Extended analytical charge modeling for permanent-magnet based devices: practical application to the interactions in a vibration isolation system

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DOI: 10.6100/IR719555

Published: 01/01/2011

Citation for published version (APA):
Extended Analytical Charge Modeling for Permanent-Magnet Based Devices
Practical Application to the Interactions in a Vibration Isolation System

PROEFSCHRIFT

ter verkrijging van de graad van doctor
aan de Technische Universiteit Eindhoven,
op gezag van de rector magnificus, prof.dr.ir. C.J. van Duijn,
voor een commissie aangewezen door het College voor Promoties
in het openbaar te verdedigen
op dinsdag 13 december 2011 om 16.00 uur

door

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geboren te Boxmeer
Dit proefschrift is goedgekeurd door de promotoren:

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This research is part of the IOP-EMVT program (Innovatiegerichte Onderzoeksprogramma's - Electromagnetische Vermogenstechniek) under IOP-EMVT 06225A. This program is funded by AgentschapNL, an agency of the Dutch Ministry of Economical Affairs, Agriculture and Innovation.


A catalogue record is available from the Eindhoven University of Technology Library.
ISBN: 978-90-386-2949-0

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This thesis was prepared with the pdfGjusteX documentation system.
Cover Design: GJ2Design.nl
Summary

Extended Analytical Charge Modeling for Permanent-Magnet Based Devices
Practical Application to the Interactions in a Vibration Isolation System

This thesis researches the analytical surface charge modeling technique which provides a fast, mesh-free and accurate description of complex unbound electromagnetic problems. To date, it has scarcely been used to design passive and active permanent-magnet devices, since ready-to-use equations were still limited to a few domain areas. Although publications available in the literature have demonstrated the surface-charge modeling potential, they have only scratched the surface of its application domain.

The research that is presented in this thesis proposes ready-to-use novel analytical equations for force, stiffness and torque. The analytical force equations for cuboidal permanent magnets are now applicable to any magnetization vector combination and any relative position. Symbolically derived stiffness equations directly provide the analytical $3 \times 3$ stiffness matrix solution. Furthermore, analytical torque equations are introduced that allow for an arbitrary reference point, hence a direct torque calculation on any assembly of cuboidal permanent magnets. Some topics, such as the analytical calculation of the force and torque for rotated magnets and extensions to the field description of unconventionally shaped magnets, are outside the scope of this thesis and are recommended for further research.

A worldwide first permanent-magnet-based, high-force and low-stiffness vibration isolation system has been researched and developed using this advanced modeling technique. This one-of-a-kind 6-DoF vibration isolation system consumes a minimal amount of energy ($< 1$W) and exploits its electromagnetic nature by maximizing the isolation bandwidth ($> 700$Hz). The resulting system has its resonance $< 1$Hz with a $-2$dB per decade acceleration slope. It behaves near-linear throughout its entire 6-DoF working range, which allows for uncomplicated control structures. Its position accuracy is around $4$µm, which is in close proximity to the sensor’s theoretical noise level of $1$µm.

The extensively researched passive (no energy consumption) permanent-magnet-based gravity compensator forms the magnetic heart of this vibration isolation system. It combines a $7.1$kN vertical force with $< 10$kN/m stiffness in all six degrees of
freedom. These contradictory requirements are extremely challenging and require the extensive research into gravity compensator topologies that is presented in this thesis. The resulting cross-shaped topology with vertical airgaps has been filed as a European patent. Experiments have illustrated the influence of the ambient temperature on the magnetic behavior, $1.7\%/\text{K}$ or $12\text{N/K}$, respectively. The gravity compensator has two integrated voice coil actuators that are designed to exhibit a high force and low power consumption (a steepness of $625\text{N}^2/\text{W}$ and a force constant of $31\text{N/A}$) within the given current and voltage constraints. Three of these vibration isolators, each with a passive 6-DoF gravity compensator and integrated 2-DoF actuation, are able to stabilize the six degrees of freedom.

The experimental results demonstrate the feasibility of passive magnet-based gravity compensation for an advanced, high-force vibration isolation system. Its modular topology enables an easy force and stiffness scaling. Overall, the research presented in this thesis shows the high potential of this new class of electromagnetic devices for vibration isolation purposes or other applications that are demanding in terms of force, stiffness and energy consumption. As for any new class of devices, there are still some topics that require further study before this design can be implemented in the next generation of vibration isolation systems. Examples of these topics are the tunability of the gravity compensator's force and a reduction of magnetic flux leakage.
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Chapter 1

Introduction

An overview on the background of the project, the dears in analytical magnet modeling, the envisaged application and the research goals that have been identified.
1.1 Background

Permanent magnets are pieces of magnetic material which are magnetized by an external magnetic field and retain a usefully large magnetic moment after it is removed. They then become a source of magnetic field which interacts with other magnetic materials or current-carrying conductors. Throughout history, permanent magnets have been used for a vast range of applications. In ancient times they were found in nature in the form of loadstones, rich in magnetite Fe₃O₄. Early and improved artificial magnets made of quench hardened iron-carbon alloys (sword steel) were discussed by W. Gilbert in 1600 and over the centuries more materials were added to increase their strength. In the 30’s of the last century the development of the Alnico-family started with the patenting of the first precipitation hardenable magnet alloy based on Fe, Ni and Al – no longer a steel – and later the alloys with SmCo and NdFeB [170].

Their practical use was initially limited to compass applications, such as that shown in Fig. 1.1 which was seen in ancient China and was first described in Europe around 1200 AD. In electromechanical devices such as early electric generators and motors [135, 170] permanent magnets in the form of soft-magnetic steel in horseshoe or bar shapes were already used in the eighteen-hundreds. After being surpassed by field-wound rotating machines the introduction of the AlNiCo materials and hard magnetic ferrites became popular again and they were used in a wide variety of applications such as DC motors, especially in automotive applications, hand tools, etc. [170].

Especially since the development of rare-earth NdFeB magnetic materials in the eighties the application areas of devices with permanent magnets have been vastly expanded. Hard ferrites became an abundant inexpensive magnet material while the rare-earth magnets raised the highest available energy products 4 to 5-fold and coercivity by an order of magnitude. Although their name suggests otherwise, the materials neodymium, iron and boron are far from rare. Nevertheless, as today most of the world’s neodymium is mined in China, despite its abundance in the earth's crust, the term ‘rare’ may not be that strange after all, as a monopoly on this popular material may lead to a depletion of its world supply. Although this is not covered in this thesis it should definitely play a role in the commercialization of a permanent-magnet based device.
Compared to the previous magnet materials, the rare-earth materials showed significantly increased energy densities and demagnetization withstand capabilities. Their improved physical properties enabled performance improvement of existing applications and the development of new applications and design concepts, especially for linear and rotary limited motion actuators. For instance, their increased coercivity further enabled devices based on repulsion between permanent magnets as opposed to attracting topologies [111, 186, 190]. Previously, the low coercivity of magnetic materials caused a great risk of demagnetization which forced designers to solely use permanent magnets in attracting configurations.

1.1.1 Trend towards increased performance and complexity

Nowadays, permanent magnets have become vital components of many highly advanced electromechanical machines and electronic devices, but they are usually hidden in subassemblies invisible to the end user. Examples are free electron lasers for physics, particle beam controllers, sensors, high-precision transducers, MRI actuators for robotics and flight control, suspension/propulsion units for magnetically levitated vehicles, high-quality loudspeaker systems, etc. [66, 170]. As designers increasingly push the performance limits, the necessity for fast and accurate modeling techniques increases accordingly.

Due to the nonlinear, hysteretic and widely varying behavior of the early magnetic materials with low coercivity, the mathematical description of permanent magnet circuits containing these magnets had always been quite difficult. The improved physical properties of permanent magnets and the simultaneous rise of advanced modeling techniques enabled more accurate physical modeling. Ever since, the development of electromechanical devices and principles has been enabled more and more by improvements in modeling accuracy and speed instead of the empirical measurements and design evolutions that were mostly seen before.

The continuously progressing technological developments have caused that electromechanical devices are ever more approaching their performance limits. An example is seen in the trend towards three-dimensional (3D) magnetic modeling rather than the two-dimensional (2D) models that are have often been used for conventional rotating or linear machines. Although any electromagnetic problem is inherently 3D, it is for many applications sufficient approximate the behavior with a 2D model. The three-dimensional phenomena in rotating machines with laminated back-steel, such as parasitic stray effects due to the end windings, are mostly separately modeled and combined with the 2D calculations afterwards. Even eddy current losses can often be approximated, with reasonable accuracy, based upon the two-dimensional modeling results. However, with the development of new types of devices such as magnetically levitated multi-DoF devices [47, 94], rotating devices with small axial lengths and therefore large end effects [60], spherical machines [188], some types of magnetic high-field cavities [38, p. 401] transverse flux machines, machines with bi-directional flux [115], etc., the simplified field modeling in two dimensions does not provide accurate results. In the afore mentioned applications
the need for accurate 3D modeling is evident. Further, when mounting permanent magnets side-by-side, i.e. on the same back-plane, knowledge of their interaction behavior is essential. Examples are found in the assembly force calculations of planar magnet arrays [162] or in glue strength calculations in beam insertion devices [66, 170].

Accurate three-dimensional modeling is a necessity to push the designs of such modern permanent-magnet devices to their respective performance limits. Over the years various modeling techniques have been developed to accomplish this. These techniques are discussed in Chapter 2.

1.1.2 Energy, field and interaction modeling of permanent-magnet based devices

Numerical methods, such as the well-known Finite Element modeling method, are very powerful in terms of high accuracy and low model abstraction. Unfortunately, they are computationally expensive due to their mesh-based implementation and are only capable of computing interaction forces and torques as post-processing on energy and field results. As a result they exhibit numerical noise and give little analytical insight into parameters such as the three-dimensional cross-coupling stiffness. Another property of such a technique is the bounded domain to which these models are restricted. This is especially useful for well bound problems with unsaturated iron edges, symmetry or periodicity. However, for problems without these properties the modeling boundaries must be sufficiently far from the studied object, which comes at the cost of computational effort. A more elaborate discussion concerning these methods is found in Chapter 2.

Mesh-free analytical modeling tools provide significantly less computationally expensive direct energy, field, force and torque equations compared to numerical methods. They are characterized by their ability to express the fields throughout the
domain with an analytical equation. However, they require more model abstractions than numerical methods and are restricted to less complicated geometries. The main assumption in the analytical technique that is used in this thesis is that the relative permeability equals unity (see Chapter 2). Today’s permanent-magnet materials enable this assumption as their relative permeability approaches that of vacuum. Previously, this assumption was not valid as a result of the high magnetic permeability and poor coercive field strength of ferrite magnets. Consequently, such analytical methods are ever more considered for modeling a variety of electromechanical devices. Their fast-solving and mesh-free solution which yields direct, analytical energy, field and interaction equations make them very suitable for fast evaluation of large numbers of topologies and for optimization purposes.

Surface charge modeling

The particular analytical field and interaction modeling method that is used in this thesis is the three-dimensional analytical surface charge method. It is based on a magnetic scalar potential derivation of Maxwell’s equations under the assumption that the relative permeability $\mu_r$ is continuous throughout the studied volume. As such, it is often used for problems that comprehend permanent magnets without highly permeable materials in their close proximity. Although the basic mathematical derivation of this method was already known for years [169] it was not until 1984 that the field of a cuboidal permanent magnet was described with this method simultaneous to the force equations for two parallel magnetized permanent magnets [3].

Although many authors have been using this technique ever since, only a limited class of problems could be described with these equations. This thesis aims to reduce this dearth and to extend the existing equations towards a more generic framework which enables a larger range of permanent-magnet devices to be accurately modeled. These extended models allow the design of more advanced electromagnetic devices, such as the vibration isolation system discussed in this thesis.

1.2 Application to a vibration isolation system

Today, an increasing number of applications require a platform or table which is extremely well isolated from vibrations [13, 49, 177, 194]. They demand a stable environment to function at their peak of accuracy and precision. The influence of typical disturbances like ground motions, personnel activities and the extensive support machinery on the isolated system needs to be reduced. A well-designed isolation system relaxes the requirements on the floor or the building in which it is placed. On the other hand, a proper floor design may reduce the demands for the vibration isolation system. The financial investment that is required is another factor that is of influence. This demands a trade-off between factors such as installation complexity, operational costs and performance that is unique for each situation.
1.2.1 Examples

Examples of extremely vibration-sensitive instruments are found in facilities such as those for metrology systems, microbiological research, optical research, space research or metrology laboratoria [177]. Scanning probe microscopes or scanning electron microscopes (Fig. 1.3) operating in for example clean rooms or even on upper floors of buildings also require a sufficiently vibration-free environment to achieve the high resolution they are designed for.

An industry in which vibration isolation is of critical importance is the semiconductor industry. This is an industry where high production speed and low failure rate are essential [8, 73, 177]. Our society has and will be ever more dependent on the use of devices and systems that are equipped with small, fast and energy-efficient integrated semiconductor chips which incorporate billions of nanometer-scale transistors. Of the many processes involved in the fabrication of microelectronics products, photolithography has traditionally been one of the most sensitive to vibration disturbance. This process takes place in high-precision lithographic machines such as the wafer scanner, which is at the heart of integrated circuit manufacturing (Fig. 1.4). This machine is used by chip manufacturers to transfer a circuit pattern from a photomask to a thin slice of silicon referred to as the wafer. A light beam passes through a complex lens system and is projected on a silicon wafer placed on the wafer stage to imprint details of a material layer. As each layer is applied the issue of positioning accuracy is critically important, since each layer must line up exactly with all preceding layers. The metrological frame on which this lens system is placed must therefore be well-isolated from vibrations which could destroy this positioning accuracy.

The semiconductor industry has continuously been striving to produce smaller features on these integrated circuits to increase the performance per chip and to increase the throughput of the machines in an effort to reduce the costs per chip. Moore's
1.3 Electromagnetic vibration isolation

A wide variety of active and passive technologies is available to accomplish vibration isolation \[49, 136, 156\]. Amongst the most commonly used techniques are elastomers \[49, 78\], pneumatics \[29, 48, 81\] and piezoelectronics \[84, 85, 116, 147\]. Although they offer a suitable solution for most applications, the technological limits of these devices is reached for some specific high-precision applications such as those discussed above. A relatively unexplored development in such vibration isolation devices is the electromagnetic vibration isolation system with passive magnetic, or contact-less, gravity compensation. In such a system a permanent magnet structure, called a gravity compensator, exhibits a passive vertical force that lifts a platform. This gravity compensator is assisted by active actuators which ensure stabilization and advanced vibration isolation. A challenge in designing such a contact-less device is to achieve a low stiffness whilst still exhibiting a relatively large vertically oriented gravity compensation force. Although very promising in terms of bandwidth, maintenance and energy consumption, their high manufacturing costs and technological immaturity currently render them defendable for use in only a very limited range of applications.

Examples of such devices found in literature are often limited to low force levels, two-dimensional topologies and non-linear responses, as will be discussed in Chapter 4. A general investigation into the suitability of a number of topologies for implementation in a gravity compensator is therefore necessary for a better understanding of this kind of devices and for the specific application that is envisaged in this thesis.
1.4 Research goals and objectives

Based on the knowledge at the beginning of this research, the dearth in literature concerning the analytical charge modeling technique and its application into vibration isolation, several research challenges have been identified for this project. This identification also has lead to a separation of the work in a theoretical and a practical part.

Part I: Extended Analytical Charge Modeling

1. To investigate and extend the field modeling of permanent magnets, based on the magnetic surface charge model. 
   Discussed in Chapters 2 and 3
   The traditional field calculation methods for permanent magnets based on the analytical surface charge or current sheet models are restricted to cuboidal, axisymmetrical or arc-shaped permanent magnets. To gain more insight into 3D magnetic structures and their fields it is therefore essential to obtain the field equations for permanent magnets with different shapes, such as triangles, parallelograms, pyramidal frusta, etc.

2. The development of analytical models describing the interaction between a permanent magnet array and another array or current carrying coils, based on the magnetic surface charge model. 
   Discussed in Chapter 3
   In many of the high-performance applications seen today the accuracy requirements for modeling techniques are rather stringent. Further, these techniques should be capable of fast and accurate evaluation of many topologies and magnet shapes, for example in an optimization process. Many existing methods however are either limited to 2D problems or suffer from computational challenges, hence, a new method or an expansion of an existing one should be found. The 3D analytical surface charge model has been utilized to derive analytical equations in previous investigations. However, it is limited to the force and energy between two parallel magnetized permanent magnets. Investigation into the force between perpendicularly magnetized magnets, the resulting stiffness and the torque provides a significant expansion of this modeling technique.

Part II: Application to a Vibration Isolation System

3. Investigation into the requirements for a vibration isolator with passive gravity compensation. 
   Discussed in Chapter 4
   The envisaged application to illustrate the versatility of the analytical equations discussed above is a vibration isolation system with passive gravity compensation. An investigation into the set of requirements of such a device is necessary to come to a suitable design. This requires an investigation into the mechanical
requirements of such a system and a projection onto the electromechanical properties.

4. **Research into feasible topologies for and the design of a passive magnetic spring.**
   
   *Discussed in Chapter 5*

   Passive magnetic gravity compensation for vibration isolation is an unexplored domain and therefore it is considered necessary to do an assessment of the feasibility for various structures. This is initially performed with a field study which provides insight in suitable topologies. Automated design optimization is used to assess the suitability of various topologies for an electromagnetic gravity compensator. The most suitable design for a magnetic spring, to be used in the vibration isolation application, can be derived and optimized from these basic topologies based on the requirements.

5. **The design of actuators which can be integrated into the design of the passive magnetic spring.**
   
   *Discussed in Chapter 6*

   The passive magnetic spring is not capable by itself to fulfill all design challenges as well as providing a stable platform. As such, active actuation is a necessity in the electromagnetic vibration isolation system. Preferably, the active actuators that are incorporated into the system will be integrated into the magnetic spring as much as possible and exhibit a near linear behavior.

6. **The realization and test of a prototype.**
   
   *Discussed in Chapter 7*

   A prototype is the best method to verify the developed modeling tools and the design of the electromagnetic vibration isolator. Therefore, a single electromagnetic vibration isolator is built and tested in a laboratory environment.

7. **The development of a custom test rig which is suitable to evaluate the performance of a prototype of the magnetic vibration isolator.**
   
   *Discussed in Chapter 7*

   To evaluate the vibration isolation performance of the system, a custom test rig is designed to evaluate its performance. An integrated shake rig provides an artificial floor environment to the isolation system which can be used to generate artificial vibrations.

### 1.5 Outline of the thesis

The first part of this thesis aims to extend the theory on magnetic surface charge modeling of permanent magnets and their interactions. Chapter 2 uses Maxwell’s equations in a magnetostatic environment to describe the basics of the electromagnetic modeling techniques and provides an overview of the techniques available. The magnetic surface charge model is discussed in Chapter 3 and extended with novel field equations for exotic magnet shapes and novel equations to model the interaction between cuboidal permanent magnets.
The second part of the thesis concerns the application of the derived equations into an electromagnetic vibration isolation system. Chapter 4 aims to isolate the specific problems that occur in the design and implementation of such an advanced vibration isolation system and provides an overview of electromagnetic vibration isolation systems realized over the years. The investigation into gravity compensator topologies and the magnetic design of the final prototype is presented in Chapter 5. The integrated actuators which complete the electromagnetic vibration isolation strut are shown in Chapter 6. In Chapter 7 these magnetic designs are translated to a prototype which is experimentally evaluated in a custom test rig. The conclusions, contributions and recommendations are presented in Chapter 8.
Part I

Extended Analytical Charge Modeling
Chapter 2

Electromagnetic modeling

A repetition of Maxwell's equations and a discussion how they are employed for the magnetostatic modeling of the permanent magnet fields and for force, numerical and analytical modeling methods and suitability of these methods for various types of electromechanical devices.
Before investigating an analytical modeling technique for magnetostatic problems it is first necessary to start at the basic Maxwell’s equations. From these equations various properties can be derived which already provide some insight in the behavior of magnetic systems. This chapter presents these exercises and discusses various modeling techniques for analyzing electromechanical systems.

2.1 Outline

Maxwell’s equations are repeated in Section 2.2 and are reduced to the magnetostatic problem that is considered in this thesis. Section 2.3 translates these equations to the modeling of a permanent magnet. With these field and energy equations the available force equations are discussed in Section 2.4. Several methods exist to implement the derived energy, field and force equations to serve as a design and optimization tool. These are discussed in Section 2.5 and followed by a brief discussion on the dearths in the analytical modeling in Section 2.6. Finally, Section 2.7 presents the conclusions of this chapter.

2.2 Maxwell’s equations

To describe the electromagnetic behavior for devices with permanent magnets, it is necessary to start with Maxwell’s equations [121]. These are generalized equations which describe the electromagnetic field phenomena and the interaction of charged matter. They are actually a collection of equations found by a number of scientists which have been assembled and linked together by Maxwell and are in differential form given by

Ampère’s law: \[ \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}, \] (2.1)

Gauss’s law for magnetism: \[ \nabla \cdot \vec{B} = 0, \] (2.2)

Faraday’s law: \[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \] (2.3)

Gauss’s flux theorem: \[ \nabla \cdot \vec{D} = \rho_c. \] (2.4)

The vector \( \vec{H} \) [A/m] is the magnetic field strength, \( \vec{E} \) [V/m] is the electrical field strength, \( \vec{B} \) [T] is the magnetic flux density, \( \vec{D} \) [C/m²] is the electric flux density and these fields are vector-valued functions of space \((x, y, z)\) [m] and time \(t\) [s]. The free current density is given by \( \vec{J} \) [A/m²] and \( \rho_c \) [C/m³] is the electric charge density. Since these equations by themselves do not provide a complete set of equations for the fields [66] they must be augmented by additional independent equations that take the form of constitutive relations which describe the material properties

\[ \vec{B} = \mu_0 (\vec{H} + \vec{M}), \] (2.5)

\[ \vec{D} = \varepsilon_0 \vec{E} + \vec{P}, \] (2.6)

\[ \vec{J} = \sigma_e \vec{E}. \] (2.7)
2.2: Maxwell’s equations

The natural constants \( \mu_0 = 4\pi \times 10^{-7} \text{[H/m]} \) and \( \epsilon_0 = 8.854 \times 10^{-12} \text{[F/m]} \) are the permeability and permittivity of free space, respectively. The magnetization is given by \( \vec{M} \text{[A/m]} \) and the polarization by \( \vec{P} \text{[C/m}^2] \). These values represent the net magnetic and electric dipole moment per unit volume, respectively. The electrical conductivity is represented by \( \sigma_e \text{[S/m]} \).

2.2.1 Magnetization

In the Sommerfeld convention the fields are related by the constitutive relation (2.5). The magnetization \( \vec{M} \) is composed of a primary and a secondary component

\[
\vec{M} = \vec{M}_{\text{prim}} + \vec{M}_{\text{sec}}. \tag{2.8}
\]

The primary magnetization \( \vec{M}_{\text{prim}} \) is the used to represent the (idealized) physical source of the magnetic field. This source is composed of magnetic dipoles (Section 2.3) which are the fundamental element of magnetism. The secondary magnetization \( \vec{M}_{\text{sec}} \) is the result of the interaction between the field \( \vec{H} \) and the magnetic dipoles. They are given for linear and homogeneous media, in which the dimensionless magnetic susceptibility \( \chi_m \) is independent of \( H \), by

\[
\vec{M}_{\text{prim}} = \frac{B_r}{\mu_0}, \tag{2.9}
\]

\[
\vec{M}_{\text{sec}} = \chi_m \vec{H}. \tag{2.10}
\]

The variable \( B_r \text{[T]} \) is the remanent flux density that remains when the field \( \vec{H} = 0 \) and \( \mu \text{[H/m]} \) is the permeability. The constitutive relation (2.5) can be written as

\[
\vec{B} = \mu_0 (\chi_m \vec{H} + B_r \mu_0^2) = \mu \vec{H} + \vec{B}_r. \tag{2.11}
\]

The permeability \( \mu \) is decomposed into \( \mu = \mu_0 \mu_r \), where \( \mu_r \) is the dimensionless relative permeability, given by

\[
\mu_r = (\chi_m + 1). \tag{2.12}
\]

Linear, homogeneous and isotropic materials have no primary magnetization \( \vec{M}_{\text{prim}} \), hence \( \vec{B}_r \) is zero. The variables \( \vec{B} \) and \( \vec{M} = \vec{M}_{\text{sec}} \) are both proportional to \( \vec{H} \).

2.2.2 Magnetostatic analysis

In electromechanical devices the field changes are generally much slower than the time required for it to propagate across the region. In other words, the fields are in the considered volume \( V \text{[m}^3] \) and time span \( t_{\text{min}} < t < t_{\text{max}} \) not a function of time as the wavelength of the electromagnetic field that permeates it is much larger. As such, the field’s finite speed of propagation is ignored and it is assumed that any change in the field is felt instantaneously across the region. Consequently, the displacement current term \( \partial \vec{D}/\partial t \) is considered negligible after which (2.1) becomes

\[
\nabla \times \vec{H} = \vec{J}. \tag{2.13}
\]
This quasi-static field theory governs an important range of applications including electrical circuit analysis, electromechanical devices and eddy current phenomena. In this thesis we focus on static field theory which has no time variation. Amazingly, this restrictive theory applies to a wide range of important phenomena involving steady currents or charges [66]. This renders it very suitable for the analytical surface charge models in Chapter 3. The application described in Part II of this thesis is a vibration isolation system, which is inherently dynamic. However, as Chapter 4 discusses, the velocity and displacement levels that are expected have such low values that dynamic electromagnetic effects, such as induced fields and eddy currents, are considered negligible.

If the time derivative $\partial \vec{B}/\partial t$ equals zero, for a static analysis, Maxwell’s equations (2.1)-(2.4) uncouple into magnetic and electric equations. The magnetostatic equations, resulting from Ampère’s law and Gauss’s law for magnetism, are written as

\begin{align}
\nabla \times \vec{H} &= \vec{J}, \\
\nabla \cdot \vec{B} &= 0,
\end{align}
(2.14) (2.15)

A direct solution of the field equations is a valid method to obtain these fields. However, it is often more convenient to obtain the fields using potential functions [66, 92, 169]. In particular, the equations are written in the form of a Poisson’s equation $\nabla^2 \varphi = f$ or, with vanishing $f$, Laplace’s equation $\nabla^2 \varphi = 0$. The Poisson’s equation may be solved with a Green’s function.

**Vector potential** The vector potential formulation starts from the magnetostatic field equations (2.14) and (2.15). A vector potential $\vec{A} [\text{Vs/m}]$ is introduced from (2.15)

$$
\vec{B} = \nabla \times \vec{A}.
$$
(2.16)

By substituting (2.16) into (2.14) and the equality $\nabla \times \nabla \times \vec{A} = \nabla (\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$, taking into account the constitutive relation (2.11), it can be derived that

\begin{equation}
\nabla^2 \vec{A} - \nabla (\nabla \cdot \vec{A}) = -\mu_0 (\mu_r \vec{J} + \nabla \times \vec{M}_{prim}),
\end{equation}
(2.17)

With the Coulomb gauge condition $\nabla \cdot \vec{A} = 0$ it follows that

\begin{equation}
\nabla^2 \vec{A} = -\mu_0 (\mu_r \vec{J} + \nabla \times \vec{M}_{prim}),
\end{equation}
(2.18)

This suggests the introduction of a fictitious equivalent magnetic volume current density

$$
\vec{J}_m \equiv \nabla \times \vec{M} [\text{A/m}^2].
$$
(2.19)

In integral form, using the Green’s function for the operator $\nabla^2$ it is expressed as [92]

$$
\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi} \int_{-\infty}^{\infty} \mu_r \vec{J}(\vec{x}') + \vec{J}_m(\vec{x}') \frac{\vec{x} - \vec{x}'}{|\vec{x} - \vec{x}'|} \, d\nu'.
$$
(2.20)
Hence, the vector potential is a function of the position, the currents and the relative permeability in the considered domain. It is often used in 2D modeling to visualize flux lines, which indicate the direction of the \( \vec{B} \)-field at any point along its length, because equal amounts of its equipotential contours coincide with these flux lines. From (2.20), the flux density \( \vec{B}(\vec{x}) \) is given by

\[
\vec{B}(\vec{x}) = \frac{\mu_0 \mu_r}{4\pi} \int_{-\infty}^{\infty} \vec{J}(\vec{x}') \times \frac{(\vec{x} - \vec{x}')}{|\vec{x} - \vec{x}'|^3} d\vec{v}' + \frac{\mu_0}{4\pi} \int_{-\infty}^{\infty} \vec{J}_m(\vec{x}') \times \frac{(\vec{x} - \vec{x}')}{|\vec{x} - \vec{x}'|^3} d\vec{v}'.
\] (2.21)

Two identities have been used here [66]

\[
\nabla \times \frac{\vec{J}(\vec{x}')}{|\vec{x} - \vec{x}'|} = -\frac{\vec{J}(\vec{x}') \times \nabla}{|\vec{x} - \vec{x}'|},
\] (2.22)

\[
\nabla \frac{1}{|\vec{x} - \vec{x}'|} = -\frac{(\vec{x} - \vec{x}')}{|\vec{x} - \vec{x}'|^3}.
\] (2.23)

**Scalar potential**  
The scalar potential formulation starts from the magnetostatic field equations for current-free regions \( \nabla \times \vec{H} = 0 \) and \( \nabla \cdot \vec{B} = 0 \). The scalar potential \( \Psi [A] \) is introduced from (2.14) as

\[-\nabla \Psi = \vec{H}.\] (2.24)

Substitution of (2.24) into (2.15) taking into account (2.11) and (2.9) yields for the scalar potential

\[\nabla^2 \Psi = \frac{\nabla \cdot \vec{M}_{\text{prim}}}{\mu_r}.\] (2.25)

In the absence of boundary surfaces it can be written in integral form using the Green's function [92]

\[\Psi = \frac{1}{4\pi \mu_r} \int_{-\infty}^{\infty} \nabla' \cdot \frac{\vec{M}_{\text{prim}}(\vec{x}')}{|\vec{x} - \vec{x}'|} d\vec{v}'.\] (2.26)

The scalar potential is a function of the position and the magnetization vectors \( \vec{M}(\vec{x}) \) in the considered domain, as well as the relative permeability \( \mu_r \). The numerator suggests the introduction of a fictitious magnetic volume charge density \( \rho_m [\text{A/m}^2] \)

\[\rho_m = -\nabla \cdot \vec{M}_{\text{prim}},\] (2.27)

\[\Psi = \frac{1}{4\pi \mu_r} \int_{-\infty}^{\infty} \rho_m(\vec{x}') \frac{\vec{x} - \vec{x}'}{|\vec{x} - \vec{x}'|} d\vec{v}'.\] (2.28)

With the help of (2.24) and identity (2.23) the resulting equation for the field \( \vec{H}(\vec{x}) \) is given by

\[\vec{H}(\vec{x}) = \frac{1}{4\pi \mu_r} \int_{-\infty}^{\infty} \rho_m(\vec{x}')(\vec{x} - \vec{x}') \frac{1}{|\vec{x} - \vec{x}'|^3} d\vec{v}'.\] (2.29)
Energy

A magnetostatic field contains energy. In the magnetostatic model this electromagnetic energy $W_{em}$ is, in the absence of dissipation mechanisms, equal to the energy that is required to create this field. The energy added to a system of currents and magnetizable materials to create a field is given by

$$W_{em} = \int_{V} \left[ \int_{0}^{B} \vec{H} \cdot d\vec{B} \right] dV. \quad (2.30)$$

If the system is linear this reduces to [66, p. 117][92]

$$W_{em} = \frac{1}{2} \int_{V} \vec{B} \cdot \vec{H} dV. \quad (2.31)$$

2.2.3 Magnetostatic boundary conditions

Boundary-value problems arise when studying problems with transitions between different media. Observing the Gaussian pill-box of Fig. 2.1(a), with $w \gg h$ such that $h \rightarrow 0$, the integral form of (2.2) is given by

$$\int_{S_1} \vec{B}_1 \cdot \vec{v} ds - \int_{S_2} \vec{B}_2 \cdot \vec{v} ds = 0, \quad (2.32)$$

where $\vec{B}_1$ and $\vec{B}_2$ are the flux densities in the respective materials and $S_1$ and $S_2$ the pillbox surfaces. If these surfaces are chosen sufficiently small this reduces to

$$(\vec{B}_1 - \vec{B}_2) \cdot \vec{v} = 0. \quad (2.33)$$

It is concluded that the normal component of $\vec{B}$ is continuous at the boundary.

The boundary condition for the tangential component of $\vec{H}$ is derived similarly. Using Fig. 2.1(b), again with $w \gg h$ such that $h \rightarrow 0$, the integral form of (2.14) yields

$$\int_{l_1} \vec{H}_1 \times \vec{v} dl - \int_{l_2} \vec{H}_2 \times \vec{v} dl = \vec{j}_s, \quad (2.34)$$
2.3 Modeling of a permanent magnet

The magnetic dipole with dipole moment \( \vec{m} \) [Am\(^2\)] is the fundamental element of magnetism. It can be thought of as a pair of closely spaced magnetic poles or equivalently as a small current loop as Fig. 2.2(a) and (b) show. These magnetic dipoles, which arise from the angular momentum of the electrons on an atomic level, form small domains within the permanent magnet as is shown in Fig. 2.2(c). Although their magnetization is generally not uniform throughout the permanent magnet as the material is composed of tiny domains \( \Delta V \), the number of domains is of such large value that statistically one can speak about a net magnetization \( \vec{M} \) of the permanent magnet.

In and around the permanent magnet the \( B \) and \( H \) fields are coupled by (2.5) or more specifically by (2.11). Assuming that the magnet is in free space, \( \vec{M} \) is zero outside the magnet. Within the magnetic material there is a magnetization and consequently \( \vec{B} \) and \( \vec{H} \) are not necessarily proportional or even parallel.

Given the equalities \( \nabla \cdot \vec{B} = 0 \) and \( \nabla \times \vec{H} = 0 \) (no free currents) the energy density is integrated over all space and it can be shown that [38, p. 2]

\[
\int_{\infty} \vec{B} \cdot \vec{H} \, dV = 0. \tag{2.36}
\]
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Figure 2.3: Idealized illustration of (a) the magnetization $\vec{M}$, (b) the $\vec{B}$-field and (c) the $\vec{H}$-field of a permanent magnet. Within the magnet body the direction of $\vec{H}$ is reversed whereas $\vec{B}$ is parallel to $\vec{M}$.

With the magnet volume $V$ this can now be written into

$$\int_{\infty-V}^{\infty-V} \vec{B} \cdot \vec{H} \, dv = - \int_{V}^{\infty} \vec{B} \cdot \vec{H} \, dv.$$  \hspace{1cm} (2.37)

The left hand side of the equation is necessarily positive and equals

$$\int_{\infty-V}^{\infty-V} \vec{B} \cdot \vec{H} \, dv = \mu_0 \int_{\infty-V}^{\infty-V} H^2 \, dv.$$  \hspace{1cm} (2.38)

This is identified as twice the potential energy associated with the field set up by the magnet outside its volume. This energy is always positive and therefore the right hand side of (2.37) must also be positive. It follows that in the permanent magnet $\vec{B}$ and $\vec{H}$ tend to be antiparallel within the body of the permanent magnet as Fig. 2.3 shows.

Given the boundary conditions on the magnet surface – the normal component of $\vec{B}$ is continuous whereas the tangential component of $\vec{H}$ is continuous – it follows that $\vec{B}$ is continuous through the studied domain and that $\vec{H}$ reverses direction within the magnet volume. More intuitively, $\vec{B}$ can be seen as if the direct result from a surface current density over the magnet’s surface, similar to a solenoid. $\vec{H}$ is more like a dipole field and may be estimated if the magnet is replaced by an equivalent surface distribution of fictitious ‘magnetic charge’ which act as sources or sinks of
2.3: Modeling of a permanent magnet

$H$. Outside the magnet it is known as stray field, but within the sample volume $V$ it is known as demagnetizing field as it tries to reduce $\vec{M}$. These current and dipole representations are especially used in the analytical surface charge and current sheet models described in Section 2.5.3.

The density of the field lines in Fig. 2.3(b) represents the flux density $\vec{B}$. The amount of magnetic flux $\varphi$ [Wb] of this field through a surface $S$ is given by

$$\varphi = \int_S \vec{B} \cdot d\vec{s}. \tag{2.39}$$

2.3.1 Hysteresis

The state of magnetization of a ferromagnetic material is changed by an external field in a nonlinear and irreversible way. Figure 2.4 shows a typical and idealized $B(H)$ hysteresis curve for such a material. Starting at the origin the material is magnetized along the virgin curve towards saturation in the first quadrant. The magnetic moments of all domains are oriented along the external magnetic field generated by a magnetizing coil. If the magnetization current is removed, the working point shifts to the second quadrant in accordance with the hysteresis loop. At $H = 0$ the flux density $\vec{B}$ attains the remanent flux density $B_r$. The point where this flux density reaches zero is called the coercivity $H_{cb}$ [A/m]. The intrinsic coercivity $H_{ci}$ [A/m] (or switching field) is the point where the hysteresis loop switches; it is a measure of the field required to magnetize or demagnetize a magnet specimen. This symmetric hysteresis loop is traced reproducibly provided that the applied field is sufficient to achieve saturation in each direction.

Permeable materials such as low-carbon iron exhibit a high remanent flux density $B_r$, up to 2T. As a result of their high relative permeability, yielding a large derivative of the $B_H$-characteristic, their coercivity is very low, meaning a small demagnetization withstand. This is a very valuable property for use as back-iron in many electrical machines as it increases the possible magnetic loading and reduces hysteresis loss. This renders them unsuitable to act as permanent magnet.

Typical permanent magnets (NdFeB, SmCo, Alnico, ferrites) exhibit remanent flux densities between approximately 0.4 T and 1.5 T. Except for Alnico magnets the second quadrant of these magnets is almost a straight line with a slope close to $\mu_0$. Ideal permanent magnets exhibit a magnetization curve that is a square loop, as a result of which the slope of the $B_H$-characteristic exhibits the same slope up to $H_c$. Especially for NdFeB magnets this can be assumed as they exhibit an intrinsic coercivity which is larger than their coercivity.
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Figure 2.4: Schematic representation of a typical $B(H)$ hysteresis curve of a permanent magnet showing the four quadrants I-IV, the coercivity $H_{cB}$, the intrinsic coercivity $H_{ci}$, the remanent flux density $B_r$ and the maximum energy point $B H_{\text{max}}$.

2.3.2 Magnetostatic energy

A magnetized specimen with volume $V$ and fixed magnetization $\vec{M}$ possesses a self-energy \[ W_{\text{self}} = -\frac{\mu_0}{2} \int_V \vec{M} \cdot \vec{H}_M \, d\nu, \] (2.40)

where $\vec{H}_M$ is the field in the specimen due to $\vec{M}$. This can be considered as the energy required to assemble a continuum of dipole moments in absence of an applied field. When subjected to an external field $\vec{H}_{\text{ext}}$ the specimen obtains a potential energy

\[ W_{\text{ext}} = -\mu_0 \int_V \vec{M} \cdot \vec{H}_{\text{ext}} \, d\nu, \] (2.41)

which can be viewed as the work required to move it from a region with zero external field to a region with field $\vec{H}_{\text{ext}}$.

The maximum potential energy is obtained at the point on the loop where the product $-B \cdot H$ is maximized. This is the maximum energy product, or the figure of merit, $(BH)_{\text{max}}$ and is shown in Fig. 2.4. It gives an indication of the potential energy that the magnet exhibits (the hatch-filled square) and, especially for magnets in structures with back-iron, it is often a design target to have the magnet working in this point. In environments without highly permeable materials it is often difficult to find a single working point, or at least a small working area, on the $BH$-characteristic as the fields may become strongly nonuniform throughout the magnet.
2.4 Force calculation

Generally, there are three main methods distinguished to obtain the interaction force in electromechanical devices, namely virtual work based on variation of energy, the Maxwell stress tensor and Lorentz force.

2.4.1 Lorentz Force

The basic Lorentz force equation describes the force \( F(\vec{x}) \) [N] on a particle of charge \( q \) [C] which moves through an external field \( \vec{B}_{\text{ext}}(\vec{x}) \) with velocity \( \vec{v} \) [m/s] in the presence of an electric field \( \vec{E}(\vec{x}) \) [V/m]

\[
F(\vec{x}) = q(\vec{E}(\vec{x}) + \vec{v} \times \vec{B}(\vec{x})).
\]  

(2.42)

Under the assumption that \( \vec{E} \) equals zero a translation of this equation to a constant volume current density \( \vec{J} \) that is located in an external static magnetic field \( B \) yields a force density \( \vec{f}(\vec{x}) \) [N/m³] that is written as [64, 66]

\[
\vec{f}(\vec{x}) = \vec{J}(\vec{x}) \times \vec{B}_{\text{ext}}(\vec{x}),
\]  

(2.43)

\[
\vec{F}(\vec{x}) = \int_V \vec{f}(\vec{x}') \, dV' = \int_V \vec{f}(\vec{x}') \times \vec{B}_{\text{ext}}(\vec{x}') \, dV' + \int_S \vec{j}(\vec{x}') \times \vec{B}_{\text{ext}}(\vec{x}') \, dS'.
\]  

(2.44)

Here, \( \vec{J}(\vec{x}) \) [A/m³] is a volume current density in the considered volume \( V \) and \( \vec{j}(\vec{x}) \) [A/m²] is a surface current density on its surface \( S \) [m²]. Similarly, the torque density \( \vec{T}(\vec{x}) \) [Nm/m³] and torque \( \vec{T}(\vec{x}) \) [Nm] are obtained by

\[
\vec{t}(\vec{x}) = \vec{r}(\vec{x}') \times \vec{f}(\vec{x}'),
\]  

(2.45)

\[
\vec{T}(\vec{x}) = \int_V \vec{t}(\vec{x}') \, dV',
\]  

(2.46)

where \( \vec{r}(\vec{x}') \) [m] is the arm. The integrals above can be performed numerically as well as analytically, depending on the method that has been selected for the field modeling, and the geometrical properties of the device under focus.

It is observed that this formulation is only directly suitable for a vector potential formulation, as the scalar potential formulation lacks the free currents \( \vec{j}(\vec{x}) \). With the help of the identity (2.19) the Lorentz force (2.44) may be written in terms of the magnetization

\[
\vec{F}(\vec{x}') = \int_V (\nabla \times \vec{M}(\vec{x}')) \times \vec{B}_{\text{ext}}(\vec{x}') \, dV' + \int_S (\vec{M}(\vec{x}') \times \vec{n}) \times \vec{B}_{\text{ext}}(\vec{x}') \, dS'.
\]  

(2.47)

This equation enables the use of the current-free scalar potential to obtain the Lorentz force technique. It can be reduced to [64]

\[
\vec{F}(\vec{x}') = -\int_V (\nabla \cdot \vec{M}(\vec{x}')) \vec{B}_{\text{ext}}(\vec{x}') \, dV' + \int_S (\vec{M}(\vec{x}') \cdot \vec{n}) \vec{B}_{\text{ext}}(\vec{x}') \, dS'.
\]  

(2.48)
2.4.2 Maxwell Stress Tensor

A more general approach to obtain the electromagnetic force between sources is the Maxwell stress tensor. The force on a body is computed by

\[ \vec{F}(\vec{x}) = \frac{1}{\mu} \int_V \nabla \cdot \mathbb{T}(\vec{x}') \, dV'. \tag{2.49} \]

Here, \( \mathbb{T} \) [N/m²] is the Maxwell stress tensor which is a matrix given by

\[ \mathbb{T}(\vec{x}) = \begin{bmatrix}
(B_x^2 - \frac{1}{2} |\vec{B}|^2) & B_xB_y & B_xB_z \\
B_yB_x & (B_y^2 - \frac{1}{2} |\vec{B}|^2) & B_yB_z \\
B_zB_x & B_zB_y & (B_z^2 - \frac{1}{2} |\vec{B}|^2)
\end{bmatrix}. \tag{2.50} \]

This volume integral can be written as a surface integral using the Divergence theorem \[66]\]

\[ \vec{F}(\vec{x}) = \frac{1}{\mu} \oint_S \mathbb{T}(\vec{x}') \cdot \hat{n} \, ds', \tag{2.51} \]

where \( \mu \) is the permeability of the medium where the integration takes place, given by \( \mu = \mu_0 \mu_r \), \( \hat{n} \) is the outward unit normal to the bounding surface and \( S \) is the integration surface immediately surrounding the body. Equally, the torque is obtained by

\[ \vec{T}(\vec{x}) = \frac{1}{\mu} \oint_S \vec{r}(\vec{x}') \times (\mathbb{T}(\vec{x}') \cdot \hat{n}) \, ds'. \tag{2.52} \]

2.4.3 Virtual work method

The virtual work principle is based on the conservation of energy and is the work resulting from either virtual forces acting through a real displacement or real forces acting through a virtual displacement. It is the most general force calculation method because it is based on the calculation of energy and is therefore on this energy level compatible with other domains, such as the mechanical, thermodynamic or electrical domain, unlike the two methods described above, which are strictly limited to the electromagnetic domain.

If an energy \( W_{in} \) [J] is added to a system it is partly dissipated in \( W_{diss} \), partly stored as electromagnetic energy in the system \( W_{em} \) and partly converted to mechanical output energy \( W_{mech} \)

\[ dW_{in} = dW_{diss} + dW_{em} + dW_{mech}. \tag{2.53} \]

If the system is treated lossless and with \( W_{in} = 0 \) it is considered that the energy is only changing form from mechanical to electromagnetic and vice versa. In other words, all mechanical and electromagnetic energy can be exchanged lossless. The force can be written as

\[ \vec{F} = -\nabla W_{em}. \tag{2.54} \]
This energy is obtained using the equations described in Section 2.3.2. E.g. for a permanent magnet with volume $V$ in an external field a torque $T$ [Nm] is produced when $\vec{B}_{\text{ext}}$ and $\vec{M}$ are not parallel or antiparallel [38, p. 6]

$$T \approx \int_V \vec{M} \times \vec{B}_{\text{ext}} \, dv'. \quad (2.55)$$

When $\vec{B}_{\text{ext}}$ is nonuniform, a net force $F$ acts on the permanent magnet along the field gradient

$$\vec{F} = \nabla \left( \int_V \vec{M} \times \vec{B}_{\text{ext}} \, dv' \right). \quad (2.56)$$

### 2.5 Magnetostatic field modeling methods

A wide variety of modeling tools, numerical or analytical, has been developed over the years to solve the fields and potentials described in the previous sections [21, 34, 55, 66]. This section provides a short overview of some of the most used methods, starting from the numerical ones and gradually working towards the analytical methods.

#### 2.5.1 Volume discretization

By discretizing the solution space volume into small volumes and solving the fields for all of these volumes an accurate representation of the electromagnetic phenomena can be obtained. The most well-known method in this is the Finite Element Method. The Magnetic Equivalent Circuit model is placed in this category too as it also represents a volume discretization, though with fewer mesh elements. In two-dimensional problems the volume discretization reduces to a surface discretization. As a result of their volume discretization these methods are especially suitable for bound problems, such as periodical structures or problems with iron boundaries, and less for unbound problems, such as magnets in free space [34].

**Finite Element Method**

The Finite Element Method (FEM) is a numerical technique for finding approximate solutions of partial differential equations (PDE) as well as of integral equations and is the most often-used tool to model electromagnetic field problems [93]. It is a powerful tool, capable of handling many different system properties, and widely available. However, it does exhibit some significant disadvantages due to its mesh-based approach.

In FEM, a solution region is decomposed into a finite number of subregions called mesh elements which are sufficiently small to assume constant fields and potentials. The density of this mesh may vary throughout the solution space and it is often
necessatory to have a-priori knowledge of the expected fields in the model to make a suitable mesh. An example mesh is found in Section 5.9.4. A (polynomial second-order) approximation function is searched for each element and is used to iteratively find the nodal potential values, from which all fields are derived. A major consequence of meshing is the inherently bound problem that results; as the number of elements is finite there must be an edge to the problem on which boundary conditions, such as those briefly described in Section 2.2.3, apply. Often these are in the form of Neumann or Dirichlet conditions [66]. The FEM solution accuracy rises with a higher mesh density, however the computational requirements may become overwhelming and in the limit numerical truncation errors may occur [30]. This method’s field calculation is also sensitive to jumps in permeability on the material boundaries. Although this method is computationally expensive, it is, unlike many other methods, able to include complex structures of magnetically permeable materials, saturation and dynamic effects.

In ironless structures, with no concentrated magnetic fields, or machines with a small displacement versus dimensions ratio implementing this method may become problematic due to the necessity of a high mesh density. The meshing of the model becomes a dominant time factor in the solving process, even though the absence of saturation effects reduces the computational efforts for solving the fields themselves. The mesh must be dense inside the active volumes as well as outside of them. Further, the solving domain with the mesh elements must be sufficiently large to accommodate for the vanishing fields. If the outer boundary, generally with an imposed potential of 0, is too close, the fields near the studied object may be significantly influenced by these boundaries. The resulting compromise between accuracy and computational efforts makes it not the best choice for fast design evaluations or advanced optimization routines with many model iterations. However, its ability to include more complexity into the model compared to other methods below, its availability and accuracy make it very suitable to be used as verification for the more simplified models and if necessary for fine-tuning the modeling variables in a very late stage.

**Finite Differences Method**

This method, abbreviated with FDM is one of the oldest numerical methods for the solution of partial differential equations. It uses a uniformly-spaced grid of nodes (mesh). The differential equation is approximated by a finite difference equation that relates the value of the solution at a given node to its values at neighboring surfaces, constructs a set of equations from this and solves this set to obtain the nodal values. Finite Difference methods exhibit the same numerical problems as FEM methods, with the addition that their mesh is generally uniformly spaced.

**Finite Volume Method**

Similar to FEM or FDM, the Finite Volume Method (FVM) makes use of a discretely meshed geometry. The term Finite volume refers to the small volume surrounding
2.5: Magnetostatic field modeling methods

Each node point of the mesh. In the finite volume method, volume integrals in a partial differential equation that contain a divergence term are converted to surface integrals, using the divergence theorem. These terms are evaluated as fluxes at the surfaces of each finite volume.

Magnetic Equivalent Circuit modeling

A MEC (Magnetic Equivalent Circuit) model is an electric equivalent representation of a magnetic circuit [38, 141, 161] and can be categorized as Finite Volume Method. The most significant difference, and simultaneously the most important pitfall, is that magnetic flux, unlike electric currents, are not confined to well-defined, analytically traceable paths. It is based on Ampere’s law 2.1 which, in its integral form, is rewritten to

\[ H_m l_m + \int_A \vec{H} \cdot d\vec{l} = \sum I_{\text{enclosed}}, \] (2.57)

where \( H_m \) is average field over the flux path \( l_m \) through the magnet, and the line integral is outside the magnet.

Compared to e.g. FEM, the MEC model is relatively simple and fast-solving, because the model is discretized in a limited number of mesh elements, called ‘flux tubes’. It is assumed that all flux enters such a tube perpendicularly through one of its surfaces, remains parallel within the tube, and exits perpendicular to the opposite surface of the tube. The magnetic reluctance, equivalent to the electrical resistance, of such a tube is obtained by

\[ R = \int L \frac{d\vec{l}}{\mu L S_{\text{avg}}}, \] (2.58)

where \( S_{\text{avg}} [\text{m}^2] \) is the average surface of the flux tube having the (flux-dependent) average permeability \( \mu_L \) over its length \( L [\text{m}] \).

This method is therefore considered as a simplification of FVM, which is also based on the calculation of flux through the surface of the volumes surrounding the mesh nodes. These flux tubes should be very well-defined and therefore a good understanding of the magnetic structure is eminent. The technique is very sensitive to geometrical changes, as flux paths tend to change and possibly require a new equivalent circuit design. Equation 2.57 shows that only the average field \( H_m \) is used in this modeling technique, or, regarding Fig. 2.4, that the whole magnet is in the same working point. This assumption may be reasonable for structures with highly permeable flux conductors and fairly constant flux through the magnets, but quickly becomes inaccurate in ironless structures with no pre-defined flux paths and highly nonuniform flux inside the magnets. Especially for this kind of models the regions, or flux tubes, are strictly bound by zero-flux boundary conditions, which renders them unsuitable for ironless applications.
In general, MEC modeling is mostly used in structures with unsaturated, soft magnetic materials which define the flux tubes with relatively high accuracy. Further, small air gaps are generally necessary to avoid inaccuracies caused by flux fringing and it is assumed that flux through a permanent magnet body is strictly defined along the magnetization vector. To come to a suitable model it is often necessary to consult a more accurate method, such as FEM, many times [97]. Even then, a MEC model is highly sensitive to geometrical model variations and quickly becomes inaccurate. In ironless structures, especially with permanent magnets in repulsion on short intermediate distances, this method is very difficult to implement due to the absence of well-defined flux paths. It is therefore only considered suitable for a very limited range of electromagnetic devices and certainly not for ironless applications which exhibit unbound fields.

2.5.2 Surface discretization

Some methods only require a discretization of boundary surfaces in the solution space instead of discretization of the whole volume of the studied problem. Two commonly used examples are the Boundary Element Method and harmonic modeling. In two-dimensional problems the surface discretization reduces to a line discretization. As for the volume discretization methods the methods with surface discretization are especially suitable for bound problems, such as periodical structures or problems with iron boundaries.

Boundary Element Method

The boundary element method (BEM) is a numerical computational method of solving linear partial differential equations which have been formulated as integral equations (i.e. in boundary integral form) [187]. In this method only the surfaces of the permeable objects in the model are meshed, instead of also those volumes themselves. The fields are solved only outside the permeable objects using the given boundary conditions at those surfaces to fit boundary values into the integral equation, rather than values throughout the space defined by a partial differential equation. Once this is done, in the post-processing stage, the integral equation can be used again to numerically calculate the solution directly at any desired point in the solution domain.

