation  (c) Droplet breakup

Figure 1.1. Principle of a rotating element
Compared to conventional cyclones the RPS is an order of magnitude smaller in size at equal separation performance, or, at equal size it separates particles ten times smaller [40]. Figure 1.2 shows some examples of RPS applications. Former applications are: ash removal from hot flue gases in small scale combustion installations [13], collection of powders and particles from gases in food and pharmaceutical processes, and dust filtering in domestic environments. More recent applications, developed for the oil and gas industry, are oil/water separation [25, 37], dehydration of natural gas [29, 30] and removing ultra fine CO$_2$ rich mists from natural gas [10, 45].

1.2 Oil and gas applications

Separation of liquid dispersions from another fluid is one of the most important unit operations in the oil and gas industry [22]. Dispersions to separate are either oil/water or gas/liquid mixtures. Gas/liquid separation (demisting) often takes place in so-called internals inside separator vessels, referred to as natural gas scrubbers [2]. Figure 1.3 shows a few examples of internals: a vane pack, a cyclone deck and the rotating element, which can also be viewed as a kind of internal. In each of these demisting is based on inertial separation, which means that the centrifugal force is employed to force droplets to a collecting wall. Whereas vane packs as well as mist mats are used for pre-separation of large mist droplets (> 15 $\mu$m), axial cyclones are standard for fine mist separation in hydrocarbon processing plants.

Axial cyclones [20] are used for water and condensate removal but can not be applied for removing condensed contaminants, such as CO$_2$ or H$_2$S. This is because cyclones can only handle condensing droplet sizes above 15 $\mu$m. When condensing contaminants from natural gas, the droplet size of the dispersed contaminant is small: typically on the order of 1 $\mu$m [2, 4]. It is well known in laboratory chemical applications that microcyclones can separate such small droplets, but then the flow is very small and orders of magnitude less than the flow in gas well applications.
1.2 Oil and gas applications

(a) Vane pack  (b) Cyclone deck  (c) Rotating element

Figure 1.3. Mist extractors in natural gas processing

The RPS, which in these type of applications stands for rotating phase separator, effectively separates droplets down to 1 micron, even at the large flows associated with natural gas processing. In a previous study RPS and cyclone have been compared on the basis of two independent process parameters: residence time and specific energy consumption, defining capital cost (separator size) and operating cost respectively. It became clear that the RPS is able to separate an order of magnitude smaller droplets than the axial cyclone at equal residence time or specific energy consumption [40]. However, as mentioned above, demisters are usually integrated as internals in a vessel. In natural gas processing industry, the maximum allowable gas velocity in such a vessel, defined through a gas load factor (GLF), is commonly used as performance indicator. The RPS has not yet been approached in such a way.

Applying RPS technology in natural gas processing installations presents an additional challenge. The high pressure (high gas density) and large volume flows lead to a high Reynolds number, inducing turbulent conditions within the rotating channels [30, 45]. To maintain laminar flow inside the channels would impose a too severe restriction on the design, especially in offshore applications, where platform space is limited and load capacity should be maximal.

An extra complication arises in applications for oil/water separation [25, 37]. Despite the fact that the bulk Reynolds number in the channels is only about 200 (usually laminar), rotation destabilizes the flow [26]. Flow instabilities start growing and cause spiral motions that tend to trap droplets, preventing them from reaching the wall [36]. Droplet separation in this unstable regime is still a largely unexplored area.

Direct numerical simulations have shown that turbulence/instabilities decrease the separation efficiency by 25% at most [23, 36]. However, these findings have never been validated by experiments. Among other things, this is due to the lack of an accurate measurement method to determine the separation efficiency curve. The expected effect of turbulence falls within the spread of previous measurements [7, 8], obtained with laminarly operating RPS units.
1.3 CRS technology

Condensed rotational separation (CRS) is a new method for separating mixtures of gases, based on RPS technology. It can be used to remove CO\(_2\) and/or H\(_2\)S from contaminated natural gas wells [11, 38, 39, 43, 46]. The gas mixture is first cooled by expansion to a temperature at which the contaminants condense. A recent study on condensing CO\(_2\) droplets concludes that the condensate is entrained in the gas stream as 1–20 micron mist droplets [3, 4] (Figure 2.6). The mist is subsequently removed in a rotating phase separator (RPS). Due to the small energy penalty as compared to amine treatment and the complete removal of the condensing mist, it is an attractive technique for sweetening heavily contaminated natural gas wells.

Lab scale experiments in a 0.05 MMSCFD* gas loop at Shell Global Solutions in Amsterdam have delivered the proof of principle [43, 46]. A drawback of testing at such a small scale is that a rotating element is actually redundant due to the inherently high efficiency of any inertial separation mechanism: micron droplets are largely separated even in a bend. Moreover, the large liquid fractions associated with the high levels of contamination result in a considerable liquid loading of the downstream RPS unit. This caused liquid hold up in the small scale lab unit [46]. Liquid removal was then recognized as one of the key design challenges.

This led to several design modifications, which were implemented in a scaled-up 80 MMSCFD design [43]. The full scale design, geared towards large liquid loadings, comprises capacious liquid collection volutes. Moreover, it was decided to go for cocurrent, downwards gas and liquid flow to minimize the thickness of the liquid film that builds up inside the channels of the rotating element (Figure 1.1b) [44].

In this study we use a prototype of this scaled-up unit [45], built for testing with air and water at atmospheric conditions (a detailed description is given in section 3.1). However, testing at near atmospheric conditions brings up the following conflict: air at 1 bar gives laminar channel flow whereas operating with 80 MMSCFD of high pressure, high density natural gas would yield turbulent conditions inside the channels.

In conclusion, we identify the following key issues of upscaling:

- Complete removal of micron droplets from large gas flows.
- Handling large liquid loadings involving high contamination levels.
- The occurrence of turbulent flow conditions inside the rotating channels.

*Million standard cubic feet per day


1.4 Goal and outline

The main focus of this thesis is on droplet collection efficiency. Following standard practice we validated the separation efficiency of demisters with air and water (mist), using an improved measurement method based on laser diffraction particle sizing. We refined existing analytic models for the separation efficiency and quantified the effects of turbulence and flow instabilities in rotating channels.

In chapter 2 we simultaneously derive the efficiency of a rotating element, a cyclone deck and a vane pack (Figure 1.3). To that end, we treated the rotating element like a demister internal as is usual for natural gas scrubbers. We show that the three demister configurations all share the same mathematical basis. This provides an objective benchmark for demister efficiency.

In chapter 3 we give a detailed description of the scaled-up RPS prototype for CRS. We further describe our experimental air/water test setup to which we connected the unit, and which models an 80 MMSCFD natural gas installation (roughly a gas well). Finally, we explain a measurement technique, based on laser diffraction particle sizing, to determine the separation efficiency as a function of droplet size.

In chapter 4 we validate the separation efficiency of vane packs and axial cyclones by measurements in smaller, dedicated experimental setups. We also refine a model for the efficiency of axial cyclones by implementing realistic vortex profiles.

In chapter 5 we discuss the RPS in case of laminar (Poiseuille type) channel flow. We measured the separation efficiency of the RPS prototype with a standard element, running at laminar flow conditions. After improving the measurement technique, we obtained good correspondence with theory in an array of flowrates and rotation speeds. More importantly, the reached accuracy is good enough to distinguish the effect of turbulence and flow instabilities on separation efficiency.

In chapters 6 we concentrate on unstable/turbulent conditions, using a threefold approach. First, we derive basic analytic formulas describing the essential mechanisms. Second, we present measurements of the separation efficiency in a rotating element which, based on scaling laws, was specially constructed with enlarged channels to achieve unstable/turbulent flow. Third, we analyze direct numerical simulations (DNS) of rotating pipe flow, in which particles are tracked under the influence of a centrifugal force. For turbulent flow, results from both experiments and DNS can be explained well in terms of two simplified models: plug flow without mixing and continuous radial mixing of droplets. For unstable flow we defined a new empirical correction factor, accounting for the loss in efficiency due to internal spiral motions.
Basic principles of inertial separation

This chapter serves as a basis for later chapters. Starting from the equation of motion (section 2.1), section 2.2 shows that, in the most basic form, separation efficiency can be described as a universal function of a dimensionless droplet diameter. Section 2.3 applies the result to the three basic types of demister internals introduced in Ch. 1. Section 5.2.3 shows how to calculate the overall separation efficiency, in particular for a lognormal input distribution. Finally, section 2.5 concentrates on the rotating element, analyzing specifically the effect of the inflow distribution.

2.1 Droplet motion

A droplet (particle), moving in a non-inertial (accelerating) frame of reference, has an apparent, external body force $\vec{F}_e$ acting on it. If it moves in a fluid (gas or liquid), it experiences two counteracting forces: the buoyancy force $\vec{F}_b$ and a drag force $\vec{F}_d$. For a spherical droplet we can write:

$$\vec{F}_e = \rho_p \frac{\pi}{6} d_p^3 \vec{a}$$

$$\vec{F}_b = -\rho g \frac{\pi}{6} d_p^3 \vec{a}$$

$$\vec{F}_d = -3\pi \mu g d_p (d\vec{x}/dt - \vec{v})$$

with $\vec{x}$ particle position, $\vec{v}$ fluid velocity and $\vec{a}$ the external acceleration vector. Further, $d_p$, $\rho_p$, $\rho g$ and $\mu g$ are droplet (particle) diameter, droplet density, fluid density and fluid dynamic viscosity respectively. The drag force (2.3) is based on Stokes’ law for small, spherical objects in a continuous viscous fluid [14, 20] (Figure 2.1). Stokes’ law is valid for small particle Reynolds number and small Knudsen number:

$$Re_p = \frac{\rho g |d\vec{x}/dt - \vec{v}|}{d_p/\mu g} < 1$$

$$Kn = \frac{2\lambda_g}{d_p} < 0.1$$
where $\lambda_g$ is mean free path of the fluid molecules. Applying Newton’s second law, we have for the motion of a droplet [14, 20]

$$\rho_p \frac{\pi}{6} d_p^3 \left( \frac{d^2 \vec{x}}{dt^2} \right) = \sum \vec{F}$$

Substituting (2.1)–(2.3), the equation of motion can be written as

$$\frac{d^2 \vec{x}}{dt^2} = -\frac{1}{\tau_p} \left( \frac{d\vec{x}}{dt} - \vec{v} \right) + \left( \frac{\rho_p - \rho_g}{\rho_p} \right) \vec{a} \quad \text{(2.7)}$$

with $\tau_p$ the particle relaxation time, defined as

$$\tau_p = \frac{\rho_p d_p^2}{18 \mu_g} \quad \text{(2.8)}$$

For the type of applications considered in this thesis, $\tau_p$ is small as compared to the residence time [20]. This means that the inertial term can be neglected. In other words, we may assume that, relative to the fluid, droplets instantly reach their terminal velocity $\vec{U}_T$, based on equilibrium between drag and body force [18, 20].

$$\vec{U}_T = \tau_p \gamma \vec{a} \quad \text{(2.9)}$$

with $\gamma = 1 - \rho_g/\rho_p$ a buoyancy correction. Even compared to the rapid, small-scale fluctuations of turbulence, $\tau_p$ turns out small [20, 23]. Therefore, even in a turbulent flow, particle inertia plays no significant role and the terminal velocity can simply be superimposed on the local fluid velocity to arrive at the actual particle velocity:

$$\frac{d\vec{x}}{dt} = \vec{v} + \vec{U}_T \quad \text{(2.10)}$$

For Stokes’ law to be applicable in turbulent flows, droplets also must be small compared to the Kolmogorov length scale. In practice, this restriction is more severe than small $\tau_p$ or small $Re_p$ [36].

Figure 2.1. A nice flow visualization of Stokes settling.
For $Re_p > 1$ (Eq. 2.4) the fluids’ inertia starts to matter, corresponding non-linear terms in the Navier-Stokes equations can no longer be neglected and drag increases. For $Kn > 0.1$ (Eq. 2.5), for which $d_p$ approaches the molecular free path $\lambda_g$, the surrounding fluid can not be treated as a continuum and no-slip is no longer correct: a slip between particle and fluid decreases the drag, which can be accounted for in the Cunningham correction factor. The drag term (Eq. 2.7) can be corrected for these effects through a correction on $\tau_p$ [8, 18, 20, 23, 36] (see Table 2.1).

**Table 2.1.** Corrections for non-Stokesian behavior [18].

<table>
<thead>
<tr>
<th>Condition</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 &lt; Re_p &lt; 1000$</td>
<td>$\tau'_p = \tau_p \left(1 + 0.15 Re_p^{0.687}\right)^{-1}$</td>
</tr>
<tr>
<td>$0.1 &lt; Kn &lt; 0.5$</td>
<td>$\tau'_p = \tau_p \left(1 + 1.26 Kn\right)$</td>
</tr>
</tbody>
</table>

### 2.2 Inertial separation efficiency

If droplets or particles are separated from a flowing medium by the centrifugal force, we speak of inertial separation. Since the droplets find themselves in a rotating frame of reference, it is convenient to use a cylindrical coordinate system $(r, \theta, z)$. The acceleration vector of a centrifugal field $\vec{a} = \left(\frac{v_\theta^2}{r}, 0, 0\right)$. For the three coordinates, Eq. (2.10) can then be written as

\[
\begin{align*}
\frac{dr}{dt} &= v_r + \tau_p \gamma \left(\frac{v_\theta^2}{r}\right) \\
\frac{r d\theta}{dt} &= v_\theta \\
\frac{dz}{dt} &= v_z
\end{align*}
\]  

(2.11)  
(2.12)  
(2.13)

In section 1.2 we introduced three types of demisters (see Figure 1.3). They are based on the principle of inertial separation in three basic geometric configurations:

- **A bend**, in which the centrifugal force is induced by forcing the gas to change direction (Figure 2.2). This is the basis of vane packs.
- **In axial cyclones**, droplets are centrifuged outwards in the cylindrical (or annular) space downstream of a swirl generator (Figure 2.3).
- **In a rotating element** the gas is led through **channels**, rotating parallel to the rotation axis (Figure 2.4). The distance to the rotation axis is far as compared to the inner (radial) channel height.

In each configuration, we defined the components of fluid velocity in Table 2.2. Using these as boundary conditions, we combined Eqs. (2.11)–(2.13) into a single *spatial* equation of motion, valid in a rotating frame of reference. Because some of the
10 Basic principles of inertial separation

Figure 2.2. Droplet separation in a bend ($v_\theta$ = constant).

Figure 2.3. Droplet separation in a cyclone ($v_z$, $v_\theta$ = constant, $r_i = 0$).

Figure 2.4. Droplet separation in a rotating channel ($v_z$ = constant).
velocity components can still be a function of $r$, they are on the left hand side of the equations of motion. In the third column we disregarded the radial variation of $v_\theta$ inside the channel as the channel is small in the radial direction. $\Omega$ is the angular velocity of channels, $R$ is used for the center distance to the rotation axis.

Table 2.2. Spatial equations of motion.

<table>
<thead>
<tr>
<th>Bend</th>
<th>Cyclone</th>
<th>Channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_r = 0$</td>
<td>$v_r = 0$</td>
<td>$v_r = 0$</td>
</tr>
<tr>
<td>$v_\theta = v_\theta (r)$</td>
<td>$v_\theta = v_\theta (r)$</td>
<td>$v_\theta = \Omega R$</td>
</tr>
<tr>
<td>$v_z = 0$</td>
<td>$v_z = v_z (r)$</td>
<td>$v_z = v_z (r)$</td>
</tr>
</tbody>
</table>

$(1/v_\theta) \, dr = \tau_p \gamma \, d\theta$ \quad $(v_z/v_\theta^2) \, rdr = \tau_p \gamma \, dz$ \quad $(1/v_z) \, dr = \tau_p \gamma \, \Omega^2 R \, dz$

The simplest assumption is to assume that each component of the fluid velocity is a constant. Practically this means that we assume plug flows, and in addition a constant swirl ratio $S = v_\theta/v_z$ for the cyclone. Using these simplifications, we derived the separation efficiency in Table 2.3.

The fraction of droplets that reaches the wall defines the separation efficiency $\eta$. A droplet’s initial radial position (at the inlet) determines whether it will be collected or not (see Figs. 2.2–2.4). The critical radius $r_c$ is the initial radial position for which a droplet will just reach the wall at the end of the separation space ($\theta = \varphi$ or $z = L$). The corresponding critical path is found by integration (Table 2.3). If droplets enter at $r > r_c$ they are collected, else they are lost.

Table 2.3. Separation efficiency for plug flow.

<table>
<thead>
<tr>
<th>Bend</th>
<th>Cyclone</th>
<th>Channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S = v_\theta/v_z$</td>
<td>$U_T = \tau_p \gamma , \Omega^2 R$</td>
<td></td>
</tr>
<tr>
<td>$dr = \tau_p \gamma , v_\theta , d\theta$</td>
<td>$rdr = \tau_p \gamma , (S^2 v_z) , dz$</td>
<td>$dr = (U_T/v_z) , dz$</td>
</tr>
<tr>
<td>$r_o - r_c = \tau_p \gamma , v_\theta , \varphi$</td>
<td>$r_o^2 - r_c^2 = 2 \tau_p \gamma , (S^2 v_z) , L$</td>
<td>$r_o - r_c = (U_T/v_z) , L$</td>
</tr>
<tr>
<td>$\eta = (r_o - r_c)/r_o - r_i$</td>
<td>$\eta = (r_o^2 - r_c^2)/r_o^2 - r_i^2$</td>
<td>$\eta = (r_o - r_c)/r_o - r_i$</td>
</tr>
</tbody>
</table>

Commonly, the droplet size that is separated for 50% is used as the typically separated droplet size [20, 40]. This droplet is referred to as $d_{p50}$ or cut-size. Setting $\eta = 0.5$, substituting $\tau_p$ and $\gamma$ (see Eqs. 2.8 and 2.9) and solving for $d_p$ yields the expressions for $d_{p50}$ listed in Table 2.4.
Table 2.4. Values of \( d_{p50} \).

<table>
<thead>
<tr>
<th></th>
<th>Bend</th>
<th>Cyclone</th>
<th>Channel</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \sqrt{\frac{9\mu_g (r_o - r_i)}{(\rho_p - \rho_g) v_\theta \varphi}} )</td>
<td>( \sqrt{\frac{9\mu_g (r_o^2 - r_i^2)}{2(\rho_p - \rho_g) S^2 v_z L}} )</td>
<td>( \sqrt{\frac{9\mu_g v_z (r_o - r_i)}{(\rho_p - \rho_g) \Omega^2 R L}} )</td>
</tr>
</tbody>
</table>

Next, we define a *dimensionless* particle size \( x \) as follows

\[
x = \frac{d_p}{d_{p50}}
\]

(2.14)

The separation efficiency, for each of the three configurations (Table 2.3), now shares the same function of this dimensionless droplet size

\[
\eta = \begin{cases} 
\frac{1}{2}x^2 & x \leq \sqrt{2} \\
1 & x \geq \sqrt{2} 
\end{cases}
\]

(2.15a)

(2.15b)

Eq. (2.15b) refers to droplets that are fully separated as they travel the maximum radial distance. The result is equally valid if droplets are lighter than the surrounding fluid (i.e. oil-water separation). In that case \( \gamma < 0 \) and droplets coming in at \( r < r_c \) are separated (a few changes in Table 2.3).

