Vibration isolation control of a contactless electromagnetic suspension system
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Vibration Isolation Control of a Contactless Electromagnetic Suspension System

PROEFSCHRIFT

ter verkrijging van de graad van doctor aan de Technische Universiteit Eindhoven, op gezag van de rector magnificus, prof.dr.ir. C.J. van Duijn, voor een commissie aangewezen door het College voor Promoties in het openbaar te verdedigen op donderdag 26 september 2013 om 16.00 uur

door

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Summary

Vibration Isolation Control of a Contactless Electromagnetic Suspension System

This thesis systematically explores the opportunities to improve the performance (transmissibility and compliance) of a 6-DoF contactless electromagnetic suspension system by system-level design, choices of sensors, identification of the static and dynamic behaviors, and development of advanced control strategies. Such a system, also referred to as the Single Electro-Magnetic Isolator System (SEMIS), combines the advantage of passive gravity force (7.2 kN) compensation and low power consumption (0.3~6 W). It is a typical multi-DoF active suspension system.

For vibration isolation control of a general multi-DoF active suspension system, this thesis proposes a design strategy of vibration isolation control which combines the optimal static decoupling and the sliding surface control. The optical static decoupling reduces a multi-DoF vibration isolation control problem to multiple 1-DoF vibration isolation control problems. Each 1-DoF vibration isolation control is to be solved by the sliding surface control.

Optimal static decoupling is studied for an LTI mechanical system with and without the assumption of proportional damping. If proportional damping may be assumed for an LTI mechanical system, two methods, modal decomposition and Owens method, are both applicable and perfect decoupling can be achieved. It is proved in this thesis that the two different pairs of input- and output-decoupling matrices derived by these two methods can be normalized to the same pair of decoupling matrices. If proportional damping may not be assumed for an LTI mechanical system, Vaes-procedure can be applied to derive a pair of optimal static decoupling matrices. This thesis develops two modifications to the Vaes-procedure to improve its numerical stability and reduce its computational complexity.

A measurement scheme, which combines metrology frame absolute acceleration and its displacement relative to its supporting base frame, is chosen for vibration isolation control of SEMIS. As such, the controller for each DoF motion has two inputs and a single output (force or torque). The sliding surface control is developed based on the frequency-shaped sliding surface approach in literature, to design a DISO vibration isolation controller for a 1-DoF active suspension system. The advantage of the sliding surface control with respect to conventional methods is that the transmissibility and the compliance are designed in two independent steps. As such, the design process is simplified. The cross-coupling of a multi-DoF system can be reduced as much as possible by static decoupling such that the decoupled system may be treated as multiple 1-DoF systems in control design.

The proposed control strategy is compared with $H_\infty$ optimization based on an identified model of the 3-DoF experimental demonstrator. It shows that the developed strategy has comparable transmissibility, better compliance, simpler design process, lower controller order, less requirement on identification, but lower robustness than the $H_\infty$ controller. The performance comparison is validated in experiments on the 3-DoF demonstrator.

For the system-level design of a general 6-DoF active suspension system, the single-gravity-compensator concept is developed. Its advantages with respect to the conventional...
multi-gravity-compensator concepts are lower stiffness, lower passive damping, lower cross-coupling, and lower cost. The SEMIS concerned in this thesis is realized based on this single-gravity-compensator concept.

Static and dynamic behaviors of SEMIS are identified in experiments. The high vertical passive force produced by the gravity compensator is measured as 7.2 kN. The temperature dependency of this vertical passive force is measured as -12.1 N/K, which is 1.7 %/K of the total 7.2 kN. The dynamic properties of SEMIS, low cross-coupling, low stiffness, low passive damping, and suitability of high-bandwidth control are experimentally validated. These dynamic behaviors are inherited from the gravity compensator. This proves the success of the system-level design.

Analysis of the dynamic measurement results shows that further decoupling SEMIS using static matrices in addition to the geometric transformation matrices is neither possible nor necessary. This also proves the success of the system-level design. As a result, the sliding surface control is applied to design a vibration isolation controller for each of the 6-DoF motions. Closed-loop performance is pushed to the limit of SEMIS hardware. Closed-loop transmissibility and compliance are experimentally validated.
### List of symbols

#### Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADC</td>
<td>analog to digital converter</td>
<td></td>
</tr>
<tr>
<td>CAD</td>
<td>computer aided design</td>
<td></td>
</tr>
<tr>
<td>CoA</td>
<td>center of actuator</td>
<td></td>
</tr>
<tr>
<td>CoM</td>
<td>center of mass</td>
<td></td>
</tr>
<tr>
<td>CPSD</td>
<td>cross power spectral density</td>
<td></td>
</tr>
<tr>
<td>DISO</td>
<td>double input single output</td>
<td></td>
</tr>
<tr>
<td>DoF</td>
<td>degrees of freedom</td>
<td></td>
</tr>
<tr>
<td>EUV</td>
<td>extreme ultra-violet</td>
<td></td>
</tr>
<tr>
<td>FEM</td>
<td>finite element method</td>
<td></td>
</tr>
<tr>
<td>FRF</td>
<td>frequency response function</td>
<td></td>
</tr>
<tr>
<td>IC</td>
<td>integrated circuit</td>
<td></td>
</tr>
<tr>
<td>LTI</td>
<td>linear time-invariant</td>
<td></td>
</tr>
<tr>
<td>MIMO</td>
<td>multiple input multiple output</td>
<td></td>
</tr>
<tr>
<td>nD</td>
<td>n-dimensional, where <em>n</em> is a positive integer</td>
<td></td>
</tr>
<tr>
<td>PI</td>
<td>proportional integral</td>
<td></td>
</tr>
<tr>
<td>PID</td>
<td>proportional integral derivative</td>
<td></td>
</tr>
<tr>
<td>PM</td>
<td>permanent magnet</td>
<td></td>
</tr>
<tr>
<td>PSD</td>
<td>power spectral density</td>
<td></td>
</tr>
<tr>
<td>RC</td>
<td>resistor-capacitor</td>
<td></td>
</tr>
<tr>
<td>RGA</td>
<td>relative gain array</td>
<td></td>
</tr>
<tr>
<td>SEMIS</td>
<td>single electromagnetic isolator system</td>
<td></td>
</tr>
<tr>
<td>SISO</td>
<td>single input single output</td>
<td></td>
</tr>
<tr>
<td>SIDO</td>
<td>single input double output</td>
<td></td>
</tr>
<tr>
<td>SVCA</td>
<td>shaker voice coil actuator</td>
<td></td>
</tr>
<tr>
<td>VCA</td>
<td>voice coil actuator</td>
<td></td>
</tr>
</tbody>
</table>

#### Symbols of signals

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_a$</td>
<td>payload absolute displacement</td>
<td>$m$</td>
</tr>
</tbody>
</table>
\( x_r \) payload relative displacement \( m \)

\( x_b \) base absolute displacement \( m \)

\( v_a \) payload absolute velocity \( m/s \)

\( a_a \) payload absolute acceleration \( m/s^2 \)

\( v_b \) base absolute velocity \( m/s \)