Although often less accurate than FEM the required number of mesh elements is significantly lower for this method, which makes it significantly faster. If all objects with meshed surfaces are infinitely permeable, the accuracy of this method is very high, however generally reduces if saturation effects occur. If an ironless device is considered there is no surface with well-defined boundary conditions that can be meshed. Consequently, only the analytical integral equations need to be solved.
2.5: Magnetostatic field modeling methods

Harmonic modeling

Harmonic modeling, sometimes referred to as subdomain or Fourier modeling, has been widely developed and used to model electromagnetic devices of various kinds [75, 92, 95, 125, 183]. In this technique the direct solution of the magnetostatic Maxwell equations is considered, which reduces to the Laplace equation in source-free regions and the Poisson equation in a magnetized or current-carrying region. It is applied under the assumptions that all materials are linear homogeneous and that the soft-magnetic materials are uniformly permeable.

The method describes a magnetic structure in terms of infinite series of space harmonics. It describes a topology with infinitely permeable iron boundaries in a periodical structure, or at least a structure which can be converted into a periodical structure by incorporating a sufficiently large amount of space around the studied object to magnetically separate the periodical structures. Perpendicular to the direction of the periodicity, several regions are stacked and are given each a different material and property (magnetized magnet, vacuum, current, etc.). Each of these regions is described by a Fourier representation in the periodical direction. Starting from the source regions, the boundary values between the various regions are coupled with each other, which yields the coefficients of the Fourier series describing each region. The general equations for the magnetic flux density distribution for a 2D model, with \( q \) along the periodical direction and \( b \) perpendicular to it can be written as

\[
\vec{B} = B_p(p, q_k)\hat{e}_p + B_q(p, q_k)\hat{e}_q, \tag{2.59}
\]

\[
B_p = \sum_{n=1}^{\infty} (B_{ps}(p) \sin(w_k q_k) + B_{pc}(p) \cos(w_k q_k)), \tag{2.60}
\]

\[
B_q = \sum_{n=1}^{\infty} (B_{qs}(p) \sin(w_k q_k) + B_{qc}(p) \cos(w_k q_k)) + B_{q0}(p). \tag{2.61}
\]

where the variables used in these equations are defined in [75] for Cartesian, polar and cylindrical coordinates. In theory, this method yields a fully analytical solution with the summation from \( n = 1 \ldots \infty \). In most situations, its practical implementation requires a limitation in the number of considered harmonics to reduce the computational efforts, as every harmonic requires a separate calculation. This technique is also suitable for use in 3D structures [125]. However, since more harmonic coefficients need to be coupled together at the region boundaries the complexity rises accordingly.

2.5.3 Mesh- and boundary-free modeling methods

Analytical field calculation techniques do not require any form of discretization and provide a fully analytical field solution. This makes such mesh-free modeling methods extremely fast since they can directly obtain the field at any given position. The number of assumptions is generally higher in such models than in the volume or surface discretization methods, hence, care must be taken that the model remains an accurate representation of the reality. The main assumptions are that the magnetization within the magnets is homogeneous (common to most modeling methods) and that the relative permeability remains constant throughout the domain (unique to these techniques).
The surface charge and current sheet magnet models

Analytical surface charge models – sometimes referred to as ‘Coulombian’ – and current sheet models – also called ‘Amperian’ – are in many cases considered suitable to obtain the 3D magnetic field components of permanent magnets in free space. These methods provide a fully 3D computation of the field, without a need for meshing, which gives an extremely accurate and time inexpensive field calculation, especially compared to numerical methods [123]. A significant advantage with respect to other techniques is the absence of the necessity for boundaries, which often increase the modeling complexity in numerical models for unbound problems [34]. Neither periodicity, iron boundaries or limited solution space are required, nor is the total considered volume of influence on the computational efforts of these models.

The main and probably the most influencing assumption, that is unique to these classical analytical models, is that throughout the studied volume it is assumed that the dimensionless relative permeability \( \mu_r = 1 \), or in other words that the susceptibility \( \chi_m = 0 \). The total permeability, \( \mu = \mu_0 \mu_r \), equals that of vacuum, producing a system which enables the use of superposition to model multiple magnets. The effects of these assumptions are discussed in Section 3.9. The surface charge model places fictitious magnetic charge distributions on the magnet using a scalar potential formulation and the current sheet current model puts fictitious currents on the magnets with a vector potential formulation. The field results of these two approaches are equivalent and therefore it is to be chosen which model is the most appropriate for calculating the three components of the magnetic field produced by permanent magnets.

The resulting analytical models are very fast, accurate and provide great insight into the parametric properties. Their linearity enables to use superposition to model the force between magnets with any magnetization direction. They are especially suitable for ironless applications, such as the gravity compensator discussed in this thesis, planar actuators or passive high-field magnetic cavities [38].

Current model

This analytical modeling technique is based on the vector potential formulation (2.20) with the assumption that the relative permeability \( \mu_r = 1 \). Under the assumption that there is no free current (\( \vec{J} = 0 \)), the magnetization vector \( \vec{M} \) is confined to the magnet volume \( V \), and falls abruptly to zero outside this volume, this vector potential becomes [66, 92][169, p. 355]

\[
\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi} \int_V \frac{\vec{J}_m(\vec{x}')}{|\vec{x} - \vec{x}'|} \, dV' + \frac{\mu_0}{4\pi} \int_S \frac{\vec{j}_m(\vec{x}')}{|\vec{x} - \vec{x}'|} \, dS'.
\] (2.62)

The fictitious magnetic volume current density \( \vec{J}_m \) was defined in (2.19) and forms together with the fictitious magnetic surface current density \( \vec{j}_m \) on the magnet surface \( S \) the equivalent currents:

\[
\text{Equivalent currents: } \begin{cases} 
\vec{J}_m = \nabla \times \vec{M} \quad \text{(Volume current density)} \\
\vec{j}_m = \vec{M} \times \vec{n} \quad \text{(Surface current density)}
\end{cases}
\] (2.63)
Figure 2.5: Schematic explanation of (a) the surface current model with the counter-clockwise surface current and (b) the magnetically charged top and bottom surface, both shown for vertical magnetization $M_z$.

If the magnetization of the permanent magnet is homogeneous and uniform within the magnet the volume current density $\vec{j}_m$ is zero and only the second term $\vec{j}_m$ remains. The magnetic flux density is written as

$$\vec{B}(\vec{x}) = \frac{\mu_0}{4\pi} \int_S \frac{\vec{j}_m(\vec{x}') \times (\vec{x} - \vec{x}')}{|\vec{x} - \vec{x}'|^3} \, d\vec{s}' .$$  \hfill (2.64)

The resulting field equations model the permanent magnet as an equivalent distribution of surface current $\vec{j}_m$ on the side surfaces of its volume. For a cuboidal magnet magnetized parallel to one of its edges this translates to a surface current density on the four sides which are parallel to the magnetization vector as is shown in Fig. 2.5(a).

**Charge model**  This analytical modeling technique is based on the scalar potential formulation (2.28) with the assumption that the relative permeability $\mu_r = 1$ which already assumes a current-free region by $\nabla \times \vec{H} = 0$. If the magnetization vector $\vec{M}$ is confined to the magnet volume $V$ with surface $S$, and falls abruptly to zero outside this volume, this vector potential becomes $[66, 92]$

$$\Psi = \frac{1}{4\pi} \int_V \frac{\rho_m(\vec{x}')}{|\vec{x} - \vec{x}'|} \, d\vec{v}' + \int_S \frac{\sigma_m(\vec{x}')}{|\vec{x} - \vec{x}'|} \, d\vec{s}' .$$  \hfill (2.65)

The equivalent magnetic volume charge density $\rho_m[A/m^2]$ and surface charge density $\sigma_m[A/m]$ are given by

$$\text{Equivalent charges:} \begin{cases} \rho_m = \nabla \cdot \vec{M} \quad \text{(Volume charge density)} \\ \sigma_m = \vec{M} \cdot \vec{n} \quad \text{(Surface charge density)} \end{cases} .$$  \hfill (2.66)

If the magnetization of the permanent magnet is homogeneous and uniform within the magnet the volume charge density $\rho_m$ is zero and only the second term $\sigma_m$ remains. The magnetic flux density is written as

$$\vec{B}(\vec{x}) = \frac{\mu_0}{4\pi} \int_S \frac{\sigma_m(\vec{x}') (\vec{x} - \vec{x}')}{|\vec{x} - \vec{x}'|^3} \, d\vec{s}' .$$  \hfill (2.67)
The resulting equations model the permanent magnet as an equivalent distribution of magnetic charges on the side surfaces of its volume as is shown in Fig. 2.5(b).

The need to model only two individual surfaces instead of four often renders the surface charge model preferred above the surface current model for 3D problems. However, for homogeneous magnetization vectors which are not perpendicular to the side surfaces or for non-cuboidal permanent magnets, this preference disappears, since there is little distinction between both models. In any case the numerical results of both modeling techniques are equal, although the symbolic representation may differ.

### 2.5.4 Conformal mapping, superposition and imaging

Conformal mapping and imaging are no modeling methods by themselves, but can be of assistance when designing an electromechanical device.

**Conformal mapping**

Conformal mapping, such as the Schwartz-Christoffel transformation, is a mathematical transformation of one domain to another and has the ability to map a complex structure, like a slotted structure, to a relative simple structure (circle, rectangle, bi-infinite strip, upper half plane, etc.) [21, 56]. A conformal mapping, also called a conformal map, conformal transformation, angle-preserving transformation, or biholomorphic map, is a transformation $w = f(z)$ that preserves local angles where the $W$-domain is the original domain and the mapping domain is the $Z$-domain. For simple topologies an analytical solution can be found, however for higher order problems the solution has to be numerically evaluated. Although suitable for many problems, such as tubular and linear machines, this method is limited to 2D problems for complexity reasons. It should be used only for problems with well-defined boundary conditions such as air-iron interfaces and is unsuitable for unbound problems.

**Superposition**

The superposition principle states that, for linear systems, the net response at a given place and time caused by two or more stimuli is the sum of the responses which would have been caused by each stimulus individually. In linear systems which exhibit some function $f(\vec{x}) = \vec{y}$ this can be written as

$$f(\vec{x}_1 + \vec{x}_2 + \ldots + \vec{x}_n) = f(\vec{x}_1) + f(\vec{x}_2) + \ldots + f(\vec{x}_n). \quad (2.68)$$

Each function on the right hand side of this equation may be a body or magnetization vector within the same body. Linear modeling techniques such as the harmonic model and the analytical surface charge and current sheet models often utilize this technique.
### Imaging

Unlike the volume-or surface discretization techniques, the meshfree analytical modeling techniques of Section 2.5.3 do not account for boundary conditions. Air-iron boundaries, for example, are in the other methods solved with Neumann boundary conditions, whereas in the analytical techniques there is no direct equivalent of this, as they are inherently unbound. A symmetrically placed second permanent magnet in the analytical model solves this problem as the field conditions at the iron-air boundary between the permanent magnets, viz. only normal flux and no tangential flux, are obeyed by the permanent magnet and its virtual image (Fig. 2.6) [92]. This technique is easily expanded with angled interfaces [76, 169] or with multiple boundaries [98], however remains limited to simple structures. It is even expandable to materials with $\mu_r \ll \infty$ for which Hague [76] derived that the imaged source $\sigma'_m$ equals

$$\sigma'_m = \sigma_m \left[\frac{\mu_r - 1}{\mu_r + 1}\right], \quad (2.69)$$

where $\sigma_m$ is the original magnetic charge source.

### 2.6 Expansion of the 3D analytical models

This thesis comprehends the three-dimensional analytical surface charge modeling technique for permanent-magnet based devices. This method is very powerful in regions free of magnetically permeable materials or, using the imaging technique, regions which have simple soft-magnetic surfaces. The main modeling assumptions of this thesis are summarized in Table 2.1. As discussed in Chapter 3 the assumption that $\mu_r = 1$ introduces errors that can be well estimated beforehand and if necessary be checked with the computationally expensive but accurate Finite Element Method. The time dependencies are considered negligible as their inclusion would complicate the modeling too much. The mesh-free and unbound domain renders the computational efforts insensitive to the dimensions or displacement of the device as they are only related to the number of magnet bodies in the model and allow for direct stiffness calculations without the need for (virtual) displacement.

The existing equations for this model, which have already been known in the first half of the last century [169] but were not implemented for 3D cuboidal magnets before 1984 [3], have for many years hardly been improved. It was initially only suitable for modeling the force between two parallel or anti-parallel magnetized
Table 2.1: The main modeling assumptions that have been used in this thesis.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Assumption</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetization of magnet</td>
<td>Homogeneous</td>
<td>Eliminates volume charge density</td>
</tr>
<tr>
<td>Relative permeability</td>
<td>Constant</td>
<td>Necessary to solve equations</td>
</tr>
<tr>
<td>Time dependencies</td>
<td>Absent</td>
<td>Ease of modeling</td>
</tr>
</tbody>
</table>

cuboidal permanent magnets. Most investigations focused on differently shaped magnets, as the next chapter discusses, and especially on field computations and in a lesser degree on force computations. The field models proposed in this thesis are based on an analytical calculation of the magnetic field created by a magnetically charged triangular plane. The interaction modeling enhancements that are presented start at the force equations for cuboidal magnets with parallel magnetization. These are extended to situations where the edges align, which is a necessity for optimization. The equations for perpendicular magnetization are added. Further, stiffness equations are derived for both magnetization combinations which provide an analytical description of the $3 \times 3$ stiffness matrix. Torque equations are proposed that provide analytical torque equations with respect to any given reference point. Together, these extended analytical equations enable to describe a significantly wider range of applications.

### 2.7 Conclusions

By eliminating the time dependencies Maxwell’s equations are reduced to magnetostatic models. From these models the potential, energy and force functions are derived. With these equations and the fundamental dipole the electromagnetic description of permanent magnets has been derived. It can be considered as a collection of closely spaced magnetic poles, leading to a $H$-field description in a scalar potential formulation, or as a collection of electrical current loops, leading to a $B$-field description in vector potential. These formulations can be used to obtain the energy and interaction. A review of some of the most frequently used numerical or analytical modeling techniques and an assessment of the suitability for ironless permanent magnet device modeling have been performed. The analytical energy, field and interaction models are considered very suitable for a large range of applications and exhibit their advantageous properties mainly in terms of computational efforts, absence of a mesh and boundaries and accuracy, despite some assumptions that are made in the model. These analytical models are yet incomplete and require expansion on force, stiffness and torque calculations.
Chapter 3

Three-dimensional analytical modeling of magnet-based devices

Analytical modeling techniques for permanent-magnet based devices and a discussion on the inaccuracies of these techniques and the manufacturing process.
This chapter is based on:


Chapter 1 discusses that many advanced modern permanent-magnet devices require accurate and fast modeling tools to maximize their performance. In Chapter 2 it is concluded that especially for permanent-magnet applications in free space analytical modeling tools are very suitable as a consequence of their fast and accurate mesh- and boundary-free solution. However, as discussed in Section 2.6 some dearths in this modeling technique have existed for a long period in terms of force, stiffness and torque modeling. This chapter proposes extensions to the analytical equations that enable more modeling insight into this class of devices.

3.1 Outline

Section 3.2 provides an overview of state-of-the-art concerning the magnetostatic analytical field and interaction modeling. The analytical field modeling is expanded in Section 3.3 with triangular magnet surfaces. This thesis continues with cuboidal magnet modeling: Their interaction force is derived in Section 3.4, the stiffness in Section 3.5 and the torque in Section 3.6, based on the magnetic surface charge model. The coordinate rotation and superposition principle, necessary to simulate the interaction, are explained in Section 3.7. Section 3.8 shows an overview of the progress in the analytical modeling technique for the interactions between cuboidal magnets and the dearths that still exist. Some of the modeling and assembly uncertainties are discussed in Section 3.9. Finally, Section 3.10 present the conclusions, contributions and recommendations of this chapter.

3.2 Prior art in analytical field and force modeling

The analytical surface charge and current sheet models of Section 2.5.3 provide a powerful tool for obtaining the fields of permanent magnets in free space. The shape of such permanent magnets is not of importance to obtain analytical field equations, as long as the magnetization is uniform and homogenous within the magnet volume and the relative permeability \( \mu_r \) can be assumed to be unity.

3.2.1 Cuboidal permanent magnets

The general field equation (2.67) describes the field of a cuboidal permanent magnet. It is repeated here

\[
\vec{B}(\vec{x}) = \frac{\mu_0}{4\pi} \int_S \sigma_m(\vec{x}') (\vec{x} - \vec{x}') \frac{d\vec{s}'}{|\vec{x} - \vec{x}'|^3},
\]

(3.1)

The outward pointing normal vector is given by \( \vec{n} \). The observation point \( \vec{x} \) is defined by \( [x, y, z]^T \). The variable \( \sigma_m \) is the surface charge density which was defined in (2.66) and is rewritten using (2.9)

\[
\sigma_m(\vec{x}) = \vec{M}_{\text{prim}} \cdot \vec{n}(\vec{x}) = \mu_0^{-1} \vec{B}_r \cdot \vec{n}(\vec{x}).
\]

(3.2)

Fig. 3.1 shows the modeled cuboidal permanent magnet PM1. The dimensions
of the permanent magnet are given by $2a_1, 2b_1$ and $2c_1$ and the origin is on the geometrical center of PM1, $O_1$. Its magnetization vector $\vec{M}$ is oriented along the vertical $z$-axis ($\hat{e}_z$) with consequently a remanent flux density $B_r \hat{e}_z$. The inproduct of (3.2) vanishes on the side planes. It equals $|B_r|/\mu_0$ at the top surface, with the outward pointing flux, and equals $-|B_r|/\mu_0$ at the bottom surface, with the inward pointing flux and (3.1) becomes

$$\vec{B}(\vec{x}) = B_r \frac{4\pi}{1} \sum_{p=0}^{1} \sum_{k=0}^{p} \sum_{l=0}^{p} (-1)^{i+k+p} \left( \frac{\log(R-T)}{\log(R-S)} \tan^{-1}\left( \frac{ST}{RU} \right) \right),$$

(3.4)

$$S = x - (-1)^i a_0, \quad T = y - (-1)^k b_0,$$

(3.5)

$$U = z - (-1)^p c_0, \quad R = \sqrt{S^2 + T^2 + U^2}.$$

(3.6)

The arctangent function $\tan^{-1}$ in the description of $B_z$ is the direct result of the integration (3.3). As discussed in Section 2.3 this technique, based on the magnetic scalar potential model, actually describes the field $\vec{H}$ according (2.24) and is rewritten to the flux density $\vec{B}$ using the constitutive relation (2.5) which is repeated here:

$$\vec{B} = \mu_0 (\vec{H} + \vec{M}).$$

(3.7)

Outside the magnet, there is no magnetization, hence $\vec{B} = \mu_0 \vec{H}$ and the results of (3.4) are correct. Inside the magnet’s volume the field $H_z$ reverses, as discussed in Section 2.3 and observed from the results of (3.4). The magnetization $M_z$ must be included to obtain a correct equation for $B_z$ inside the magnet and (3.4) does not take this into account.

The four-quadrant implementation ‘atan2’ of the arctangent function directly provides the flux density $B_z$ both inside and outside the cuboidal magnet. The vertical
flux density is given by

\[ B_z(\vec{x}) = \frac{B_r}{4\pi} \sum_{i=0}^{1} \sum_{k=0}^{1} \sum_{p=0}^{1} (-1)^{i+k+p} \tan(\frac{ST}{RU}) \]. (3.8)

Outside the magnet's volume this equation yields the same results as (3.8) and inside the magnet the flux density is directly obtained. As this is more general than (3.8) it is considered a more elegant method.

The equations for other magnetization directions along \( \vec{e}_x \) or \( \vec{e}_y \) are obtained using coordinate rotation (Section 3.7.1) or a direct calculation similar to (3.3)-(3.6). Using superposition such multiple orthogonal magnetization vector directions can be combined to simulate a cuboidal magnet with a magnetization vector that is not along one of its edges. Surface current models, based on vector potential, present an alternative modeling method. In two dimensions they are easy to derive and very similar to those for the charge model [66]. Three dimensional current equations for a single current sheet are given in [42, 178] and can easily be expanded to the four current sheets required to represent the magnet sides.

**Interaction force**

Simplified two-dimensional analytical models to obtain the interaction force between magnets in radial bearings and couplings were initially proposed using superposition of the interaction force between an integer number of magnetic dipoles [190, 191] and with a magnetic surface charge distribution in [38, 89, 193]. The problem was treated in the Cartesian coordinate system, hence, without relative rotation between the permanent magnets. Therefore, a correction factor for this relative rotation in bearings or couplings was necessary. Using the magnetic imaging technique [76] a soft-magnetic slotless back-iron was incorporated in the models of the PM coupling [193].

Although the resulting 2D analytical equations are not complicated and require little computational effort, their accuracy is insufficient for many applications. Investigation into 3D solutions performed in [18, 176] resulted in semi-analytical equations with a numerical integration of some of the remaining logarithmic terms. The current sheet model used in [130, 176] employed Lorentz force calculation in a simple topology, although the last integration step needed to be performed numerically. Akoun [3, 38] was the first to pose fully three-dimensional analytical equations of the field and force between cuboidal magnets. These equations have since then been used by numerous researchers [2] but have not been improved until the start of this investigation.

This investigation has lead to a number of publications on the subject of 3D analytical force [101, 105], stiffness [105] and torque [102, 109] modeling of permanent magnets, which are summarized in this chapter. Simultaneously, similar force and torque equations have been derived in another research group [5, 6].
Chapter 3: Three-dimensional analytical modeling of magnet-based devices

Equations for rotated magnets  The 3D models for parallel and perpendicular magnetized permanent magnets proposed in [3, 17] do not incorporate relative rotation. Especially for rotating magnetic couplings it is often not sufficient to assume that no rotation exists, especially if the radius becomes small with respect to the magnet dimensions, as shown in Fig. 3.2(a).

Analytical equations for the circumferential force component of magnetic couplings with radial and tangential magnetization were proposed in [60, 61]. Semi-numerical equations for axial magnetization were discussed in [35, 61]. For the design and analysis of multi-DoF magnetic suspensions with 6DoF movements these equations for rotated magnets also prove to be helpful. These equations describe the effects of rotation around a single axis such as that shown in Fig. 3.2(a) and are partially semi-numerical. Fully analytical equations with rotation around multiple axes, such as shown in Fig. 3.2(b), would be more generally applicable but have not been found.

3.2.2 Cylinder-, ring- and arc-shaped permanent magnets

These kinds of magnets have been used in a wide variety of applications such as magnetic axial or radial couplings, bearings, wiggles, undulators, etc. [66, 170]. Although they are not elaborated on in more detail, an overview of the state-of-the-art in field modeling for these magnet types is discussed below to form a more complete overview of analytical current sheet and surface charge modeling.

Axial magnetization

Analytical field models of axially magnetized cylinders have already been known for a long time [43, 169] and were refined with analytical field models of axially magnetized ring-magnets [14, 15, 57, 143, 152]. An equation for cylinder-shaped coil conductors is found in [179]. From these equations semi-analytical parametric force
3.3: Field of the triangular-shaped charged surface

Any triangular shape can be written as a combination of rectangular triangles. The field equations for the magnetically charged surface $\triangle 1$ with the dimensions shown in Fig. 3.4(e) are derived. Implementation of coordinate rotation allows to obtain the
The magnets shown in (a), (b) and (c) can be composed of a combination of (d) cuboids, however it is possible to combine field descriptions of square and triangular (e) magnets too.

3.3.1 Derivation of the magnetic field components

The general flux equation in the magnetic surface charge model (3.1) has three components along $\hat{e}_x$, $\hat{e}_y$ and $\hat{e}_z$.

$$\vec{x} - \vec{x}' = (x-x')\hat{e}_x + (y-y')\hat{e}_y + (z-z')\hat{e}_z.$$  \hspace{1cm} (3.9)

The analytical equations of the horizontal flux density components $\Delta^1 B_x$ and $\Delta^1 B_y$ for charged surface $\Delta 1$ are obtained by [100]

$$\Delta^1 B_x(\vec{x}) = \frac{\sigma m \mu_0}{4\pi} \int_{-b}^{b} \int_{y=-a}^{y=a} \frac{x-x'}{|\vec{x} - \vec{x}'|^3} \, dx' \, dy',$$

$$= \frac{\sigma m \mu_0}{4\pi} \sum_{j=0}^{\infty} (-1)^j \left[ -\log(G_j + I_F) - \frac{b \log \left( b \left( I_p a^2 + b^2 - ax + (-1)^j a^2 \right) - b^2 G_j \right)}{\sqrt{a^2 + b^2}} \right], \hspace{1cm} (3.10)$$

$$\Delta^1 B_y(\vec{x}) = \frac{\sigma m \mu_0}{4\pi} \int_{-a}^{a} \int_{-b}^{b} \frac{y-y'}{|\vec{x} - \vec{x}'|^3} \, dx' \, dy',$$

$$= \frac{\sigma m \mu_0}{4\pi} \sum_{j=0}^{\infty} (-1)^j \left[ +\log(F_j + I_G) + \frac{a \log \left( a \left( I_p a^2 + b^2 - by + (-1)^j b^2 \right) - a^2 F_j \right)}{\sqrt{a^2 + b^2}} \right], \hspace{1cm} (3.11)$$

$$F = x-a, \hspace{1cm} F_j = x - (-1)^j a,$$

$$G = y+b, \hspace{1cm} G_j = y - (-1)^j b,$$

$$H = z-z', \hspace{1cm} I_F = \sqrt{F_j^2 + G_j^2 + H^2},$$

$$I = \sqrt{F_j^2 + G_j^2 + H^2}, \hspace{1cm} I_G = \sqrt{F_j^2 + G_j^2 + H^2}. \hspace{1cm} (3.12)$$
3.3: Field of the triangular-shaped charged surface

The vertical field component, $\Delta^1 B_z$, is separated into the variables $\Delta^1 A B_z(\vec{x})$ and $\Delta^1 B^1 z(\vec{x})$ to simplify its derivation

$$\Delta^1 A B_z(\vec{x}) = \frac{\sigma m \mu_0}{4\pi} \int_{-b}^{a} \frac{\vec{b} \cdot \vec{y}'}{[\vec{x} - \vec{x}']^3} \, dy' \, dx' = \Delta^1 A B_z(\vec{x}) - \Delta^1 B_z(\vec{x}), \tag{3.13}$$

$$\Delta^1 B^1 z(\vec{x}) = \frac{\sigma m \mu_0}{4\pi} \int_{-b}^{a} \frac{-(y - \frac{b}{2} x') (z - z')}{((x - x')^2 + (z - z')^2)^{\frac{3}{2}}} \, dx', \tag{3.14}$$

$$\Delta^1 B^1 z(\vec{x}) = \frac{\sigma m \mu_0}{4\pi} \int_{-b}^{a} \frac{-(y + b)(z - z')}{((x - x')^2 + (z - z')^2)^{\frac{3}{2}}} \, dx'. \tag{3.15}$$

The solution to $\Delta^1 B^1 z(\vec{x})$ in Eq. (3.15) is given by

$$\Delta^1 B^1 z(\vec{x}) = \sum_{j=0}^{1} (-1)^j \tan^{-1} \left[ \frac{F_j G}{I_j H} \right]. \tag{3.16}$$

The analytical solution to $\Delta^1 A B_z(\vec{x})$ in (3.14) is more complicated and is obtained by rewriting this equation to a standard integral found in [74].

$$\Delta^1 A B_z(\vec{x}) = \frac{\sigma m \mu_0}{4\pi} \sum_{j=0}^{1} (-1)^j \left[ \sum_{i=0}^{L} \frac{(2N + M(T - P))}{\sqrt{2TW}} \log \left( \frac{T - P - 2U}{V} \right) \right], \tag{3.17}$$

$$V = P^2(R + 2SU) + T(4Q - RP - 2PSU + 2RU) - 4O(R + 2SU) + 2\sqrt{2}Q + U(R + SU)TW, \tag{3.18}$$

$$W = \sqrt{SP^2 - (R + ST)P + 2Q + TR - 2OS}. \tag{3.19}$$

$$M = \frac{b}{a} z, \quad N = -(-1)^i y z,$$

$$O = x^2 + z^2, \quad P = -2(-1)^i x,$$

$$Q = x^2 + y^2 + z^2, \quad R = (-1)^i \left[ -2x - 2\frac{b}{a} y \right], \tag{3.20}$$

$$S = 1 + \left( \frac{b}{a} \right)^2, \quad T = 2\sqrt{P^2 - 4\bar{O}},$$

$$U = (-1)^{i+j} a.$$

It is observed that the intermediate variable $T$ in (3.20) becomes an imaginary number. However, due to the summations in (3.17) the value of $\Delta^1 A B_z(\vec{x})$ remains real as the imaginary parts appear to cancel each other.

Similar to the analytical description cuboidal magnets, the equations presented above are not continuous for all values of $x$, $y$, and $z$. For example, at $z = 0$ the value of variable $B_z$ is undefined, however it can be shown that $\lim_{z \to 0} B_z = 0$. The equations are also undefined above the vertical side planes $(x = \pm a, y = \pm b)$ and $(y = b/a, x)$. A linear interpolation around those singular points suffices in many situations and is not difficult to implement. However, it is a relatively slow and less accurate method.
Chapter 3: Three-dimensional analytical modeling of magnet-based devices

Figure 3.5: Pyramidal-frustum shaped (a) as a wire-frame showing the center $G\bar{O}$ and (b) implementation in a quasi-Halbach array.

Figure 3.6: Definition of the variables in the pyramidal frustum.

compared to the more elegant analytical calculation which provides an exact solution in these points. This research has not been performed for this triangular permanent magnet surface. A validation of these field equations for triangular-shaped charged surfaces is found in [104].

3.3.2 Application to pyramidal-frustum shaped permanent magnets

The field equations for the magnetically charged triangular surface are suitable to model the field of the pyramidal frustum [100] such as that shown in Fig. 3.5(a) which may be implemented in the quasi-Halbach array of Fig. 3.5(b).

The pyramidal frustum, or 3D trapezoidal permanent magnet, is defined in Fig. 3.6 which shows its top- and side-views. The dimensions of the top surface are defined by $a_t$ and $b_t$, and those for the bottom surface by $a_b$ and $b_b$, respectively. By reducing $a_t$ and $b_t$ towards zero, a pyramidal shape is obtained. The angles $\alpha_1 \ldots 4$ are defined counterclockwise, hence $\alpha_1$ and $\alpha_4$ are positive, and $\alpha_2$ and $\alpha_3$ are negative. The rectangular bottom and top surfaces are parallel but do not necessarily share the same symmetry axis, hence $\alpha_1$ and $\alpha_3$ are not automatically equal. The same is valid for $\alpha_2$ and $\alpha_4$.

The origin of the reference coordinate system, $G\bar{O}$, is at the center of mass of the
field of the triangular-shaped charged surface

Figure 3.7: Unfolded pyramidal frustum with fourteen segments.

Superposition of analytical fields

The analytical equation for the magnetic field of the pyramidal-frustum shaped permanent magnet is derived by identifying fourteen individual geometrical surfaces. Each trapezoidal side plane is cut into one rectangular and two triangular planes, as is shown in the unfolded pyramidal frustum of Fig. 3.7. For each surface, the analytical field is obtained individually. Two surface types, rectangular and triangular, are now distinguished.

The field equations for the rectangular charged surface apply to surfaces 1, 2, 4, 7, 10 and 13 are found in Section 3.2.1. Those for the triangular charged surface apply to surfaces 3, 5, 6, 8, 9, 11, 12 and 14. Figure 3.4(e) shows the location of the origin of the triangular surface, \( \hat{O}_\Delta \). The magnetic charge density on each surface is defined by the inproduct

\[
\sigma = \vec{M} \cdot \hat{n} = \mu_0^{-1} \vec{B}_r \cdot \hat{n}, \tag{3.21}
\]

where \( \hat{n} \) is the local outward pointing normal vector of the respective surface. The vectors \( \vec{M} \) and \( \vec{B}_r \) are the magnetization and remanent flux density of the permanent magnet, respectively, and \( \hat{n} \) is the normal vector of the surface. By applying this equality to all surfaces, the condition that the net magnetic charge of the permanent magnet must be zero is satisfied.

At a random observation point in global coordinates \( \vec{x} \), the magnetic field induced by surface \( m \) (with \( m = 1 \ldots 14 \)) is obtained by

\[
G \vec{B}_m (\vec{x}) = M_m^{-1} L \vec{B}_m \left( M_m \left( G \vec{x} - G \hat{O}_m \right) \right). \tag{3.22}
\]

The vector \( G \vec{x} - G \hat{O}_m \) defines the relative position of the observation point \( G \vec{x} \) with respect to the reference point \( G \hat{O}_m \) of surface \( m \) in global coordinates. Table 3.1 summarizes these reference points and the appropriate rotation matrix \( M_{1\ldots4} \). Multiplication with the rotation matrix \( M_{1\ldots4} \) transforms the global coordinates into the local coordinates of surface \( m \), after which \( L \vec{B}_m \) is obtained. Finally, multiplication of
Chapter 3: Three-dimensional analytical modeling of magnet-based devices

Table 3.1: Rotation matrix and reference point in global coordinates for each surface of the pyramidal frustum.

<table>
<thead>
<tr>
<th>Surface</th>
<th>( M_m )</th>
<th>( ^G\vec{O}_m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( I )</td>
<td>( \begin{bmatrix} 0 &amp; 0 &amp; -c \end{bmatrix}^T )</td>
</tr>
<tr>
<td>2</td>
<td>( I )</td>
<td>( \begin{bmatrix} 0 &amp; 0 &amp; c \end{bmatrix}^T )</td>
</tr>
<tr>
<td>3</td>
<td>( M_1 )</td>
<td>( \begin{bmatrix} -a_m &amp; -b_m &amp; 0 \end{bmatrix}^T )</td>
</tr>
<tr>
<td>4</td>
<td>( M_1 )</td>
<td>( \begin{bmatrix} -a_m &amp; 0 &amp; 0 \end{bmatrix}^T )</td>
</tr>
<tr>
<td>5</td>
<td>( M_1 )</td>
<td>( \begin{bmatrix} -a_m &amp; b_m &amp; 0 \end{bmatrix}^T )</td>
</tr>
<tr>
<td>6</td>
<td>( M_2 )</td>
<td>( \begin{bmatrix} -a_m &amp; b_m &amp; 0 \end{bmatrix}^T )</td>
</tr>
<tr>
<td>7</td>
<td>( M_2 )</td>
<td>( \begin{bmatrix} 0 &amp; b_m &amp; 0 \end{bmatrix}^T )</td>
</tr>
<tr>
<td>8</td>
<td>( M_2 )</td>
<td>( \begin{bmatrix} a_m &amp; b_m &amp; 0 \end{bmatrix}^T )</td>
</tr>
<tr>
<td>9</td>
<td>( M_3 )</td>
<td>( \begin{bmatrix} a_m &amp; b_m &amp; 0 \end{bmatrix}^T )</td>
</tr>
<tr>
<td>10</td>
<td>( M_3 )</td>
<td>( \begin{bmatrix} a_m &amp; 0 &amp; 0 \end{bmatrix}^T )</td>
</tr>
<tr>
<td>11</td>
<td>( M_3 )</td>
<td>( \begin{bmatrix} a_m &amp; -b_m &amp; 0 \end{bmatrix}^T )</td>
</tr>
<tr>
<td>12</td>
<td>( M_4 )</td>
<td>( \begin{bmatrix} a_m &amp; -b_m &amp; 0 \end{bmatrix}^T )</td>
</tr>
<tr>
<td>13</td>
<td>( M_4 )</td>
<td>( \begin{bmatrix} 0 &amp; -b_m &amp; 0 \end{bmatrix}^T )</td>
</tr>
<tr>
<td>14</td>
<td>( M_4 )</td>
<td>( \begin{bmatrix} -a_m &amp; -b_m &amp; 0 \end{bmatrix}^T )</td>
</tr>
</tbody>
</table>

\( ^L\vec{B}_m \) with the inverse rotation matrix \( M^T \) transforms the magnetic field equation from local to global coordinates, hence, gives \( ^G\vec{B}_m \).

The rotation matrices \( M_{1..4} \) and the variables \( a_m \) and \( b_m \) from Table 3.1 are given by the equations below. The matrix \( I \) is the 3-by-3 eye matrix.

\[
M_1 = \begin{pmatrix}
0 & -1 & 0 \\
\cos(\alpha_1) & 0 & \sin(\alpha_1) \\
-\sin(\alpha_1) & 0 & \cos(\alpha_1)
\end{pmatrix},
M_2 = \begin{pmatrix}
-1 & 0 & 0 \\
0 & -\cos(\alpha_2) & -\sin(\alpha_2) \\
0 & -\sin(\alpha_2) & \cos(\alpha_2)
\end{pmatrix},
M_3 = \begin{pmatrix}
0 & 1 & 0 \\
-\cos(\alpha_3) & 0 & -\sin(\alpha_3) \\
-\sin(\alpha_3) & 0 & \cos(\alpha_3)
\end{pmatrix},
M_4 = \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos(\alpha_4) & \sin(\alpha_4) \\
0 & -\sin(\alpha_4) & \cos(\alpha_4)
\end{pmatrix}.
\]

\[
a_m = \frac{a_t + a_b}{2},
\]

\[
b_m = \frac{b_t + b_b}{2}.
\]

Vectorial summation of the magnetic fields obtained by (3.22) over the fourteen surfaces provides exact analytical equations for the magnetic field, \( ^G\vec{B} \), of the pyramidal frustum. A comparison of this model with FEM results is presented in [100]. It is found that there is high correspondence between the results obtained analytically, and those of FEM.
3.4 Extension to the interaction force equations

The rest of this chapter is dedicated to modeling the cuboidal permanent magnets of Section 3.2.1. The analytical field model for a cuboidal magnet from this section is expanded with a second cuboidal permanent magnet, PM2. Both permanent magnets, PM1 and PM2, are studied in the Cartesian coordinate system as shown in Fig. 3.8. It is assumed that these PMs are parallel to each other and that the respective magnetization vectors $\vec{M}_1$ and $\vec{M}_2$ are homogeneous and constant. The dimensions of PM1 are again defined by $2a_1, 2b_1, 2c_1$ and those of PM2 are given by $2a_2, 2b_2, 2c_2$. The center of PM1, $\vec{O}_1$, is again centered in the global coordinate system at $[0, 0, 0]^T$ and that of PM2, $\vec{O}_2$, is displaced by the vector $\vec{x}$ given by $[\alpha, \beta, \gamma]^T$.

Two magnetization combinations are discussed: in the first case both permanent magnets are magnetized along $z$ and in the second case one magnet is magnetized along $z$ and the other along $y$. Hereafter, these cases are referred to as parallel magnetization and perpendicular magnetization, respectively. All other parallel or perpendicular magnetization combinations can be derived from these equations using superposition and coordinate rotation (Section 3.7.1).

3.4.1 Parallel magnetization

Akoun [3] was proposed fully three-dimensional analytical equations of the field and force between cuboidal magnets with magnetizations as shown in Fig. 3.9(a). The analytical equations for the interaction force for $zz$-magnetization were obtained using the virtual work method

$$
\vec{F}(\vec{x}) = -\nabla W(\vec{x}),
$$

$$
W(\vec{x}) = \iiint \frac{\sigma_{m_1}\sigma_{m_2}}{4\pi\mu_0 r} dX dY dx dy,
$$

$$
r = \sqrt{(\alpha + X - x)^2 + (\beta + Y - y)^2 + \gamma^2},
$$

where $\vec{x}$ is the position vector of the point of interest.
Figure 3.9: Definition of the parallel (a) magnetization with both vectors along $\hat{e}_z$ and the perpendicular (b) magnetization with PM1 along $\hat{e}_z$ and PM2 along $\hat{e}_y$.

where $\sigma_{m1}$ and $\sigma_{m2}$ are the respective charge densities of PM1 and PM2. The resulting force equations have the form

$$\vec{F}(\vec{x}) = \frac{B_{r1} B_{r2}}{4\pi \mu_0} \sum_{i=0}^{1} \sum_{j=0}^{1} \sum_{k=0}^{1} \sum_{l=0}^{1} \sum_{m=0}^{1} \sum_{n=0}^{1} (-1)^{i+j+k+l+m+n} \vec{\xi}(u,v,w). \tag{3.30}$$

In this equation $B_{r1}$ and $B_{r2}$ are the respective remanent flux densities of the permanent magnets along the considered axes, $B_{r1} \hat{e}_z$ and $B_{r2} \hat{e}_z$, respectively. The constant $\mu_0$ is the permeability of vacuum ($4\pi \cdot 10^{-7}$ Hm$^{-1}$). The intermediate variable $\vec{\xi}$ [m$^2$] is an analytical function which is obtained in [3] using the virtual work method and is given by

$$\xi_x = \frac{1}{2} \left( v^2 - w^2 \right) \log(r - u) + uv \log(r - v) + vw \arctan \frac{uv}{rw} + \frac{1}{2} ru, \tag{3.31}$$

$$\xi_y = \frac{1}{2} \left( u^2 - w^2 \right) \log(r - v) + uv \log(r - u) + uw \arctan \frac{uv}{rw} + \frac{1}{2} rv, \tag{3.32}$$

$$\xi_z = -uw \log(r - u) - uv \log(r - v) + uv \arctan \frac{uv}{rw} - rw, \tag{3.33}$$

The intermediate variables $u$, $v$, $w$ and $r$ depend on the dimensions and displacement between the magnets

$$u = \alpha - (-1)^i a_1 + (-1)^j a_2, \quad v = \beta - (-1)^k b_1 + (-1)^l b_2, \quad w = \gamma - (-1)^m c_1 + (-1)^n c_2, \quad r = \sqrt{u^2 + v^2 + w^2}. \tag{3.34}$$

Special cases of the equations

Detailed research of the force equations and specifically (3.34)-(3.35) shows that they are not defined when the edges of two permanent magnets are aligned [105]. For certain combinations of $i...n$, $a_1...c_2$ and $\alpha...\gamma$ the variables $u$, $v$ or $w$ become zero. Two of such situations are shown in Fig. 3.10. As a result, the analytical equations become undefined as more elaborately discussed in [105]. An exact solution for the
analytical equations in these cases is necessary to provide continuous and reliable force and stiffness data.

Especially for planar magnetic isolators or other machines that are based on the interaction between permanent magnets such undefined equations may become problematic, e.g., during optimization routines which cannot handle undefined numbers. Although linear interpolation around those discontinuous points is not difficult to implement, it is a relatively slow and less accurate method compared to the direct analytical calculation. The accuracy of the linear interpolation can be increased by taking more sample points, however at the cost of computational efforts. Therefore, the analytical equations for $\vec{\xi}(u, v, w)$ in the case that $u = v = 0$, $u = w = 0$ or $v = w = 0$ are obtained by the limits of $\vec{\xi}(u, v, w)$ [105].

\[
\lim_{u \to 0 \atop v \to 0} \vec{\xi}(u, v, w) = \begin{pmatrix}
-\frac{1}{4} w^2 \log(w^2) \\
-\frac{1}{4} w^2 \log(w^2) \\
-ww
\end{pmatrix}, \quad (3.36)
\]

\[
\lim_{u \to 0 \atop w \to 0} \vec{\xi}(u, v, w) = \begin{pmatrix}
\frac{1}{4} v^2 \log(v^2) \\
vw \\
0
\end{pmatrix}, \quad (3.37)
\]

\[
\lim_{v \to 0 \atop w \to 0} \vec{\xi}(u, v, w) = \begin{pmatrix}
\frac{1}{4} u^2 \log(u^2) \\
uw \\
0
\end{pmatrix}. \quad (3.38)
\]

By inserting these equations in the appropriate cases, as described in [105], a fully analytical and continuous force calculation method for cuboidal permanent magnets is obtained, which is especially suitable for optimization routines. It is also possible to obtain the interaction force when the magnets are touching, for which the equations all reduce to zero as shown in [106].

### 3.4.2 Perpendicular magnetization

Instead of a virtual work method [5] it is possible to derive the force with the Lorentz force of Section 2.4 [101] as is shown below. The force on PM2 is directly obtained by integrating the virtual surface force density $\vec{f}_s$ [N/m²].

\[
\vec{F}(\vec{x}) = \int_{S_2} \vec{f}(\vec{x}') \, ds', \quad \vec{f}(\vec{x}) = \sigma_{m_2}(\vec{x}) \cdot \vec{B}_1(\vec{x}). \quad (3.39)
\]
The integration surface $S_2$ is the surface of the PM2 which for perpendicular magnetization reduces to the magnet sides in the $xz$-plane which are highlighted in grey Fig. 3.9(b). $\sigma_{m2}$ represents the surface charge density of PM2 and $\vec{B}_1$ is the analytically obtained flux density from PM1.

As for parallel magnetization (3.30) is the basic form of the force equation. In this case $B_{r1}$ and $B_{r2}$ are the respective remanent flux densities of the permanent magnets along the perpendicular axes, $B_{r1}\hat{e}_z$ and $B_{r2}\hat{e}_y$, respectively (Fig. 3.9(b)). The vector $\vec{\xi}$ is defined by

$$\xi_x = \frac{1}{2}\left[\left(\tan^{-1}\left(\frac{u}{w}\right) + \tan^{-1}\left(\frac{v}{w}\right)\right) + 2v\log(w + r)\right] + w\left(\tan^{-1}\left(\frac{u}{w}\right) + \tan^{-1}\left(\frac{v}{w}\right)\right) - 2v\log(u + r) - 2u\log(r - v)\right],$$

$$\xi_y = 2\left[\left(\tan^{-1}\left(\frac{u}{w}\right) + \tan^{-1}\left(\frac{v}{w}\right)\right) + 2v\log(w + r)\right] + 2u\left(\tan^{-1}\left(\frac{u}{v}\right) + \tan^{-1}\left(\frac{w}{v}\right)\right) + 2u\log(r - u)\right],$$

$$\xi_z = 2\left[\left(\tan^{-1}\left(\frac{u}{w}\right) + \tan^{-1}\left(\frac{v}{w}\right)\right) + 2v\log(w + r)\right] + 2u\left(\tan^{-1}\left(\frac{u}{v}\right) + \tan^{-1}\left(\frac{w}{v}\right)\right) + 2u\log(r - u)\right].$$

The variables $u$, $v$, $w$, and $r$ are again defined by (3.34)-(3.35).

**Special cases of the equations**

The vector $\vec{\xi}(u, v, w)$ for perpendicular $zy$-magnetization is in the discontinuous points given by

$$\lim_{u\to 0} \lim_{v\to 0} \vec{\xi}(u, v, w) = \left(\frac{1}{2} \frac{uw}{w^2 \log(w^2)} \right),$$

$$\lim_{u\to 0} \lim_{w\to 0} \vec{\xi}(u, v, w) = \left(\frac{1}{2} \frac{v^2 \log(v^2)}{v^2} \right),$$

$$\lim_{v\to 0} \lim_{w\to 0} \vec{\xi}(u, v, w) = \left(-\frac{1}{2} \frac{u^2 \log(u^2)}{v^2} \right).$$

**3.4.3 Mathematical abstraction**

The analytical force calculation methods described above are based on a mathematical approach and as such have no direct physical meaning. In other words, the equivalent magnetic surface charge is in practice non-existent and is only a mathematical trick to derive the necessary energy, field or interaction equations.
3.4: Extension to the interaction force equations

Geometrical symmetry in the cuboidal permanent magnets enables the use of variables as defined in (3.34)-(3.35), which correspond to the distance between the cube corners and their respective projections on the axes. This has been used in [17] to propose an abstraction known as ‘magnetic nodes’, where the magnetic nodes are the respective corners of the magnets. Summing the magnetic node forces between every combination of corners of both magnets yields the total force between them. Such abstraction is mathematically convenient, but has no physical meaning. In [124], FEM was used to show that this method based on equivalent sources does not represent the actual force distribution on magnets, although the total force is correct. This is for the magnetic nodes representation explained in [192]: although the ‘local’ force calculation between the magnetic nodes, i.e. the corners of the permanent magnets, shows no symmetry in the case where this could be expected from a physical point of view, the ‘global’ force calculation, obtained by the addition of all the contributions of the magnetic nodes, is correct. This is caused by the mathematical abstraction that is introduced while solving the necessary integrals.

As opposed to the energy-based force calculation of [192] this thesis uses the Lorentz force equation to obtain the interaction. This force equation in terms of magnetization (2.48) shows that the external field $\vec{B}_{\text{ext}}(\vec{x})$ is integrated using the source term $\vec{M}(\vec{x})$ of the considered magnet. As the first term disappears this mathematical abstraction suggests that the field of the first permanent magnet $\vec{B}_{\text{ext}}(\vec{x})$ integrated over the surface of the second magnet determines the force. The stiffness and torque equations that are derived in Sections 3.5 and 3.6 have the same mathematical abstraction. In reality this is not the case, as the force acts on the whole magnet volume instead of only its surface [124]. Nevertheless, this mathematical abstraction may help to gain more a-priori insight into the geometrical properties and requirements of a permanent-magnet device. Such a discussion is found in Section 4.7 for the vibration isolation system envisaged in this thesis.

3.4.4 Validation of the force equations

The analytical force equations proposed above have been validated using an FEM model, numerical Maxwell Stress integration on the analytically obtained field equations and with experimental measurements. These methods are described in more detail in Section A. Table 3.2 shows the magnet dimensions, the relative displacement and the respective magnetization remanent flux density vectors. These vectors have been provided by the magnet manufacturer, however, have not been verified by in-house measurements.

The results in Fig. 3.11 show the high correspondence between the various methods. Especially the analytical equations, FEM and Maxwell Stress exhibit a high correspondence. The slightly deviating value of the experimental results can be addressed to sensor inaccuracy, misalignment in the magnetization directions and geometrical inaccuracies in the simple test setup.
### 3.5 Stiffness equations and their extensions

Stiffness is an important property in the design process of many 6-DoF permanent-magnet devices. Earnshaw’s theorem \[58\], discussed in Appendix C, states that a magnet-based device is never passively stable along these all directions simultaneously. The calculation of the stiffness between permanent magnets was briefly discussed in \[3, 157, 191\], however only for $K_{zz} = \frac{dF_z}{dz}$ or $K_{xx} = \frac{dF_x}{dx}$. Conversely, the stiffness for a 6-DoF magnetic isolator should be given by a $3 \times 3$ Jacobian matrix $J$ of the force vector $\vec{F}(\vec{t})$. The Jacobian matrix describes all first-order partial derivatives of the vector-valued force function with respect to the displacement vector. In a Cartesian coordinate system without rotations around the axis the stiffness matrix $K[\text{N/m}]$ is given by

$$K(\vec{x}) = \frac{-1}{J(\vec{F}(\vec{t}))} = \begin{pmatrix}
\frac{\partial F_x}{\partial x} & \frac{\partial F_x}{\partial y} & \frac{\partial F_x}{\partial z} \\
\frac{\partial F_y}{\partial x} & \frac{\partial F_y}{\partial y} & \frac{\partial F_y}{\partial z} \\
\frac{\partial F_z}{\partial x} & \frac{\partial F_z}{\partial y} & \frac{\partial F_z}{\partial z}
\end{pmatrix} = \begin{pmatrix}
K_{xx} & K_{xy} & K_{xz} \\
K_{yx} & K_{yy} & K_{yz} \\
K_{zx} & K_{zy} & K_{zz}
\end{pmatrix}. \quad (3.46)
$$

Such stiffness matrix is symmetric around its diagonal and is a symmetric positive-semidefinite matrix that generalizes the stiffness of Hooke’s law $F = -kx$ to a matrix. The equations of the entries in this stiffness matrix have a form which is similar to the force equations from which they have been derived

$$\vec{K}(\vec{t}) = \frac{\vec{B}_{11} \cdot \vec{B}_{22}}{4\pi \mu_0} \sum_{i=0}^{1} \sum_{j=0}^{1} \sum_{k=0}^{1} \sum_{l=0}^{1} \sum_{m=0}^{1} \sum_{n=0}^{1} \Xi(u, v, w), \quad (3.47)$$

$$\Xi(u, v, w) = -j|\tilde{\xi}(u, v, w)|, \quad (3.48)$$

with $\tilde{\xi}$ defined in Section 3.4 for parallel and perpendicular magnetization.
3.5: Stiffness equations and their extensions

Intermedeiate variable $\Xi$ which is used in analytical equations of the stiffness matrix $K$ in the case that both magnets are magnetized along the $z$-axis [105]

$$
\Xi_{1,1} = -\frac{\nu u^2}{u^2+w^2} - \nu \log(r-v) - r, \quad (3.49)
$$

$$
\Xi_{1,2} = -\frac{v^2 - w^2 v + 2(v^2 + w^2)\left(\frac{u v}{u^2+w^2}\right) + v \log(r-u) + u \log(r-v)}{2(v^2 + w^2)}, \quad (3.50)
$$

Figure 3.11: Interaction force results obtained by the analytical surface charge model (solid), the semi-numerical Maxwell Stress method (dashed), Finite Element analysis (square) and experimental measurements (dash-dot) for (a) parallel and (b) perpendicularly magnetized magnets.

