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Kuerten, J.G.M.; van der Geld, C.W.M.; Geurts, B.J.

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Turbulence modification and heat transfer enhancement by inertial particles in turbulent channel flow

J. G. M. Kuerten,1,2,a) C. W. M. van der Geld,1 and B. J. Geurts2,3
1Department of Mechanical Engineering, Eindhoven University of Technology, P.O. Box 513, NL-5600 MB Eindhoven, The Netherlands
2Faculty EEMCS, University of Twente, P.O. Box 217, NL-7500 AE Enschede, The Netherlands
3Department of Technical Physics, Eindhoven University of Technology, P.O. Box 513, NL-5600 MB Eindhoven, The Netherlands

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We present results of direct numerical simulation of turbulence modification and heat transfer in turbulent particle-laden channel flow and show an enhancement of the heat transfer and a small increase in the friction velocity when heavy inertial particles with high specific heat capacity are added to the flow. The simulations employ a coupled Eulerian-Lagrangian computational model in which the momentum and energy transfer between the discrete particles and the continuous fluid phase are fully taken into account. The effect of turbophoresis, resulting in an increased particle concentration near a solid wall due to the inhomogeneity of the wall-normal velocity fluctuations, is shown to be responsible for an increase in heat transfer. As a result of turbophoresis, the effective macroscopic transport properties in the region near the walls differ from those in the bulk of the flow. To support the turbophoresis interpretation of the enhanced heat transfer, results of simulations employing no particle-fluid coupling and simulations with two-way coupling at considerably lower specific heat, or considerably lower particle concentration are also included. The combination of these simulations allows distinguishing contributions to the Nusselt number due to mean flow, turbulent fluctuations and explicit particle effects. We observe an increase in Nusselt number by more than a factor of two for heavy inertial particles, which is the net result of a decrease in heat transfer by turbulent velocity fluctuations and a much larger increase in heat transfer stemming from the mean temperature difference between the fluid and the particles close to the walls. © 2011 American Institute of Physics. [doi:10.1063/1.3663308]

I. INTRODUCTION

Scalar transport in turbulent flows of a fluid and particles with diameters in the millimeter range is important in the processing of food or minerals, pulverized coal combustion, air pollution control, and energy conversion industry.1 A key element is to understand the interaction between turbulence, heat transfer, and particle inertia for such particle-laden flows. Of course, the heat conductivity of the particles affects heat transfer of the mixture flow. For particle sizes up to 0.3 μm, enhanced conductivity was measured. Recent experimental data show that the thermal conductivity of these so-called nanofluids increases with increasing particle size.2 Although flows of bigger particles display settling in horizontal flows, as well as an increased pressure drop, it is interesting to investigate whether the heat transfer augmentation observed for micro-sized particles larger than 0.3 μm continues to considerably larger particles with diameters in the millimeter range in turbulent flow. This is the focus of the present study, which aims at a clear interpretation of the effect of turbulence on heat transfer in particle-laden flows.

Gravity is neglected to highlight the interaction of turbulence, heat transfer, and particles without the additional effect of particle deposition. Further details on the effect of settling velocity of particles in a suspension on heat transfer can be found in Blanchette et al.3

Particle size in the present study is typically taken to be 120 μm and particle relaxation time, τp, is such that inertia effects are noticeable. We find that heat transfer is significantly enhanced in particle-laden flow that involves particles with strong inertial effects. In both laminar and turbulent flows, inertia effects are known to cause a preferential localization of particles with respect to the wall. In a Poiseuille flow, a high concentration of high-density solid particles occurs at about 60% of the radius of the tube4,5 by the combined action of particle inertia and lift force. In turbulent flow, turbophoresis causes peak concentrations closer to the wall.5,7 For a homogeneous particle distribution at the inlet of a channel, it takes about 6 s to reach a fully developed concentration profile in the typical case of a channel width of 4 cm, a friction velocity of 0.12 m/s and a kinematic viscosity of 1.57 × 10⁻⁵ m²/s. However, particle concentrations near the wall will be seen to be noticeably higher already 1 s after entry or, typically, beyond a channel length of 1.80 m for the various heat transfer cases investigated. The flows studied are, therefore, of direct practical importance.