\( a_b \) base absolute acceleration \( m/s^2 \)

\( \tilde{x}_r \) measured payload relative displacement \( m \)

\( \tilde{v}_a \) measured payload absolute velocity \( m/s \)

\( \tilde{a}_a \) measured payload absolute acceleration \( m/s^2 \)

\( \tilde{v}_b \) measured base absolute velocity \( m/s \)

\( \tilde{a}_b \) measured base absolute acceleration \( m/s^2 \)

\( n_x \) displacement sensor noise \( m \)

\( n_a \) absolute sensor noise \( m/s^2 \) or \( m/s \)

\( r \) reference \( m \)

\( f_d \) direct-acting disturbance force \( N \)

\( f_a \) control force \( N \)

\( \sigma \) sliding surface variable 

\( f \) force \( N \)

\( t \) torque \( N \cdot m \)

---

**Symbols of frequency domain criteria**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T )</td>
<td>transmissibility ( (x_b \rightarrow x_a) )</td>
</tr>
<tr>
<td>( C )</td>
<td>compliance ( (f_d \rightarrow x_a) )</td>
</tr>
<tr>
<td>( R )</td>
<td>relative sensitivity ( (n_x \rightarrow x_a) )</td>
</tr>
<tr>
<td>( S )</td>
<td>absolute sensitivity ( (n_a \rightarrow x_a) )</td>
</tr>
<tr>
<td>( L )</td>
<td>loop gain</td>
</tr>
<tr>
<td>( L_r )</td>
<td>relative loop gain</td>
</tr>
<tr>
<td>( L_a )</td>
<td>absolute loop gain</td>
</tr>
<tr>
<td>( R )</td>
<td>regulator</td>
</tr>
<tr>
<td>( \Lambda_1 ) and ( \Lambda_2 )</td>
<td>the two transfer functions which determine the sliding surface</td>
</tr>
</tbody>
</table>

---

**Symbols of subscripts**
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>translation along x-axis</td>
</tr>
<tr>
<td>$y$</td>
<td>translation along y-axis</td>
</tr>
<tr>
<td>$z$</td>
<td>translation along z-axis</td>
</tr>
<tr>
<td>$\phi$</td>
<td>roll, rotation around x-axis</td>
</tr>
<tr>
<td>$\theta$</td>
<td>pitch, rotation around y-axis</td>
</tr>
<tr>
<td>$\psi$</td>
<td>yaw, rotation around z-axis</td>
</tr>
<tr>
<td>$a$</td>
<td>absolute or active</td>
</tr>
<tr>
<td>$p$</td>
<td>passive</td>
</tr>
<tr>
<td>$d$</td>
<td>disturbance or decoupled</td>
</tr>
<tr>
<td>$r$</td>
<td>relative</td>
</tr>
<tr>
<td>$b$</td>
<td>base</td>
</tr>
<tr>
<td>$s$</td>
<td>sensor</td>
</tr>
<tr>
<td>$\sim$</td>
<td>is derived in experimental measurement</td>
</tr>
</tbody>
</table>

Symbols of signal vectors

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\vec{w}_a$</td>
<td>active control wrench (6D: $[f_x, f_y, f_z, t_x, t_y, t_z]^T$, 3D: $[f_z, t_x, t_y]^T$)</td>
<td>$N$ or $N \cdot m$</td>
</tr>
<tr>
<td>$\vec{w}_d$</td>
<td>disturbance wrench (6D: $[f_{dx}, f_{dy}, t_{dz}, t_{dx}, t_{dy}, f_{dz}]^T$, 3D: $[f_{dz}, t_{dx}, t_{dy}]^T$)</td>
<td>$N$ or $N \cdot m$</td>
</tr>
<tr>
<td>$\vec{w}_p$</td>
<td>total passive wrench (6D: $[f_{px}, f_{py}, t_{pz}, t_{px}, t_{py}, f_{pz}]^T$, 3D: $[f_{pz}, t_{px}, t_{py}]^T$)</td>
<td>$N$ or $N \cdot m$</td>
</tr>
<tr>
<td>$\vec{t}_a$</td>
<td>current of Lorentz actuator: $[i_{1h}, i_{2h}, i_{3h}, i_{1v}, i_{2v}, i_{3v}]^T$</td>
<td>$A$</td>
</tr>
<tr>
<td>$\vec{f}_a$</td>
<td>actuator force: (6D: $[f_{1h}, f_{2h}, f_{3h}, f_{1v}, f_{2v}, f_{3v}]^T$, 3D: $[f_{1h}, f_{2h}, f_{3h}]^T$)</td>
<td>$N$</td>
</tr>
<tr>
<td>$\vec{q}_s$</td>
<td>position sensor output: (3D: $[q_1, q_2, q_3]^T$, 6D: $[q_{1h}, q_{2h}, q_{3h}, q_{1v}, q_{2v}, q_{3v}]^T$)</td>
<td>$m$</td>
</tr>
<tr>
<td>$\vec{a}_s$</td>
<td>accelerometer output: (3D: $[a_1, a_2, a_3]^T$, 6D: $[a_{1h}, a_{2h}, a_{3h}, a_{1v}, a_{2v}, a_{3v}]^T$)</td>
<td>$m/s^2$</td>
</tr>
<tr>
<td>$\vec{q}_a$</td>
<td>metrology-frame/payload absolute displacement: (3D: $[q_2, q_\phi, q_\theta]^T$, 6D: $[q_2, q_x, q_y, q_\psi, q_\phi, q_\theta, q_z]^T$)</td>
<td>$m$ or $rad$</td>
</tr>
<tr>
<td>$\vec{q}_r$</td>
<td>metrology-frame/payload relative displacement to shaker: (6D: $[q_{rx}, q_{ry}, q_{r\psi}, q_{r\phi}, q_{r\theta}, q_{rz}]^T$, 3D: $[q_{rz}, q_{r\phi}, q_{r\theta}]^T$)</td>
<td>$m$ or $rad$</td>
</tr>
<tr>
<td>$\vec{q}_b$</td>
<td>shaker absolute displacement: (3D: $[q_{bx}, q_{by}, q_{b\psi}, q_{b\phi}, q_{b\theta}, q_{bz}]^T$, 6D: $[q_{bx}, q_{by}, q_{b\psi}, q_{b\phi}, q_{b\theta}, q_{bz}, q_{b\phi}, q_{b\theta}, q_{b\psi}]^T$)</td>
<td>$m$ or $rad$</td>
</tr>
</tbody>
</table>
The metrology-frame/payload absolute acceleration is given as:

\[
\vec{a}_a = \begin{bmatrix} a_x, a_y, a_\psi, a_\phi, a_\theta, a_z \end{bmatrix}^T, \quad 3D: \begin{bmatrix} a_z, a_\phi, a_\theta \end{bmatrix}^T
\]

And the shaker absolute acceleration is:

\[
\vec{a}_b = \begin{bmatrix} a_{bx}, a_{by}, a_{b\psi}, a_{b\phi}, a_{b\theta}, a_{bz} \end{bmatrix}^T, \quad 6D: \begin{bmatrix} a_{bz}, a_{b\phi}, a_{b\theta} \end{bmatrix}^T
\]