### 2.3 Demisting internals

In the natural gas processing industry, demisters are integrated in gas/liquid separator vessels, called scrubbers. The sections that take care of mist removal (Fig. 1.3) are called demisting internals. In a vessel with radius \( R_v \), the mean gas velocity in the internals can be defined as

\[
v_m = \frac{Q}{\lambda \pi R_v^2}
\]

(2.16)

with \( Q \) total volume flow and \( \lambda \) the *effective* area fraction. Table 2.5 defines \( \lambda \) for the three types of demisters introduced in section 1.2. We used \( \varepsilon \) to indicate blind area occupied by plating material. \( N \) stands for the number of cyclones, \( R_i \) and \( R_o \) are inner and outer radius of the rotating element. Further, \( h = (r_o - r_i) \) and in the cyclones we took \( r_i = 0 \).

Common values for \( \lambda \) with a cyclone deck or rotating element are 25–30%. A rotating element made of winded corrugated sheets \([9]\) gives \( \varepsilon \approx 12\% \). An element of wax fastened tubes yields \( \varepsilon \approx 30\% \) because of some extra loss of area to interspacing* and irregular packing.

\*Densest packing of circles gives \( \varepsilon = \left(1 - \frac{\pi}{2\sqrt{3}}\right) \approx 9.31\% \).
### Table 2.5. Values of $d_{p50}$ for demisting internals.

<table>
<thead>
<tr>
<th>Vane pack</th>
<th>Cyclone deck</th>
<th>Rotating element</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Vane Pack Diagram" /></td>
<td><img src="image" alt="Cyclone Deck Diagram" /></td>
<td><img src="image" alt="Rotating Element Diagram" /></td>
</tr>
</tbody>
</table>

$$
\lambda = (1 - \varepsilon) \\
\frac{d_{p50}}{\sqrt{\frac{9 \mu_g h}{(\rho_p - \rho_g) v_m N \varphi}}} = \sqrt{\frac{9 \mu_g r_o^2}{2 (\rho_p - \rho_g) S^2 v_m L}} = \sqrt{\frac{9 \mu_g v_m h}{(\rho_p - \rho_g) \Omega^2 R_e L}}
$$

Ideally, the separation efficiency is constant within a demister: each vane, cyclone or channel has the same $d_{p50}$. In that case Eq. (2.15) can be also be used for a complete demister, using the mean gas velocity $v_m$ (2.16). Inside a rotating element, separation efficiency is only constant if channel velocity $v_z$ is proportional to the channels’ radial position: $v_z = v_m (R/R_e)$ with $R_e$ the radial position of a channel in which the gas velocity $v_z$ equals the mean velocity $v_m$

$$R_e = \frac{2}{3} \left( \frac{R_o^3 - R_i^3}{R_o^2 - R_i^2} \right) \quad (2.17)$$

For the ideal case of constant separation efficiency, we defined $d_{p50}$ for our three types of demisters (see Table 2.5). Because $v_m$ is used as the gas velocity, the channel located at $R_e$ should be taken for the $d_{p50}$ of a rotating element. In Table 2.5, $N$ is used for the number of sequential vanes as well as for the number of cyclones.

While $d_{p50}$ is commonly adopted as the typically separated droplet size [20, 40], the gas capacity of scrubber vessels is specified by the so-called $K$-value or gas load factor (GLF). After correcting for pressure (gas density), the Souders-Brown equation gives the maximum allowable gas velocity $V$ in the vessel [16]:

$$V = K \sqrt{\frac{\rho_p - \rho_g}{\rho_g}} \quad (2.18)$$

Thus, for a given volume flow and pressure, the $K$-value tells what the vessel’s footprint should be. Today’s cyclonic scrubbers have $K$-values ranging from 0.1–0.3 m/s (see Fig. 2.5). Since the rotating element is a newcomer in natural gas processing, its gas load factor has not been established yet. However, we can relate the $K$-value to $d_{p50}$ theoretically by using $\lambda = V/v_m$. Figure 2.5 shows this for typical element
14 Basic principles of inertial separation

Figure 2.5. K-values with a fixed-speed rotating element: \( h = 4 \) mm, \( \lambda = 0.3 \), \( L = 2R_o \), \( R_i = 0.4R_o \), \( v_0(R_o) = 35 \) m/s (\( \Omega = 35/R_o \) rad/s), \( \mu = 10^{-5} \) Pa s, \( \rho_p = 1000 \) kg/m\(^3\). Red: methane at -60 °C (ideal gas, \( M = 16.04 \) g/mol). Blue: air (\( \rho_\text{g} = 1.2 \) kg/m\(^3\)). Dashed: Stokes regime. Solid: corrected for non-Stokesian behavior (see Table 2.1).

dimensions. At ordinary K-values (up to 0.3 m/s), the rotating element separates very fine mist droplets around 1 micron. Separating 5–10 micron droplets, the element allows K-values up to 10 m/s, implying a very compact separator. We further see that K-values are underestimated in an atmospheric air/water equivalent.

2.4 Overall efficiency

So far we have defined the separation efficiency as a function of (dimensionless) droplet size, that is, for monodisperse droplets. However, the input to practical separators is always polydisperse. In the end it is the overall efficiency of all droplets together that counts. For a polydisperse mist with known cumulative distribution \( F(d_p) \), the overall efficiency can be derived as

\[
\eta_\infty = \int_0^\infty \eta f \, dd_p
\]

(2.19)

with \( f(d_p) = dF/dd_p \) the mass or volume based probability density function. Mist distributions often resemble the lognormal distribution [18]

\[
\frac{dF}{d \ln d_p} = \frac{1}{\sigma \sqrt{2\pi}} \exp \left( -\frac{(\ln d_p - \mu)^2}{2\sigma^2} \right)
\]

(2.20)

where \( \mu \) and \( \sigma \) are mean and standard deviation of the natural logarithm of droplet size (\( \ln d_p \)). The properties of a lognormal distribution are more readily treated using
the geometric mean $\mu_g$ and the geometric standard deviation $\sigma_g$ (GSD):

$$\mu_g = e^\mu$$  \hspace{1cm} (2.21)

$$\sigma_g = e^\sigma$$  \hspace{1cm} (2.22)

The geometric mean $\mu_g$ of a lognormal distribution is equal to its mass (or volume) median diameter (MMD). Having efficiency defined as a function of dimensionless droplet size $x = d_p/d_{p50}$, it is convenient now to make MMD ($\mu_g$) dimensionless [7]

$$\bar{x} = \frac{\mu_g}{d_{p50}}$$  \hspace{1cm} (2.23)

and to write the lognormal distribution (2.20) as

$$\frac{dF}{dx} = \frac{1}{x \sigma \sqrt{2\pi}} \exp \left( - \frac{\ln^2 (x/\bar{x})}{2\sigma^2} \right)$$  \hspace{1cm} (2.24)

Eq. (2.19) can then be integrated over $x$ according to

$$\eta_\infty = \int_0^\infty \eta \left( \frac{dF}{dx} \right) dx$$  \hspace{1cm} (2.25)

For the basic expression (2.15), this yields

$$\eta_\infty (\bar{x}, \sigma) = \sum_i a \bar{x}^n \frac{1}{2} \exp \left( \frac{1}{2} (n\sigma)^2 \right) \left( - \text{erf}(y_1) + \text{erf}(y_2) \right)$$  \hspace{1cm} (2.26)

with coefficients as defined in Table 2.6 and where $y_{1,2}$ is related to $x_{1,2}$ through

$$y_{1,2} = \frac{\ln \left( x_{1,2} / \bar{x} \right) - n\sigma^2}{\sigma \sqrt{2}}$$  \hspace{1cm} (2.27)

We will now apply Eq. (2.26) in a practical example. Recently, Bansal [3, 4] measured droplet size distributions of condensing CO$_2$ droplets in CH$_4$/CO$_2$ mixtures. Cooling occurred by isenthalpic expansion in a Joule-Thomson valve. We fitted one typical measurement (21 mole% CO$_2$) to a lognormal distribution (Figure 2.6). Based on the MMD and GSD of the fit [18], we calculated the overall efficiency by Eq. (2.26). Figure 2.6 shows both the distribution and the overall efficiency in one plot. We see that a $d_{p50}$ of at most 1 micron is required to remove all mist droplets from the gas. As the $d_{p50}$ of practical cyclones ranges from 6–10 micron (see Ch. 4), one needs more efficient demisting equipment. To remove condensing CO$_2$ droplets, a rotating element is a good solution [43, 46].

### Table 2.6. Coefficients in (2.26), based on (2.15).

<table>
<thead>
<tr>
<th>$i$</th>
<th>$a$</th>
<th>$n$</th>
<th>$x_1$</th>
<th>$x_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\frac{1}{2}$</td>
<td>2</td>
<td>0</td>
<td>$\sqrt{2}$</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0</td>
<td>$\sqrt{2}$</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>
2.5 The rotating element

In this section we focus on the rotating element, comprising a bundle of channels that extends from an inner radius \( R_i \) to an outer radius \( R_o \). In section 2.3 we assumed that the separation efficiency in a rotating element is constant. We mentioned that this is only possible if the velocity \( v_z \) in the channels is proportional their center distance \( R \) to the rotation axis. To obtain \( d_{p50} \) for the element, we took the channel at \( R_e \), in which the gas velocity equals the mean velocity in the element. We will now look at non-ideal inflow distributions, and in specific a uniform flow distribution.

Weighing the channel efficiency \( \eta \) for the local channel velocity \( v_z \), the element efficiency \( \vartheta \) can be obtained for an arbitrary flow distribution \( v_z (R) \) [8]

\[
\vartheta = \frac{1}{Q} \int_{R_i}^{R_o} \eta v_z (1 - \varepsilon)^2 \pi R dR
\]  

(2.28)

with \( Q \) the total volume flow and \( \varepsilon \) the element’s blind area fraction, covered by the channel walls. Mean velocity through the element is defined as

\[
v_m = \frac{Q}{(1 - \varepsilon) \pi \left( R_o^2 - R_i^2 \right)}
\]  

(2.29)

Plugging in (2.15a) into (2.28), it appears that for any flow distribution the efficiency is equal to that of an equivalent channel, which is flown through with the element’s mean velocity \( v_m \) and which is located at radial position

\[
R_e = \frac{2}{3} \left( \frac{R_o^3 - R_i^3}{R_o^2 - R_i^2} \right)
\]  

(2.30)
2.5 The rotating element

Limiting condition is that (2.15a) is valid in all the channels. As soon as the efficiency reaches 100% (2.15b) in a part of the element, its efficiency will start to deviate from the equivalent channel. Taking the equivalent channel as basis, we defined the element cut-size as \( d_{p50}(v_m, R_e) \) in Table 2.5. A natural choice for the dimensionless particle diameter of an element is then \( X = \frac{d_p}{d_{p50}(v_m, R_e)} \) (2.31)

Defining dimensionless parameters according to

\[
v_z^* \quad R^* = \frac{R}{R_o} \quad R_e^* = \frac{2}{3} \left[ 1 - \frac{R_i^{*3}}{1 - R_i^{*2}} \right]
\]

we can rewrite Equation (2.28) as

\[
\vartheta = \frac{2}{1 - R_i^{*2}} \int_{R_i^*}^{1} \eta v_z^* R^* dR^*
\]

(2.33)

If course we want to have the element efficiency defined as \( \vartheta = \vartheta(X) \). However, the channel efficiency (2.15) is written as \( \eta = \eta(x) \). \( X \) relates to \( x \) as

\[
x = X \sqrt{\frac{1}{v_z^*} \frac{R^*}{R_e^*}}
\]

(2.34)

Ideally, all channels have the same efficiency. As we saw before, this happens when the gas velocity is proportional to the radius: \( v_z^* = R^*/R_e^* \) \( (x = X) \). The equivalent channel then indicates which channel has \( v_z = v_m \). Analogous to (2.15) we have

\[
\vartheta = \begin{cases} 
\frac{1}{2} X^2 & X \leq \sqrt{2} \\
1 & X \geq \sqrt{2}
\end{cases}
\]

(2.35a, 2.35b)

In practice, the rotating element acts as a flow straightener. The larger the pressure drop, the more uniform the flow distribution. We now consider a fully \textbf{uniform} flow distribution: \( v_z^* = 1 \). Using (2.34), the channel efficiency (2.15) in case of uniform inflow is as follows

\[
\eta = \begin{cases} 
\frac{1}{2} X^2 (R^*/R_e^*) & R^* \leq 2 R_e^*/X^2 \\
1 & R^* \geq 2 R_e^*/X^2
\end{cases}
\]

(2.36a, 2.36b)

The channel efficiency starts off proportional to the radius (2.36a), until it reaches a plateau (2.36b). As long as the plateau starts beyond \( R_o \), case (2.36a) holds for the complete element. As soon as it starts at \( R_i \), the efficiency is 100% in all channels (2.36b). Substituting (2.36) into (2.33), one obtains the element efficiency for uniform inflow conditions

\[
\vartheta = \begin{cases} 
\frac{1}{2} X^2 & X \leq X_o \\
\left(1 - R_i^{*2}\right)^{-1} \left( 1 - \frac{4}{3} R_e^* X^{-4} - \frac{1}{3} R_i^{*3} R_e^{-1} X^2 \right) & X_o \leq X \leq X_i \\
1 & X \geq X_i
\end{cases}
\]

(2.37a, 2.37b, 2.37c)
with $X_o = \sqrt{2R_c^*}$ and $X_i = \sqrt{2R_c^*/R_i^*}$. For an element which extends all the way to the rotation axis one has $R_i^* = 0$ and $R_c^* = 2/3$ so that (2.37) reduces to

$$\vartheta = \begin{cases} \frac{1}{2} X^2 & X \leq \sqrt{4/3} \\ 1 - \frac{16}{27} X^{-4} & X \geq \sqrt{4/3} \end{cases}$$

(2.38a) (2.38b)

Figure 2.7a shows the element efficiency $\vartheta(X)$ for the two different flow distributions in case $R_i^* = 0$. Figure 2.7b illustrates, for the large dots (●) in Fig. 2.7a, how the channel efficiency depends on radial position within the element. By definition, the ideal curve (solid line) in Fig. 2.7a is also the efficiency of the equivalent channel. As from $\eta = 66.7\%$ the element efficiency with uniform flow (dashed curve) starts to deviate from the equivalent channel, because $\eta$ reaches 100% in the outer channels (Eq. 2.36b). The latter can be clearly seen in Fig. 2.7b. Lastly, with a uniform inflow the element never separates for 100% due to a lack of centrifugal force at $R = 0$.

![Graph](image-url)

(a) Element efficiency for two different flow distributions according to (2.35) and (2.38).

(b) Channel efficiency in case of a uniform flow distribution, see Eq. (2.36).

**Figure 2.7.** Separation efficiency of a rotating element ($R_i^* = 0$).

In section 5.2.3 we derived an expression for the overall efficiency with a lognormal input distribution (Eq. 2.26). The coefficients in this expression, as listed Table 2.6, were based on (2.15). Hence, they are only valid for rotating elements at ideal inflow conditions (Eq. 2.35). Table 2.7 gives the coefficients in case of uniform inflow (based on Eq. 2.37). We defined overall efficiency as a function of the rather abstract parameter $\varpi = \text{MMD}/d_{50}$, denoting the dimensionless mass (or volume) median diameter (MMD) of the input distribution. For rotating elements, $\varpi$ can be instructively viewed as a dimensionless rotation speed

$$\varpi = \frac{\Omega}{\Omega_0}$$

(2.39)

based on a nominal rotation speed

$$\Omega_0 = \frac{1}{\text{MMD}} \sqrt{\frac{9\mu g v_m h}{(\rho_p - \rho_g) R_c L}}$$

(2.40)
2.5 The rotating element

Figure 2.8. Overall efficiency as a function of dimensionless rotation speed for uniform inflow and with $R^* = R_o/R_i = 0.5$ (Eqs. 2.26–2.27 & Table 2.7, see m-file in Appendix B).

Figure 2.8 shows the overall efficiency of a rotating element with uniform inflow for varying geometric standard deviation (GSD). The case GSD = 1 ($\sigma = 0$) corresponds to a monodisperse distribution for which (2.26) reduces to (2.37). Clearly, the wider the distribution (larger GSD) the harder it gets to remove all of the mist from the gas stream. For GSD = 2, which is roughly the geometric standard deviation of condensing CO$_2$ droplets (see Fig. 2.6), it takes about 5 to 6 times the nominal speed to remove all mist. For convenience, we included a piece of Matlab code which evaluates Eq. (2.26) in Appendix B.

<table>
<thead>
<tr>
<th>i</th>
<th>a</th>
<th>n</th>
<th>$x_1$</th>
<th>$x_2$</th>
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</thead>
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<td>2</td>
<td>0</td>
<td>$X_o$</td>
</tr>
<tr>
<td>2</td>
<td>$(1 - R^*_i^2)^{-1}$</td>
<td>0</td>
<td>$X_o$</td>
<td>$X_i$</td>
</tr>
<tr>
<td>3</td>
<td>$-\frac{4}{3} R^<em>_e R^</em>_i (1 - R^*_i^2)^{-1}$</td>
<td>-4</td>
<td>$X_o$</td>
<td>$X_i$</td>
</tr>
<tr>
<td>4</td>
<td>$-\frac{1}{3} R^<em>_e^3 R^</em>_i^{-1} (1 - R^*_i^2)^{-1}$</td>
<td>2</td>
<td>$X_o$</td>
<td>$X_i$</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>0</td>
<td>$X_i$</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>
2.6 Closure

Based on – among other things – the simplifying assumption of plug flow, we derived a general expression for inertial separation efficiency as a function of dimensionless droplet size (Eq. 2.15). Dropping individual assumptions, this model is refined in later chapters. In chapter 4 we introduce a cyclone model based on Rankine vortex profiles, and we improve the definition of the swirl ratio \( S \). Chapters 5 and 6 focus on rotating channels. In chapter 5 we discuss the effect of Poiseuille type velocity profiles and in chapter 6 we are concerned with turbulent flow, in which plug flow is all right but mixing (dispersion) can play a role. Till now we assumed an invariant channel height, which is only true for rectangular channels. Later on we also look at tubes, in which the channel height varies internally.

Subsequently, we used the model to characterize the separation efficiency of industrial gas/liquid scrubber vessels, equipped with internals for demisting. We also estimated potential \( K \)-values if a rotating element is used (Fig. 2.5). With regard to the latter, we emphasize that actual \( K \)-values should be based on field tests at real conditions. One should also realize that, in design choices, pressure drop and reentrainment issues have to be considered as well. Van Wissen et al. [40] compared rotating element and cyclone, taking also the pressure drop into account.

We also looked in more detail to rotating elements. We saw that the inflow distribution only affects the upper part of the separation efficiency curve (Fig. 2.7a). Based on the concept of the ‘equivalent channel’, we can state that in case of an ideal flow distribution \( v_z = v_m (R/R_e) \) any expression for the channel efficiency is also valid for an element (in that case \( x = X \), see section 2.5). For other distributions the element’s efficiency can be calculated by Eqs. (2.33) and (2.34).

Assuming lognormal input distributions, we derived an additional expression for the overall efficiency (Eq. 2.26). For condensing CO\(_2\) droplets, which give a nearly lognormal distribution, we illustrated that the expression can be useful as a first indicator of separator performance (Fig. 2.6). We saw that the cut-size \( d_{50} \) would have to be 5–6 times smaller than the mass median diameter (MMD) to remove the full droplet distribution of condensing carbon dioxide.
Experimental setup

We built an air/water test setup, operating at near atmospheric conditions, to test a scaled-up rotating phase separator (refer to section 1.3). The setup, which models an 80 MMSCFD* natural gas installation (see Table 3.1), covers two aspects:

**Liquid capacity:** To simulate the large CO₂ liquid loading extracted from a heavily contaminated gas well, a large amount of water is sprayed into the rotating element. Behavior and removal of the liquid can be observed through windows in the separator casing.

**Separation efficiency:** The separation efficiency is determined separately with a smaller loading. Mist nozzles inject micron sized droplets into the air inlet pipe. By means of laser diffraction particle sizing, the droplets’ volume concentration and size distribution is measured in the air stream leaving the unit.

