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Simultaneous Quality and Reliability Optimization for Microengines Subject to Degradation

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Abstract—Micro-Electro-Mechanical Systems (MEMS) represent an exciting new technology, but to achieve more widespread usage and wider adoption within more industrial applications, they must be highly reliable, and manufactured to stringent quality standards. Many challenging manufacturing issues are of concern during the fabrication of MEMS, such as precise dimensional inspection, reliability modeling, burn-in scheduling, avoiding stiction, and maintenance strategies. However, only limited mathematical tools for improving MEMS reliability, quality, and productivity are currently available. This paper proposes a mathematical model to jointly determine inspection & preventive replacement policies for surface-micromachined microengines subject to wear degradation, which is a major failure mechanism for certain MEMS devices. The optimal specification limits for inspection, and the replacement interval are determined by simultaneously optimizing MEMS quality and reliability. The proposed model can be used as a tool for decision-makers in MEMS manufacturing to make sound economical and operational decisions on reliability, quality, and productivity. While illustrated considering one specific microengine design, the proposed model can be applied to a broader range of MEMS devices that experience wear degradation between rubbing surfaces.

Index Terms—Burn-in, MEMS reliability, preventive replacement, quality and reliability optimization, specification limits, wear degradation.

ACRONYM¹

pdf	probability density function
cdf	cumulative distribution function
MEMS	Micro-Electro-Mechanical Systems
NDE	Non-Destructive Evaluation
SQP	Sequential Quadratic Programming

NOTATION

t	Number of revolutions to failure
$X(t), X(t; \beta)$	Wear volume of material at t (sometimes expressed as a function of model coefficients β)

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¹The singular and plural of an acronym are always spelled the same.

H	Critical wear volume or failure threshold
r	Radius of the pin joint
c	Model parameter proportional to the wear coefficient, and inversely proportional to the hardness of the material
F	Force between the contacting surfaces
t_0	Burn-in time, or number of revolutions to burn-in
η	Upper specification limit
$L(X(t_0))$	Quality loss function after burn-in
$C_Q(\eta, t_0)$	Expected quality loss after burn-in
s	Scrap cost per unit
$C_S(\eta, t_0)$	Expected scrap cost
C_I	Inspection cost per unit
QC	Expected quality-related cost
f_c	Cost of failure per unit
FC	Expected failure cost
RC	Replacement cost
τ	Replacement time
B_τ	Upper bound of the replacement interval
$\phi(\cdot)$	pdf of a standard normally distributed variable
$\Phi(\cdot)$	cdf of a standard normally distributed variable

I. INTRODUCTION

TO ACHIEVE WIDESPREAD usage, Micro-Electro-Mechanical Systems (MEMS) must be highly reliable, and manufactured to stringent quality standards. MEMS technology shows great promise for many critical applications in aerospace, biological/medical, nuclear, and weapons areas. In addition to new applications enabled by MEMS technology, existing applications are enhanced by miniaturized, low-cost, high-performance, and “smart” MEMS technology. MEMS devices have been effectively used in many commercial products, such as accelerometers in automotive airbag deployment systems [13], and inkjet print heads [29]. With more widespread commercialization of MEMS products, many challenging manufacturing/fabrication issues are of concern including precise dimensional control and inspection, reliability testing and modeling, avoiding stiction, and maintenance strategies. These reliability, quality, and productivity issues are dominant factors that impact the process of MEMS moving from the laboratory into the mainstream market. Decision-makers in MEMS manufacturing need tools to optimize these operational decisions.

However, such mathematical tools to improve MEMS reliability, quality, and productivity are currently lacking.

This study proposes a mathematical model to jointly determine inspection and preventive replacement policies for the surface-micromachined microengines subject to wear degradation, which is a major failure mechanism in MEMS devices [24], [25]. The optimal specification limits for inspection, and the replacement interval are determined by optimizing MEMS quality and reliability simultaneously.

A. Failure Analysis of MEMS

Reliability, and quality are important factors for MEMS to evolve from prototypes to commercialization. These issues for MEMS are complicated due to both electronic and mechanical parts, and their interactions [14]. Sufficient understanding of failure mechanisms is required to improve reliability and quality of MEMS devices in critical applications. However, knowledge is still somewhat limited on MEMS failures, and failure causes, at least in the public domain. According to their operational interactions, MEMS devices can be categorized into four classes [22], [23], [30]: Class I devices have no freely moving parts, but may have parts which stretch, compress, or bend, such as accelerometers, pressure sensors, or strain gauges; Class II devices have moving parts without rubbing or contacting surfaces, such as gyros, resonators, and filters; Class III devices have moving parts with contacting surfaces, such as relays, and valve pumps; and Class IV devices have moving parts with rubbing, contacting surfaces, such as shutters, scanners, and optical switches.