Symbols of matrices:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w^T_f$</td>
<td>geometric transformation ($\vec{f}_q \rightarrow \vec{w}_q$)</td>
</tr>
<tr>
<td>$q^T_s$</td>
<td>geometric transformation ($\vec{q}_s \rightarrow \vec{q}_r$)</td>
</tr>
<tr>
<td>$a^T_s$</td>
<td>geometric transformation ($\vec{a}_s \rightarrow \vec{a}_a$)</td>
</tr>
<tr>
<td>$M$</td>
<td>inertia matrix</td>
</tr>
<tr>
<td>$D$</td>
<td>damping matrix</td>
</tr>
<tr>
<td>$K$</td>
<td>stiffness matrix</td>
</tr>
<tr>
<td>$T_y$</td>
<td>output decoupling matrix</td>
</tr>
<tr>
<td>$T_u$</td>
<td>input decoupling matrix</td>
</tr>
</tbody>
</table>
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Chapter 1

Introduction

1.1 Background of vibration isolation applications

1.1.1 Introduction to vibrations

Vibration is a common physical phenomenon in daily life. The sound/voice signal we hear is produced by mechanical vibrations and transmitted by propagation of the vibrations of the air molecules. The term vibration in this thesis is specifically referred to as mechanical vibration which is defined as the mechanical position oscillations of a (rigid) body about an equilibrium point. Vibrations are desirable in some cases. For example, hearing the ground vibrations were used in ancient China as the first warning of the strike attack by horse-riders; vibrations produced by many massage devices are used to relax the human muscles to increase the feeling of comfort; vibrations of musical instruments, microphone, loudspeaker are used to create sound/voice signals which are used for communication or entertainment purpose. However, in most engineering fields, vibrations are not desired. The possible reasons are:

1. Vibrations can reduce the life time of a mechanical component/system. For example, vibrations can accelerate the progress of material fatigue; vibrations can cause loose mechanical connections (for example, the thread line connection) which would eventually lead to a system failure.
2. Vibrations in extreme conditions could directly cause physical damages. For example, earthquakes could damage or even destroy a building [103]; seat vibrations in a traveling vehicle could physically injure the passenger; vibrations of a bridge (excited by wind, transportation vehicles, human activities, etc.) could intensively increase the bridge tension force which could break the bridge.
3. The kinetic energy of the vibrations is converted by damping elements to heat which might rise the temperature to an unacceptable level.
4. Vibrations could compromise many high-precision applications, for example, high-precision positioning or measuring, optical experiments, medical devices, etc.

Vibration reduction/isolation has to be considered in these cases.

In most engineering fields, vibration reduction from the root sources is very difficult. Therefore, the vibration-sensitive equipments/systems need to be isolated from the vibra-
tion root sources. A typical natural vibration source is the earth shell. Geological activities like earthquakes and volcano eruptions create intense ground vibrations. To reduce such powerful vibrations from the source is almost impossible. Vibration reduction from most artificial vibration sources is also difficult. For example, the vibrations of engines of aircraft or ground transportation could be reduced by lowering the power output which is unacceptable in most cases; the vibrations of a rotary motor could be reduced by increasing the manufacturing accuracy of the rotor components which requires additional cost; the vibrations of the transportation vehicle could be reduced by improving the road flatness which requires additional cost or by lowering the traveling speed which is not preferred.

The technique of vibration isolation has been extensively studied and applied in many engineering fields in the past a few decades. Seismic vibration isolation is considered to protect an architecture (houses, buildings, bridges, etc.) from earthquakes and other types of seismic vibrations [22, 103, 94, 105, 95, 62, 82], especially at earthquake hot zones, like Japan. To improve ride comfort and to shield the passenger from the vehicle vibrations, trains [102, 101] and cars [32, 45] are all equipped with vibration isolation systems. The vibration transmission from the vibration sources (e.g. the engine or motor of ships, washing machines, helicopters, airplanes, and space facilities, etc.) to the structure which it is mounted has to be reduced to prevent structural damage or unwanted noise [31]. Many vibration isolation systems [7, 114, 115] are developed for specific precision machines.

1.1.2 Introduction to suspension systems

A suspension system, or vibration isolation system, is an auxiliary system that separates the object, or the payload, which is vibration-sensitive and the source of vibration excitation [90]. The purpose of a suspension system is to keep the payload free of vibrations despite of vibrations transmitted from the source and force disturbances acting directly on the payload. As such, the performance of a suspension system is evaluated by two frequency domain criteria: the transmissibility, defined by the transfer function from the source vibration to the payload vibration; and the compliance, defined by the transfer function from the force disturbance directly acting on the payload to the payload vibration. A general requirement on transmissibility and compliance is to make their magnitudes as low as possible. Lower magnitude transmissibility/compliance indicates lower system response to floor-vibrations/direct-disturbances. The detailed requirements and preferences of these two criteria are described in Chapter 3.

From the control point of view, suspension systems can be classified into three categories:

1. Passive suspension systems;
2. Semi-active suspension systems;
3. Active suspension systems.

Passive suspension systems employ only passive elements, for instance, mechanical springs and dampers. Passive suspension systems are usually of low cost. However, passive suspension systems have limited performances [90].

Semi-active suspension systems execute the control effort using, for example, Magneto-Rheological (MR) dampers [96, 71]. The MR-damper damping ratio is adjustable by a
voltage signal. However, due to the nature of the damping force, the direction of the MR-damper force output is not controllable. Therefore, the MR-damper is in lack of control degrees of freedom and this is why the corresponding closed-loop system is called semi-active. In semi-active control, the MR-damper is controlled to have high damping ratio when the MR-damper force is preventing the vibrations of the isolated system and low damping ratio otherwise. Due to the feedback control actions, the semi-active suspension systems perform better than passive suspension systems in general but the cost is higher. The semi-active suspension systems are the most commonly seen in vehicle suspension [63, 3, 121, 96, 46, 32].

Due to the lack of control degree of freedom, the performance of the semi-active suspension systems still leaves room for further improvement. Active suspension systems employ actuators that provide fully controllable output force, for example, hydraulic actuators, piezo-electric actuators, or Lorentz actuators. Active suspension systems are of higher cost but can perform better than semi-active suspension systems.

1.2 Suspension in lithography machine

High-performance active suspension systems are crucial for many high-precision machines, including optical tables [118], space telescopes [12], micro-biological setups, high-resolution microscopes [49, 67, 68, 83, 84, 15], medical machines [51, 80], and lithography machines [41, 13], etc. In this thesis, the lithography machine is taken as an application example for an advanced active suspension system. In this section, design preferences of a high-performance active suspension system are summarized. Subsequently, the functionality of a lithography machine and the reason why an active suspension system is necessary in a lithography machine are described. Finally, the pneumatic active suspension system that is currently applied in a lithography machine is introduced. Its advantages and disadvantages are analyzed.

1.2.1 Design preferences of advanced active suspension

A few design preferences for advanced active suspension systems intended for precision-machines are summarized:

- Contactless design;
- Large inertia;
- Low stiffness;
- Low passive damping.