As argued in section 1.3, the change in flow regime is an important aspect of upscaling. To simulate turbulent flow conditions in our atmospheric setup, we constructed a second element with enlarged channels.

Section 3.1 gives a detailed description of the prototype, section 3.2 describes the setup and section 3.3 explains the method to determine separation efficiency. Further, section 3.4 goes shortly into some aspects of liquid drainage.

<table>
<thead>
<tr>
<th>Table 3.1. Our laboratory conditions, next to a contaminated natural gas well.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gas</strong></td>
</tr>
<tr>
<td>Contaminant</td>
</tr>
<tr>
<td>Contaminant</td>
</tr>
<tr>
<td>Pressure</td>
</tr>
<tr>
<td>Temperature</td>
</tr>
<tr>
<td>Product gas flow rate</td>
</tr>
<tr>
<td>Liquid waste loading</td>
</tr>
</tbody>
</table>

*Million standard cubic feet per day
3.1 Prototype

A scaled-up RPS has been designed for high pressure and semi-cryogenic temperatures (Table 3.1). Special materials are needed for corrosion protection and hermetic sealing is taken care of by magnetic bearings and a magnetic coupling [43]. For this thesis we used a downgraded, full scale version to test with air and water at atmospheric pressure and room temperature. Flanges and casing walls are thinner and made out of ordinary stainless steel. We further used ball bearings and common rubber seals. A unique feature of the test unit is that it has 6 PETG windows all around which allow to see what is going on inside. The unit is schematically depicted in Figure 3.1. Appendix A.1 shows a photograph and a construction drawing.

![Figure 3.1. Schematic of the scaled-up prototype (gray components rotate).](image)

**Pre-separator:** Gas with mist enters the unit via a tangential inlet. Coarse droplets (> 10 μm) are separated in a pre-separator section, which acts as a cyclone. This liquid collects in the upper volute, which, at the same time, is used as a housing for the rotating element.

**Rotating element:** The gas stream, containing the remaining mist of mainly 1–20 micron droplets, enters the rotating element, which acts as a droplet coalescer. While traveling in the axial direction, the centrifugal force drives droplets to the channel walls, where they coagulate into a thin film (see Figure 1.1). For optimal film behavior and minimal pressure drop the flow direction through the element is downwards [44]. Due to gravity and shear forces, the film breaks up into droplets of typically 50 μm at the end of the channels.
Post-separator: At that moment we enter the post-separator section, where the liquid is actually separated from the gas stream. Droplets that break up (A) are centrifuged outwards (B) in a solid body field, which is enforced by blades. The liquid collects (C) on an extension of the element outer wall, from where it drains away (D) to the lower stationary volute. The water in the bucket (E) has a curved surface due to the large angular momentum passed on by the rotating element. An intrusion (F) keeps the liquid separated from the product gas flow and prevents re-entrainment by splashing in the volute.

Since a large water load is sprayed axially onto the rotating element (see section 3.2), the water misses the angular momentum that is normally brought along through the tangential inlet. As a consequence the unit is equipped with a relatively large, 15 kW electric motor. A frequency converter gives full control over the shaft speed.

The liquid leaves via pipes connected tangentially to the two volutes. One needs large diameter pipes because it is not possible to build up pressure like in a pump housing. The pipes are partially filled with liquid and also serve as a gas back-vent.

To realize different types of flow regimes within the rotating channels, we needed two different channel sizes (see Table 3.2). Aside from the standard element A, as would be used for natural gas, we constructed a second element B with larger channels. To preserve the channels’ $L/h$ ratio, element B is much longer (70 cm).

Element A is made according to the manufacturing process of corrugated paper [9]. Two foils, of which one is corrugated, are wound up and spot welded around an axis. Corrugation gives the channels a trapezoidal, nearly rectangular shape. Element B is wax fastened: a bundle of loose tubes (circular channel shape) is molded at both ends in resin, hardened and trued up afterwards on a lathe.

We constructed the axis-element assembly in a modular way in order for any newly constructed element to be easily mountable to the axis. Even blades and extended collecting wall of the element are removable. To fit in element B, we had to lengthen shaft and casing of the original unit (Appendix A.1 shows the elongated unit). The gap between rotating element and its housing is filled by a mechanical seal.

| Table 3.2. Dimensions (mm) of both rotating elements. |
|---------------------------------|---------|---------|
| Channel dimensions              | A       | B       |
| $1.9 \times 2.1$ ($h \times b$) | 19.0    | 6.6     |
| Length $L$                      | 152.5   | 700     |
| Inner diameter $2R_i$           | 180     | 168.3   |
| Outer diameter $2R_o$           | 340     | 350     |
| Blind fraction $\varepsilon$    | 12.1%   | 30.5%   |
3.2 Test rig

We connected the prototype (see section 3.1) to an atmospheric test setup, using air and water as working fluids. Table 3.1 shows that the setup is based on a natural gas feed of 80 MMscfd (24 m³/s), contaminated with 30 mole% of CO₂. Expansion of such a gas mixture to 20 bar and -60 °C would lead to a methane rich gas flow of 0.45 m³/s (actual) and a CO₂ rich liquid flow rate of 7.5 l/s in the form of micron sized condensate droplets that are entrained in the gas stream. On our test rig similar volume flows of air (up to 0.5 m³/s) and water (up to 9 l/s) can be provided to the RPS unit. But because we could not achieve a high liquid loading with micron sized water droplets, coarse spray nozzles were utilized to simulate the effect of high loadings on liquid removal in the post-separator volute. The separation efficiency is measured separately using a smaller amount of micron sized droplets.

Figure 3.2 shows a schematic of the experimental setup. For a more detailed P&I diagram refer to Appendix A.2. Details on the components are given below:

**Air supply:** Pressurized air from an 8 bar supply expands over a globe valve to approximately 1 bar. In this way an actual flow rate up to 0.5 m³/s can be provided to the RPS unit. The valve is actuated via pressurized air and operated manually. A Coriolis-type mass flow meter is used to measure the mass flow. With air temperatures ranging from 18-22 °C (density ρₐ = 1.2 ± 0.75%), the volume flow rate can be controlled within about ±2%.

**Water supply – coarse spray:** A large storage tank (capacity 1 m³) is filled with ordinary tap water. A 5.5 kW multistage centrifugal pump provides up to 9 l/s of water to three large spray nozzles, mounted in the top flange (see Figure 3.3),

![Figure 3.2. Schematic of the test rig](image-url)
which spray the large water load directly onto the RPS element in the axial
direction. Produced droplet sizes are in the millimeter range. The volume flow
is monitored by a variable area (VA) flow meter, and can be adjusted by a
frequency converter controlling rotational speed of the pump motor.

Figure 3.3. Close-up of the Eindhoven lab unit

Water supply – fine mist: Demineralized water from another, smaller storage tank
is pressurized by a plunger pump. A pressure reducing control valve keeps the
pressure in the upstream line at 70 bar, bypassing the surplus of water back to
the tank. Mist is generated in a mechanical way by 28 so-called pin jet nozzles
(manufactured by Bete, type PJ6), and injected in the airflow entering the RPS
unit. These pin jet nozzles force water through an orifice of 152 µm, and the
resulting jet breaks up against a metal pin that is situated right in front of the
orifice. A substantial amount of the generated droplets is in the micron-range.
From the specifications of the manufacturer it can be estimated that, at 70 bar,
water is injected at a rate of approximately 3.2 l/min. To prevent blockage of
the orifice, demineralized water was used.

Water discharge: Water from the post-separator is discharged directly into the
main storage tank, whereas the pre-separator discharge ends as a dip pipe in
a secondary tank. The dip pipe acts as a siphon water lock to force the entire
airstream to flow through the RPS element. The flowrate $Q_1$ coming from the
pre-separator (Figure 3.2) can be derived from the rising water level in the sec-
ondary tank. $Q_1$ is subtracted from the feed $Q_0$ to obtain the flowrate coming
from the post-separator $Q_2 = Q_0 - Q_1$. Adequate drainage is ensured by using
large diameter discharge pipes, connected tangentially to the separator casing
and running to the tanks over a short horizontal distance.

3.3 Measurement method

A constant amount of mist is injected into the airstream entering the RPS unit (sec-
tion 3.2). Mist droplets that are not separated in the RPS remain in the airstream
leaving the unit. We measured the volume concentration and size distribution of
the nonseparated droplets in the air outlet (Figure 3.2) by means of laser diffraction
particle sizing (3.3.1) and derived the separation efficiency from it (3.3.2).
3.3.1 Laser diffraction particle sizing

The technique of laser diffraction is based on the principle that particles (droplets) passing through a laser beam scatter light in a way that depends on their size. Large particles scatter at narrow angles with high intensity, whereas small particles scatter at wider angles but with low intensity [27, 28].

MALVERN particle sizers capture the scattered light on a series of detectors. After fitting a spline through the detector signal histogram, the particle size distribution is calculated by comparing the sample’s scattering pattern with an optical model (Mie theory) using a mathematical inversion process. Mie theory uses the refractive index difference between particle and surrounding medium to predict how presumed spherical particles scatter. MALVERN particle sizers measure two quantities:

Size distribution: Configured to ensure equal volumes of droplets (of different sizes) yield a similar measured signal, the instrument reports the distribution as a histogram of volume fractions $\Delta f_i$, distributed over intervals equally spaced on a logarithmic scale. Logarithmic spacing suits the detector arrangement best.

The representative diameter for each interval should be taken as the geometric mean of the size band limits $\sqrt{d_{p,i-1} d_{p,i}}$. The geometric mean is the midsize on a logarithmic scale. Of course the sum of all fractions equals unity $\sum_i \Delta f_i = 1$. Similar to Eq. (2.20), the probability density function (PDF) now should also be defined in fractions per logarithmic size interval $df/d (\ln d_p)$ [18].

When we derive the separation efficiency (see section 3.3.2), we shall stick to the logarithmic spacing. For that reason measurements of the separation efficiency in this thesis are displayed on a logarithmic x-axis.

Concentration: The instrument carries out a separate measurement of the laser beam extinction. The software uses the Beer-Lambert law [18] to calculate the total volume concentration $c_v$ of the spray. In order to do so, it needs the measured size distribution (see above). It also requires an estimate of the laser extinction path length from the user. In this thesis, the concentration $c_v$ is the total volume fraction occupied by water droplets in air.

Note that the total volume concentration is not an independent measurement. Disadvantage is that any error in the measured size distribution will propagate in the $c_v$ value. Advantage on the other hand is that everything is determined in one single measurement. As opposed to counting based techniques [4], laser diffraction particle sizing is volume based. Volume based methods are very sensitive to the appearance of a few large droplets as these have comparatively huge volumes.

In this thesis we make use of two different types of particle sizers. Table 3.3 gives the settings used with both instruments. Three issues must be dealt with carefully:

Vignetting: Light scattered at wide angles (small drops) is cut off from the lens if a sample is far away. The MASTERSIZER has a small lens: even though we located the sample between 25 and 135 mm in front of the lens, it is not possible to
3.3 Measurement method

Table 3.3. Settings of both Malvern particle sizers used.

<table>
<thead>
<tr>
<th></th>
<th>Mastersizer S 300</th>
<th>Spraytec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data acquisition rate (Hz)</td>
<td>500</td>
<td>50</td>
</tr>
<tr>
<td>Background duration (sec)</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Sample duration (sec)</td>
<td>20</td>
<td>5</td>
</tr>
</tbody>
</table>

properly capture the scattering pattern of distributions having a range of 1–20 micron. The main advantage of the Spraytec is its much larger lens: within 150 mm in front of the lens any distribution is measured accurately.

**Beam steering:** Small variations in air density produce an artificial, large signal on the first (small scattering angle) detectors. The higher the velocity of the air passing by, the worse it gets. To solve this we always measured the background scattering signal after setting the required airflow. In this way the noise is taken up in the background signal.

**Lens contamination:** Though it seems to go without saying, it is very important to keep the lens clean. Lacking a purge, we used tissues to do the job.

With the Mastersizer, we let the laser beam pass perpendicularly through open holes (φ 35 mm) in the air outlet pipe (φ 110 mm). The Spraytec was simply placed after the open outflow. The laser extinction path length was approximately the pipe diameter. We further used some cardboard to stop light from outside. After switching on mist injection we waited about 1 minute before starting a measurement. Finally, the RPS unit was flushed at maximum air throughput before each measurement.

### 3.3.2 Determining the efficiency curve

The laser beam of a laser diffraction particle sizer (section 3.3.1) passed through the airstream leaving the RPS unit, where it measured the total volume concentration \(c_v\) and the size distribution of non-separated mist droplets.

In order to exclude pre-separator action from the separation efficiency measurement, we first measured a reference \(0\) with the element standing still. Then we drove up the rotation speed \(\Omega\) to find the separation efficiency due to solely the rotating element. At a fixed gas flow rate its overall efficiency (all droplets sizes together) is then

\[
\eta = 1 - \frac{(c_v)\Omega}{(c_v)_0}
\] (3.1)

Note that this is different from the separation efficiency of the complete unit. Further, as we shall see in section 5.2, even the stationary element itself has a removal effect due to droplet impaction at the channel entrance. However, at sufficiently high rotation speed droplet removal in the channels dominates and any other removal effects are negligible (see section 5.2.1).
To derive the efficiency as a function of the droplet size, we have to compare concentrations $c_v \Delta f_i$ of the individual, monodisperse fractions

$$\eta(d_{p,i}) = 1 - \frac{(c_v \Delta f_i) \Omega}{(c_v \Delta f_i)_0}$$

(3.2)

where subscript 0 refers to the reference $\Omega = 0$ rpm and with $d_{p,i}$ the geometric mean of the limits of interval $i$ (see section 3.3.1).

Figure 3.4 shows an example measurement with an overall efficiency of 74% (Eq. 3.1). The size distributions are shown as droplet concentrations per logarithmic size interval $c_v \frac{df}{d(\ln d_p)}$. These normalized droplet concentrations can be read out on the right ordinate axis. Clearly the right part of the distribution is cut off first, because large droplets are separated more efficiently than small droplets. This is also reflected in the separation efficiency curve (left ordinate axis), obtained according to Eq. (3.2).

For droplet sizes $< 2 \mu m$, the measured efficiency increases with decreasing particle size. Such a ‘fish-hook’ is a fairly common observation [20]. The accuracy for $d_p < 2 \mu m$ is too low to draw meaningful conclusions since the efficiency (Eq. 3.2) is based upon the far left tail of both size distributions. It is virtually impossible for them to match exactly at the start. In the end we are more interested in how the rest of the curve is located. In the remainder of this thesis we therefore omitted results $< 2 \mu m$.

**Figure 3.4.** Example measurement of a separation efficiency curve at airflow 120 g/s (grams per second) and rotation speed 600 rpm (element A). Separation efficiency is indicated on the left vertical axis, normalized droplet volume concentrations are displayed on the right hand side. Volume concentrations of individual droplet fractions $c_v \Delta f_i$ are normalized as follows: for each interval $i$, ranging from $d_{p,i-1}$ to $d_{p,i}$ (in $\mu m$), the value $c_v \Delta f_i / (\ln d_{p,i-1} - \ln d_{p,i})$ is plotted against the geometric mean $\sqrt{d_{p,i-1} d_{p,i}}$.
The experiments on liquid drainage do not fall within the scope of this thesis. A separate publication shows that the large collection volutes in combination with the large diameter, tangentially connected liquid outlets (see section 3.1) enable the RPS of handling large liquid loadings [45]. Apart from this we have been asking ourselves the question whether large amounts of separated liquid CO$_2$ will be susceptible to foaming due to its low surface tension. To that end we did a number of experiments in an autoclave in a laboratory within Shell Exploration & Production, Rijswijk.

We cooled CH$_4$/CO$_2$ mixtures in the autoclave. Stirring the condensed CO$_2$ (which always has some CH$_4$ dissolved in it) heavily, we did not observe any signs of foaming. Some further investigation revealed that, although foam formation by shearing indeed is easier with a low surface tension, an additional factor is absolutely necessary: foam stability. This can only be obtained with a suitable surfactant. Repulsion between heads of the surfactant on opposite interfaces of a liquid film have to prevent the foam film from collapsing. However, surfactant-like substances are not present in contaminated natural gas and if even if they would, CRS is a ‘clean’, once-through process: trace contaminants do not accumulate like in the recirculating solvents of amine treaters. Foaming is therefore no longer regarded an issue of concern.

As mentioned in section 1.4, the focus of this thesis is on droplet collection efficiency. Chapter 4 discusses vane pack and axial cyclone efficiency. Separation efficiency of elements A and B (Table 3.2) is covered in chapters 5 and 6 respectively.
Chapter 4

Vanes and axial cyclones

In this chapter we validate the separation efficiency of vane packs and axial cyclones by experiments. Applying the laser diffraction method of Chapter 3, we tested commercial separators of the above types in smaller, dedicated setups. We also refined the universal model of section 2.2 for axial cyclones.

Questions about the radial profile of tangential velocity in axial cyclones have been answered in a recent paper [21]. Velocity field measurements with a neutrally buoyant tracer have clearly shown that the flow develops into a solid body core, surrounded by a loss free vortex. However, existing models for the separation efficiency are based on either complete solid body or complete loss free rotation [2].

We propose a new model for the separation efficiency of axial cyclones, based on realistic tangential velocity profiles. We assume plug flow in the axial direction and disregard mixing. For a Rankine vortex profile we derived an explicit expression. The latter can be used to vary the vortex core size between the two extremes of complete solid body rotation and complete loss free rotation.

Another open question is how to relate the vortex strength to the geometry of the swirl generator in an unambiguous way. We suggest to relate the vortex circulation to the tangential velocity within the swirl element. This is consistent with common practice for tangential cyclone inlets [20].

Section 4.1 shortly discusses the vane experiments. In section 4.2 we present the new cyclone model. Section 4.3 describes the cyclone setup and 4.4 discusses the results obtained with it. Section 4.5 gives conclusions and evaluates how to proceed.

4.1 Vane type separators

The very first time that we measured separation efficiency by means of laser diffraction particle sizing was to characterize the efficiency curve of panel filters [10]. The panels, available from DONALDSON, are bend or vane type separators, used to demoisturize the air inlet to gas turbines. Here they are used as a substitute for the vane packs
that are used in natural gas scrubbers. In Table 2.4 we defined for vane packs

\[ d_{p50} = \sqrt{\frac{9\mu_g h}{(\rho_p - \rho_g)v_m N\phi}} \]  

with \( h \) the inner spacing of the vanes, \( N \) the number of sequential vanes and \( \phi \) their angle. Further, \( \rho_p, \rho_g \) and \( \mu_g \) are particle density, gas density and gas dynamic viscosity respectively. The average gas velocity \( v_m \) was defined in Eq. (2.16). Eq. (2.15) predicts the efficiency curve for the ideal case of a constant flow distribution over the panel and plug flow inside the vanes.

The panels were installed in a \( 220 \times 220 \text{ mm} \) square test duct (Figure 4.1), which fits in the bench of a MASTERSIZER S (see section 3.3.1). A downstream fan simulated the suction of a gas turbine. The MASTERSIZER is located 300 mm downstream of the panel outlet. Gas velocity is measured in the middle of the duct by hot wire velocimetry. The separation efficiency is determined by comparing two measurements with and without the panel in place (see also section 3.3.2).