Designed with no rubbing surfaces, the first three classes of MEMS devices can achieve a high level of reliability if they are properly manufactured, and packaged. For Class IV MEMS devices, in which rubbing surfaces are unavoidable, failure analysis, and reliability assessment must be performed to further advance the growing commercialization of MEMS. Failure modes, and reliability models of Class IV MEMS were investigated by researchers at Sandia National Laboratories, a leader of MEMS technology [24], [25]. They conducted their research by performing many experiments on a reliability testing infrastructure. The MEMS device used in the reliability testing is a surface-micromachined microengine, developed at Sandia. As shown in Fig. 1, the microengine consists of orthogonal linear comb drive actuators that are mechanically connected to a rotating gear. The linear displacement of the comb drives is transformed to the gear via a pin joint. The gear rotates about a hub that is anchored to the substrate [26].

The dominant failure mechanism is identified as visible wear on rubbing surfaces, which often results in either seized microengines, or microengines with broken pin joints [21], [30]. Wear can be defined as the removal of material from solid surfaces as a result of mechanical actions. Wear degradation is a very complex phenomenon, involving both the mechanical and chemical properties of the bodies in contact, and also the pressure and interfacial velocity under which the bodies make contact.

B. Literature Review on Degradation Processes

Wear processes are degrading phenomena that have been studied in electronics, and other engineering fields. Lu & Meeker [16], and Meeker *et al.* [19] developed general statis-

Fig. 1. Scanning electron microscopy image of a microengine [26] (courtesy of SPIE).

tical models to estimate the time-to-failure distribution from degradation measures. A general model, and several examples were provided for the degradation path model. Bae & Kvam [2] introduced a log-linear degradation model with an unknown change point to characterize nonlinear degradation paths representing incomplete burn-in during the manufacturing process of plasma display panels. Kharoufeh [11] derived the explicit probability distribution of the random failure time for single-unit systems that deteriorate continuously and additively due to the influence of a random environment modeled as a general, finite-state Markov process. Kharoufeh & Cox [12] presented a degradation-based procedure for the estimation of full, and residue lifetime distribution for single-unit systems using real sensor data. Boulanger & Escobar [3], Tseng *et al.* [27], Yu & Chiao [31], and Joseph & Yu [10] used experimental design to improve reliability for degradation processes.

For degradation processes, preventive replacement (PR), or preventive maintenance (PM) is often considered as a policy to reduce the number of failures. Many different PR or PM approaches for degrading systems have been studied in the literature. Grall *et al.* [8] developed a maintenance cost model for determining the optimal inspection schedule and replacement threshold for a single unit degrading system. To reduce the uncertainty in cost estimates, Liao *et al.* [15] proposed a condition-based maintenance model for a continuous degradation process by considering imperfect maintenance, and a short-run availability constraint. Lu *et al.* [17], and Lu *et al.* [18] presented a preventive condition-based maintenance approach based on monitoring, modeling, and predicting a system's deterioration. Drapella & Kosznik [4] developed a model to seek for equilibrium of burn-in, and preventive replacement periods. Jiang & Jardine [9] examined the effectiveness of a jointly applied burn-in and preventive replacement policy for situations where the failure time follows a mixture distribution.

C. Research Objectives and Contribution

Although many different preventive replacement approaches for degrading systems have been studied, a comprehensive approach to jointly determine the inspection and preventive replacement policies is not available in the literature. For the microengine wear degradation, this paper proposes a mathematical model to jointly determine the parameters for *inspection*,

and *preventive replacement* policies. For systems with degradation characteristics, manufacturing decisions should be determined by taking into account *quality at the manufacturing phase*, and *reliability during system operation*, simultaneously. By rewarding high system reliability, and penalizing the quality loss due to variation, the proposed model can determine the optimal specification limits for inspection, and an optimal replacement interval. Although the idea of integrating quality and reliability into system design is not a new concept, there is no existing approach for determining specifications on degradation characteristics to optimize quality and reliability simultaneously. This integrated strategy can reduce warranty cost, and repair cost, while increasing customer satisfaction in the long run.