In the present study, we treat the particles effectively as point particles. Regarding temperature, this implies that it is taken uniformly over the particle. This may readily be motivated, realizing that even if the thermal properties of water are taken for the particles, the particle Biot number is typically 2.4 × 10⁻⁴ and the Fourier number at time τp, which is
about 35 ms, is about 1.4. Heat diffusion inside the particle is, therefore, sufficiently large to result in a homogeneous particle temperature.\(^8\)

A study of Mansoori et al.\(^9\) with an Eulerian/Lagrangian four-way coupled model showed that inter-particle heat transfer has little effect on mean temperatures of gas and particles. The present study applies two-way coupling only and ignores the direct inter-particle interactions. Results of Vreman et al.\(^7\) can be used to show that the effect of particle collisions is still small for particle volume fractions below 1%, which motivates our use of two-way coupling as the dominant mechanism for particle-fluid interaction.

The physical properties of particles and carrier fluid chosen here have bearings to a vertical water droplet laden flow of air. Measurements of such a flow were performed by Hishida et al.\(^10\). A considerable heat transfer enhancement in comparison with single-phase flow was found, even for a rather low droplet mass-loading ratio of 4.1%. Results proved to be well correlated with the particle inertia as expressed by its Stokes number. Hetsroni et al.\(^11\) considered particles with a mass density close to that of the embedding fluid and observed enhancement of heat transfer in particle-laden turbulent flow in a flume. Two-way coupled DNS (direct numerical simulation) computations were validated with experiments. The wall-normal turbulent heat flux was found to increase with increasing particle diameter and correlations of temperature fluctuations and velocity fluctuations were related to heat transfer and “film scraping.”

To date, no Eulerian-Lagrangian DNS simulations of inertial particle-laden turbulent channel flow with two-way coupling and heat transfer have been reported in the literature. A recent study of Abdolzadeh and Mehrabian\(^12\) employed an Eulerian-Eulerian 2D approach with one-way coupling to model deposition of particles near a single wall. Yuge and Hagiwara\(^13\) applied a modified VOF (volume of fluid) algorithm to model the motion of four drops carried along with a turbulent upward flow between two heated walls for a short period of time. Enhancement of heat transfer was reported and related to the product of wall-normal velocity fluctuations and temperature fluctuations. The same quantity will be computed in the present study, but now at a much higher particle number density of \(1.6 \times 10^9 \text{ m}^{-3}\).

The structure of the paper is as follows. After introduction of the governing equations, the numerical method is described. Then, full solutions are presented for four cases, one of which is defined as a reference case in which two-way coupling of momentum and energy transfer is studied, involving particles with high specific heat capacity and at a relatively high mass density. These results will be analyzed with the aid of an analytical solution of Reynolds-averaged equations. The other three simulations quantify the effects of separately varied particle number density and particle specific heat. It will be shown that the four cases selected exhibit a parameter variation that is sufficient to foster a clear understanding of the interaction of various heat transfer augmentation trends that may occur.

### II. GOVERNING EQUATIONS

In this section, we present the mathematical model for turbulent non-isothermal flow of an incompressible gas laden with particles in a channel, i.e., between two infinite flat plates. An Eulerian-Lagrangian formulation is adopted, distinguishing a continuous fluid phase, and a discrete particle phase in which each particle is described by a set of equations for its position, velocity, and temperature.

Temperature variations in the gas are assumed to be sufficiently small so that their effect on the mass density of the gas and on the flow is negligible. Moreover, it is reasonable to assume the gas to be incompressible. The governing equations for the gas consist of the incompressibility condition, the Navier-Stokes equation and a convection-diffusion equation for gas temperature

\[
\nabla \cdot \mathbf{u}_g = 0, \quad (1)
\]

\[
\frac{\partial \mathbf{u}_g}{\partial t} + \frac{1}{\rho_g} \nabla p = -\omega \times \mathbf{u}_g + \frac{\mu}{\rho_g} \nabla^2 \mathbf{u}_g + \mathbf{a}_p, \quad (2)
\]

\[
\frac{\partial T_g}{\partial t} + \mathbf{u}_g \cdot \nabla T_g = \frac{k}{\rho_g C_{p,g}} \nabla^2 T_g + q_p. \quad (3)
\]

In these equations, \(\rho_g\) and \(\mathbf{u}_g\) denote gas mass density and velocity, \(p\) total pressure, \(\omega\) vorticity, \(T_g\) gas temperature, and \(t\) time. Moreover, \(\mu, k, C_{p,g}\) are the dynamic viscosity, heat conduction coefficient, and specific heat at constant pressure of the gas, respectively. The equations are made non-dimensional using the gas mass density, half the channel height, \(H\), and the friction velocity, \(u_c\), as mass, length, and velocity scale, so that \(Re_f\) is the friction Reynolds number given by

\[
Re_f = \frac{\rho_g H u_c}{\mu}. \quad (4)
\]

In the results, time is made dimensionless with timescale \(\tau_g = \mu/\rho_g u_c^2\). The Prandtl number is defined as

\[
Pr = \frac{C_{p,g} H}{k}. \quad (5)
\]

The friction Reynolds number of the flow is kept fixed by prescribing the mean pressure gradient per unit mass \(f\) in the direction parallel to the plates. Finally, the terms \(a_p\) and \(q_p\) describe the feedback by the particles on the momentum and temperature of the gas, which we will specify momentarily.