3.5.1 Parallel magnetization

Intermediate variable $\Xi$ which is used in analytical equations of the stiffness matrix $K$ in the case that both magnets are magnetized along the $z$-axis [105]
\[ \Xi_{1,3} = \frac{1}{2} w \left( -\frac{2r^2}{u^2 + w^2} - \frac{2uw}{u^2 + w^2} + 2\log(r-w) + 1 \right) - rv \tan^{-1} \left( \frac{uw}{wr} \right), \quad (3.51) \]

\[ \Xi_{2,1} = -u^3 - u^2 w + 2(u^2 + w^2) \left( w \tan^{-1} \left( \frac{uw}{wr} \right) + v \log(r-w) + u \log(r-v) \right), \quad (3.52) \]

\[ \Xi_{2,2} = -\frac{u^2 v}{u^2 + w^2} - u \log(r-u) - r, \quad (3.53) \]

\[ \Xi_{2,3} = \frac{1}{2} w \left( -\frac{2r^2}{u^2 + w^2} - \frac{2vu}{v^2 + w^2} + 2\log(r-v) + 1 \right) - ru \tan^{-1} \left( \frac{uv}{wr} \right), \quad (3.54) \]

\[ \Xi_{3,1} = \frac{uwv + (u^2 + w^2) \left( w \log(r-u) - v \tan^{-1} \left( \frac{uw}{wr} \right) \right)}{u^2 + w^2}, \quad (3.55) \]

\[ \Xi_{3,2} = \frac{uvw + (v^2 + u^2) \left( v \log(r-v) - u \tan^{-1} \left( \frac{uw}{wr} \right) \right)}{v^2 + u^2}, \quad (3.56) \]

\[ \Xi_{3,3} = \left( \frac{u}{u^2 + w^2} + \frac{v}{v^2 + u^2} \right) w^2 + u \log(r-u) + v \log(r-v) + 2r. \quad (3.57) \]

**Extension to the stiffness equations**

The matrix \( \Xi(u, v, w) \) for parallel magnetization is in the discontinuous points discussed in Section 3.4.1 given by

\[ \lim_{w \to 0} \Xi(u, v, w) = \begin{bmatrix} -r & 0 & \frac{1}{2} w(1 + \log(w^2)) \\ 0 & -r & \frac{1}{2} w(1 + \log(w^2)) \\ \frac{1}{2} w(1 + \log(w^2)) & \frac{1}{2} w(1 + \log(w^2)) & 2r \end{bmatrix}, \quad (3.58) \]

\[ \lim_{v \to 0} \Xi(u, v, w) = \begin{bmatrix} -u & \frac{1}{2} v \log(v^2) & 0 \\ -\frac{1}{2} v \log(v^2) & -r & 0 \\ 0 & 0 & v + 2r - |v| \log(r + |v|) \end{bmatrix}, \quad (3.59) \]

\[ \lim_{u \to 0} \Xi(u, v, w) = \begin{bmatrix} -u & \frac{1}{2} w(1 + \log(u^2)) & 0 \\ -\frac{1}{2} w(1 + \log(u^2)) & -u + r + |u| \log(r + |u|) & 0 \\ 0 & 0 & u + 2r - |u| \log(r + |u|) \end{bmatrix}. \quad (3.60) \]

### 3.5.2 Perpendicular magnetization

Intermediate variable \( \Xi \) which is used in analytical equations of the stiffness matrix \( K \) in the case that the first magnet is magnetized along the \( z \)-axis and the second along the \( y \)-axis:

\[ \Xi_{1,1} = -\log(r-v) w + w - u \left( \tan^{-1} \left( \frac{w}{u} \right) + \tan^{-1} \left( \frac{uw}{wr} \right) \right) + v \log(w+r), \quad (3.61) \]

\[ \Xi_{1,2} = \frac{uv^2 - u(v^2 + w^2)}{u^2 + w^2} + 2v \tan^{-1} \left( \frac{u}{v} \right) - ru \tan^{-1} \left( \frac{uw}{wr} \right), \quad (3.62) \]

\[ \Xi_{1,3} = \frac{u^2 + (u + v)w^2}{u^2 + w^2} - w \left( 2 \tan^{-1} \left( \frac{u}{w} \right) + \tan^{-1} \left( \frac{uw}{wr} \right) \right) + v \log(u+r) - u \log(r-v), \quad (3.63) \]

\[ \Xi_{2,1} = \frac{v^3}{u^2 + w^2} + \log(w+r) u - \frac{u}{2} + w - v \left[ \tan^{-1} \left( \frac{w}{v} \right) + \tan^{-1} \left( \frac{uw}{wr} \right) \right] + v \log(w+r), \quad (3.64) \]

\[ \Xi_{2,2} = \frac{(u-v)v(u+v) - 2(u^2 + v^2) \left[ u \left( \tan^{-1} \left( \frac{w}{u} \right) + \tan^{-1} \left( \frac{uw}{wr} \right) \right) \right] + v \log(w+r)}{2(u^2 + v^2)}. \quad (3.65) \]
Interaction torque equations and their extensions

\[ \Xi_{2,3} = -u \log(r-u) - r, \]  
\[ \Xi_{3,1} = \frac{u^3}{u^2 + w^2} + \log(v + r) u - \frac{u}{2} + \nu - w \left( \tan^{-1} \left( \frac{v}{w} \right) + \tan^{-1} \left( \frac{\nu}{w} \right) \right), \]
\[ \Xi_{3,2} = -u \log(r-u) - r, \]
\[ \Xi_{3,3} = \frac{(u - w) w (u + w) - 2 (u^2 + w^2) (u \left( \tan^{-1} \left( \frac{v}{w} \right) + \tan^{-1} \left( \frac{\nu}{w} \right) \right) + w \log(v + r))}{2 (u^2 + w^2)}. \]

Extension to the stiffness equations

The matrix \( \Xi(u, v, w) \) for perpendicular magnetization is in the discontinuous points discussed in Section 3.4.1 given by
\[ \lim_{u \to 0, v \to 0} \Xi(u, v, w) = \begin{pmatrix} u - \frac{1}{2} w \log(w^2) & \frac{1}{2} w \log(w^2) & -\frac{1}{2} \pi r \\ w - \frac{1}{2} w \log(w^2) & 0 & -r \\ 0 & -r & -\frac{1}{2} w (1 + \log(w^2)) \end{pmatrix}, \]
\[ \lim_{u \to 0, w \to 0} \Xi(u, v, w) = \begin{pmatrix} \frac{1}{2} v \log(v^2) & 0 & \frac{1}{2} v \log(v^2) \\ 0 & 0 & -r \\ v - \frac{1}{2} v \log(v^2) & -r \end{pmatrix}, \]
\[ \lim_{v \to 0, w \to 0} \Xi(u, v, w) = \begin{pmatrix} \frac{1}{2} w (1 + \log(w^2)) & 0 & -u + \frac{1}{2} u \log(u^2) \\ 0 & 0 & -r + |u| \log(|u| + r) \\ \frac{1}{2} w (1 + \log(w^2)) & -r + |u| \log(|u| + r) & 0 \end{pmatrix}. \]

As for the force equations above, these equations enable fully analytical and continuous stiffness results for cuboidal permanent magnets for any position.

Validation

Since it is not possible to directly measure the stiffness - especially considering the 3×3 stiffness matrix - the analytical stiffness equations have been validated with place-derivatives of the force equations. In [105] a magnetic spring incorporating two 7-by-7 Halbach arrays has been used to validate the stiffness equations. Firstly, the force results from the Maxwell stress method described in A and the analytical equations in Section 3.4 have been compared. Secondly, a displacement along the \( x \)-axis has been made, over which the displacement-derivatives \( \partial F_x / \partial x, \partial F_y / \partial x \) and \( \partial F_z / \partial x \) has numerically been obtained using a numerical derivation of the Maxwell Stress force results. The comparison of these results with the direct analytical stiffness equations of this chapter has demonstrated the validity of the equations.

Interaction torque equations and their extensions

When designing 6-DoF devices, which exhibit both translational and rotational displacements, the force and stiffness provide limited insight into the device’s behavior. The analytical calculation of torque provides significantly more insight into the behavior of the device.
The force equations can be expanded towards analytical torque calculation using the Lorentz force method [102, 109] or with virtual work as has been performed in [6, 7] simultaneous to the work in this thesis. Similar to the derivation of the force equations, both permanent magnets PM1 and PM2 are studied in the Cartesian coordinate system of PM1. It is assumed that these permanent magnets have parallel side surfaces (i.e. no rotation) and that the respective magnetization vectors $\mathbf{M}_1$ and $\mathbf{M}_2$ are homogeneous and constant. The dimensions of the permanent magnets are again defined by $2a_1, 2b_1, 2c_1$ and $2a_2, 2b_2, 2c_2$, respectively as shown in Fig. 3.12. The center of PM1, $\mathbf{O}_1$, is located at $[0, 0,0]^T$ and that of PM2, $\mathbf{O}_2$, at $[\alpha, \beta, \gamma]^T$ represented by the vector $\mathbf{x}$. The reference point for the torque calculation, $\mathbf{O}_T$, is located at $[\delta, \epsilon, \zeta]^T$. This point could be the center point of a torque sensor which is used in an experiment or could be any point on a structure around which the torques of the permanent magnets are calculated.

The torque calculation is based on the same equations as the force calculation and is obtained for PM2. The analytical equations result from integration of a virtual surface torque density $\mathbf{t}[\text{Nm/m}^2]$ over the magnet’s surface. This gives

$$\mathbf{T}(\mathbf{x}) = \int_S \mathbf{t}(\mathbf{x}')\, dS', \quad \mathbf{t}(\mathbf{x}) = \mathbf{r}(\mathbf{x}) \times \mathbf{f}(\mathbf{x}).$$  \hspace{1cm} (3.73)

The arm $\mathbf{r}(\mathbf{x})[\text{m}]$ is given by $\mathbf{x} - \mathbf{O}_T$ and (3.73) becomes

$$\mathbf{T}(\mathbf{x}) = \int_S \begin{pmatrix} \sigma_m(x') (r_y B_z(x') - r_z B_y(x')) \\ \sigma_m(x') (r_z B_x(x') - r_x B_z(x')) \\ \sigma_m(x') (r_x B_y(x') - r_y B_x(x')) \end{pmatrix} \, dS. \hspace{1cm} (3.74)$$

After symbolically solving (3.74) the resulting torque equations for PM2 obtain the general form

$$\mathbf{T}(\mathbf{x}) = \frac{B_{r_1} B_{r_2}}{4\pi \mu_0} \sum_{i=0}^{1} \sum_{j=0}^{1} \sum_{k=0}^{1} \sum_{l=0}^{1} \sum_{m=0}^{1} \sum_{n=0}^{1} (-1)^{i+j+k+l+m+n} \lambda. \hspace{1cm} (3.75)$$

The vector $\lambda[\text{m}^3]$ is an intermediate variable used in the torque computations.
3.6.1 Parallel magnetization

In the case of parallel magnetization the variables \( B_{r1} \) and \( B_{r2} \) in (3.75) are the remanent flux density components along the \( z \)-axis of the respective PMs. The vector \( \vec{A} \) contains the integrand and is separated in three Cartesian components \( \lambda_x, \lambda_y \) and \( \lambda_z \). These integrands are given in (3.76)-(3.78) shown at the top of the next page and are substituted in (3.75) to obtain the torque [102].

\[
\begin{align*}
\lambda_x &= \frac{1}{\delta} \left[-2C_w \left(-4\nu u + s^2 + 2v \right) - w \left(-8\nu u + s^2 + 8C_u s + 6v \right) \right. \\
&\quad + 4 \left[2C_w \nu \nu \coth^{-1} \left( \frac{u}{s} \right) + w \left(s^2 + 2C_w w - w(2C_w + w) \right) \coth^{-1} \left( \frac{u}{s} \right) \right] \\
&\quad - u \left[2w(C_w + w) \tan^{-1} \left( \frac{u}{w} \right) + 2w(C_w + w) \log(s - u) \right] \\
&\quad + \left[w^2 + 2C_w w - (2C_w + v) \right] \tan^{-1} \left( \frac{w}{v} \right)] + 2(2C_w + w) \left[\nu^2 + w^2 \right] \log(v + s) \right] \tag{3.76} \\
\lambda_y &= \frac{1}{\delta} \left[2(2C_w + w)u^2 - 8uv(C_w + w) + 8uv(C_w + w) \log(s - v) + 4C_u w s + 6uv \right] \\
&\quad + 2 \left[w^2 + 2w(C_w w - u(2C_w + u) \right] \tan^{-1} \left( \frac{u}{w} \right) \\
&\quad + \left[w^2 + 2C_w w - u(2C_u + u) \right] \tan^{-1} \left( \frac{u}{w} \right)] - 2(2C_w + w) \left[\nu^2 + w^2 \right] \log(u + s) + 8C_u w s \right] \tag{3.77} \\
\lambda_z &= \frac{1}{\delta} \left[-18uv^2 - 6w \left[w^2 + 6C_u w - 3v(2C_u + v) + 3C_u s \right] + v \left[\nu^2 + 6 \left[w^2 + 3C_u s \right] \right] \right. \\
&\quad - u^2 + 6w \left[w^2 - 3v(2C_u + v) \right] \tan^{-1} \left( \frac{u}{w} \right) - 6w \left[\nu^2 - 3u(2C_u + u) \right] \tan^{-1} \left( \frac{u}{w} \right) \\
&\quad - 9w \left[w^2 + 2C_u w - u(2C_w + w) \right] \tan^{-1} \left( \frac{u}{w} \right)] - 2w(2C_u + u) \log(s - u) \\
&\quad - (2C_u + v) \left[\nu^2 - w^2 \right] \log(u + s) + 2uv(2C_u + v) \log(s - v) + (2C_u + u) \left[\nu^2 - w^2 \right] \log(v + s) \right] \tag{3.78}
\end{align*}
\]

Variables \( \vec{C}, u, v, w \) and \( s \) are defined by

\[
\begin{align*}
\vec{C} &= \begin{pmatrix} C_u \\ C_v \\ C_w \end{pmatrix} = \begin{pmatrix} (-1)^{j}a_1 - \delta \\ (-1)^{k}b_1 - \epsilon \\ (-1)^{m}c_1 - \zeta \end{pmatrix},
\end{align*}
\]

\[
\begin{align*}
u &= \alpha - (-1)^{j}a_1 - (-1)^{j}a_2, \\
v &= \beta - (-1)^{k}b_1 - (-1)^{l}b_2, \tag{3.80}
\end{align*}
\]

\[
\begin{align*}
w &= \gamma - (-1)^{m}c_1 - (-1)^{n}c_2, \\
s &= \sqrt{u^2 + v^2 + w^2}, \tag{3.81}
\end{align*}
\]

with the dimensions of Fig. 3.12. Although these equations seem rather complicated, they enable a fast and accurate calculation of the torque on a cuboidal permanent magnet in the presence of a magnetic field of another magnet.

**Extension to the analytical interaction torque equations**

In the cases that \( u = v = 0 \), \( u = w = 0 \) or \( v = w = 0 \) the analytical equations for \( \vec{A} \) are obtained by taking its respective limit points. This results in equations which are
suitable for those particular positions, given by

\[
\begin{align*}
\lim_{\mu \to 0} \lambda_x &= \frac{1}{8} \mu \left( -\mu^2 - 2C_w w - 8C_r \sqrt{w^2} + 2w(2C_w + w) \log(\sqrt{w^2}) + u \right), \\
\lim_{\mu \to 0} \lambda_y &= \frac{1}{8} \mu \left( \mu^2 + 2C_w w + 8C_r \sqrt{w^2} - 2w(2C_w + w) \log(\sqrt{w^2}) + u \right), \\
\lim_{\mu \to 0} \lambda_z &= \frac{1}{4} (C_u - C_v) w^2 \log(u^2), \\
\lim_{\mu \to 0} \lambda_x &= -\frac{1}{4} C_u \nu \left( \nu^2 - 1 \right), \\
\lim_{\mu \to 0} \lambda_y &= -\frac{1}{4} C_u \nu \left( \log(v^2) - 1 \right), \\
\lim_{\mu \to 0} \lambda_z &= \frac{1}{72} \nu \left( 2 \left( \nu^2 + 18C_u \sqrt{w^2} \right) + 9 \nu(2C_v + \nu) \log(v^2) \right), \\
\lim_{\mu \to 0} \lambda_x &= \frac{1}{4} C_w u^2 \log(u^2) - 1, \\
\lim_{\mu \to 0} \lambda_y &= \frac{1}{4} C_w u \left( u + 2\sqrt{w^2} \right), \\
\lim_{\mu \to 0} \lambda_z &= -\frac{1}{72} \nu \left( 2 \left( \nu^2 + 18C_u \sqrt{w^2} \right) + 9 \nu(2C_u + \nu) \log(v^2) \right).
\end{align*}
\]  

(3.82)  \hspace{1cm} (3.83)  \hspace{1cm} (3.84)  \hspace{1cm} (3.85)  \hspace{1cm} (3.86)  \hspace{1cm} (3.87)  \hspace{1cm} (3.88)  \hspace{1cm} (3.89)  \hspace{1cm} (3.90)

It is observed that the logarithmic terms in (3.94) and (3.95) produce imaginary terms. Due to the symmetric summations in (3.75) these imaginary terms are canceled, hence, the obtained torque only consists of real numbers.

### 3.6.2 Perpendicular magnetization

In the case of perpendicular magnetization the variables \(B_{r1}\) and \(B_{r2}\) in (3.75) are the remanent flux density components along the \(z\)-axis and \(y\)-axis of the respective permanent magnets. The vector \(\vec{\Lambda}\) contains the integrand and is separated in three Cartesian components \(\lambda_x, \lambda_y, \) and \(\lambda_z\). These integrands are given in (3.76)-(3.78) [102] shown at the top of the next page and are substituted in (3.75). Another derivation is given in [7] using Virtual Work and has been developed simultaneous to the equations in this document.

\[
\begin{align*}
\lambda_x &= \frac{1}{12} \left[ -2w^3 + 3 \left( \nu^2 + \nu^2 \right) + 12 \nu (C_r + \nu) \coth^{-1} \left( \frac{w}{r} \right) + 2 \left( \nu^2 + 3C_r \nu - 2w^2 \right) \right] r \\
&\quad + 2\nu \nu^2 - 6 \nu^2 + w(2C_w + w) \log(r - w) - 3C_w \left( \nu^2 - 4w + \nu^2 + w(w + 2r) \right) \\
&\quad - 6(C_r + \nu) \left( \nu^2 - \nu^2 \right) \coth^{-1} \left( \frac{w}{r} \right) - 12 \nu C_w \coth^{-1} \left( \frac{w}{r} \right) \nu^2 + w \nu(C_u \nu \tan^{-1} \left( \frac{w}{r} \right)) \\
&\quad - (C_r + \nu) \nu \tan^{-1} \left( \frac{w}{u} \right) + C_u \nu \tan^{-1} \left( \frac{vr}{u} \right) + 6C_w \left( \nu^2 + \nu^2 \right) \log(w + r)
\end{align*}
\]  

(3.91)
3.6: Interaction torque equations and their extensions

\[ \lambda_y = -\frac{1}{72} \left[ 2 \log \left( v^2 + w^2 \right) + 2 \log \left( u + r \right) - 2 \log \left( v^2 + u^2 + 2u(u + r) \right) - 3 \right] \nu^3 \\
+ 36 \left( u - 2Cu \tan^{-1} \left( \frac{v}{u} \right) + C_w \tan^{-1} \left( \frac{u}{v} \right) \right) w^2 + 36u(2Cu + u) \coth^{-1} \left( \frac{v}{u} \right) v \\
- 9 \left( v^2 - 8Cu \nu + 8Cu \log (w + r) + u^2 + 2u(4Cu + w) \log (u + r) \right) \nu - 36Cu \nu v \\
- 30u \nu + 36 \left( u^2 (Cu + u) - Cu \nu \right) \coth^{-1} \left( \frac{v}{u} \right) + 2 \left( 123Cu + w \right) \tan^{-1} \left( \frac{u}{w} \right) w^2 \\
+ 6 \left( u^2 + 3Cw \nu - 3u(2Cu + u) \right) \tan^{-1} \left( \frac{u}{w} \right) w + 18Cw \left( u^2 + \nu^2 \right) \tan^{-1} \left( \frac{u}{w} \right) \\
+ u \left[ 3 \log \left( v^2 + w^2 \right) w^2 - 3 \log \left( v^2 + w^2 + 2v(v + r) \right) \nu^2 + 18Cw \tan^{-1} \left( \frac{u}{w} \right) u \\
- 5u^2 - 3u(18Cw + 7w) + 6 \left( u^2 + 3u(2Cw + w) \right) \log (r - \nu) \right] \right) \] (3.92)

\[ \lambda_z = \frac{1}{36} \left[ \nu^3 + 6 \left( v^2 - 3u(2Cu + u) \right) w + 18u(2Cu + u) \log (r - u) w \\
- 3 \left( 3 \nu^2 + v^2 \right) \log (u + r) + 3(6Cu + u) \nu w - 6r \left( v^2 - 3u(2Cu + u) \right) \tan^{-1} \left( \frac{w}{v} \right) \right] \nu^2 + 3(6Cu + u) \nu^2 - 2 \log (u + r) \nu \\
+ 6r \left( 3 \nu^2 + u(2Cu + u) \right) \tan^{-1} \left( \frac{w}{v} \right) - 6 \left( 2u^3 + 3Cu \left( \nu^2 - \nu^2 \right) \right) \log (w + r) \\
- 18(Cw + v) \left( \tan^{-1} \left( \frac{w}{u} \right) + \tan^{-1} \left( \frac{w}{v} \right) \right) u^2 + 2vu - 3w[u - 2vlog(w + r)]w \\
- 2v \nu \left( \tan^{-1} \left( \frac{w}{u} \right) u^2 + v \left( 2v \log (u + r) + 2v \log (r - u) \right) \right) + w \nu \left( 2 \log (u + r) + 2 \log (r - u) \right) \right] \right) \] (3.93)

Extension to the analytical interaction torque equations

In the cases that \( u = v = 0, u = w = 0 \) or \( v = w = 0 \) the analytical equations for \( \lambda \) are obtained by taking their respective limit points. This results in equations which are suitable for those particular positions, given by

\[ \lim_{u \to 0} \lambda_x = \frac{1}{24} \left( 3Cu \nu^2 \log \left( v^2 \right) - 2 \left( 3\nu^2 \log \left( -\sqrt{\nu^2} \right) \nu^2 + 3Cu \left( \nu^2 + 2\sqrt{\nu^2} \nu + 4 \left( \nu^2 \right)^{3/2} \right) \right) \] (3.94)

\[ \lim_{u \to 0} \lambda_y = -\frac{1}{8} Cu \nu \log \left( v^2 \right) - 2 \left( 3\nu^2 \log \left( -\sqrt{\nu^2} \right) \nu^2 + 3Cu \nu \log \left( -\sqrt{\nu^2} \right) \right) \] (3.95)

\[ \lim_{u \to 0} \lambda_z = \frac{1}{12} \left( v^2 \log \left( v^2 \right) + 3Cu \nu \log \left( v^2 \right) \right) \] (3.96)

\[ \lim_{u \to 0} \lambda_x = \frac{1}{12} \left( 3Cu \nu^2 \log \left( v^2 \right) - 3 \left( v^2 + 3Cu \nu \log \left( v^2 \right) \right) \right) \] (3.97)

\[ \lim_{u \to 0} \lambda_y = \frac{1}{24} \left( v^2 \log \left( v^2 \right) - 3 \left( v^2 + 3Cu \log \left( -\sqrt{\nu^2} \right) \right) \right) \] (3.98)

\[ \lim_{u \to 0} \lambda_z = \frac{1}{24} \left( 3Cu \nu^2 \log \left( v^2 \right) \right) \] (3.99)

\[ \lim_{u \to 0} \lambda_x = \frac{1}{24} \left( v^2 \log \left( v^2 \right) - 3 \left( v^2 + 3Cu \nu \log \left( v^2 \right) \right) \right) \] (3.100)

\[ \lim_{u \to 0} \lambda_y = \frac{1}{24} \left( 3Cu \nu^2 \log \left( v^2 \right) - 3 \left( v^2 + 3Cu \nu \log \left( v^2 \right) \right) \right) \] (3.101)
In these equations the symmetric summations of (3.75) cancel the imaginary terms, hence, the obtained torque only consists of real numbers.

3.6.3 Validation

The torque equations have been validated in the same setup as the force equations of Section 3.4. The dimensions of Table 3.2 are again used to compare the torque results. The reference for the torque is chosen at \([0,0,−47] mm\) in the coordinate system of PM1. In the experimental setup, described in Appendix A, this is the geometrical center of the 6-DoF Force/Torque transducer. The results are given in Fig. 3.13. These characteristics show that especially the correspondence between the analytical equations and FEM modeling is high, and that again the experimental results differ slightly. This can be explained with measurement inaccuracies, such as sensor noise, geometrical tolerances and magnetization tolerances of the magnets.

3.7 Coordinate rotation and superposition

The analytical equations for the interaction force, stiffness and torque proposed in Sections 3.4-3.6 are specifically intended for the case that both permanent magnets are magnetized strictly along the \(z\)-axis (parallel magnetization) and with the magnetization vectors along the \(z\)- and \(y\)-axis (perpendicular magnetization), respectively. The superposition principle and appropriate coordinate rotation enable the simulation of magnet structures with other magnetization vector combinations.

3.7.1 Multiple magnetization directions

The two magnetization configurations that have been discussed in the previous sections severely limit design and analysis of 6-DoF magnet-based devices. Other magnetization combinations are of interest and must be included in the model. One way of performing this is to derive new equations for all magnetization combinations. Another method, used in this thesis, is the use of coordinate rotation, which eliminates the need to recalculate the analytical force, stiffness and torque equations. It enables to write any parallel or perpendicular magnetization, such as the example shown in Fig. 3.14(a) into the \(zz\) or \(zy\) form.

A 3-by-3 coordinate rotation matrix \(M\) rotates the coordinate system in such a way that their respective magnetization vectors correspond with the parallel and perpendicular magnetization situations described in Section 3.4. A rotation back into the original coordinate system provides the correct force, stiffness and torque results. This process can be mathematically described by writing the force, stiffness and torque as a function of the displacement \(\vec{x}\) and the vectorized dimensions of both permanent magnets \(\vec{d}_1\) and \(\vec{d}_2\), respectively. The correct analytical results in the
The matrix $M^T$ is here the transpose matrix of $M$. 

The original coordinate system are given by

\[ F(\vec{x}, \vec{d}_1, \vec{d}_2) = M^T \tilde{F}_r, \quad \tilde{F}_r = \tilde{F}(\vec{M^T}\tilde{M}\tilde{\vec{d}}_1, |\vec{M}|\tilde{\vec{d}}_2), \quad (3.103) \]

\[ K(\vec{x}, \vec{d}_1, \vec{d}_2) = M^T (K_r \vec{M}) , \quad K_r = K(\vec{M^T}\tilde{M}\tilde{\vec{d}}_1, |\vec{M}|\tilde{\vec{d}}_2), \quad (3.104) \]

\[ \bar{T}(\vec{x}, \vec{d}_1, \vec{d}_2) = M^T \tilde{T}_r , \quad \tilde{T}_r = \tilde{T}(\vec{M^T}\tilde{M}\tilde{\vec{d}}_1, |\vec{M}|\tilde{\vec{d}}_2). \quad (3.105) \]
3.7.2 Nonclassical magnetization vector

Within the permanent magnets the magnetization vectors are not necessarily aligned with one of the Cartesian vectors, or the magnet edges, as discussed in Section 3.4. This could be caused by misalignment during the magnetization process or it may be a design consideration. Considering the assumption that the relative permeability $\mu_r$ equals 1 anywhere, analytical surface charge model is a linear model. The superposition principle can be used to obtain field, force, stiffness and torque for arrays with multiple magnets. As Fig. 3.14 shows, these nonclassical magnetization vectors may be seen as the superposition of two classical, or Cartesian, magnetization vectors

$$\vec{M} = M_x \hat{e}_x + M_y \hat{e}_y + M_z \hat{e}_z.$$ (3.106)

With this property the permanent magnet can be considered as the superposition of three permanent magnets having the same position and dimensions as the original, however with the magnetization parallel to the magnet edges (Fig. 3.14). Using the coordinate rotation described above it is possible to analytically model the interaction between cuboidal permanent magnets with any magnetization vector as long as it is homogeneous.

3.8 Progress in analytical 6-DoF modeling

Very few scientists have contributed to the development of the analytical surface charge interaction modeling technique for cuboidal magnets. It can even be stated that since 1984 very little progress has been made on this subject. Around 1999, analytical equations for the tangential force of magnets in rotating magnetic couplings were derived, as discussed in Section 3.2.1. However, closed-form equations that describe all dependencies of the force vector on the magnet rotation have not been found in literature. The same holds for the torque vector as function of rotation. Recently, the analytical modeling technique has gained more attention and has been extended with force equations for perpendicular magnetization and with the torque components as function of translational displacements.

Table 3.3 summarizes the progress in the analytical surface-charge force and torque modeling technique that have been made during the last decades. The force and torque vectors, which together form the so-called wrench, are set out horizontally against the translational displacements along the three axes $[x, y, z]^T$ and the rotations
3.8: Progress in analytical 6-DoF modeling

Table 3.3: Overview of the 6-DoF analytical force and torque modeling achievements for displacements and rotations of cuboidal magnets. The areas that require further research are marked by ×.

<table>
<thead>
<tr>
<th></th>
<th>Parallel magnetization</th>
<th>Perpendicular magnetization</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$F_x$</td>
<td>$F_y$</td>
</tr>
<tr>
<td>$x$</td>
<td>[3]</td>
<td></td>
</tr>
<tr>
<td>$y$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r_x$</td>
<td>Partially</td>
<td></td>
</tr>
<tr>
<td>$r_y$</td>
<td>Partially</td>
<td>in [35, 60, 61]</td>
</tr>
</tbody>
</table>

Table 3.4: Overview of the 6-DoF analytical stiffness modeling achievements for stiffness between cuboidal magnets. The areas that require further research are marked by ×.

<table>
<thead>
<tr>
<th></th>
<th>Parallel magnetization</th>
<th>Perpendicular magnetization</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$\partial F_x$</td>
<td>$\partial F_y$</td>
</tr>
<tr>
<td>$\partial x$</td>
<td>This thesis</td>
<td></td>
</tr>
<tr>
<td>$\partial y$</td>
<td>[105]</td>
<td></td>
</tr>
<tr>
<td>$\partial z$</td>
<td>(2009)</td>
<td></td>
</tr>
<tr>
<td>$\partial r_x$</td>
<td>×</td>
<td></td>
</tr>
<tr>
<td>$\partial r_y$</td>
<td>×</td>
<td></td>
</tr>
<tr>
<td>$\partial r_z$</td>
<td>×</td>
<td></td>
</tr>
</tbody>
</table>

around these axes $[r_x, r_y, r_z]^T$ vertically. Although the recent developments have filled some of the dearths in Table 3.3, a need for research into in the modeling of the force and torque as function of rotation around the Cartesian axes remains.

The translational stiffness matrices of the force, which are an important design parameter in many 6-DoF devices, have been proposed in this thesis as summarized in Table 3.4. The translational stiffness of the torque has not been presented, albeit that these equations can be easily obtained by a derivative of the torque equations in Section 3.6. The rotational stiffness of the force and torque follow from the equations that describe their dependency on rotation. However, as Table 3.3 indicates, only a very small part of these equations have been proposed and as such the stiffness cannot be derived at this point.

As mentioned in Chapter 2, the analytical surface-charge based modeling methods discussed above are an idealized representation of the reality. Modeling inaccuracies and manufacturing tolerances cause discrepancies between the model and the reality. Therefore, it is important to estimate their influence, especially for applications such as the gravity compensator that is discussed in Chapter 5.
3.9 Modeling and manufacturing inaccuracies

Each modeling technique inherently has certain assumptions that introduce errors in the predictions made with it. Further, the manufacturing process of any device is subject to certain tolerances. The inaccuracies that are discussed in this section are those that are expected to have the strongest influence on a permanent-magnet based device’s behavior. They are summarized in Table 3.5, which shows the page on which they are discussed too.

3.9.1 Global effects

The global effects are defined as those that can be considered equal for a permanent magnet body or even a batch of multiple magnets. A small number of samples from a batch can therefore predict their magnitude for the full batch with reasonable accuracy.

Variations in remanent flux density

As shown in [180] the remanent flux density $B_r$ of permanent magnets may vary and as such is presented by its typical and minimum value. Typically, this tolerance mainly occurs between different batches; within a batch of magnets, i.e. magnets that were made simultaneously, this variation is significantly lower. Tolerances in the composition and pressing of the magnetic materials during the magnetization process have an effect on the remanent flux density which is difficult to predict prior to manufacturing, however can relatively simply be measured after or even during the manufacturing process. The device that is used to perform these measurements is a hystograph or hysterigraph, which characterizes the $BH$-characteristic of the material.

The variation in remanent flux density $\vec{B}_r$ on the interaction can by directly incorporated with all field modeling methods. More elaborate research of the general force equation \ref{eq:force} shows that any change in remanent flux density exhibits a square relationship to the exhibited interaction force. This also holds for the stiffness and torque, according \ref{eq:stiffness} and \ref{eq:torque}, respectively. As such, these relations are approximated by $F \sim B_r^2$, $K \sim B_r^2$, and $T \sim B_r^2$. 

<table>
<thead>
<tr>
<th>Cause of inaccuracy</th>
<th>Page</th>
<th>Cause of inaccuracy</th>
<th>Page</th>
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<tbody>
<tr>
<td>Remanent flux density</td>
<td>74</td>
<td>Geometrical effects</td>
<td>76</td>
</tr>
<tr>
<td>Relative permeability</td>
<td>75</td>
<td>Hot-cold effect</td>
<td>76</td>
</tr>
<tr>
<td>Magnetization misalignment</td>
<td>75</td>
<td>Partial demagnetization</td>
<td>77</td>
</tr>
</tbody>
</table>
Relative permeability

The most eminent assumption of the analytical magnetic surface charge or current modeling technique is that the relative permeability \( \mu_r \) equals unity everywhere. In reality, this value is never exactly 1 in high-grade permanent magnets, where this value is approximately 1.03 or higher. It is difficult to derive the exact modeling error that is introduced by this assumption of the analytical surface charge model. Hence, experimental measurements or simulations with other modeling techniques should be conducted to verify it. From the measurement results shown in this chapter (Fig. 3.11 and Fig. 3.13) this influence is almost impossible to extract, as geometrical or other magnetic effects have an effect on the measurements too.

In [94, p. 128] the interaction between a coil and a permanent-magnet array is simulated with a surface charge model and with a harmonic model which includes a relative permeability of \( \mu_r = 1.03 \). It is concluded that this difference in relative permeability yields an error of 2%. This suggests that a rough approximation of the force error \( F_{\text{err}} \) could be \( F_{\text{err}} \approx |1-\mu_r|F_{\text{ana}} \). However, this error strongly depends on the working point, or working region, of the permanent magnets and may therefore not be used as rule of thumb. A better approach is the use of a modeling technique that does account for the permeability of the materials. In Section 5.10 a FEM model of a magnetic gravity compensator is used to find the influence of the relative permeability on the vertical force. It is varied between 1 and 1.08 and its influence on the vertical force seems to be near-linear for the considered topology. As mentioned, this relation is not necessarily valid for all topologies.

Misalignment of the magnetization angle

Permanent magnets are magnetized using large magnetizing coils. If for some reason the geometrical axis of the permanent magnet is not precisely aligned with the magnetic axis of the domains in the permanent magnet the resulting magnetization will exhibit a small inclination with respect to the geometry. Further, the magnetic field of the magnetizing coil is not totally uniform, as Fig. 3.15(a) shows in an exaggerated way. Under the assumptions that the magnetized material is cut into eight pieces as Fig. 3.15(b) shows and that the magnetization is uniform within these pieces, the magnetization of each of them is different, i.e. misalignment of the magnetization vectors occurs. Such misalignment is modeled using the superposition principle which was described in Section 3.7.2.

3.9.2 Local effects

Local effects are those effects that may vary within the volume of a permanent magnet and are unique for each magnet. This makes them often more difficult to predict and to measure.
Chapter 3: Three-dimensional analytical modeling of magnet-based devices

Figure 3.15: A magnetization coil produces a magnetic field which is not fully uniform and homogeneous. Figure (a) shows a schematic and exaggerated magnetizing field distribution, and (b) the resulting magnetization pattern.

Geometrical tolerances

As for any other machined product, the dimensions of permanent magnets are accurate within certain tolerances. However, these are generally of such low value (tens of micrometers) compared to the magnet dimensions that they are not of significant influence on its behavior. Further, the edges are subject to chipping: As magnet material is extremely brittle often have sharp edges, small chips may break off. As for the dimensional tolerances, the effect of these chips on the magnet characteristic is estimated to be minimal.

The placement of the permanent magnets is subject to sub-millimeter tolerances too. As for the dimensional tolerances it is expected that these only have a minimal effect as a result of the orders-of-magnitude difference between the tolerances and the magnet dimensions. Both effects can be easily modeled in the analytical surface charge model.

Local magnetization misalignment

Most magnetic field modeling techniques assume that the local magnetization within the permanent magnet volume is homogeneous and uniform, although this is often not the case for permanent magnets. As discussed above, the magnetic field during the magnetization process is never exactly parallel and the batch of permanent magnet being magnetized is oriented as shown in Fig. 3.15(a). The result of this is that the individual magnets of the batch exhibit a non-uniform magnetization, additional to the misalignment discussed above. Such non-uniformity is often referred to as the hot-cold effect [185] and results in a magnetization distribution as shown in Fig. 3.16. One of the magnet’s pole surfaces has more magnetic flux than the other pole surface as illustrated in Fig. 3.16(a). Furthermore, when multiple magnets are cut from one piece of magnetized material, this hot-cold effect is not evenly distributed over the permanent magnets.

The hot-cold effect is measured using Hall-sensors which are placed at a defined measuring distance of the magnet's North- and South pole [26, 185]. It is expressed as percentage of the difference between and amplitude of the absolute flux values on the
3.9: Modeling and manufacturing inaccuracies

In \[185\] this hot-cold effect is modeled in two dimensions by dividing a magnet into 6 sections, where at the cold side two of those sections have an inclined magnetic orientation, as shown in Fig. 3.16(b). A translation to three dimensions requires a redistribution of the magnet into 10 new permanent magnets. The superposition that results from the off-vertical magnetization causes that in total 22 idealized magnets represent a single magnet with hot-cold effect.

To simplify the modeling, the hot-cold effect has been modeled by redistributing the charge densities of the surface charge model. On the hot side (the bottom side in Fig. 3.16(c)) the magnetic charge remains unchanged. On the top side, however, the charge is reduced by a certain percentage and is distributed over the four side surfaces. In Fig. 3.16 this is schematically illustrated in 2D with 14 ‘negative’ charges on the bottom side and only 10 ‘positive’ charges at the top side with the 4 remaining ‘positive’ charges redistributed over the four side surfaces. The total charge of the permanent magnet must always equal zero, in accordance with \( \nabla \cdot \vec{B} = 0 \). With the force, torque and stiffness equations derived in this chapter the interaction can be obtained for each surface individually, rather than for each magnet. The percentage of charge that needs to be redistributed is iteratively determined based on (3.107).

**Local demagnetization**

In most applications the permanent magnets experience external factors such as heat, opposing magnet fields and ageing \([59, 138]\) which may impair their magnetic properties. Local demagnetization occurs when the local working point of (a part of) the permanent magnet is driven below the knee point of its \( B(H) \)-characteristic. This may happen when the load line shifts or when the temperature changes. The remanent flux density reduces as a result of this and the performance of the device will decrease. In active devices this is often compensated by increasing the current, however this may lead to a vicious circle with even more demagnetization.

Consider the working point \( P_1 \) on load line 1 of Fig. 3.17(a) which is on the linear part of the magnet’s hysteresis curve. If the load line, e.g. due to a reversing magnetic
Figure 3.17: Schematic representation of demagnetization of a permanent magnet as a result of (a) a changing load line and (b) a change in operating temperature.

Field of a coil or other permanent magnet, changes to load line 2 the working point shifts over the hysteresis curve to $P_2$. When the magnetic load is reduced again to load line 1, the working point, instead of returning to $P_1$ moves to $P_3$ as $P_2$ was underneath the knee point of the characteristic. Accordingly, the remanent flux density has decreased from $B_r_0$ to $B_r_1$. If the load line is reversed the original working point $P_1$ can only be reproduced if $P_2$ is within the linear section of the demagnetization curve.

Temperature variation

The demagnetization curves of permanent magnets are temperature dependent. This dependence is characterized by the temperature coefficients of the remanent flux density and the coercivity given in [180, p. 56]. A temperature increase of the permanent magnet could be the result of an ambient temperature rise, or could be local, for example as a result of locally induced eddy currents.
Starting from the same situation as the example above the temperature in Fig. 3.17(b) rises from $T_0$ to $T_1$. As a result the $B(H)$-characteristic changes and the coercivity and intrinsic coercivity decrease. The working point shifts over the load line from $P_1$ to $P_2$ and is below the new knee point of line $T_1$. Cooling the magnet does not result in the original $B(H)$-characteristic, hence, irreversible demagnetization occurs which can only be canceled out by remagnetization of the magnet. To avoid irreversible changes in the flux density through temperature fluctuations, the working point must remain within the linear section of the demagnetization curve over the entire temperature range in which the magnet is to be used.

3.10 Conclusions

This chapter has researched and improved the analytical surface charge modeling of permanent magnets. The field equations for rectangular, or cuboidal, permanent magnets have been repeated and those for triangular charged surfaces have been proposed and validated. With the help of these equations the magnetic field of any permanent magnet with straight edges and flat surfaces can be modeled with closed-form 3D analytical equations.

The focus of the chapter is further on the analytical interaction modeling of cuboidal magnets based on the surface charge model. The existing force equations for parallel magnetized permanent magnets have been extended with equations for perpendicular magnetization. A superposition of these equations enables the modeling of any magnetization vector combination. The necessary extension to these equations for the case where the magnet edges align is necessary and has been proposed. Based upon the force equations, the 3-by-3 stiffness matrix has been successfully derived analytically for all magnetization directions, including the equations for the special situations described above. Finally, the surface charge model and Lorentz force have been utilized to derive analytical torque equations for any magnetization and position. Experimental results have validated these analytical models. The models that have been derived in Part I of this thesis enable the modeling of a wide range of permanent-magnet structures. One of the possible applications is electromagnetic vibration isolation with permanent-magnet based gravity compensation. The following part of this thesis discusses this application and the design of an appropriate vibration isolation system.
3.10.1 Contributions

- **Section 3.3** - The analytical field equations for triangular charged surfaces and magnets incorporating such surfaces have been proposed and validated. The high accuracy and low numerical noise exhibited by these models has been confirmed by comparison with FEM results. With these analytical field equations the field of any permanent magnet shape with straight edges can be modeled with closed-form 3D analytical equations, as an example with the pyramidal frustum shows.

- An extensive investigation of the analytical interaction models for cuboidal permanent magnets has been performed. This has resulted in
  - **Section 3.4** - Equations for the interaction force between perpendicularly magnetized permanent magnets. Further, the discontinuities in these and the existing equations for parallel magnetized magnets have been investigated. The result is a continuous function for the interaction force between permanent magnets.
  - **Section 3.5** - An analytical description of the $3 \times 3$ stiffness matrix, including the removal of the discontinuities.
  - **Section 3.6** - Continuous analytical equations for the torque of cuboidal magnets in each other’s proximity.

- **Section 3.9** - Characterization of the manufacturing and modeling tolerances of permanent magnets with respect to the design of permanent-magnet devices.

3.10.2 Recommendations

Based on the results and experience gained in this Chapter some recommendations can be made on the future analytical modeling of permanent-magnet devices.

- The relative permeability $\mu_r$ is considered unity in the analytical surface charge and current sheet model. For simple structures with near-infinite permeability it is possible to use the method of imaging for incorporating these materials [76, 98]. The analytical models would be significantly stronger if they could incorporate more complex soft-magnetic structures or if they could incorporate a relative permeability that is somewhat above 1. An estimation of this effect is discussed in Section 5.10, however is a post-processing analysis and as such not incorporated in a design process.

  The inclusion of complex structures is most probably far away, and may be left being privileged to FEM models. However, the relative permeability of a permanent magnet – or magnet array – seems somewhat more realistic. For instance, the harmonic models of Section 2.5.2 can model a layer of permeable materials by solving the boundary conditions at the magnet’s edges. The solving of this boundary condition at the edges, either by connecting two models with a different $\mu_r$, or apply a form of the basic imaging of Section 2.5.4, would be a very
interesting subject of research. Parallels can be drawn to the electric domain, rather than the magnetic one, where such calculations may have been initiated or even performed for field calculation around capacitors with varying relative permittivity.

• As was briefly mentioned in Section 3.2.1 there still exists a dearth in the analytical interaction models. Rotation between the permanent magnets has only partially been solved [35, 60, 61] but may be important for many magnet-based devices, such as the magnetic gravity compensator discussed the Chapters 4 and 5. Inclusion of these rotations in the analytical form of the force, stiffness and torque equations would be a significant step forward in the analytical modeling of the interaction between permanent magnets.

• The modeling assumptions and manufacturing tolerances which are discussed in Section 3.9 are based on theoretical assumptions. This list is by no means exhaustive and more effects could be present that influence a device's electromagnetic behavior. Although some of them have been documented and investigated (e.g. relative permeability and temperature effects), others are difficult to measure and predict. Additional research into their effects on the performance could help to improve future models. For instance, it could be researched which tolerance has the most significant effect on the fields and forces between permanent magnets.
Chapter 3: Three-dimensional analytical modeling of magnet-based devices
Part II

Application to a Vibration Isolation System
Chapter 4

Electromagnetic vibration isolation

Vibration isolation in a nutshell, a translation of required mechanical properties to electromechanics and the results of a literature search into electromagnetic vibration isolation.
Part I discusses the analytical models that describe the force between permanent magnets. The remainder of this thesis, Part II, applies these models in the design of an electromagnetic vibration isolation system with permanent-magnet based gravity compensation. The high accuracy and low computational costs of the analytical modeling technique are considered very suitable to model a magnet-based vibration isolation device.

Structural vibrations are a problem of all times in a vast range of applications such as those in Fig. 4.1. As discussed in Chapter 1, many techniques and technologies have been developed over time and are being used today, all with their own unique properties in terms of costs, performance, maturity, complexity, etc.

As opposed from the more established technologies such as hydraulic, mechanical or pneumatic isolation systems, fully electromechanical vibration isolation systems are a relatively new technology to fulfill the vibration isolation requirements. Despite their reduced force density, technological immaturity and design complexity they offer the advantages of a high control bandwidth and low static energy consumption. The analytical models described and expanded in Chapters 2 and 3 provide an excellent tool to accomplish this.

4.1 Outline

Section 4.2 discusses the vibration sources that are commonly distinguished. The passive isolation system, its performance evaluation methods and limits are shown in Section 4.3. The suitability of active isolation systems is discussed in Section 4.4. Section 4.5 introduces negative stiffness that can be employed to reduce an isolator system's stiffness. The use of electromagnetics for vibration isolation systems is discussed in Section 4.6 along with their stability issues. Section 4.7 translates the mechanical requirements of the system to the electromagnetic domain followed by a literature review in Section 4.8. The framework and design goals that result from it are discussed in Section 4.9. Finally, the conclusions are presented in Section 4.10.

4.2 Structural vibration isolation

A completely vibration-free environment is unachievable, just like any other idealized abstractions such as immovable objects, irresistible force or a perfect vacuum. Practically feasible challenges and design targets therefore need to be set for the vibration isolation system. Fortunately, in practice it often suffices to provide a platform that is adequately vibration-free, i.e. one that does not exceed suitably selected vibration limits [177]. The floor vibrations in buildings have sub-millimeter amplitudes in most situations, however these are not the only source of vibrations.
4.2: Structural vibration isolation

Figure 4.1: Vibration-sensitive devices such as (a) optics (Courtesy Australian National University) and (b) lithography machines (Courtesy of ASML Veldhoven) require advanced vibration isolation.

Figure 4.2: The three sources of vibrational noise that are generally distinguished: floor vibrations, acoustic noise and direct platform forces.

4.2.1 Sources of vibration

Three main disturbance sources are generally distinguished when considering a high-performance vibration isolation system. These sources are ground vibration, acoustic noise and force disturbances acting directly onto the isolated mass [49, 194] (Fig. 4.2). These sources are continuously present, although careful precautions may already reduce these disturbances even before an active or passive suspension system is considered [8, 73, 177].

Ground vibrations

Ground vibrations exist in all environments and are caused by natural (waves crashing on shore lines, tectonic activity, etc.) or man-made (machinery, traffic, etc.) phenomena [177]. Establishment of appropriate limits for these vibrations is crucial to the successful design of a sensitive facility. Given a certain acceptable vibration limit at the isolated platform there is a compromise to be made between floor requirements and isolator performance. Floor vibration limits that are insufficiently stringent lead to an increase in the performance requirements of the vibration isolation system that is placed on top of it. Although the financial investment in building and floor can be
Figure 4.3: Generic floor vibration criterion (VC) curves for vibration-sensitive equipment [8]. Please note that velocity is used as a vibration measure in this figure instead of acceleration.

kept low in this case, the costs of the vibration isolation system increases as a result. If the floor limits are too stringent they may lead to excessive complexity and increased costs of the building, although for some applications an advanced vibration isolation may even become obsolete. For some of the most sensitive systems, e.g. those that work at sub-nanometer scales, even a well-designed floor may impose intolerable vibrations that need to be isolated by an advanced vibration isolation system.

The fact that vibration amplitudes on the floor of the toolmaker’s laboratory are generally much less than the vibration amplitudes on the fabrication process floor can mean that a tool that works perfectly prior to delivery may not work satisfactorily after delivery [72]. Therefore, manufacturers often command floor vibration criteria to ensure the functionality of their device. The floor specifications based upon the standards proposed by Bolt, Beranek and Newman (BBN) have been used for many years now. They specified curves with different vibration levels suited for different environments. The curves are indicated as BBN-A to BBN-E, or VC-A to VC-G (VC stands for Vibration Criteria) [8]. Figure 4.3 shows a graph of such VC criteria which are described in [8, 73].

**Acoustic noise**

Acoustic noise often comes from similar sources as the ground vibration. Instead of transmitting through the suspension it acts directly on the isolated mass as Fig. 4.2 shows. This acoustic noise is not necessarily audible sound, but may occur in the
form of pressure waves from air vents or an acoustic coupling between the floor and the isolated platform (Fig. 4.2). This noise source is difficult to measure and as such it is often treated as an unknown direct force disturbance on the isolated platform.

**Direct disturbance forces**

Force disturbances acting directly on the isolated platform are often the result of a vibrating mechanical coupling such as oil or water hoses, strong time-varying currents in electrical wires or moving objects located on the isolated platform itself. For example, in semiconductor equipment the metrological frame supports a watercooled lens system which becomes a source of vibration. People may be another direct disturbance source when they work with isolated optical tables or microscopes.

**4.2.2 Environmental and isolator requirements: a compromise**

As was mentioned, the establishment of appropriate limits for the disturbance sources is crucial to the successful design of a sensitive facility. In general, it can be said that the most severe disturbances that may occur should be isolated such that the transmitted vibration does not exceed the most stringent limitation set by the isolated equipment. The floor design and the vibration isolator design determine the amount of transmitted vibrations. For each situation there is an optimum between floor and isolator design in terms of financial costs and performance.

**4.3 Passive vibration isolation**

In a passive vibration isolation system three basic components are generally identified. Figure 4.4 shows these components in a simplified 2D single-DoF representation. A passive vibration isolation system generally consists of a passive spring with stiffness $k$ [N/m] and a damper $b$ [Ns/m] which together isolate a platform with mass $m$ [kg].

The (vertical) floor vibrational displacement is represented by $z_0$ and that of the isolated platform by $z_1$. An external force acting on the isolated platform is represented by $F_{ext}$. This simplified representation of the actual system, generally a local working-range linearization, is often used to derive the desired system properties.

**4.3.1 Evaluation of the vibration isolation performance**

The transmissibility and the compliance are two commonly used transfer functions in assessing the vibration isolation performance of such system. The former is a measure for the ability of the system to isolate floor vibrations and the latter for the ability to reject force disturbances that directly act on the isolated platform. It is possible to express them in terms of displacement, velocity or acceleration of the platform. For high-performance systems with large isolation bandwidths it is common practice to measure the absolute velocity or acceleration $[171, 181]$. These quantities remain measurable at elevated frequencies and are measured with respect
Figure 4.4: A passive vibration isolation system with a spring $k$ and a damper $b$. The isolated mass $m$ experiences an external force $F_{\text{ext}}$. The vertical displacements of the floor and the isolated platform are defined by $z_0$ and $z_1$, respectively.

to an inert reference. The transmissibility $H_t$ and the compliance $H_c$ are defined as

$$H_t = \frac{\ddot{z}_1}{\ddot{z}_0}, \quad (4.1)$$

$$H_c = \frac{\ddot{z}_1}{F_{\text{ext}}}. \quad (4.2)$$

A detailed derivation of these transfer functions is found in Appendix B.

These transfer functions are often written in terms of the undamped natural frequency and the damping ratio. The undamped natural frequency $\omega_n$ [rad/s], or resonance frequency, on which the system vibrates when excited is given by

$$\omega_n = \sqrt{\frac{k}{m}}. \quad (4.3)$$

This frequency is reduced by increasing the mass $m$ or reducing the stiffness $k$ and vice versa. The dimensionless damping ratio $\zeta$ is given by

$$\zeta = \frac{b}{2m\omega_n}. \quad (4.4)$$

This ratio provides a mathematical means of expressing the level of damping in a system relative to critical damping. This critical damping is the level of damping at which the system will just fail to overshoot when being excited, i.e. without oscillating. The system converges to an equilibrium in the minimum amount of time (otherwise it is called overdamped).

### 4.3.2 Transmissibility

The amount of floor vibrations being transmitted to the isolated platform is expressed by the dimensionless transmissibility transfer function $H_t(s)$. It can be seen as the ratio between the isolation performance between the considered system and when the load is fixed to the floor. Intuitively it becomes clear that a soft spring leads to an improved transmissibility; a floor movement leads to a compression of the spring rather than to a movement of the isolated platform. Such low stiffness yields a low
4.3: Passive vibration isolation

Table 4.1: The compromise between transmissibility and compliance that are inevitable for a vibration isolation system.

<table>
<thead>
<tr>
<th>Transfer function</th>
<th>Stiffness requirements</th>
<th>Damping requirements</th>
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<tbody>
<tr>
<td>Transmissibility</td>
<td>As low as possible</td>
<td>Reasonably low</td>
</tr>
<tr>
<td>Compliance</td>
<td>As high as possible</td>
<td>As high as possible</td>
</tr>
</tbody>
</table>

Some form of damping is required at the resonance frequency as a system with low damping exhibits large oscillations. However, it is shown in Appendix B that a high damping yields a poor transmissibility above the resonance frequency as floor vibrations are transmitted to the isolated platform. Hence, the damping ratio $\zeta$ provides a trade-off between suppression of resonances and good floor vibration isolation.