Only limited mathematical tools for improving MEMS reliability, quality, and productivity are currently available. This paper proposes a model that can be used as a tool for decision-makers in MEMS manufacturing to economically optimize operational decisions on reliability, quality, and productivity, which are critical factors during the fabrication of the micro-engine. While illustrated using Sandia-developed microengines as examples [24], [25], the proposed model can be applied to a broad range of MEMS devices that experience wear degradation between rubbing surfaces.

II. MODEL FORMULATION

Consider a MEMS system containing one microengine that is subject to wear. Furthermore, failure of the microengine causes failure of the system. The failure of the microengine occurs when the wear volume of material reaches a critical threshold, H [24]. This type of failure is referred to as “soft” failures, as opposed to “hard” failures when systems or components stop functioning abruptly. The critical threshold on a wear volume is assumed to be a constant in this study, although it may vary from unit-to-unit. The wear volume of material can be estimated by measuring the volume of wear debris, or the missing volume in the worn device. For example, a Focused Ion Beam system is effective to evaluate the amount of wear debris by producing cross sections of the precise area of interest in MEMS structures [25].

To simultaneously improve quality and reliability over the lifetime of MEMS systems, a systematic inspection and preventive replacement procedure has been developed, as depicted in Fig. 2. The initial wear volume of material after the completion of manufacturing is assumed to be zero, i.e., $X(0) = 0$. A burn-in procedure following MEMS manufacturing is used to detect, and remove defective, and early-failed parts. Burn-in is an important process to achieve reliable components and systems, but it also exposes all units to stresses. For the burned-in units, the nondestructive inspection is implemented to screen-out the fraction of units whose wear volumes exceed a certain specification limit. The screened units, with high quality level, are then released for field operation until reaching the periodic replacement time, where the cost of an impending failure makes it economical to replace it with a new one. The preventive replacement procedure is used to prevent failure due to the wear-out of typical operating units.

Fig. 2. Burn-in, inspection, and preventive replacement procedures for MEMS.

A. Wear Degradation Model

Let $X(t; \beta)$ denote the actual degradation path of a degrading characteristic over time t , where β is a vector of model coefficients. The choice of a degradation model requires not only specification of the form of the $X(t; \beta)$ function, but also specification of fixed and random parameters in β [7]. Typically, degradation paths are described by a model with up to four parameters. Some of the parameters in β are random from unit-to-unit, and one or more parameters could be modeled as constant across all units [19].

For the wear degradation of microengines, the degradation model is derived based on physical theory to quantify the functional relationship between the wear volume, $X(t; \beta)$, and the number of revolutions to failure, t [26]. Given the radius of the pin joint, r (shown in Fig. 1), the coefficient related to wear and hardness of the material, c , and the force between the contacting surfaces, F , the linear degradation path, $X(t; r, c, F)$, is shown in Fig. 3, and can be expressed as

$$X(t; r, c, F) = 2\pi r c F t. \quad (1)$$

c is a parameter that is directly proportional to the wear coefficient, and inversely proportional to the hardness of material. The radius of pin joint, r , is random from unit-to-unit with mean μ_r , and standard deviation σ_r . For a sinusoidal drive signal, the force applied between rubbing surfaces, F , varies with drive frequency as the critical frequency for resonance is approached. At a given drive frequency, the force applied between rubbing surfaces is random among units with nominal value μ_F , and standard deviation σ_F .

The wear volume at any time t , $X(t)$, is random from unit-to-unit, and can be reasonably assumed to follow a s -normal distribution. There are many combinations of distributions for r , and

Fig. 3. Linear wear degradation path.

F that will result in a s -normally distributed $X(t)$. If a s -normal distribution is not appropriate, then transformations can be performed to obtain a s -normally distributed random variable. Assuming s -independence between r and F , it is demonstrated that

$$\mu_t = 2\pi c \mu_r \mu_F t, \quad \text{and} \quad (2)$$

$$\sigma_t = 2\pi c t \sqrt{\sigma_r^2 \sigma_F^2 + \sigma_r^2 \mu_F^2 + \mu_r^2 \sigma_F^2}, \quad (3)$$

which indicates that the mean μ_t changes linearly, and the standard deviation σ_t increases linearly, over time. At the completion of MEMS manufacturing, but prior to burn-in, $X(0) = 0$.