The gas velocity satisfies no-slip conditions at the two plates, whereas the gas temperature has prescribed constant, but different values at the two plates. In the other two directions, periodic boundary conditions are applied for all quantities. The length of the domain equals \(4\pi\) times half the channel height in streamwise direction and \(2\pi\) times half the channel height in the spanwise direction.

The motion of a particle is determined by the forces acting on it. If the particles are small and have a mass density that is high compared to the gas mass density, by far the most important force acting on a particle is the drag force exerted by the gas.\(^14,15\) Moreover, in this paper, gravity is not taken into account. Therefore, the governing equations for each particle are

\[
\frac{dx_p}{dt} = \mathbf{u}_p, \quad (6)
\]
\[
\frac{du_p}{dt} = \frac{u_g(x_p(t), t) - u_p}{\tau_p} (1 + 0.15 \text{Re}_p^{0.687}),
\]
(7)
\[
\frac{dT_p}{dt} = \frac{T_g(x_p(t), t) - T_p}{\tau_t} (1 + 0.3 \text{Re}_p^{1/2} \text{Pr}^{1/3}),
\]
(8)
where \(x_p, u_p, \) and \(T_p\) denote particle position, velocity, and temperature, \(\text{Re}_p\) is the particle Reynolds number, given by
\[
\text{Re}_p = \frac{\rho_p d_p |u_g - u_p|}{\mu}
\]
(9)
and \(\tau_p \) and \(\tau_t\) denote the relaxation time for particle velocity and temperature. They are given by
\[
\tau_p = \frac{\rho_p d_p^2}{18 \mu}
\]
(10)
and
\[
\tau_t = \frac{3 C_{p,p}}{2 C_{p,g}} \text{Pr} \tau_p,
\]
(11)
where \(d_p, \rho_p \) and \(C_{p,p}\) denote diameter, mass density, and specific heat of a particle. The terms between parentheses in Eqs. (7) and (8) are well known correlations for drag coefficient \(^{16}\) and Nusselt number \(^{17}\) for spherical particles. In the simulations, reported here the ratio between the thermal relaxation time and the velocity relaxation time varies between 1 and 4.

Direct particle-particle interaction is disregarded under the assumption that the particle concentration is low. However, the particle volume fraction and mass load are considered high enough to include two-way coupling between the discrete particle phase and the continuous gas. \(^{18,19}\) The terms \(a_p, q_p\) in Eqs. (2) and (3) follow from the requirement that the interaction between gas and particles conserves momentum and energy. In particular, the interaction between the two phases should vanish in the equation for the total momentum and total internal energy of the two phases. For the momentum, this equation is obtained as the sum of the integral of Eq. (2) over the domain and Eq. (7) summed over all particles. For the internal energy, the equation is the sum of the integral of Eq. (3) over the domain and Eq. (8) summed over all particles. If we assume that the interaction terms are localized at the position of a particle, the result is
\[
a_p = -\sum_i \frac{m_i}{\rho_g} \frac{u_i(x_{i}(t), t) - u_p}{\tau_p} \delta(x - x_{pi})(1 + 0.15 \text{Re}_p^{0.687})
\]
(12)
and
\[
q_p = -\sum_i \frac{m_i C_{p,p} T_g(x_{i}(t), t) - T_p}{\tau_t} \delta(x - x_{pi})
\]
\times (1 + 0.3 \text{Re}_p^{1/2} \text{Pr}^{1/3}),
\]
(13)
where the sum is taken over all particles and \(m_i\) is the mass of a particle. In Sec. III, the way these terms are calculated numerically is described. Particles collide elastically with the walls if their center point is at a distance from the wall equal to their radius. During a collision, the particle temperature is unchanged. If a particle leaves the computational domain through a periodic boundary, it reenters at the other side.