4.3.3 Compliance

The compliance transfer function describes the relation between forces acting directly on the isolated platform and its acceleration. Such forces may be the result of reaction forces on moving parts of the isolated platform, such as cabling and moving equipment on the platform as discussed in Section 4.2.1. Intuitively, a high stiffness is necessary to reject these disturbances as in this case these external forces only lead to a minimal compression of the spring. This yields a high value of the undamped natural frequency $\omega_n$ according (4.3) which is summarized in Table 4.1.

As for the transmissibility, a high damping, or high damping ratio $\zeta$, reduces oscillations around the resonance frequency. As seen in Appendix B a high damping seems to have no negative effect on the compliance as the platform is coupled to the floor.

4.3.4 Design compromise

The transmissibility and compliance exhibit conflicting requirements as Table 4.1 shows. A low undamped natural frequency is required for the former and a high undamped natural frequency for the latter. Further, a good transmissibility, or floor rejection, performance requires a low damping, whereas a high damping leads to an improved compliance, or direct-force rejection. A more detailed discussion on this matter is found in Appendix B. This compromise between transmissibility and compliance is fundamental to the passive suspension: it is impossible to simultaneously reject both floor disturbances and direct force disturbances [49].
4.3.5 Linearization

The transmissibility and the compliance are not only used in the performance evaluation of the passive system, but control engineers also tend to create their control schemes based on the measured transfer functions of the system. These functions linearize the system, hence assume that each combination of input parameters creates a unique combination of output parameters. Especially for magnet-based systems, with their inherent nonlinear interaction behavior, this is not by definition true [25, 46, 158] as nonlinear stiffness or damping, hysteresis and other nonlinear effects are not incorporated. Linearized systems enable a reduced control-system complexity and a projection of these transfer functions on the physical properties of the system would benefit the analysis and design process. With a-priori knowledge of the required system properties, extracted from the transfer functions, the physical designer can specifically focus on achieving them throughout the working range.

4.4 Active vibration isolation

Active vibration isolation systems (Fig. 4.5) are generally more expensive than a passive solution due to their sensors, controllers, actuators and power amplifiers. Nevertheless, they are suitable for a wide range of applications [22, 81, 86, 87, 171, 194]. An external actuator exhibiting a force $F_{\text{act}}[\text{N}]$, a feedback controller $G(s)$, and sensors $n$ are added to the passive system. This enables an improvement in both transmissibility and compliance as is discussed in more detail in Appendix B. However, this theoretical performance improvement must be handled with care, as actively generated noise may harm the vibration isolation performance.

4.4.1 Active noise generation

Generally, the manufacturers of active isolation systems specify the isolation performance using the compliance and transmissibility graphs described above. To measure these characteristics the floor vibration and direct disturbance force are chosen large enough to obtain clear transfer functions. However, the actual system is often placed on a much more vibration-less floor which could be of such a low value, that noise generated by the sensors in the control loop becomes dominant. In active systems the actively generated noise (sensors, amplifiers, etc.) leads to disturbance forces on the
platform. This noise may become the limiting factor in the isolation performance and even cause the active system to perform not better or even worse than the passive system. With this knowledge one could say that passive systems are sufficient in most cases and that active systems add more costs and troubles than improvement in performance.

Nevertheless, it is especially common in lithographic systems to use active isolation. This is due to the accelerations and forces that act on the systems: accelerations of 10 [m/s²] are quite normal [181]. Active actuation is in such cases a necessity to pre-act on these known forces rather than to let it effect the passive system. Improvement of the isolation bandwidth of such system enables more detailed structures at higher throughput.

4.5 Utilization of nonlinear behavior

With very few exceptions, passive and active vibration isolation topologies are primarily based on either repelling or attracting elastic elements such as springs, (electro)magnets, and, to smaller extent, pneumatic suspensions. Given the need for low-frequency isolation systems in high-tech applications [181] this may lead to problems with respect to the pre-compression as a result of the gravitational force of the isolated platform. The isolation system must either accommodate a highly pre-compressed linear spring, a non-linear spring that is sufficiently soft in its working range or a nonlinear device of which the nonlinearities are compensated by forceful actuators which artificially reduce the system’s resonance frequency. The first option requires a large volume, the third a large investment in active elements, whereas the second option combines the low actuator requirements of the first with the reduced volume of the third.

Instead of using either repulsion forces or attraction forces between the environment (floor, ceiling, etc.) and the isolated platform, Platus [144, 145] proposed to combine negative stiffness with positive stiffness as Fig. 4.6(a) shows. From the total stiffness \( k_{\text{tot}} = k_{\text{pos}} + k_{\text{neg}} \) it follows that both springs (at least partially) cancel each other’s stiffness. An increased vibration isolation performance is obtained, especially when the targeted resonance frequency is in the sub-hertz region.

Such quasi-zero stiffness topology, sometimes referred to as a system having high-static and low-dynamic stiffness [4, 31] may be implemented with elastomeric springs as Fig. 4.6(b) shows. Due to the parallel placement of the springs each one of them supports a only part of the gravitational force which reduces the (pre-)tension (Fig. 4.6(c)). As such they become suitable for vibration isolation systems with good passive floor vibration rejection [4, 31, 90, 156, 189]. Additional measures need to be taken to improve the impaired compliance of such system that results from the low natural undamped resonance frequency. In this thesis this is accomplished with active elements.
Figure 4.6: A parallel configuration of positive and negative springs (a) yields a quasi-zero stiffness system. Such negative stiffness can be achieved by (b) a re-orientation of the springs and the resulting summation of stiffness (c) approaches zero in a narrow working range.

4.6 Electromagnetic vibration isolation

A wide variety of active and passive technologies is available to accomplish vibration isolation [49, 136, 156]. These include hydraulics, elastomers [49, 78], pneumatics [29, 48, 81] and piezoelectronics [84, 85, 116, 147]. Each of these technologies has its unique properties, such as stiffness, energy consumptions, technical maturity, installation and operational costs, bandwidth, etc. The variety in applications and specifications is as large as the variety of implementations and combinations of these technologies. This thesis concerns a vibration isolation system fully based on the physical electromagnetic domain.

As electromagnetic vibration systems we classify systems that utilize interaction forces in the electromagnetic domain to achieve stable suspension of a body. Such forces may be based on static phenomena such as permanent magnet or DC current fields, dynamic, e.g. based on eddy currents, or a combination of both [23, 49, 111, 112]. Some of their advantages are low noise, reduced maintenance, vacuum compatibility, reduced parasitic effects and contactless nature [142]. This last property eliminates the need for a mechanical path between floor and isolated platform, i.e. a path for vibrations to pass through or parasitic resonances to take place. This makes this technology also suitable for e.g. vacuum conditions, although care must be taken with respect to the so-called out-gassing of the materials.

As any other technology, electromagnetic vibration isolation systems exhibit some specific disadvantages. Especially compared to hydraulic or some pneumatic suspensions, the achievable force densities are limited. Further, they are generally more costly as a result of their technical immaturity and low production volumes. Heat is also an important factor, as it could disrupt the proper functioning of the system. The contactless properties make it difficult to remove any heat from the isolated platform. If permanent magnets are used to compensate the gravitational force of the isolated platform the active heat generation may be reduced significantly. The high potential of such a magnet-based electromagnetic solution for vibration isolation systems and low commercial maturity renders it very suitable for an investigation, especially in combination with the analytical models of Chapter 3.
4.6.1 Magnetic suspension versus magnetic levitation

In literature a distinction between the terms magnetic suspension and magnetic levitation is often made. Magnetic suspensions refer to systems which are based on attractive magnetic forces and magnetic levitation refers to systems which are based on repelling magnetic force [23, 111, 112]. However it may be disputed which name should be used for a topology with a combined attraction and repulsion often seen in a quasi-zero stiffness system discussed in Section 4.5. To avoid confusion this thesis refers to the device as vibration isolator or vibration isolation system.

4.6.2 Stability

Magnet-based electromagnetic vibration isolation systems have a common instability problem. Earnshaw [58] stated that objects in the influence of fields that apply forces with an inverse-square \(1/r^2\) relation to displacement, such as the force between permanent magnets, cannot form configurations of stable levitation. Earnshaw’s theorem was originally formulated for electrostatics (point charges) to show that there is no stable configuration of a collection of point charges but is also suitable for magnetic charges, or dipoles, or arrays of magnets. In other words, a permanent-magnet device is only capable of producing a saddle-shaped energy in space which generates a metastable equilibrium instead of a local minimum, comparable to balancing a ball on a saddle-shaped body. A more detailed discussion on Earnshaw’s theorem and its effects on stability is found in Appendix C.

Only a few years after Earnshaw’s theorem, Lord Kelvin theoretically showed that diamagnetic substances could levitate in a magnetic field [174]. This work was further elaborated on by Braunbek [27, 28] and is discussed in Appendix C. In fact, Earnshaw’s theorem does not apply to induced magnetism and it is possible for the total potential (gravitational and magnetic) to exhibit a minimum. Over the years, many methods have been found to achieve stability for permanent-magnet based levitation systems. These can be summarized in a number of techniques [12, 24, 27, 28, 111, 112]. These techniques are an inclusion of elastic elements, diamagnetics, superconducting, spin-stabilized magnetic levitation, oscillating fields, tuned LCR circuits and feedback-controlled time-varying magnetic fields, which are explained in Appendix C.

4.7 Mechanical requirements in the electromagnetic domain

As was discussed in Section 4.3.1 it is important for the physical designer of the electromagnetic vibration isolation system to express the force, stiffness and damping requirements in properties that can be directly evaluated in the electromagnetic domain. Such information is of vital importance in the design of these systems as it helps to gain a-priori insight into the behavior of possible topologies.

From the field equation (2.67) and Fig. 2.5(a) of Section 2.5.3 it is seen that the magnetic surface charge model is a mathematical abstraction. In reality, the
permanent magnet is a piece of material with a net magnetization vector throughout
its volume and not a collection of two surfaces with a magnetic surface charge.
These equivalent magnetic surface charges are a mathematical manner to derive the
necessary energy, field or interaction equations [124, 192]. As it has been shown
with experiments that these interactions are correctly described [3, 101, 102, 109] it
is concluded that this abstraction is a very elegant method to describe them. A further
abstraction discussed in [17, 192] uses the symmetry within the cuboidal magnets to
simplify these magnetic surface charges to so-called ‘magnetic nodes’ on the magnet
corners.

4.7.1 Force level, $F$

The integrand in the Lorentz force equation (3.39) concerns a multiplication of the
flux density $\vec{B}_1$ with the second magnet’s surface charge density $\sigma_m$. It is discussed
below for a parallel magnetization, however, holds for perpendicular magnetization
too. The Lorentz force equation is repeated here:

$$\vec{F} = \int_{S_2} \sigma_m \vec{B}_1 \, ds'. \quad (4.5)$$

The flux density $\vec{B}_1$ of PM1 in Fig. 3.9(a) is analytically obtained with (3.4). Accord-
ing (2.66) the surface charge density $\sigma_m$ is defined by the dot product of PM2’s source
term $\vec{M}_2$ with the normal of its surface. Consequently, (4.5) reduces to a surface
integral over the top and bottom surfaces of PM2 in Fig. 3.9(a), on which the charge
density $\sigma_m$ attains a constant value. As such, the flux density $\vec{B}_1$ is in the integrand
of (4.5) the only variable that varies at the integration surface. The magnitude of the
three resulting force components $F_{x, y, z}$ is defined by $\sigma_m$, $B_{x, y, z}$, and the integration
limits of (2.48), which are the dimensions of PM2. In other words, this abstraction
suggest that an evaluation of PM1’s field and only the dimensions of PM2 are sufficient
to predict the force between two permanent magnets.

Given the surface charge density $\sigma_m$, the integral is performed for each component
individually. As a result the force component $F_x$ depends on $B_x$, $F_y$ on $B_y$ and $F_z$
depends on $B_z$. With the superposition principle these conclusions can be expanded
to the magnet arrays which form the gravity compensator in a vibration isolation
system. For a high vertical force level it is necessary to have a topology that focuses
the flux of both permanent magnet arrays towards the airgap between them as this
maximizes the interaction.

A positive side-effect of a force maximization, or force density maximization if the
volume is constrained, is magnetic shielding. If the magnet arrays focus the magnetic
flux towards the airgap, hence towards each other, the leakage of this flux outside the
structure automatically reduces. As such, the influence on nearby systems that may
be sensitive to magnetic field reduces.
4.7.2 Undamped natural frequency, $\omega_n$

As discussed in Section 4.3.1 the undamped resonance frequency, given in (4.3), is an important design objective for high-performance vibration isolation systems. Often, this requires an adaptation of the stiffness as the mass that is isolated can often not be changed. Stiffness is the sensitivity of the force to displacements and therefore the sensitivity of the integral (4.5) to variations in the integration limits. If PM2 is displaced, these limits shift accordingly. A field gradient around the considered line therefore causes a force gradient, or stiffness. If this stiffness is to be minimized, the designer should minimize the field gradients, especially around the edges of this line as is shown below. As is shown in the next chapter, the placement of the second magnet’s edges in a low-magnitude part of the field results in a minimal change in surface underneath the integrand of (4.5) and in this way helps to minimize stiffness.

4.7.3 Damping ratio, $\zeta$

Damping is a mechanism that helps stabilizing a system, for instance around its resonance frequency, but may impair its vibration isolation properties as Section 4.3 shows. It is a dissipative mechanism occurring according to Faraday’s law (2.3). Its magnitude is often low for electromagnetic systems, especially compared to mechanical damping mechanisms. Nevertheless, it is related to the magnetic field gradients inside the conductive materials of the design; such gradient induce damping if there is relative movement of the magnetic field. This suggests the use of magnet arrays with large magnet dimensions relative to the magnitude of movement or at least with a low number of poles along the direction of movement [96, 146]. A more detailed investigation of the passive damping of electromagnetic vibration isolation systems is necessary to verify this hypothesis.

4.8 Prior art

Many publications concerning vibration isolation systems with feedback-controlled electromagnetic components, hereafter referred to as magnetic vibration isolation systems, have been reported in literature. An extensive overview of electromagnetic vibration isolation systems developed until the eighties is given in [111, 112]. Bleuler [23] classifies the isolation systems in categories, however, gives an incomplete overview of the methods available today.

4.8.1 Elastical gravity compensation

As was mentioned the inclusion of elastic elements may help to stabilize the unstable degrees of freedom of a permanent magnet device [175] or to provide the vertical gravitational force compensation fully or partially [148]. These elements may also act as positive spring which, combined with permanent magnets having negative stiffness, exhibit a high-static and low-dynamic stiffness [32, 140].
4.8.2 Active gravity compensation

Magnetic vibration isolation systems with active compensation of the gravitational force were already described in the beginning of the 19th century [111, 112]. Due to the vertical force being actively generated, the required current grows approximately linear with the mass, hence, the static power consumption grows quadratically. This makes these systems practically only suitable for levitation of small masses such as seen in [12, 136, 142, 163, 182].

4.8.3 Magnet-biased reluctance-based gravity compensation

The basic active isolation concept of [12] was further developed in [128] with a permanent-magnet biased reluctance to compensate the gravitational force of a 6-DoF isolated payload of several kilograms. This eliminated the need for active gravity compensation, which significantly reduced the system's power consumption. Reluctance-based gravity compensators with permanent-magnet bias have also been reported in [50, 62, 97, 132] in single-axis systems and in [10, 131, 136, 164] for multi-DoF systems. The reluctance topology exhibits very high force densities, although the downside of such actuators is their strongly nonlinear force/current/air-gap relationship which makes it hard to achieve high isolation performance and linearity in the working range.

A 3-DoF reluctance-based permanent-magnet biased device with infinite stiffness was proposed in [86, 127]. Both springs are placed in series and the resulting stiffness is semi-infinite, which is suitable for good compliance but not for transmissibility [189]. Although the addition of a mass support spring system reduces this problem as discussed in [86, 88], it makes the system complex to design and fabricate.

4.8.4 Ironless magnet-biased gravity compensation

Although the force density of ironless magnetic isolators is generally lower than their reluctance-based counterparts, they still offer many advantages in terms of e.g. low or no (nonlinear) hysteresis effects and reduced eddy currents, although their force density and magnetic shielding are generally less. Examples with single-DoF control are given in [36, 45, 137, 149, 158] and with multi-DoF control in [37]. Analyses on permanent magnet interaction devices, being either single-DoF or multi-DoF; in
4.9: Framework for the electromagnetic vibration isolation system

4.9: Framework for the electromagnetic vibration isolation system

Many authors [43, 51, 99, 160, 176] proposed topologies with a single or multiple magnets in an array which were designed such that the force, or force density, was maximized. Agashe and Arnold [2] published a study on scaling effects and force density differences between cuboidal and cylindrical magnets. A similar goal was pursued in [9], however, this paper also investigated the influence of the horizontal displacement. These papers all resulted in structures similar to those presented in Fig. 4.7.

As mentioned in Section 4.3.2 it is beneficial for transmissibility to exhibit a low undamped natural frequency instead of the high stiffness found in the topologies discussed above. Several authors investigated topologies with low stiffness, however often for applications with relatively limited force levels up to some hundreds of newtons [36, 37, 83, 132, 158]. Such low stiffness is often achieved by a combination of repulsion-based and attraction-based topologies, i.e. a vertically floating magnet is ‘sandwiched’ between two others as shown in Fig. 4.8.

Another approach, proposed in [36], places the floating magnet above two spaced permanent magnets instead of between them. Fig. 4.9(a) shows this topology. Its stiffness reduction capabilities are less intuitive than for the repulsion-attraction topologies described above and are discussed in Chapter 5. Vertical airgaps have been reported for axisymmetric systems [82, 83] where several concentric permanent-magnet rings interact in such a way that a vertical force is generated with low stiffness. A single permanent magnet in a vertical airgap topology was proposed in [37] and is shown in Fig. 4.9(b). It is claimed that this topology exhibits a four times larger force density due to the use of the quasi-Halbach magnetization pattern [77, 119].

4.9 Framework for the electromagnetic vibration isolation system

This thesis aims to develop a vibration isolator which exhibits low stiffness and a high isolation bandwidth to isolate advanced systems from vibrations. An ironless
topology has been chosen for the system for its hysteresis-free behavior and suitability to be described by the analytical models of Chapter 3. These three-dimensional, mesh- and boundary-free models are capable to describe such non-periodic structure with its position-dependent magnetic fields, energy and interactions. The aim is to achieve high force and force density levels as well as low stiffness and parasitic torque with the help of the conclusions drawn in Sections 4.7.

This thesis uses the metrological frame that is often encountered in an industrial application, such as seen in lithography, microscopy or telescopy, as a framework. It is a heavy platform with moving systems placed on top of it and is located in an environment where these devices may move with high accelerations and, for lithography, near or even in vacuum. Nevertheless, it is of utmost importance that the vibrations of this platform must remain within certain strict boundaries to guarantee an accurate operation of the devices mounted on top of it. Furthermore, the metrological frame acts as a reference frame for many of the systems in the machine. A qualitative overview of the properties that are pursued for such a high-performance system is given below. The objective values for force, stiffness, volume, etc. of the gravity compensator in this thesis are summarized in Section 5.2.

**High force** Many accurate systems exhibit heavy metrology frames which are used as references for all subsystems. Such a frame may weigh up to thousands of kilograms, depending on the specific application it is used for. In [52] it is stated that the mass in the isolated frame of the lithographic machine is approximately 1600kg whereas [184] states a total mass of approximately 3200kg. With three supporting units the mass per unit equals 1030kg. This thesis focuses on an electromagnetic vibration isolation for a mass of approximately 700kg. A system with three units would therefore isolate a mass of 2100kg. Although this is a scaled representation compared with [184] it is believed that the working principle of the system can be sufficiently demonstrated with this mass.

**Six degrees of freedom** Since vibrations and disturbances do not only exist in vertical direction, the isolation system should be capable of supporting and isolating the platform in six degrees of freedom. This is to be performed with three support struts which exhibit two degrees of freedom each.

**Low stiffness** A low stiffness reduces the undamped resonance frequency and in this
way results in a good transmissibility performance. The objective is to achieve a sub-hertz resonance frequency. The platform exhibits six degrees of freedom and therefore all entries in the stiffness matrix should have low values.

**Low passive damping** A damper is a velocity-dependent force between the two frames. Although some damping is necessary to suppress the system’s resonance frequency, it degrades the transmissibility performance as discussed in Section 4.3. As such, the passive damping should be minimized, and a form of active damping could be applied around the low-frequency resonance frequency.

**Small movement range** The movement range, or working envelope must be larger than that of the vibrational source. In the case of floor motion the necessary stroke is therefore in the order of a millimeter (not considering a possible earthquake). The small stroke also poses a design challenge; combined with the low envisaged stiffness and high mass, the resulting system should exhibit a very high vertical force of several kilonewtons which only has a very small variation in force over its range.

**High isolation bandwidth** A high isolation bandwidth of hundreds of hertz means better suppression of high-frequency vibrations. Especially for high-end production facilities, such as the lithographic machine, this enables the production of smaller features in the future at higher throughputs. Although it is difficult to achieve in reality, the various components are designed and selected for a theoretical isolation bandwidth of one thousand hertz.

**High positioning accuracy** Many applications, such as the metrological frame in lithography, require that the (low-frequency) positioning accuracy of system is very high. The position accuracy of the vibration isolation system in this thesis is defined at ±10 µm.

**Limited volume** As in almost any system, there is a limited volume constraint in which the isolation device should fit. This envisaged volume has been set to 300 mm along x, y and z. This constraint is close to the dimensions of the pneumatic mounts which are in use in many lithographic devices today.

**Low energy consumption** Energy consumption per produced volume is an ever more important design criterion for many systems. With this in mind the energy consumption of the electromagnetic vibration isolation system should be kept as low as possible, especially considering the high envisaged mass of the isolated platform.

**Low heat injection** The metrology frame isolated by the advanced isolation system provides a reference for many high-precision positioning and measurement systems. A heat injection, or temperature rise, of this platform could destroy the performance in the form of thermal expansion or other thermally induced effects. One of the challenges therefore is not only to minimize the required energy of the system as described above. The amount of heat that transfers to the isolated platform must be minimized too.
4.10 Conclusions

The analytical models that are extended and developed in Chapter 3 are very suitable to analyze and design a magnet-based vibration isolation system. This chapter describes the requirements and the framework for such a system. Three main vibration sources are identified and their influence on the vibration of an isolated platform are measured with the transmissibility and compliance transfer functions. These quantities are conflicting: for good transmissibility (floor vibration rejection) performance the natural resonance frequency and damping should be low. For good compliance (direct force rejection) a high undamped natural frequency and high damping are the properties to be pursued. Passive vibration isolation systems are inherently restricted to a compromise between these two performance functions. Active systems theoretically offer a better performance as they are not bound to the same compromise, however, may suffer from actively generated noise. Nevertheless, they are very attractive for applications with severe disturbances where the degree of vibration isolation must be high.

Electromagnetic vibration isolation systems are an alternative to commercially available mechanical, pneumatic or hydraulic isolation systems as a result of their high isolation bandwidth, low energy consumption and low noise injection. An investigation into the electromechanical objectives for vibration isolation systems shows that the force level is related to the magnetic field strength in the magnet arrays. A low stiffness is accomplished by a minimization of the field and its gradient around the magnet edges and the damping is related to the number of poles along the direction of movement. The utilization of the nonlinear behavior of such an electromagnetic topology enables an increase in force density whilst the stiffness is kept to a minimum.

A literature review has shown that only limited research has been conducted on electromagnetic vibration isolation systems and has been focused on either high-force, high-stiffness systems or low-force, low-stiffness systems. The combination of high force with low stiffness is investigated in this thesis. The analytical models of Chapter 3 are very suitable to accurately model the nonlinear behavior of a permanent-magnet based vibration isolator. As such, an advanced six-DoF ironless magnet-based vibration isolator, that could serve in industrial applications such as lithography, microscopy or telescopy, is investigated in this thesis.
Chapter 5

The passive electromagnetic gravity compensator

An investigation into the suitability of various gravity compensator topologies for the vibration isolation system. Its design is presented along with an analysis of modeling errors and manufacturing tolerances.

This chapter is based on:


The permanent-magnet based gravity compensator, or magnetic spring, is a key component of the electromagnetic vibration isolation system. To obtain a design that satisfies the requirements for the vibration isolator it is necessary to investigate the suitability of various topologies. A-priori insight into their electromechanical behavior is an aid to predict the performance of these topologies before optimization and to set some of the optimization constraints. As a result, the optimization is utilized to accelerate the design process instead of serving as a tool to find suitable topologies. The modeling errors and manufacturing tolerances discussed in Chapter 3 are individually evaluated for the final design of the gravity compensator. The results help to predict the minimum force that the realized device will produce, which is of importance for the design of the test setup.

5.1 Outline

The design objectives and constraints for the electromagnetic gravity compensator are discussed in Section 5.2. A basic analysis of some basic magnet topologies in Section 5.3 provides an estimation of their performance prior to an topology optimization. Section 5.4 investigates the suitability of topologies with a horizontal airgap, followed by an investigation into topologies with multiple horizontal gaps in Section 5.6. Section 5.7 presents the gravity compensator with vertical airgaps which meets the requirements for the vibration isolation system. In Section 5.8 the cross-shaped gravity compensator is presented, which reduces the parasitic torques that occur and is optimized in Section 5.9. Section 5.10 discusses an estimation of the modeling errors and manufacturing tolerances that can be expected and is followed by a discussion on methods to compensate them in Section 5.11. Finally, Section 5.12 presents the contributions, conclusions and recommendations of this chapter.

5.2 Design objectives

From the project objectives of Section 4.9 a number of constraints and design objectives have been derived which are summarized in Table 5.1. The pursued vertical force $F_z$ is higher than the 7.00 kN target of the previous chapter to anticipate on the modeling errors and manufacturing tolerances discussed in Chapter 3, which tend to reduce this vertical force in practice. The target stiffness is increased accordingly and applies to $K_{zz}$ as well as the other entries in the stiffness matrix. The vertical resonance frequency $f_r$ [Hz] is obtained from the vertical force $F_z$ and the stiffness $K_{zz}$ by using the gravitational acceleration $g$ to obtain the equivalent mass $m = gF_z$. This yields for the resonance frequency

$$f_r = \frac{1}{2\pi} \sqrt{\frac{K_{zz}g}{F_z}}$$  \hspace{1cm} (5.1)

where $g = 9.81 \text{ [m/s}^2\text{]}$ is the gravitational acceleration. The resonance frequency and force level given in Table 5.1 correspond to a stiffness of approximately 7.50 kN/m. The gravity compensator, the active actuators that are discussed in Chapter 6 and the
5.3 Analysis of simple topologies

Section 4.7 discusses the translation of the mechanical requirements into the magnetic domain. This section describes this translation in more detail for the envisaged electromagnetic vibration isolation system. Based on the 2D field of a single magnet the suitability of three basic topologies is discussed and verified [108]. The high force level in Table 5.1 and the resulting force density requirement suggest a high airgap flux according Section 4.7. However, the need for low stiffness complicates the topology choice, as the field gradients and field magnitudes around the magnet edges must be sufficiently small.

5.3.1 Magnetic field

Figure 5.1 shows the 2D equipotential lines outside a vertically magnetized permanent magnet. The width of this magnet is 40 mm and its height is 10 mm. The potential formulation that is used is the 2D form of (2.20) [66, pp. 211]. The magnitude of the magnetic flux density $\vec{B}$ is related to the density of these equipotential lines and their intermediate distance is a measure for the gradient.

As discussed in Section 4.7.1 the force is the result of an integration of the flux density over the surface charge density of a second magnet. As such, the magnitude and direction of the interaction force are related to these flux density components. Figure 5.2(a) and (b) show the 2D magnetic flux density components on the line indicated in Fig. 5.1. The airgap between this line and the magnet is varied between 0 mm and 6 mm. At small airgap values, the vertical flux density component in Fig. 5.2(a) maximizes above the surface of the magnet and changes sharply around the magnet edges. Fig. 5.3(a) shows a basic topology with equally sized magnets that

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical force</td>
<td>$\geq 7.50, \text{kN}$</td>
</tr>
<tr>
<td>Resonance frequency</td>
<td>0.5 Hz</td>
</tr>
<tr>
<td>Degrees of Freedom DoF</td>
<td>6</td>
</tr>
<tr>
<td>Maximum width and maximum length</td>
<td>300 mm</td>
</tr>
<tr>
<td>Maximum height</td>
<td>300 mm</td>
</tr>
<tr>
<td>Maximum volume</td>
<td>$27.0 \cdot 10^6, \text{mm}^3$</td>
</tr>
<tr>
<td>Displacement volume envelope</td>
<td>$1, \text{mm} \times 1, \text{mm} \times 1, \text{mm} ,(x, y, z)$</td>
</tr>
</tbody>
</table>
Figure 5.1: The equipotential contours of a permanent magnet modeled in 2D. The magnet is colored grey and is magnetized along the vertical axis. The thick black line is used to evaluate the field results in Fig. 5.2.

Figure 5.2: The (a) vertical and (b) horizontal flux density components above the magnet on the thick horizontal line in Fig. 5.1. The distance between this line and the magnets is indicated by ‘airgap’.

are aligned, which yields the integration limits summarized in Table 5.2. As these enclose this broad peak in flux density, such a topology maximizes the vertical force according (4.5). The horizontal field component in Fig. 5.2(b) is odd-symmetric and as a result the integral yields no net horizontal force component. A vertical displacement (increased airgap in Fig. 5.2) yields a strong change in the flux density enclosed by the integration limits, especially for small airgap values, which results in a high stiffness. Only for large intermediate distances, hence, reduced field and thus force levels, the field gradients decrease to such a level that the stiffness is minimal. As such, this commonly used topology is prone to a strong coupling between position and force. Although an increase of the airgap length reduces these gradients it simultaneously impairs the force density level.

In the topology of Fig. 5.3(b) the top permanent magnet is displaced horizontally,
5.3. Analysis of simple topologies

Figure 5.3: Possible topologies with (a) two magnets on top of each other, (b) horizontally displaced magnets and (c) unequal magnet sizes.

<table>
<thead>
<tr>
<th>Topology</th>
<th>Lower limit</th>
<th>Upper limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fig. 5.3(a)</td>
<td>$x = -20$ mm</td>
<td>$x = 20$ mm</td>
</tr>
<tr>
<td>Fig. 5.3(b)</td>
<td>$x = 0$ mm</td>
<td>$x = 40$ mm</td>
</tr>
<tr>
<td>Fig. 5.3(c)</td>
<td>$x = -10$ mm</td>
<td>$x = 10$ mm</td>
</tr>
</tbody>
</table>

hence the force integration limits become $0 \leq x \leq 40$ mm as Table 5.2 summarizes. The integration of the vertical flux density yields a reduced force compared to Fig. 5.3(a) as the flux density over which is integrated is lower. That of the horizontal flux density in Fig. 5.3(b) induces a horizontal force component as there is no longer an odd symmetry of the integrand. Especially the horizontal stiffness is expected to be low: the integral (4.5) has a low sensitivity to horizontal displacement due to the low flux density values and gradients at the integration limits. The vertical force does not show such low stiffness due to the large vertical flux density component at the integration limit $x = 0$ which influences its position dependency.

Another method to reduce the stiffness is shown in Fig. 5.3(c) in which the top magnet has reduced horizontal dimensions. It combines the purely vertical force of Fig. 5.3(a) with the reduced stiffness of Fig. 5.3(b). Although the vertical force has reduced, due to the narrower integration limits summarized in Table 5.2, the integration limits of the vertical field component in Fig. 5.2(a) remain within the relatively constant field region. The integral (4.5) is therefore relatively insensitive to horizontal displacements. The sensitivity of both field components to the airgap length has reduced too and as such it is expected that this topology exhibits significantly lower stiffness values compared to that of Fig. 5.3(a).

5.3.2 Interaction force

The predictions above, which are based on an analysis of the 2D magnetic field, are verified with the 3D analytical models of Chapter 3 for two permanent magnets. Figure 5.4(b) shows the three force components when the two $40 \times 40 \times 10$ mm magnets of Fig. 5.4(a), placed directly above each other, (the airgap length is 0 mm) are moved along the $x$ direction relative to each other. At $x = 0$ mm in Fig. 5.4(b) the topology of Fig. 5.3(a) is represented. Figure 5.4(c) shows the influence of a vertical movement for this configuration. The topology in Fig. 5.3(b) is obtained at $x = 20$ mm in Fig. 5.3(b). Fig. 5.4(d) shows the force components for a vertical movement of this topology.
Chapter 5: The passive electromagnetic gravity compensator

Figure 5.4: When two permanent magnets (a) are horizontally moved horizontally above each other a force characteristic as shown in (b) is obtained ($z = 0$ mm). The force in (c) results from the vertical movement for $x = 0$ mm and (d) from vertical movement for $x = 20$ mm.

As predicted, the topology without horizontal displacement exhibits the higher vertical force of the two topologies as is observed from Fig. 5.4(c) for $z = 0$ mm. However, both Fig. 5.4(b) Fig. 5.4(c) show that it exhibits a high sensitivity to displacements. The amplitude of the horizontal force component $F_x$ at $x = 20$ mm is significantly lower. However, its stiffness to vertical and to horizontal displacements is has reduced too, as Fig. 5.4(b) Fig. 5.4(d) show. Despite the reduced force levels, it seems that the topology with horizontally displaced permanent magnets is more promising than that with vertically centered magnets. The following sections investigate this hypothesis.

Figure 5.5(a) shows the topology with unequal arrays. The top magnet’s horizontal dimensions have been reduced to $20 \times 20 \times 8$ mm. It is shown in Fig 5.5(b) that the dependency of both force components on horizontal movement is low, although the force level has reduced too. The same behavior is observed in Fig 5.5(c) that shows the vertical force for $x = 0$. For small vertical displacements the stiffness is very low and may even become negative if the magnet dimensions are chosen well. As such, this basic topology seems suitable to achieve the low stiffness that is envisaged in this thesis, although the force level is compromised.

From the analysis above it is concluded that the topologies in Figs. 5.3(b) and 5.3(c) seem the most promising for a low-stiffness magnetic gravity compensator. A parametric search into their optimal dimensions would be an extremely time consuming process due to the large number of variables.
The following sections describe the extensive investigation into various topologies that has been performed using constrained nonlinear optimization. The results confirm the predictions regarding the suitability of the three studied topologies.

### 5.4 Single-airgap topologies with equal arrays

Repulsion-based gravity compensators, such as the two facing permanent magnets schematically shown in Fig. 5.6 are intuitive topologies for the gravity compensator. In this orientation, with the displacement between two permanent magnets being parallel to their magnetization vectors, the force between these two magnets is the largest one. However, as discussed before, this large force may come at the cost of stiffness and resonance frequency.

#### 5.4.1 Two permanent magnets

Several authors have investigated magnetic suspension systems which utilize the repulsion force between two permanent magnets to provide a vertical gravity compensation force [43, 160, 176]. However, the focus in these publications is often solely on force level and not on stiffness. Figure 5.6 shows a topology with two magnets in which the pole pitch \( \tau_p [\text{m}] \) equals the magnet pitch \( \tau_m [\text{m}] \), the height of both magnets is \( h_m [\text{m}] \) and the airgap length is \( h_g [\text{m}] \).
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Figure 5.6: Schematic (a) 3D view and cross-section with dimensions of the gravity compensator with (b) two repelling and (c) two attracting permanent magnets.

Topology optimization

A topology optimization provides more insight into the limits of this configuration. The topology of Fig. 5.6(b) is optimized using a nonlinear constrained multi-variable optimization (the sequential quadratic programming or SQP in Matlab’s fmincon) method to find a vector of variables \( \bar{x}^* \) which obeys

\[
\bar{x}^* = \min_{\bar{x} \in \bar{X}} \{ g_K(\bar{x}) \mid \bar{c}(\bar{x}) \leq 0, \bar{c}_{eq}(\bar{x}) = 0, \bar{x} \in \mathbb{R}^n \}, \tag{5.2}
\]

The same optimization algorithm is used for the optimizations described in the remainder of this chapter. The nonlinear inequality constraints are described by \( \bar{c}(\bar{x}) \) and the nonlinear equality constraints by \( \bar{c}_{eq}(\bar{x}) \). They are described below. The objective function that is minimized \( g(\bar{x}) \) is a function of the vector of variables \( \bar{x} \) which contains the optimization variables. These are found in Fig. 5.6(b). According \( \bar{x} \in \bar{X} \subset \mathbb{R}^n \) this vector is subject to linear constraints

\[
\bar{x} = \begin{bmatrix} \tau_p/250 \text{mm} \\ h_m/\tau_p \\ h_g/15 \text{mm} \end{bmatrix}, \quad \begin{bmatrix} 0.5 \\ 0.2 \\ 0.1 \end{bmatrix} \leq \bar{x} \leq \begin{bmatrix} 1 \\ 1 \\ 10 \end{bmatrix}. \tag{5.3}
\]

The pole pitch \( \tau_p \) is related to a maximum width \( w_{\max} [\text{m}] \) of 250 mm. This value is lower than that observed in Table 5.1 as some room has been reserved for the structural support. The magnet height \( h_m \) is related to this pole pitch and constrained such that it does not become larger than this pitch. The airgap length \( h_g \) is related to 15 mm to bring the three components of \( \bar{x} \) into the same order of magnitude and has a minimum of 1.5 mm according the working range in Table 5.1 added with 1 mm of margin.

For ease of comparison between different topologies the objective function minimizes stiffness and the force becomes a nonlinear inequality constraint. As such, the stiffness, or resonance frequency according (5.1), are minimized for fixed force levels. The objective function \( g_K(\bar{x}) \) is given by the vertical stiffness \( K_{zz}(\bar{x}) \) divided by 1.00 kN/m;

\[
g_K(\bar{x}) = \frac{K_{zz}}{1.00 \text{kN/m}}. \tag{5.4}
\]
The nonlinear equality constraint \( \vec{c}_{eq}(\vec{x}) \) applies to the vertical force \( F_z(\vec{x}) \) which must equal the target force which is varied between 1 kN and 10 kN in steps of 500 N

\[
\vec{c}_{eq}(\vec{x}) = [F_t - F_z(\vec{x})] = 0, \quad F_t = 500i, \quad i = 2 \ldots 20. \tag{5.5}
\]

The nonlinear inequality constraint vector \( \vec{c}(\vec{x}) \) is given by

\[
\vec{c}(\vec{x}) = \begin{bmatrix} 1.00 \text{kN/m} - K_{zz} \\ \frac{1.00 \text{kN/m}}{(2h_m + h_g) - 250 \text{mm}} \end{bmatrix} \leq 0. \tag{5.6}
\]

The first term ensures that the vertical stiffness remains positive and above 1.00 kN/m. This stiffness corresponds to a resonance of 0.5 Hz at a force of 1.00 kN according (5.1). The variables \( h_m \) and \( h_g \) are shown in Fig. 5.6(b) and the maximum spring height of 250 mm is derived from Table 5.1.

**Optimization results**

The results of the optimization form a front which is shown in Fig. 5.7. The stiffness is minimized for a fixed force level in each point and the resonance frequency is obtained using (5.1). As such, any optimization result with this topology is located somewhere in the ‘Solution space’ in Fig. 5.7 and not in the ‘Infeasible space’. It can be seen that this 1-by-1 topology produces a vertical force of 7.50 kN with a resonance frequency of 1.9 Hz. For any simulated force level its resonance frequency does not meet the target frequency set in Table 5.1.

Table 5.3 shows the optimization results for force levels of 1.00 kN and 7.50 kN, respectively. From these data it can be observed that the maximum volume of 250 \( \times \) 250 \( \times \) 250 mm is almost fully occupied in both situations. Especially the 1.00 kN topology requires a large airgap of 130 mm to minimize the stiffness. This clearly induces a high leakage of magnetic flux as can also be observed from the very poor force density levels. A reduced airgap improves this force density [99], but simultaneously leads to a high stiffness and resonance frequency. The sensitivity of the resonance to this airgap length has not been investigated. Manufacturability becomes an issue with the magnet dimensions from Table 5.3, as well as the integration of the actuators. As such, this rather straight-forward topology is infeasible for the envisaged vibration isolation system, according the conclusions of Section 5.3.

### 5.4.2 Checkerboard arrays

The poor force density of the topology described in the previous section has been acknowledged by various authors who have investigated linear alternating arrays (containing \( 1 \times n \) magnets) [159] or square arrays (containing \( n \times n \) magnets) [99], such as those shown in Fig. 5.8(a). This configuration is hereafter referred to as the checkerboard array.

Figure 5.8 shows dimensional variables for a topology in which the number of poles along the horizontal directions \( n \) equals three. The cross-section in Fig. 5.8(b) with the
Figure 5.7: Optimization results for the topology of Fig. 5.6. The force is varied with steps of 500 N and the stiffness is minimized. The front is composed of the optimization results and shows the achievable resonance frequency for each force level.

Table 5.3: Optimization results for 1.00 kN and 7.50 kN.

<table>
<thead>
<tr>
<th>Description</th>
<th>Variable</th>
<th>Value</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( F_z = 1.00 \text{kN} )</td>
<td>( F_z = 7.50 \text{kN} )</td>
</tr>
<tr>
<td>Stiffness</td>
<td>( K_{zz} )</td>
<td>11.8 kN/m</td>
<td>111.9 kN/m</td>
</tr>
<tr>
<td>Resonance frequency</td>
<td>( f_r )</td>
<td>1.72 Hz</td>
<td>1.92 Hz</td>
</tr>
<tr>
<td>Magnet pitch</td>
<td>( \tau_m )</td>
<td>248.2 mm</td>
<td>250.0 mm</td>
</tr>
<tr>
<td>Magnet height</td>
<td>( h_m )</td>
<td>48.2 mm</td>
<td>97.7 mm</td>
</tr>
<tr>
<td>Airgap height</td>
<td>( h_g )</td>
<td>130.6 mm</td>
<td>45.5 mm</td>
</tr>
<tr>
<td>Total volume</td>
<td>( V )</td>
<td>1.4 \cdot 10^6 \text{mm}^3</td>
<td>15.1 \cdot 10^6 \text{mm}^3</td>
</tr>
<tr>
<td>Force density</td>
<td>( f )</td>
<td>71.4 kN/m$^3$</td>
<td>498.0 kN/m$^3$</td>
</tr>
</tbody>
</table>

dimensional variables shows the \( zx \)-plane and is equal for the \( yz \)-plane. The arrays are full-pitch, hence, \( \tau_p = \tau_m \). The magnet height is again represented by \( h_m \) and the airgap length by \( h_g \). A parametric search into the achievable force and force densities of such checkerboard magnet arrays has been conducted in [99]. Although the force density levels found in [99] reach maxima of 6.6 MN/m$^3$, the stiffness levels have such high values that an additional minimization is required, although this is at the cost of force and force density.

**Topology optimization**

The optimization for this checkerboard pattern topology is defined by

\[
\tilde{x}^* = \min_{\tilde{x} \in \tilde{X}} \{ \varphi_K(\tilde{x}) \mid \tilde{z}(\tilde{x}) \leq 0, \tilde{c}_{eq}(\tilde{x}) = 0, \tilde{x} \in \mathbb{R}^n \},
\]

(5.7)
5.4: Single-airgap topologies with equal arrays

![Diagram of gravity compensator with planar checkerboard arrays](image)

Figure 5.8: Schematic (a) 3D view and (b) cross-section with dimensions of the gravity compensator with planar checkerboard arrays. This figure exhibits three poles in each horizontal direction, hence \( n = 3 \).

with

\[
\begin{align*}
\varphi_K(\vec{x}) &= \frac{K_{zz}}{1.00 \text{kN/m}}, \\
\vec{x} &= \begin{bmatrix} nt \tau_p/250 \text{mm} \\ h_m/\tau_p \\ h_g/15 \text{mm} \end{bmatrix}, \\
0.2 \leq x \leq \begin{bmatrix} 1 \\ 1.1 \\ 5 \end{bmatrix}, \\
\vec{c}_{eq}(\vec{x}) &= F_t - F_z(\vec{x}) = 0, \\
\vec{c}(\vec{x}) &= \begin{bmatrix} 1.00 \text{kN/m} - K_{zz} \\ 1.00 \text{kN/m} \\ (2h_m + h_g) - 250 \text{mm} \end{bmatrix} \leq 0.
\end{align*}
\]

The number of magnets in each direction, \( n \), is varied between 3 and 11 in steps of two although it is not necessarily an odd number. It is introduced in the first component of \( \vec{x} \).

The optimization results in Fig. 5.9 show the inverse relationship between number of magnets and the resonance frequency: an increase in \( n \) leads to a higher resonance frequency, given the same force level. Table 5.4 shows some key properties of the gravity compensators with \( n \in [3,5,9] \) at the rated force of 7.50 kN. Compared to the results in Section 5.4.1 the resonance frequencies are higher but the volumes in which the forces are generated are significantly lower. This is beneficial in terms of force density, flux leakage and available volume for the active actuators.

At a force level of 7.50 kN the \( n = 9 \) topology exhibits a resonance frequency which
is approximately twice that of the \( n = 3 \) topology, although the same vertical force is produced in less than a third of the volume. This is a result of the smaller pole pitch \( \tau_p \) that is exhibited by topologies with higher pole numbers, as was already observed in [99] in terms of force density. A reduction of this pitch focuses the magnetic flux in a smaller region and in this way the force rises. As the magnets become smaller, the gradient of their individual and collective magnetic field rises. As a result, the distance between the arrays must be decreased to achieve the same vertical force with a given magnet surface. Table 5.4 shows that the airgap length \( h_g \) indeed decreases as the number of poles increases. Due to this increased gradient of the field, the stiffness and with it the resonance frequency rise. This is in agreement with the conclusions of Section 5.3.

Although the force density of this topology is larger than a simple topology with only two permanent magnets, it does not meet the required specifications. An intermediate topology, with checkerboard arrays exhibiting a spacing between the
magnets ($\tau_m < \tau_p$) would be a hybrid between the $n = 1$ arrays and the $n > 1$ arrays. However, such topology would still be subject to the field gradient problem discussed in Section 5.3. It is not likely that this would increase the performance and as such it is considered necessary to investigate other topologies.

### 5.4.3 The planar quasi-Halbach array

The quasi-Halbach array [77, 119] may be considered more suitable for magnetic springs, as this topology focuses the magnetic fields towards the airgap, hence a high force and magnetic shielding [99, 157, 159]. Its (ideal) sinusoidal magnetization is approximated with two permanent magnets per pole pitch as shown in Fig. 5.10(a). A preliminary investigation performed in [99] shows an increase in force density by as much as 40% with respect to the checkerboard array. However, considering the results of the previous section, an increase of stiffness is a serious risk of this topology, as the magnetic fields are even more focused and therefore increase the position dependency of the field through the magnets.

Figure 5.10(b) shows that the magnet pitch $\tau_m$, the width of the vertically oriented permanent magnets, is smaller than $\tau_p$ as was the case above. The special case where $\tau_m = \tau_p$ would yield the checkerboard array.
Topology optimization

The optimization is defined by

$$\hat{x}^* = \min_{\vec{x} \in \mathbb{X}} \{ g_K(\vec{x}) | \vec{c}(\vec{x}) \leq 0, \vec{c}_{eq}(\vec{x}) = 0, \vec{x} \in \mathbb{R}^n \}, \quad (5.12)$$

with

$$g_K(\vec{x}) = \frac{K_{zz}}{1.00 \text{kN/m}}, \quad (5.13)$$

$$\vec{x} = \begin{bmatrix} n \tau_p / 250 \text{mm} \\ \frac{\tau_m}{\tau_p} \\ \frac{h_m}{\tau_p} \\ h_g / 15 \text{mm} \end{bmatrix}, \quad 0.5 \leq x \leq \begin{bmatrix} 1.5 \\ 0.8 \\ 4 \end{bmatrix}, \quad (5.14)$$

$$\vec{c}_{eq}(\vec{x}) = F_t - F_z = 0, \quad F_t = 500 i, \quad i = 2 \ldots 20, \quad (5.15)$$

$$\vec{c}(\vec{x}) = \begin{bmatrix} 1.00 \text{kN/m} - K_{zz} \\ \frac{1.00 \text{kN/m}}{n \tau_m + (n - 1)(\tau_p - \tau_m) - 250 \text{mm}} \\ \frac{1.00 \text{kN/m}}{(2h_m + h_g) - 250 \text{mm}} \\ \frac{1.00 \text{kN/m}}{1.1 \tau_m - h_m} \end{bmatrix} \leq 0. \quad (5.16)$$

The pitch ratio $\tau_m / \tau_p$ is added to $\vec{x}$ as, opposed to the previous topologies, the quasi-Halbach topology is not full-pitch. The minimum stiffness and maximum height constraint in $\vec{c}(\vec{x})$ are no longer sufficient. An additional nonlinear width constraint is introduced as well as a constraint that limits the magnet height to no more than $1.1 \tau_m$. These constraints could not be implemented as linear constraints as a result of the variables in $\vec{x}$.

Figure 5.11 shows the characteristic for topologies with $n \in \{2, 3, 4, 5, 9\}$ poles and Table 5.5 shows some key properties of the topologies at 7.50kN. The figure demonstrates that the capabilities of this topology to achieve a lower stiffness have hardly improved with respect to the planar checkerboard array topology. This is confirmed in Table 5.5 which summarizes some optimized properties. Compared with those obtained for the checkerboard array in Table 5.4 it shows that the quasi-Halbach topology performs marginally better than the checkerboard topology. The stiffness and the resonance frequency are somewhat lower for the quasi-Halbach topology and are produced within a marginally smaller volume. The quasi-Halbach array only partially fills this volume with hard-magnetic material as opposed to the checkerboard topology. In terms of force per magnet volume the Halbach array therefore is more efficient and less expensive. Nevertheless, the Halbach topology too is subject to a trade-off between volume and resonance frequency and seems incapable of exhibiting the desired properties within the set constraints. Furthermore, it is more difficult to be manufactured.

The investigations above show that the checkerboard topology and the quasi-Halbach topology with a single horizontal airgap are limited in producing simultaneously a high vertical force and a low vertical stiffness, which corresponds to the
5.5 Arrays with unequal dimensions

In most investigations on magnetic suspension devices all permanent magnets and the arrays containing them are given the same dimensions and spacing, i.e. both arrays are geometrically equal \[36, 51, 99, 149, 157, 160\]. If the high forces that are obtained need to be combined with low stiffness it may be more optimal to use differently sized magnets \[176\] or magnet arrays.
Chapter 5: The passive electromagnetic gravity compensator

Choi et al. [36] uses three equally sized permanent magnets in unequal arrays to reduce the stiffness. The proposed topology, shown in Fig. 5.12(a) places a single magnet between and above two oppositely magnetized permanent magnets, thus creating a ‘low stiffness well’.

The design goal in [36] is 37.5N which is two orders of magnitude below the force levels envisaged in this thesis. Further, the topology of Fig. 5.12(a) exhibits an asymmetry between its behavior along $\hat{e}_x$ and $\hat{e}_y$. An investigation in [110] shows that this topology exhibits improved properties with respect to the previously investigated topologies.

5.5.1 Fractional pitch checkerboard array

To obtain the same behavior along $\hat{e}_x$ and $\hat{e}_y$ the topology of Fig 5.12(a) is expanded into both directions. This yields a single top magnet above four magnets on the bottom of Fig. 5.12(b). These four magnets are merged into a single bottom magnet which reduces the number of magnets in the bottom array by a factor four as shown in Fig. 5.12(c). Figure 5.13 shows the unequal checkerboard array that is investigated for the case where $n = 3$.

The bottom magnet array is full-pitch ($\tau_{m_1} = \tau_{p_1}$) and the upper array is fractional pitch ($\tau_{m_2} < \tau_{p_2}$). Further, both pole pitches, $\tau_{p_1}$ and $\tau_{p_2}$, are not necessarily equal and the magnet height $h_{m_2}$ may vary with respect to that of the bottom array, $h_{m_1}$.

Topology optimization

With the dimensions of Fig. 5.13(b) the optimization is defined by

$$\overline{x}^* = \min_{\overline{x} \in \overline{X}} \left\{ \mathcal{G}_K(\overline{x}) \mid \overline{c}(\overline{x}) \leq 0, \overline{c}_{eq}(\overline{x}) = 0, \overline{x} \in \mathbb{R}^n \right\},$$

with

$$\mathcal{G}_K(\overline{x}) = \frac{K_{zz}}{1.00 \text{ kN/m}^2},$$

(5.17)
5.5: Arrays with unequal dimensions

Figure 5.13: Schematic (a) 3D view and (b) cross-section with dimensions of the gravity compensator with unequal planar checkerboard arrays. This example has three pole pitches in each horizontal direction, hence \( n = 3 \).

\[
\vec{x} = \begin{bmatrix}
    nh_{m1}/250 \text{mm} \\
    h_{m2}/h_{m1} \\
    \tau_{p2}/\tau_{p1} \\
    \tau_{m2}/\tau_{p2} \\
    h_g/15 \text{mm}
\end{bmatrix}, \quad \begin{bmatrix}
    0.07 \\
    0.9 \\
    0.75 \\
    0.4 \\
    0.1
\end{bmatrix} \leq \vec{x} \leq \begin{bmatrix}
    1 \\
    1.1 \\
    1.25 \\
    0.95 \\
    1
\end{bmatrix}, \quad (5.19)
\]

\[
\vec{c}_{eq}(\vec{x}) = F_t - F_z = 0, \quad F_t = 500i, \quad i = 2 \ldots 20, \quad (5.20)
\]

\[
\vec{c}(\vec{x}) = \begin{bmatrix}
    \frac{1.00 \text{kN/m} - K_{zz}}{1.00 \text{kN/m}} \\
    \frac{(2h_m + h_g) - 250 \text{mm}}{250 \text{mm}}
\end{bmatrix} \leq 0, \quad (5.21)
\]

\[
\tau_{p1} = \frac{250 \text{mm}}{n}, \quad \tau_{m1} = \tau_{p1}. \quad (5.22)
\]

It is a design choice, based on optimization results of preliminary investigations, that the bottom array is full pitch, hence \( \tau_{m1} = \tau_{p1} = 250 \text{mm}/n \text{mm} \). The vector of variables \( \vec{x} \) has changed form with respect to the previous optimizations. The first term in (5.19) normalizes the magnet height \( h_{m1} \) to the pole pitch \( \tau_{p1} = 250 \text{mm}/n \text{mm} \) and the second determines the ratio of both magnet heights. The third term describes the ratio between both pole pitches and the fourth describes the pitch ratio of the top array. The last term investigates the airgap height \( h_g \).
The results of the optimization for this topology are shown in Fig. 5.14. A first comparison with the characteristics of Fig. 5.9 and Fig. 5.11 shows that this topology performs significantly better than the other two topologies. The minimum resonance frequency for particular pole number \( n \) and given force \( F_t \) is lower and, for force levels up to 2.50 kN, even allows to achieve the 0.5Hz resonance target. Lower resonance frequency values are possible, however the stiffness constraint in (5.20) prevents this. The volume is used more efficiently as the airgap no longer needs to be increased to reduce the stiffness and even is minimized as observed in Section 5.3.2 and Table 5.6. Nevertheless, at elevated force levels it still seems not possible to fulfill the design specifications.