Contactless design indicates that the suspended object is fully floating and it has no mechanical contact with the environment. Mechanical connections between the suspended object and the environment can induce undesired flexible modes. These flexible modes are usually limiting the control bandwidth. The recent-developed hummingbird system [7] isolates vibrations at frequencies as low as 0.2 Hz. However, the control bandwidth is no higher than 40 Hz due to the flexible modes of the mechanical connection between the suspended object and the vibration source. By eliminating mechanical connections, the induced flexible modes can be removed. As a result, the control bandwidth can be increased to higher frequency which leads to better control performance. As a fully floating rigid body, the suspended object is subjected to six Degrees-of-Freedom (DoF) motions:
three translational motions \((x, y, \text{ and } z)\) along the three Cartesian axes and three rotational motions \((\phi, \theta, \text{ and } \psi)\) around the same three Cartesian axes. As such, all the 6-DoF motions of the suspended object have to be stabilized if contactless design applies.

The \textit{inertia} (mass or moment of inertia) of the suspended payload acts as a passive filter against all disturbances. The larger the payload inertia, the less influence of external disturbances. Therefore, the suspension system which has higher payload inertia has less payload vibrations under the same conditions of external disturbances. However, if the suspension system operates on the vertical direction, larger payload mass demands higher vertical force from the suspension system to compensate the payload gravity.

The \textit{stiffness} is defined as the position derivative of the \textit{wrench} (a vector of forces and torques) produced by the passive elements which connect the payload and the vibration source, e.g. mechanical or air springs. For a closed-loop controlled suspension system, the control effort has to counteract the passive wrench. Lower stiffness requires less control effort which relaxes the requirements on actuator capacity and power consumption.

The \textit{passive damping} is referred to as the derivative of the passive wrench with respect to the relative velocity between the payload and the vibration source. The same aforementioned reasoning about low stiffness also applies to low passive damping. Additionally, a suspension system with lower passive damping has better passive transmissibility at higher frequencies than the natural frequency. The natural frequency of a suspension system is defined as the square-root of the ratio stiffness over inertia. Active control always has limited bandwidth in practice. Beyond the control bandwidth, the closed-loop transmissibility always converges to the passive transmissibility. Therefore, a suspension system with lower passive damping has better closed-loop transmissibility at frequencies beyond the control bandwidth. The high magnitude peak at the natural frequency, which is a result of the low passive damping, can be suppressed by active control.

\textbf{Remark 1.2.1.} The term "passive damping" is used here to distinguish it from the term "active damping", which is commonly seen in literature as a damping force induced by active control. The term "stiffness" is used in this thesis in stead of the term "passive stiffness" because the term "active stiffness" is not commonly seen in literature.

\section*{1.2.2 Introduction to lithography machine}

The lithography machine \cite{13} is one of the core equipments used to manufacture integrated circuits (ICs) in the semiconductor industry. The dynamic architecture of a lithographic machine shown in Fig. 1.1 is described in \cite{14}. The designed IC details up to nanometer accuracy are reproduced in a pattern of transparent and opaque areas on the surface of a mask on a reticle stage. The light passes through this mask and an image of this mask pattern is formed. This image is focused by a complex optical system (lens or mirror) and projected onto the surface of the silicon wafer which is coated with photosensitive material. The designed IC will be created by further chemical processes of the silicon wafer. The complex optical system, together with many sensor systems, are rigidly fixed to a \textit{metrology frame}. The vibrations of the metrology frame could cause incorrect projections on the silicon wafer rendering the silicon wafer unusable.

The vibration source of the metrology frame is its supporting platform, or \textit{base frame}. This base frame, installed on the factory floor, is exposed to seismic vibrations which have
broad band spectrum and unpredictable waveform. The causes of these seismic vibrations are human activities, machines motions, transportation around the manufacturing facility, and micro-earthquakes. The base frame is subjected to six Degrees-of-Freedom (DoF) vibrations, including translational vibration (x, y, and z) along the three Cartesian axes and the rotational vibration (ϕ, θ, and ψ) about the three Cartesian axes.

Additionally, there are also disturbance forces acting directly on the metrology frame, such as internal motions of the metrology frame, the water/air flow in the cooling system of the metrology frame, the power/signal cables connecting the metrology frame and the base frame, and acoustic noises.

The base vibrations lower than a certain frequency-threshold (0.1~1 Hz) may be transmitted to the metrology frame. The reasons are explained as follows. First of all, these extremely-low-frequency metrology-frame vibrations can be easily followed by the position-controlled silicon wafer. In other words, these extremely-low frequency vibrations are not affecting the aforementioned lithographic process. Second, these extremely-low-frequency base vibrations usually have much larger amplitudes than that of higher frequencies. If the metrology frame is perfectly isolated at these extremely-low-frequency vibrations, the relative displacement between the metrology frame and the base frame could be larger than the maximal displacement range so that these two frames would collide. This collision must be avoided by making the metrology frame to follow the base frame vibrations at these extremely-low frequencies. An extensive description of the performance criterion of an active suspension system is given in Section 3.2.2.

### 1.2.3 Pneumatic suspension

In industry, an active suspension system is mounted on the base frame to support the metrology frame and to suppress the metrology frame vibrations [13, 42]. The suspended payload is the metrology frame and the complex optical system which has a total mass of a few thousands of kilogram. The huge gravity force of this suspended payload is a great challenge to the supporting device, or gravity compensator. In Fig. 1.1, pneumatic isolators [78, 41], or air-mounts [13], are used as gravity compensators to support the heavy payload.
1.3. Electromagnetic suspension

The pneumatic isolator is a metal cannister with the dimensions in the order of 0.1 m. The air is pressurized by an air compressor, subsequently flows through a control valve into the cannister at the entrance, and is finally released to the atmosphere from the cannister at the exit. The valve control is used to maintain the air pressure in the cannister for stabilization. The pneumatic isolator uses the air-bearing to eliminate mechanical contact. Lorentz actuators are actively controlled to provide position-independent force for further performance improvement. This system has many desired properties.

- High capacity gravity compensation with contactless design (except the air);
- Low stiffness gravity compensator;
- Ensured stability by maintaining the air pressure;
- Stiffness is not sensitive to the payload mass;
- Long life, low cost, and easy maintenance.

There are also disadvantages of the pneumatic isolators. First of all, these devices suffer from nonlinear dynamic behavior due to the dependency of the stiffness on the volume [39]. Second, there are difficulties in accurate modeling and control due to the high frequency dynamics of the air. The pump and valve for the pneumatic isolators would induce unnecessary disturbances. Third, certain passive damping is unavoidable due to the air, which is not preferred for vibration isolation. Finally, the suitability of the pneumatic isolators for use in deep vacuum is limited because of the continuous air release from the cannister. Vacuum operation of the lithography machine is preferred for three reasons:

- To eliminate acoustic noises;
- For better light transmission;
- To keep dust away from the wafer.

As the Extreme Ultra-Violet (EUV) radiation [66, 100] is applied in the next generation of lithography machine, vacuum operation becomes a necessity because the air absorbs the EUV photons.