### III. NUMERICAL METHOD

In this section, the numerical methods for the gas and particle phases are described. Moreover, some results for flow without particles are shown, which serve as a validation for the numerical method.

Periodic boundary conditions are applied in two coordinate directions—this makes the use of a pseudo-spectral method very convenient. In these two periodic directions, a Fourier-Galerkin approach is chosen, whereas the wall-normal direction is treated by a Chebyshev-tau method. In order to satisfy the incompressibility constraint, the wall-normal component of the vorticity vector and the Laplacian of the wall-normal velocity component are used as dependent variables, instead of the velocity components. Hence, the spatial discretization of the problem for the gas velocity closely follows the method by Kim et al. \(^{20}\) Nonlinear terms are calculated in physical space by fast Fourier transform (FFT) with application of the 3/2 rule in both periodic directions. For integration in time, a combination of a second-order accurate three-stage Runge-Kutta method and the implicit Crank-Nicolson method is chosen according to Spoelart et al. \(^{21}\) In this way, the nonlinear terms are treated in an explicit way, whereas the linear terms are treated implicitly. For the Chebyshev-tau method, this implicit step is particularly simple: all Fourier modes are decoupled and in the wall-normal direction, a tri-diagonal matrix has to be inverted for the even and for the odd coefficients separately. Moreover, the particular solutions of the problem for the wall-normal velocity component are calculated only once at the beginning of the computation. By subtracting a linear temperature profile, the boundary conditions for gas temperature are also made homogeneous, so that the same matrices result as for the velocity components.

The motion of the particles is simulated using the explicit Runge-Kutta method as used for the gas phase. For evaluation of the drag force and heat exchange in Eqs. (7) and (8), the fluid velocity and temperature are interpolated from grid points to the particle positions using tri-linear interpolation. Although fourth-order interpolation is more accurate for individual particle tracks, it requires substantially more computing time and results in negligible differences in statistical particle properties. \(^{22}\)

The calculation of the gas-particle interaction terms \(a_p\) and \(q_p\) in Eqs. (12) and (13) requires special attention if a spectral method is adopted. Fourier transform of the delta functions gives rise to large contributions at high wave numbers. This induces numerical instabilities in case only few particles are present in the flow. The contributions to high wave numbers are for a large part canceled if the number of particles is of the same order of magnitude as the number of grid points. Moreover, extra smoothing of the source terms is obtained by distributing the contribution from a particle over
the eight neighboring grid points. For this distribution, the same weights are used as for the tri-linear interpolation of the velocity and temperature to the particle positions. The coupling terms are integrated explicitly in time, similar to the treatment of the nonlinear terms in the governing equations.

The initial solution for the gas velocity and temperature fields is obtained by performing first a simulation without particles at the same Reynolds and Prandtl number until a statistically stationary state of fully developed turbulence is reached. Then particles are inserted at random positions, homogeneously distributed over the entire computational domain and with initial velocity and temperature equal to the gas velocity and temperature at the position of the particle. In all simulations shown in this paper, the Reynolds number based on half the channel height and the friction velocity is equal to 150, whereas the Prandtl number has a value of 0.7, which agrees with the physical properties of air. In both periodic directions, 128 Fourier modes are employed and the number of Chebyshev polynomials in wall-normal direction equals 129.

The program is parallelized with the message passing interface mpi for usage on a distributed memory computer. The Fourier modes in the spanwise direction are distributed over the different processors. In this way, parallelization of the linear terms in the equations is perfect for our solver. The FFT in the wall-normal direction can also be parallelized in this way. Before the two-dimensional FFT’s in the periodic directions are carried out the velocity and temperature fields are redistributed over the processors, so that this FFT and the subsequent calculation of the nonlinear terms are parallelized over the wall-normal direction. Calculation of the momentum and heat exchange terms between gas and particles is performed on the processor where the gas properties at the position of the particle are stored.

Results of the flow without two-way coupling between particles and gas have been validated in several ways. Gas and particle velocity statistics and particle concentration have been compared with results from the DNS benchmark for turbulent channel flow at the same Reynolds number. The agreement with other spectral methods is very good. Results for gas temperature have been validated by examination of the terms in the budget equations for mean and fluctuating temperature variance. In our case, the budget equation for the fluctuating temperature variance reads in non-dimensional form

\[
\frac{\partial \langle T_g'^2 \rangle}{\partial y} + \frac{1}{Re Pr} \frac{d^2}{dy^2} \langle |\nabla T_g'^2| \rangle = 0.
\]