The results in Table 5.6 show some key properties at 7.50 kN for \( n \in (3, 5, 9) \). When compared to Table 5.4 and Table 5.5 it becomes clear that as well the respective resonance frequencies as the used volumes are significantly lower. The latter is due to the significantly reduced airgap lengths and the smaller magnet heights. A study of the results in Table 5.6 shows that the optimized pole pitch of the top array is almost equal to that of the bottom array. The same can be concluded for the array heights. The main difference between both arrays is therefore in the magnet pitch \( \tau_m \). It is therefore concluded that a future optimization of such array for minimal stiffness should equalize both pole pitches, magnet heights and exhibit a low airgap, which in this case has been restricted to a minimum of 1.5mm according the conclusions in Section 5.3.
5.6: Multi-airgap topologies

Table 5.6: Optimization results for \( n \in \{2,3,4,5\} \) at the rated force of 7.500 kN.

<table>
<thead>
<tr>
<th>Description</th>
<th>Name</th>
<th>( n=3 )</th>
<th>( n=5 )</th>
<th>( n=9 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiffness</td>
<td>( K_{zz} )</td>
<td>211.7 kN/m</td>
<td>390.1 kN/m</td>
<td>812.6 kN/m</td>
</tr>
<tr>
<td>Resonance frequency</td>
<td>( f_r )</td>
<td>2.65 Hz</td>
<td>3.60 Hz</td>
<td>5.19 Hz</td>
</tr>
<tr>
<td>Top pole pitch</td>
<td>( \tau_{p2} )</td>
<td>85.4 mm</td>
<td>50.5 mm</td>
<td>27.9 mm</td>
</tr>
<tr>
<td>Top magnet pitch</td>
<td>( \tau_{m2} )</td>
<td>52.0 mm</td>
<td>30.2 mm</td>
<td>15.9 mm</td>
</tr>
<tr>
<td>Top magnet height</td>
<td>( h_{m2} )</td>
<td>31.0 mm</td>
<td>19.5 mm</td>
<td>13.1 mm</td>
</tr>
<tr>
<td>Bottom pole pitch</td>
<td>( \tau_{p1} )</td>
<td>83.3 mm</td>
<td>50.0 mm</td>
<td>27.8 mm</td>
</tr>
<tr>
<td>Bottom magnet pitch</td>
<td>( \tau_{m1} )</td>
<td>83.3 mm</td>
<td>50.0 mm</td>
<td>27.8 mm</td>
</tr>
<tr>
<td>Bottom magnet height</td>
<td>( h_{m1} )</td>
<td>31.0 mm</td>
<td>19.5 mm</td>
<td>13.1 mm</td>
</tr>
<tr>
<td>Airgap height</td>
<td>( h_g )</td>
<td>1.5 mm</td>
<td>1.5 mm</td>
<td>1.5 mm</td>
</tr>
<tr>
<td>Total volume</td>
<td>( V )</td>
<td>( 3.96 \cdot 10^6 ) mm(^3)</td>
<td>( 2.54 \cdot 10^6 ) mm(^3)</td>
<td>( 1.73 \cdot 10^6 ) mm(^3)</td>
</tr>
</tbody>
</table>

5.6 Multi-airgap topologies

As opposed to the checkerboard and the quasi-Halbach topology, the design with unequal magnet arrays can exhibit a resonance frequency as low as 0.5 Hz. However, this criterion is only achieved at force levels below 2.50 kN and not at the envisaged force of 7.50 kN. Series- or parallel placement of such magnetic springs could be a feasible alternative to achieve these requirements as they combine magnetic springs which may individually not be suitable for the gravity compensator, but once combined exhibit the correct properties. This is discussed in mechanical terms in Section 4.5 and Appendix B and is discussed here for electromechanical magnetic springs.

5.6.1 Series topologies

By placing two or more magnetic springs in series a reduction in stiffness can be achieved. The diagram in Fig. 5.15(b) illustrates such series topology, of which the cumulative stiffness \( k_{zz} \) is obtained by

\[
\frac{1}{k_{zz}} = \frac{1}{k_1} + \frac{1}{k_2}.
\]

All springs bear the same full gravity force exhibited by the load, provided that the mass of the 'floating' permanent magnet is ignorable compared to the isolated mass.

Topologies with either repulsion or attraction The results of Table 5.6 show that to achieve e.g. 0.5 Hz at a force level of 7.50 kN it would be necessary to stack 28 units of \( n = 3 \), 52 units of \( n = 5 \) or even 107 units of \( n = 9 \) to achieve the desired stiffness of 7.50 kN/m. Obviously, this is infeasible with the given volume constraints. An additional problem that would occur is the stabilization of each platform individually, which is a practically impossible task. This is schematically shown in Fig. 5.15(a) for
two airgaps with repulsive forces only. The grey magnet is attached to the isolated platform, although the unstable middle magnet also requires stabilization by itself.

**Topologies with a combination of repulsion and attraction** An example of such topology with two airgaps is shown in Fig. 5.15(c). According to (5.24) the total stiffness rises significantly if the stiffness of both individual airgaps is opposite and equal in amplitude. This principle was used in [86–88, 127] for a system with semi-infinite stiffness for high compliance. For systems pursuing low- or zero-stiffness this series connection is not suitable.

### 5.6.2 Parallel topologies

By placing springs in parallel as shown in the diagram of Fig. 5.16(b) the vertical load force is distributed over the springs. Their respective stiffness is summed

$$k_{z_{\text{tot}}} = k_1 + k_2,$$  \hspace{1cm} (5.25)

where \( p \) is the number of series springs each with the stiffness \( k_o \). Hence, there is a gain in load force per magnetic springs, although at the cost of their individual stiffness which should be significantly lower.

**Topologies with either repulsion or attraction** Figure 5.16(a) schematically shows this parallel topology, where both airgaps are repulsion-based. All topologies discussed above belong to this category, as they contain \( n^2 \) of \( 1 \times 1 \) magnetic springs with two permanent magnets each. These magnetic springs are placed in large patterns with alternating magnetization to decrease flux leakage, hence, increase the force density. From Figs. 5.9, 5.11 and 5.14 it is observed that the resonance frequency increases with \( n \). This effect is observed in Fig. 5.17, which shows the force directly versus stiffness for the studied topologies instead of the resonance frequency. There seems to be an almost linear relation between the minimum stiffness and the vertical
5.6: Multi-airgap topologies

Figure 5.16: Parallel topologies of magnets (a) with two airgaps exhibiting repulsive forces, (b) an equivalent mechanical diagram and (c) an airgap with repulsive force in parallel with an airgap exhibiting attractive force.

Figure 5.17: Force versus vertical stiffness $K_{zz}$ of (a) the $n = 1$ topology, (b) the planar checkerboard array, (c) the planar quasi-Halbach topology and (d) the checkerboard topology with unequal arrays. Please note that contrary to previous figures the vertical axes show stiffness instead of resonance frequency to show the relation between force and stiffness instead of the resonance frequency.

force, given the constraints of the various optimizations. The derivative of this line depends on $n$ and all lines seem to originate from (0, 0). The topology with unequal arrays in Fig. 5.17(d) exhibits a semi-asymptotic behavior at low frequencies, which is the result of the nonlinear inequality constraint for the stiffness in (5.22).
From the results in Fig. 5.17 it becomes clear that placing two or more of these topologies in parallel is no feasible alternative to meet the requirements. Two planar 2.50 kN topologies in parallel produce the same passive force and stiffness as a single 5kN spring as a result of the linearity of the topology. The analysis in Section 5.3 confirms this, as all the parallel magnetic springs exhibit the same force and stiffness behavior, which equally sum and as such do not allow for a significant resonance frequency reduction.

**Topologies with a combination of repulsion and attraction**  According (5.25) the parallel placement of positive and negative springs (Fig. 5.16(b)) renders a system with reduced stiffness values. It is a method to let the position-dependency of force components of Section 5.3 cancel each other, which yields a low stiffness. As opposed to the topologies discussed above this solution seems feasible for magnetic gravity compensation at many kilonewtons and sub-hertz resonance frequencies.

The topology of Fig. 5.16(b), i.e. with one or two magnets per array, was studied in [132, 158, 158]. These publications focus on a limited force of tens of newtons and single-DoF movement. If the force level increases up to 10kN such a simple topology obviously does not suffice in terms of flux leakage, force density, etc. and a planar magnet array must be used as seen in [99]. The vertical force in such a topology generated between the magnets must be guided towards the floor and the isolated platform, respectively. As permanent magnets are a ceramic, brittle material which fails under severe compressive and small tensile stresses [180] this necessitates a support material. This support is utilized to guide the force acting on the magnets to the isolated platform and should be sufficiently stiff not to resonate in the frequency band of interest. Such structure could look like that in Fig. 5.18 which shows the cross-sections in the \( zx \) and \( yz \) planes. In Fig. 5.18(a) it is shown that downward forces to the floor are guided around the structure and in Fig. 5.18(b) how the upward forces on the levitated magnets are guided to the isolated platform. Such structure may pose design and manufacturing challenges when being built as well as the placement of integrated active elements.

Figure 5.19 shows simulation results for a structure, such as that in Fig. 5.16(b), with the assumption that the attraction-based (negative) spring and the repulsion-based (positive) spring are magnetically decoupled. Both characteristics exhibit an inverse exponential relationship between vertical force and vertical displacement. The horizontal axes in Fig. 5.19 show the vertical displacement of the isolated part in millimeters. In this topology the forces sum up and the stiffness of both parallel springs is canceled for zero displacement. Although the vertical stiffness is zero in the middle, this stiffness is position dependent and increases rapidly. At \( x = 1 \text{ mm} \) the stiffness is 6.5 kN/m which is high compared to the force level of 116 N and zero stiffness at \( x = 0 \text{ mm} \). A topology optimization in the full working range may reduce this position-dependency of the stiffness. Nonetheless, the magnet edges remain in a region with high flux density amplitudes and gradients, which locally induces high stiffness values as discussed in Section 5.3. A small manufacturing inaccuracy (for example, assembly tolerance) may cause that the cancelation of these high stiffness
Figure 5.18: When large forces are considered it is necessary to guide the force around the planar magnet arrays of the parallel springs. A solution is to use two square tubes for both frames as shown in (a) the \( z \times \) plane and (b) the \( x \times y \) plane.

Figure 5.19: When two repelling magnets of dimensions 20 mm \( \times \) 20 mm \( \times \) 10 mm are placed in parallel with a topology with equally sized magnets in attraction, (a) their forces sum up and (b) the stiffness amplitudes are subtracted.

values by the parallel topology is non-perfect. If these stiffness values are large, the residual stiffness that remains may be unacceptable.

An optimization of the multi-airgap topology to achieve a more linear behavior has not been performed here. It is therefore not exactly known what the opportunities and limitations of this particular topology are. However, the results of the preliminary investigation above have lead to the investigation of different topologies. A further investigation into these multi-airgap topologies is recommended for future research.

5.7 Vertical airgaps

Electromagnetic gravity compensators with vertical airgaps have not been researched as intensively as their counterparts with horizontal airgaps. Da Silva and Horikawa [44] propose a table with permanent attached to its lateral ends which is
located between two magnets fixed on a base frame as Fig. 5.20(a). The attraction force between the permanent magnets on each side results in a vertical force component. Another example is found in [37] in which a single permanent magnet, connected to a mechanical structure, exhibits a vertical force inside a Halbach array as Fig. 5.20(b) shows. At first sight such topologies seem intuitively less suitable to produce large forces: the force is oriented parallel to the airgap, instead of perpendicular, and in the examples above the main magnetization directions are horizontal too.

As observed from Figs. 5.4(b) and 5.4(d) the $F_x$ and $F_z$ components exhibit comparable force levels for this configuration. The total force is maximized if the vectorial sum of these components is taken. In this case, the two magnets should be rotated around the $y$-axis such that there is only a net vertical force component and zero horizontal force. Such rotation is not investigated in this thesis as it complicates the design and manufacturability of the envisaged gravity compensator.

### 5.7.1 Modularity and symmetry

As was shown in Section 5.6 it proves beneficial for the performance of the gravity compensator to place repelling and attracting magnetic springs in parallel to reduce the stiffness. In the topology with vertical airgaps this is performed by stacking permanent magnets vertically as shown in Fig. 5.21(a). The black force vectors indicate the $zx$-force between all individual permanent magnets and show that in each airgap their force vertical components sum up and horizontal components virtually cancel each other. As a result of the stacking, this topology is modular: the addition of an extra vertical layer yields an increase in force and the same relative increase in stiffness, hence, has almost no effect on the resonance frequency. The horizontal force components that occur as a result of horizontal movement are decreased by using two vertical airgaps, or gravity compensators, placed in parallel.

If the isolated middle structure of Fig. 5.21(a) moves along $x$, the horizontal force component on the right hand side will rise and that on the left hand side will decrease with an almost equal value. As a result, the total horizontal force hardly changes and thus the stiffness remains low. A similar effect is seen for the vertical force when moving horizontally: a rightward movement increases the vertical force on the right hand side and decreases it on the left side with almost the same amount. A such the
5.7: Vertical airgaps

Figure 5.21: Topology with (a) vertically stacked magnets implemented in two parallel gravity compensators and (b) a square topology with symmetrical behavior along \( x \) and \( y \). In (a) the magnetization vectors are grey and force vectors are black.

The checkerboard and the quasi-Halbach of Sections 5.4.2 and 5.4.3 perform almost equally. The former is less complicated in manufacturing and for this reason the latter is not further investigated in this thesis for suitability in vertical-airgap topologies.

5.7.2 Vertical checkerboard topology

The vertical checkerboard topology is schematically shown in Fig. 5.22. The arrays that are investigated are \( 1 \times n \), hence have only one pole in the horizontal direction. This reduces the stiffness for horizontal movement according the findings in Section 5.3. As shown in Section 5.3 this topology exhibits near-zero stiffness when the vertical offset \( z_0 = \tau_p/2 \text{[m]} \). As a resonance frequency of 0.5Hz is therefore achievable for any given force level, a research as performed in the previous sections is considered superfluous. The maximum force level that is achieved within the same constraints as the horizontal airgap topologies is 10.9kN.

Topology optimization

With the dimensions of Fig. 5.22(b) the optimization is defined by

\[
\bar{X}^* = \min_{\bar{X} \in \mathbb{X}} \{ \mathcal{G}_K(\bar{X}) \mid \bar{c}(\bar{X}) \leq 0, \bar{c}_{eq}(\bar{X}) = 0, \bar{X} \subset \mathbb{R}^n \},
\]

(5.26)

with

\[
\mathcal{G}(\bar{X}) = \frac{K_{zz}}{1.00 \text{kN/m}},
\]

(5.27)
Figure 5.22: 3D impression (a) of the vertical-airgap topology with a fixed magnetwidth and (b) the dimensions that define the topology.

\[
\vec{x} = \begin{bmatrix}
  nr_p/160\text{ mm} \\
  \tau_m/\tau_p \\
  h_m/\tau_m \\
  h_g/15\text{ mm} \\
  z_0/\tau_p
\end{bmatrix}
\begin{bmatrix}
  0.5 \\
  0.7 \\
  0.05 \\
  0.1 \\
  0.47
\end{bmatrix} \leq \begin{bmatrix}
  1 \\
  1 \\
  0.4 \\
  1 \\
  0.53
\end{bmatrix}, \quad (5.28)
\]

\[
c(\vec{x}) = 1 - \frac{K_{zz}}{7.50\text{kN/m}}, \quad (5.29)
\]

\[
e_{eq}(\vec{x}) = \frac{F_z}{7.50\text{kN}} - 1. \quad (5.30)
\]

The magnet width \(w_m = 250\text{ mm}\) and the height \(h_{\text{max}}\) has been reduced to 160\text{ mm}\ since four of these springs are used in parallel as Fig. 5.22(a) shows. A further minimization of this height has not been researched, although it is interesting in terms of material costs. To reduce the computational costs, which increase exponentially with the number of magnets, it is assumed during the optimization that the four parallel springs of Fig. 5.21(b) are magnetically decoupled. Hence, it suffices to optimize one of these springs and to use the superposition principle and coordinate rotation to include the other three. The optimized variables for a topology with \(n = 5\) are shown in Table 5.7.

Figure 5.23 shows a number of stiffness and force components for displacements through the working envelope. As opposed to the quarter model used in the optimization, the full topology is modeled here, with all sides included, and therefore with all magnetic cross-couplings. The volume envelope \(-1\text{ mm} \leq [x, y, z]^T \leq 1\text{ mm}\) is studied with eleven points along each direction. From Fig. 5.23(a) and (b) it becomes clear that the stiffness has a strong position dependency throughout the working range.
Table 5.7: Results for a \( n = 5 \) topology that has been optimized to \( F_z = 7.50 \text{kN} \) and \( f_r = 0.5 \text{Hz} \).

<table>
<thead>
<tr>
<th>Description</th>
<th>Variable</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pole pitch</td>
<td>( \tau_p )</td>
<td>26.1 mm</td>
</tr>
<tr>
<td>Magnet pitch</td>
<td>( \tau_m )</td>
<td>23.6 mm</td>
</tr>
<tr>
<td>Magnet height</td>
<td>( h_m )</td>
<td>8.3 mm</td>
</tr>
<tr>
<td>Airgap height</td>
<td>( h_g )</td>
<td>7.5 mm</td>
</tr>
<tr>
<td>Vertical offset</td>
<td>( z_0 )</td>
<td>13.2 mm</td>
</tr>
</tbody>
</table>

Figure 5.23: The stiffness and force behavior that results from an optimization in a single working point located at \((x, y, z)^T = 0\). Shown are (a) the stiffness \( K_{zz} \) as function of \((0, 0, z)^T\), (b) \( K_{zy} \) as function of \((0, y, 0)^T\), (c) \( F_x \) (d) \( F_z \) as function of \( x \) and \( y \) while the height is varied between \( z = -1 \text{ mm} \) (red) and \( z = 1 \text{ mm} \) (yellow).

The horizontal and vertical force components are affected by this stiffness variation as shown in Fig. 5.23(c) for \( F_x \) and Fig. 5.23(d) for \( F_z \). The horizontal force \( F_x \) hardly depends on \( y \) but strongly varies with \( x \) and \( z \). A similar variation of tens of newtons is observed for \( F_z \). For this reason, the gravity compensator has been optimized throughout its working envelope to minimize the overall stiffness instead of only in one point.
5.7.3 Minimization of the torque

Besides stiffness and force variations throughout the working envelope the torque is another property that is of importance. The equation for the torque vector $\vec{T}$ is repeated

$$\vec{T}(\vec{x}) = \int_{V} \vec{r}(\vec{x}') \times \vec{f}(\vec{x}') \, dV', \quad (5.31)$$

where $\vec{f}(\vec{x}')$ is the position dependent force density and $\vec{r}(\vec{x}')$ is the arm to a certain point. In this case this torque reference point is chosen in the middle of the gravity compensator at $(0, 0, 0)^T$. As the force density is difficult to change – it is related to the total force – the minimization of the arm is used to reduce the torque. In the horizontally aligned situation of Fig. 5.24(a) the vertical force component in both (equal) airgaps are equal. Under the assumption that all vertical force is produced in these airgaps the total torque is $\vec{r} \times \vec{F}_r = -\vec{r} \times \vec{F}_l$.

Figure 5.24(b) schematically shows the situation in which the floating structure is displaced horizontally. Both forces $F_l$ and $F_r$ change their magnitude in opposite directions as is seen from the results in Section 5.3. As these change are approximately equal in amplitude, the total vertical force $F_l + F_r$ remains approximately equal. This is a result of the changed airgap, which on the right hand side has decreased yielding a higher force and vice versa on the left hand side. Under the assumption that their arm $r$ towards the center of the gravity compensator remains approximately equal, the two torques differ according $\vec{r} \times \vec{F}_r > -\vec{r} \times \vec{F}_l$. A minimization of this difference leads to a low value of the net torque that is to be compensated by active means. Therefore it is concluded that the value of $r$, hence, the distance between the airgaps should be minimized to reduce this displacement torque.

One of the design objectives is to exhibit rotation symmetrical behavior along $\vec{e}_x$ and $\vec{e}_y$, and for this reason the topology should be rotational symmetric. This can be accomplished with one of the topologies of Fig. 5.25(a)-(d) where the behavior is symmetric around respectively $120^\circ$ for three legs (a), $90^\circ$ for four legs (b), $72^\circ$ for five legs (c) and $60^\circ$ for six legs (d). The use of six or even more legs as shown in Fig. 5.25(d) impairs the active volume available for the magnet arrays as the angle between the
5.8: Cross-shaped arrays

The topology with vertical airgaps and a reduced arm can be implemented in different ways. This figure shows gravity compensator topologies with (a) three, (b) four, (c) five and (d) six legs.

Figure 5.25: The topology with vertical airgaps and a reduced arm can be implemented in different ways. This figure shows gravity compensator topologies with (a) three, (b) four, (c) five and (d) six legs.

legs decreases. This thesis focuses on the topology of Fig. 5.25(b), referred to as the cross-shaped topology and is documented in a patent [107].

5.8 Cross-shaped arrays

The dimensional variables of the cross-shaped topology are shown in Fig. 5.22(b) and Fig. 5.26. The thickness of each leg of the support structure that is referred to as ‘inner cross’, as it has the shape of a solid cross, is defined by $d_i \text{[m]}$. The thickness of the support structure around the grey-colored magnets is referred to as ‘outer cross’ as it is cross-shaped too (Fig. 5.27). This material has a thickness $d_o \text{[m]}$. The variable $r_1 \text{[m]}$ represents the distance between the geometrical center of the leg and the back side of the corresponding magnet array, hence

$$r_1 = \frac{d_i}{2} + 2h_m + h_g.$$ (5.32)

The magnets are placed on the leg at a distance $r_1 + w_i$ from this geometrical center, where $w_i \geq 0$. The width of the magnet in each leg is given by $w_m$ and exhibits a separation $w_o \text{[m]}$ to the support structure of the outer cross. This separation is necessary to allow for the horizontal movement. The variable $r_2 \text{[m]}$ is half the width of the gravity compensator and equals

$$r_2 = r_1 + w_i + w_m + w_o + d_o.$$ (5.33)

The maximum outer dimensions of the gravity compensator, summarized in Table 5.1, are 300 mm-by-300 mm. With a margin of 10 mm on each side this results in a constraint for $r_2$.

$$r_2 = \frac{d_i}{2} + 2h_m + h_g + w_i + w_m + w_o + d_o \leq 140 \text{ mm}.$$ (5.34)

5.8.1 Optimization of a single leg

The initial optimizations in this thesis are performed with a topology in which the four legs are assumed to be magnetically decoupled. Although this introduces an additional modeling error it significantly reduces the computational effort as this
Figure 5.26: The dimensions of the cross-shaped topology in a top view. The remaining variables are defined in Fig. 5.22(b).

Figure 5.27: Coordinate rotation and superposition enables the modeling of only one of the four legs to reduce the computational effort.

effort is related to the square of the number of magnets. Especially in an early stage of optimization such simplification allows for a quick analysis of many topologies. As a result, the analytical model incorporates only one leg with two parallel springs. Any horizontal or vertical displacements are modeled with appropriate coordinate rotation. This is shown in Fig. 5.27 where each leg has been numbered. From the interaction results obtained for leg 1 the contribution of the other legs has been obtained by coordinate rotation from these results.

The combined force $\vec{F}_{\text{tot}}(\vec{x})$ and stiffness $K_{\text{tot}}(\vec{x})$ of the four magnetically decoupled
5.8: Cross-shaped arrays

![Figure 5.28](image)

**Figure 5.28:** It is possible to give (a) the second array one vertical pole less than the first array or (b) give both arrays an equal number of poles.

legs are obtained by

\[
\tilde{F}_{\text{tot}}(\tilde{x}) = \sum_{i=1}^{4} M_i^T \tilde{F}(M_i \tilde{x}) ,
\]

(5.35)

\[
K_{\text{tot}}(\tilde{x}) = \sum_{i=1}^{4} M_i^T (K(M_i \tilde{x}) M_i) ,
\]

(5.36)

where \(M_{1...4}\) are the respective transformation matrices for each leg as derived in Section 3.7.1 and shown in Fig. 5.27.

Besides the continuous dimensional variables that have been used in the optimization there are some discrete properties that also require investigation. Such properties are the use of unequal magnet arrays, symmetry of the legs and the selection of the permanent-magnet material which are discussed below.

### 5.8.2 Unequal magnet arrays

An intrinsic property of the proposed vertical-airgap topology is the vertical displacement of approximately \(\tau_p/2\) of one of the permanent magnet arrays with respect to the other. However, the number of poles in the vertical direction is not necessarily equal for both arrays, as Fig. 5.28 shows. Both topologies generate a vertical force with low stiffness, but it is difficult to predict which of the two is the most suitable.

### 5.8.3 Magnetic symmetry

There is magnetic coupling between the parallel magnetic springs in each leg as a result of the limited thickness \(d_i\). An even symmetry, such as that shown in Fig. 5.29(a), places the magnetic fields of both gravity compensators 'in series', with normal fields through the symmetry plane in the middle. The odd symmetry of Fig. 5.29(b) puts the springs magnetically opposite, hence, minimizes flux through the symmetry axis. Such even or odd symmetry may alter the characteristics of the gravity compensator and as such have been investigated.
5.8.4 Magnetic periodicity

The cross-shaped topology exhibits a geometrical periodicity of 90° as Fig. 5.26 shows. This periodicity can be even or odd on the symmetry lines shown in Fig. 5.29(c). An even rotational periodicity results in mainly normal flux through the symmetry lines whereas an odd periodicity yields mostly tangential flux. Figure 5.29(c) does not show magnetization vectors, as these depend on the symmetry within the leg of Section 5.8.3.

5.8.5 Magnet material selection

From the force and stiffness equations in Chapter 3, especially the general equations (3.30) and (3.47), it becomes clear that the gravity compensator’s force and stiffness are related to the magnet's remanent flux density by $F \sim B_r^2$ and $K \sim B_r^2$. This suggests the use of magnets with a high $B_r$, as the resonance frequency is hardly affected. The main modeling assumption and thus error is that the relative permeability is unity which is not fully true. To minimize the modeling error it is therefore important to select a material that exhibits a low relative permeability.

Table E.1 in Appendix E.4 shows the remanence and coercivity values of several hard-magnetic materials. Based on their typical and minimum values the relative permeability and its tolerance have been derived for these materials. It has been decided to use Vacodym 854TP material [180] for the gravity compensator. This decision is based on the stability of the typical and maximum magnetic properties, as well as manufacturability of the magnets and the complexity of the gluing onto the aluminum supports [39]. The material exhibits a typical remanent flux density $B_r = 1.32$ T and a typical coercivity $H_{cb} = 1020$ kA/m. The abbreviation TP stands for transverse-field pressed and is further discussed in Section 5.10.
5.9 Topology optimization

The variables that have been optimized are shown in Fig. 5.26 and Fig. 5.22(b). The optimization focuses on the force and stiffness characteristics in the working point and considers the working envelope of the device too. Figure 5.30 shows this working envelope: point $A$ is the center of the working envelope cube that has sides of 2 mm. Points $B$ and $C$ represent a vertical movement, along $\vec{e}_z$, of 1 mm in each direction. The points $D$, $E$, $F$ and $G$ represent a displacement in the horizontal plane at $z = 0$.

Preliminary optimizations have shown that an objective function with a minimization of the force variation throughout the working envelope tends to be faster than an optimization with stiffness minimization. This is due to the calculation time that, when only a force vector is computed, is almost an order of magnitude lower than that required to obtain both a force vector and a 3-by-3 stiffness matrix.

The optimization and the objective function $G(\vec{x})$ are given by

$$\vec{x}^* = \min_{\vec{x} \in \mathbb{R}^n} \left\{ G(\vec{x}) \mid \vec{c}(\vec{x}) \leq 0, A\vec{x} \leq \vec{b}, \vec{x} \in \mathbb{R}^n \right\},$$

$$G(\vec{x}) = |F_z(A) - F_z(C)| + |F_z(D) - F_z(A)| + |F_x(D)|.$$  \hspace{1cm} (5.37)  

The objective function $G(\vec{x})$ consists of three terms. The first term minimizes the vertical stiffness $K_{zz}$ for a purely vertical movement, the second term minimizes the cross-couplings $K_{zx}$ and $K_{zy}$ and the third term minimizes the horizontal stiffness $K_{xx}$ and $K_{xy}$.

The optimized vector of variables $\bar{x}$ and its constraints are characterized by

$$\bar{x} = \begin{pmatrix} w_m \\ h_m \\ h_g \\ z_0/r_p \\ d_i \end{pmatrix}, \quad \begin{pmatrix} 1 \text{ mm} \\ 2 \text{ mm} \\ 1.5 \text{ mm} \\ 0.8 \\ 17 \text{ mm} \end{pmatrix} \leq \bar{x} \leq \begin{pmatrix} 100 \text{ mm} \\ 30 \text{ mm} \\ 10 \text{ mm} \\ 1.2 \\ 25 \text{ mm} \end{pmatrix}.$$  \hspace{1cm} (5.39)

The linear inequality constraint $A\bar{x} \leq \vec{b}$ applies to the total width $r_2$ and is given by (5.34). This constraint and the dimensional variables that have been kept constant...
Table 5.8: Dimensional variables that have been kept constant during the topology optimization of the cross-shaped topology.

<table>
<thead>
<tr>
<th>Description</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum array height</td>
<td>$h_{\text{max}}$</td>
<td>160 mm</td>
</tr>
<tr>
<td>Maximum total width</td>
<td>$2r_2$</td>
<td>280 mm</td>
</tr>
<tr>
<td>Inner offset</td>
<td>$w_i$</td>
<td>2 mm</td>
</tr>
<tr>
<td>Outer offset</td>
<td>$w_o$</td>
<td>2 mm</td>
</tr>
<tr>
<td>Outer support thickness</td>
<td>$d_o$</td>
<td>18 mm</td>
</tr>
</tbody>
</table>

Table 5.9: Discrete variables for which optimizations have been performed.

<table>
<thead>
<tr>
<th>Description</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of horizontal poles</td>
<td>$n_h$</td>
<td>$\in {1, 2}$</td>
</tr>
<tr>
<td>Number of vertical poles</td>
<td>$n_v$</td>
<td>$\in {2, 3, 4, 5, 6, 7}$</td>
</tr>
<tr>
<td>Difference in number of vertical poles</td>
<td></td>
<td>$\in {0, 1}$</td>
</tr>
<tr>
<td>Magnetic symmetry within the legs</td>
<td></td>
<td>$\in {\text{even, odd}}$</td>
</tr>
<tr>
<td>Magnetic rotational symmetry between the legs</td>
<td></td>
<td>even</td>
</tr>
<tr>
<td>Remanent flux density of the magnets</td>
<td>$B_r$</td>
<td>1.23 T</td>
</tr>
</tbody>
</table>

are summarized in Table 5.8. As mentioned, the array height is limited to 160 mm, although a further minimization of this height has not been investigated and is recommended for future research. The variables $w_i$, $w_o$ and $d_o$ have been given their respective values for manufacturability reasons.

The nonlinear inequality constraint vector $c(\vec{x})$ is given by

$$c(\vec{x}) = \begin{bmatrix} 1 - \frac{F_z(A)}{7.50 \text{kN}} \\ \frac{K_{zz}(A)}{k_t} - 1 \\ -K_{zz}(A) \end{bmatrix} \leq 0 \quad (5.40)$$

The first component ensures that the minimum target force is achieved. The second and third inequality constraints ensure that the stiffness is positive and below the value $k_t$ that corresponds to a resonance frequency of 0.5 Hz according (5.1).

To validate the assumption of Section 5.3 that only one pole along the horizontal direction results in a better performance than multiple poles the optimization has been performed with this horizontal pole number $n_h \in \{1, 2\}$. The vertical pole number $n_v$ has been varied according $n_v \in \{2, 3, \ldots, 7\}$. These and the discrete properties described in Sections 5.8.2-5.8.4 are summarized in Table 5.9.

### 5.9.1 Optimization results for a single leg

All combinations of the variables in Table 5.9 require an individual optimization and as such at least $2 \cdot 6 \cdot 2 \cdot 2 = 48$ optimizations. To compare the optimized results
of all these combinations the optimized topologies have been re-simulated with the full gravity compensator included, as opposed to the single leg that has been modeled during the optimization. This has an effect on the interaction force, stiffness and torque which are now influenced by the cross-coupling between the four legs. Consequently, there is a risk that the topology selection which is based on these results eliminates the designs which are influenced the most by this cross-coupling instead of selecting the most suitable topology. An optimization of these 48 topologies based on a full gravity compensator model instead of one fourth is therefore more elegant. However, the additional computational effort that results from such optimization may become problematic and requires further investigation.

Selection of suitable topologies

The simulation results of the 48 optimized topologies including the cross-couplings, hence with four legs, are shown in Fig. 5.31(a)-(d). The vertical force $F_z$ and stiffness $K_{zz}$ in the working point $A$ of Fig. 5.30 are shown in Fig. 5.31(a). The straight vertical and horizontal lines represent the nonlinear inequality constraints of (5.40). Due to the cross-coupling between the four legs, which is included in the results of Fig. 5.31 and not in the optimizations that lead to these results, a number of topologies does not obey the nonlinear constraints anymore. Considering the large amount of different topologies that still satisfy the constraints it is suggested to add additional objectives, such as the magnet volume or the number of magnets. Further, the optimization of a full gravity compensator instead of a quarter would lead to more suitable topologies.

Figure 5.31(b)-(d) show for the optimized topologies the maximum value of $|K_{xx}|$, $|K_{yz}|$ and $|K_{zz}|$, respectively, when the position is varied throughout the working envelope of Fig. 5.30 in 5 steps of 0.5mm along each axis. These figures show that the maximum stiffness values are rather large for some of the topologies. The number of feasible topologies has been reduced by applying the following criteria to the simulated topologies:

- The vertical force $F_z$ must be larger than 7.50kN,
- The maximum value in the stiffness matrix in the working envelope must remain below 9kN/m.

The number of feasible topologies has reduced to two by applying these rules. Some properties for these topologies are summarized in Table 5.10. For both topologies the magnet arrays on the inner and the outer cross are equal, they exhibit one horizontal pole and three and respectively four vertical poles vertically. Especially the reduced magnet thickness, the lower expected flux leakage and the smaller magnet dimensions, which improve the manufacturability, suggest the use of the topology with four vertical poles rather than that with thee poles. As such, this topology has been further investigated.
Chapter 5: The passive electromagnetic gravity compensator

5.9.2 Optimization results for a full gravity compensator

Based on the optimization results discussed above a new optimization has been performed. It is based on the $1 \times 4$ topology of Table 5.10, with the difference that the optimization is based on the full gravity compensator model, instead one quarter. The in-leg symmetry and periodicity between the legs have been re-investigated.

Table 5.10: Characteristics of the topologies that remain after reduction of the optimization results.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Variable</th>
<th>$1 \times 3$</th>
<th>$1 \times 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of horizontal poles</td>
<td>$n_h$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>No. of vertical poles</td>
<td>$n_v$</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Difference in number of vertical poles</td>
<td>equal</td>
<td>equal</td>
<td></td>
</tr>
<tr>
<td>Magnetic symmetry</td>
<td></td>
<td>odd</td>
<td>odd</td>
</tr>
<tr>
<td>Magnetic periodicity</td>
<td></td>
<td>even</td>
<td>even</td>
</tr>
<tr>
<td>Remanent flux density</td>
<td>$B_r$</td>
<td>1.32 T</td>
<td>1.32 T</td>
</tr>
<tr>
<td>Magnet thickness</td>
<td>$h_m$</td>
<td>14.9 mm</td>
<td>9.8 mm</td>
</tr>
<tr>
<td>Airgap length</td>
<td>$h_g$</td>
<td>5.4 mm</td>
<td>4.2 mm</td>
</tr>
</tbody>
</table>
Table 5.11: The optimization properties that have been used for the 1-by-4 topology optimization which is based on the full gravity compensator model.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimization vector</td>
<td>$\vec{x}$</td>
</tr>
<tr>
<td>Linear inequality constraint</td>
<td>$A\vec{x} = b$</td>
</tr>
<tr>
<td>Nonlinear inequality constraint</td>
<td>$\tilde{c}(\vec{x}) \leq 0$</td>
</tr>
<tr>
<td>Objective function</td>
<td>$\mathcal{G}(\vec{x})$</td>
</tr>
<tr>
<td>Remanent flux density of the magnets</td>
<td>$B_r$</td>
</tr>
<tr>
<td>Minimum spacing between adjacent magnets</td>
<td>0.2 mm</td>
</tr>
<tr>
<td>Magnetic symmetry within legs</td>
<td>$\in {\text{even, odd}}$</td>
</tr>
<tr>
<td>Magnetic rotational symmetry between legs</td>
<td>$\in {\text{even, odd}}$</td>
</tr>
</tbody>
</table>

Table 5.12: The optimization results of the 1-by-4 topology which has been optimized using the full gravity compensator models.

<table>
<thead>
<tr>
<th>Magnetic symm. within legs</th>
<th>Magnetic rot. symm. between legs</th>
<th>even</th>
<th>even</th>
<th>odd</th>
<th>odd</th>
<th>odd</th>
<th>odd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnet width</td>
<td>$w_m$ mm</td>
<td>91.1</td>
<td>86.7</td>
<td>74.9</td>
<td>77.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Magnet thickness</td>
<td>$h_m$ mm</td>
<td>7.1</td>
<td>8.3</td>
<td>9.9</td>
<td>8.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Airgap height</td>
<td>$h_g$ mm</td>
<td>3.4</td>
<td>3.8</td>
<td>4.2</td>
<td>2.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative vertical offset</td>
<td>$z_0/\tau_p$</td>
<td>0.48</td>
<td>0.48</td>
<td>0.47</td>
<td>0.35</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inner cross thickness</td>
<td>$d_i$ mm</td>
<td>18.5</td>
<td>21.6</td>
<td>38.15</td>
<td>20.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Active magnet volume</td>
<td>$V \cdot 10^6$ mm$^3$</td>
<td>1.66</td>
<td>1.85</td>
<td>1.90</td>
<td>1.60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertical force</td>
<td>$F_z$ kN</td>
<td>7.55</td>
<td>7.54</td>
<td>7.54</td>
<td>7.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertical stiffness</td>
<td>$K_{zz}$ kN/m</td>
<td>0.0</td>
<td>1.8</td>
<td>3.0</td>
<td>1.4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

New properties that have been included are the minimum distance between adjacent magnets of 0.2 mm and the separation of the rather long magnets into two parallel segments for ease of manufacturing. Both properties have been introduced for manufacturability reasons.

Table 5.11 summarizes the optimization that has been performed. The optimization results are shown in Table 5.12 with the magnet volume and the vertical force and stiffness in the center of the working envelope (Point $A$ in Fig. 5.30). Although the dimensions, especially $d_i$, $z_0/\tau_p$, and $w_m$, vary between these topologies, they exhibit comparable force and stiffness properties which are all within the constraints of (5.40). As such, they are all considered suitable for implementation in the magnetic gravity compensator.

The first topology in Table 5.12, with even magnetic symmetry within the four legs and even periodicity, has been selected for implementation in a prototype. It exhibits a limited inner cross thickness $d_i$, magnet thickness $h_m$ and airgap length $h_g$, which reduce the leg thickness $r_1$ that was introduced in Fig. 5.26(a). This leaves more room to place the external actuators in the four empty corners of the cuboidal-shaped
Figure 5.32: In the gravity compensator (a) the outer cross, consisting of four pieces (exploded view), falls over the inner cross to form the gravity compensator. The magnetization of the vertical cross-section of a leg is shown in (b). The magnetization directions that are drawn illustrate that the symmetry within the legs is as well as the rotational symmetry of the gravity compensator are even.

The cumulative mass of permanent-magnet material is 12.7 kg, given a mass density of 7.7 g/cm$^3$ [180].

Figure 5.32 shows the magnetization vectors on the inner and the outer cross. It is a design choice to let the inner cross be mounted to the floor and the outer cross to the isolated platform. The segmentation of all permanent magnets into two parallel magnets is clearly shown in Fig. 5.32(a). Further, it shows the even symmetry between the four legs. The outer cross, which falls over the inner cross, is composed of four
sections for manufacturability reasons. The magnetization of each leg when observed from the outside is shown in more detail in Fig. 5.32(b).

5.9.3 Electromechanical behavior

Figure 5.33 and Fig. 5.34 show the force, torque and stiffness behavior of the selected topology. The corresponding colors of the working envelope, related to the height, are shown in Fig. 5.35. Throughout the working range the simulated force remain within a band of 3 N as is shown in Figs. 5.33(a)-(c). Considering the vertical force in Table 5.12 this variation of ±3 N is very small at less than 0.5%. This effect is also observed in Fig. 5.34 which shows the stiffness matrix: all theoretical stiffness components remain within the ±7.50 kN/m band that defines the 0.5 Hz resonance frequency that is envisaged. Although there is a position dependency in the stiffness,
the resulting amplitudes are sufficiently low to only have a minor influence on the force level. The theoretical torque around the horizontal axes, shown in Figs. 5.33(d)-(f), remains below 7.5 Nm and around the vertical axis remains virtually zero. The reference point for the torque calculation is here in the geometrical center of the magnet array on the outer cross assuming no displacement.

The characteristics in Fig. 5.33 and Fig. 5.34 show that the gravity compensator exhibits the same behavior along \( \hat{e}_x \) and \( \hat{e}_y \), i.e. the characteristics of \( F_x \) versus \( F_y \) seem rotated by 90° with respect to each other. The same is observed for \( T_x \) versus \( T_y \), \( K_{xx} \) versus \( K_{yy} \) and for \( K_{xx} = K_{zx} \) versus \( K_{yy} = K_{zy} \). This is in accordance with the symmetric topology of the gravity compensator.
Fig. 5.36: The FEM model (a) is a quarter model with periodic boundary conditions and (b) its mesh.

5.9.4 Design comparison with FEM

A 3D FEM model of the passive gravity compensator has been implemented in the FE software package Flux 3D [33]. The gravity compensator exhibits an even rotational periodicity of 90° around the $\hat{e}_z$-axis, which allows for modeling a quarter of the device as is shown in the top view of Fig. 5.36(a). The dash-dotted lines are the symmetry lines. Figure 5.36(b) shows some of the mesh elements in the quarter 3D FEM model from the viewpoint indicated in Fig. 5.36(a). This magnetostatic model incorporates 319,592 second order volume elements and 774,042 nodes.

The relative permeability in the FEM model equals unity as this is one of the most important assumptions in the analytical surface charge model. The vertical force obtained with the FEM model is 7402N. This is a reduction of only 1.9% with respect to the analytically obtained value. Although the mesh density in the FEM model is high, as Fig. 5.36 shows, the numerical truncation resulting from this meshing is most likely the cause this small deviation. For this model, with $\mu_r = 1$, the analytical results are assumed to be more accurate than the FEM results. The high correspondence between the two validates the implementation of the analytical model.

5.9.5 Discussion

The gravity compensator characteristics that have been presented and validated with FEM are based on simulations which exhibit certain assumptions and shortcomings. The relative permeability that equals unity, uniform and homogeneous magnetization, amongst others, are the most important modeling assumptions. Especially the theoretical force variation throughout the working envelope that is less than 0.5% of the passive vertical force poses a very tight margin for error. As a result of the low stiffness a modeling error in the vertical force of the gravity compensator is not compensated passively. For this reason, it is necessary to make an estimation on the force and stiffness tolerances that result from the modeling assumptions and manufacturing tolerances. Such estimation helps to design the actuators of Chapter 6 and the test rig of Chapter 7.
5.10 Modeling inaccuracies and manufacturing tolerances

Section 3.9 described the main modeling uncertainties of the analytical models used throughout this dissertation. This section projects them on the design of the gravity compensator to predict the force tolerances of the prototype. All tolerances have been modeled separately instead of combined for simplicity of modeling.

Variations in remanent flux density

A variation in remanent flux density exhibits a squared relationship to the interaction force, stiffness and torque as is observed from the general force equation (3.30). The resonance frequency of the gravity compensator is not affected by variations in remanent flux density as its definition (5.1) incorporates the ratio between stiffness and force, which increase proportionally. This remanent flux density, as well as the relative permeability discussed below, may be measured on a number of samples from a production batch of permanent magnets, as discussed in Section 3.9.

The material properties of the used material Vacodym 854TP state that its remanent flux density $B_r$ is typically 1.32 T and that the minimum value is 1.28 T [180]. Unfortunately, a maximum value is not provided. The typical remanent flux density value 1.32 T has been used in Section 5.9 to obtain the design of the gravity compensator. The worst-case remanent flux density of 1.28 T is 97.0% of the typical value. The passive vertical gravity compensation force is multiplied by 0.970$^2$ which yields a decrease of 6%, corresponding to a force reduction of 450 N. This value is significantly larger than the passive 3 N force variation of the gravity compensator through its range and should be compensated externally.

Relative permeability

As discussed in Section 2.5.3, the main modeling assumption of the analytical modeling technique used in this thesis is that the relative magnetic permeability $\mu_r$ equals unity throughout the model. As discussed in Section 2.5.3 this results in a linear model which enables fast calculation of electromagnetic properties in an ironless structure. However, in reality the relative permeability of permanent magnets around their operating point is never exactly equal to unity. This permeability is derived from the hysteresis curve of the permanent magnet. This curve, shown in Appendix E.4, is linear in the second quadrant as $H_{ci} \ll H_{cb}$ and therefore the permeability can be obtained by

$$\mu_r = \frac{B_r}{H_{cb}} \frac{1}{4\pi \cdot 10^{-7}}$$

(5.41)

The characteristic magnetic properties of Vacodym 854TP are expressed in Table 5.13 as typical and minimal values. These values have been used to derive a theoretical tolerance for the relative permeability. From the typical values a relative
5.10: Modeling inaccuracies and manufacturing tolerances

Table 5.13: Magnetic properties of VACODYM 854TP as defined in the datasheet [180].

<table>
<thead>
<tr>
<th>Name</th>
<th>Quantity</th>
<th>Min. value</th>
<th>Typ. value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Remanence</td>
<td>$B_r$</td>
<td>1.28 T</td>
<td>1.32 T</td>
</tr>
<tr>
<td>Coercivity</td>
<td>$H_{cb}$</td>
<td>970 kA/m</td>
<td>1020 kA/m</td>
</tr>
</tbody>
</table>

Figure 5.37: The relative vertical force obtained with FEM as function of the relative permeability for the gravity compensator.

permeability of 1.03 can be derived. The steepest curve based on $B_{r_{typ}}$ and $H_{cb_{min}}$ yields a maximum permeability of 1.08. The flat curve composed with $B_{r_{min}}$ and $H_{cb_{typ}}$ would give $\mu_r = 1.00$, however is only theoretical and will not be achieved in practice. The same can be said for the maximum value of 1.08 as $B_r$ and $H_{cb}$ are not independent of each other. A realistic value for this relative permeability would therefore be between 1.02 and 1.05. As such, the surface charge model would require some kind of modification to incorporate permeable materials. Such modification has not been found to date. The 3D FEM model used in Section 5.9.4 as verification tool does allow inclusion of this relative permeability.

Figure 5.37 shows simulation results obtained with FEM where the vertical force has been studied as function of the relative permeability. Under the assumption that $1.02 \leq \mu_r \leq 1.05$ the change in interaction force will be $-230 \text{ N} \leq \Delta F_z \leq -550 \text{ N}$ or a reduction between 3.1% and 7.4%. It must be noted that this characteristic is unique to the topology under investigation, as it depends on the working points of the permanent magnets.

Misalignment of the magnetization angle

The analytical modeling of misalignment of the magnetization vector was discussed in Section 3.7.2. The angles of misalignment of this manufacturing tolerance are not necessarily equal for all permanent magnets that are mounted in the gravity compensator. Therefore, the effect of such misalignment is simulated with randomly misaligned magnetization vectors for the permanent magnets in the gravity compensator.

The magnetization vector for TP material exhibits a tolerance of approximately ±2° and that for AP materials approximately ±4° [39]. TP and AP relate to the pressing
method (transversal or axial) that is used in the manufacturing of the permanent magnet [180]. To investigate the boundaries of its effect on the vertical force four extreme situations have been simulated which are shown in Figs. 5.38(a)-(d) of which the force results are shown in Table 5.14. These results show that the influence of this misalignment is significantly lower than that of the permeability or remanence. Especially when the distribution of the misalignment is random, unlike the uniform misalignment in these simulations, its effect is considered negligible.

**Geometrical tolerances**

A placement tolerance of ±0.1mm and a dimension tolerance of ±0.05 mm are considered realistic numbers in the manufacturing and assembly process of the various aluminum and hard-magnetic components of the gravity compensator. As these tolerances are unique for each magnet their cumulative effect on the vertical force are calculated in a different way than the extreme-case scenarios seen above. The position and dimensions of each permanent magnet in the gravity compensator have been varied randomly (uniformly distributed pseudo-random numbers) within the given tolerances. The magnets may overlap in this case as the maximum cumulative tolerance is larger than the magnets separation of 0.2mm that was mentioned in Section 5.9.2. This is not considered as a problem as the results are meant to be indicative and therefore have not been further investigated. One thousand topologies with such randomly displaced and re-dimensioned magnets form the force distribution which is shown in Fig. 5.39. From this figure it is concluded that the influence of these geometrical tolerances is less than 0.5%.

**Local magnetization misalignment**

The hot-cold effect, discussed in Section 3.9.2, is difficult to predict for each individual permanent magnet. It is for TP material approximately ±3% and for AP material approximately ±5% as a result of the manufacturing process [39]. A measurement of this effect is very time-consuming as it is unique to each magnet.
As such, it has been investigated for the for extreme situations. Therefore, the four extreme situations have been simulated, as has been performed for the misaligned magnetization vector. The maximum force deviation that have been found are an increase of 5.5 % and a decrease of 6.9 % for the extreme situations where the hot sides of one magnet array and the cold sides of the other are located at the same airgap. Thus, the influence of this effect is considered significantly larger than that of misalignment, which is the main reason to choose for TP material. It must be noted that these extremes are very unlikely as the magnets never exhibit their maximum hot-cold effect and placement in this manner.

**Working point variation**

Due to the absence of flux-focusing materials in the vibration isolator, it cannot be assumed that the permanent magnets are bound to the same load line, hence operate in the same working point. Throughout the magnet volume this working point may vary, especially near the edges of the magnet. The demagnetization characteristic shown in Appendix E.4 shows that the used 854TP material [180] is withstands a field of more than 2000 kA/m at room temperature.

A geometrical 3D mesh with a mesh distance of 0.5mm has been implemented in the permanent magnets of the gravity compensator. The $B(H)$ values that have been obtained for these points are shown in Appendix D.1. A comparison of these results with the knee point of the material shows that demagnetization is no issue.

**Temperature variation**

The magnetic gravity compensator is a passive device and as such is subject to ambient temperature variations. The magnet properties summarized in Appendix E.4 show that the remanent flux density depends on the temperature $T$ by $-0.105\%/K$ and the coercivity by $-0.60\%/K$. This suggests a force reduction of $\approx 2\%/K$ according the squared relationship that was discussed in Section 3.9.1.

If the lab temperature cannot be maintained constant, as is the case in this thesis,
Table 5.15: Relative force tolerances as a result of the various physical tolerances.

<table>
<thead>
<tr>
<th>Name</th>
<th>Lower limit</th>
<th>Upper limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Remanent flux density</td>
<td>-6.0%</td>
<td>0.0%</td>
</tr>
<tr>
<td>Relative permeability</td>
<td>-7.4%</td>
<td>-3.1%</td>
</tr>
<tr>
<td>Magnetization vector misalignment</td>
<td>-1.2%</td>
<td>+0.75%</td>
</tr>
<tr>
<td>Geometrical effects</td>
<td>-0.5%</td>
<td>+0.5%</td>
</tr>
<tr>
<td>Ambient temperature variation</td>
<td>-0.5%</td>
<td>+0.5%</td>
</tr>
<tr>
<td>Local magnetization misalignment</td>
<td>-6.9 %</td>
<td>+5.5 %</td>
</tr>
</tbody>
</table>

the variation of this temperature may affect the vertical force. As such, it is necessary to monitor this ambient temperature during the static measurements on the gravity compensator that are described in Section 7.6.

5.10.1 Cumulative effects

Table 5.15 summarizes in terms of percentage the expected modeling errors that have been derived. Especially the relative permeability is a pre-known error and its impact of more than 3.1% force reduction must be incorporated a-priori. The remanent flux density is another error which may exhibit significant influence and can only be determined from hysteresis measurements on magnets that are mounted in the gravity compensator. The other errors are mostly stochastic, hence, are less likely to have much influence on the total force.