1.3 Electromagnetic suspension

Besides the principle of air-bearing, electromagnetic suspension or levitation is an alternative technique to realize contactless design. An extensive overview of magnetic suspension or levitation techniques is given in [61]. Electromagnetic levitation or suspension is a technique to suspend an object with no other support than electromagnetic fields [52]. This definition indicates that the suspended object of an electromagnetic suspension or levitation system does not have any mechanical contact with the environment. In literature, some automotive suspension systems which employ both electromagnetic actuators and mechanical connection components (for example, springs) are also called electromagnetic suspension systems [35, 36, 37] but they are not within the scope of this thesis.

In literature, there are two different definitions to distinguish the two terminologies *levitation* and *suspension*:

- In [61], the term suspension indicates that attractive forces are used, and the term levitation indicates that repulsive forces are used.
Chapter 1. Introduction

- In [65], the two terms are distinguished by their application categories. The objective of a suspension system is to keep the suspended object inertially fixed against base vibrations and directly acting disturbance forces. A levitation system is to make the levitated object move with a varying reference with minimal error.

The first definition cannot distinguish some complex magnetic systems which uses both attractive forces and repulsive forces [91, 92]. Therefore, the second definition for suspension and levitation is adopted in this thesis.

Many contactless multi-DoF electromagnetic levitation systems [65, 17, 52, 116, 20, 28, 47, 81, 86, 18] have been developed for accurate positioning applications. These electromagnetic levitation systems are out of the scope of this thesis because the design aspects, measurement schemes, and control methods are all different from that of electromagnetic suspension systems. For instance, the mass of a levitated object in an electromagnetic levitation system is usually kept as small as possible to increase the achievable acceleration. On the other hand, the suspended payload in an electromagnetic suspension system is preferred to have large mass, as explained in Section 1.2.1.

To apply an electromagnetic suspension system in a lithography machine, a straightforward solution is to design an electromagnetic isolator to replace the pneumatic isolator. Such an electromagnetic isolator should function as a passive gravity compensator to support the heavy metrology frame. In addition, linear actuators must be integrated for active control. The gravity compensator is required to possess the following properties:

- Contactless design;
- Low stiffness;
- High force density (the ratio of force over volume);
- Low passive damping.

To be an alternative to the pneumatic isolators, the gravity compensator should have similar dimensions (0.1 m). To support the heavy payload, the gravity compensator should produce a large vertical force. As such, the gravity compensator should possess the property of high force density. The rest of the three properties have been described in Section 1.2.1.

In a recent research [59, 55, 56], which is a companion research to this thesis, an ironless electromagnetic isolator has been modeled, designed, and realized. The photo and drawings of this device [54] are shown in Fig. 1.2 and Fig. 1.3, respectively. It combines:

![Figure 1.2: A photo of the realized electromagnetic isolator [54].](image-url)
1.3. Electromagnetic suspension

Figure 1.3: The 3D drawings of the designed electromagnetic isolator [54].

Figure 1.4: In the gravity compensator (a) the outer cross, consisting of four pieces (exploded view), falls over the inner cross to form the gravity compensator. The magnetization of the vertical cross-section of a leg is shown in (b) [54].
Chapter 1. Introduction

1. Vertical and horizontal Lorentz actuators, which provide position-independent force output;
2. Fully-passive Permanent-Magnet (PM) based gravity compensator.

The outer dimensions of this electromagnetic isolator are $300 \times 300 \times 200$ mm. The magnetic topology of this gravity compensator, shown in Fig. 1.4, is taken from [54]. It consists of two cross-shaped parts with PMs that interact with each other. It is hysteresis-free due to the absence of soft-magnetic materials. The PM structure is designed such that it combines a high vertical force density to compensate the gravity force with a minimized stiffness. These properties have been predicted by means of the analytical surface charge model [60, 53, 57], which is a dedicated modeling method for ironless devices. The stroke range of this gravity compensator is $\pm 1$ mm along all the three Cartesian axes.

This realized gravity compensator is designed to possess the following properties [54]:

1. High vertical passive force;
2. Low stiffness;
3. Contactless design;
4. Feasibility for high-bandwidth control;
5. Low passive damping.

The first two properties, high vertical passive force and low stiffness, are predicted using the aforementioned modeling method for ironless PM-based devices [54]. The passive vertical force, induced by the interactions between the two parts of this gravity compensator, is predicted as 7.5 kN. At the center of stroke range, the horizontal passive forces and the three passive torques are predicted as zero. Within the stroke range, the variation of the passive forces and the passive torques are predicted to be within a range of $\pm 3$ N or $\pm 7.5$ Nm; the maximal value in the stiffness matrix is less than 7.5 kN/m, which is lower than that of the pneumatic isolators (in the order of 10 kN/m). These two properties need to be measured in experiments to verify the aforementioned analytical modeling tools, which is one of the goals of the companion research [54] to this thesis.

Contactless design indicates that the two parts of the gravity compensator do not have any mechanical contact during operation. Experimental demonstration is necessary to verify this concept. Because of the contactless design, the bandwidth-limiting resonances, which are induced by the flexible modes of the suspended platform, are above 1 kHz [54], which is a great improvement compared to many commercially available active suspension systems, for example, the recent-developed hummingbird system [7]. This dynamic property is predicted by the Finite Element Method (FEM) model constructed by the CAD software used for mechanical design. It is a promising prediction according to experience but still needs to be verified in experiments. The low passive damping is an estimated property from experience, however, it is mentioned in [54] that this estimation is still a theoretical hypothesis. It also needs experimental verification as a proof.

An advanced test rig, the Single Electro-Magnetic Isolator System (SEMIS), shown in Fig. 1.5, is developed to perform the following tasks:

1. To verify the aforementioned static and dynamic properties that are predicted for the realized PM-based gravity compensator;
1.3. Electromagnetic suspension

2. To establish a feasibility proof of a contactless electromagnetic suspension system with its full performance potential.

![Figure 1.5: Photo of SEMIS.](image)

![Figure 1.6: Schematic of the single-gravity-compensator concept designed for SEMIS.](image)

The PM-based gravity compensator, which acts as a contactless magnetic spring, is used to support a fully-floating rigid metrology frame of 730 kg. As a rigid body, this floating metrology frame is subjected to 6-DoF motions: translations \((x, y, z)\) along the three Cartesian axes and the rotations \((\phi, \theta, \psi)\) about the three Cartesian axes. According to the Earnshaw’s theory [23], at least one of the 6-DoF motions is inherently unstable which leads to the inherent instability of the total system. Six additional Lorentz actuators are employed for stabilization of the 6-DoF motions. Six position sensors are mounted between
the metrology frame and the supporting base frame to measure the relative displacement between the two frames and six acceleration sensors are mounted on the metrology frame. This will be necessary to minimize independently both the transmissibility and the compliance, which will be explained later in Section 3.3.

1.4 Research objective

The goal of this thesis is for systematically exploring opportunities to improve the performances (transmissibility and compliance) of the aforementioned Single Electro-Magnetic Isolator System (SEMIS) by system-level design, choices of sensors, identification of the static and dynamic behaviors, and applying advanced control strategies.