Here, \(y\) is the wall-normal coordinate and \(v\) the wall-normal velocity component; the brackets indicate Reynolds averaging and a prime the fluctuating part of a quantity. The first term on the left-hand side is the production of turbulent fluctuating temperature, the second term is turbulent diffusion in wall-normal direction, the third term is the dissipation of temperature fluctuations, and the fourth term is the molecular diffusion. Reynolds averages of the DNS results are computed by averaging over a long time and over the two homogeneous directions. The four terms in Eq. (14) are shown in Fig. 1 as functions of the wall-normal coordinate. The terms are non-dimensionalized with the temperature difference between the two plates, the friction velocity and the kinematic viscosity. Averages are taken over a time interval of length 33 000 in wall units. The figure shows that outside the near wall region, the production and dissipation terms are dominant, whereas close to the wall, the molecular diffusion plays a role. The sum of the four terms, the residual, which should be equal to zero, is a measure for the error in the simulation. It appears to be more than two orders of magnitude smaller than the maximum term. A further increase in the time averaging interval does not decrease the residual further. Therefore, we can conclude that the main source of the error is the spatial discretization. However, for the present resolution, the residual is already sufficiently small to estimate the error in statistical flow quantities as on the order of 1%.

IV. RESULTS

In this section, we present results of the simulations and compare them with results for flow of gas without particles. Four different cases are considered, see Table I. In the sequel, we will distinguish between one-way and two-way coupling (1W and 2W) and within the two-way coupled cases, we will add a reference to the specific parameter that was varied. In the reference simulation, (case 2W) 2 000 000 particles are present in the flow, each with diameter \(1.2 \times 10^{-4} \text{ m}\), mass density 1000 kg/m\(^3\), and specific heat 4186 J/kg K. For the gas phase, air has been taken with mass density equal to 1.3 kg/m\(^3\) and specific heat 1007 J/kg K. Moreover, the channel has a height of 4 cm. This leads to a Stokes number, the particle relaxation time in wall units, \(St = \tau_p/\tau_s\), equal to 34.6 and a thermal Stokes number, \(\tau_p/\tau_s\), of 151. The particle number density, \(n\), equals \(1.6 \times 10^9 \text{ m}^{-3}\) and the particle volume fraction is \(1.4 \times 10^{-3}\). The particle

mass load, i.e., the ratio of the total particle mass and total fluid mass in the computational domain, equals 1.1 for this case. The parameters correspond to particles with a high specific heat to serve as a point of reference for variations in these parameters for the other simulations.

One simulation has been performed in which all properties are the same except the particle specific heat, which is smaller by a factor of four (case 2Wcp), and one simulation in which the number of particles is reduced to 1 000 000 (case 2Wn). Apart from these three cases with two-way coupling, a simulation has been performed in which the particle concentration is so low that two-way coupling can be disregarded (case 1 W). This makes it possible to investigate the effect of two-way coupling on particle concentration. In order to obtain sufficiently accurate particle statistics, the number of particles in this case has been set equal to 100 000 and the two-way coupling terms \( a_p \) and \( q_p \) have been set to zero.

### A. Velocity and particle concentration

In case 2Wcp, the velocity statistics is exactly the same as in case 2 W; only the thermal properties are different and they do not influence the velocity in our model. Figure 2 shows the concentration of particles close to the wall as a function of time for the three cases 2 W, 2Wn, and 1 W normalized with the initial particle concentration. To this end, the wall-normal direction of the channel is divided into 40 slabs of uniform height and particles in the first and last slab are counted. Case 1 W, without two-way coupling, shows the effect of turbophoresis: due to the inhomogeneity of the fluctuations of the wall-normal gas velocity component, particles experience statistically a net force towards the walls and the particle wall concentration increases until a stationary state is reached.24

Fig. 2 shows that the turbophoresis effect is strongly dependent on the particle concentration. In case 2 W, 2Wcp, and 2Wn, the particle concentration is so large that the particles considerably affect the gas flow, which leads to reduced wall-normal velocity fluctuations, and correspondingly a reduced turbophoresis effect. This can also be seen in Fig. 3, where for cases 2 W, 2Wn, and 1 W, the wall-normal component of the gas velocity fluctuations is shown as a function of the wall-normal coordinate. The lower the gradient of the fluctuating velocity r.m.s. near the wall, the smaller the effect of turbophoresis and the smaller the resulting increase in particle concentration close to the walls. This effect has been found before in DNS and LES (large-eddy simulation) of wall-bounded particle-laden flow at high mass loading.7,26 In case 2Wn, the number of particles is smaller than in cases 2 W and 2Wcp. This results in a smaller reduction of the fluctuating wall-normal velocity r.m.s. Hence, the turbophoresis is between that of case 2 W and 1 W.