5.11 Adjustability of the force

The vertical force of the gravity compensator has been set to 7.50 kN, however the gravitational force of the isolated platform may differ significantly per application. Large variations in the gravitational force may be compensated by using the modularity of the gravity compensator design. The cross-shaped gravity compensator design proposed in this chapter exhibits four vertically stacked layers in each magnet array. The addition or removal of one of these layers changes the vertical force by approximately 25% and is in this way a method to adjust the passive vertical force. Its effects on stiffness are expected to be limited, as the force and the stiffness change equivalently, but requires additional research.

The previous section discussed how the vertical force of the gravity compensator is only predictable within a couple of percents. On a passive force of 7.50 kN this means a margin of hundreds of newtons, which is difficult to compensate with active actuators considering the limited available volume that is left from the volume envelope of Table 5.1. A small variation in the gravitational force of the isolated platform may also lie in this range, which is too small to be compensated by the use of the design's modularity described above. As the magnetic properties of the magnets cannot be changed it seems logical to adjust the geometry of the device. For example, a change
in airgap length adjusts the force, but it influences the stiffness and the torque too. Further, some method to mechanically vary this airgap and to maintain this during operation should be developed. Such adjustability has not been investigated in this thesis. Instead, it is proposed to make the isolated platform adjustable: the addition or removal of mass on this platform is easier accomplished than to develop an adjustable magnetic gravity compensator.

5.12 Conclusions

This chapter presented a feasibility study on various topologies for a magnetic gravity compensator. Based on this investigation a gravity compensator design has been proposed that combines a high force with low force and torque stiffness. The analyses presented in this chapter are based on the interaction equations that have been derived in Chapter 3.

It has been shown how, based on the 2D potential and field equations for a rectangular magnet, that it is possible to predict the performance of a simple topology. By observing the flux density the force is estimated and its gradient around the magnet edges is a measure for stiffness. This has been validated with simulations on some basic topologies.

Based on the classical magnetic spring with two opposing permanent magnets has been an investigation into the use of planar quasi-Halbach arrays and unequal fractional pitch checkerboard arrays has been performed. It is concluded that these topologies, with one horizontal airgap, are unsuitable to meet the requirements which have been set. Topologies with two airgaps, which combine attraction force through one airgap with repulsion force through the other airgap, meet the requirements in the working point. This is due to the equal force and opposite stiffness that is produced in both airgaps, which yields a low-stiffness topology. However, the manufacturability and the expected nonlinear behavior of such topology have lead to the design choice to abandon it. Nevertheless, further investigation is necessary to determine the limits of such a topology and to investigate an integration of the active actuators.

In an effort to reduce the device’s resonance frequency, topologies with vertical airgaps have been studied. In these topologies the main force vector lies in the plane of the airgap instead of perpendicular to it. It is concluded that the force density of such topology, provided that a high vertical force and low stiffness are pursued, is not lower than that for the topologies with horizontal airgaps. Further, it has been shown that such a topology even combines these conflicting properties better than the topologies with horizontal airgaps. To minimize any nonlinear behavior the device must be optimized throughout its working envelope instead of only in a single working point. To reduce parasitic torques the vertical arrays need to be placed on smaller intermediate distances which has resulted in a cross-shaped topology. The four legs of this cross house two parallel gravity compensators which are placed in such close proximity.
An investigation throughout its working envelope confirms the near-linear, low-stiffness and low-torque behavior of the cross-shaped magnetic gravity compensator. A direct design comparison with FEM has shown that there is a discrepancy of 1.9% between the results. However, it is concluded that when $\mu_r = 1$ throughout the studied volume, the meshfree analytical models are more accurate than FEM. The resulting design exhibits a simulated force of 7.50 kN with a stiffness below 7.50 kN/m throughout its working envelope, hence, minimizes the stiffness. It is concluded that the gravity compensator that has been designed exhibits a high vertical force, low stiffness and low parasitic force and torque, a modular design which can be expanded to >10.00 kN within the available volume envelope. The influence of the manufacturing tolerances and modeling errors on the passive vertical force have been estimated and are found to be sufficiently low to be passively compensated in the test setup.

With the extensive analyses of various gravity compensator topologies in this chapter a designer of a magnetic gravity compensator is able to pre-select the topologies that are suitable for his or her envisaged application. Based on the required force and stiffness level a selection between the investigated topologies or a prediction of another configuration, using Section 5.3, can be made.

### 5.12.1 Contributions

- **Section 5.3** - This section presents an analysis of the electromechanical properties that are predicted from various magnet configurations, based on the 2D field equations for a single cuboidal permanent magnet. The locations of the magnet edges play a key role in the force, stiffness and torque calculations and with this knowledge it is possible to predict the performance of a certain topology beforehand.

- **Section 5.4** - An investigation into various topologies with one horizontal airgap has been conducted. Starting from the basic two-magnet topology checkerboard patterns and quasi-Halbach arrays are investigated for their suitability to exhibit both a high force and low stiffness. Similar topologies found in literature investigate either a maximization of the vertical force or reduce the stiffness at the cost of a low force level.

- **Section 5.5** - The gravity compensator topology with unequally sized magnet arrays has been proposed. In this topology both magnet arrays have the same pole pitch and different magnet pitches. As the magnet edges do not align, the stiffness of this topology is reduced significantly for a given force level.

- **Section 5.7** - This chapter proposes the extended design of magnet springs with vertical airgaps and multiple magnets, which combine a high vertical force level with an inherently low stiffness. The use of multiple of these springs reduces the required device height and minimizes parasitic force components even more.

- **Section 5.8** - The cross-shaped arrays that are proposed exhibit share high-force and low-stiffness properties of the vertical-airgap topology and combine this
5.12: Conclusions

with a very low parasitic torque. Further, the four corners are very suitable to house the integrated 2-DoF external actuators.

• **Section 5.10** - Based on the analysis of errors and tolerances in Chapter 3 an analysis of the expected tolerance in force level between the analytical model and the realized prototype is performed.

### 5.12.2 Recommendations

Based on the analysis in this chapter several recommendations for future work have been formulated

- The topologies with two parallel horizontal airgaps of Section 5.6 have only been briefly investigated in their working point. Although they have been abandoned in this thesis based on preliminary optimizations, this does not mean that they are of no interest for vibration isolation systems. A further investigation into their suitability for magnetic gravity compensators in terms of resonance frequency, linear range, force density, suitable force levels, manufacturability, etc. would give more insight into their advantages and disadvantages.

- This chapter has investigated only topologies with vertically or horizontally oriented airgaps. An investigation into a combination of these two, or even tilted airgaps, is has not been performed, although it may yield interesting results.

- The topology optimization of Section 5.9 has been performed with multiple objectives that have not been weighted. Such weighting may result in a different design than that presented in this chapter. Further, it is recommended to include more objectives, such as a minimization of the (magnet) volume, field leakage or costs.

- The optimizations of the cross-shaped gravity compensator that have led to the choice for a 1-by-4 topology are based on simulations of a quarter of the gravity compensator. As a result of this, the magnetic cross-coupling between the four legs have not been taken into account during this optimization. The topology selection from these optimized results has been performed with simulations of the full gravity compensator. It is therefore possible that the topology with the lowest amount of cross-coupling has been selected, rather than the best performing topology. For this reason, it is recommended to include the full gravity compensator, including all cross-couplings, in the optimization, although this exponentially increases the computational efforts.

- Despite their increased complexity with respect to assembly, the investigation of quasi-Halbach topologies for use in the cross-shaped gravity compensator is considered an interesting expansion.

- An inclusion of iron parts may improve the force density and the magnetic shielding of the magnetic gravity compensator. However, such inclusion yields more complicated modeling, with nonlinear effects and hysteresis, and as
such has not been investigated. It is a recommendation for future research to investigate this inclusion and its effects on the device's performance.

- The analysis presented in this chapter is based on magnetostatics and as such does not include dynamic effects such as eddy currents within the magnets or support material. Such investigation would provide insight into the damping that occurs, although it is expected that this damping is low considering the limited velocities that occur in floor vibration.

- The design of the gravity compensator is based on a fixed force level and exhibits extremely low stiffness. As such, a mass variation of the platform that is isolated cannot be handled by the magnetic circuit. Consequently, the gravity compensator needs to be designed for the maximum mass and the isolated platform must be equipped with weights such that it always exhibits this mass. An adjustable vertical force would therefore be beneficial. A possible solution would be a variable airgap length between the magnets, although that this has not been investigated in this chapter and is left for a future research.
Chapter 6

The integrated actuators

The electromagnetic design of the ironless actuators that are integrated into the electromagnetic vibration isolator.

This chapter is based on:


Chapter 6: The integrated actuators

Figure 6.1: Three vibration isolation units, each with 2-DoF actuation (a) stabilize the vibration isolation system. Two of the free corners in the vibration isolation strut (b) are used for vertical actuation and two corners for horizontal actuation.

The gravity compensator described in Chapter 5 is passively unstable according to Earnshaw’s theorem [58] that is discussed in Appendix C. This chapter discusses the 2-DoF actuation integrated into this gravity compensator. The electromechanical and electrical properties of the actuators are modeled and optimized. This is undertaken for the horizontal and the vertical actuators, which both fit in the volume that is not used by the cross-shaped gravity compensator. Further, it is shown that the influence of these actuators on the behavior of the gravity compensator is minimal.

6.1 Outline

From the general design objectives and the gravity compensator design Section 6.2 extracts the design objectives for the actuators. Section 6.3 describes the electromechanical models that have been used to model their behavior and Section 6.4 derives the electrical properties. The design of the actuators for horizontal actuation is discussed in Section 6.5 and for vertical actuation in Section 6.6. The validation of the various models is performed in Section 6.7 and is followed by a discussion on the integration of the actuator magnets and their interaction with the other permanent magnets in Section 6.8. Finally, Sections 6.9 and 6.10 present the contributions, conclusions and recommendations.

6.2 Integration of the actuators

The cross-shaped gravity compensator proposed in the previous chapter leaves four rectangular-shaped corner sections available for the active actuators. A vibration isolation unit exhibits one 6-DoF gravity compensator, one vertical active degree and a horizontal active degree. Three of these isolators placed in a triangle result in a system with 6 controlled degrees of freedom (3x2DoF) as Fig. 6.1(a) shows. Figure 6.1(b) shows that two of the corners in the gravity compensator are equipped with vertical actuators and two with horizontal actuators. Given this information and the gravity compensator design of the previous chapter a number of boundary conditions for the design of the actuators are derived.
6.2: Integration of the actuators

Table 6.1: Design objectives for the integrated Lorentz actuators.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actuator type</td>
<td>Ironless Lorentz actuator</td>
</tr>
<tr>
<td>Magnet material</td>
<td>Vacodym 854 TP</td>
</tr>
<tr>
<td>Maximum horizontal dimensions</td>
<td>70 mm x 70 mm</td>
</tr>
<tr>
<td>Maximum vertical dimensions</td>
<td>130 mm</td>
</tr>
<tr>
<td>Amplifier current limit</td>
<td>2 A</td>
</tr>
<tr>
<td>Amplifier voltage limit</td>
<td>52 V</td>
</tr>
<tr>
<td>Max. operating frequency</td>
<td>1 kHz</td>
</tr>
</tbody>
</table>

![Diagram of Lorentz actuators](image)

**Figure 6.2:** Artist impression of the Lorentz actuators.

6.2.1 Boundary conditions

The design constraints for the integrated actuators are summarized in Table 6.1. An important design choice is the use of ironless Lorentz actuators instead of actuators with back-iron which are more common. These actuators consist of a rectangular coil interacting with a permanent-magnet field. The reason for this choice is the ironless topology of the gravity compensator and the influence that the proximity of soft-magnetic materials could have on its performance. In [103] it was shown that the ironless voice coil actuator with quasi-Halbach magnetization performs similar to a voice coil actuator with back-iron. An impression of the resulting actuator is shown in Fig. 6.2. As these actuators are integrated in the gravity compensator, the hard-magnetic material is the same as for the gravity compensator. The volume constraint for the actuator is based on the volume constraint of Table 5.1 and the design of the gravity compensator. The limits of the amplifier that feeds the actuators determine the current and voltage limit and follow from Appendix E.5.

The actuators have been designed to produce their maximum force over a large frequency range which is shown in the table. Considering the voltage limits of the amplifiers, this puts limits on the resistance and inductance of the coils as is discussed in Section 6.4.
6.3 Modeling of the electromechanical properties

An accurate and fast force prediction of the actuator properties is a necessity for optimization purposes. The electromechanical modeling tools that have been used are based on the analytical surface charge model discussed in Chapter 3 combined with a piecewise continuous function of the coil current that is numerically integrated.

6.3.1 Dimensional variables

Figure 6.3 shows the dimensions of the coil and magnets of the actuator shown in Fig. 6.2. The force that is produced is oriented along \( \hat{e}_x \). The dimensions of the vertical magnets are characterized by \( [l_m, h_m, w_m]^T [m] \) and those of the horizontally oriented Halbach magnets by \( [l_h, h_m, w_m]^T [m] \). Their magnetization vectors are oriented along the arrows shown in Fig. 6.3. The coil bundle width and height are \( l_b [m] \) and \( h_b [m] \), respectively. The outer horizontal dimensions of the coil are \( l_c [m] \) and \( w_c [m] \), whereas their inner dimensions are \( l_{ci} [m] \) and \( w_{ci} [m] \). The inner radius of the coil edges is given by \( r_c [m] \). The center of actuation (CoA) is defined here as the geometrical center of the coil. This variable is used in Chapter 7 in the measurements on the prototype.

6.3.2 Piecewise continuous function of the coil current density

The coil current is represented by a 3D piecewise continuous function representing the 8 sections shown in Fig. 6.4 [41]. The corresponding current density vector \( \vec{J} \) for each section is defined in Table 6.2. The variable \( J \) is the scalar amplitude of the current density, which is considered to be constant within the coil. The influence of the coil's lead wires is considered to be negligible in this calculation due to the large number of turns.
6.4: Modeling of the electrical properties

Figure 6.4: Eight pieces form the piecewise continuous function of the current density in the coil. The numbers 1…8 and angles $\theta_{2,4,6,8}$ correspond with those in Table 6.2.

Table 6.2: Piecewise continuous current density inside the coil. The vertical component $J_z$ is zero everywhere. The scalar amplitude of the current is given by $J$.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J_x$</td>
<td>$-J$</td>
<td>$J\cos\theta_2$</td>
<td>0</td>
<td>$J\cos\theta_4$</td>
<td>$J$</td>
<td>$J\cos\theta_6$</td>
<td>0</td>
<td>$J\cos\theta_8$</td>
</tr>
<tr>
<td>$J_y$</td>
<td>0</td>
<td>$J\sin\theta_2$</td>
<td>$-J$</td>
<td>$J\sin\theta_4$</td>
<td>0</td>
<td>$J\sin\theta_6$</td>
<td>$J$</td>
<td>$J\sin\theta_8$</td>
</tr>
</tbody>
</table>

6.3.3 Numerical integration

With the analytical field equations and the piecewise continuous current density formulation the Lorentz force on the coil has been obtained using the force, which was discussed in Section 2.4.1. As in [41, 95] the necessary volume integration of $\vec{J} \times \vec{B}$ is performed numerically by means of trapezoidal integration. The coil has been discretized into a cuboidal mesh. To minimize numerical truncation errors the mesh element size has been chosen such, that an integer number of elements fits in the bundle width $l_b$ and in the bundle height $h_b$ [103]. The maximum mesh element size is 0.6 mm.

6.4 Modeling of the electrical properties

The electromechanical parameters are certainly not the only important parameters to be included in an optimization of the actuator. As summarized in Table 6.1, the amplifier that drives the actuator exhibits certain limitations in current, voltage, and bandwidth which determine the maximum static and dynamic force, respectively. Especially when the actuator is designed to exhibit force at elevated frequencies of hundreds of hertz, as discussed in Section 4.9, these current and voltage limits influence the actuator design.

From the Lorentz force equation it can be derived that a high current density $\vec{J}$ increases the static force of the device. As the current $i$ of the feeding amplifier is limited, the use of many turns maximize the achievable force with the given
current. However, an increase in force gives rise to an increased coil resistance $R$ and consequently elevated power dissipation $P_{\text{diss}} \text{W}$. An increase of the number of turns also effects the coil inductance $L$, which affects the high-frequency behavior of the device. This is summarized in the general machine voltage equation

$$V = Ri + L \frac{di}{dt} + \frac{d\Phi}{dx} \frac{dx}{dt}.$$  \hspace{1cm} (6.1)

The voltage is here given by $V \text{[V]}$, the electrical resistance by $R \text{[\Omega]}$, the current by $i \text{[A]}$, the inductance by $L \text{[H]}$, the magnetic flux linkage by $\Phi \text{[Wb]}$ and the displacement by $x \text{[m]}$. At low frequencies the ohmic voltage component $Ri$ is dominant and at elevated frequencies the inductive voltage component $L \frac{di}{dt}$ becomes the largest voltage component. The component related to the flux linkage remains very low for the chosen type of actuator.

### 6.4.1 Number of windings and fill factor

The number of turns of the rectangular coil is necessary to determine its resistance and inductance. With the orthocyclic winding technique the number of turns in a specific coil cross section is maximized \([40, 118]\) as the wire is carefully placed in the grooves of the layer underneath. Figure 6.5 shows two filling patterns that may occur. The variable $G_i$ determines if each layer contains the same number of turns

$$G_i = \begin{cases} 0 & \text{if } \frac{h_b}{d_{\text{wo}}} - \frac{h_b}{d_{\text{wo}}} \leq 0.5 \quad \text{Fig. 6.5(a)}, \\ 1 & \text{otherwise} \quad \text{Fig. 6.5(b)}. \end{cases}$$  \hspace{1cm} (6.2)

The variable $d_{\text{wo}} \text{[m]}$ is the outer diameter of the copper wire, including the electrical insulation. Following, the maximum number of turns $N$ is obtained.

$$N_{\text{hor}} = \left[ \frac{h_b}{d_{\text{wo}}} \right]$$  \hspace{1cm} (6.3)

$$N_{\text{ver}} = \left[ 1 + \frac{h_b-d_{\text{wo}}}{0.5d_{\text{wo}} \sqrt{3}} \right]$$

$$N = N_{\text{hor}} N_{\text{ver}} - \left[ \frac{N_{\text{ver}}}{2} \right] (1 - G_i).$$  \hspace{1cm} (6.3)

The fill factor, $\zeta$, of the coil is the ratio between the copper area and the total area of the coil bundle cross-section. With the number of windings $N$, the copper diameter
6.4: Modeling of the electrical properties

of the wire $d_w$ [m] and the dimensions $l_b$ and $h_b$ the fill factor is given by

$$\zeta = \frac{N\pi (d_w/2)^2}{l_b h_b}. \quad (6.4)$$

6.4.2 Resistance and inductance

With the physical parameters derived in the previous section the electrical resistance and inductance are derived.

Resistance

The resistance $R$ of the rectangular coil is obtained by the standard equation

$$R = \rho_e \frac{l_w}{\pi (d_w/2)^2}. \quad (6.5)$$

$\rho_e$ [Ωm] is the electrical resistivity of copper ($1.72 \cdot 10^{-8}$ Ωm) and $l_w$ [m] is the length of the copper wire in the coil. As not all turns have the same length the total length of the coil wire is estimated by

$$l_w = \left(2(l_c + w_c - 4r_c) + 2\pi r_c + l_b \right) N. \quad (6.6)$$

Inductance

The general equation for the inductance $L$ of a volume $V$ is given by [66, Chap. 3.2.5]

$$L = \frac{1}{i^2} \int_V \vec{A} \cdot \vec{J} \, dV, \quad (6.7)$$

where $\vec{A}$ is the magnetic vector potential, $\vec{J}$ is the volume current density and $i$ is the coil current. In [179] the magnetic vector potential $\vec{A}$ for a current-carrying straight beam with rectangular cross-section has been derived analytically. This thesis proposes a different formulation as that found in [179] to obtain a formulation which is of the format of the force, stiffness and torque equations found in Chapter 3.

The vector potential is derived for the bar-shaped volume shown in Fig. 6.6(a) with volume current density $\vec{J}$ and dimensions $(2a, 2b, 2c)$. Starting with the law of Biot-Savart the vector potential in this bar can be written as

$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi} \int_V \frac{\vec{J}(\vec{x}')}{|\vec{x} - \vec{x}'|} \, dV'. \quad (6.8)$$

The current density $\vec{J}(\vec{x}')$ is uniform and homogeneous within the bar (Fig. 6.6(a)) and therefore can be removed from the integrand. This homogeneity causes the two vector potential components perpendicular to the volume current to be zero. In the bar of
A bar-shaped volume current density (a) is used for the inductance calculation. The coil is replaced by four bars (b) in the inductance calculation.

Fig. 6.6(a) this would give \( A_y = A_z = 0 \). The vector potential is now obtained and with the intermediate variable \( \Lambda \) it is given by

\[
\vec{A} = \frac{\mu_0 J}{4\pi} \sum_{i=0}^{1} \sum_{j=0}^{1} \sum_{k=0}^{1} (-1)^{i+j+k} \Lambda . \tag{6.9}
\]

\[
\Lambda = \frac{1}{2} \left( (v^2 + w^2)\tan^{-1}\left( \frac{v}{w} \right) - w \left[ 2u + 3v + w\tan^{-1}\left( \frac{uw}{vr} \right) \right] \right.
+ u \left( 2u\tan^{-1}\left( \frac{w}{u} \right) - u\tan^{-1}\left( \frac{uw}{ur} \right) + 2v\log(v + r) + 2v\log(w + r) \right)
- \left. v \left( 2v\tan^{-1}\left( \frac{w}{v} \right) - v\tan^{-1}\left( \frac{uw}{vr} \right) + 2w\log(u + r) \right) \right) . \tag{6.10}
\]

\[
u = (-1)^i a - x , \tag{6.11}
\]

\[
v = (-1)^j b - y , \tag{6.12}
\]

\[
w = (-1)^k c - z , \tag{6.13}
\]

\[
r = \sqrt{u^2 + v^2 + w^2} . \tag{6.14}
\]

By representing the coil as four overlapping straight beams (Fig. 6.6(b)) the vector potential for the rectangular coil has been obtained analytically. Numerical integration of (6.7) over the cuboidal mesh discussed in Section 6.3.3 provides the inductance.

**Electrical time constant**

The resistance and the inductance determine the electrical time constant \( \tau_e [s] \) of the actuator which equals

\[
\tau_e = \frac{L}{R} . \tag{6.15}
\]
Physically, the constant represents the time it takes the current to reach \(1 - \frac{1}{e} \approx 63.2\%\) of its final (asymptotic) value for step in the voltage level. As the amplifier is current controlled this electrical time constant is a measure for the required voltage at elevated frequencies.

### 6.5 Horizontal actuators

A constrained nonlinear multivariable optimization function has been employed to find the optimized form \(\bar{x}^*\) of the actuator dimensions, which are enclosed in vector \(\bar{x}\), such that

\[
\bar{x}^* = \min_{\bar{x} \in \bar{X}} \left\{ \mathcal{G}(\bar{x}) \mid \bar{c}(\bar{x}) \leq 0, A\bar{x} \leq \bar{b}, \bar{x} \in \mathbb{R}^n \right\}.
\] (6.16)

#### 6.5.1 Objective function

Two objective functions have been compared for these actuators. In the first objective function the objective \(\mathcal{G}_1(\bar{x})\) is the inverse of the actuator steepness \(S [N^2/W]\). It is defined as

\[
\mathcal{G}_1(\bar{x}) = \frac{1}{S},
\] (6.17)

\[
S = \frac{F(\bar{x})^2}{P(\bar{x})} = \frac{i^2(\bar{x})k^2(\bar{x})}{i^2(\bar{x})R(\bar{x})} = \frac{k^2(\bar{x})}{R(\bar{x})} [N^2/W],
\] (6.18)

where \(F\) is the generated force, \(P\) the dissipated power, \(i\) is the current through the coil linked to the force by the motor force constant \(k = F/i [N/A]\) and \(R\) is its electrical resistance. As the squared current is present in the numerator as well as in the denominator this steepness provides a current-independent optimization means to maximize the produced force with minimized power dissipation. It also is a means to compare different actuator designs.

The second objective function is a maximization of the force constant of the actuator, or

\[
\mathcal{G}_2(\bar{x}) = \frac{i}{F(\bar{x})}.
\] (6.19)

Both objectives are conflicting, as the first incorporates power loss into the objective and the second does not. Nevertheless, a compromise needs to be found, as besides a low power loss one of the objectives for these actuators is to maximize the force in the given volume.

#### 6.5.2 Optimization variables and linear constraints

Some of the variables in Fig. 6.3 have been maintained constant during the optimization. The airgap length \(h_g\) has been minimized by an optimization to maximize
Table 6.3: The linear constraints of the actuator optimization.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_g = 3.0\text{ mm}$</td>
<td>Airgap thickness</td>
<td>$10 \leq l_m \leq 30$</td>
<td>Magnet width</td>
</tr>
<tr>
<td>$w_{co} = 130\text{ mm}$</td>
<td>Coil depth</td>
<td>$5 \leq h_m \leq 30$</td>
<td>Magnet height</td>
</tr>
<tr>
<td>$r_c = 3.0\text{ mm}$</td>
<td>Inner coil radius</td>
<td>$10 \leq l_b \leq 30$</td>
<td>Coil bundle width</td>
</tr>
<tr>
<td>$w_m = w_c - l_c$</td>
<td>Magnet depth</td>
<td>$5 \leq h_b \leq 20$</td>
<td>Coil height</td>
</tr>
<tr>
<td>$w_{ci} = w_{co} - 2l_c$</td>
<td>Coil gap width</td>
<td>$5 \leq l_h \leq 25$</td>
<td>Halbach magnet length</td>
</tr>
<tr>
<td>$2h_m + h_b + h_g \leq 70$</td>
<td>Actuator height</td>
<td>$6 \leq l_c \leq 30$</td>
<td>Coil core length</td>
</tr>
<tr>
<td>$2l_m + l_b \leq 70$</td>
<td>Actuator width</td>
<td>$0.5 \leq d_{wo} \leq 1.2$</td>
<td>Wire thickness</td>
</tr>
<tr>
<td>$l_{ci} \leq 63$</td>
<td>Coil width</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

the force, but must be maintained at a minimum of 3mm to allow for sufficient movement in all directions as the envisaged application is a 6-Dof system. Further, the coil length $w_{c}$ occupies the full available length of the actuator. The variables that are optimized are enclosed in vector $\vec{x}$ which is given by

$$\vec{x} = [l_m \ h_m \ l_b \ h_b \ l_h \ l_{ci}]^T.$$  \hspace{1cm} (6.20)

The constraints for $\vec{x}$ as well as those the vectors $\vec{A}$ and $\vec{b}$ are summarized in Table 6.3. The maximum actuator dimensions are 70mm along $x$, 130mm along $y$ and 64mm along $z$. It is a design choice to maintain the coil width $l_{ci}$ at least 7mm below the width of the magnet array to reduce the sensitivity of the actuator to displacements along $x$. The outer wire thickness $d_{wo}$ has been optimized continuously, although it is restricted to discrete wire diameter values that are commercially available (IEC 60317).

### 6.5.3 Nonlinear inequality constraints

The nonlinear inequality constraints that have been imposed are related to the dynamic behavior of the actuator. Table 6.1 summarizes the amplifier’s voltage and current limits. At the maximum frequency $f_{\text{max}} = 1\text{ [kHz]}$ the inductive voltage component of (6.8) becomes dominant. The actuator has been been designed to exhibit its maximum force at this frequency, and as such the maximum theoretical terminal voltage has been limited to 45V. The electrical time constant is limited to comply with amplifier requirements. The nonlinear inequality constraint $c(\vec{x})$ is defined as

$$c(\vec{x}) = \begin{bmatrix} \tau_e - 4\text{ ms} \\ i(R + 2\pi f_{\text{max}} L) - 45\text{ V} \end{bmatrix}.$$  \hspace{1cm} (6.21)

### 6.5.4 Optimization results

Table 6.4 summarizes the optimization results for the two objective functions described in Section 6.5.1. It shows the large difference in steepness between both objectives. Further, the second objective results in a minimization of the coil height and maximization of the magnet height.
Table 6.4: Optimization results for the two objective functions.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S = 860 \text{ N}^2/\text{W}$</td>
<td>Steepness</td>
<td>$S = 462 \text{ N}^2/\text{W}$</td>
<td>Steepness</td>
</tr>
<tr>
<td>$k = 28.7 \text{ N/A}$</td>
<td>Motor force constant</td>
<td>$k = 34.6 \text{ N/A}$</td>
<td>Motor force constant</td>
</tr>
<tr>
<td>$l_m = 25.0 \text{ mm}$</td>
<td>Magnet width</td>
<td>$l_m = 25.2 \text{ mm}$</td>
<td>Magnet width</td>
</tr>
<tr>
<td>$h_m = 22.1 \text{ mm}$</td>
<td>Magnet height</td>
<td>$h_m = 27.5 \text{ mm}$</td>
<td>Magnet height</td>
</tr>
<tr>
<td>$l_b = 25.5 \text{ mm}$</td>
<td>Coil bundle width</td>
<td>$l_b = 15.5 \text{ mm}$</td>
<td>Coil bundle width</td>
</tr>
<tr>
<td>$h_b = 10.4 \text{ mm}$</td>
<td>Coil height</td>
<td>$h_b = 5.0 \text{ mm}$</td>
<td>Coil height</td>
</tr>
<tr>
<td>$l_h = 20.1 \text{ mm}$</td>
<td>Halbach magnet</td>
<td>$l_h = 16.7 \text{ mm}$</td>
<td>Halbach magnet</td>
</tr>
<tr>
<td>$l_{c1} = 12.0 \text{ mm}$</td>
<td>Coil core width</td>
<td>$l_{c1} = 12.4 \text{ mm}$</td>
<td>Coil core width</td>
</tr>
<tr>
<td>$d_{wo} = 1.16 \text{ mm}$</td>
<td>Wire thickness</td>
<td>$d_{wo} = 0.68 \text{ mm}$</td>
<td>Wire thickness</td>
</tr>
</tbody>
</table>

Table 6.5: The properties of the horizontal actuators that have been realized.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_m = 25.0 \text{ mm}$</td>
<td>Magnet width</td>
<td>$S = 625 \text{ N}^2/\text{W}$</td>
<td>Steepness</td>
</tr>
<tr>
<td>$h_m = 22.0 \text{ mm}$</td>
<td>Magnet height</td>
<td>$k = 30.3 \text{ N/A}$</td>
<td>Motor force constant</td>
</tr>
<tr>
<td>$l_b = 22.9 \text{ mm}$</td>
<td>Coil bundle width</td>
<td>$\tau = 2.3 \text{ ms}$</td>
<td>Electrical time constant</td>
</tr>
<tr>
<td>$h_b = 8.4 \text{ mm}$</td>
<td>Coil height</td>
<td>$k_{fcu} = 0.71$</td>
<td>Coil filling factor</td>
</tr>
<tr>
<td>$l_h = 20.0 \text{ mm}$</td>
<td>Halbach magnet</td>
<td>$R = 1.5 \Omega$</td>
<td>Resistance</td>
</tr>
<tr>
<td>$l_{c1} = 11.8 \text{ mm}$</td>
<td>Inner coil width</td>
<td>$L = 3.3 \text{ mH}$</td>
<td>Inductance</td>
</tr>
<tr>
<td>$d_{wo} = 0.959 \text{ mm}$</td>
<td>Wire thickness</td>
<td>$N = 216$</td>
<td>Number of turns</td>
</tr>
</tbody>
</table>

The properties of the topology that has been realized are summarized in Table 6.5. Some of the variables result from preliminary design fixes throughout the design process and as such this topology is not Pareto-optimal. Further, the frequency band for which this particular actuator has been optimized is a worst-case situation. An improvement in both the steepness and the motor force constant could be achieved by changing the voltage constraint in (6.21) to a higher value. The validation of the analytically obtained parameters for this horizontal actuator is shown in Section 6.7.

6.6 Vertical actuators

The vertical actuators in the gravity compensator should fit in the same volume as the horizontal actuators. Their actuation axis is rotated 90° as Fig. 6.1(b) shows and as a result the coil length is limited to 70 mm instead of 130 mm. This limitation has resulted in the topology shown in Fig. 6.7 in which two coils are placed in parallel between two quasi-Halbach arrays. These coils are placed electrically in series to ensure the same current through both of them. The dimensions of both coils and the two magnet types are defined similar to Fig. 6.3 and are therefore not repeated here.
Chapter 6: The integrated actuators

Figure 6.7: Artist impression of the vertical actuators.

Table 6.6: The linear constraints of the vertical actuator optimization which are different from those in Table 6.3.

<table>
<thead>
<tr>
<th>Variable Description</th>
<th>Variable Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_{c_v} = 70$ mm</td>
<td>$d_{w_v} = 0.959$ mm</td>
</tr>
<tr>
<td>Actuator width</td>
<td>Wire thickness</td>
</tr>
<tr>
<td>$4l_m + 3l_R \leq 140$ mm</td>
<td></td>
</tr>
</tbody>
</table>

The linear inequality constraints of Table 6.3 have been kept the same except a limited amount of differences that are summarized in Table 6.6. The coil depth and actuator width constraints result from the rotation of the actuator in the available volume. The wire thickness has been fixed to that obtained for the horizontal actuator. The nonlinear constraints (6.21) are equal too, however must hold for the series connection of the two coils.

6.6.1 Optimization results

As for the horizontal actuators the steepness and the force constant have been investigated in separate optimizations. The results are summarized in Table 6.7 and show the difference in performance. As was seen for the horizontal actuator, the force maximization objective minimizes the coil height at the cost of the steepness.

The realized dimensions and properties of the vertical actuators are summarized in Table 6.8. They are different from the results of Table 6.4 which is a result of a number of design fixes during the design process.

6.7 Model validation

The electromechanical and electrical properties of the actuators that have been obtained with the models of Sections 6.3 and 6.4 have been validated using FEM and experimental measurements. In the experimental measurements the coils shown in Fig. 6.8(a) have been potted into stainless steel supports as shown in Fig. 6.8(b). These assemblies are discussed in more detail in Chapter 7 and are mentioned here as they
Table 6.7: Optimization results for the two objective functions. The results are obtained for the two coils electrically in series.

<table>
<thead>
<tr>
<th>Variable Description</th>
<th>$\mathcal{G}_1(\vec{x}) = 1/S$</th>
<th>Variable Description</th>
<th>$\mathcal{G}_2(\vec{x}) = 1/k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S = 514 \text{ N}^2/\text{W}$</td>
<td>Steepness</td>
<td>$S = 367 \text{ N}^2/\text{W}$</td>
<td>Steepness</td>
</tr>
<tr>
<td>$k = 18.8 \text{ N/A}$</td>
<td>Motor force constant</td>
<td>$k = 31.7 \text{ N/A}$</td>
<td>Motor force constant</td>
</tr>
<tr>
<td>$l_m = 19.4 \text{ mm}$</td>
<td>Magnet width</td>
<td>$l_m = 16.3 \text{ mm}$</td>
<td>Magnet width</td>
</tr>
<tr>
<td>$h_m = 22.4 \text{ mm}$</td>
<td>Magnet height</td>
<td>$h_m = 27.5 \text{ mm}$</td>
<td>Magnet height</td>
</tr>
<tr>
<td>$l_b = 18.1 \text{ mm}$</td>
<td>Coil bundle width</td>
<td>$l_b = 11.1 \text{ mm}$</td>
<td>Coil bundle width</td>
</tr>
<tr>
<td>$h_b = 10.1 \text{ mm}$</td>
<td>Coil height</td>
<td>$h_b = 5.0 \text{ mm}$</td>
<td>Coil height</td>
</tr>
<tr>
<td>$l_h = 20.8 \text{ mm}$</td>
<td>Halbach magnet</td>
<td>$l_h = 19.5 \text{ mm}$</td>
<td>Halbach magnet</td>
</tr>
<tr>
<td>$l_{ci} = 16.9 \text{ mm}$</td>
<td>Coil core width</td>
<td>$l_{ci} = 17.8 \text{ mm}$</td>
<td>Coil core width</td>
</tr>
</tbody>
</table>

Table 6.8: The properties of the vertical actuator as they have been realized. The results are obtained for the two coils electrically in series.

<table>
<thead>
<tr>
<th>Variable Description</th>
<th></th>
<th>Variable Description</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_m = 20.0 \text{ mm}$</td>
<td>Magnet width</td>
<td>$S = 455 \text{ N}^2/\text{W}$</td>
<td>Steepness</td>
</tr>
<tr>
<td>$h_m = 22.0 \text{ mm}$</td>
<td>Magnet height</td>
<td>$k = 27.6 \text{ N/A}$</td>
<td>Motor force constant</td>
</tr>
<tr>
<td>$l_b = 17.8 \text{ mm}$</td>
<td>Coil bundle width</td>
<td>$\tau = 2.0 \text{ ms}$</td>
<td>Electrical time constant</td>
</tr>
<tr>
<td>$h_b = 9.8 \text{ mm}$</td>
<td>Coil height</td>
<td>$k_{fcu} = 0.73$</td>
<td>Coil filling factor</td>
</tr>
<tr>
<td>$l_h = 20.0 \text{ mm}$</td>
<td>Halbach magnet</td>
<td>$R = 3.0 \Omega$</td>
<td>Resistance</td>
</tr>
<tr>
<td>$l_{ci} = 14.8 \text{ mm}$</td>
<td>Inner coil width</td>
<td>$L = 3.1 \text{ mH}$</td>
<td>Inductance</td>
</tr>
<tr>
<td>$d_{wu} = 0.959 \text{ mm}$</td>
<td>Wire thickness</td>
<td>$N = 200$</td>
<td>Turns per coil</td>
</tr>
</tbody>
</table>

The force of the actuator has been compared with FEM and the results, summarized in Table 6.5, show high correspondence between both methods. The somewhat higher estimation of the FEM results is due to numerical truncation errors resulting from the mesh size and the simple integration in the semi-analytical model. Although the analytical field prediction is more accurate, the numerical coil approximation introduces truncation errors. A different meshing, integration method or mesh size could improve the semi-analytical results. Direct force measurements of the active actuators have not been performed.

The resistance and inductance of the coil have been compared with FEM results and experimental results. The experimental data have been obtained using an Agilent 4294A impedance analyzer on the coil shown in Fig. 6.8. The analytically obtained resistance and inductance of Table 6.5 correspond well with the FEM and experimental data. It must be noted that the measured resistance includes 3 m of lead wire which is not accounted for in the simulations and is therefore higher.
Figure 6.8: Picture of (a) the coil horizontal actuator coil and (b) the support bodies in which they have been potted.

Table 6.9: The dimensions of the vertical actuator as they have been implemented. The results for the vertical actuators are obtained for the two coils electrically in series.

<table>
<thead>
<tr>
<th></th>
<th>Analytical</th>
<th>FEM</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Horizontal actuator</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Force constant</td>
<td>30.3 N/A</td>
<td>32.2 N/A</td>
<td>-</td>
</tr>
<tr>
<td>Inductance</td>
<td>3.3 mH</td>
<td>3.31 mH</td>
<td>3.38 mH</td>
</tr>
<tr>
<td>Resistance</td>
<td>1.5 Ω</td>
<td>1.46 Ω</td>
<td>1.92 Ω</td>
</tr>
<tr>
<td><strong>Vertical actuator</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Force constant</td>
<td>27.6 N/A</td>
<td>28.6 N/A</td>
<td>-</td>
</tr>
<tr>
<td>Inductance</td>
<td>3.08 mH</td>
<td>3.03 mH</td>
<td>3.02 mH</td>
</tr>
<tr>
<td>Resistance</td>
<td>1.53 Ω</td>
<td>1.55 Ω</td>
<td>2.00 Ω</td>
</tr>
</tbody>
</table>

6.8 Magnetic cross-coupling

The actuators have been placed in the gravity compensator such that their vertical center is aligned with that of the magnets of the inner cross, i.e. at 80mm from the bottom of the inner cross. This is shown in Fig. 6.9 where the magnets of the inner cross, the outer cross and the actuators are marked with different grey tones in a scaled top view and side view. The distance between the permanent magnets of the actuators and those of the outer cross is 28mm. The distance between these actuator magnets and the magnets of the inner cross is therefore more than 38mm. Simulations performed with the analytical model have shown that the actuator magnets interacting with the inner cross causes a force of less than one newton and it is concluded that their influence is minimal.

6.9 Contributions

- **Section 6.2.1** - The publications related to this chapter have demonstrated the feasibility to use ironless Lorentz actuators with quasi-Halbach arrays compared to more conventional Lorentz actuators with back-iron. Their performance in terms of the steepness $S$ and force constant $F$ are comparable.

- **Section 6.4** - Renewed analytical equations to calculate the inductance of a current-carrying bar have been proposed to better integrate with the interaction
6.10 Conclusions

This chapter discussed the design of the ironless Lorentz actuators for integration into the gravity compensator. The four corners of the gravity compensator have been used to house two horizontally oriented and two vertically oriented actuators. The current density in the coil is described by a piecewise continuous function and numerical integration provides the Lorentz force on the coil. From the coil data the number of windings and the resulting resistance, inductance and electrical time constant have been obtained. The inductance calculation is based on analytical equations that have been derived for a bar-shaped conductor. The actuators have been optimized for a maximized steepness and actuator force constant.

The realized actuators’ dimensions differ somewhat from the optimized results as a result of intermediate design fixes. The performance of these actuators have been validated with FEM and in-house measurements. The correspondence between the various models is high, although some discrepancy in the force calculation has been found which is due to the meshing in the semi-numerical model. A simulation of the gravity compensator’s performance including the translator magnets has shown their limited influence on the gravity compensator’s force.

6.10.1 Recommendation

- The bandwidth set for the actuators in Section 6.2.1 is 1 kHz and is meant as a worst-case scenario. The inductive voltage that results from this bandwidth has been an important constraint on the actuator design. A design with less constraints on this bandwidth enables an increase in as well steepness S as force constant k.

- The permanent magnets that have resulted from the optimizations are rather
thick. An investigation into the sensitivity of the actuator performance to this magnet thickness may result in a design which exhibits thinner magnets.

- The proposed analytical vector potential equations for the coil-carrying bar are integrated numerically to obtain the inductance. A further step would be the fully analytical calculation of the coil inductance.

- The magnetic shielding of the actuator has not been investigated. Partly as a result of the large magnets and partly as a result of the absence of flux focusing materials the flux leakage of this topology is relatively high. Another feature which could improve the shielding and force density is the addition of an extra Halbach magnet at the sides of the magnet arrays.

- A more advanced meshing and integration in the semi-analytical actuator model would improve its force results. They are too low in the current model, which is due to an integration that is one mesh element smaller than the actual coil dimensions.
Chapter 7

Experimental setup

About the realization of the electromagnetic vibration isolator, the test rig to evaluate its performance and the measurements that have been conducted to characterize validate the design and implementation.
The passive gravity compensator with integrated 2-DoF actuators has been realized as a prototype. It is integrated in a test rig with inertial accelerometers for an accurate measurement of the vibration isolator’s performance. A shake rig enables to control the floor vibrations for accurate transmissibility measurements. By means of static measurements the proof-of-principle of the electromagnetic vibration isolation system is shown.

7.1 Outline

Section 7.2 describes the test setup that facilitates the characterization and evaluation of the electromagnetic vibration isolator. The design and realization of the prototype of this vibration isolator is shown in Section 7.3 and the external actuators that are necessary for stabilization purposes in Section 7.4. The remaining components of the test rig are described in Section 7.5. The static measurements, described in Section 7.6, include the necessary calibration of the actuators and gravity compensator and describe the results obtained from a static identification and Section 7.7 describes the measurement of the passive wrench. The dynamic simulations in Section 7.8 are then used to obtain an estimation for the stiffness and vertical force of the gravity compensator. Section 7.9 presents the conclusions, contributions and recommendations of this chapter.

7.2 The test setup

Instead of the three vibration isolators placed on the symmetry lines of the triangular setup, as discussed in Section 6.2, a single vibration isolator has been realized. Its gravity compensator exhibits the necessary passive vertical force, however its two active degrees of freedom are insufficient to guarantee six-DoF stable operation. For this reason, external actuators are included that provide the 6-DoF actuation that is necessary to stabilize the system. The resulting vibration isolation system is placed on a shake rig that may be used for evaluation of the floor vibration rejection. By means of leaf springs this rig may be attached to the lab floor for compliance measurements.

Figure 7.1(a) shows the simplified 1-DoF free body diagram of the test rig with the various components indicated. As the three external two-DoF actuation units are placed on the symmetry angles of the triangular setup their horizontal actuation axes have 60° intermediate angles as Fig. 7.1(b) shows. In an ideal situation, where the gravity compensator’s force exactly matches the gravitational force of the isolated platform, these actuators only have to deliver dynamic force and no static force, i.e. no gravity compensation or parasitic static force compensation. These components are placed on a shake table which rests on mechanical springs and is excited by three vertical actuators, hence, is three-DoF controllable. Position sensors measure the distance between the isolated platform and the shake table, necessary for stabilization. Accelerometers are used to measure the absolute acceleration of both platforms. The components in Fig. 7.1 are discussed in the following sections.
7.3 Vibration isolator

The vibration isolator comprises the cross-shaped gravity compensator of Chapter 5 with the integrated 2-DoF actuation discussed in Chapter 6.

7.3.1 Gravity compensator

Section 5.8 introduces the inner cross and outer cross that are distinguished in the gravity compensator. Fig. 7.2 shows an artist impression of the gravity compensator with the inner cross (grey) and the outer cross (white) indicated. As well the inner cross as the outer cross have 64 permanent magnets (Vacodym 854TP) glued onto a non-magnetic (industrial grade Al 6082) support. For the inner cross this is a single cross-shaped structure. The magnets of the outer cross are glued onto four sections which are bolted together as Fig. 7.2(a) shows. The aluminum cross-shaped support of the inner cross is mounted to an aluminum base block which raises the gravity compensator's geometrical center. This reduces the demands on the actuators control system as this brings the center of the gravity compensator closer to the center of gravity (CoG) of the isolated platform, which is discussed in Section 7.5.4. The outer cross supports the isolated platform through an aluminum flange which is mounted on its top surface.

As discussed in Section 3.9 and Section 5.10, the manufacturing process of permanent magnets is subject to certain tolerances. Measurements have been undertaken to
determine the hysteresis curve of the permanent-magnet material prior to assembly. It is found that the mean remanent flux density of the five samples from the total batch is 1.309T and that the relative permeability equals 1.03. Both values are close to the theoretical values that have been used in the design of the gravity compensator.

7.3.2 Integrated actuators

The actuators proposed in Chapter 6 are integrated into the corners of the gravity compensator. Their translators with the permanent magnets are integrated into the outer cross as discussed in Section 6.2. Fig. 7.2(b) shows an impression with both parallel vertical actuators visible. The horizontal actuators are located at the other side of the device and are therefore not visible in this figure. The stator coils of all actuators are mounted to the same aluminum base as the inner cross by means of stainless steel support bodies. These supports are manufactured of 316 stainless steel and the coils are potted into these bodies. They are mounted to the ‘floor’ platform as this is the most suitable place to remove any heat produced in the coils. Especially if the system isolates sensitive equipment or if it is placed in vacuum conditions it is difficult to remove heat from the isolated platform.

One of the design purposes set in Section 4.9 is the minimization of the heat injection into the isolated platform and as such the actuator coils are equipped with watercooling. Fig. 7.2(b) shows the entries of the cooling ducts in the aluminum base.
Figure 7.3: The top end-plates of the inner and outer cross (a) create the pre-tension that is required and align the arrays when the device is unloaded. The structure in (b) aligns both parts when the spring is unloaded.

which are internally distributed to the stator support structures. These channels are integrated into the coil support bodies. The coil temperature is monitored by means of 4-wire PT100 temperature sensors which are potted into the cores of the actuator coils.

### 7.3.3 End-stops

The magnetic gravity compensator exhibits a vertical force of more than 7kN and requires an end-stop to pre-load the magnetic spring. An additional functionality of this end-stop is the alignment of the gravity compensator when its passive force exceeds the gravity force that loads it.

The pre-tension of the magnetic gravity compensator is accomplished by end-plates on the inner cross and outer cross, which are shown in Fig. 7.3. The end-plate of the inner cross prevents the upward movement of the outer cross. The alignment structure, shown in Fig. 7.3(b), consists of a Delrin cylinder mounted to the outer cross which falls into a V-shaped slot in the end-plate of the inner cross. Although that this alignment structure mechanically over determines the system in this top position it is considered suitable for this system as it is not part of the working envelope of the device. Landing pads made of the same material are glued to the aluminum base and serve as end-stop for downward movement of the device. These bottom end-stops are shown in Fig. 7.4(a).

The aluminum support structures of the inner and outer cross have been designed such, that the brittle magnet material can not touch other magnets or supports. The gravity compensator’s airgap length (Section 5.9.2) is significantly larger than the envisaged stroke of $\pm 1$ mm which minimizes the risk of collision. Removable Delrin guidance strips, shown in Fig. 7.3(a), limit the horizontal movements of the gravity
compensator and as such provide an extra safety margin.

### 7.3.4 Assembly

Figure 7.4 shows an impression of the inner and outer cross with the integrated actuators that have resulted from the considerations above. A photo of the assembled vibration isolator is shown in Fig 7.5.

### 7.4 External actuators

As discussed, three 2-DoF actuator units are placed on the three corners of the test rig. Figure 7.1(b) shows that these units are placed on the symmetry axes of the regular triangle and incorporate a vertical actuator and one actuator in the circumferential
7.5: Test rig

The magnetic design of both integrated actuators is equal to that of the horizontal actuator discussed in Section 6.5. For the vertical external actuator this design is simply rotated 90°.

Figure 7.6 shows the external actuators after assembly of the stator and translator parts. The translator parts in Fig. 7.6(a) consist of an aluminum structure with two magnet arrays glued into it. The magnetization of the array belonging to the horizontal actuator is shown and that of the vertical actuator is rotated by 90°. The stator coils, shown in Fig. 7.6(b), are potted into 316-grade stainless steel supports. These are mounted on a central adapter that is fixed to the floor, or in this case the shake rig.

7.5 Test rig

The test rig incorporates the vibration isolator, the external actuators that stabilize it and the payload that is isolated.

7.5.1 Isolated platform

The isolated platform is levitated by the gravity compensator and stabilized by the actuators. It incorporates the components shown in Fig. 7.7 and summarized in Table 7.1. According Fig. 7.1(b) the top view of the test rig is based on a regular triangle. The gravity compensator is placed in the middle of this triangle and is vertically close to the center of gravity (CoG) of the isolated platform (Section 7.5.4). Further, the corners of the triangles are cut to improve the dynamic properties and the actuators are placed at the bottom of the isolated platform. Hence, their CoA is...
Figure 7.7: The isolated platform incorporates a granite table, the outer cross, the flange that connects them, the translator parts of the external actuators and the stainless steel disks on top of it.

Table 7.1: The individual and cumulative mass in the isolated platform, and that on the shake table.

<table>
<thead>
<tr>
<th>Component</th>
<th>Material</th>
<th>Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Granite table + flange</td>
<td>Granite</td>
<td>600 kg</td>
</tr>
<tr>
<td>Outer cross</td>
<td>Aluminum / Vacodym 854</td>
<td>42.2 kg</td>
</tr>
<tr>
<td>Ext. actuator translator</td>
<td>Aluminum / Vacodym 854</td>
<td>3·10.7 kg</td>
</tr>
<tr>
<td>Additional mass</td>
<td>Stainless steel</td>
<td>3·18 kg</td>
</tr>
<tr>
<td><strong>Isolated mass</strong></td>
<td></td>
<td><strong>728.3 kg</strong></td>
</tr>
<tr>
<td>Inner cross</td>
<td>Aluminum / Vacodym 854</td>
<td>27.7 kg</td>
</tr>
<tr>
<td>Aluminum base</td>
<td>Aluminum</td>
<td>19.7 kg</td>
</tr>
<tr>
<td>Ext. actuator stator</td>
<td>Aluminum / Stainless steel / Copper</td>
<td>3·4.0 kg</td>
</tr>
<tr>
<td>Shake table</td>
<td>Aluminum</td>
<td>285.7 kg</td>
</tr>
<tr>
<td>Translator of shake actuator</td>
<td></td>
<td>3·11 kg</td>
</tr>
<tr>
<td><strong>Shaker mass</strong></td>
<td></td>
<td><strong>378 kg</strong></td>
</tr>
<tr>
<td>Total mass of the magnets in the gravity compensator</td>
<td></td>
<td>12.7 kg</td>
</tr>
</tbody>
</table>

located significantly lower than the CoG of the isolated platform. These centers are discussed more elaborately in Section 7.5.4. The granite table has been designed to exhibit its first internal resonance at a frequency of more than 1 kHz.

**Mass**

The cumulative mass of the isolated platform has been chosen such, that the chance that the gravity force exceeds the passive vertical force of the gravity compensator is minimized. Stainless steel disks, placed on the three symmetry axes of the granite table, are used to minimize the difference between the gravity force of the isolated
platform and the passive vertical force of the gravity compensator. They are shown in Fig. 7.7. The flange, the outer cross, the external actuator translators and the sensors mounted on this platform add to the isolated mass. As such, their cumulative mass is subtracted from the minimum passive vertical spring force to obtain the mass of the granite table. Given the modeling inaccuracies and manufacturing tolerances of the gravity compensator, discussed in Section 5.10, and the expected manufacturing tolerances of the granite stone, the minimum achievable force level has been estimated at 6.8 kN. After subtraction of the components mentioned above, the mass of the granite stone has been determined. The necessary additional mass of the stainless steel disks has been obtained empirically.

### 7.5.2 Shake rig

The shake rig generates controlled vibrations to a shake table, which acts as an artificial floor for the vibration isolation system. This feature is especially useful in dynamic transmissibility measurements as the vibrations can be monitored and controlled. The rig consists of a shake table which is placed on mechanical coil springs and is excited by commercially available voice coil actuators. This shake rig is placed on a steel floor platform which is fixed onto the lab floor as Fig. 7.8 shows.