For different degradation characteristics in MEMS or other applications, the functional form of $X(t; \beta)$, and the approximate transformation may be suggested by physical or chemical theory, past experience, or the available data. When the degradation path is not linear, the scales of $X(t)$, and t can be chosen to simplify the form of the degradation model. For many problems, the Box-Cox family of transformations will be useful, especially the log transformation of degradation and/or time [19].

B. Burn-In Procedure

It is observed through experiments that the occurrence of microengine failures consistently decreases at the early stage of testing, which indicates infant mortality caused by the early failures of defective parts [24]. This implies that a burn-in procedure should be applied following the manufacturing process to effectively remove weak devices from the population. The burn-in process is an extension of manufacturing processes where manufactured units are operated for a short period of time to screen-out defective parts. The burn-in time, t_0 , must be determined to prevent the early failures. Selection of the burn-in time is made based on a combination of test data, industry standards, and time restrictions. In this paper, the burn-in time is determined prior to the optimization of the quality tolerance level, and replacement time.

The need for shorter production cycles drives MEMS manufacturers to reduce the burn-in time. However, if the burn-in is incomplete, the microengine may experience unacceptable early failures. An effective burn-in schedule should be determined that it is long enough to induce the defective units to fail, but not too long to impinge on the required lifetime of engines. Further study on burn-in procedure is necessary to incorporate the cost of burn-in procedure into the model, which includes the operating cost of the burn-in equipment, the failure cost during

the burn-in process, and marketing losses caused by increased production lead time.

C. Nondestructive Evaluation, and Specification Limits

As wear is the most critical failure mode for the microengine, the wear volume should be carefully evaluated at the end of the burn-in procedure. The units whose wear volumes are beyond a certain specification limit are prone to fail early, and they should be screened to ensure that the wear volume does not unsatisfactorily reduce the lifetime of microengines. Nondestructive evaluation (NDE) systems can be implemented to provide 100% inspection capabilities, such as Focused Ion Beam systems.

At the end of the burn-in procedure, t_0 , the wear volume is given as $X(t_0) = 2\pi r c F t_0$, which follows a s -normal distribution, $X(t_0) \sim N(\mu_0, \sigma_0^2)$, where $\sigma_0^2 = (2\pi c)^2 (\sigma_r^2 \sigma_F^2 + \sigma_r^2 \mu_F^2 + \mu_r^2 \sigma_F^2) t_0^2$, and $\mu_0 = 2\pi c \mu_r \mu_F t_0$, respectively. During the NDE, an upper specification limit (USL) should be applied based on the wear volume, to screen the units that have a large amount of wear after the burn-in process. The selection of the USL is a crucial decision that is usually determined by optimizing the quality of a system [5], [6]. However, this may be inefficient for the manufacturing of new technologies such as MEMS because of the degradation process, and reliability concerns. Therefore, quality and reliability should be integrated in the optimization of specification limits.

During the NDE, three quality-related costs are considered: quality losses due to the deviation from the ideal value, scrap or rework cost, and inspection cost [5]. The quality loss of each unit can be measured by a quality loss function, which can be chosen based on the type of the quality characteristic: the smaller the better (S-type), the larger the better (L-type), or the target the best (T-type). The wear volume is clearly an S-type quality characteristic, and its quality loss function is expressed as

$$L(X(t_0)) = kX(t_0)^2, \quad (4)$$

where k is the coefficient that transforms deviations into economic values. The quality loss can be estimated using the expected value of $L(X(t_0))$. Based on the derivation of the expected quality loss for the T-type characteristic provided in Feng & Kapur [6], the expected quality loss for the S-type characteristic, $C_Q(\eta)$, is proven to be

$$\begin{aligned} C_Q(\eta) &= \int_0^{USL} L(x; t_0) f_{X(t_0)}(x; t_0) dx = \int_0^{\eta} kx^2 f_{X(t_0)}(x; t_0) dx \\ &= -k\sigma_0[\mu_0 + \eta] \phi\left(\frac{\eta - \mu_0}{\sigma_0}\right) + \sigma_0 \mu_0 k \phi\left(\frac{-\mu_0}{\sigma_0}\right) \\ &\quad - k[\sigma_0^2 + \mu_0^2] \Phi\left(\frac{-\mu_0}{\sigma_0}\right) + k[\sigma_0^2 + \mu_0^2] \Phi\left(\frac{\eta - \mu_0}{\sigma_0}\right), \end{aligned} \quad (5)$$

where $f_{X(t_0)}(x; t_0)$ is the pdf of the wear volume at the end of the burn-in process.