It is known that four-way coupling plays a role if the particle volume fraction is large, see for example Vreman et al.,7 where the particle volume fraction is larger than 1%. Turbophoresis leads to increased particle volume fractions close to the walls, but this effect is significantly reduced by the two-way coupling, so that also close to the walls, the particle volume fraction stays well below 1%, motivating our use of two-way coupling as the dominant mechanism throughout the domain.

The wall-normal and spanwise velocity fluctuations are reduced by the presence of particles, but the streamwise velocity fluctuations slightly increase. The net effect of this is a decrease of the turbulent kinetic energy. The effect of the two-way coupling on the total gas flow rate can be understood from a study of Eq. (2). Double integration of the Reynolds-averaged streamwise component of this equation over the wall-normal direction gives the following expression for the friction velocity:

### TABLE I. Definition of the test cases. The particle mass load is denoted by \( m \).

<table>
<thead>
<tr>
<th>Case</th>
<th>( n ) [m(^{-3})]</th>
<th>( m )</th>
<th>( C_{\nu_p} ) [J/kg/K]</th>
<th>( \tau_f/\tau_s )</th>
<th>Coupling</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 W</td>
<td>( 1.6 \times 10^9 )</td>
<td>1.1</td>
<td>4186</td>
<td>151</td>
<td>2 way</td>
</tr>
<tr>
<td>2Wcp</td>
<td>( 1.6 \times 10^9 )</td>
<td>1.1</td>
<td>1000</td>
<td>36</td>
<td>2 way</td>
</tr>
<tr>
<td>2Wn</td>
<td>( 0.8 \times 10^9 )</td>
<td>0.55</td>
<td>4186</td>
<td>151</td>
<td>2 way</td>
</tr>
<tr>
<td>1 W</td>
<td>( \approx 0 )</td>
<td>( \approx 0 )</td>
<td>4186</td>
<td>151</td>
<td>1 way</td>
</tr>
</tbody>
</table>

![FIG. 2. Particle concentration normalized with the initial particle concentration close to the walls as a function of time in wall units; solid: case 2 W, dash-dotted: case 2Wn, and thin solid: case 1 W.](image1)

![FIG. 3. Root-mean square of wall-normal component of gas velocity fluctuations as a function of wall-normal coordinate in wall units; solid: case 2 W, dash-dotted: case 2Wn, and thin solid: case 1 W.](image2)
\[ u_i^2 = H f_i - \frac{1}{2H} \int_{-H}^{H} (u_i v_i) dy + \frac{1}{2H} \int_{-H}^{H} \int_{-H}^{H} \langle a_{p,i} \rangle dy ds dy, \]

(15)

where \( u_i, f_i \), and \( a_{p,i} \) denote the streamwise components of the gas velocity, the mean pressure gradient, and the feedback force from the particles on the gas. The notation \( \langle \cdot \rangle_L \) is used to indicate an ensemble average over all particles on a certain wall-normal coordinate. Because of anti-symmetry, the second term on the right-hand side equals zero. This equation shows that without two-way coupling, the friction velocity is determined by the mean pressure gradient. If the effects of finite particle Reynolds number in Eq. (12) are neglected, substitution of Eq. (12) in Eq. (15) yields

\[ u_i^2 = H f_i - \frac{3}{2H} \frac{\mu_d}{\rho_g} \int_{-H}^{H} \int_{-H}^{H} n(s) \langle u_i - u_p \rangle dy ds dy, \]

(16)

where \( n(s) \) is the local particle number density. Note that \( \langle u_i \rangle_L \) differs from the mean Eulerian fluid velocity \( \langle u_i \rangle \). In the region close to the walls, the transport of particles from the more central regions of the channel, which have a higher streamwise velocity, results in a negative value of \( \langle u_i - u_p \rangle \). This shows that the presence of particles leads to a small increase of the friction velocity and hence of the gas flow rate, as can be seen in Fig. 4, where the mean streamwise gas velocity is plotted as a function of the wall-normal coordinate.