The aluminum shake table has been designed to exhibit its first resonance above 1 kHz and as such is sufficiently stiff to be treated as a rigid body. Additionally, its mass has been minimized to reduce the actuator forces. It is equipped with a mounting hole pattern on the top, as shown in Fig. 7.8(a), to be suitable for testing other devices.
Chapter 7: Experimental setup

The test rig includes the shake rig, the external actuators, the vibration isolator and the isolated platform. The cable bridge is shown, which leads power and sensor cabling to the shake table.

Figure 7.9: The test rig includes the shake rig, the external actuators, the vibration isolator and the isolated platform. The cable bridge is shown, which leads power and sensor cabling to the shake table.

The shake table is sprung at approximately 10Hz - 30Hz (six DoF's) by custom coil springs which exhibit their first internal (parasitic internal) resonance above 500Hz (see Appendix E.1). The high stiffness of these coil springs also reduces leveling, pre-compression and parasitic resonance problems associated with soft springs. For measurements of the static behavior and of the compliance it is necessary to fix the shake table. This is done by placing steel leaf springs, shown in Fig. 7.8(a), parallel to the coil springs, which increases the resonance frequency of the shake table.

The actuators that excite the shake table are commercial LA50-65-001Z actuators driven by Prodrive PADC Quad 260/50 current amplifiers (Appendix E.5). According to their specifications these actuators are capable of producing 500N continuous and a peak force of 1kN. Three vertically placed actuators excite the system in three degrees of freedom as Fig. 7.8 shows and as such can generate a peak force of 3kN in the vertical direction up to almost 1kHz. Although there are no horizontal actuators in the current shake rig, mounting facilities have been foreseen underneath the shake table and on the floor platform to accommodate these actuators if necessary.

7.5.3 Sensors

The relative position between both parts of the gravity compensator must be known to stabilize the vibration isolator. These sensors are not fully integrated in the topology but are placed as close as possible to the CoAs of the gravity compensator and the external actuators. To minimize the interference with the strong magnetic fields and for their excellent resolution for a large stroke, optical sensors are used to measure the relative position. These sensors (Philtec RC100-AELN) measure the amount of reflected light on a specular target surface and as such provide absolute distance measurement.
Table 7.2: CoA coordinates with respect to the isolated platform’s CoG that have been obtained from the CAD model of the test rig.

<table>
<thead>
<tr>
<th>Object</th>
<th>Property</th>
<th>Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>External horizontal actuators</td>
<td>CoAₜ</td>
<td>$r_τ = 485 \text{ mm}$ $h_τ = -136 \text{ mm}$</td>
</tr>
<tr>
<td>External vertical actuators</td>
<td>CoAᵥ</td>
<td>$r_ν = 412 \text{ mm}$ $h_ν = -136 \text{ mm}$</td>
</tr>
<tr>
<td>Gravity compensator</td>
<td>CoAₙₑ</td>
<td>$r = 0 \text{ mm}$ $h_{g_e} = -49 \text{ mm}$</td>
</tr>
</tbody>
</table>

An inertial measurement is necessary to measure the absolute vibration of the floor and isolated platform. The 1-DoF capacitive accelerometers (Kistler 8330M04) that have been used provide such inertial data. Each platform accommodates six of these sensors which measure the acceleration in the vertical and circumferential directions. As a result of their capacitive-based measurement principle these sensors are capable of measuring also low frequency components, whilst their bandwidth is still suitable to measure up to 1 kHz. The A/D convertor of the data acquisition system has a limited resolution and therefore the acceleration signals are amplified. More detailed sensor data for the accelerometers and the position sensors is found in Appendix E.5.

Figure 7.9 shows the test rig that has been realized. The figure also shows the cable bridge that carries sensor and power cables to the shake table. This bridge is mounted to the 19” racks and is not connected to the test setup. More detailed information on the components used in the test rig is found in Appendix E.

7.5.4 Pre-determined coordinates

Table 7.2 summarizes the centers of actuation (CoAₙₑ) of the gravity compensator and of the external actuators with respect to the pre-determined center of gravity of the isolated platform. The external actuators’ CoAᵥ (vertical actuators) and CoAₜ (horizontal actuators) are defined in the center of the coil, as discussed in Chapter 6. It has been deemed most suitable to place three 2-DoF actuators on the symmetry planes of the test rig. The CoAₙₑ of the gravity compensator is the geometrical center of its magnet assembly. The center of gravity, or CoG, of the isolated platform includes the granite table, the outer cross with its flange and the translator parts of the external actuators. This CoG moves with the outer cross if it is displaced with respect to the inner cross.

Figure 7.10 shows a top view and cross-section of the test setup. A number of angles are indicated in Fig. 7.10(a) as well as symmetry lines $aa'$, $bb'$ and $cc'$ and the positive force directions of the various actuators. The cross-section in Fig. 7.10(b) is through these symmetry lines and shows that the CoG of the isolated platform, the CoA of the gravity compensator and the actuators and the radii of these actuators. The gravity compensator is schematically shown as a spring.
To prove the principle of passive magnetic gravity compensation for high forces and low stiffness, i.e. the design of chapter 5, static measurements have been performed. During these static measurements, the passive gravity compensator lifts the granite table and three 2-DoF external actuators provide active stabilization. Six static-decoupled and low-bandwidth PID controllers stabilize the six degrees of freedom. The static decoupling is proposed in [53, 54] and is not part of this thesis.

### 7.6.1 Equilibrium of wrenches

The transformation from the individual forces and locations of the various components in the setup to the wrench $\vec{w}$ is described in Appendix E.3. Further details, including the transformations of the various sensors and the decoupling, are found in [53]. The wrench $\vec{w}$ includes the three force components along the Cartesian axes and three torque components around these axes and is defined as

$$\vec{w} = [F_x, F_y, F_z, T_x, T_y, T_z]^T.$$  

(7.1)

The wrench of the gravity compensator $\vec{w}_{mag}$, that of the gravity force acting on the isolated platform $\vec{w}_{grav}$ and the actively generated wrench $\vec{w}_{act}$ are in assumed to be in equilibrium

$$\vec{w}_{mag} + \vec{w}_{grav} + \vec{w}_{act} = 0.$$  

(7.2)

They are related to the same point that is used in Section 5.9.2. As such, the wrench of the isolated platform $\vec{w}_{grav}$ has a vertical gravity force $F_{grav} = -mg\hat{z}$ acting on its CoG. As a result of its horizontal movement this force exhibits a net torque $T_{grav}$
with respect to the non-moving coordinate system that equals

$$\vec{T}_{grav} = \begin{pmatrix} yF_{grav} \\ -xF_{grav} \\ 0 \end{pmatrix}, \quad (7.3)$$

where $x$ and $y$ are the displacement along $\vec{e}_x$ and $\vec{e}_y$, respectively. The resulting wrench of the isolated platform is given by

$$\vec{w}_{grav} = \begin{bmatrix} 0, 0, -mg, -ymg, xmg, 0 \end{bmatrix}^T. \quad (7.4)$$

The vertical force component is compensated by the vertical force of the gravity compensator, whereas the torque components of this wrench are compensated by the actuators.

During the static measurements, the system is considered to be settled if the average position error $|\epsilon_{pos}| \leq 0.5 \mu m$ for all DoF’s and the noise level is within the position sensor noise. The center of the horizontal displacements have been determined empirically as the middle of the available stroke. Based on the manufacturing tolerances this is a sufficiently accurate estimation.

### 7.6.2 Characterization of the force constant and temperature dependency

A first and necessary step in the measurement of the vibration isolator is the calibration of the various components. The actuators which are discussed in Chapter 6 have only been compared to FEM results as no direct measurements, i.e. with a load sensor, of the actuator force constant have been undertaken. A characterization of this force constant has been performed indirectly. It is necessary for the static and dynamic measurements of the vibration isolator.

Further, the field strength of permanent magnets is temperature-dependent as Sections 3.9.2 and 5.10 show. As such, the vertical passive force is affected by the temperature of the permanent magnets. Both of these properties have been characterized to increase the accuracy and precision of further measurements.

#### Actuator force constant

The force constant $k$ of the external vertical actuators has been measured indirectly by means of the actuator current. For a given 6-DoF position/rotation setpoint various masses were added to the isolated platform of which the vertical force $\Delta F_z$ was compensated fully by the actuators (no position change). The change in current $\Delta I$ was measured and the force constant obtained by

$$k = \frac{\Delta F_z}{\Delta I} = \frac{\Delta m g}{\Delta I}, \quad (7.5)$$

where $\Delta m$ represents the added mass and $g$ is the gravitational acceleration. Here, it
is assumed that $k$ is equal for the actuators. The results are summarized in Table 7.3 and show that the results for the three different loads correspond within 0.5%. As such, the actuator force constant for the vertical actuators is defined at 31.75 N/A. It is assumed that the force constant of the horizontal actuators, considering that they have the same magnetic design, is equal.

**Temperature dependency**

Section 5.10 predicts that the temperature sensitivity of the gravity compensator’s vertical force is approximately $2\%/K$. Variations in the ambient temperature influence the vertical passive force and are actively compensated by the actuators if the 6-DoF setpoint is maintained at a constant value. This principle was applied in this measurement in which the ambient temperature was recorded using the 4-wire PT100 temperature sensors potted in all actuator coils. As the actuators inside the vibration isolator were not active – only the external actuators were used for stabilization – it was assumed that their temperature sensors were a close representation of the gravity compensator’s ambient temperature. As a result of the heat capacity of and airgap between the sensors and the magnets of which the temperature was estimated the temperature gradient was minimized. As such, the ambient lab temperature was used as input parameter instead of an active heating system, as its variation in time tends to be very low.

The results of these measurements are shown in Fig. 7.11. The curve is composed of six measurements performed at different days. Apart from some discontinuities in the characteristic such as that at 22.8°C the characteristic is very linear. Such discontinuity could be the result of a temperature gradient in time during a measurement. The absence of measurement results between 23.3°C and 23.7°C is due to the fact that the ambient lab temperature could not be controlled during the measurements. A first order polynomial approximation of the results seems a good approximation and yields a temperature constant $k_T$ of 12.1 N/K for the vertical gravity compensator’s passive force. This corresponds to 1.7%/K temperature influence on the vertical force. Although this is small in comparison with the gravity compensator’s passive force, it is certainly not negligible in comparison to the force range of the active actuators.

**Table 7.3: Measurement of the actuator force constant.**

<table>
<thead>
<tr>
<th>$\Delta m$</th>
<th>$\Delta i$</th>
<th>Force constant $k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.003 kg</td>
<td>0.3095 A</td>
<td>31.79 N/A</td>
</tr>
<tr>
<td>2.006 kg</td>
<td>0.6182 A</td>
<td>31.83 N/A</td>
</tr>
<tr>
<td>8.028 kg</td>
<td>2.4879 A</td>
<td>31.66 N/A</td>
</tr>
</tbody>
</table>
7.7: Verification of the passive wrench

In reality, the measurements for the actuator force constant and the temperature dependency are coupled with each other. On one hand, the ambient temperature should be part of the actuator force constant measurement to compensate for temperature variations during this measurement. Conversely, the temperature dependency of the vertical force is scaled with actuator force constant as the force is measured indirectly with the current. As such, they cannot be fully decoupled. In this thesis the actuator force constant was measured first, subsequently the temperature dependency was measured and with this data the actuator force constant has been determined again. The resulting error is considered to be small enough to be regarded as measurement noise.

The temperature dependency of the force is only applied to the vertical offset force of the gravity compensator as it’s magnitude of $1.7 \% / \text{K}$ is only observable in relation to this large offset force. The other forces and torques of the wrench (7.1) are of such low value that its influence becomes of negligible importance.

7.7 Verification of the passive wrench

The active wrench $\vec{u}_{\text{act}}$ has been used to obtain the passive wrench of the gravity compensator $\vec{u}_{\text{mag}}$ according (7.2). This active wrench has been measured for a number of position and rotation setpoints and the results are described below.

The active wrench of the system was measured through the working envelope shown in Fig. 7.12. This working envelope has sides of 1 mm in the three Cartesian
directions. The rotations around the respective axes were kept zero during these measurements. The wrench was measured in the three horizontal planes for $z \in \{-0.5, 0, 0.5\}$ mm where $\hat{e}_x$ and $\hat{e}_y$ were varied in steps of 0.1 mm, yielding 120 points per plane and 360 points in total.

Data processing

The active wrench is obtained from the individual actuator forces using the transformation matrix $E.1$ in Appendix E.3. The point to which they are related is the same as in Section 5.9.2, i.e. the geometrical middle of the gravity compensator. According (7.2) the wrench of the gravity compensator is obtained by

$$\vec{w}_{mag} = -\vec{w}_{grav} - \vec{w}_{act}. \quad (7.6)$$

The position-dependent wrench $\vec{w}_{grav}$, given by (7.4), has been transformed to the actuator forces using the inverse of matrix (E.1) (with $h_{gc} = 0$ as the CoG is concerned). It is transformed to the gravity compensator coordinates with (E.1). The result of (7.6) is shown in Fig. 7.13 and Fig. 7.14. The vertical control force $F_{z,act}$ has been corrected for the ambient temperature as discussed in Section 7.6.2.

Results

The measurement results of Fig. 7.13 and Fig. 7.14 are compared with the theoretical results of Section 5.9.2. It stands out that the vertical displacement along $\hat{e}_z$ hardly has an influence on the force and torque components as in all figures the three planes are close and parallel. In fact, the three measured planes of the vertical force component in Fig. 7.13 are of such equal value, that a vertical stiffness $K_{zz}$ is difficult to determine from these static measurements. This confirms the low envisaged vertical stiffness that is one of the main design objectives of this vibration isolation system.

The horizontal force components in Fig. 7.13(a)-(b) exhibit a low sensitivity to displacement along $\hat{e}_z$, hence $\partial F_x/\partial z$ and $\partial F_y/\partial z$ are low. However, $F_x$ has a strong dependency on displacement along $\hat{e}_x$, and similarly $F_y$ is related to the displacement along $\hat{e}_y$. Compared to Fig. 5.33 the resulting position dependency that
7.7: Verification of the passive wrench

Figure 7.13: The measured force components $[F_x, F_y, F_z]$ from the wrench $\vec{u}_{\text{mag}}$. The measured planes are at $z = -0.5\text{mm}$ (red), $z = 0.0\text{mm}$ (orange) and $z = 0.5\text{mm}$ (yellow).
Figure 7.14: The measured torque components $[T_x, T_y, T_z]$ from the measured wrench $\mathbf{w}_{\text{mag}}$. The measured planes are at $z = -0.5\text{ mm}$ (red), $z = 0.0\text{ mm}$ (orange) and $z = 0.5\text{ mm}$ (yellow).
has been measured is too high. It is possible that there are unaccounted mechanical and or magnetic inaccuracies in the assembly which are the cause of this high stiffness. For example, the actuator coils are potted in 316-stainless steel supports, which theoretically have almost no magnetic behavior. Nevertheless, some residual magnetism remains in the material after processing. This has been observed during the assembly of the setup in the form of an attraction force between these supports and the permanent magnets of the actuators. This (unstable) attraction force may be influencing the gradient of these characteristics. Further, assembly tolerances may be due to these discrepancies, which are within the tolerances discussed in Section 5.10. It is concluded that the horizontal force components are not behaving as specified, however, remain sufficiently low and are almost linear. Further, the required compensation force level remains well below the actuator and amplifier limits. Such errors may seem large in comparison with the limited actuator force levels, however are minute when compared with the vertical passive force of 7.1 kN.

The three measurement planes shown in Fig. 7.13(c) show that the absolute values of the cross-couplings $\partial F_z/\partial x$ and $\partial F_z/\partial y$ remain low. The torque around the $z$-axis in Fig. 7.14(c) has only a minor dependency on displacements, although it does have an offset of approximately $-5\text{Nm}$. Figure 7.13(c) and Fig. 7.14(c) exhibit a slightly asymmetrical behavior with respect to $\hat{\delta}_x$ and $\hat{\delta}_y$, however with a very low magnitude. As such, this asymmetry is most probably the result of assembly tolerances.

The force characteristics in Fig. 7.13 exhibit an offset force. It is possible that this offset is caused by tolerances in the assembly of the vibration isolator, as these tolerances sum up when various components are bolted together. $F_x$ becomes zero for $x \approx -0.2\text{mm}$ and $F_y$ becomes zero for $y \approx 0.1\text{mm}$. This suggests that the real center position of the gravity compensator is at this point, instead of the center points in the figures, which have been derived from the available stroke.

The average position dependencies of the forces in Fig. 7.13 have been collected in the stiffness matrix $K$

$$K = \begin{pmatrix}
\frac{\partial F_x}{\partial x} & \frac{\partial F_x}{\partial y} & \frac{\partial F_x}{\partial z} \\
\frac{\partial F_y}{\partial x} & \frac{\partial F_y}{\partial y} & \frac{\partial F_y}{\partial z} \\
\frac{\partial F_z}{\partial x} & \frac{\partial F_z}{\partial y} & \frac{\partial F_z}{\partial z}
\end{pmatrix} = \begin{pmatrix}
-60\text{kN/m} & 1\text{kN/m} & 1\text{kN/m} \\
0\text{kN/m} & -75\text{kN/m} & -0.5\text{kN/m} \\
2\text{kN/m} & -0.5\text{kN/m} & 0\text{kN/m}
\end{pmatrix} \quad (7.7)
$$

This matrix confirms that, except for the large values of $\frac{\partial F_x}{\partial x}$ and $\frac{\partial F_y}{\partial y}$ all cross-couplings have low values. It is not fully symmetrical along its diagonal as the low stiffness values are near the measurement noise.

The torque components in Fig. 7.14 have offsets, which are presumably mainly related to assembly tolerances in the test rig. Their dependencies on horizontal displacement are more in the line of expectation than the observed forces. However, the offset in the torque $T_z$ is significantly higher than expected, especially since the gravity compensator should be symmetrical and as such exhibit a very low torque in at least its center point. As for the force components discussed above, it is concluded that the torque levels remain sufficiently low and linear.
Discussion

The force and torque components that have been measured remain sufficiently low and linear to be actively compensated by the actuators. However, the sensitivity of some components to horizontal displacement is larger than expected. Although a variety of modeling errors and manufacturing tolerances are discussed in Section 5.10, this list is by no means exhaustive. It could well be possible that there are other inaccuracies which have their influences on the force and torque components of the gravity compensator as discussed above. Further, it is possible that a summation of these modeling inaccuracies and manufacturing and/or placement tolerances may have led to the inaccuracies seen in the measurement. Even a manufacturing error in the gravity compensator is not unthinkable, considering the large amount of magnets involved. Other variations could, for example, be the relative permeability of permanent magnets which is different perpendicular to the magnetization vector or the presence of stainless steel coil supports as discussed above. Additionally, due to the manufacturing tolerances and the bolted connections it is possible that there are displacements in the position and intermediate angles of the sensors, actuators, etc. Further, linearization inaccuracies and model errors could be present in the transformation matrices. Nevertheless, it is concluded that the discrepancies that are seen are sufficiently low to have a minimal effect on the closed-loop system. Although not presented in this chapter, measurements have been conducted which show that the influence of such relative rotation on the wrench is at least an order of magnitude lower than the discrepancies seen above.

7.7.1 Power consumption

The dissipated power of the actuators has been estimated from the wrench measurements described above. For each actuator the dissipated power is obtained by...
7.8: Dynamic measurements

The static measurements do not provide a full characterization of the electromagnetic vibration isolator and as such mainly serve as a proof-of-principle of the gravity compensator of Chapter 5. Simple dynamic measurements have been conducted to gain more insight into the behavior of the gravity compensator. In these measurements the vibration isolation system was stabilized with the same controller as in the static measurements. The dynamic behavior of the system along its vertical axis was identified by means of noise injection, i.e., a compliance measurement, into the vertical axis of its active wrench. From the results the passive behavior of the plant has been identified. The results have been used to verify the mass of the isolated payload, and to estimate the damping and stiffness of the vibration isolation system. An elaborate 6-DoF system identification and advanced isolation performance evaluation is not part of this thesis, which focused on the modeling, design and proof-of-principle of the electromagnetic vibration isolator.

7.8.1 Measurement results

Three separate measurements have been conducted to identify the passive behavior of the vibration isolator between 0.01Hz and 1kHz. For very low frequencies ranging from 0.01Hz - 0.5Hz a multi-sine identification has been used (measurement A) in which the transfer function is measured with the optical position sensors. A 10Hz low-pass filtered white noise injection has lead to the results of measurement B, in which the transfer function has been measured with the position sensors too. At elevated frequencies displacement amplitude disappears in the noise level of the position sensors. As such, the results of measurement C are based on accelerometer measurements.

The respective compliance characteristic of the plant from the measurements A (crosses), B (light green) and C (dark red) are shown in Fig. 7.16 for the vertical displacement. The frequency responses of the three measurements are overlapping and consistent. The first major resonance in the system occurs above 700Hz. The black solid line is a curve fit of the plant which has been used to derive the mass from this dynamic measurement. Its transfer function is similar to (B.5) with the difference that the displacement is studied and not the acceleration, which introduces a scaling of $s^2$. The isolated mass $m$ of the fitted system is 730kg, its vertical stiffness is approximately 500N/m and its damping is $1.1 \cdot 10^3$ Ns/m. The mass corresponds to the value in Table 7.1 and stiffness corresponds to the values obtained in Section 5.9.2.
Figure 7.16: Passive frequency response of the vibration isolator (compliance of the vertical displacement).

The damping is relatively high, as can be observed from the absence of a resonance peak in the frequency response. Pneumatics between the shake table and the isolated platform or (damping) eddy currents that may occur in the various materials could be due to this. Although this has not been investigated in detail – an estimation in Chapter 5 based on the geometry predicted a ‘low’ damping value – it is expected that only a modest part of this damping is of electro-dynamic nature. The distance between the magnets on one part of the gravity compensator and the conductive materials on the other part is large and the vibration velocities are small.

The measurement results presented in this chapter have demonstrated the proof-of-principle of the electromagnetic vibration isolation system by means of static measurements and a simply dynamic measurement. Further experiments should be undertaken to investigate cross-coupling within the vibration isolator, transmissibility, improved vibration isolation by inclusion of accelerometers in the loop, etc. These are found in the recommendations.

7.9 Conclusions, contributions and recommendations

7.9.1 Conclusions

This chapter has presented the realization of the electromagnetic vibration isolation system prototype and the design and realization of a test rig suitable to validate its performance. An indirect verification of the actuator force has confirmed the conclu-
sions of Chapter 6. Although the ambient temperature of the gravity compensator has an effect of $1.7 \% / K$ on the vertical force of the gravity compensator, this effect is in the same order of magnitude as the control force and as such must be accounted for in the measurement results.

The static measurements have shown that a magnet-based gravity compensator is capable of levitating 730 kg with a power consumption of only 0.3 W. Further, this gravity compensator exhibits a vertical stiffness $K_{zz}$ below 1 kN/m, which yields a resonance frequency in the sub-hertz region. The electromechanical behavior of the system seems to be not fully equal for $\hat{e}_x$ and $\hat{e}_y$, which is most probably caused by manufacturing and assembly tolerances. The measured horizontal force components $F_x$ and $F_y$ exhibit a strong dependency on the displacement along $\hat{e}_x$ and $\hat{e}_y$, respectively. It is expected that this dependency is mostly the result of mechanical transformation errors, such as a position tolerance on the estimation of the exact centers of actuation or gravity, and assembly tolerances. Nevertheless, it proofs to be difficult to fully eliminate these effects, partly due to their large influence with respect to the small expected parasitic force levels of the gravity compensator. Compared to the vertical offset force their influence is marginal. More advanced measurements, such as the frequency response of all individual actuators on the measured degrees of freedom, would help to make a better, empirical estimation of the various coordinates. A dynamic measurement on the system learns and a curve fit on the results have confirmed the stiffness of the gravity compensator and the mass of the isolated platform. The damping of the system is quite high and most probably due to pneumatic effects.

It is concluded that the electromagnetic vibration isolation system is a feasible alternative for the solutions seen today, considering its high force with low stiffness, low energy consumption and the high bandwidth that it exhibits, even with the simple position-controllers used in this thesis. The system that has been realized is a first-of-a-kind six-DoF electromagnetic vibration isolation system with passive gravity compensation. It exhibits a vertical passive force of 7.1 kN at a vertical stiffness that is of such low value that a sub-Hertz vibration isolation system is formed. Preliminary measurements have shown that it passively isolates vibrations up to 700 Hz and that it dissipates as less as 0.3 W to stabilize this system. To the author’s knowledge, a vibration isolation system with such properties is unprecedented.

7.9.2 Contributions

- Sections 7.3 and 7.6 to 7.8 - The magnetic design of the gravity compensator (Chapter 5) has been integrated with 2-DoF actuation (Chapter 6) to form an electromagnetic vibration isolator. This novel design has been realized as a prototype. Its static characteristics have been verified with the use of an advanced test rig and dynamic measurements have been conducted to verify the stiffness, mass and damping, and to gain insight into its vibration isolating properties. As such, this chapter presents a proof-of-principle of the cross-shaped gravity compensator with vertical airgaps.
• **Section 7.5** - A test rig has been designed and realized to evaluate the performance of the electromagnetic vibration isolation system. Its individual components are designed to exhibit their first resonance at high frequencies, to facilitate vibration isolation measurements over a large bandwidth. It incorporates a shake rig with a 20 Hz-sprung shake table, capable of injecting artificial floor vibrations up to at least 1 kHz into the system placed on top of it. The modular design and its extensive mounting hole pattern make this shake rig suitable to be used for a wide range of applications instead of only the vibration isolator of this thesis.

7.9.3 Recommendations

• The analysis of Section 7.6.2 shows that the influence of the ambient temperature on the gravity compensator may certainly not be considered negligible as it attains levels comparable to the control force. It is now only accounted for in the passive vertical force component, however its influence on the other force components, or those in the active actuators have not been fully characterized. Further, the actuators in the vibration isolator have not been used during the measurements discussed in this chapter. This validates the implementation of forced water-cooling of these actuators, as any heat generated in the coils that transfers to the nearby gravity compensator has an effect on its performance. In a future measurement, with three vibration isolator units with 2-DoF actuation, the cooling requirements of the actuators should be investigated in more detail. Ideally, the gravity compensator should be operated in a temperature-controlled room, to rule out any temperature effects in the measurement results. Further, the cooling of the nearby actuators should be such, that the ambient temperature rise of the gravity compensator is minimized.

• The damping that is found in Section 7.8 is relatively large as the presented frequency response even has no peak at its resonance. As discussed it is expected that only a small amount of this damping is of electro-dynamic nature, i.e. due to eddy currents, and that it is mainly a pneumatic effect. A measurement in a vacuum environment could validate this, however is quite laborious. Simple adjustments, such as a design with more attention to air-pockets in the vibration isolator and inclusion of air-venting channels, or even an active air injection to create a turbulent air-flow, could improve the vibration isolator's performance. Further, as the full system is measured, an air-cushion effect between the shake table and the isolated platform is another possible cause of damping, and could for example be influenced by a vibration-damping foam material on the shake table. Besides extra damping, a change in stiffness is another possible effect of these pneumatic properties. It is unlikely that this has an effect on the sub-hertz measurements that have been undertaken.

• The dynamic measurements of Section 7.8 have been used to estimate the mass, stiffness and damping of the realized vibration isolation system. It is found that the first major resonances in the system occur above 700 Hz. Further investigation into the dynamic behavior, i.e. the vibration isolation
properties of the system is a logical continuation on the work in this thesis, which serves as a proof-of-principle for the electromagnetic vibration isolation. For example, only the accelerometers mounted on the isolated platform were used in the measurements of Section 7.8. The shake table was locked to the floor and considered an inertial platform. A true dynamic evaluation of the isolation performance should relate these payload accelerations to an acceleration measurement on the shake table.

- The static and dynamic measurement results have been obtained with low-bandwidth PID controllers which were aimed at stabilization of the system, based on a relative position measurement. The use of the accelerometers in the control loop would improve the vibration isolation properties according the conclusions in Appendix B.2. The current system, with only active stabilization, behaves as a passive vibration isolation system and for instance has difficulties with compliance. Further, a more advanced control structure could improve the isolation characteristics, although a possible increase in complexity should not overwhelm such performance improvement.

- From the static measurement results in Section 7.6 it becomes clear that it is difficult to extract the exact behavior of the gravity compensator from the measurement results using post-processing on the actuator currents. The estimation of the various centers of actuation and the center of gravity and the linearization of these relations under horizontal movement seems not trivial. Transformation matrices, based on linearized relations between the various coordinates and coordinate systems, are used to convert the individual actuator forces into global coordinates. The results may suffer from the assumptions and tolerances that are made and it is expected that this is the underlying cause of the discrepancies that have been observed. A future investigation could focus on a more accurate estimation of these coordinates and transfer relations, for example by means of measurements, and in this way improve the accuracy of the results.

The main drawback of the static Newton approach used in this thesis is that it considers the individual components of a system separately, which necessitates the calculation of interacting force components resulting from connections between the various bodies in the system [13]. Furthermore, it needs to be extended with Newton dynamics if the frequency response is considered. A different approach to this is to consider the system as a whole rather than its individual components, thus eliminating the need to calculate all individual interaction forces. A Lagrangian approach formulates the problems in dynamics in terms of two scalar functions, the kinetic energy and the potential energy, and so-called generalized nonconservative forces. The last represent losses such as friction and externally applied forces or torques. The physical coordinates are replaced with more abstract generalized coordinates, which do not necessarily have a physical meaning. The dynamics of the system are then derived by considering the virtual work that is done by virtual variations of the generalized coordinates as a result of the non-conservative generalized forces.
Such a dynamic approach is considered necessary for the dynamic modeling of the vibration isolation system. Further, it is expected that it helps in the characterization of the static properties of the gravity compensator.
Part III

Closing
Chapter 8

Conclusions and recommendations

This chapter summarizes the most important conclusions of this thesis. Further, the scientific contributions are presented and recommendations for future research are summarized.

8.1 Conclusions

This thesis has presented an extension to an analytical modeling technique for permanent magnets and has used it to develop an advanced vibration isolation system. It is shown that for large, unbounded electromagnetic problems the analytical models are more accurate as their numerical counterparts, although certain material assumptions are essential for this. This analytical modeling technique, with its assumptions, enables the modeling, design and realization of applications that comprehend complex magnet interaction devices. The system that has been developed is a first-of-a-kind six-DoF electromagnetic vibration isolation system with passive gravity compensation, low energy consumption and a high bandwidth. It exhibits a vertical passive force of 7.1 kN at a vertical stiffness that is of such low value that a sub-Hertz vibration isolation system is formed. Preliminary measurements have shown that the first resonance measured on the isolated platform occurs above 700Hz and that it costs as less as 0.3W to stabilize this system.

8.1.1 Modeling

Maxwell’s equations are a solid basis to obtain magnetic (scalar or vector) potential, energy and interaction equations in a magnetostatic environment, especially for
models with permanent magnets. Various modeling methods are discussed, and it is found that the magnetostatic analytical surface charge modeling technique is an excellent method to describe the strongly non-linear force and torque behavior permanent-magnet based devices, such as vibration isolation systems. This analytical modeling technique is considered very suitable for a large range of applications and exhibits its advantageous properties mainly in terms of computational efforts, and a highly accurate noise-free solution due to the absence of a mesh or boundaries.

Based on the charge modeling technique the analytical field equations for triangular charged surfaces and magnets incorporating such surfaces have been proposed and validated. With these new analytical field equations the magnetic field of any permanent magnet shape with straight edges can be modeled with closed-form 3D analytical equations, as an example with the pyramidal frustum has shown.

Analytical equations for the force between permanent magnets which are magnetized along the same axis are well-known in literature. Based on the analytical surface charge model new equations have been proposed in this thesis to model any other magnetization combination. Experimental results and numerical modeling have validated their high accuracy. Analytical extensions have been added and validated for situations in which the magnet edges are aligned and the standard equations do not suffice and produce discontinuous results. Especially optimization routines, which have difficulties in handling discontinuities, profit from these extended analytical equations.

Novel equations to analytically derive the stiffness matrix of a magnet-based system, including the extension to the discontinuous cases, have been proposed and validated. This stiffness matrix, which includes cross-coupling, is an aid in the design of six-DoF permanent-magnet based devices. It eliminates the need for multiple force calculations to obtain the values in the stiffness matrix. The novel analytical torque equations around any given point in space that have been proposed are another necessity in the design of six-DoF permanent-magnet based devices. The analytical force, stiffness and torque equations are combined by classical coordinate rotation and superposition techniques to describe large arrays of permanent magnets. These models are excellent to describe the non-linear force-displacement relations seen in such devices.

An investigation into the modeling errors and manufacturing tolerances has identified a number of causes for discrepancies between theoretically modeled interaction force and its experimentally obtained values. They are separated in global effects, that apply to the whole permanent magnet volume, and local effects, which may vary throughout the magnet’s volume. Under global effects properties such as remanence, relative permeability and misalignment of the magnetization angle are categorized. The local effects consider dimensional and placement tolerances, local magnetization misalignment, working point variation and local heating. Some of these errors and tolerances can be easily identified for a batch of magnets whereas others are of such local nature that an individual characterization of all permanent magnets in a device would be necessary to accurately predict their influence.
8.1.2 Design

Vibration isolation is a promising application of permanent-magnet based devices. The application benefits from their inherently high bandwidth solution with low energy consumption. Further, the nonlinear behavior of permanent-magnet interactions can be utilized to obtain a high-force, low-stiffness vibration isolation design. A dearth of literature on this subject facilitates the necessity to perform an investigation into suitable topologies for the gravity compensator in this vibration isolation system.

An investigation into the electromechanical objectives for vibration isolation systems has shown that their force level is related to the magnetic field strength of the magnet arrays. The mathematical abstraction of the analytical modeling technique can be utilized to predict the performance of various magnet topologies, based on the field of a single magnet. The force is maximized with a high flux density, a low stiffness is accomplished by a minimization of the field amplitude and gradient around the magnet edges and the damping is related to the number of magnetic poles along the direction of movement. These predictions have been validated with constrained nonlinear multi-variable optimization and as such it is found that they are very suitable to select suitable topologies in an early stage.

It is concluded that topologies with one horizontal airgap and geometrically equal magnet arrays are unsuitable to meet the envisaged specifications as the force and stiffness are strongly coupled in such topologies. Since both magnet arrays are equal in size, their magnet edges align and as such the position dependency of the force rises. The use of geometrically unequal magnet arrays is an improvement, however, still not suitable to fulfill the high requirements that have been set. The combination of attraction- and repulsion-based topologies, with multiple airgaps, decouples the force and stiffness, at least in the working point. If these airgaps are horizontal, nonlinear behavior and manufacturability may become an issue, although further investigation on this subject is necessary to determine the limits of such a topology in terms of volume and linearity.

A gravity compensator with vertical airgaps produces a high force at low stiffness and decouples the vertical force from the stiffness in a similar manner to the double-airgap topologies. Here, the vertical passive force vector lies in the plane of the airgap instead of perpendicular to it. The force density, provided that a high vertical force and low stiffness are pursued, is not lower than that for the topologies with horizontal airgaps. To minimize any nonlinear behavior the device must be designed for its working envelope instead of only in a single working point. Parasitic torques are minimized by placing two of these gravity compensators in parallel on a small intermediate distance which has resulted in a patented cross-shaped topology. This topology has been established in a patent and yields a high vertical force at very low stiffness levels for all entries of the stiffness matrix and throughout the working envelope.

A simulation throughout the working envelope has confirmed the near-linear, low-stiffness and low-torque behavior of the cross-shaped magnetic gravity compensator. A direct design comparison with FEM has shown a small difference which is allocated
to numerical truncation errors in FEM. It is concluded that the gravity compensator that has been designed is a modular design that exhibits the envisaged high vertical force, low stiffness and low parasitic force and torque. The influence of the manufacturing tolerances and modeling errors on the passive vertical force have been estimated, and are found to be sufficiently low to be compensated in the test setup by other components.

The vibration isolator is completed by two-DoF integrated Lorentz actuators, which are placed in the volume not used by the gravity compensator. With a combination of analytical and numerical techniques their electromechanical and electrical properties have been identified. A separate maximization of the actuator steepness and of the actuator force constant show that the force constant objective yields a design with larger magnets and a smaller coil. The correspondence between the various models and the experiments is high, although some discrepancy in the force calculation has been found which is due to the meshing in the semi-numerical model. A simulation of the gravity compensator’s performance including the translator magnets has shown their limited influence on the gravity compensator’s force.

8.1.3 Realization

The gravity compensator with its integrated actuators has been realized as a prototype which has been evaluated in a test rig. Three 2-DoF external actuator units, with a magnetic design equal to that of the integrated actuators, are part of this test rig for the necessary stabilization purposes. The ambient temperature of the gravity compensator has an effect of $1.7 \, ^\circ C/K$ on the vertical force of the gravity compensator. This is small in relative terms, although its absolute value is in the same order of magnitude as the active control force and as such must be accounted for in the measurement results.

The measurements have shown that a magnet-based gravity compensator levitates 730 kg with a power consumption of only 0.3 W, necessary for stabilization. Especially compared with pneumatic or hydraulic systems, which require a continuously pressurized system, this is a major advantage. Further, this gravity compensator exhibits a vertical stiffness $K_{zz}$ below 1 kN/m, which yields a resonance frequency in the sub-hertz region, as is confirmed with dynamic measurements. The system behaves somewhat asymmetric for displacements along $\hat{e}_x$ and $\hat{e}_y$, which is related to manufacturing tolerances and non-ideal materials. The measured horizontal force components $F_x$ and $F_y$ show a significant dependency on the displacement along $\hat{e}_x$ and $\hat{e}_y$, respectively. It is expected that this dependency is mostly the result of idealized assumptions, such as a position tolerance on the estimation of the exact centers of actuation and gravity, unforeseen assembly tolerances of the magnetic spring itself and magnetic behavior of the coil supports. Nevertheless, it proofs to be difficult to fully eliminate these effects, partly due to their large influence with respect to the small expected parasitic force levels of the gravity compensator. More advanced measurements, such as the frequency response of all individual actuators on the measured degrees of freedom, could help to make a better, empirical estimation of
8.2: Thesis contributions

the various coordinates. Dynamic measurements on the system and a curve fit on the results have confirmed the stiffness of the gravity compensator and the mass of the isolated platform. The damping of the system is larger than expected and requires further investigation.

Overall, it is concluded that the electromagnetic vibration isolation system is a feasible alternative for the solutions seen today, considering its high force with low stiffness, low energy consumption and the high bandwidth that it exhibits, even with the simple position-controllers used in this thesis.

8.2 Thesis contributions

- **Chapter 3** - Novel analytical equations for the field of magnets with triangular faces have been proposed and validated. They are an addition to the existing analytical field models of cuboidal, cylindrical, ring- and arc shaped magnets.

- **Chapter 3** - An extensive investigation of the analytical interaction models for cuboidal permanent magnets has been performed. This has resulted in equations for the interaction force between perpendicularly magnetized permanent magnets. Further, the discontinuities in these and the existing equations for parallel magnetized magnets have been investigated. The result is a continuous function for the interaction force between permanent magnets.

- **Chapter 3** - An analytical description of the $3 \times 3$ stiffness matrix, including the removal of the discontinuities.

- **Chapter 3** - Continuous analytical equations for the torque of cuboidal magnets in each other's proximity. The point of reference for this torque may be chosen freely.

- **Chapter 3** - An identification of the manufacturing tolerances and modeling errors and their effects on the force variation in permanent-magnet based devices, especially the magnetic gravity compensator proposed in this thesis.

- **Chapters 4 and 5** - This thesis has pursued a translation of the mechanical requirements of the vibration isolation system into electromechanical objectives. It is shown that, using the mathematical abstraction in the interaction equations, it is possible to predict the performance of a certain topology based on the a study of the field of a single magnet. For low-stiffness, high-force vibration isolation applications it is concluded that a high magnetic field should be combined with a topology that does not align the magnet edges.

- **Chapter 5** - An extensive investigation into topologies with horizontal airgaps and classical magnetization patterns has shown their limitations in terms of force maximization and stiffness minimization. The main difference with other investigations is the high force level and low stiffness level that are pursued here.

- **Chapter 5** - The gravity compensator topology with multiple unequally sized magnet arrays has been investigated. In this topology both magnet arrays have
almost the same pole pitch and different magnet pitches. The magnet edges in both arrays do not align and as a result the stiffness of this topology significantly reduces.

• **Chapter 5** - This thesis proposes the extended design of a gravity compensator with vertical airgaps and multiple magnets, which combine a high vertical force level with an extremely low stiffness. As a result of the vertical airgap, the magnet volume is used optimally. The used of a cross-shaped gravity compensator reduces parasitic effects even more and is very suitable for integration of active elements. This topology has been recorded in a patent application.

• **Chapter 6** - Ironless voice coil actuators with rectangular coils and double-sided quasi-Halbach magnet arrays have been designed for integration into the gravity compensator. Their electromechanical (force constant, steepness) and electrical (resistance, inductance) properties have been modeled and optimized to meet the requirements and constraints set for these actuators. Renewed analytical equations to calculate the vector potential of a current-carrying bar have been proposed to better integrate with the interaction formulations of Chapter 3.

• **Chapter 7** - The first working prototype of an electromagnetic vibration isolation system with passive gravity compensation and active stabilization, capable of isolating 730kg at near-zero stiffness and near-zero energy consumption, has been realized. Its static characteristics have been verified with the use of an advanced test rig and dynamic measurements have been conducted to verify the stiffness, mass and damping, and to gain insight into its vibration isolating properties. A compliance measurement of the passive vertical isolation properties show that the vibration isolation system has its first resonance above 700Hz. As such, this chapter presents a proof-of-principle of the cross-shaped gravity compensator with vertical airgaps.

• **Chapter 7** - A test rig has been designed and realized to evaluate the performance of the electromagnetic vibration isolation system. Its individual components are designed to exhibit their first resonance at high frequencies, to facilitate vibration isolation measurements over a large bandwidth. It incorporates a shake rig with a 20Hz-sprung shake table, capable of injecting artificial floor vibrations up to at least 1kHz into the system placed on top of it. The modular design and its extensive mounting hole pattern make this shake rig suitable to be used for a wide range of applications instead of only the vibration isolator of this thesis.

### 8.3 Recommendations

• **Chapter 3** - The relative permeability \( \mu_r \) is considered unity in the analytical surface charge and current sheet model. For simple structures with near-infinite permeability it is possible to use the method of imaging for incorporating these materials [76, 98]. The analytical models would be significantly stronger if
they could incorporate more complex soft-magnetic structures or if they could incorporate a relative permeability that is above 1.

The inclusion of complex structures is most probably far away, and may be left being privileged to FEM models. However, the relative permeability of a cuboidal permanent magnet – or magnet array – seems more realistic. For this, the boundary conditions must be solved at the magnet edges, for example by connecting two models with a different $\mu_r$, or apply a form of the magnetic imaging.

- **Chapter 3** - The interaction equations for rotated permanent magnets have only partially been solved but may be important for many magnet-based devices. Inclusion of such rotations in the analytical formulation of the force, stiffness and torque would be a significant step forward.

- **Chapter 5** - The topologies with two parallel horizontal airgaps have only been briefly investigated in their working point. Although they have been considered unsuitable for the application envisaged in this thesis based on preliminary optimizations, this does not mean that they are of no interest for vibration isolation systems. A further investigation into their suitability for magnetic gravity compensators in terms of resonance frequency, linear range, force density, suitable force levels, manufacturability, etc. would give more insight into their advantages and disadvantages.

- **Chapter 5** - The optimization of the gravity compensator could be performed with additional objectives, such as magnet volume, field leakage or general costs. Further, such optimization should incorporate all cross-couplings between the various parallel springs throughout the optimization process instead of at the end of this process.

- **Chapter 5** - An inclusion of iron parts may improve the force density and the magnetic shielding of the magnetic gravity compensator. However, such inclusion yields more complicated modeling, with saturation and hysteresis, which has not been investigated. It is a recommendation for future research to investigate this inclusion and its effects on the device's performance. For this, either a different set of modeling techniques should be used, or the method used in this thesis should be extended to include soft-magnetics and possibly hysteresis effects.

- **Chapter 5** - The analyses presented in this thesis are based on magnetostatic equations and as such does not include dynamic effects such as eddy currents within the magnets or support material. Such investigation would provide insight into the damping that occurs, although it is expected that this damping is low considering the limited velocities that occur in floor vibration.

- **Chapter 5** - The design of the gravity compensator is based on a fixed force level and exhibits extremely low stiffness. As such, a mass variation of the platform that is isolated cannot be handled by the magnetic circuit. Consequently, the gravity compensator needs to be designed for the maximum mass and the
isolated platform must match this mass. An investigation into adjustability of this vertical force would therefore be beneficial.

- **Chapter 6** - The actuators have been designed to exhibit their worst-case maximum force over a large frequency range. Additional research on the test rig should be conducted to determine if this is really necessary, or that a lower force capability at elevated frequencies is sufficient for the vibration isolation system. Such reduction in bandwidth would increase the force density of the actuators. Further, flux leakage is an issue to be investigated, as it was not considered in the current design.

- **Chapter 7** - The influence of the ambient temperature on the gravity compensator may certainly not be considered negligible as it attains levels comparable to the control force. It is now only accounted for in the passive vertical force component, however its influence on the other force components, or those in the active actuators have not been fully characterized. Further research could focus on the thermal behavior of the gravity compensator and the active actuators, especially those integrated in the vibration isolator.

- **Chapter 7** - The damping of the vibration isolation has been estimated from a simplified electromagnetic model and has been estimated from the dynamic measurements. Other possible causes of this damping are pneumatic effects. Further research into this damping should be conducted for a better prediction of the effects that play a role.

- **Chapter 7** - According to the preliminary dynamic measurements of the vibration isolation system's behavior along the vertical axis the first major resonances in the system occur above 700Hz. Further investigation into the dynamic behavior, i.e. the vibration isolation properties, of the system is a logical continuation on the work in this thesis, which serves as a proof-of-principle for the electromagnetic vibration isolation. Such additional research includes an investigation of the transmissibility next to the studied compliance, the dynamic behavior and cross-couplings in all degrees of freedom, decoupling methods and more advanced control schemes.

- **Chapter 7** - The static and dynamic measurement results have been obtained with low-bandwidth PID controllers which were aimed at stabilization of the system, based on a relative position measurement. The use of the accelerometers in the control loop would improve the vibration isolation properties according the conclusions in Appendix B.2, for example applied in a skyhook damping. Further, a more advanced control structure could improve the isolation characteristics, although a possible increase in complexity should not impair the performance improvement.

- **Chapter 7** - From the static measurement results in Section 7.6 it becomes clear that it is difficult to extract the exact behavior of the gravity compensator from the measurement results using post-processing on the actuator currents. The estimation of the various centers of actuation and the center of gravity
and the linearization of these relations under horizontal movement seems not trivial. Transformation matrices, based on linearized relations between the various coordinates and coordinate systems, are used to convert the individual actuator forces into global coordinates. The results may suffer from model assumptions and assembly tolerances that are made and it is expected that this is the underlying cause of the discrepancies that have been observed. A future investigation could focus on a more accurate estimation of these coordinates and transfer relations, for example by means of dynamic measurements, and in this way improve the accuracy of the results.

- **Chapters 5 and 7** - The static Newton modeling approach used in this thesis considers the individual components of a system separately, which necessitates the calculation of interacting force components resulting from connections between the various bodies in the system [13]. Furthermore, it needs to be extended with Newton dynamics if the frequency response is considered. A different approach to this is to consider the system as a whole rather than its individual components, thus eliminating the need to calculate all individual interaction forces. A Lagrangian approach formulates the problems in dynamics in terms of two scalar functions, the kinetic energy and the potential energy, and so-called generalized nonconservative forces. The last represent losses such as friction and externally applied forces or torques. The physical coordinates are replaced with more abstract generalized coordinates, which do not necessarily have a physical meaning. The dynamics of the system are then derived by considering the virtual work that is done by virtual variations of the generalized coordinates as a result of the non-conservative generalized forces.
Appendix
Appendix A

Validation methods of the analytical models

Several methods have been used to verify the analytical equations for force, stiffness and torque which were derived in Chapter 3.

A.1 Experimental setup

In this experimental setup two magnets are mounted with either parallel or perpendicular magnetization. One of the magnets is moved along one of the Cartesian axes while the force and torque are measured using a 6-DoF transducer below the other magnet. The center of this 6-DoF transducer is located 47 mm below the center of PM1. The force and torque are obtained with intervals of 1 mm.

High-grade Vacodym 655 HR [180] permanent magnets have been used to experimentally verify the analytical models. The average remanent flux density of the Batch was estimated to be 1.23 T. The dimensions are 10 mm × 26 mm × 14 mm ($x \times y \times z$) and the magnets are magnetized along the $z$-direction (14 mm). From the catalogue [180] it can be derived that these magnets exhibit a typical relative permeability, $\mu_r$, of $1.03 \pm 0.02$ although its exact value has not been verified for the used magnets.

The magnets are mounted on non-magnetic aluminum adapters shown in Fig. A.1. During the measurements one of the two adapters was fixed to a base frame with the components of Table A.1 as shown in Fig. A.2. A vertical $z$-elevator stage in series with a horizontal $y$-stage fixed the bottom magnet. The other adapter was attached to a linear drive. A 6-DoF strain-gauge load cell was used to measure the forces and torques on the permanent magnet above, mounted on an aluminum structure with a height of 36 mm to minimize the influence of soft-magnetic components. As such, the
resulting position of the transducer’s center was located 47 mm below the magnet’s center.

The linear synchronous permanent-magnet machine moved the second magnet along \( x \). To eliminate any transient effects in the measurements, the speed of the linear drive was limited to 1 mm/s. The encoder signal of this machine with its high resolution was used to determine the displacement along the \( x \)-axis.

### A.1.1 Finite Element Modeling

The FEM method is a reasonably reliable comparison for the analytical models. Nevertheless care should be taken with meshing the problem, especially when the airgap between the permanent magnets decreases to a few millimeters. Due to its mesh-based field and force calculation, a very large number of mesh elements is required to provide results with sufficient accuracy. As such, it is a slow method, especially for permanent magnets in air, and is therefore only used as verification purpose. The verification was performed with a 2\(^{nd}\) order mesh with 417,634 nodes in 309,267 volume elements.

Advantageous to this method compared to the proposed analytical models is the
A.1: Experimental setup

Table A.1: Components used in the experimental setup to validate the analytical equations.

<table>
<thead>
<tr>
<th>Component</th>
<th>Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical elevator stage</td>
<td>OWIS HV 100</td>
</tr>
<tr>
<td>Scaling resolution</td>
<td>5 µm</td>
</tr>
<tr>
<td>Load capacity</td>
<td>220 N</td>
</tr>
<tr>
<td>Horizontal displacement stage</td>
<td>OptoSigma 122-0115</td>
</tr>
<tr>
<td>Scaling resolution</td>
<td>10 µm</td>
</tr>
<tr>
<td>Load capacity</td>
<td>190 N</td>
</tr>
<tr>
<td>Force/Torque load cell</td>
<td>ATI MINI40 SU-80-4</td>
</tr>
<tr>
<td>Resolution $F_x, F_y$</td>
<td>5 mN</td>
</tr>
<tr>
<td>Resolution $F_z$</td>
<td>10 mN</td>
</tr>
<tr>
<td>Resolution $T_x, T_y, T_z$</td>
<td>125 µNm</td>
</tr>
<tr>
<td>Linear drive</td>
<td>Philips</td>
</tr>
</tbody>
</table>

Figure A.3: Finite Element mesh elements projected on the outside of the magnets and a virtual box around the magnets.

ability to include the relative permeability $\mu_r$ in the 3D model. As the validity of the analytical equations need to be shown, and not the validity of the assumption that the relative permeability equals 1, this assumption is also made in the FEM results that have been shown in this chapter.

The 3D FEM package that has been used is Flux3D version 10.3.3 [33]. Figure A.3 shows a projection of mesh elements directly around the permanent magnet. Displaying the full mesh in this figure would lead to an incomprehensible figure due to the large number of mesh elements that have to be used. The figure also demonstrates the high density of mesh elements in the permanent magnets themselves. The model has a so-called infinite box, which forces the magnetic potential at infinity to zero and enables a reduction in model size and as such computational effort. It is a transformation of the open exterior modeling domain into a closed exterior domain [33].

A.1.2 Maxwell Stress Method

The Maxwell Stress method was described in Section 2.4.2. It can be implemented in a variety of ways, even fully analytical to obtain equations similar to those proposed
Appendix A. Validation methods of the analytical models

in this chapter. In this thesis it is based on the analytical field equations of Section 3.2 on which numerical integration is applied to obtain a semi-numerical equation for the force. It can therefore not be used as a direct verification for the analytical equations as it uses exactly the same field calculation method. However, for configurations with multiple magnets this method solves faster than the FEM method and is therefore considered more suitable to verify the analytical equations for large arrays. Furthermore, it can be used to obtain the interaction force between odd-shaped permanent magnets, such as the pyramidal frustums in [100]. The results from Maxwell Stress method are based on a cuboidal mesh of 0.7 mm at 0.5 mm distance around one of the magnets, resulting in a total of 3668 elements for the magnets considered in the validation process.
Chapter 4 presents the concept of vibration isolation systems and focuses on the electromagnetic solutions. This Appendix shows a derivation of the transmissibility and compliance, that are discussed in Chapter 4. Further, it provides a short overview of other vibration isolation techniques.

B.1 Passive vibration isolation

The classical mass and spring system of Fig. B.1 is often used to describe a vibration isolation system. The vertical stiffness is given by $k$ [N/m], the damping constant of the damper by $b$ [Ns/m] and the mass of the isolated platform by $m$ [kg]. The transmissibility and the compliance are described here in more detail than in Chapter 4 and the influence of the natural undamped resonance frequency and damping ratio is discussed.