If an observed measurement is outside the USL, the unit will be reworked or scrapped. Let $q(\eta)$ be the fraction of

conforming units, which corresponds to the area under the pdf curve bounded by the USL.

$$q(\eta) = \int_0^\eta f_{X(t_0)}(x; t_0) dx = \Phi\left(\frac{\eta - \mu_0}{\sigma_0}\right) - \Phi\left(-\frac{\mu_0}{\sigma_0}\right). \quad (6)$$

If the scrap/reworked cost per unit is denoted as s , then the scrapped portion of $(1 - q(\eta))$ results in an expected scrap cost of $(1 - q(\eta))s$. Thus, the expected scrap cost is

$$C_S(\eta) = (1 - q)s = \left(1 - \Phi\left(\frac{\eta - \mu_0}{\sigma_0}\right) + \Phi\left(-\frac{\mu_0}{\sigma_0}\right)\right) s. \quad (7)$$

The inspection cost per unit is denoted as C_I , which is a constant independent of η . Therefore, the total expected quality cost per unit incurred at the manufacturing is expressed as

$$QC(\eta) = C_Q(\eta) + C_S(\eta) + C_I. \quad (8)$$

D. Preventive Replacement, and Cost of Failure

A preventive periodic-replacement policy is used to prevent failure due to the wear-out of typical operating units. As the system ages, it is more economical to replace an aged system because the cost of a planned replacement is less than the associated cost of unscheduled maintenance. The microengine fails when the wear volume of material reaches a critical threshold, H . Therefore, the reliability of a microengine at any time t (or number of cycles) can be assessed by the probability that the wear volume is less than the failure threshold, i.e.,

$$R(t) = P(X(t) < H) = \int_0^H f_{X(t)}(x; t) dx = \Phi\left(\frac{H - \mu_t}{\sigma_t}\right). \quad (9)$$

This equation is valid when the wear volume follows a s -normal distribution, i.e., $X(t) \sim N(\mu_t, \sigma_t^2)$. If another distribution is more suitable, an analogous relationship can be derived. The reliability at any time $t(t_0 < t < \tau)$, is measured as a conditional reliability given the probability that the wear volume during the burn-in process is less than H , or

$$R(t|t_0) = \frac{P(X(t) < H)}{P(X(t_0) < H)}, \quad t_0 < t < \tau. \quad (10)$$

To be consistent with the monetary measure of quality costs measure, the system reliability can be evaluated considering the cost-of-failure approach [28]. The cost of failure per unit is assumed to be a constant, f_c , which is s -independent of the time to failure, and can be estimated by a one-year warranty cost, or a one-time repair cost. The system reliability at the time of replacement is then evaluated by the expected failure cost:

$$FC(\tau) = f_c(1 - R(\tau|t_0)). \quad (11)$$

If the system fails prior to τ , then it must be replaced by an operational replacement, and the cost is $f_c + RC$, where RC is the replacement cost. Alternatively, if it has not failed by τ , it should be replaced based on economic considerations, and the

cost is just RC . Thus, the expected total failure plus replacement cost at τ is $FC(\tau) + RC$.

E. Simultaneous Quality and Reliability Optimization Model

By simultaneously rewarding high reliability during system operation, and penalizing quality loss during manufacturing, a comprehensive model is proposed to determine the specification limit for inspection, η , and the replacement interval, τ . The expected total system cost includes the expected quality cost, failure cost, and replacement cost, which should be minimized over the expected usage time of a microengine. The expected usage time, $E[U|t_0, \tau]$, is demonstrated to be (see the Appendix)

$$\begin{aligned} E[U|t_0, \tau] &= \int_0^{\tau-t_0} R(t+t_0|t_0) dt \\ &= \frac{1}{R(t_0)} \left(\tau R(\tau) - t_0 R(t_0) + \int_{t_0}^{\tau} t f_T(t) dt \right), \end{aligned} \quad (12)$$

where $f_T(t)$ is the pdf of the failure time with the form

$$f_T(t) = \frac{H}{\sqrt{2\pi}bt^2} e^{-\frac{(H-at)^2}{2b^2t^2}}, \quad (13)$$

with $\frac{a}{b} = \frac{2\pi c\mu_r\mu_F}{\sqrt{(2\pi c)^2(\sigma_r^2\sigma_F^2 + \sigma_r^2\mu_F^2 + \mu_r^2\sigma_F^2)}}$. This is the pdf for a two-parameter Bernstein distribution [1].