![Figure 4.1. Experimental setup for vane-type mist extractors.](image)

We tested three panel types having slightly different geometries. All panels essentially depend on two bends (\( N = 2, \phi = 90^\circ \)) for the removal of droplets. Each panel was tested at gas velocities of 3 m/s and 5 m/s. Figure 4.2 shows the results, plotted against the dimensionless droplet size \( d_p/d_{p50} \). The curves fall together, which indicates that \( d_{p50} \) (Eq. 4.1) correctly accounts for geometry and velocity. The only exception is panel type 3 at the highest velocity (5 m/s), where re-entrainment occurred due to flooding. The ideal curve (2.15) is followed up to \( \eta \approx 70\% \). Apparently, departure from the ideal assumptions mainly affects the top part of the curve.

## 4.2 Axial cyclone model

To model droplet separation in an axial cyclone, we approach it as a cylindrical space with axisymmetric flow. The point at which the swirl generator vanes start to twist can be taken as \( z = 0 \) (see Figure 4.3).

Due to axisymmetry there is no radial gas motion (\( v_r = 0 \)). Since we have no information on the axial velocity profile in axial cyclones, we also conform to the common
assumption of constant \( v_z \) (plug flow). In section 2.2 we also assumed a constant tangential velocity \( v_\theta \) through a swirl ratio \( S = v_\theta / v_z \). Of course, this is not very realistic if we look at real vortex profiles which feature a solid body core, surrounded by a loss free vortex.

To refine the definition of \( d_{50} \), we assume at first that we are in the vortex periphery, dominated by so-called irrotational or loss free flow [20]

\[
v_\theta = \frac{\Gamma}{2\pi r}
\]

(4.2)

The circulation \( \Gamma \) is a measure for the strength or intensity of a vortex [1]. We assume that the decay is negligible, so that the \( v_\theta \)-profile is independent of axial position \( z \). Because we assume plug flow, particle influx simply goes with cross-sectional area so we can write for the efficiency (as in Table 2.3)

\[
\eta = \frac{R^2 - r_c^2}{R^2}
\]

(4.3)

with \( R = D/2 \) the cyclone radius and \( r_c \) the critical radius for which a droplet just reaches the wall at the end of the cyclone \( z = L \). If a droplet enters outside of the critical radius it will be separated, otherwise not (Fig. 4.3c). The value of \( r_c \) follows from integration of the equation of motion (see Table 2.2)

\[
\int_{r_c}^{R} \left( \frac{r v_z}{v_\theta^2} \right) \, dr = \tau_p \gamma \int_{0}^{L} \, dz
\]

(4.4)

To obtain \( d_{50} \) we set \( \eta = 0.5 \) which gives \( r_c = R/\sqrt{2} \). After substituting \( r_c \) together with (4.2) in (4.4) and solving for the particle size, we have

\[
d_{50} = \frac{\pi R^2}{\Gamma} \sqrt{\frac{27 \mu_g v_z}{2 (\rho_p - \rho_g) L}}
\]

(4.5)
We can now define three dimensionless variables as follows

\[
x = \frac{d_p}{d_{p50}} \quad r^* = \frac{r}{R} \quad v^*_\theta = \frac{v_\theta R}{\Gamma/2\pi}
\]  

(4.6)

In terms of these dimensionless parameters, the cyclone efficiency (4.3) is given by

\[
\eta = 1 - r^*_c^2
\]  

(4.7)

while for the critical radius (4.4) we can write

\[
\int_{r^*_c}^{1} \left( \frac{r^*}{v^*_\theta} \right) dr^* = \frac{3}{16} x^2
\]  

(4.8)

To evaluate (4.8) we need the full radial profile of the tangential velocity \(v_\theta (r)\). Due to viscosity, the core of concentrated vortices approaches solid body rotation. This is reflected in the popular Rankine vortex model [1]

\[
v^*_\theta = \begin{cases} 
  \frac{r^*}{r^*_c}^2 & r^* \leq r^*_c \\
  \frac{1}{r^*} & r^* \geq r^*_c
\end{cases}
\]  

(4.9)

which has a finite solid body core (eye) of radius \(r_c\), surrounded by the free vortex of Eq. (4.2). Substituting (4.9), Eq. (4.8) yields the dimensionless critical droplet position \(r^*_c\) (for \(r^*_c < r^*_c\) the integral must be split in two). Using (4.7), we find the following expression for the efficiency

\[
\eta = \begin{cases} 
  1 - \sqrt{1 - \frac{3}{4} x^2} & x \leq \sqrt{\frac{4}{3} \left(1 - r^*_c^4\right)} \\
  1 - r^*_c^2 \exp \left(\frac{1}{2} \left(1 - \frac{3}{4} x^2\right) / r^*_c^4 - 1\right) & x \geq \sqrt{\frac{4}{3} \left(1 - r^*_c^4\right)}
\end{cases}
\]  

(4.10)
Figure 4.4. Rankine vortex profiles (Eq. 4.9) and corresponding separation efficiency curves (Eq. 4.10) for varying vortex core size $r_e$.

Figure 4.4 shows the relation between vortex core size $r_e$ and separation efficiency. The case $r_e^* = 0$ represents a coreless, free vortex (the ideal case), resulting in a very sharp separation curve. For increasing core radius, the right hand side of the curve falls off. If the core radius $r_e^* = 1/\sqrt{2}$ the efficiency already comes off halfway the ideal curve (at $x = 1$, $\eta = 0.5$). As soon as the vortex core covers the whole cross-section ($r_e^* = 1$), we have complete solid body rotation. This can be seen as the worst case scenario for which $\eta = 1 - \exp\left(-\frac{3}{8}x^2\right)$.

The Rankine vortex is rather simplistic. A Lamb (or Burgers) vortex model features a more smooth tangential velocity profile [1]

$$v^*_\theta = \frac{1}{r^*} \left(1 - \exp\left\{-\left(r^*/r_e^*\right)^2\right\}\right)$$

(4.11)

With this profile it is not possible any more to derive an explicit expression for the efficiency. The integral in Eq. (4.8) must be integrated numerically. It is easiest to use $r_e^*$ as the running variable, after which (4.8) gives $x$ and (4.7) gives $\eta$.

We still have to relate the circulation $\Gamma$ to the swirl element, in which the vortex is generated in the first place. In line with common practice for tangential cyclone inlets [20], we propose to use $\alpha$ as the ratio of angular momentum in the swirl element to vortex circulation $\Gamma$ as follows

$$\Gamma = 2\pi \left(\frac{v_{\theta sw} R_{sw}}{\alpha}\right)$$

(4.12)

with the swirl radius taken in the middle of the vanes

$$R_{sw} = \frac{1}{2} (R + R_{body})$$

(4.13)

Due to the presence of a central body (see Fig. 4.3a), the gas inside the swirl element is accelerated axially by a factor $R^2/(R^2 - R_{body}^2)$. At the same time, the vanes force
the gas to flow at an angle $\varphi$ with respect to the vertical direction. Taking the axial velocity $v_m$ in the separation space as basis, the tangential velocity within the swirl element then becomes

$$v_{\theta_{sw}} = \left(\frac{v_z}{1 - (R_{\text{body}}/R)^2}\right) \tan \varphi$$

(4.14)

Substituting (4.12)–(4.14), the definition of $d_{p50}$ (4.5) can now be rewritten in terms of the swirl geometry

$$d_{p50} = \frac{\alpha (R - R_{\text{body}})}{\tan \varphi} \sqrt{\frac{13.5 \mu_g}{(\rho_p - \rho_g) v_z L}}$$

(4.15)

It is also possible to write $d_{p50}$ in the form of Tables 2.4 and 2.5, but then $S$ should be defined according to

$$S = \frac{\Gamma}{\pi R v_z \sqrt{3}} = \frac{\tan \varphi}{\alpha (1 - R_{\text{body}}/R) \sqrt{3}}$$

(4.16)

The swirl constant $\alpha$ sets $d_{p50}$, whereas the core size $r_c^*$ only affects the shape of the efficiency curve. These constants can be found by fitting a tangential velocity profile as obtained from a measurement or numerical simulation. Unfortunately, such data is rare in literature. Recently though, the tangential velocity profile has been measured accurately in a perspex axial cyclone, through flow visualization by means of a neutrally buoyant tracer [21]. As an example, we fitted that profile in Figure 4.5. The data clearly represent a Lamb vortex. The Rankine model gives a bad fit: we

---

**Figure 4.5.** Fit of a measured tangential velocity profile halfway the separation space [21], plus corresponding efficiency curve: $v_a = 3.8$ m/s, $\varphi = 45^\circ$, $R = 25$ mm, $R_{\text{body}} = 15$ mm, $L = 185$ mm, $\mu_g = 1.8 \cdot 10^{-5}$ kg/ms, $\rho_p = 1000$ kg/m$^3$, $\rho_g = 1.2$ kg/m$^3$ ($d_{p50} = 6.5$ µm).
oversized the vortex core to get a somewhat reasonable approximation. Nonetheless, the corresponding efficiency curves are quite similar (see Fig. 4.5). We would expect the swirl constant $\alpha$ to be around unity, which is indeed the case. The fact that it is slightly larger ($\alpha = 1.1$) could be explained by the drainage slits in the cyclone wall [21], which may have generated some extra friction. As a final remark, we point out that the region $R/\sqrt{2} < r < R$ approximates a loss free vortex indeed.

### 4.3 Cyclone experiments

Figure 4.6 shows the setup that we used to determine the efficiency curve, i.e. the separation efficiency as a function of droplet size. Two mist heads, containing 7 BETE PJ6 pin jet nozzles each, injected a constant amount of demineralized water (1.6 l/min) into an adjustable airstream. For details regarding the air and water supplies, refer to section 3.2.

The tested axial cyclone is available from Frames Separation Technologies as scrubber internal. We enclosed the cyclone in a bigger pipe ($\varnothing$ 185 mm), simulating a scrubber vessel with upwards gas flow (similar area ratio). The geometry of the cyclone, regarded company intellectual property, is represented in the value of $d_{p50}$

![Figure 4.6. Measurement setup](image-url)
(Eq. 4.15). Liquid was drained via a cap at the end of the cyclone tube (annular co-
axial drainage system). The SPRAYTEC (see section 3.3.1) is situated directly above
the open outflow (see Fig. 4.6).

Brunazzi et al. [12] have done similar measurements with axial cyclones. However,
they used a MASTERSIZER S, which suffers from vignetting: distributions having a
range around 1–20 micron are inaccurate, as light scattered at wide angles is cut off
from the lens (see also section 5.2.2). The cyclone efficiency curve, which we try to
determine, lies exactly in that range.

As reference measurement (see section 3.3.2) we took a dummy cyclone at the same
airflow setting as the operational cyclone. The dummy – a cyclone with the swirl
generator removed – makes sure that we measure only the effect of the swirl generator.
Similar to section 3.3.2 (see Eq. 3.2), the efficiency is obtained as

\[ \eta(d_{p,i}) = 1 - \frac{(c_v \Delta f_i)_{\text{cyclone}}}{(c_v \Delta f_i)_{\text{dummy}}} \]  (4.17)

As usual, we measured the background scattering signal after setting the required
airflow, to prevent beam steering (section 3.3.1). To prevent lens contamination, we
did the measurement immediately after switching on mist injection. Data acquisition
rate was 50 Hz, the background signal was measured for 20 sec and the actual mea-
surement lasted 5 sec. With dummy, the laser extinction path length was about 75
mm. Above the operational cyclone the strong swirl widened the mist laden airstream,
increasing the extinction path length to about 90 mm. In between measurements the
lens was cleaned, and the inside of the pipe was made dry.

In operation, the cyclone cap lost some droplets due to reentrainment which are not
present in the ‘dry’ background measurement. Since these droplets are relatively large,
a high signal is obtained on the first (small scattering angle) detectors (see section
3.3.1). The MALVERN software has trouble to fit a line trough both the powerful
signal of these first detectors and the weaker signal of the later (large scattering
angle) detectors that record the actual mist scattering. It is not uncommon to “kill”
a number of small scattering angle detectors [28]. This does not affect the \( c_v \) value
of the measurement: large droplets have a very small surface to volume ratio so they
hardly contribute to the laser extinction (obscuration). We had to remove between 8
and 21 detectors to obtain a proper fit through the mist scattering signal.

### 4.4 Results and discussion

Figure 4.7 shows the results at an axial gas velocity \( v_z = 38.2 \) m/s. The size distribu-
tions are shown as droplet concentrations per logarithmic size interval \( c_v df/d(\ln d_p) \).
The injected droplets were in the range 1–100 micron, whereas the cyclone only al-
 lows some droplets in the range 1–20 micron to pass. The efficiency curve, calculated
according to Eq. (4.17), therefore lies on the tail of the reference distribution.
Figure 4.7. Axial cyclone measurement at $v_z = 38.2\text{ m/s}$. $d_{50}$ (Eq. 4.15) is evaluated with $\alpha = 1$, $\mu_g = 1.8 \cdot 10^{-5}\text{ kg/ms}$, $\rho_p = 1000\text{ kg/m}^3$ and $\rho_g = 1.2\text{ kg/m}^3$.

The efficiency seems to go up towards the finest droplets. Such a ‘fish-hook’ is a fairly common observation [20]. Here it is probably due to measurement error: below 2 micron the efficiency is based on the far left tail of both droplet distributions, which makes it subject to a lot of uncertainty. A physical explanation has been given for hydrocyclones [15, 31]: at sufficiently large concentrations $c_v$ large droplets tend to take along small ones – not in their direct path, but due to relatively weak entrainment around their larger periphery. We omitted any results < 2 $\mu$m in our analysis.

Figure 4.7 further shows the same result as a function of the dimensionless droplet size $x = d_p/d_{50}$. The cut-size $d_{50}$ is calculated on the basis of Eq. (4.15), taking $\alpha = 1$ for the swirl constant. The theoretical curve of Eq. (4.10), which was based on a Rankine vortex, is shown for three cases: a free vortex ($r_e^* = 0$), a vortex core of 80% the cyclone radius ($r_e^* = 0.8$), and, finally, a complete solid body ($r_e^* = 1$). Clearly, the measurement is predicted closely for $r_e^* = 0.8 R$.

Next, we lower the flowrate three steps in Figure 4.8. As a result the efficiency, plotted against the dimensionless droplet size, seems to go up. But if we look at the dummy size distributions, now shown as fractions per logarithmic size interval $df/d(\ln d_p)$, we see a strange trend: towards lower flowrates one has comparatively more droplets < 10 $\mu$m and less droplets > 10 $\mu$m. Indeed we expect that the reference distribution depends on flowrate, because some inertial separation of droplets typically > 10 $\mu$m takes place in the bend of the pipe and at the transition from pipe to cyclone (see Fig. 4.6). However, inertial separation works the other way around: we would then expect to have (relatively) more droplets > 10 $\mu$m at lower flowrates. In conclusion, the cyclone did not improve but the reference increased disproportionally.

Something causes the tail of the dummy distribution to be lifted up, thus causing an unrealistic separation efficiency curve. Because we injected a constant amount of mist, the droplet concentration increases with decreasing volume flows. In cases where the droplet concentration is high, scattered light is being re-scattered by other droplets before it reaches the detector, known as multiple scattering [28]. A patented software
algorithm by MALVERN is supposed to correct for this, allowing the SPRAYTEC to continue to operate where older versions like the MASTERSIZER would fail [27, 28]. However, the multiple scattering analysis was switched on in all our measurements. It seems that it did not correct the dummy distributions properly.

To get some idea of the internal flow pattern, we reverted to CFD. Using a Reynolds stress model in ANSYS CFX, a simulation was done at $v_z = 28.4$ m/s. Examining the tangential velocity profile, a swirl constant $\alpha = 1$ turns out appropriate. Also the vortex core is rather large, justifying $r_c = 0.8R$. CFD further shows an axial velocity profile with backflow in the cyclone center. A strong pressure dip in the eye causes a suction of about 10% of the gas to recirculate. Based on an indicative calculation, we expect the effect on the efficiency to be limited: the negative effect of an increase in axial velocity $v_z$ in the periphery is amply compensated by the fact that droplets also enter the cyclone nearer to the wall (the center will be unused).

### 4.5 Conclusions and recommendations

We validated the efficiency of vane packs and axial cyclones. Results obtained with laser diffraction particle sizing are in good correspondence with models and design formulae. The cyclone swirl ratio (Eq. 4.16) as used for $d_{p50}$ in Tables 2.4 and 2.5 turns out $S = 1.2$ for the experiments and $S = 1.3$ in the example of Figure 4.5.

Unfortunately, we encountered an annoying bias in the reference measurement of the cyclone. In subsequent chapters we solve this problem by applying a pre-separation step. This has two advantages:

1. It lowers concentrations, thus preventing multiple scattering.
2. The peak of the reference shifts towards $d_{p50}$. Reference and actual measurement then lie in the same range, which prevents that inaccuracies in the tail of the distribution affect the main result.
Apart from a pre-separation step, we give some other points for attention in future cyclone experiments. The widening of the swirling gas stream above the cyclone requires a more objective observation of the laser extinction path length (we now estimated it using a tape measure). For example, one could take photographs of the extinction path (from aside) in each measurement. Since cyclones tend to lose droplets due to reentrainment at high liquid loadings or very high flowrates, it might be better also to use a lens purge.

The presented axial cyclone model successfully connects the radial profile of tangential velocity to a separation efficiency curve as function of droplet size. It unifies previous approaches, based on either solid body or free vortex rotation. The model can be further improved by implementing a profile of axial velocity, instead of the rather bold plug flow assumption. This can be done by assuming a so-called helical vortex model [1], which couples the axial velocity profile to that of the tangential velocity. Even backflow in the cyclone center could be implemented in this way.

An alternative approach to derive the separation efficiency is to assume continuous radial mixing of droplets over the cyclone cross-section [2]. This gives an expression of exactly the same form as we obtained for complete solid body rotation \((r_e = R)\) without mixing: \(\eta = 1 - \exp \left( -\text{constant} \cdot x^2 \right) \) [20, 24]. Basically, a solid body rotation affects droplet paths in such a way that it distributes them evenly over the cross-section, thus resembling full radial mixing. In chapter 6 we shall implement radial mixing in rotating channels. Unlike rotating channels, the length to diameter ratio of cyclones is limited. Therefore, if mixing dominates in the radial plane, it will also take place in the axial direction. We expect that it is unlikely that dispersion in axial cyclones is so severe that droplets are fully mixed up both axially and radially.
Rotating phase separators with laminar channel flow

Former applications of the RPS (e.g. dust filtering from air, ash removal from flue gas) usually operated in the laminar regime. The separation efficiency curve of a number of prototypes has been determined experimentally [7, 8]. The removal of micron sized particles was thus proven. However, there is quite some spread (±20%) on the obtained curves. Till now, there has been no effort to improve the accuracy of the measurement method itself. Moreover, the efficiency was generally measured at a nominal operating point, not in a range of flowrates and rotation speeds.

In this chapter we measured the separation efficiency of an RPS operating with laminar channel flow. The unit removes a micron sized water mist from a near-atmospheric airstream. By means of laser diffraction particle sizing, we determined efficiency curves in a range of flowrates and rotation speeds. In section 5.2 we address a number of issues to obtain a good measurement accuracy: channel entrance effects, side leakage along the rotating element and vignetting in the laser diffraction measurement. The latter refers to the situation that light scattered at wide angles (fine droplets) is cut off from the lens, which is very important if distributions having a range around 1–20 micron must be determined accurately. In section 5.3 we compare the new measurements with previous (laminar) results from literature.

Theoretically, the separation efficiency for laminar channel flow is well established. By definition, particle trajectories follow a deterministic path, so that analytical expressions can be found for the efficiency. Brouwers [8] has derived a set of expressions for annuli, for circular channels (tubes) and for triangular and sinusoidal channels in which the radial height is much smaller than the base width. In section 5.1 we complete the analysis by deriving the efficiency for rectangular channel cross-sections with a variable aspect ratio. Keeping the same channel height, we vary the width of the channels between an annulus and the situation that the annulus is split up by partitioning walls into an infinite number of ‘lamellae’.
5.1 Efficiency for laminar channel flow

In section 2.2 we derived a basic expression (Eq. 2.15) for the collection efficiency of rotating channels, under the premise of plug flow and constant channel height. Laminar flow however is characterized by Poiseuille type, often parabolic velocity profiles. Further, channel height is not constant for other than rectangular cross-sections. In the following we implement Poiseuille profiles as well as channel shape within single (rows of) channels, rotating at a far distance from the rotation axis.