The system-level design of a mechatronic system is distinguished from subsystem-level design and component-level design (detailed design) in [79]. For a general mechatronic system, the objective of system-level design is to increase the achievable closed-loop performance (constrained by the hardware limits) as much as possible. The objective of control design is to push the closed-loop performance to the hardware limits as much as possible under the constraint of stability and robustness. In addition to the general objective, the system-level design for SEMIS should also facilitate the measurements that are necessary to perform the aforementioned property-verification tasks.

For control design, this thesis proposes a strategy which combines the sliding surface control and optimal static decoupling to design the vibration isolation controller. Optical static decoupling is applied to reduce a multi-DoF suspension system to multiple single-DoF suspension systems. The sliding surface control is applied to each single-DoF suspension system to design a vibration isolation controller.

1.4.1 Single-gravity-compensator concept for system-level design

The single-gravity-compensator concept is proposed in this thesis as the system-level design. The subsystem-level and component-level design are described in [54].

As explained in Section 1.3, the Single Electro-Magnetic Isolator System (SEMIS) serves two tasks. A system-level design is needed to complete these tasks as good as possible. In the starting phase of this project (the research described in this thesis and its companion research [54]), the intention is to investigate the achievable performance of a contactless electromagnetic suspension system using three electromagnetic isolators and whether or not such a system is a possible alternative to the aforementioned pneumatic suspension system [41, 13]. However, only one electromagnetic isolator is realized due to limited time and budget. For this reason, a contactless electromagnetic suspension system using a single isolator becomes a necessity.

The single-gravity-compensator concept is shown in Fig. 1.6. The electromagnetic isolator which acts as a passive magnetic spring is vertically aligned to compensate the gravity force of a rigid metrology frame. Lorentz actuators provide position-independent force to execute active control. SEMIS is constructed on a shaker table, which is designed to excite artificial vibrations to measure the vibration isolation performance of SEMIS. The metrology-frame absolute-acceleration and its position relative to the shaker table are both measured. The necessity of this measurement scheme is explained in Section 3.3. As the electromagnetic isolator and the Lorentz actuators are of electromagnetic nature, the floating metrology frame does not have any mechanical contacts with the shaker table.
1.4. Research objective

In this system-level design, there are only three wrenches (a vector of forces and torques) acting on the floating metrology frame during steady state:

1. Passive interactions between the two parts of the gravity compensator induced by the integrated PMs;
2. Passive wrench induced by the gravity of the metrology frame;
3. Stabilizing active wrench produced by the Lorentz actuators.

We assume the inertia of the metrology frame and its center of gravity are known parameters. As such, the second wrench vector is assumed to be known. The third wrench vector is measurable because it is exactly the control effort at steady state. As a result, the first wrench vector can be calculated from the second and the third wrench vector.

By coupling the shaker table on the floor, the Frequency Response Function (FRF) of SEMIS can be measured. The stiffness and the passive damping can be estimated from the measured FRF as SEMIS is expected to be linear. As a summary, this system-level design is expected to be feasible for measurement of all static and dynamic properties of the gravity compensator.

The measurement scheme of absolute acceleration and relative position is necessary for implementing vibration isolation control, as described in Section 3.3. The vibration isolation control strategy proposed in the next subsection will be applied to SEMIS to explore its achievable performance.

1.4.2 Vibration isolation control of a multi-DoF active suspension system

The Single Electro-Magnetic Isolator System (SEMIS) concerned in this thesis is a 6-DoF contactless electromagnetic suspension system. The objective of the control design is for simultaneously achieving stabilization, vibration isolation (transmissibility), and direct disturbance force rejection (compliance) for all six Degrees-of-Freedom. According to the prediction and estimation in [54], this system exhibits a unique combination of many dynamic behaviors, including inherent instability, cross-coupled motions, low stiffness, and a weak position-dependent nonlinearity, low passive damping, and high-frequency parasitic resonance induced by flexible modes of the suspended metrology frame. Two characteristics, low stiffness and low passive damping, are inherited from the PM-based gravity compensator. Both are indeed advantages for a suspension system, as explained in Section 1.2.1. The position-dependent nonlinearity is predicted to be very weak such that the linearized model around an equilibrium may be used for control design. As a result, the control design for SEMIS stays in a linear framework. Of course, this prediction has to be verified in experiments. The parasitic resonance induced by flexible modes of the suspended metrology frame is shifted to be above 1 kHz, which allows higher control bandwidth than systems without contactless design [7, 115]. The expected challenges for control design are inherent instability, cross-coupled motions, isolation of base vibrations, and rejection of direct-acting force disturbances. These challenges will be analyzed in the following subsections.

Control design for a general MIMO system using static decoupling

SEMIS is a Multi-Input-Multi-Output (MIMO) system. The input is a six-dimensional wrench vector (a vector of forces and torques) and the output is a six-dimensional relative
position vector and a six-dimensional absolute acceleration vector. The necessity of this measurement scheme is explained in Section 3.3.

The 6-DoF motions of SEMIS are cross-coupled due to the passive interactions of the PM-based gravity compensator. For example, a vertical force will not only produce vertical translation but also other motions like rotations. According to the results of aforementioned analytical modeling [54], the passive interactions between the two parts of the gravity compensator produce position-dependent wrench and this dependency is weakly nonlinear. Each passive force or torque varies with all the three relative translations between the two parts. According to analytical calculations in [54], the cross-coupling induced by the PM-based gravity compensator is weak. However, without experimental verification, the possibility of significant cross-coupling has to be considered. Furthermore, there are other factors that could also induce cross-coupled motions, e.g. imperfect sensors, imperfect actuators, and imperfect electrical/mechanical components. The cross-coupling of SEMIS has to be quantified by experimental measurements in this thesis. Before knowing the true cross-coupling strength of SEMIS, methodologies how to deal with the cross-coupling in control design have to be studied in this thesis.

For a MIMO system, the cross-coupling is referred to as the phenomenon that each output is influenced not only by its corresponding input but also by some or all of the other inputs. Almost all practical Multi-Input-Multi-Output (MIMO) systems have cross-coupling between inputs and outputs. If a MIMO system has absolutely no cross-coupling, this system would be equivalent to a group of Single-Input-Single-Output (SISO) systems. In that case, the control design for such a MIMO system would be easily solved by several classic SISO control design methods.

In this thesis, the term MIMO system indicates that the cross-coupling is not zero by default unless it is clearly defined otherwise. The term direct MIMO control is referred to as the control design for a complete model of the MIMO system. A decentralized controller is a group of SISO controllers applied to a MIMO system. Fig. 1.7 shows the block diagram of a decentralized control loop. The block $P$ represents the $n \times n$ plant. The group of SISO controllers $C_1, C_2, \ldots, C_n$ is the decentralized controller, which has a diagonal structure.