In this way, the expected total system cost per unit expected usage time is given as

$$TC(\eta, \tau) = \frac{QC(\eta) + FC(\tau) + RC}{E[U|t_0, \tau]}. \quad (14)$$

In practice, the upper bound of the replacement interval is usually specified, and is denoted as B_τ . Thus, the constrained optimization model that minimizes the expected total system cost rate due to quality loss, and unreliability during the system life cycle, can be expressed as

$$\begin{aligned} (\eta^*, \tau^*) &= \arg \min \left\{ TC(\eta, \tau) = \frac{QC(\eta) + FC(\tau) + RC}{E[U|t_0, \tau]} \right\} \\ \text{subject to } &\eta > \mu_0, \quad t_0 \leq \tau \leq B_\tau. \end{aligned} \quad (15)$$

We implemented a sequential quadratic programming (SQP) method to solve the constrained nonlinear problem, because it outperforms many other methods in terms of efficiency, accuracy, and percentage of successful solutions [20].

III. NUMERICAL EXAMPLES

Consider a MEMS system with a microengine subject to wear degradation. As given in Tanner *et al.* [24], the coefficient c in (1) is $3 \times 10^{-4} \mu\text{m}^2/\text{N}$, the mean value of the radius r of the pin joint is $1.5 \mu\text{m}$, and the nominal value of the force applied between rubbing surfaces is $3 \times 10^{-6} \text{ N}$. The standard deviations of the radius, and the applied force are assumed to be 5% of their respective mean values. The burn-in period, t_0 , is assumed to be 1,000 revolutions. Using (2) and (3), the mean, and standard deviation of the wear volume at the end of the burn-in

Fig. 4. Sensitivity analysis of TC , and τ , on f_c/RC . Fig. 5. Sensitivity analysis of TC , and η , on $\ln(k/f_c)$.

procedure are $8.4823 \times 10^{-6} \mu\text{m}^3$, and $6.0016 \times 10^{-7} \mu\text{m}^3$, respectively. The microengine experiences “soft” failures when the wear volume of material reaches a critical threshold, H , which is $0.00125 \mu\text{m}^3$. The following cost parameters are used to illustrate this example. The coefficient, k , in the quality loss function is set to be 10^{10} . The cost to inspect the microengine at the manufacturing phase is assumed to be \$0.1 per unit, and the scrap/rework cost of a nonconforming unit is \$20. The replacement cost of the microengine is \$50, and the cost of failure is assumed to be \$1,000. The microengine has to be replaced before 10^5 revolutions, which is the upper bound of the replacement interval.

Using SQP methods provided by MATLAB, the optimal solution is obtained, which indicates that the upper specification limit should be set at $1.5461 \times 10^{-5} \mu\text{m}^3$, and the microengine should be replaced every 1.2068×10^5 revolutions. The resultant minimum total cost per expected revolution, TC , is about $\$4.32 \times 10^{-4}/\text{revolution}$.

A. Sensitivity Analysis

Sensitivity analysis was also performed to observe the effects of model parameters on optimal solutions. The parameters that we are interested in include the ratio between the cost of failure per unit and the replacement cost, f_c/RC ; the ratio between the coefficient in the quality loss function and the cost of failure per unit, k/f_c ; the critical threshold value, H ; and the burn-in time, t_0 . The results are shown in Figs. 4 to 7, respectively. It can be observed how the optimal solution changes as each parameter changes.

The ratio f_c/RC indicates the relative magnitude of the failure cost to the replacement cost. When f_c/RC increases from 1 to 100 (keeping RC as a constant) as shown in Fig. 4,

Fig. 6. Sensitivity analysis of TC , and τ , on H . Fig. 7. Sensitivity analysis of TC , and τ , on $\ln(t_0)$.

the expected total system cost rate increases from $\$4.06 \times 10^{-4}$ to $\$4.43 \times 10^{-4}$, and the optimal replacement interval decreases from 1.2978×10^{-5} to 1.1737×10^{-5} revolutions. It suggests that microengines should be replaced more frequently as the failure cost increases, while the replacement cost keeps the same.