In section 5.1.1 we show that, in a longitudinal radial plane (slice), Eq. (2.15) is valid for any arbitrary axial velocity profile. Using the concept of parallel planes, section 5.1.2 gives a general outline to derive laminar channel efficiency, which we apply at the same time to circular channels. Section 5.1.3 deals with rectangular channels with a variable aspect ratio. Appendix C.1 gives a blueprint for equilateral triangles. By definition, expressions for single channels are also valid for a complete element in case of an ideal flow distribution (see section 2.5). Eqs. (2.33) and (2.34) can be used to derive the efficiency for a uniform flow distribution over the element.

5.1.1 Efficiency in a longitudinal plane

In rotating laminar flow, the fluid rotates as a solid body (in the plane perpendicular to the rotation axis). Since droplets migrate radially at their terminal velocity $U_T$ (Eq. 2.9, Table 2.3) their radial position as a function of time is

$$r = r_0 + U_T t$$

(5.1)

with $r_0$ the initial radial position. For an arbitrary axial velocity profile $v_z(r)$, the corresponding axial particle position can be found by integration

$$z = \int_0^t v_z dt$$

(5.2)

Now we consider radial droplet motion in a longitudinal section of a channel. Based on Eqs. (5.1) and (5.2), Figure 5.1 shows particle trajectories for a parabolic velocity profile, as found within pipes (Hagen-Poiseuille flow) or between parallel plates. In section 2.2 we defined the critical position $r_c$ as the initial radial position for which a droplet just reaches the wall at $z = L$. Since particle influx goes with local axial inflow velocity, the plane efficiency is given by [8]

$$E = \frac{\int_{r_c}^{r_o} v_z dr}{\int_{r_i}^{r_o} v_z dr}$$

(5.3)

The nominator is found by substitution of $dr = U_T dt$, taking into account that the critical droplet starts at $r = r_c$ on $t = 0$ and ends at $r = r_o$ at $t_c = (r_o - r_c) / U_T$.

$$\int_{r_c}^{r_o} v_z dr = U_T \int_0^{t_c} v_z dt = U_T L$$

(5.4)
The denominator is found considering the definition of mean velocity

\[ \langle v_z \rangle = \frac{1}{r_o - r_i} \int_{r_i}^{r_o} v_z \, dr \]  

(5.5)

The plane efficiency (5.3) then becomes

\[ E = \frac{U_T L}{\langle v_z \rangle (r_o - r_i)} \]  

(5.6)

which is exactly the same result as for plug flow (Table 2.3). Apparently, Eq. (2.15) is not restricted to plug flow. It is valid for an arbitrary velocity profile [8], provided that the average velocity \( \langle v_z \rangle \) is used in \( d_{p50} \) (Table 2.4). We now write (2.15) as

\[
E = \begin{cases} 
\frac{1}{2} \chi^2 & \chi \leq \sqrt{2} \\
1 & \chi \geq \sqrt{2}
\end{cases}
\]  

(5.7a)

(5.7b)

with \( \chi = d_p/d_{p50} \) the dimensionless droplet size of Eq. (2.14) if \( d_{p50} \) (see Table 2.4) is based on the mean velocity \( \langle v_z \rangle \) in the plane of height \( h = (r_o - r_i) \).

### 5.1.2 Circular channels

In a complete channel, built up from parallel slices like Figure 5.1, \( d_{p50} \) (refer to Table 2.4) is based on channel mean velocity \( v_m \) and maximum channel height \( h_0 \)

\[ d_{p50} = \sqrt{\frac{9 \mu_g v_m h_0}{(\rho_p - \rho_g) \Omega^2 R L}} \]  

(5.8)

The channel’s dimensionless particle size \( x \) (Eq. 2.14) is related to that of the individual planes \( \chi \) (in Eq. 5.7) as follows

\[ x = \frac{d_p}{d_{p50}} = \chi \sqrt{\frac{h}{h_0}} \frac{\langle v_z \rangle}{v_m} \]  

(5.9)
We now map the channel cross-section to a local dimensionless coordinate system \((\zeta, \xi)\), corresponding to the \((\theta, r)\) directions in the global coordinate system. Figure 5.2 shows this for Hagen-Poiseuille flow in circular channels. In dimensionless form the channel height, local velocity and mean plane velocity are respectively

\[
\delta = 2\sqrt{1 - \zeta^2} \tag{5.10}
\]

\[
v^* = \frac{v_z}{v_m} = 2\left(1 - \zeta^2 - \xi^2\right) \tag{5.11}
\]

\[
\langle v^* \rangle = \frac{\langle v_z \rangle}{v_m} = \frac{1}{\delta} \int_\xi v^* d\xi = \frac{4}{3}(1 - \zeta^2) \tag{5.12}
\]

![Figure 5.2. Coordinate system and velocity profile in circular channels](image)

In dimensionless local coordinates the maximum channel height \(\delta_0 = 2\). Using (5.9), Eq. (5.7) can be rewritten as a function of coordinate \(\zeta\) (the tangential direction if seen from the relatively far away axis of rotation) as follows

\[
E(\zeta) = \begin{cases} 
\frac{x^2}{\delta\langle v^* \rangle} & x \leq \sqrt{\delta\langle v^* \rangle} \\
1 & x \geq \sqrt{\delta\langle v^* \rangle} 
\end{cases} \tag{5.13a}
\]

\[
E(\zeta) = \begin{cases} 
\frac{|\zeta|^2}{\delta\langle v^* \rangle} & |\zeta| \leq \zeta_o \\
1 & |\zeta| \geq \zeta_o \text{ or } x \geq \sqrt{2\langle v_0^* \rangle} 
\end{cases} \tag{5.13b}
\]

As long as \(x \leq \sqrt{2\langle v_0^* \rangle}\) with \(v_0^*\) the maximum velocity in the middle, it is possible to solve \(x = \sqrt{\delta\langle v^* \rangle}\) for \(\zeta\). Defining the answer as \(\zeta_o\), (5.13) can be written as

\[
E(\zeta) = \begin{cases} 
\frac{x^2}{\delta\langle v^* \rangle} & |\zeta| \leq \zeta_o \\
1 & |\zeta| \geq \zeta_o \text{ or } x \geq \sqrt{2\langle v_0^* \rangle} 
\end{cases} \tag{5.14a}
\]

\[
E(\zeta) = \begin{cases} 
\frac{|\zeta|^2}{\delta\langle v^* \rangle} & |\zeta| \leq \zeta_o \\
1 & |\zeta| \geq \zeta_o \text{ or } x \geq \sqrt{2\langle v_0^* \rangle} 
\end{cases} \tag{5.14b}
\]

with \(\zeta_o\) the solution of \(x^2 = \delta\langle v^* \rangle\).
This yields for a circular channel

\[
E(x, \zeta) = \begin{cases} 
\frac{3}{8} x^2 (1 - \zeta^2)^{-3/2} & |\zeta| \leq \zeta_0 \\
1 & |\zeta| \geq \zeta_0 \text{ or } x \geq \sqrt{\frac{8}{3}} 
\end{cases} \tag{5.15a}
\]

with \( \zeta_0 = \sqrt{1 - \left(\frac{3}{8} x^2\right)^{2/3}} \). The reason that \( \zeta_0 \) does not exist if \( x \geq \sqrt{\frac{8}{3}} \) is that the efficiency is then 100\% in the entire channel. Figure 5.3 clearly shows, for droplet size \( x = 1 \), how to interpret Eqs. (5.12) and (5.15). Notice that, near the side walls, the mean velocity in the vertical planes \( \langle v^* \rangle \) is so low that the efficiency reaches 100\%.

Weighing the contribution of all planes (slices) according to their local velocity \( \langle v^* \rangle \) we obtain the efficiency of the entire channel \[8\]

\[
\eta = \frac{1}{\alpha} \int_{-1}^{1} E \langle v^* \rangle \delta d\zeta \tag{5.16}
\]

with \( \alpha \) the cross-sectional area in the dimensionless units. Plugging in Eq. (5.14), taking into account the two regions separated by \( \zeta_0 \) and exploiting symmetry with respect to \( \zeta = 0 \) (see Eq. C.7), the efficiency of rotating channels in general is

\[
\eta = \begin{cases} 
\frac{2}{\alpha} \zeta_0 x^2 + \frac{2}{\alpha} \int_{\zeta_0}^{1} \langle v^* \rangle \delta d\zeta & x \leq \sqrt{2\langle v_0^* \rangle} \\
1 & x \geq \sqrt{2\langle v_0^* \rangle} 
\end{cases} \tag{5.17a}
\]

For circular channels \( \alpha = \pi \) and the integral in (5.17a) is some standard calculus integral (see Eq. C.8). The efficiency in circular channels (tubes) is then

\[
\eta = \begin{cases} 
\frac{2}{\pi} \zeta_0 x^2 + \frac{2}{\pi} \arcsin \zeta_0 - \frac{4}{3\pi} \zeta_0 \left(\frac{5}{2} - \zeta_0^2\right) \sqrt{1 - \zeta_0^2} & x \leq \sqrt{\frac{8}{3}} \\
1 & x \geq \sqrt{\frac{8}{3}} 
\end{cases} \tag{5.18a}
\]

which is a result originally derived by Brouwers \[8\].
5.1.3 Rectangular channels

In a rectangular channel of width to height aspect ratio $\beta = b/h$, the velocity profile is given by an infinite series [6]. Mapping the channel irrespective of its width to the local dimensionless coordinates $\zeta$ and $\xi$ (see Fig. 5.4) the velocity is given by

$$\hat{v}(\zeta, \xi) = \frac{1}{A} \left( \frac{3}{2} (1 - \xi^2) - 6 \sum_{n=0}^{\infty} \frac{(-1)^n}{k^3} \frac{\cosh k\beta \zeta}{\cosh k\beta} \cos k\xi \right)$$

(5.19)

with

$$k = \frac{(2n + 1) \pi}{2} \quad A = 1 - \frac{6}{\beta} \sum_{n=0}^{\infty} \frac{\tanh k\beta}{k^5}$$

(5.20)

Figure 5.4 shows the profile in the middle ($\zeta = 0$) of a square cross-section ($\beta = 1$), compared to a concentric channel ($\beta = \infty$) and a circular channel (Eq. 5.11).

![Figure 5.4](image)

**Figure 5.4.** Coordinates in a rectangular channel, and axial velocity profiles at $\zeta = 0$.

Similar to Eq. (5.12) for circular channels, the mean velocity in the planes (averaged over $\xi$) within rectangular channels is obtained as

$$\bar{v}(\zeta) = \frac{1}{\delta} \int_{\xi} v^* d\xi = \frac{1}{2} \int_{-1}^{1} \hat{v} d\xi = \int_{0}^{1} \hat{v} d\xi = \frac{1}{A} \left( 1 - 6 \sum_{n=0}^{\infty} \frac{1}{k^4} \frac{\cosh k\beta \zeta}{\cosh k\beta} \right)$$

(5.21)

where we used a rule which says that $(-1)^n \sin k = 1$. For $\zeta = 0$ (the middle of the rectangular channel), Eq. (5.21) yields

$$\bar{v}_0 = \frac{1}{A} \left( 1 - 6 \sum_{n=0}^{\infty} \frac{1}{k^4 \cosh k\beta} \right)$$

(5.22)

Figure 5.5 shows Eq. (5.21) for a number of width/height aspect ratios $\beta$. The case $\beta = \infty$ corresponds to a concentric annulus for which $\bar{v} = 1$. A ratio $\beta = 0$ reflects...
the case that the annulus is split up into an infinite number of \textquote{lamellae}, which gives parabola in cross direction $\overline{v} = \hat{v} = \frac{3}{2} \left(1 - \zeta^2\right)$. Because a rectangular channel has a constant height, $\overline{v}$ can also be seen as the volume flow per unit of width. In that sense it is remarkable that the distribution of $\overline{v}$ in a square channel ($\beta = 1$) is very close to that of a lamellar channel ($\beta = 0$).

The solution of $\delta \langle v^* \rangle = x^2$ (see Eq. 5.14) yields $\zeta_o$, the position outside of which the efficiency is 100\% (recall the example of Figure 5.3). We therefore need to solve

$$6 \sum_{n=0}^{\infty} \frac{1}{k^4} \frac{\cosh k\beta \zeta_{\text{100}}}{\cosh k\beta} = 1 - \frac{A}{2} x^2$$

(5.23)

Isolating the first term of the summation and solving for $\zeta_o$ gives

$$\zeta_o = \frac{2}{\pi \beta} \arccosh \left[ \frac{\pi^4 \cosh \pi \beta / 2}{96} \left(1 - \frac{A}{2} x^2 - 6 \sum_{n=1}^{\infty} \frac{1}{k^4} \frac{\cosh k\beta \zeta_o}{\cosh k\beta} \right) \right]$$

(5.24)

which can be solved by a quickly converging iteration (the cosh terms are very large). The efficiency of the rectangular channel as a whole can now be derived according to Eq. (5.17). First we calculate the second term on the right hand side of (5.17a), substituting (5.21) together with $\delta = 2$ and $\alpha = 4$

$$\frac{2}{\alpha} \int_{\zeta_o}^{1} \overline{v} \delta d\zeta = \int_{\zeta_o}^{1} \overline{v} d\zeta = \frac{1}{A} \left(1 - \zeta_o - \frac{6}{\beta} \sum_{n=0}^{\infty} \frac{1}{k^5} \left[ \tanh k\beta - \frac{\sinh k\beta \zeta_o}{\cosh k\beta} \right] \right)$$

(5.25)

After substituting $A$ from Eq. (5.20) into (5.25), we can evaluate the full Eq. (5.17). Finally, we obtain for the rectangular channel efficiency

$$\eta = \begin{cases} \frac{1}{2} \zeta_o x^2 + 1 - \frac{1}{A} \left(\zeta_o - \frac{6}{\beta} \sum_{n=0}^{\infty} \frac{\sinh k\beta \zeta_o}{k^5 \cosh k\beta} \right) & \text{if } x \leq \sqrt{2\overline{v}_0} \\ 1 & \text{if } x \geq \sqrt{2\overline{v}_0} \end{cases}$$

(5.26a, b)
with $\tau_0$ and $\zeta_o$ according to (5.22) and (5.24) respectively. One can identify two extreme cases for Eq. (5.26):

**Annulus:** If $\beta \to \infty$ channels have infinite width $b$. The cross-section forms two concentric rings, spaced at distance $h$. The velocity profile reduces to that of parallel plates $\hat{v} = \frac{3}{2} (1 - \zeta^2)$ and $\overline{v} = \tau_0 = 1$. The efficiency then corresponds to the ideal curve Eq. (2.15)

$$\eta = \begin{cases} \frac{1}{2} x^2 & x \leq \sqrt{2} \\ 1 & x \geq \sqrt{2} \end{cases}$$

(5.27a)  (5.27b)

**Lamellae:** In the limit $\beta \to 0$ channel width $b$ is negligible with respect to channel height $h$. This means that channels can be thought of as lamellae. The velocity profile is that of parallel plates, rotated by a quarter turn: $\hat{v} = \overline{v} = \frac{3}{2} (1 - \zeta^2)$ and $\tau_0 = \frac{3}{2}$. Further $\zeta_o = \sqrt{1 - x^2/3}$ (see Eq. 5.14) and $\eta = 1 - \zeta_o^3$ for $x \leq \sqrt{3}$ (Eq. 5.17a). The efficiency, representing the worst case, reduces to

$$\eta = \begin{cases} 1 - (1 - \frac{1}{3} x^2)^{3/2} & x \leq \sqrt{3} \\ 1 & x \geq \sqrt{3} \end{cases}$$

(5.28a)  (5.28b)

Figure 5.6 compares a square channel ($\beta = 1$) to the two extreme cases. Surprisingly, the efficiency of square channels is very close to the worst case of lamellae. The explanation is found in Figure 5.5, where we saw that the volume flow per unit of width in a square channel is very close to that of a lamellar channel. In fact, for $\beta \leq 1$ we can use Eq. (5.28) as an excellent approximation of Eq. (5.26).

![Figure 5.6](image-url)

**Figure 5.6.** Separation efficiency curve of square channels, compared to the ideal case of concentric channels and the worst case of lamellar channels.
Figure 5.7 zooms in on the top right corner of Figure 5.6, showing curves for varying aspect ratio. Figure 5.8 compares circular and square channels (Eqs. 5.18 and 5.26 respectively). Because \( d_{50} \) (see Eq. 5.8) is based on mean velocity \( v_m \) and maximum channel height \( h_0 \), the circular channel is smaller. The cross-section of the circular channel so to say fits into the square channel. It not surprising that if we then plot against \( x = \frac{d_p}{d_{50}} \), tubes have a somewhat higher efficiency. After all, the corresponding rotating element (bundle of tubes) would have more channels.

![Figure 5.7](image1.png)

**Figure 5.7.** Rectangular channels of varying width/height aspect ratio \( \beta \).

![Figure 5.8](image2.png)

**Figure 5.8.** Efficiency curve of square channels, compared to circular channels (tubes).
5.2 Results and discussion

We tested a large scale RPS prototype (section 3.1) with a standard rotating element (Table 3.2, element A) for removing a water mist from air at near-atmospheric pressure (see the setup in section 3.2). Due to the low gas density (1.2 kg/m³) and small channel height (1.9 mm), the bulk Reynolds number does not exceed 500 and the rotation Reynolds number (see section 6.2.2) does not exceed 7. This guarantees a laminar, Poiseuille type flow inside the channels of the rotating element (see also section 6.2.3). We use the method of section 3.3.2 to measure separation efficiency in a range of flowrates and rotation speeds.

5.2.1 Accessible operating range

To derive the separation efficiency, we measure downstream, taking the nonrotating element as reference (see section 3.3.2). Big advantage is that we the isolate collection efficiency due to the rotating element in this way. As we discussed in a preliminary publication [45], a drawback of this way of measuring is that even a stationary element has a removal effect due to droplet impaction at the channel entrance.

In the pre-separator (Figure 3.1) the airflow rotates due to the tangential air inlet. When the tangential velocity in the pre-separator does not match with the tangential velocity of the element, the airflow undergoes a sharp bend at the entrance of the channels (see Figure 5.9). As discussed in section 2.2, droplets are separated by inertial separation in such a bend. This effect, similar to the principle of an inertial impactor, is most significant when the element is stationary. As soon as the element starts rotating the mismatch in tangential velocity between the pre-separator airflow and the element becomes smaller (smoother transition), and impaction reduces.

Because impaction tends to reduce the reference, it leads to an underestimation of the measured separation efficiency, based on Eqs. (3.1) and (3.2). The higher the tangential velocity mismatch, the stronger the inertia at the channel entrance. In other words, impaction lowers measurements at high flowrates or low rotation speed.
5.2 Results and discussion

Figure 5.10 shows a set of overall measurements at a relatively high flowrate, together with the predicted overall efficiency (explained in section 5.2.3). Clearly, measurements at low rotation speed are affected. Initially, we even have an artificial negative efficiency: at those points the combined efficiency of element plus impaction onto the rotating channels is even less than impaction alone in the stationary (0 rpm) reference. At Ω > 500 rpm, the channels separate an order of magnitude better, so that the result is less troubled by the abnormality of the reference.