B.1.1 Transmissibility

The amount of floor vibrations being transmitted to the isolated platform is expressed by the dimensionless transmissibility transfer function $H_t(s)$ [80]. It can be seen as the ratio between the isolation performance between the considered system and when the load is fixed to the floor, as function of the frequency. It is for the simplified system of Fig. B.1 given by

$$H_t(s) = \frac{\ddot{z}_1(s)}{\ddot{z}_0(s)} = \frac{bs + k}{ms^2 + bs + k} = \frac{2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}.$$  \hspace{1cm} (B.1)
Appendix B. Passive and active vibration isolation

![Figure B.1: A passive vibration isolation system with a spring $k$ and a damper $b$. The vertical displacements of the floor and the isolated platform are defined by $z_0$ and $z_1$, respectively.](image)

The variable $\omega_n$ is the natural frequency of the passive system and $\zeta$ defines the damping ratio. They are given by

$$\omega_n = \sqrt{\frac{k}{m}}, \quad (B.2)$$

$$\zeta = \frac{b}{2m\omega_n}. \quad (B.3)$$

Figure B.4 shows this transmissibility of a system with $\omega_n = 2\pi$ and $\zeta = 1$.

**Undamped natural frequency, $\omega_n$** The combination of the isolated mass and the spring acts a mechanical low-pass filter. This is shown in the frequency response in Fig. B.2(a). An increase of the mass of the isolated platform or reduction the stiffness leads to a decreased resonance frequency $\omega_n$. As a result of such low undamped natural frequency the transmissibility improves. For this reason, many high-performance isolation systems exhibit natural frequencies below 5 [Hz], or 10$\pi$ [rad/s] [181]. However, when using linear passive spring elements the gravity-induced static pre-compression may become problematic. This static deflection is given by

$$x = \frac{mg}{k} = \frac{g}{\omega_n^2}, \quad (B.4)$$

where $g = 9.81$ [m/s$^2$] is the acceleration due to gravity. Furthermore, a low resonance frequency may lead to leveling problems and more importantly it causes excessive error motions during direct isolated platform disturbances. The elasticity of the springs needs to be created with a relatively small mass if the internal resonances are to be separated significantly from the isolation frequency. This would cause high mechanical stress in such springs and may become incompatible with linearity requirements.

**Damping ratio, $\zeta$** If the damping ratio is low the transmissibility exhibits a high resonance peak as Fig. B.2(b) shows. An increased damping ratio $\zeta$ attenuates this resonance peak, although at the cost of transmissibility at frequencies above $\omega_n$. It is therefore observed that such a simple viscous damping mechanism is limited in
B.1: Passive vibration isolation

Figure B.2: Transmissibility of the passive single degree of freedom system in Fig. B.1. In (a) the damping ratio $\zeta = 0.707$ and the natural frequency $\omega_n$ is varied and in (b) the natural frequency $\omega_n = 2\pi \text{[rad/s]}$ and the damping ratio $\zeta$ is varied.

performance at either the resonance frequency (due to low damping ratios) or at high frequencies (due to high damping ratios).

B.1.2 Compliance

The compliance transfer function which describes the transfer of isolated platform forces to the acceleration of the isolated platform. Such forces may be the result of reaction forces on moving parts of the isolated platform. This compliance transfer function $H_c(s)$ is defined as

$$H_c(s) = \frac{\ddot{u}_l(s)}{F_{\text{ext}}(s)} = \frac{s^2}{ms^2 + bs + k} \Rightarrow \frac{s^2/m}{s^2 + 2\zeta\omega_n s + \omega_n^2}.$$  \hspace{0.5cm} (B.5)

Undamped natural frequency $\omega_n$ Figure B.3(a) shows the compliance of the system in Fig. B.1 as a function of the undamped natural frequency. To do this, (B.5) is multiplied with the mass to define the function with only the natural frequency and damping ratio. This damping ratio is kept at $\zeta = 0.707$ and the natural frequency $\omega_n$ is varied. This form is often referred to as the passive sensitivity function (Section B.1.3). Contrary to the transmissibility, which requires a low resonance frequency for floor vibration rejection, the rejection of external forces only takes place below the resonance frequency, hence, a high natural frequency results in a good force rejection of the system. Above the resonance frequency the compliance is maximum, which means that high-frequency force disturbances are directly influencing the payload acceleration.

Damping ratio $\zeta$ A high damping, or damping ratio $\zeta$, reduces oscillations around the resonance frequency. Further, it helps minimizing the platform vibration that results from this direct force as the platform is coupled to the floor. This is shown in Fig. B.3(b), where the damping mainly helps to reduce the resonance but on other frequencies hardly affects the performance.
Appendix B. Passive and active vibration isolation

B.1.3 Trade-off

By multiplying (B.5) with the mass \( m \) the passive sensitivity function \( P(s) = \frac{m \ddot{z}}{F_{ext}(s)} \) is obtained \([171, \text{Chap. 3}, \text{172}]\). The ratio \( Q(s) = \frac{2 - 1(s)}{\ddot{z}_0(s)} \) is then called the passive complementary sensitivity function, and is equal to the transmissibility function discussed above. These functions are then given by

\[
P(s) = \frac{m \ddot{z}_1(s)}{F_{ext}(s)} = \frac{Ms^2}{ms^2 + bs + k} = \frac{s^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \tag{B.6}
\]

\[
Q(s) = \frac{\ddot{z}_1(s)}{\ddot{z}_0(s)} = \frac{bs + k}{ms^2 + bs + k} = \frac{2\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}. \tag{B.7}
\]

From these equations it can be shown that \([171]\)

\[
P(s) + Q(s) = 1. \tag{B.8}
\]

Figure B.4 shows the transmissibility and the passive sensitivity function of a system with \( \omega_n = 2\pi \) and \( \zeta = 1 \). This shows the compromise between transmissibility and compliance that is fundamental to the passive suspension: it is impossible to both have good rejection of both floor disturbance and isolated platform disturbances \([49, 80]\). Good floor vibration isolation requires the isolated platform to be mounted on very soft springs which gives that the transmissibility to be small across the frequencies of interest. However, this increases the sensitivity to isolated platform disturbances and herewith impairs the compliance. Subrahmanyan \([171]\) shows that (B.8) can be generalized to multiple degree of freedom problems.

B.2 Active vibration isolation

An active vibration isolation is generally more expensive than a passive solution because it requires sensors, a controller, actuators and power amplifiers. However, it not subject to the same trade-off as a passive mount and is therefore often considered more suitable; it a theoretical improvement of the transmissibility and the compliance with respect to passive isolation. In \([22]\) four actuator configurations were identified.
to modify the passive transmissibility, viz. parallel actuation, series actuation, momentum compensation and force compensation. The latter two use a reaction mass to modify the passive transmissibility. Because the use of such a reaction mass is out of scope for this thesis these types not further elaborated on in this thesis. The former two configurations are shown in Figs. B.5(a) and B.5(b), respectively. The actuators may be of any type, such as piezoelectric, hydraulic, pneumatic or electromechanical.

The series configuration requires an actuator which is capable of carrying the weight of the isolated platform and to exert additional dynamic control forces. In the parallel configuration, the weight of the isolated platform is carried by a passive spring, and the actuator only produces dynamic control forces. Contactless electromagnetic actuators are especially suitable for such configuration, because of their low internal stiffness and damping, hence not disturbing the spring, and the limited amount of force that is needed with respect to the series actuator. An additional advantage with respect to many other actuator types is the fully passive compensation
of the gravity force, which considerably reduces the consumed amount of energy for applications with a high mass.

Fig. B.6 illustrates the advantages of feedback control on the isolation performance of the parallel topology. It extends the basic configuration of Fig B.5(a) with a sensor that measures absolute velocity \( \dot{z}(s) \) and with a controller \( G(s) \). The transmissibility and force compliance of the closed-loop system can now be given by [171]

\[
H_t G(s) = \frac{\ddot{z}_1(s)}{\ddot{z}_0(s)} = \frac{bs + k}{ms^2 + (b + G(s))s + k} = S(s)H_c(s), \tag{B.9}
\]

\[
H_c G(s) = \frac{\ddot{z}_1(s)}{F_{ext}(s)} = \frac{1}{ms^2 + bs + k} = S(s)H_c(s). \tag{B.10}
\]

The function \( S(s) \) is the sensitivity transfer function given by

\[
S(s) = \left( I + s^{-1}H_c(s)G(s) \right)^{-1}, \tag{B.11}
\]

and can be derived by defining \( F_{act} = -G\dot{z}_1 \) and rewriting the equations of motions which results in

\[
\ddot{z}_1(s) = S(s)H_c(s)\ddot{z}_0 + S(s)H_c(s)\frac{F_{ext}}{m}. \tag{B.12}
\]

This simple example shows that by using velocity feedback damping is added which is proportional to the absolute velocity. This is known in literature as skyhook damping [113]. The fundamental limit of (B.8) becomes [171]

\[
S(s)P(s) + S(s)Q(s) = 1. \tag{B.13}
\]

This demonstrates that the sensitivity function of the loop has the same effect on both the conflicting disturbance sources. When the sensitivity function is less than unity a performance improvement over passive isolation is obtained. This enables a lot more freedom in shaping the sensitivity function and thus both the seismic vibration transmissibility and the disturbance force rejection through proper design of the compensator \( G(s) \). By applying advanced control schemes the isolation performance can be tuned quite accurately. However, as discussed in Section 4.4, factors such as sensor noise limit the performance improvement and may even cause the isolation properties to deteriorate compared to passive systems. Further, the actively generated force may have amplitude limitations that become dominating in the performance of the system.
Appendix C

Earnshaw’s theorem and stability

C.1 Stable levitation from an energy perspective

Stable levitation of any object in any manner has certain conditions which should be met beforehand. First of all the vertical upward force exhibited by the suspension should be equal and opposite to the gravity force acting on the levitated body. Another requirement is that the energy must be at a minimum, i.e. the force $\vec{F}$ must be restoring, which gives the necessary condition for stability

$$\oint_S \vec{F}(\vec{x}) \cdot ds < 0,$$  \hspace{1cm} (C.1)

where the integral is over any small closed surface surrounding the equilibrium point [20]. From the divergence theorem this implies that $\nabla \cdot \vec{F}(\vec{x}) < 0$. In terms of energy $W$ this gives the stability condition

$$\nabla \cdot \vec{F} = \nabla \cdot (-\nabla W) = -\nabla^2 W < 0.$$ \hspace{1cm} (C.2)

Hence, the energy can only have a minimum at points where the Laplacian of the energy is greater than zero. This equality is necessary for stable levitation, however is not sufficient because also with a positive $\nabla^2 W$ a negative second derivative is well possible. The energy must increase in all directions from an equilibrium point [20, 27], that is

$$\begin{align*}
\frac{\partial^2 W(\vec{x})}{\partial x^2} &> 0 \\
\frac{\partial^2 W(\vec{x})}{\partial y^2} &> 0 \\
\frac{\partial^2 W(\vec{x})}{\partial z^2} &> 0
\end{align*}$$

Sufficient conditions for stability.  \hspace{1cm} (C.3)
This energy $W$ is composed of the gravitational potential $W_{\text{grav}} = mg$, where $m$ is the mass and $g$ the gravitational acceleration of 9.81 m/s$^2$, and the magnetic potential $W_{\text{mag}}$. It was shown by Braunbek [27] that for the gravitational potential holds

$$\nabla^2 W_{\text{grav}} = 0.$$  \hspace{1cm} (C.4)

The stability criteria should therefore be achieved with the magnetic potential.

## C.2 Magnetic dipole in a conservative field

Earnshaw [58] originally proved the instability of an electric charge in an electrostatic field. However, this result can be easily be expanded to the magnetic domain. A magnetic dipole with a magnetic dipole moment $\vec{m}$ subjected to an external field $\vec{B}$ acquires an energy

$$W_{\text{mag}} = -\vec{m} \cdot \vec{B} = - (m_x B_x + m_y B_y + m_z B_z).$$  \hspace{1cm} (C.5)

If $\vec{m}$ is constant the energy only depends on the components of $\vec{B}$ [168]. The Laplacian of this energy is written as

$$\nabla^2 W_{\text{mag}} = - \frac{\partial^2 (m_x B_x + m_y B_y + m_z B_z)}{\partial x^2} - \frac{\partial^2 (m_x B_x + m_y B_y + m_z B_z)}{\partial y^2} - \frac{\partial^2 (m_x B_x + m_y B_y + m_z B_z)}{\partial z^2}.$$  \hspace{1cm} (C.6)

Keeping in mind that $\vec{m}$ is constant this can be written as

$$\nabla^2 W_{\text{mag}} = - \{m_x \nabla^2 B_x + m_y \nabla^2 B_y + m_z \nabla^2 B_z\}.$$  \hspace{1cm} (C.7)

The Laplacian for the magnetic field component $B_x$ is obtained by

$$\nabla^2 B_x = \frac{\partial^2 B_x}{\partial x^2} + \frac{\partial^2 B_x}{\partial y^2} + \frac{\partial^2 B_x}{\partial z^2} = \frac{\partial}{\partial x} \left( \frac{\partial B_x}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\partial B_x}{\partial y} \right) + \frac{\partial}{\partial z} \left( \frac{\partial B_x}{\partial z} \right).$$  \hspace{1cm} (C.8)

Because $\nabla \times \vec{B} = 0$ (no currents or changing electric fields) it holds that

$$\frac{\partial B_y}{\partial y} = \frac{\partial B_z}{\partial z},$$  \hspace{1cm} (C.9)

$$\frac{\partial B_x}{\partial x} = \frac{\partial B_z}{\partial z},$$  \hspace{1cm} (C.10)

and (C.8) is written as

$$\nabla^2 B_x = \frac{\partial}{\partial x} \left( \frac{\partial B_x}{\partial x} + \frac{\partial B_x}{\partial y} + \frac{\partial B_x}{\partial z} \right) = \frac{\partial}{\partial x} (\nabla \cdot \vec{B}).$$  \hspace{1cm} (C.11)
Because $\nabla \cdot \vec{B} = 0$ (no magnetic monopoles) it is concluded that $\nabla^2 B_x = 0$. Similarly, the same can be deducted for the other two field components. When this result is used in (C.7) it is found that $\nabla^2 W_{\text{mag}} = 0$. The energy is therefore incapable of reaching a local minimum, which rules out stable levitation but only allows for metastable levitation as the energy obtains a saddle-shaped position dependency [58, 111, 166–168].

C.3 Beyond Earnshaw

Earnshaw’s theorem assumes that all magnetic fields are conservative, hence, time invariant. Further, all magnetic materials considered are hard-magnetic materials. This leaves room for configurations in which stable levitation based on magnetic fields is feasible [27]. These are not really exceptions to any theorem but are ways around it which violate the assumptions.

Ferro- and paramagnetic substances align with the magnetic field and move towards field maxima. This rules out the possibility to achieve stable levitation of paramagnets in free space, because field maxima only occur at the sources of the field. Unlike the assumption above that the magnetic dipole is invariant of the field in reality paramagnets and diamagnets are dynamic in the sense that their magnetization changes with the external field [20, 168]. Diamagnets are repelled by magnetic fields and attracted by field minima. Because these minima can exist in free space, unlike the field maxima, stable levitation is possible for diamagnets. It was shown in (C.5)-(C.11) that there are no local minima for any vector component of the magnetic field. However, the magnitude of the field can have local minima.

The field-dependent dipole moment $\vec{m}$ is approximated by [20]

$$\vec{m}(\vec{x}) = -m_d \vec{B}(\vec{x}), \quad (C.12)$$

where $m_d$ is a constant greater than zero for paramagnetic materials and less than zero for diamagnetic materials. The magnetic energy from (C.5) and the Laplacian of the magnetic field are then written as

$$W_{\text{mag}} = m_d |\vec{B}|^2 = -m_d \left( B_x^2 + B_y^2 + B_z^2 \right). \quad (C.13)$$

$$\nabla^2 |\vec{B}|^2 = \nabla^2 \left( B_x^2 + B_y^2 + B_z^2 \right),$$

$$= 2 |\nabla B_x|^2 + |\nabla B_y|^2 + |\nabla B_z|^2$$

$$+ B_x \nabla^2 B_x + B_y \nabla^2 B_y + B_z \nabla^2 B_z,$$

$$= 2 \left[ |\nabla B_x|^2 + |\nabla B_y|^2 + |\nabla B_z|^2 \right]. \quad (C.14)$$

The last equation follows from (C.11). Because the square of a magnitude is always positive it follows that

$$\nabla^2 |\vec{B}|^2 \geq 0. \quad (C.15)$$
This means that the Laplacian of the energy is never positive for paramagnetic materials [20]. As a result stable levitation in all directions simultaneously is not possible with these materials. For diamagnetic materials this Laplacian of the energy is never negative and as a result instability in all directions simultaneously is not possible.

A more elaborate mathematical discussion on stable magnetic levitation in free space is given in [19, 20, 27, 28, 166–168]. This extends beyond the scope of this Appendix, which only is intended to briefly show what Earnshaw’s law exactly proves and how stable levitation can be achieved by adjusting the assumptions.

These and other articles describe a number of implementation methods, active and passive, to achieve stable levitation with permanent magnets. In essence, the levitation system must be implemented such, that (C.2)-(C.4) are obeyed.

C.4 Techniques with stable passive levitation

Over the years, many methods have been found to achieve stability for permanent-magnet based levitation systems. These can be summarized in a number of techniques [12, 24, 27, 28, 111, 112].

Inclusion of elastic elements Elastic materials connecting the two frames in the suspension may be used to stabilize the system in the unstable degrees of freedom. In other words, the magnetic fields stabilize only a few degrees of freedom, where mechanical connections ensure stability in the other directions. One could think of a leaf spring assembly connecting two repelling permanent magnets or the use of sliding bearings. Such elements provide a physical contact between the frames and via this way may induce parasitic effects into the system such as eigenmodes of the springs and the transmission of unwanted vibrations.

Diamagnetics As shown in Appendix C passive stable levitation is possible when diamagnetic materials are placed in conservative permanent magnetic fields. Contrary to ferro- or paramagnetic materials, which have tendency to align with the magnetic field and move towards field maxima, diamagnetic materials are repelled by magnetic fields and attracted to field minima, hence, provide for a stable equilibrium position. Experiments of stable levitation using such diamagnetic materials have been described by several authors [24, 28, 166, 168]. Natural diamagnetic materials as graphite or bismuth exhibit relative permeability values lower than, but close to one, and therefore the repelling effect is very weak [20, 168]. Mainly for this reason diamagnetic levitation is considered unsuitable for the application under focus because of the large envisaged masses and is even considered of no more than academic interest [111, 112].

Superconduction Ideal diamagnetic behavior is exhibited by superconducting materials, which exhibit zero relative permeability, or \( \mu_r = 0 \) [63]. As a result, the magnetic field is expelled by the superconductor (the Meissner-Ochsenfeld...
effect [63, 126]) and passive levitation with permanent magnets and supercon-ductors becomes feasible [27]. After the first example of superconducting lev-itation achieved in 1947 by Arkadiev [11] many examples of superconduction-based suspension have been published. In most cases it is applied in Maglev systems for heavy transport vehicles although some applications in magnetic suspension are also known [91, 111, 112]. The high degree of cooling that is required to obtain superconduction leads to a high energy consumption, which cannot in all cases be rationalized.

Spin-stabilized magnetic levitation A stable form of levitation is spin-stabilized mag-netic levitation of a spinning magnet top over a permanent magnetic field, originally proposed by Harrigan [79] in the form of the so-called Levitron. The principal mechanism of stability is static equilibrium in a potential energy field $W$, arising dynamically from the adiabatic coupling of the spin with the magnetic field $\vec{B}$ of the base and involving the magnitude $|\vec{B}|$ of this field [19, 167]. $W$ is close to a harmonic potential, that is, one whose Laplacian is zero, for which Earnshaw’s theorem would forbid stable equilibrium. Therefore its minimum is very shallow, and requires the mass of the top to be adjusted delicately so that it hangs within a small interval of height. The stability interval is increased by a post-adiabatic dynamic coupling of the velocity of the top to $\vec{B}$, through an effective ‘geometric magnetic field’ constructed from the spatial derivatives of $\vec{B}$; this effect gets stronger as the top is spun faster [19].

Actually, the levitron can also be considered as a sort of diamagnet. By rota-tion, one stabilizes the direction of the magnetic moment in space (magnetic gyroscope). Then this magnet is placed with the fixed magnetization in an anti-parallel magnetic field and it levitates.

Oscillating fields An oscillating magnetic field will induce an alternating current in a conductor and thus generate a levitating force. This oscillating field causes a conducting body to behave like a diamagnetic material as it generates a current on its surface. Due to a finite resistance, the induced changes in electron trajectories disappear after a short time but it is possible to create a permanent screening current at the surface by applying an oscillating field, thus creating a kind of induction machine [111, 112]. Although the force capabilities of such system are quite high, the induced heat as result of the eddy currents quickly becomes rather high due to the material resistance.

Tuned LCR circuits This method utilizes the variation of inductance of an electro-magnet in the proximity of a ferromagnetic body, depending on the separation between the two, to regulate the current and hence an attraction force. This is achieved by incorporating an electromagnet within an LCR-circuit which is tuned such that it becomes resonant when the airgap increases, thus the current and consequently the force rises. Conversely, when the body moves towards the electromagnet the current and the force of attraction diminish. Okress et al. [139] used the principle to levitate metal specimens of several grams and reports that the high frequencies. Systems based on this principle exhibit large time constants and consequently go into a divergent oscillation once disturbed
unless some means are employed to speed up the current changes or add more damping. The main disadvantages of such system is the fact that the circuit is predominantly inductive and hence the reactive power input is rather large and that the iron structure including the object to be suspended must be laminated and therefore has not resulted in any practical applications [111, 112].

**Feedback-controlled time-varying magnetic fields** This method is considered the most developed and most widely used method to gain stable magnetic suspension systems. It disobeys Earnshaw's conclusions, who solely discussed static fields, by producing local minima of total energy in space with the use of feedback-controlled time-varying magnetic fields. Such time-dependent magnetic field may be created by incorporating electrical conductors with variable current in the system. An overview of the early developments in these systems is found in [111, 112]. If all magnetic fields in such a device are generated by electromagnets rather than permanent magnets, the energy consumption and resulting thermal issues become of significant influence on the machine design, especially for large vertical forces. Permanent-magnet based systems, where a dynamically controlled field interacts with a static permanent-magnetic field, are less effected by these issues, however, are generally more costly.
Appendix D

Gravity compensator

D.1 Working points of the magnets

The flux density levels in the magnets of the gravity compensator are obtained with the equations in Section 3.2.1. Subsequently, (2.11) is used to obtain the corresponding field. This is performed for all magnets in one of the four gravity compensator legs on a 0.5mm mesh. Figure E.3 in Appendix E.4 shows that the material has a high demagnetization withstand. The results in Fig. D.1 show that demagnetization is no issue in the gravity compensator.

![Figure D.1: Working points on a 0.5mm mesh within the magnets of the gravity compensator.](image)
Appendix D. Gravity compensator

D.2 Estimation of the electromagnetic damping

The passive damping of the gravity compensator described in Chapter 5 is estimated with a 3D semi-analytical model. This technique, described in [146], is based on a scalar potential formulation, which yields the surface charge model of a cuboidal permanent magnet that is located above a moving conductive plate. The electric field in this moving conductor is obtained using Faraday’s law (2.3)

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}. \quad (D.1)$$

The eddy current density $\vec{J}_e$ is obtained by the Lorentz force (2.42) on this conductor

$$\vec{J}_e = \sigma_e \left( \vec{E} + \vec{v} \times \vec{B} \right). \quad (D.2)$$

If this is written into the primed coordinates of the moving frame [66, 146] it simplifies to

$$\vec{J}_e' = \sigma_e \vec{E}', \quad (D.3)$$

where (D.1) is written in primed coordinates to obtain $\vec{E}'$. Under the assumption that the eddy current density in the primed coordinate system, $\vec{J}_e'$, equals that of the global coordinate system, $\vec{J}_e$, the damping force is obtained by a numerical integration of

$$\vec{F} = \int_V \vec{J}_e' \times \vec{B} \, dv', \quad (D.4)$$

where $V$ is the volume of the conductive plate. The method of images (see Section 2.5.4) is used to account for the finite dimensions of the conductive plate. This method, as well as a more detailed description and validation of the above mentioned technique to model the eddy current damping, is found in [146]. As it does not solve the diffusion equation, i.e. the magnetic field that is induced by the
eddy currents, is not incorporated, this method may become inaccurate at elevated velocities. However, the relative velocities that are expected in this thesis are below those simulated in [146] and as such the method is considered sufficiently suitable.

The gravity compensator is simplified to a single magnet above a conductive plate moving vertically as Fig. D.2 shows. The magnet dimensions are \((7.1 \hat{e}_x \times 91 \hat{e}_y \times 40 \hat{e}_z)\) mm. The aluminum conductor \((\sigma_e = 36 \cdot 10^6 \text{ S/m})\) is \((18 \hat{e}_x \times 100 \hat{e}_y \times 200 \hat{e}_z)\) mm. The airgap between the magnet and the conductor is 10.5 mm. It is found that the damping for this simplified model is 4.3 Ns/m. The gravity compensator has 64 of such poles, hence a multiplication of the damping yields a total damping of 3.0kNs/m. It must be noted that this is a rough estimation, based on the model of Fig. D.2. Nevertheless, it provides the order of magnitude of the eddy current damping constant.
Appendix E

Test setup
### E.1 Shake rig

Figure E.1 shows the simulated eigenmodes of the shake rig described in Chapter 7 as well as the first internal resonance frequency of the shake springs.

![Eigenmodes and resonance frequencies](image)

**Figure E.1:** The simulated first six eigenmodes of the shake rig (with dummy mass for the inner cross and actuator stators) and the internal resonance of the shake springs.
The active wrench as measured in Section 7.7 result from the actuator forces according (E.1). The directly measured active wrench is shown in Fig. E.2.

**Figure E.2:** The measured active wrench $\vec{\alpha}_{\text{act}}$. The measured planes are at $z = -0.5\,\text{mm}$ (yellow), $z = 0.0\,\text{mm}$ (orange) and $z = 0.5\,\text{mm}$ (red).
E.3 Transformation matrices

The transformation matrix that has been used to transform the individual actuator forces $F_{1h} \ldots F_{3v}$ to a wrench on the isolated platform's CoG is defined by

$$
\begin{bmatrix}
F_x \\
F_y \\
F_z \\
T_x \\
T_y \\
T_z
\end{bmatrix}
= \begin{bmatrix}
\cos(\frac{\pi}{4}) & \cos(\frac{\pi}{6}) & -1 \\
\sin(\frac{\pi}{4}) & -\sin(\frac{\pi}{6}) & 0 \\
0 & 0 & 0 \\
-\sin(\frac{\pi}{4})(h_h - h_{gc}) & \sin(\frac{\pi}{4})(h_h - h_{gc}) & \ldots \\
\cos(\frac{\pi}{4})(h_h - h_{gc}) & \cos(\frac{\pi}{4})(h_h - h_{gc}) & \cos(\frac{\pi}{4})(h_h - h_{gc}) \\
r_h & r_h & r_h
\end{bmatrix}
\begin{bmatrix}
F_{1h} \\
F_{2h} \\
F_{3h} \\
F_{1v} \\
F_{2v} \\
F_{3v}
\end{bmatrix}
$$

(E.1)

The variables are defined in Section 7.5.4.

E.4 Material properties

Table E.1 summarizes some important typical and minimum remanence and coercivity values of transverse-field pressed material. The typical remanent flux density and typical coercivity have been used to obtain the relative permeability $\mu_{\text{typ}}$ of the last column, which is approximately 1.03 for all materials. The material properties of Vacodym 854 TP are shown in Table E.3 and Fig. E.3.
Table E.1: Characteristic properties of transverse-field pressed hard magnetic materials [180] and the typical relative permeability obtained from these data.

<table>
<thead>
<tr>
<th></th>
<th>( B_{\text{typ}} )</th>
<th>( B_{\text{min}} )</th>
<th>( H_{c,b_{\text{typ}}} )</th>
<th>( H_{c,b_{\text{min}}} )</th>
<th>( \mu_{\text{typ}} )</th>
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<tr>
<td>745 TP</td>
<td>1.41</td>
<td>1.37</td>
<td>1090</td>
<td>1035</td>
<td>1.0294</td>
</tr>
<tr>
<td>764 TP</td>
<td>1.37</td>
<td>1.33</td>
<td>1060</td>
<td>1005</td>
<td>1.0285</td>
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<td>776 TP</td>
<td>1.32</td>
<td>1.28</td>
<td>1020</td>
<td>970</td>
<td>1.0298</td>
</tr>
<tr>
<td>837 TP</td>
<td>1.37</td>
<td>1.33</td>
<td>1060</td>
<td>1010</td>
<td>1.0285</td>
</tr>
<tr>
<td>854 TP</td>
<td>1.32</td>
<td>1.28</td>
<td>1020</td>
<td>970</td>
<td>1.0298</td>
</tr>
<tr>
<td>863 TP</td>
<td>1.29</td>
<td>1.25</td>
<td>995</td>
<td>950</td>
<td>1.0317</td>
</tr>
<tr>
<td>872 TP</td>
<td>1.25</td>
<td>1.21</td>
<td>965</td>
<td>915</td>
<td>1.0308</td>
</tr>
<tr>
<td>890 TP</td>
<td>1.19</td>
<td>1.15</td>
<td>915</td>
<td>865</td>
<td>1.0349</td>
</tr>
<tr>
<td>633 TP</td>
<td>1.32</td>
<td>1.28</td>
<td>1020</td>
<td>970</td>
<td>1.0298</td>
</tr>
<tr>
<td>655 TP</td>
<td>1.26</td>
<td>1.22</td>
<td>970</td>
<td>925</td>
<td>1.0337</td>
</tr>
<tr>
<td>669 TP</td>
<td>1.22</td>
<td>1.17</td>
<td>940</td>
<td>875</td>
<td>1.0328</td>
</tr>
<tr>
<td>677 TP</td>
<td>1.18</td>
<td>1.13</td>
<td>915</td>
<td>860</td>
<td>1.0262</td>
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<tr>
<td>688 TP</td>
<td>1.14</td>
<td>1.09</td>
<td>885</td>
<td>830</td>
<td>1.0251</td>
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</table>

Table E.2: Physical properties of aluminum and stainless steel.

<table>
<thead>
<tr>
<th>Description</th>
<th>Aluminum</th>
<th>Stainless steel</th>
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</thead>
<tbody>
<tr>
<td>Mass density</td>
<td>(2.7 \cdot 10^3) kg/m(^3)</td>
<td>(8.0 \cdot 10^3) kg/m(^3)</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>235 W/(mK)</td>
<td>14.6 W/(mK)</td>
</tr>
<tr>
<td>Resistivity (20(^\circ))</td>
<td>(2.8 \cdot 10^{-8}) Ωm</td>
<td>(74 \cdot 10^{-8}) Ωm</td>
</tr>
</tbody>
</table>

Table E.3: Characteristic properties of Vacodym 854TP [180].

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Remanence</td>
<td>(B_r) [T]</td>
<td>1.28</td>
<td>1.32</td>
<td></td>
</tr>
<tr>
<td>Coercivity</td>
<td>(H_{c,b}) [kA/m]</td>
<td>970</td>
<td>1020</td>
<td></td>
</tr>
<tr>
<td>Intrinsic coercivity</td>
<td>(H_{ci}) [kA/m]</td>
<td>1670</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Remanence temperature coefficient</td>
<td>(B_r) [%/°C]</td>
<td>-0.105</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coercivity temperature coefficient</td>
<td>(H_{ci}) [%/°C]</td>
<td>-0.60</td>
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</tr>
<tr>
<td>Mass density</td>
<td>(\rho) [g/cm(^3)]</td>
<td>7.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Electrical resistivity</td>
<td>(\rho) [Ωm]</td>
<td>1.2</td>
<td>1.6</td>
<td></td>
</tr>
</tbody>
</table>
Figure E.3: Typical demagnetization curves for Vacodym 854TP [180].
E.5 Equipment properties

Table E.4: Equipment used in the measurements.

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<th>Description</th>
<th>Manufacturer</th>
<th>Type</th>
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<td>Data acquisition system</td>
<td>Speedgoat</td>
<td>xPC Target system</td>
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<tr>
<td>Amplifiers of the shake actuators</td>
<td>Prodrive</td>
<td>QUAD 260/50</td>
</tr>
<tr>
<td>Amplifiers of the vibration isolator</td>
<td>Prodrive</td>
<td>3AX 52/6</td>
</tr>
<tr>
<td>Optical position sensors</td>
<td>Philtec</td>
<td>RC100-AELN</td>
</tr>
<tr>
<td>Accelerometers</td>
<td>Kistler</td>
<td>8330M04</td>
</tr>
<tr>
<td>Shake actuators</td>
<td>BEI Kimco</td>
<td>LA50-65-01Z</td>
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<tr>
<td>PT100 temp. readout</td>
<td>Agilent</td>
<td>34970A</td>
</tr>
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</table>

Table E.5: Physical properties of the data acquisition system.

<table>
<thead>
<tr>
<th>Description</th>
<th>Type</th>
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<tbody>
<tr>
<td>Core</td>
<td>Core2Duo 3.33GHz / 2048MB RAM</td>
</tr>
<tr>
<td>Analog inputs</td>
<td>IO106 64-channel 16-bit parallel conversion</td>
</tr>
<tr>
<td>Serial communication</td>
<td>IO504 8-channel RS485</td>
</tr>
</tbody>
</table>

Table E.6: Physical properties of the power amplifiers.

<table>
<thead>
<tr>
<th>Description</th>
<th>3AX</th>
<th>QUAD</th>
</tr>
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<tbody>
<tr>
<td>Converter type</td>
<td>PWM</td>
<td>PWM</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>4 kHz</td>
<td>6 kHz</td>
</tr>
<tr>
<td>Current (rms)</td>
<td>2 A</td>
<td>A</td>
</tr>
<tr>
<td>Current (max)</td>
<td>±6 A</td>
<td>±12.5 A</td>
</tr>
<tr>
<td>Voltage (max)</td>
<td>±52 V</td>
<td>±260 V</td>
</tr>
<tr>
<td>Interface</td>
<td>RS485 (5.5 Mbps)</td>
<td>RS485 (5.5 Mbps)</td>
</tr>
<tr>
<td>DC-link</td>
<td>GEN60-40-1P230</td>
<td>GEN500-30-MD-3P400</td>
</tr>
<tr>
<td></td>
<td>(60 V - 40 A)</td>
<td>(500 V - 30 A)</td>
</tr>
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Table E.7: Physical properties of the optical position sensors.

<table>
<thead>
<tr>
<th>Description</th>
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<tbody>
<tr>
<td>Type</td>
<td>Philtec RC100-AELN</td>
</tr>
<tr>
<td>Range</td>
<td>0-5 mm</td>
</tr>
<tr>
<td>Spot size Ø</td>
<td>2.5 mm</td>
</tr>
<tr>
<td>Resolution</td>
<td>0.75 µm</td>
</tr>
<tr>
<td>Nominal sensitivity</td>
<td>1.3 mV/µm</td>
</tr>
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</table>
### Table E.8: Physical properties of the accelerometers.

<table>
<thead>
<tr>
<th>Description</th>
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<tbody>
<tr>
<td>Type</td>
<td>Kistler 8330M04</td>
</tr>
<tr>
<td>Range</td>
<td>±3 g</td>
</tr>
<tr>
<td>Resolution</td>
<td>&lt; 1.3 µg</td>
</tr>
<tr>
<td>Nominal sensitivity</td>
<td>1200 mV/g</td>
</tr>
<tr>
<td>Linearity</td>
<td>±0.1%</td>
</tr>
<tr>
<td>Frequency response ±5%</td>
<td>0…1000 Hz</td>
</tr>
<tr>
<td>Frequency response ±3 dB</td>
<td>0…2500 Hz</td>
</tr>
</tbody>
</table>

### Table E.9: Physical properties of the shake actuators.

<table>
<thead>
<tr>
<th>Description</th>
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<tbody>
<tr>
<td>Type</td>
<td>BEI Kimco LA50-65-001Z</td>
</tr>
<tr>
<td>Force constant</td>
<td>124.1 N/A</td>
</tr>
<tr>
<td>Back EMF constant</td>
<td>125 V/(m/s)</td>
</tr>
<tr>
<td>DC Resistance</td>
<td>4.6 Ω</td>
</tr>
<tr>
<td>AC inductance (1 kHz)</td>
<td>5.3 mH</td>
</tr>
<tr>
<td>Actuator constant</td>
<td>57 N/√W</td>
</tr>
<tr>
<td>Coil assembly weight</td>
<td>1.45 kg</td>
</tr>
<tr>
<td>Field assembly weight</td>
<td>11 kg</td>
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</tbody>
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### Table E.10: Physical properties of the amplifiers used for the accelerometer signals.

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<tr>
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<tr>
<td>Type</td>
<td>Krohn-Hite Model 3364</td>
</tr>
<tr>
<td>Mode</td>
<td>Low-pass 4-pole</td>
</tr>
<tr>
<td>Attenuation</td>
<td>24 dB/dec</td>
</tr>
<tr>
<td>Pre-filter gain</td>
<td>0 dB-50 dB</td>
</tr>
<tr>
<td>Post-filter gain</td>
<td>0 dB-20 dB</td>
</tr>
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### Table E.11: Physical properties of the readout device of the PT100 sensors.

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<thead>
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<tbody>
<tr>
<td>Type</td>
<td>Agilent 34970A</td>
</tr>
<tr>
<td>Measurement</td>
<td>4-wire PT100</td>
</tr>
<tr>
<td>Multiplexer</td>
<td>Agilent 34901A, 20 channels</td>
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</table>
## Nomenclature

### Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Quantity</th>
<th>Page</th>
</tr>
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<tbody>
<tr>
<td>( \vec{A} )</td>
<td>[Vs/m]</td>
<td>Magnetic vector potential</td>
<td>26</td>
</tr>
<tr>
<td>( \vec{B} )</td>
<td>[T]</td>
<td>Magnetic flux density</td>
<td>24</td>
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<tr>
<td>( \vec{B}_r )</td>
<td>[T]</td>
<td>Remanent magnetic flux density</td>
<td>25</td>
</tr>
<tr>
<td>( b )</td>
<td>[Ns/m]</td>
<td>Damping constant</td>
<td>89</td>
</tr>
<tr>
<td>( \vec{c} )</td>
<td>[\text{--}]</td>
<td>Nonlinear inequality constraint vector</td>
<td>110</td>
</tr>
<tr>
<td>( \vec{c}_{eq} )</td>
<td>[\text{--}]</td>
<td>Nonlinear equality constraint vector</td>
<td>110</td>
</tr>
<tr>
<td>( \vec{D} )</td>
<td>[C/m(^2)]</td>
<td>Electrical flux density</td>
<td>24</td>
</tr>
<tr>
<td>( d_i )</td>
<td>[m]</td>
<td>Inner cross support thickness</td>
<td>131</td>
</tr>
<tr>
<td>( d_o )</td>
<td>[m]</td>
<td>Outer cross support thickness</td>
<td>131</td>
</tr>
<tr>
<td>( d_{w_i} )</td>
<td>[m]</td>
<td>Copper diameter of wire</td>
<td>159</td>
</tr>
<tr>
<td>( d_{w_o} )</td>
<td>[m]</td>
<td>Outer diameter of wire</td>
<td>158</td>
</tr>
<tr>
<td>( \vec{E} )</td>
<td>[V/m]</td>
<td>Electrical field strength</td>
<td>24</td>
</tr>
<tr>
<td>( F )</td>
<td>[N]</td>
<td>Force</td>
<td>33</td>
</tr>
<tr>
<td>( f )</td>
<td>[N/m(^3)]</td>
<td>Force density</td>
<td>33</td>
</tr>
<tr>
<td>( f_s )</td>
<td>[N/m(^2)]</td>
<td>Surface force density</td>
<td>59</td>
</tr>
<tr>
<td>( f_r )</td>
<td>[Hz]</td>
<td>Resonance frequency</td>
<td>104</td>
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<td>( g )</td>
<td>[m/s(^2)]</td>
<td>Acceleration due to gravity</td>
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</tr>
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<td>[\text{--}]</td>
<td>Transmissibility transfer function</td>
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</tr>
<tr>
<td>( H_c(s) )</td>
<td>[1/kg]</td>
<td>Compliance transfer function</td>
<td>215</td>
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<tr>
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<td>[A/m]</td>
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<td>Intrinsic coercivity</td>
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<td>[m]</td>
<td>Height</td>
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<td>[m]</td>
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<td>[m]</td>
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<td>[m]</td>
<td>Magnet height</td>
<td>109</td>
</tr>
<tr>
<td>( i )</td>
<td>[A]</td>
<td>Electrical current</td>
<td>158</td>
</tr>
<tr>
<td>( J )</td>
<td>[A/m(^2)]</td>
<td>Electrical volume current density</td>
<td>24</td>
</tr>
<tr>
<td>Symbol</td>
<td>[Unit]</td>
<td>Quantity</td>
<td>Page</td>
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<td>----------</td>
<td>------</td>
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<td>$\vec{j}_m$</td>
<td>[A/m$^2$]</td>
<td>Equivalent magnetic volume current density</td>
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<tr>
<td>$\vec{j}$</td>
<td>[-]</td>
<td>Jacobian</td>
<td>62</td>
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<td>[N/m]</td>
<td>Stiffness</td>
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<td>[m]</td>
<td>Outer coil length</td>
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<td>[m]</td>
<td>Halbach magnet length</td>
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</tr>
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<td>[m]</td>
<td>Magnet length</td>
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</tr>
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<td>[m]</td>
<td>Wire length</td>
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</tr>
<tr>
<td>$\vec{M}$</td>
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<td>Magnetization</td>
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<td>Rotation matrix</td>
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<td>[kg]</td>
<td>Mass</td>
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<td>$\vec{m}$</td>
<td>[Am$^2$]</td>
<td>Magnetic dipole moment</td>
<td>29</td>
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<td>Number of turns</td>
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<td>[m]</td>
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<td>$R$</td>
<td>[\Omega]</td>
<td>Electrical resistance</td>
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<td>[N$^2$/W]</td>
<td>Actuator steepness</td>
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<td>$S$</td>
<td>[m$^2$]</td>
<td>Surface</td>
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<td>$\vec{T}$</td>
<td>[Nm]</td>
<td>Torque</td>
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<td>Torque density</td>
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<td>[J]</td>
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<td>Wrench</td>
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<tr>
<td>$w_o$</td>
<td>[m]</td>
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<td>$w_{ci}$</td>
<td>[m]</td>
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<tr>
<td>Symbol</td>
<td>[Unit]</td>
<td>Quantity</td>
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<td>--------</td>
<td>----------</td>
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</tr>
<tr>
<td>( w_c )</td>
<td>[m]</td>
<td>Outer coil width</td>
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<td>[m]</td>
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<tr>
<td>( \epsilon_{hc} )</td>
<td>[-]</td>
<td>Hot-cold effect</td>
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<td>[-]</td>
<td>Copper fill factor</td>
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<tr>
<td>( \zeta )</td>
<td>[-]</td>
<td>Damping ratio</td>
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<td>[H/m]</td>
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<td>Relative permeability</td>
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<tr>
<td>( \tilde{\lambda} )</td>
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<td>Intermediate torque computation variable</td>
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</tr>
<tr>
<td>( \Xi )</td>
<td>[m]</td>
<td>Intermediate stiffness computation variable</td>
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</tr>
<tr>
<td>( \xi )</td>
<td>[m²]</td>
<td>Intermediate force computation variable</td>
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<td>( \rho_e )</td>
<td>[Ωm]</td>
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<td>[C/m³]</td>
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<td>[A/m²]</td>
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<td>( \tau_e )</td>
<td>[s]</td>
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<td>Pole pitch</td>
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<td>( \Psi )</td>
<td>[Wb]</td>
<td>Flux linkage</td>
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<td>( \phi )</td>
<td>[Wb]</td>
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<td>[-]</td>
<td>Objective function</td>
<td>110</td>
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<td>( T )</td>
<td>[N/m²]</td>
<td>Maxwell stress tensor</td>
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### Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Explanation</th>
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<td>3D</td>
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</tr>
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<td>Analytical</td>
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<td>Axial pressed</td>
</tr>
<tr>
<td>BEM</td>
<td>Boundary Element Method</td>
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<td>Center of Actuation</td>
</tr>
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<td>Center of Gravity</td>
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<tr>
<td>CoM</td>
<td>Center of mass</td>
</tr>
<tr>
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<td>Degree of Freedom</td>
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<td>Finite Difference Method</td>
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<td>FEM</td>
<td>Finite Element Method</td>
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### Operators

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<td>·</td>
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<td>×</td>
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<td>∇·</td>
<td>Divergence</td>
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### General definitions

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Samenvatting

Extended Analytical Charge Modeling for Permanent-Magnet Based Devices
Practical Application to the Interactions in a Vibration Isolation System

Dit proefschrift onderzoekt de analytische ‘surface charge’ modelleringstechniek die een snelle, discretisatie-vrije en nauwkeurige beschrijving geeft van complexe en onbegrensde elektromagnetische problemen. Tot op heden is de methode slechts weinig gebruikt om passieve en actieve permanent-magneet apparaten te ontwerpen, aangezien de beschikbare kant-en-klare vergelijkingen gelimiteerd waren tot slechts enkele magnetische structuren. Hoewel beschikbare publicaties in de literatuur het potentieel van deze modelleringstechniek hebben gedemonstreerd, lichten zij slechts een tipje van de sluier op wat betreft de toepassingsmogelijkheden.

Het onderzoek dat in deze thesis wordt gepresenteerd biedt nieuwe kant-en-klare analytische vergelijkingen voor kracht, stijfheid en koppelberekeningen. De analytische vergelijkingen voor de kracht tussen balkvormige magneten zijn nu toepasbaar voor iedere combinatie van magnetisatievectoren en iedere relatieve positie. Symbolisch afgeleide vergelijkingen bieden een directe analytische uitdrukking voor de $3 \times 3$ stijfheidsmatrix. Verder worden analytische koppelvergelijkingen geïntroduceerd die een vrije keuze in referentiepunt toestaan. Hiermee kan het koppel op een structuur met meerdere magneten direct worden berekend. Enkele onderzoeksgebieden, zoals de analytische berekening van de kracht en het koppel op geroerde magneten en de uitbreiding van de veldbeschrijvingen naar onconventioneel gevormde magneten, liggen buiten het doelgebied van dit proefschrift en worden als zodanig aangeraden voor verder onderzoek.

Een vibratie isolatie systeem, dat wereldwijd de eerste in zijn soort is en gebaseerd is op permanente magneten, is onderzocht en ontwikkeld met behulp van deze geavanceerde modelleringstechniek. Dit systeem combineert een hoge verticale kracht met zeer lage stijfheid. Dit unieke 6-DoF vibratie isolatie systeem verbruikt een minimale hoeveelheid energie ($< 1 \text{ W}$) en maakt gebruik van zijn elektromagnetische eigenschappen door de isolatie-bandbreedte te maximaliseren ($> 700 \text{ Hz}$). Het resulterende systeem is afgeveerd op een frequentie $< 1 \text{ Hz}$ en heeft een acceleratie-helling van $-2 \text{ dB per decade}$. Het bijna lineaire gedrag in het gehele 6-DoF werkgebied
maakt het mogelijk een ongecompliceerde regelaar te gebruiken. De positioneringsnauwkeurigheid ligt rond 4 \mu m en daarmee dichtbij de theoretische sensorruis van 1 \mu m.

De uitvoerig onderzochte passieve (geen energieverbruik) zwaartekracht compensator, die gebaseerd is op permanente magneten, vormt het magnetische hart van dit vibratie isolatiesysteem. Deze zwaartekracht compensator combineert een verticale kracht van 7.1 kN met een veerstijfheid van < 10 kN/m in alle zes de vrijheidsgraden. Deze conflictende eisen zijn extreem uitdagend en daarom wordt het uitvoerige onderzoek naar geschikte topologieën voor de zwaartekracht compensator in dit proefschrift gepresenteerd. De resulterende kruisvormige topologie met verticale luchtspleten is geregistreerd als Europees patent. Experimenten hebben de invloed van de temperatuur op het elektromechanisch gedrag (1.7%/K of 12 N/K) aangetoond. De zwaartekracht compensator heeft twee geïntegreerde voice coil actuatoren die ontworpen zijn om een hoge kracht te combineren met een lage energieconsumptie (een steepness van 625 N^2/W en een kracht-constante van 31 N/A) binnen de opgelegde spannings- en stroomlimieten. Drie van zulke vibratie isolatoren, elk met een passieve 6-DoF zwaartekracht compensator en geïntegreerde 2-DoF actuatoren, kunnen de zes vrijheidgraden stabiliseren.

De experimentele resultaten demonstreren de haalbaarheid van passieve magneetgebaseerde zwaartekracht compensatie voor een geavanceerd vibratie isolatiesysteem voor hoge massa’s. De modulaire topologie laat een eenvoudige schaling van de kracht en veerstijfheid toe. Het onderzoek dat in dit proefschrift wordt gepresenteerd laat het hoge potentieel zien van deze nieuwe klasse elektromagnetische apparaten voor vibratie isolatie of andere toepassingen die hoge eisen stellen op het gebied van kracht, veerstijfheid en energie consumptie. Zoals bij iedere nieuwe klasse zijn er nog deelproblemen die een verdere studie vereisen vooraleer ze geïmplementeerd kunnen worden in de volgende generatie vibratie isolatiesystemen. Voorbeelden van dergelijke deelproblemen zijn een afstellings mogelijkheid van de verticale kracht van de zwaartekracht compensator en een reductie van de magnetische lekflux.
Acknowledgements

The research presented in this thesis was performed within the framework of the IOP-EMVT program (Innovatiericht Onderzoeksprogramma - Elektromagnetische Vermogenstechniek) under IOP-EMVT 06225, funded by Agentschap NL – an agency of the Dutch Ministry of Economic Affairs. I would like to express my gratitude to the following people:

André Vandenput, for recognizing my capabilities to perform research on a PhD level and encouraging me to start this research at the EPE group. Unfortunately, he passed away in 2008 after an unfortunate illness;

Elena and Johan, for your encouragements to apply for a PhD position after my graduation, the many technical and non-technical conversations that we enjoyed, the great professional developments that I have made and the pleasant conference trips that we made together;

The people from the Control Systems group whom I worked with: Chenyang Ding for his input and cooperation in this project. Further, Paul van den Bosch and Ad Damen for showing confidence in our skills and the discussions we had;

My roommates Bart (koop toch een fiets) en Koen (je wint nog wel een keer), for keeping me sharp with countless jokes – that we made with each other but more than often about each other –, the ever lasting discussions about who would get the coffee this time, the in-room ‘squash’, the many technical discussions we had in front of our white-board and of course enjoying a nice beer together after work on a regular basis;

Marijn, for your great suggestions, discussions and practical contributions to the test setup and of course the many enjoyable non-technical discussions we had;

Patrick de Laat (GTD) for the many hours of discussion on the design of the test setup and the efforts in design and drawing of various components of the test setup. Further, Erwin Dekkers for the contributions in the early stage of prototyping;

The core committee members Kay Hameyer, Jean-Paul Yonnet and Maarten Steinhuch for proofreading this thesis and their valuable suggestions to improve it; also the advisors André Preumont and Michiel Antal for being part of my committee;

The members of the EPE group for the pleasant time I had working in this stimulating environment, and the fun we had in our yearly trips;

Hans Butler and his colleagues from ASML for the many useful technical advices and feedback that we received during all stages of the research presented in this thesis;

The suppliers who made special efforts to contribute to the various parts of the test setup: Gemeenschappelijke Technische Dienst, Vacuumschmelze, Prodrive and
Tecnotion.

Kevin Pluk for his ‘courage’ to graduate under the supervision of two people in the last stage of their PhD research and for his last-minute efforts for one of the appendices;

Armin, Ilse, Chris, Taylor, Scooter, Fabrizio and KT for their seemingly effortless musical support during the many hours of work in our office;

Geert voor het ontwerpen van mijn omslag en Leo voor de leuke cartoon;

Alle vrienden van thuis en uit mijn middelbare school- en studententijd die ik gedurende de afgelopen jaren steeds minder heb gezien. Ik ben vastberaden om hier in de toekomst verbetering in te brengen! Verder de mensen van de fanfare die er toch voor zorgden dat in ieder geval de zaterdagavond nog een rustpunt in de week is gebleven en een goede reden om even na te borrelen;

Pap, mam, Geert en Thijs voor hun steun in de afgelopen jaren, waaraan ik veel heb gehad. Wanneer de druk even goed op de ketel stond, stonden ze altijd klaar om de zaken die er bij in schoten voor me te regelen;

Mien leeve Karlijn, veur de geweldige steun die se bós gewaes, auch wanneer ut Limburgs kwarteerke zeker op mich van toepassing woor, of nog erger, det ich pas deep in d’n aovend thoes koom nao een daag sjrieve of det ich ut ganse weekend lang zoot te tikken; natuurlijk auch veur de heerlijke magnetron-diners op maondigaovend op de TU.
Curriculum Vitae

Jeroen Janssen was born in Boxmeer, The Netherlands, in 1982. He attended secondary school at Scholengemeenschap St. Ursula, Horn. He received the M.Sc. degree in Electrical Engineering from Eindhoven University of Technology (TU/e) in 2006. The title of this work was "Design of Active Suspension using Electromagnetic Devices". His graduation work was one of the thirteen nominees which were selected from all TU/e graduation projects for the Mignot Graduation Prize 2006 of that year.

In 2007 he started as a PhD student in the Electromechanics and Power Electronics (EPE) group at Eindhoven University of Technology where his research subject was electromagnetic vibration isolation. In this project he has modeled, designed and realized a high-performance vibration isolation system with permanent-magnet based gravity compensation, which has resulted in this thesis. He is author or co-author of more than 12 journal publications, 25 conference publications, one book chapter and one patent application. In 2009 he received the award for best presentation by a young researcher at the 2009 International Symposium on Electromagnetic Fields (ISEF) in Arras, France. Since 2011 he is employed at Philips Innovation Services.

His technical interests include the modeling, design and implementation of electromagnetic actuators and drives, as well as related areas of expertise.