The ratio between k and f_c represents the relative magnitude between quality loss and failure cost. As shown in Fig. 5, when the ratio between k and f_c increases from 10^4 to 10^9 (keeping f_c as a constant), the expected total system cost rate increases from $\$4.26 \times 10^{-4}$ to $\$7.76 \times 10^{-4}$, and the upper specification limit of wear volume reduces from $1.5820 \times 10^{-5} \mu\text{m}^3$ to $8.4823 \times 10^{-6} \mu\text{m}^3$. The result indicates that, as k increases, a larger fraction needs to be scrapped or reworked to lower down the cost due to quality loss.

When the critical threshold value, H , increases from $0.001 \mu\text{m}^3$ to $0.01 \mu\text{m}^3$ as shown in Fig. 6, the expected total system cost rate decreases from $\$5.41 \times 10^{-4}$ to $\$5.36 \times 10^{-5}$, and the replacement interval increases linearly from 9.6547×10^4 to 9.6530×10^5 revolutions. This result suggests that the threshold value has a significant effect on the determination of the replacement interval.

As presented in Fig. 7, when the burn-in time, t_0 , increases from 500 to 10,000 revolutions ($\ln(t_0)$ increases from 2.7 to 4), then the replacement interval increases slightly from 1.2065×10^5 to 1.2230×10^5 revolutions, and the total cost per unit increases from $\$4.26 \times 10^{-4}$ to $\$8.38 \times 10^{-4}$. It implies that a shorter burn-in period should be applied to minimize the total system cost, while the burn-in cost is not incorporated into the system cost. It suggests a potential research direction to simultaneously determine the burn-in time while considering the associated cost.

IV. CONCLUSIONS

This study proposes a mathematical model to jointly determine inspection and preventive replacement policies for the surface-micromachined microengines subject to wear degradation, which is a major failure mechanism in MEMS devices. For the microengine example, the optimal specification limit for the inspection & the replacement interval are determined by optimizing MEMS quality and reliability simultaneously.

While illustrated using one specific microengine for die-level reliability, the proposed model can be extended to a broader range of MEMS devices that experience wear degradation between rubbing surfaces. For example, a MEMS system with several homogenous or heterogeneous degradation components presents more challenging issues on modeling the interactions between components and system reliability. Furthermore, for the reliability of a final MEMS product, all aspects of fabrication, packaging, system integration, and manufacturing must be considered.

APPENDIX

DERIVING EXPECTED USAGE TIME, AND THE pdf OF FAILURE TIME

The expected usage time, $E[U|t_0, \tau]$, is

$$\begin{aligned} E[U|t_0, \tau] &= \int_0^{\tau-t_0} R(t+t_0|t_0)dt \\ &= \frac{1}{R(t_0)} \int_0^{\tau-t_0} R(t+t_0)dt \\ &= \frac{1}{R(t_0)} \int_{t_0}^{\tau} R(t')dt' \\ &= \frac{1}{R(t_0)} \left(\tau R(\tau) - t_0 R(t_0) + \int_{t_0}^{\tau} t f_T(t)dt \right). \end{aligned}$$

Using (9), the pdf of the failure time, $f_T(t)$, is derived as

$$\begin{aligned} f_T(t) &= -\frac{dR(t)}{dt} \\ &= -\phi\left(\frac{H-\mu(t)}{\sigma(t)}\right)\left(\frac{1}{\sigma(t)^2}(bt(-a)-(H-at)(b))\right) \\ &= \frac{Hb}{b^2t^2}\phi\left(\frac{H-\mu(t)}{\sigma(t)}\right) \\ &= \frac{H}{t\sigma(t)}\phi\left(\frac{H-\mu(t)}{\sigma(t)}\right) \\ &= \frac{H}{\sqrt{2\pi}bt^2}e^{-\frac{(H-at)^2}{2b^2t^2}} \end{aligned}$$

where $a = \frac{2\pi c\mu_r\mu_F}{b}$ and $b = \sqrt{(2\pi c)^2(\sigma_r^2\sigma_F^2 + \sigma_r^2\mu_F^2 + \mu_r^2\sigma_F^2)}$.

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