The question arises where we can expect accurate measurements with the current procedure. Taking the velocity mismatch and assuming that droplets are bent off at a radius equal to the channel height, estimations (based on Eq. 2.15) confirm that indeed flowrates below 500 rpm are affected if element A is used (Table 3.2). For element B, to be discussed in section 6.3, the bend is much less sharp due to the larger channels and there is not such a strict limit on the rotation speed.

5.2.2 Separation efficiency curve

We now discuss separation efficiency as a function of droplet size in a small array of 3 flowrates × 3 rotation speeds (9 settings). Table 5.1 shows the corresponding values of \( d_{p50} \). The latter follows from Eq. (5.8), which, after substituting element mean velocity \( v_m \) (Eq. 2.28) and element equivalent radius \( R_e \) (Eq. 2.29), is written as

\[
d_{p50} = \left( \frac{13.5 \mu_g h Q}{(\rho_p - \rho_g) \pi (1 - \varepsilon) (R_o^3 - R_i^3) L \Omega^2} \right)^{1/3}
\]

with \( \rho_p = 1000 \text{ kg/m}^3 \), \( \rho_g = 1.2 \text{ kg/m}^3 \) and \( \mu_g = 1.8 \cdot 10^{-5} \text{ kg/ms} \). Table 3.2 gives channel height \( h \), length \( L \), inner radius \( R_i \), outer radius \( R_o \) and blind area fraction \( \varepsilon \) of element A. Because \( d_{p50} \) includes the operational parameters flowrate \( Q \) and rotation speed \( \Omega \), all 9 curves can be displayed on the same axis if we plot as a function of the dimensionless droplet size \( x = d_p/d_{p50} \).
Table 5.1. Values of $d_{p50}$ (µm).

<table>
<thead>
<tr>
<th>Flowrate $(m^3/s)$</th>
<th>Speed (rpm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.075</td>
<td>2.2 1.7 1.3</td>
</tr>
<tr>
<td>0.100</td>
<td>2.6 1.9 1.5</td>
</tr>
<tr>
<td>0.125</td>
<td>2.9 2.2 1.7</td>
</tr>
</tbody>
</table>

Because theoretical predictions are defined as a function of $x$ too, this way of plotting also provides the means to compare all curves to a single theoretical line. The channels of element A were manufactured by corrugation, resulting in more or less trapezoidal channels (section 3.1). Because the shape is very close to rectangular, we used Eq. (5.26) with an aspect ratio $\beta = 1.1$. We further assumed that the flow distribution over the rotating element is uniform (constant axial velocity). Because this means that the efficiency varies with radius, we used Eqs. (2.33) and (2.34) to integrate over the element. Summarizing, the predictions are for laminar flow in rectangular channels and a uniform flow distribution over the element.

Figure 5.11 shows preliminary measurements, obtained with the Malvern Mastersizer S (see section 3.3.1). The left figure shows that the results are about 10–20% lower than the predicted curve. We anticipated that this could be due to a gas leak through the gap between housing and rotating element [46], which at that moment was 4 to 5 mm due to difficulties in making the housing (roughly 8% of the element area). We estimated that over 10% of the air, including unseparated mist, might have bypassed the rotating element. The fact that in the end the curve still reaches 100% efficiency, we attributed to the idea that relatively large droplets are still separated due to rotation inside the gap [46].

Figure 5.11. Measured separation efficiency curves (see Table 5.1) without sealing (left) and with a mechanical seal (right). Results < 2 µm are not shown (see section 3.3.2).
To check this hypothesis we applied a simple flat seal (a rubber ring on the housing and a nylon ring on the element) to close the complete gap. The new result, shown on the right hand side of Figure 5.11, clearly solves the problem. This illustrates the importance of a proper seal in industrial applications. In Appendix D we provided the formulae for two types of sealing: a simple bushing (gap) and a labyrinth seal. For a matter of fact, Figure 5.10 was also obtained with additional sealing.

Still, the measurements in Figure 5.11 feature a very large spread of about 20% in terms of separation efficiency. The reduction in efficiency due to mild turbulence is expected at 25% at most [23, 36]. Therefore, the accuracy still falls short to differentiate properly between laminar and turbulent flow. The reason is found in the hardware of the laser diffraction system. The lens of a particle sizer has to capture the scattered light from a sample, which is located at some distance from the lens. If the sample is too far away, light scattered at wide angles (small drops) is cut off from the lens. Therefore, the smaller the size of the particles to be measured (wider scattering angles), the shorter the allowed working distance.

The MASTERSIZER S which we used in the above measurements has a small lens (section 3.3.1). The SPRAYTEC however has a much larger lens and therefore a much larger working range. Taking a typical measurement, located between 25 and 135 mm in front of the lens, we compared the detector signals and corresponding size distributions of both instruments in Appendix E. The MASTERSIZER does not capture all scattered light on the last 4 detectors. As a result, one obtains an erroneous, somewhat peaked distribution. The SPRAYTEC captures the full pattern, which we checked by moving the spray closer to and further from the lens.

Figure 5.12 shows that using the SPRAYTEC does not change the main trends of Figure 5.11 (right hand side), but it considerably reduces the spread. Each of the 9 measurements now almost lies on one line.

![Figure 5.12](image)

**Figure 5.12.** Measured separation efficiency curves (see Table 5.1) using the SPRAYTEC. Results < 2 µm are omitted (see section 3.3.2).
5.2.3 Overall separative performance

We injected a constant amount of mist droplets in the airstream entering the RPS unit, and measured the size distribution and concentration of droplets in the airstream after the unit (Figure 3.2 and section 3.3.2). Figure 5.13 shows measured droplet volume concentrations per logarithmic size interval with a stationary (nonrotating) element. We normalize by the interval width of the natural logarithm of the droplet diameter (see the caption of Figure 3.4). In other words, we plot the total volume concentration \( c_v \) multiplied by the probability density function (PDF).

\[
Q = 0.075 \text{ m}^3/\text{s} \\
\text{cv} = 4.0 \times 10^{-6} \text{ v/v} \\
\text{MMD} = 4.1 \text{ micron} \\
Q = 0.100 \text{ m}^3/\text{s} \\
\text{cv} = 2.8 \times 10^{-6} \text{ v/v} \\
\text{MMD} = 3.8 \text{ micron} \\
Q = 0.125 \text{ m}^3/\text{s} \\
\text{cv} = 1.8 \times 10^{-6} \text{ v/v} \\
\text{MMD} = 3.6 \text{ micron}
\]

Figure 5.13. Droplet concentration distributions at \( \Omega = 0 \text{ rpm} \). The caption of Figure 3.4 explains how the volume concentrations of the individual fractions are normalized.

We see that the remaining droplets after a stationary element were mainly in the range 1–20 \( \mu \text{m} \). Further, if we compare the total droplet volume concentrations (ranging from \( 1.8 \times 10^{-6} \) to \( 4.0 \times 10^{-6} \text{ v/v} \)) to the amount of water injected (3.2 l/min), it appears that only 4–6\% of the injected volume remains. This means that approximately 99.5\% was already separated, even when the element was not rotating.

Apparently, just droplets in the range 1–20 \( \mu \text{m} \) are not separated effectively by cyclone action in the pre-separator. Shortly after expansion of natural gas, a significant fraction of the condensed contaminant droplets falls within this size range [4]. This means that we can simulate droplet sizes of the field application. The fact that it concerns only 5\% of our total injected mist volume explains why a large number of high pressure nozzles (28) is needed.

Taking Figure 5.13 as reference, Figure 5.14 (left hand side) shows the measured overall efficiency \( \eta_\infty \) for increasing rotation speed (according to Eq. 3.1). In contrast to the pre-separator cyclone, the rotating element is capable of effectively removing micron sized droplets at large gas volume flows. However, we have to measure at very low rotation speeds in order to have some droplets left to measure.

Taking the distributions of Figure 5.13 as input, we used Eq. (2.19) to also predict the overall efficiency, based on the theoretical curve shown in Figure 5.12. It appears that in this way the measurements are predicted very closely.
Figure 5.14. Overall separation efficiency due to the rotating element, using the situation with a stationary element (Ω = 0 rpm) as reference.

On the right hand side we made the rotation speed dimensionless with the nominal speed of Eq. (2.40). Nominal speed Ω₀ corresponds to the rotation speed at which droplets that have a size equal to the mass median diameter of the input distribution are separated for 50%. After nondimensionalization not only the measurements are on one line, even the three predictions fall together. This is due to the fact that the input distributions are nearly lognormal, and have very similar geometric standard deviations (see section 2.5, Figure 2.8).
5.3 Conclusions

Measurements show that the RPS is capable of effectively removing micron sized droplets, which distinguishes it from cyclones. It proves initially that the CO$_2$ droplet removal step in the concept of CRS (section 1.3) is feasible. However, under pressure we have to consider turbulent conditions in the channels.

We were able to measure separation efficiency curves accurately (down to 2 $\mu$m). For a range of flowrates and rotation speeds, results were in good agreement with theoretical predictions. Figure 5.15 shows the present result (blue triangles), compared to earlier results for laminar flow [7, 8]. With the improved accuracy, we are now ready to measure the effect of turbulence, for which direct numerical simulations (DNS) predict a decrease of 25% at most in the separation efficiency [36].

![Figure 5.15. The 3 curves at 0.125 m$^3$/s (9), compared to former measurements [7, 8].](image)
The (semi) turbulent rotating phase separator

In chapter 5 we tested a full-scale prototype rotating phase separator (section 3.1) using air and water at near-atmospheric pressure. Similar to early RPS applications [7, 8], the low gas density (1.2 kg/m$^3$) and small channel height (1.9 mm) guaranteed a laminar Poiseuille type flow inside the channels of the rotating element. In reality the unit would be operating with high pressure, high density natural gas (Table 3.1). This leads to a large Reynolds number, corresponding to fully turbulent conditions. That is to say, the test unit has been scaled up on the basis of volume flow, but due to the atmospheric pressure in the test setup, the unit operated in a different flow regime than it would in a natural gas processing installation.

Rotation destabilizes the flow at otherwise laminar conditions. Poiseuille pipe flow is normally stable for (bulk) Reynolds numbers $Re < 2100$. But if a pipe rotates fast enough [26, 32], it becomes unstable already at $Re = 83$. Prior to the turbulent state, there exists another stable, time-dependent laminar flow regime [36]. Tests with RPS units for oil-water separation [25, 37], operating in this ‘semi-turbulent’ regime, demonstrate an unexpectedly poor separation efficiency. With increasing rotation speed, the efficiency does not improve or even gets worse. It is still unclear what causes this behavior, and whether the internal channel flow is a limiting factor.

In (semi)turbulent flows, it is not possible to describe a droplet’s path analytically, like in a stable Poiseuille flow (section 5.1). To compute the separation efficiency, we rely on direct numerical simulations (DNS), in which point particles are tracked under the influence of a centrifugal force [23, 36]. On average, the velocity field features a counterrotating vortex, induced by the Coriolis force. Hypothetically, such a vortex traps droplets, thus preventing their separation. Yet, the separation efficiency decreases by 25% at most in considered cases of DNS [23, 36]. Apparently, droplets are able to escape the internal vortex due to turbulent dispersion, or, in a time-dependent laminar (semi-turbulent) flow, due to variations of the mean velocity [36].

However, these results have never been confirmed by experiments. This is not surprising in view of the accuracy: a decrease of 25% lies within the spread of former (stable laminar) measurements [7, 8]. Using the new method developed in chapters 3
and these variations are now clearly perceptible. To get out of the stable laminar regime, we artificially increased the Reynolds number by means of enlarged channels in a customized element (Table 3.2, element B). The element was dimensioned in such a way that we could cover both turbulent flow and unstable, semi-turbulent flow within the accessible operating range (flow and rotation speed).

Existing models for the separation efficiency in rotating channels do not take into account mixing [8]. For cyclones there exist models based on continuous mixing of droplets in the radial plane [2, 20, 24]. We adopted this approach to rotating channels. It gives the efficiency in case mixing dominates the separation.

Basic models for the separation efficiency are presented in section 6.1. Focussing on pipe flow, section 6.2 discusses the stability of a rotating Hagen-Poiseuille flow (RHPF) and the nature of the flow after the onset of instabilities. In section 6.3 we present the very first measurements of separation efficiency with (semi)turbulent flow. In section 6.4, results from DNS are analyzed. Section 6.5 finishes off with a comparison of measurements and DNS, and an overall assessment.

![Figure 6.1. Osborne Reynolds’ 1883 historic pipe flow experiment [33]. Reynolds detected a spontaneous transition to turbulence only at \( Re = 13000 \). Mackrodt [26] suggested that, instead of a disturbance, a small amount of rotation might have escaped Osborne’s notice, ruining his beautiful parabolic flow profile.](image)

### 6.1 Basic models

Current models for the separation efficiency of long channels, rotating at a far distance from the rotation axis, disregard mixing [8]. They are based on either plug flow or Poiseuille type flow (for the latter see section 5.1). Because previous RPS applications operated with Poiseuille flow, the plug flow models have never been of much use. However, as we shall see later, it is allowed to disregard mixing even in specific cases of turbulent flow. And for the flat velocity profile in turbulent flows, plug flow is a reasonable assumption.
Till date there has been no attempt to devise a model that characterizes the effect of mixing (turbulent dispersion) on separation efficiency in rotating channels. A common approach is to assume continuous mixing of droplets in the radial direction [20, 24]. Only recently, this technique has been applied successfully to axial cyclones [2].

In the following, we apply the two basic approaches to rotating channels:

1. Plug flow without mixing (section 6.1.1).
2. Continuous mixing of droplets in the radial direction (section 6.1.2).

In section 6.1.3 we discuss the application to various channel cross-sections and to a bundle of channels, rotating around a common axis (a rotating element).

### 6.1.1 Plug flow without mixing

If mixing plays no role, the radial velocity of droplets is simply given by their terminal settling velocity \( U_T \) (Eq. 2.10). Droplets of one size \( d_p \) all move at the same radial velocity \( U_T \). In case of plug flow, the axial velocity \( V \) is also constant. Figure 2.4 showed a side view of the straight droplet trajectories.

Instead, we now look at a cross-section, viewed along the \( z \)-axis (Fig. 6.2). Downstream, at a distance \( z \) from the entrance, we find the nonseparated droplets in the overlap of two channel cross-sections at offset \((U_T/V)z\) (see Fig. 6.2). Because droplets cover the full cross-section at entry, the rest has been collected upstream on the wall. The separation efficiency is defined as [8]:

\[
\eta = 1 - F
\]  

(6.1)

with \( F \) the area ratio of the overlap to the total cross-section. Brouwers [8] constructed explicit expressions for a few different channel shapes (Table 6.1). Separation efficiency is defined as a function of the dimensionless droplet size (see Eq. 2.14)

\[
x = \frac{d_p}{d_{p50}}
\]  

(6.2)

based on the typical size (from Table 2.4)

\[
d_{p50} = \frac{1}{\Omega} \sqrt{\frac{9\mu g V h}{(\rho_p - \rho_g) RL}}
\]  

(6.3)

that corresponds to a droplet that has traveled an offset of half the channel height \( h/2 \) at the end of a channel with length \( L \), rotation speed \( \Omega \) and (center) distance \( R \) from the rotation axis. Further, \( \rho_p, \rho_g \) and \( \mu_g \) are particle density, fluid density and fluid dynamic viscosity respectively. Droplets that travel the maximum radial distance \( h \) within the axial space \( L \) are fully separated (for \( x \geq \sqrt{2} \) one has \( \eta = 1 \)).
6.1.2 Continuous radial mixing

Again we take a small slice $dz$, which we follow through the channel at the mean axial velocity $V$. But due to continuous mixing, we now assume that droplets are evenly distributed over the cross-section at all times. But then again, the centrifugal force causes an outflow of droplets at the wall, taking place at the terminal settling velocity $U_T$ (see Fig. 6.2). The rate of change of the amount of droplets inside the control volume equals the outflow at the wall [2]

$$\frac{d}{dt} (CAdz) = -CU_Tbdz$$

(6.4)

with $C$ the local droplet concentration, $A$ the cross-sectional area and $b$ the width of the channel. $(CAdz)$ is the number of droplets in volume $Adz$. $(CU_Tbdz)$ is the flux through the (projected) lateral surface $bdz$. Note that $dz$ cancels out. Separation of variables then gives

$$\frac{dC}{C} = -\frac{b}{A}U_T dt$$

(6.5)

In the end we are interested in the concentration as a function of axial distance, not as a function of time. Because we agreed that the control volume moves at the mean velocity $V = dz/dt$, we can translate time to position substituting $dt = dz/V$. Integrating over axial distance $z$, we then obtain

$$\ln \frac{C}{C_0} = -\frac{b}{A} \frac{U_T}{V} z$$

(6.6)

with $C_0$ the inlet concentration. Since $C$ concerns nonseparated droplets, we have for the collected fraction or separation efficiency [2]

$$\eta(z) = 1 - \frac{C}{C_0} = 1 - \exp \left( -\frac{bz}{A} \frac{U_T}{V} \right)$$

(6.7)

![Figure 6.2. Control volume: a 'slice' of thickness $dz$, perpendicular to the axis of rotation. The gray area indicates where non-separated droplets are.](image)
The efficiency asymptotically reaches 100%. We now wish to relate the result to the dimensionless droplet size \( x \) (Eq. 6.2). It is known from Table 2.3 and Eq. 2.15 that
\[
(U_T/V) \left( \frac{L}{h} \right) = \frac{1}{2} x^2.
\]
The efficiency (at \( z = L \)) then becomes
\[
\eta = 1 - \exp \left( - \psi \frac{1}{2} x^2 \right)
\] (6.8)
with a geometric factor \( \psi = bh/A \) that accounts for the channel shape. The equation is valid regardless of the shape of the axial velocity profile in the channel.

### 6.1.3 Application

Table 6.1 summarizes the results for three different channel shapes. Figure 6.3 shows a plot. Note that, in contrast to a Poiseuille type flow (see section 5.1.3), the aspect ratio of a rectangular channel has no influence in the present models.

<table>
<thead>
<tr>
<th>Profile</th>
<th>Plug flow ((x \leq \sqrt{2}))</th>
<th>Continuous mixing</th>
</tr>
</thead>
<tbody>
<tr>
<td>rectangular</td>
<td>( \frac{1}{2} x^2 )</td>
<td>( 1 - e^{-x^2/2} )</td>
</tr>
<tr>
<td>circular</td>
<td>( \frac{2}{\pi} \left( \frac{1}{2} x^2 \sqrt{1 - \frac{1}{4} x^4} + \arcsin \frac{1}{2} x^2 \right) )</td>
<td>( 1 - e^{-2x^2/\pi} )</td>
</tr>
<tr>
<td>triangular</td>
<td>( x^2 - \frac{1}{4} x^4 )</td>
<td>( 1 - e^{-x^2} )</td>
</tr>
</tbody>
</table>

By definition the single channel results are also valid for a complete rotating element at ideal inflow conditions (see section 2.5). The channel at the equivalent radial position \( R_e \) (Eq. 2.30) should then be taken with the element mean flow velocity \( v_m \). The situation complicates with a uniform flow distribution over the element.

Looking at the plug flow models combined with a uniform flow distribution, Eq. (2.37) already gave an expression for rectangular channels. For triangular channels, see [9]. For tubes it is not possible to derive a formula.

For uniform flow distributions combined with continuous mixing, we can repeat the procedure of section 2.5. To that end, Eq. (6.8) is first written as a function of the radial distance to the rotation axis (similar to Eq. 2.36)
\[
\eta = 1 - e^{-\Psi R_e^*} \quad \text{with} \quad \Psi = \psi \frac{1}{2} X^2 R_e^*^{-1}
\] (6.9)
with \( X \) (see Eq. 2.31) the dimensionless droplet size of the equivalent channel. Evaluating (2.33) by means of partial integration (see Appendix C.3), we obtained
\[
\vartheta = 1 + 2 \left( 1 - R_i^* \right)^{-1} \Psi^{-1} \left[ \left( 1 + \Psi^{-1} \right) e^{-\Psi} - \left( R_i^* + \Psi^{-1} \right) e^{-\Psi R_i^*} \right]
\] (6.10)
The (semi) turbulent rotating phase separator

Figure 6.3. Channel efficiency for three different channel profiles (see Table 6.1).