It is instructive to try to compare the predictions of the present paper with experimental findings for isothermal particle-laden turbulent flow of air in a channel. The experimental results of Kulick et al.\textsuperscript{19} are the mean and r.m.s. of streamwise and wall-normal components of the particle velocity as well as effects of the particles on fluid turbulence. Comparison with their results for different Stokes numbers and particle mass loads is only possible in a qualitative way since, for example, the experimental frictional Reynolds number was as high as 650 and since the value of the Stokes number (in wall units) was larger by at least a factor of 10. More importantly, however, is the absence of gravity in the simulations. In the simulations, the mean streamwise particle velocity is almost equal to the mean streamwise fluid velocity whereas the experimental results exhibit a substantial velocity slip, in particular close to the walls. It is, therefore, both satisfying and notable that despite these differences some of the features found in the experiments are the same as in the simulations. The attenuation of the fluid velocity r.m.s. in the wall-normal direction by the presence of particles, for example, occurs both in experiments and simulations.

B. Temperature

Next, we study the effect of particles on gas temperature. Fig. 5 shows the mean temperature as a function of the wall-normal coordinate. For practical applications, an important quantity is the Nusselt number, which is a measure for the heat flux from the hot to the cold wall. We define it as

\[ Nu = \frac{2H d \langle T_g \rangle / dy}{\Delta T_g}, \]

(17)

where the wall-normal derivative is evaluated at a wall and \( \Delta T_g \) is the temperature difference between the two plates which equals twice the difference between a wall and the mean bulk gas temperature. For case 1 W, i.e., in case two-way coupling is not incorporated, \( Nu = 5.1 \). Heat transfer is augmented if the gradient of the mean temperature at the wall increases. Figure 5 shows that all test cases considered lead to increased heat transfer, relative to case 1 W. Case 2 W results in \( Nu = 10.7 \), case 2Wcp gives \( Nu = 5.6 \), and case 2Wn: \( Nu = 9.5 \).

The most striking result is the increase in heat transfer by the presence of particles in case 2 W. The explanation for this can be found by studying the difference between case 2 W and 2Wcp. In these two cases, the particle concentration and the statistics of the gas velocity are the same, but nevertheless, the increase in Nusselt number with respect to the flow without two-way coupling is significantly reduced in case 2Wcp. Apparently, the specific heat of the particles is a major factor in the augmented heat transfer. The effect of heat transported to other places by relative motion of particles was previously named "particle convective heat transfer."\textsuperscript{11} Heat is not only transferred from one wall to the other by turbulent diffusion in

FIG. 4. Mean streamwise gas velocity as a function of wall-normal coordinate; solid: case 2 W, dash-dotted: case 2Wn, and thin solid: case 1 W.

FIG. 5. Mean gas temperature as a function of wall-normal coordinate; solid: case 2 W, dashed: case 2Wcp, dash-dotted: case 2Wn, and thin solid: case 1 W.
the gas but also through interaction between particles and gas. The strength of this second mechanism increases with increased particle specific heat and with the particle concentration close to the walls. In case 2Wcp, the particle concentration is equal to that in case 2 W, but the reduction in particle specific heat leads to reduced heat transfer relative to case 2 W.

Additional insight is gained from the following analysis of the governing equation for the gas temperature, Eq. (3). Reynolds averaging of this equation yields

\[ \frac{d}{dy} \langle v' T' \rangle_g = -\frac{k}{\rho_g C_{p,g} \Delta T_g} \frac{d^2}{dy^2} \langle T_g \rangle + \langle q_p \rangle. \]  

Integration of this relation twice over the wall-normal coordinate and application of the boundary conditions for the gas velocity leads to an expression for the Nusselt number

\[ Nu = 1 - \frac{1}{\alpha \Delta T_g} \int_{-H}^{H} \langle v' T' \rangle_g dy + \frac{1}{\alpha \Delta T_g} \int_{-H}^{H} \int_{-H}^{H} \langle q_p \rangle_L dy dx, \]  

where \( \alpha = k \rho_g C_{p,g} \) is the thermal diffusivity of the gas. We see that there are three contributions to Eq. (19). The first, equal to 1, is the result for laminar flow without particles. In that case, the temperature profile is linear. The second contribution involves the correlation between the wall-normal gas velocity component and the gas temperature and is due to turbulent diffusion, while the third contribution contains the effect of the particles. Note, however, that the presence of the particles also has a large effect on the second contribution.