![Block diagram of decentralized control.](image)

The decentralized control could be the first MIMO control strategy one might think of. It has a straightforward design process, and low implementation cost. On the other hand, the disadvantages are also obvious. Due to the cross-coupling, the transfer function matrix model of a MIMO system is not diagonal. The off-diagonal entries of the transfer function model are ignored during the decentralized control design as the SISO controllers are designed based on only the diagonal entries. In that case, these ignored off-diagonal entries might compromise the decentralized control performance or even destabilize the
The direct MIMO control design methods, like $H_{\infty}$ or $\mu$-synthesis, are based on a model derived by MIMO identification. Contrary to the decentralized control, the closed-loop performance of the direct MIMO control design is not limited by cross-coupling. Therefore, the direct MIMO control design is theoretically able to provide better closed-loop performance. However, the MIMO identification and the direct MIMO control design require high implementation cost and are usually complex. This complexity increases polynomially with the number of variables that need to be controlled.

An alternative control strategy that is widely applied in industry is to first reduce or even eliminate the cross-coupling and subsequently to apply decentralized control design to the decoupled system. This strategy, referred to as the decoupling-based decentralized control, simply provides a trade-off between cost and performance. Due to the reduction of the cross-coupling, the achievable closed-loop performance is expected to be better than the decentralized control without decoupling. It also possesses the advantages of the decentralized control: straight-forward design process and low-cost implementation. However, due to the fact the decoupling is never perfect in practice, the remaining cross-coupling still limits the achievable closed-loop performance. Therefore, the performance of this strategy is heavily dependent on the quality of decoupling.

There are two types of decoupling methods: dynamic decoupling [40] is to decouple a MIMO system using transfer function matrices and static decoupling is to decouple a MIMO system using constant real matrices. Dynamic decoupling is theoretically able to achieve better performance but the complexity and the implementation cost could be comparable to the direct MIMO control. On the other hand, the static decoupling has simpler structure and requires relatively lower implementation cost. For this reason, only static decoupling is interested in this thesis. Although SEMIS has six inputs and twelve outputs, SEMIS may be treated as two square systems (the number of input signals is equal to the number of output signals) during MIMO control design. The two square systems can be decoupled using the same pair of matrices in practice. The reason is explained by Remark 6.1.1 in Chapter 6.

Methodologies of vibration isolation control

The terminology vibration isolation control in this thesis specifically refers to as the control design for an active suspension system which is aiming to simultaneously achieve stabilization, isolation of floor vibrations (transmissibility), and rejection of direct-acting disturbance forces (compliance). SEMIS is weakly nonlinear, according to the analytical modeling results [54]. The study of vibration isolation control is therefore confined within linear framework.

To study design methods of vibration isolation control for SEMIS, the measurement scheme has to be defined. SEMIS is an inherently unstable system, according to the Earnshaw’s theory [23] as well as the analytical model [54]. As a result, the relative position measurement is necessary for stabilization. Chapter 3 will show that with only relative position measurement it is not possible to simultaneously improve vibration isolation and disturbance force rejection. The measurement scheme of SEMIS is finally chosen as a combination of relative position and payload absolute motion (acceleration) in Chapter 3. For the motion on each DoF, there is only one control effort (either force or torque). Therefore,
the vibration isolation controller for the motion on each DoF is a Double-Input-Single-Output (DISO) controller.

There are four closed-loop criteria concerned in vibration isolation control: transmissibility, compliance, and the sensitivity functions to the two type of sensors (relative displacement sensor and the absolute motion sensor). It will be proved in Chapter 3 that it is not possible to simultaneously improve all of them. The DISO control design is studied in this thesis to make an optimal trade-off between the performance criteria (transmissibility and compliance) and the performance constraint (the two sensitivity functions).

In literature, most control design methods for a linear vibration isolation controller can be classified into four categories:

1. SISO control design based on payload absolute motion feedback;
2. DISO control design using heuristic and iterative tuning of the two loops;
3. Simultaneous DISO control design;
4. The frequency-shaped sliding surface approach.

The first category is only applicable to inherently stable active suspension systems, which are most commonly seen in literature. The isolator employed in these systems acts as a passive PD controller to stabilize the relative position loop. Only the absolute motion loop is actively controlled. For this reason, the control design has limited degrees of freedom. As a result, desired performance might not be possible. For control design of this absolute motion loop, many classic SISO design tools are found in literature. The skyhook control [63] is in fact proportional control of the payload absolute velocity. The PI [64] and PID [89] control can be treated as an extension of the skyhook control. Other methods are the conventional sliding mode control [123, 120, 72], state feedback with observer [64], the quasi-optimal control [2], $H_\infty$-optimization [30, 64, 119] and $\mu$-synthesis [6].

The second category is to iteratively and heuristically tune the position controller and absolute motion controller toward desired performance [89]. In theory, it is feasible and is applicable to both inherently stable and unstable systems. For inherently unstable systems, the position controller can be designed first for stabilization and subsequently apply the iterative tuning of the two loops. But heuristic tuning is a cumbersome process and quite experience-demanding. The difficulty of the tuning is that any changes will effect both transmissibility and compliance. To achieve desired performance by heuristic tuning is theoretically possible but practically difficult.

The third category is to simultaneously design the DISO controller using robust control design methods, like $H_\infty$-optimization [112] and $\mu$-synthesis. Furthermore, the robust control is capable of directly solving vibration isolation control design problem for a multi-DoF suspension system. The four performance criteria are optimized by weighting filters design. Both robust stability and robust performance are guaranteed. However, the resultant controller is usually of high order which increases implementation cost.

The fourth category originates from conventional sliding mode control and is first exploit by L. Zuo and J.J.E. Slotine [128]. In conventional sliding mode control, the sliding surface is constructed using only one type of feedback variable. The sliding surface in [128] is constructed using two types of feedback variables, displacement and velocity. In this way, the constructed sliding surface is physically connected to the transmissibility.
However, sensor and actuator dynamics are ignored in the control design, which makes optimal performance impossible.

A relatively unexplored development in vibration isolation control is the fourth category method (frequency-shaped sliding surface approach). The constructed sliding surface in [128] is physically connected to the transmissibility. Taking advantage of this connection, the transmissibility may be designed independent of the compliance as well as the plant dynamics. This thesis explores the sliding surface control, which is an extensive study of the frequency-shaped sliding surface approach toward optimal performance. The sensor dynamics and estimated floor vibration spectrum are taken into account.

Beside the aforementioned four categories, there are also other linear vibration isolation control approaches. For example, a Kalman filter based vibration control has been developed [25] for a stable mechanical suspension system. The measurement scheme combines acceleration measurement for both the payload and the base structure. It is only applicable to inherently stable suspension systems because the relative displacement is not measured. Even the relative displacement measurement is added, using two accelerometers for one DoF motion doubles the implementation cost as a single accelerometer is sufficient. This will be further explained in Section 3.3. Therefore, this method is only of academic interest.

1.5 Thesis outline

This thesis can be separated into four parts, described in Table 1.1. Part I studies methodologies regarding vibration isolation control of a general multi-DoF active suspension system. This thesis proposes an approach to first reduce a multi-DoF suspension system to multiple single-DoF suspension systems by optimal static decoupling and subsequently apply vibration isolation control to the single-DoF suspension systems. In Chapter 2, methodologies concerning static decoupling of an LTI mechanical system are studied. In Chapter 3, the sliding surface control is developed for vibration isolation control of a single-DoF suspension system. Possible measurement schemes for suspension systems are generally studied and a measurement scheme combining relative displacement and payload absolute acceleration is chosen for the contactless electromagnetic suspension system.