For example, a tube bundle (circular channels) that extends all the way to the center ($R_i^* = 0$, $R_e^* = \frac{2}{3}$, $\psi = 4/\pi$, $\Psi = 3x^2/\pi$) has, with a uniform flow distribution

$$\vartheta = 1 + \frac{2\pi}{3X^2} \left[ \left( 1 + \frac{\pi}{3X^2} \right) e^{-3X^2/\pi} - \frac{\pi}{3X^2} \right]$$

(6.11)

With this we conclude the derivation of simplified models. In sections 6.3 and 6.4 we will validate the models against results from experiments and DNS respectively.

6.2 Characteristics of rotating pipe flow

We now concentrate on the properties of the flow itself. To that end, consider a pipe (tube), rotating around an axis parallel to its own (Fig. 6.4).

6.2.1 Navier-Stokes equations

In the pipe, the fluid velocity $\vec{v}$ is dictated by the Navier-Stokes equations for incompressible flow. In a rotating frame of reference, these look like

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} + 2\vec{\Omega} \times \vec{v} = -\frac{1}{\rho_g} \nabla P_r + \nu_g \nabla^2 \vec{v}$$

(6.12)

with $\vec{\Omega}$ the angular velocity vector and $\nabla$ and $\nabla^2$ the Nabla and Laplace operators. Since we find ourselves in a rotating frame of reference, two fictitious forces appear: the Coriolis force and the centrifugal force. The Coriolis force gives rise to an additional term in the Navier-Stokes equations. The centrifugal force causes a radial pressure gradient, which is proportional to the distance from the rotation axis. In
6.2 Characteristics of rotating pipe flow

Figure 6.4. Rotating pipe

Eq. (6.12), this gradient is deducted from the total pressure gradient by the use of a reduced pressure [9]

\[ P_r = P - \frac{1}{2} \rho_g (\tilde{\Omega} \times \vec{x})^2 \]  

(6.13)

with \( P \) the static pressure and \( \vec{x} = \vec{R} + \vec{r} \) the position vector with respect to the rotation axis. Thus correcting the pressure field for centrifugal pressure buildup, the solution does not depend on the distance \( R \) of pipe center to rotation axis. The problem is therefore indifferent to that of a pipe rotating around its own axis [9].

6.2.2 Dimensionless numbers

In addition to the usual bulk Reynolds number \( Re \), the flow is characterized by a so-called rotation Reynolds number \( Re_{\Omega} \) [26]. Their ratio is the swirl parameter [17]

\[ S = \frac{Re_{\Omega}}{Re} \]  

(6.14)

Equivalent to the inverse Ekman and Rossby numbers respectively, \( Re_{\Omega} \) and \( S \) give a measure of the ratio of Coriolis forces to viscous forces and of Coriolis to inertial forces. Table 6.2 gives the definition of the three relevant numbers, with \( V \) mean axial velocity, \( D \) pipe diameter, \( \Omega \) angular velocity (rotation rate) and \( \nu_g \) kinematic viscosity of the fluid.

<table>
<thead>
<tr>
<th>Dimensionless numbers</th>
<th>Formula</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulk Reynolds number</td>
<td>( Re = \frac{VD}{\nu_g} )</td>
<td>inertia</td>
</tr>
<tr>
<td>rotation Reynolds number</td>
<td>( Re_{\Omega} = \frac{\Omega D^2}{4\nu_g} = Ek^{-1} )</td>
<td>Coriolis viscosity</td>
</tr>
<tr>
<td>Swirl parameter</td>
<td>( S = \frac{\Omega D}{4V} = Ro^{-1} )</td>
<td>Coriolis inertia</td>
</tr>
</tbody>
</table>
6.2.3 Flow stability

In a viscous flow, we can omit the left hand side of Eq. (6.12), resulting in a rotating Hagen-Poiseuille flow (RHPF). Figure 6.5 indicates the stability of RHPF in the $(Re_\Omega, Re)$ plane, as it follows from stability analysis [17, 26].

In a convectively unstable flow, spiral wave packets propagate downstream from a perturbation source with growing amplitude. Though RHPF is unstable, stable nonlinear solutions do exist in the form of (periodic) traveling spiral waves [5, 35]. Depending on the initial conditions, it may take a very long pipe to reach the final state [34]. As disturbances only travel downstream, an (infinitesimal) source must be maintained to keep RHPF from being restored.

As soon as perturbations start traveling upstream as well, we speak of an absolutely unstable flow. This is essentially different because the flow does not need axial length to develop any more, only time. Only a temporal perturbation will change the flow state forever. In the final state, a standing spiral wave sets in, composed of both up- and downstream traveling spiral waves [34].

In a stationary pipe ($Re_\Omega = 0$), RHPF is inherently stable according to stability analysis (see Fig. 6.5). As infinitesimal disturbances have no effect, the transition to turbulence (at $Re \approx 2100$) is inevitably attributed to finite amplitude disturbances [26]. It takes so long for disturbances to damp out, that in practical circumstances the flow is always turbulent. According to the latest insights, even a turbulent state is organized around a few dominant nonlinear traveling waves [19].

![Figure 6.5. Stability of Poiseuille pipe flow with superimposed solid body rotation [17]](image-url)
6.3 Experimental results

For unstable/turbulent flow at low rotation Reynolds numbers, we extensively measured the droplet separation efficiency in a rotating tube bundle. We used the air/water test setup described in section 3.2. Measuring concentrations and distributions of water droplets in the air outlet, the separation efficiency at a fixed airflow setting $Q$ was determined for increasing rotation speed $\Omega$, by taking the 0 rpm measurement as reference (see section 3.3.2). To be able to go across the stability boundary within the accessible operating range, we constructed a rotating element with $\varnothing$ 6.6 mm tubes (Table 3.2, element B). Except for the element, the situation is equal to chapter 5.

In this section we discuss the results of two sets of measurements (see Figure 6.6). The first set ($\star$) lies largely in the convectively unstable regime, and the second ($\circ$) goes into the turbulent regime (see section 6.2.3).

The operating range ($\Omega$, $Q$) of set 1 is equal to section 5.2. Channel entrance effects (refer to section 5.2.1) however play a much smaller role because the current channel height (6.6 mm) is much larger than the previous (1.9 mm). Measurements in the upper left corner of set 1 were left out because the channel efficiency in these points was too small to be measured accurately (these points are also affected a bit by channel entrance effects). Set 1 was repeated four times, with similar results.

Set 2 covers much larger flowrates, which causes some additional problems. First, due to the high gas volume flow, the mist (a constant amount) becomes considerably diluted. The background signal is already high at large gas flow velocities, so the

![Figure 6.6](image_url)  
**Figure 6.6.** Two sets of measurements: set 1 ($\star$) and set 2 ($\circ$). Numbered points (●) correspond to DNS (see Figure 6.13). $Re$ is based on mean element velocity.
signal to noise ratio becomes low. We therefore installed an extra ring of nozzles at the top flange to have more mist (to free the inlet, we also mounted the existing mist heads to the top flange). A second problem is that the mist saturates the air with water, wetting the internal walls of the setup within a few minutes. Even though we shortened the duration to only five successive rotation speeds, some of this liquid was entrained in the outflow for flowrate settings \( Q > 0.25 \text{ m}^3/\text{s} \). It was swept against the lens, so we had to clean it constantly. The entrained drops (0.5–1 mm in size), which are not in the ‘dry’ background signal, also cause a large signal on the first (low scattering angle) detectors (see sections 3.3.1 and 4.3). The MALVERN software has trouble to fit a line through both the powerful signal of these first detectors and the weaker signal of the later (large scattering angle) detectors that record micron size droplets. To solve this we had to start the fit at a detector quite far from the center (we “killed” a large number of detectors, see also section 4.3). Lastly, at the highest airflow setting, the TU/e campus compressor station had a hard time to keep up the large flowrate (600 g/s), so it fell slightly during the measurement. Notwithstanding these problems, set 2 shows only limited spread and gives clear curves.

All measurements in this section were done with the SPRAYTEC (large lens) to prevent vignetting (see sections 3.3.2 and 5.2.2). Side leakage (section 5.2.2) was estimated at only a few percent, which is too small to be noticed in the measured efficiency. Again we disregarded all results < 2 \( \mu \text{m} \) because the accuracy falls short on the left tail of the distributions (see section 3.3.2). Unfortunately, an unbalance in the rotating element stopped us from measuring above 1500 rpm.

In the following we discuss the results and analyze the effect of spiral waves/ turbulence on separation efficiency. As usual, all results are made dimensionless with \( d_{50} \) (Eq. 6.3). For the velocity \( V \) we use the element mean velocity (Eq. 2.29) and for the radius \( R \) we take the radial position of the equivalent channel (Eq. 2.30). Predictions in this section are always for a \textit{uniform} flow distribution over the rotation element (using Eqs. 2.33 and 2.34).

### 6.3.1 Convectively unstable flow

Figure 6.7 shows a selection of measurements from set 1 (indicated in red in Fig. 6.6). At four different bulk Reynolds numbers \( Re \), the curves are shown for increasing rotation Reynolds number \( Re_\Omega \) (RR). Compared to a stable Hagen-Poiseuille flow (section 5.1.2, Eq. 5.18), the efficiency decreases when \( Re_\Omega \) increases. Nonlinear spiral waves, which travel downstream through the rotating tubes (section 6.2.3), affect the efficiency in a negative sense.

The lower the bulk Reynolds number \( Re \), the stronger the drop in efficiency (at equal \( Re_\Omega \)). We therefore need a parameter that comprises both Reynolds numbers. The question arises how the efficiency depends on the swirl parameter \( S = Re_\Omega / Re \) (Eq. 6.14). To answer this, we grouped measurements with equal \( S \), and plotted the result in Figure 6.8. It turns out that, indeed, curves of equal \( S \) more or less match. To quantify this trend, we plotted the ratio of measured \( d_{50} \) (50% efficiency) to
6.3 Experimental results

Theoretical $d_{p50}$ (Eq. 6.3) as a function of $S$ for the complete set 1 (Figure 6.9). There turns out to be an almost linear relationship. Applying a simple fit, the original value of $d_{p50}$ (Eq. 6.3) can be corrected according to

$$d_{p50 \ (corr)} = (0.7 + 8S) \ d_{p50 \ (Eq. \ 6.3)} \quad (6.15)$$

Figure 6.10 shows all 40 measurements of set 1, made dimensionless with the new, corrected value of $d_{p50}$. We see that, after applying a relatively simple correction, the theoretical curve for stable Poiseuille flow (Eq. 5.18) can now also be used for (convectively) unstable flow.

Substituting the definitions of $S$ (Table 6.2) and $d_{p50}$ (Eq. 6.3), the corrected value of $d_{p50}$ (Eq. 6.15) is defined as follows:

$$d_{p50 \ (corr)} = \left( 0.7 \Omega + \frac{2D}{V} \right) \sqrt{\frac{9\mu_0 Vh}{(\rho_p - \rho_g) RL}} \quad (6.16)$$

The most striking aspect of this result is that, when the rotation speed $\Omega \to \infty$, the cut size $d_{p50}$ approaches a minimum value. To understand what this means for

Figure 6.7. Measured efficiency at four different flowrates (set 1). RR means $Re_{\Omega}$. The prediction is for a stable Hagen-Poiseuille flow (Eq. 5.18).
Figure 6.8. Measured efficiency curves (set 1), classified by $S = Re_\Omega/Re$.

Figure 6.9. Ratio of measured to theoretical value of $d_{p50}$, as a function of $S$ (set 1).

Figure 6.10. Measurements of set 1 (40 curves), corrected according to Eq. (6.15).
practical applications, we look at the overall efficiency. Figure 6.11 shows the overall efficiency as a function of dimensionless rotation speed $\Omega/\Omega_0$, with nominal speed

$$\Omega_0 = \frac{1}{\text{MMD}_0} \sqrt{\frac{9\mu_g V h}{(\rho_p - \rho_g) RL}}$$

(6.17)

and MMD$_0$ the mass (or volume) median diameter of the 0 rpm reference distribution. In section 2.5 (Eqs. 2.39–2.40) we have seen that $\Omega/\Omega_0$ is the same as MMD$_0/d_{p50}$: the reference median diameter made dimensionless with the original $d_{p50}$ (Eq. 6.3).

Figure 6.11 clearly shows that the efficiency levels off, and approaches 100% much slower than a stable Poiseuille flow (the prediction). The lower Re (higher $S$), the stronger the effect. In terms of rotation speed, a large effort is needed to get out the last 10% ($v/v$) of the mist droplets. In a practical separator it will feel as if the efficiency reaches a ceiling and does not increase above a certain ‘critical’ rotation speed. This is the practical implication of having a minimum value to $d_{p50}$ (Eq. 6.16).

Somewhat surprisingly, we did not observe a sharp difference upon crossing the stability boundary at $Re_{\Omega} = 27$ (e.g. in Fig. 6.7, $Re_{\Omega} = 15$ is stable, 30 is just over the boundary). However, one has to realize that the stability boundary (Fig. 6.6) is nothing more than the limit at which perturbations neither damp out, nor amplify. The closer to this “neutral curve”, the longer it takes for a disturbance to either damp out or amplify. If we expect that disturbances will damp out in the end, it does not mean the stable state is reached within the length of our channels ($L = 106D$). After all, the gas enters the channels with a large disturbance, due to a mismatch in tangential gas velocity between pre-separator and element (section 5.2.1).

Summarizing, the occurrence of traveling spiral waves has a price, which rises with the swirl parameter $S = Re_{\Omega}/Re$. The swirl parameter is a measure of the ratio of Coriolis forces to inertial forces (see Table 6.2). To design, (6.16) can be used for $d_{p50}$.

### 6.3.2 Turbulent flow

In this section we discuss set 2 (see Fig. 6.6). In Figure 6.12 we ordered the results in groups of equal flowrate (or bulk Reynolds number $Re$). As long as $Re < 2100$, we are still in the convectively unstable regime, and the efficiency is below the prediction for Poiseuille flow. Going from $Re = 856$ to 1713, the $Re_{\Omega}$-range remains the same, so that $S = Re_{\Omega}/Re$ decreases and the efficiency gets closer to a stable Hagen-Poiseuille flow. We discussed this in section 6.3.1. From $Re = 856$ to 2569, we pass the laminar/turbulent transition (around $Re = 2100$). Surprisingly, we see that the laminar efficiency is fully restored.

Contrary to what we expect, for turbulent flow (the highest three flowrates) the efficiency is closer to the plug flow model without mixing than to the continuous mixing model (see section 6.1). At the measured conditions, turbulent fluctuations hardly affect the separation, causing the apparatus to run almost as if it were in the stable laminar regime.
6.4 Direct numerical simulations

In order to obtain a better understanding of droplet separation in unstable/turbulent flow, we calculated the separation efficiency by computer simulations. This also allowed us to raise the rotation Reynolds number.

To compute the flow field, we made use of a computer program based on so-called direct numerical simulation (DNS) of the Navier-Stokes equations. Originally, the code has been built to compute the statistics of nonrotating turbulent pipe flow [41], for which the output agrees well with experiments and other DNS [42]. It was extended later to rotating pipes [23]. In short, the program solves Eq. (6.12) in cylindrical coordinates and in a vorticity formulation, imposing no-slip at the wall and fixing the axial pressure gradient so as to keep constant volume flow. The solution is independent on center distance $R$ to the rotation axis (see section 6.2.1). For details refer to [23, 36].

The code computes the flow in a finite domain ($\ell = 5D$ in turbulent flows), using periodic boundary conditions in the streamwise direction. Starting from a ‘large disturbance’ (the turbulent field $Re = 5300$, $Re_\Omega = 980$ [36]), a statistically stationary state is reached after a large number of time steps. By definition, this solution is periodic in the axial direction. It is therefore impossible to compute a spatially developing flow. The code always tries to find a final, fully developed flow state, which can be ‘glued’ to itself an infinitesimal number of times.

To compute the separation efficiency, passive point particles are inserted in the flow. They neither influence each other, nor the flow (so-called one-way coupling). We solved the complete equation of motion (Eq. 2.7) for each particle (Lagrangian method), including particle inertia and a correction for non-Stokesian behavior (see Table 2.1). In contrast to the flow field, the centrifugal (body) force in the equation of motion does depend on distance $R$ to the rotation axis.

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<table>
<thead>
<tr>
<th>$Re$</th>
<th>MMD0</th>
<th>Overall Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>214</td>
<td>7.4</td>
<td>0.62</td>
</tr>
<tr>
<td>428</td>
<td>6.2</td>
<td>0.80</td>
</tr>
<tr>
<td>642</td>
<td>5.3</td>
<td>0.88</td>
</tr>
<tr>
<td>856</td>
<td>4.8</td>
<td>0.90</td>
</tr>
<tr>
<td>1070</td>
<td>4.6</td>
<td>0.92</td>
</tr>
</tbody>
</table>

Figure 6.11. Overall efficiency (set 1) as a function of dimensionless rotation speed. Predictions, based on measured input distributions (Eq. 2.25), hardly change with $Re$. 

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Initially, 25000 particles of equal size are homogeneously distributed throughout the domain. Traversing the periodic domain as many times as needed, they are tracked until they either reach the wall, or travel a distance \( L \). Since separation efficiency goes with particle influx, each particle is given a weight according to the axial velocity at its initial position \( v_0 = v_z (\bar{x}_0) \). Counting and weighing the separated particles gives the efficiency in a rotating pipe of length \( L \):

\[
\eta = \frac{1}{25000} \sum \left( \frac{v_0}{V} \right)
\]

This is repeated for 16 different particle sizes \( d_p = 1, 2, \ldots, 16 \times 0.2d_{p50} \), where \( d_{p50} \) (Eq. 6.3) can be written in terms of \( Re \) and \( Re_\Omega \) as follows

\[
d_{p50} = \frac{\sqrt{9 Re}}{16 \left( \frac{\rho_p}{\rho_g} - 1 \right) Re_\Omega^2 \left( R/D \right) \left( L/D \right)}
\]

Recall that \( D \) is pipe diameter, \( R \) center distance to rotation axis, \( \rho_p \) the particle

\[\text{Figure 6.12. Measurements (set 2), going into the fully turbulent regime.}\]
(droplet) density and \( \rho_g \) the fluid density. The simulation parameters \((R = 20.4D, L = 106D, \rho_p/\rho_g = 833)\) are chosen to correspond to the equivalent channel (Eq. 2.30) of the experimental element (Table 3.2). We analyzed the outcome at a number of different conditions (see Figure 6.13). Points 1–5 result in a time-dependent laminar flow, dominated by traveling nonlinear spiral waves (see section 6.2.3). Points 6–12 correspond to a turbulent flow.

![Figure 6.13. Conditions of simulations, and the three most unstable modes with \( n \) the modes’ azimuthal wave number [17]. The box is the axis limit of Figure 6.6.](image)

### 6.4.1 Spiral wave flows

A simulation domain of length \( \ell = 5D \) is sufficient for the statistics of turbulent flows [23, 42]. If the flow is not yet turbulent but nevertheless unstable (see section 6.2.3), this choice is not so straightforward. For example, consider point 1 in Fig. 6.13. Here the flow ends up as stable RHPF if we take \( \ell = 5D \). Apparently, the axial wavelength of none of the unstable modes at point 1 fits in a domain of length \( \ell = 5D \). Figure 6.13 shows the first three unstable modes, corresponding to the azimuthal wave numbers \( n = -1, -2, -3 \) [17, 34], \( n = -1 \) being the most unstable mode of Figure 6.5.