We write \( Nu = Nu_{lam} + Nu_{turb} + Nu_{part} \), in order to distinguish and name the three contributions of Eq. (19). Table II shows the three different contributions for all four cases considered in this paper. If we neglect the effect of finite particle Reynolds number (the term between parentheses in Eq. (13)), a simplified expression for \( Nu_{part} \) can be found

\[ Nu_{part} \approx -\int_{-H}^{H} \int_{-H}^{H} \frac{2 \pi n(s) d_p (T_g - T_p)}{\Delta T_g} dy dx, \]  

where \( n(s) \) is the local particle number density, which depends on the wall-normal coordinate, and the gas temperature is evaluated at the particle position. In Fig. 6, the mean relative temperature difference between gas and particles, which is the most important part of the integrand in Eq. (20), is plotted as a function of the wall-normal coordinate for the three cases with two-way coupling. Transport of the inertial particles from the center of the channel to the walls results in a lower mean particle temperature than mean gas temperature close to the hot wall and a higher mean particle temperature than mean gas temperature close to the cold wall.

Table II shows that the contribution to heat transfer from turbulent diffusion is significantly reduced for all cases with two-way coupling. The reason for this is that the wall-normal velocity fluctuations are damped by two-way coupling. For case 2 W and 2Wn, this reduction is more than compensated by the particles. In case 2Wcp, the contribution from the particles is smaller than in case 2Wn because \( \langle T_g - T_p \rangle \) is smaller near the wall (Fig. 6). The reason for this is that the mean relative temperature difference between gas and particles is approximately proportional to the thermal relaxation time and hence to the specific heat of the particles. The value of \( Nu_{part} \) of case 2Wn is below that of case 2 W, but not as much as could be expected from the number of particles. In case 2Wn, the mean particle concentration is smaller than in case 2 W by a factor of two, but the higher turbophoresis almost compensates this in the regions close to the walls, where the mean relative temperature differs from zero.

In order to assess the accuracy of the results shown in Table II, several sources of error have been investigated. As already stated before, the spatial resolution of the simulations leads to errors in the prediction of statistical flow quantities on the order of 1%, which is sufficiently small to conclude that the increase in Nusselt number is not caused by insufficient resolution. Deviations by changing the averaging time interval are even smaller, provided that the initial transient behavior is left out of the time averaging. Since the interaction between fluid and particles leads to more elongated flow structures in the streamwise direction, we investigated the influence of the length of the simulation domain. To that end, we repeated simulations 1 W and 2 W in a geometry with length equal to 16 \( \pi \) times half the channel height, i.e., four times longer than the standard length, and with the same particle number density. For the simulation without two-way coupling, the Nusselt number differed by less than 1% from the simulation in the original domain. The resulting Nusselt number for the simulation with two-way coupling turned out to be equal to 10.0. This is somewhat smaller than the value found in the original domain, but the
increase caused by the two-way coupling is large enough to conclude that it is not an artifact caused by a too small domain length.

In order to investigate whether this effect of inertial particles on heat transfer persists at higher Reynolds numbers, similar simulations have been performed at Reₜ = 395. Although the particle volume fraction was considerably lower than in the other simulations reported here, the presence of particles results in an increase in Nusselt number by almost a factor of 2. A more detailed investigation at higher Reynolds numbers will be published in a forthcoming paper.

V. CONCLUSIONS

Inertial particles in turbulent channel flow were found to enhance heat transfer. It has been shown that the Nusselt number can be split in three contributions: a constant laminar part, a turbulence contribution, Nuₜₜ, and a particle contribution, Nuₚₚ. The product of wall-normal velocity fluctuations and temperature fluctuations, \( \langle v'_w T'_p \rangle \), that occurs in Nuₜₜ is of course also affected by the presence of particles—nevertheless, the distinction between a contribution primarily due to turbulence and one due to particles is helpful in the interpretation of the underlying enhancement mechanisms. Comparing the four cases, we noticed that a reduction in the number of particles in a two-way coupled simulation results in a higher turbophoresis effect and correspondingly in a higher Nuₜₜ. Moreover, at lower particle heat capacity, \( C_{p,\text{p}} \), the mean gas temperature remains closer to the mean particle temperature and leads to a lower value of Nuₚₚ, as explained by Eq. (20). However, in all cases studied, the convective particle heat transfer accounted for by Nuₚₚ is considerable and the increase of Nu due to Nuₚₚ with increasing particle density exceeds the decrease due to the corresponding decrease of Nuₜₜ associated with the turbophoresis effect.

Also the effect of the presence of the particles on the gas flow are studied and explained. To this end, an integral relation between the friction velocity, the mean pressure gradient, and the mean relative velocity between gas and particles is derived. This relation explains the slight increase in mean streamwise gas velocity observed in the simulations.

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