The grid has been optimized for the small fluctuations of turbulent flow in a domain of length \( \ell = 5D \). But for spiral wave solutions (section 6.2.3), we need not such a fine grid resolution. This allows us to stretch the complete domain to a larger length, and study the effect on the computed separation efficiency. As shown in Fig. 6.14, elongating the domain to \( \ell = 10D \) indeed destabilizes the flow and affects the separation efficiency. A further increase to \( \ell = 20D \) causes the efficiency to
6.4 Direct numerical simulations

Figure 6.14. Influence of domain length $\ell$ at point 1 (see Figure 6.13). $Re = 2005$, $Re_\Omega = 150$, $L = 106D$, $R = 20.4D$, $\rho_p = 833\rho_g$ (RHPF see Eq. 5.18).

Figure 6.15. Separation efficiency at points 2–5. $Re = 430$, $\ell = 21.2D$, $L = 5\ell = 106D$, $R = 20.4D$, $\rho_p = 833\rho_g$ (RHPF and mixing by Eqs. 5.18 and 6.8).
decrease even more, indicating that we have called in (an)other unstable mode(s) (see Fig. 6.13). As an increase to \( \ell = 30D \) has no effect on the separation, it seems that all the unstable modes have been activated at \( \ell = 20D \).

Next, we investigated the effect of an increasing rotation Reynolds number \( Re_\Omega \) at constant bulk Reynolds number \( Re = 430 \) (points 2–5 in Fig. 6.13) with \( \ell = 21.2D \). Fig. 6.16 shows the distribution of vorticity. At 2–4 the vorticity distribution is very similar, but increases in strength. The distribution pertains to a downstream traveling spiral wave, probably corresponding to the most unstable mode \( n = -1 \) [34]. At point 5 we enter the absolutely unstable regime, where a standing spiral wave sets in. The vorticity distribution at 5 \( (Re_\Omega = 600) \) also seems slightly more complicated.

The effect of the nonlinear spiral waves on droplet separation is rather unpredictable, as shown in Figure 6.15. Affected by alternating spiral fluid motions (both positive and negative vorticity, see Fig. 6.16), droplets approach the wall in a zigzag slinging motion [36]. Compared to a laminar RHPF (radially straight droplet trajectories, see section 5.1), this affects the efficiency in a negative sense.

Increasing \( Re_\Omega \) from 76 to 150 (2–3), the separation efficiency levels off at large droplet sizes. Surprisingly, increasing it further to 300 and 600 (4 and 5), the efficiency seems to be restored. At \( Re_\Omega = 600 \) (5), the efficiency exactly coincides with the mixing model (section 6.1.2). It appears that the spiral motions in the absolutely unstable flow of point 5 cause continuous mixing of droplets in the transection.

### 6.4.2 Turbulent flow

Subsequently, we proceed to turbulent flow. At first approximation, the velocity profile resembles a plug flow (see Figure 6.17). Figure 6.18 clearly shows that the nature of the flow (at \( Re = 3440 \)) is very different from the spiral wave type regime (compare Fig. 6.16). The vorticity distribution is chaotic, and features the typical turbulent ‘spots’. When \( Re_\Omega \) accrues the vorticity fortifies, indicating that the fluctuating velocities, responsible for dispersion (mixing), intensify [36]. Figure 6.19 shows the corresponding separation efficiency (at points 6–8).

![Image of fluid flow](image.png)

**Figure 6.16.** Instantaneous plot of the distribution of vorticity \( \omega \) (s\(^{-1}\)) at 2–5. \( Re = 430 \).
At $Re_\Omega = 1000$ and 2000 ($7$ and $8$), the efficiency more or less follows Eq. (6.8), indicating that it satisfies the assumption of continuous mixing in the radial plane (section 6.1.2). Despite the fact that $Re_\Omega$ doubles from $7$ to $8$, the efficiency does not decrease. This is not surprising in view of continuous mixing: it is not possible to have ‘more than full mixing’. More violent dispersion therefore does not affect the efficiency any further. On that score, Eq. (6.8) declares a lower limit to the efficiency.

At $Re_\Omega = 103$ ($6$) the efficiency lies higher. Even though we have a turbulent flow, dispersion seems too weak to dominate the separation efficiency. The result starts to approach plug flow without mixing (section 6.1.1). Note that without mixing the efficiency becomes sensitive to the profile of axial velocity, which always deviates somewhat from plug flow (see Fig. 6.17).

Instead of looking at the separation of large sets (25000) of droplets of the same size between entrance and exit of a channel of length $L$, we can also follow the deposition of droplets of one size throughout the channel. We actually used this approach in section 6.1. Accordingly, we recorded the axial location $z$ at which each of the 25000 droplets of size $2d_{p50}$ reached the wall, and plotted the cumulative efficiency (after weighing for $v_0$) in Fig. 6.20. The droplet deposition at $7$ and $8$ almost exactly follows

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig6.17}
\caption{Axial velocity profile at (6). $Re = 3440$, $Re_\Omega = 103 (\ell = 5D)$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig6.18}
\caption{Instantaneous plot of the distribution of vorticity $\omega$ (s$^{-1}$) at 6–8. $Re = 3440$.}
\end{figure}
Figure 6.19. Separation efficiency at points 6–8. $Re = 3440, \ell = 5D$, $L = 106D, R = 20.4D, \rho_p = 833\rho_g$ (predictions see Table 6.1).

Figure 6.20. Axial deposition of droplets of size $x_L = 2$ (see Fig. 6.19).
Figure 6.21. Separation efficiency in a square channel (point 9). $Re = 5110$, $Re_\Omega = 205$, $\ell = 6D$, $R = 26.7D$, $\rho_p = 22.5\rho_g$ (predictions see Table 6.1).

Figure 6.22. Axial deposition of droplets of size $x_L$ based on $L = 101D$ (see Fig. 6.21).
that of continuous mixing (see Eq. 6.7), whereas 6 again demonstrates a behavior that is more towards plug flow without mixing.

As a further validation, we considered a square channel (9). We used a (commercial) DNS code for turbulent flow in square channels, and added 10000 particles per size category. This time we followed the droplets over a very large axial distance (up to 1000D). Again we saved the location at which each particle is separated, so that we can later ‘chop’ the channel at any point. Figure 6.21 shows the result for channels of varying length. Clearly, short channels \((L = 25D)\) approach the plug flow model and longer channels follow the mixing model. At \(L = 600D\) we reach full mixing: the (normalized) efficiency drops no further if the channel length increases.

Figure 6.22 shows the accompanying axial deposition for varying droplet size. Note that, in a square channel, plug flow gives a constant droplet deposition rate. We see that relatively small droplets listen to the mixing model, whereas large droplets tend towards plug flow without mixing. In view of Fig. 6.21 this is logical, because large droplets are separated at short length. And if long channels satisfy the mixing model, so do small droplets. Basically, the results of Figs. 6.21 and 6.22 are interchangeable.

We also assessed the results of earlier publications [23, 36] (points 10–12) in Fig. 6.23. They agree well with the current mixing model (Eq. 6.8). Also in these cases, continuous mixing dominates the separation of droplets, causing it to obey Eq. (6.8) virtually irrespective of the Reynolds numbers. Even though the strength of the internal ‘counterrotating vortex’, which is present in the flow, increases proportional to \(Re\Omega\), the efficiency is hardly affected [36].

![Figure 6.23. Separation efficiency from [23, 36] (points 10–12).](image)

\(Re = 5300, \ell = 5D, L = 133.5D, R = 26.7D, \rho_p = 22.5\rho_g.\)
6.5 Discussion

We started this chapter with introducing two basic models (section 6.1). In the first we assumed plug flow and straight droplet paths, in the second we assumed continuous mixing or a uniform droplet concentration in the channel cross-section. In case mixing plays no role, the velocity profile in the channels also matters. In that sense, the Poiseuille models of section 5.1 are essentially a special case of the non-mixing approach (section 6.1.1). Figure 6.24 shows all three types of predictions for circular channels (tubes). In section 6.2.3 we identified different types of unstable flow: convectively unstable flow, absolutely unstable flow and turbulent flow.

Subsequently, we determined the separation efficiency in rotating channels using experiments (section 6.3) and direct numerical simulations (section 6.4). In the experiments, the rotation Reynolds number was relatively low ($Re_{\Omega} < 120$). Because DNS is not limited to a certain operating range, we used it to extend the range of $Re_{\Omega}$. At points 2 and 6 (compare Figs. 6.13 and 6.6) we have both measurements and DNS. Experiments are done with a bundle of tubes, whereas the DNS models a single pipe. The simulation parameters correspond to the equivalent channel (see section 2.5).

In the convectively unstable regime, we identified a very clear trend in the experimental results. We defined a new, corrected value of $d_{p50}$ (Eq. 6.16), which can be used to design. Figure 6.24 shows that DNS overestimates the efficiency in convectively unstable flow (point 2). Most likely, this is due to the periodic boundary conditions of the DNS. It yields a (developed) flow field that is periodic in the axial direction, while in reality (measurements) the flow is developing axially within the channel. Moreover, the gas is bent off when it enters the channel (section 5.2.1), causing a large disturbance that grows/decays downstream.

![Figure 6.24. DNS, experiments and models at points 2 and 6 (see Figs. 6.6 and 6.13).](image-url)
Periodicity in the DNS also caused another, unexpected difficulty. Each unstable mode has its own axial wavelength, and in order to nudge an unstable mode, its length has to fit in the simulation domain (section 6.4.1). What we would really need to do is to calculate the wavelength of each unstable mode [17], and to give the domain at least that length. An even better alternative would be to simulate a long finite pipe, in which the flow is left to evolve freely after a pressure difference is set. Sanmiguel-Rojas and Fernandez-Feria [34] applied this technique to rotating pipes of length $100D$, which is very similar to the channels in our test unit ($L = 106D$). They started from very small disturbances (numerical noise). To study droplet separation, we would have to add Lagrangian particle tracking, and, if possible, an option to introduce a large disturbance at the pipe entrance.

Despite these numerical shortcomings and the departure from measurement results, we used the DNS to increase $Re_\Omega$ up to absolutely unstable flow (section 6.4.1). Interestingly, when we entered the absolutely unstable regime, the efficiency was restored and corresponded to the mixing model (Fig. 6.15). Apparently, the complex standing spiral wave pattern in absolutely unstable flow [34] starts causing continuous mixing of droplets. Unfortunately, we were not able to verify this by measurements, as we were limited in the rotation speed of the element. This leaves the absolutely unstable regime as an unexplored, interesting subject for further research.

Reverting to the experiments, when we drove up the flowrate into the turbulent regime, this yielded counterintuitive results (section 6.3.2). Though we would expect mixing to dominate, the curves corresponded better to the plug flow model, in which mixing was disregarded. Resorting to DNS, which is very accurate for turbulent flows, this is confirmed: Figure 6.24 shows a striking resemblance of DNS and measurement at point 6. Even though the flow is turbulent, we may conclude that dispersion is too weak to affect the efficiency. Or, the other way around, the settling velocity is so large that the path of droplets is hardly affected by dispersion/mixing.

However, when we increased $Re_\Omega$ with turbulent flow in the DNS, the efficiency coincided with the continuous mixing model (Fig. 6.19). Apparently, at these moderate bulk Reynolds numbers, the rotation Reynolds number must be large enough to get enough mixing. After increasing $Re_\Omega$ further, the efficiency, as a function of the dimensionless droplet size $d_{p}/d_{p50}$, does not further decrease. This is in line with the assumption of continuous mixing: once the droplets are mixed up uniformly throughout the cross-section, more mixing has no effect. We conclude that, as soon as mixing dominates, the continuous mixing model sets a lower limit for the efficiency. Again, we could not verify this experimentally, because the current setup does not reach such high rotation Reynolds numbers.

A more detailed analysis of the DNS results, in which we followed the deposition of individual droplets, provided an even closer view on mixing behavior (section 6.4.2). We saw that, at any combination of ($Re$, $Re_\Omega$), mixing plays a role for relatively small droplets, or, equivalently, at relatively large channel lengths. Basically, fluctuations of the fluid velocity have to be large enough to affect the settling velocity of the typically separated droplet size (around $d_{p50}$).
Conclusions
Drawings and pictures

A.1 Prototype

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A.2 Setup
Matlab code

Matlab m-file to evaluate Eq. (2.26): the efficiency of a rotating element.

```matlab
Ri = 0.5;
GSD = [1 1.5 2 2.5];
x = linspace(0,10,301)';

Re = 2/3*(1−Ri^3)/(1−Ri^2);
Xo = sqrt(2*Re);
Xi = sqrt(2*Re/Ri);

% IDEAL INFLOW
% coef = [0.5 2 0 sqrt(2)
%          1 0 sqrt(2) inf];

% UNIFORM INFLOW
coef = [0.5 2 0 Xo
        1/(1−Ri^2) 0 Xo Xi
       −4/3*Re^2/(1−Ri^2) −4 Xo Xi
       −1/3*Ri^3/Re/(1−Ri^2) 2 Xo Xi
          1 0 Xi inf];

[X,sigma,a] = ndgrid(x,log(GSD),coef(:,1));
n = repmat(shiftdim(coef(:,2),−2), [size(a,1) size(a,2) 1]);
x1 = repmat(shiftdim(coef(:,3),−2), [size(a,1) size(a,2) 1]);
x2 = repmat(shiftdim(coef(:,4),−2), [size(a,1) size(a,2) 1]);

y1 = (log(x1./X) − n.*sigma.^2)./(sigma*sqrt(2));
y2 = (log(x2./X) − n.*sigma.^2)./(sigma*sqrt(2));

EFF = sum(a.*X.^n.*exp(0.5*(n.*sigma).^2).*(-erf(y1)+erf(y2)),3);
EFF(1,:) = 0;

plot(x,EFF)
xlabel('\Omega/\Omega_0')
ylabel('\eta_{\infty}', 'Rotation', 0.0)
```

Appendix

Matlab code

Matlab m-file to evaluate Eq. (2.26): the efficiency of a rotating element.
Extended derivations

C.1 Triangular channels (laminar)

For equilateral triangles $b = 2h/\sqrt{3}$ [see 6, p. xx]

$$
\hat{v}(\zeta, \xi) = \frac{15}{8} (\xi + 1) \left( \xi^2 - 2\xi + 1 - 4\zeta^2 \right)
$$

(C.1)

Redefine coordinates $(\varphi, \psi)$ as $\varphi = 1 - |\zeta|$ and $\psi = (\xi + 1)$ so that the origin moves to the two bottom corners and $0 \leq \varphi \leq 1$ and $0 \leq \psi \leq 2$

$$
\hat{v}(\zeta, \xi) = \frac{15}{8} \psi \left( \psi^2 - 4\psi + 8\varphi - 4\varphi^2 \right)
$$

(C.2)

Plane velocity

$$
\bar{v}(\zeta) = \frac{1}{2} \int_{-1}^{1-2|\zeta|} \hat{v}d\xi = \frac{1}{2} \int_{0}^{2\varphi} \hat{v}d\psi = \frac{15}{16} \psi^2 \left( \frac{1}{4} \psi^2 - \frac{4}{3} \psi + 4\varphi - 2\varphi^2 \right) \bigg|_0^{2\varphi} = \frac{5}{4} (4 - 3\varphi) \varphi^3 = \frac{5}{4} (1 + 3|\zeta|) (1 - |\zeta|)^3
$$

(C.3)

And since $\delta = 2 - 2|\zeta| = 2\varphi$

$$
\int_{\zeta_{100}}^{1} \bar{v}\delta d\zeta = -\int_{0}^{\varphi_{100}} \bar{v}\delta d\varphi = \frac{5}{2} \int_{0}^{\varphi_{100}} (3\varphi - 4) \varphi^4 d\varphi = \left( \frac{5}{4} \varphi_{100} - 2 \right) \varphi_{100}^5
$$

(C.4)
Now because $\alpha = 2$ and $\zeta_{100} = 1 - \varphi_{100}$

$$
\eta_{ch} = \begin{cases} 
(1 - \varphi_{100}) x^2 + (5 \varphi_{100}/4 - 2) \varphi_{100}^5 & \text{if } x < \sqrt{5/2} \\
1 & \text{if } x \geq \sqrt{5/2}
\end{cases}
$$

(C.5a)

(C.5b)

and according to (??) with $\delta = 2\varphi$

$$(4 - 3 \varphi_{100}) \varphi_{100}^4 - 2x^2/5 = 0
$$

(C.6)

### C.2 Circular channels (laminar)

$$
\eta = \frac{1}{\alpha} \int_{-1}^{1} E\langle v^* \rangle \delta d\zeta = \frac{2}{\alpha} \int_{0}^{1} E\langle v^* \rangle \delta d\zeta
$$

$$
= \frac{2}{\alpha} \left[ \int_{0}^{\zeta_o} x^2 d\zeta + \int_{\zeta_o}^{1} \langle v^* \rangle \delta d\zeta \right]
$$

$$
= \frac{2}{\alpha} \zeta_o x^2 + \frac{2}{\alpha} \int_{\zeta_o}^{1} \langle v^* \rangle \delta d\zeta \quad \text{for } x < \sqrt{2\langle v_0^* \rangle}
$$

(C.7)

$$
\int_{\zeta_o}^{1} \langle v^* \rangle \delta d\zeta = \int_{\zeta_o}^{1} \frac{8}{3} (1 - \zeta^2)^{3/2} d\zeta
$$

$$
= \left[ \frac{2}{3} \zeta \sqrt{1 - \zeta^2} \left( \frac{5}{2} - \zeta^2 \right) + \arcsin \zeta \right]_{\zeta_o}^{1}
$$

$$
= -\frac{2}{3} \zeta_o \sqrt{1 - \zeta_o^2} \left( \frac{5}{2} - \zeta_o^2 \right) + \frac{\pi}{2} - \arcsin \zeta_o
$$

(C.8)

### C.3 Continuous mixing (uniform flow distribution)

Using partial integration to evaluate Eq. (2.33), with $\eta$ according to Eq. (6.9)

$$
\psi = \frac{2}{1 - R_i^*} \int_{R_i^*}^{1} \eta r^* dr^*
$$

$$
= 1 - \frac{2}{1 - R_i^*} \int_{R_i^*}^{1} r^* e^{-\Psi r^*} dr^*
$$

$$
= 1 + \frac{2}{1 - R_i^*} \Psi^{-1} \left( r^* e^{-\Psi r^*} \bigg|_{R_i^*}^{1} - \int_{R_i^*}^{1} e^{-\Psi r^*} dr^* \right)
$$

$$
= 1 + \frac{2}{1 - R_i^*} \Psi^{-1} (r^* + \Psi^{-1}) e^{-\Psi r^*} \bigg|_{R_i^*}^{1}
$$

$$
= 1 + 2 \left( 1 - R_i^* \right)^{-1} \Psi^{-1} \left[ (1 + \Psi^{-1}) e^{-\Psi} - (R_i^* + \Psi^{-1}) e^{-\Psi R_i^*} \right]
$$

(C.9)
For $R_i^* = 0$ this simplifies to

$$\vartheta = 1 + 2\Psi^{-1} [(1 + \Psi^{-1}) e^{-\Psi} - \Psi^{-1}]$$

(C.10)
Leak flow design formulae
Vignetting

Mastersizer S

illuminated spray volume in our setup
36 mm (working range)

Spraytec

150 mm
Airflow 150 g/s, rotational speed 800 rpm

- Spraytec
- Mastersizer S

Detector number

Fraction (% v/v) vs. $d_p$ (μm)
References