Part II validates the developed methodologies experimentally on a 3-DoF demonstrator, which is cross-coupled because of its non-symmetric design. It will be shown in Part III that the cross-coupling of the contactless electromagnetic suspension system (SEMIS) has been minimized by system-level design. For this reason, the optimal static decoupling is not applied. Validation on this 3-DoF demonstrator provides experimental proof that the proposed methodology of vibration isolation control can be generally applied to multi-DoF active suspension systems. In Chapter 4, optimal static decoupling is validated by comparing the cross-coupling of the decoupled and the original system. The identification process, control design for tracking, and closed-loop performance of the SISO control combing the optimal static decoupling, is compared with the direct MIMO control ($H_{\infty}$-optimization) on the 3-DoF demonstrator. Chapter 5 describes the design process for vibration isolation control of 1-DoF and 3-DoF suspension systems based on models experimentally derived from the 3-DoF demonstrator. Two methods are studied: the $H_{\infty}$-optimization and the sliding surface control. In Chapter 6, the two vibration isolation control methods are implemented on the 3-DoF demonstrator. Closed-loop performances are validated.
Chapter 1. Introduction

Part III describes system-level design, identification of static and dynamic behaviors, comparison of predicted and achieved closed-loop performances of the contactless electromagnetic suspension system (SEMIS). Chapter 7 develops the single-gravity-compensator concept which serves as the system-level design of SEMIS. Subsystems of SEMIS as well as its test rig are briefly described. Static measurements are carried out to identify the temperature sensitivity of the passive vertical force produced by the electromagnetic isolator, and the passive wrench-position relation. The $6 \times 6$ stiffness matrix is also derived. Chapter 8 focuses on the dynamic behavior of SEMIS, including FRF, frequency range of rigid body motion, nonlinear behavior, identification of inertia and passive damping, bandwidth-limiting resonance, and cross-coupling. In Chapter 9, vibration isolation control is designed and implemented on SEMIS. Closed-loop performances are validated in experiments.

Part IV closes this thesis by conclusions and recommendations for future research. Main contributions of this thesis are summarized.

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Table 1.1: Thesis outline.
Part I

Methodologies for vibration isolation control of a general multi-DoF active suspension system
Chapter 2

Methodologies of static decoupling for a general MIMO system

As explained in Chapter 1, the Single Electro-Magnetic Isolator System (SEMIS) concerned in this thesis is a Multi-Input-Multi-Output (MIMO) contactless suspension system. The six Degrees-of-Freedom (DoF) motions, the three translations and three rotations, are expected to be cross-coupled. This thesis explores the control strategy combining static decoupling and sliding surface control. Static decoupling decouples the MIMO system as much as possible using constant real matrices. Subsequently, the sliding surface control can be applied to design a vibration isolation controller for a single-DoF suspension system. Predicted in [54], SEMIS is only weakly nonlinear. Therefore, SEMIS may be assumed as a Linear Time-Invariant (LTI) mechanical system in control design.

Static decoupling concerns the procedure to derive constant real matrices to decouple a MIMO system. The theories and methodologies concerning static decoupling of an LTI mechanical system are studied in this chapter. The background of decoupling and the motivation to study static decoupling are described in Section 2.1. For an LTI mechanical system, the property that the passive damping matrix is proportional to the inertia matrix or the stiffness matrix or both is referred to as proportional damping. This chapter studies static decoupling with and without the assumption of proportional passive damping. There are two reasons:

• It is possible that SEMIS possesses the property of proportional damping;
• The property of proportional damping facilitates static decoupling of an LTI mechanical system.

These two reasons are explained separately as follows.

SEMIS is hypothetically assumed to have low passive damping in [54]. If zero passive damping of SEMIS may be assumed, SEMIS possesses the property of proportional damping because the passive damping is then proportional to anything with coefficient zero.

According to the Dyadic Transfer function Matrix (DTM) theory [87, 70, 5], a system which can be described by a real-coefficient transfer function matrix is called dyadic if and only if there exist two constant matrices which can diagonalize this transfer function matrix. A more complete definition and discussion follows later. The Owens method [88] is a procedure to derive two decoupling matrices for a dyadic system.
Modal decomposition [43, 26, 74] is a procedure to derive two decoupling matrices for an LTI mechanical system with the property of proportional damping. It will also be shown in this chapter that an LTI mechanical system with proportional damping is a dyadic system. For this reason, perfect decoupling is theoretically possible for an LTI mechanical system with proportional damping. Furthermore, the two static decoupling methods, modal decomposition and Owens method, are both applicable to an LTI mechanical system with proportional damping. It implies that the property of proportional damping facilitate static decoupling of an LTI mechanical system.

If zero passive damping may be assumed, SEMIS is a dyadic system and both modal decomposition and Owens method are applicable to derive the decoupling matrices. These two static decoupling methods are compared in Section 2.2.

If zero passive damping may not be assumed, SEMIS may not be treated as a dyadic system during static decoupling. In theory, modal decomposition and Owens method can not be directly applied. Nevertheless, an extension to the Owens method, the Vaes-procedure [109] is a numerical procedure to develop a pair of constant real decoupling matrices which minimizes cross-coupling. The Owens method is used to derive the initial values which is necessary to apply the Vaes-procedure. Section 2.3 improves the Vaes-procedure in terms of computational complexity and numerical stability. This chapter is concluded in Section 2.4.

2.1 Introduction to decoupling

This section describes the motivation why static decoupling is studied. A brief review of decoupling procedures and interaction measures is provided.

2.1.1 Background

The contactless electromagnetic suspension system (SEMIS) concerned in this thesis is a typical Multi-Input-Multi-Output (MIMO) system. The inputs are three forces and three torques along the three Cartesian axes. The outputs are 6-DoF motions: three translational motions \((x, y, z)\) along the three Cartesian axes and three rotational motions \((\phi, \theta, \psi)\) around the same three Cartesian axes. These 6-DoF motions are measured repetitively: both 6-DoF position and 6-DoF acceleration are measured. The necessity of this measurement scheme is explained in Section 3.3. Although SEMIS has twelve output signals, the two types of measured variables (6-DoF position and the 6-DoF acceleration) are correlated because they represent the same 6-DoF motions. Therefore, SEMIS is still treated as a square system (the number of input signals is equal to the number of output signals) during MIMO control design.

For a practical MIMO system, each input would influence not only the corresponding output but also all the other outputs. The influence of one input to all the other outputs in comparison the influence of this input to its corresponding output is called the interaction or cross-coupling [108].

For the control design of a MIMO system, there are two strategies:

- Direct MIMO control design based on a complete MIMO model;
- Decentralized control design based on decoupling.
2.1. Introduction to decoupling

The most simple way of MIMO control design is to design a Single-Input-Single-Output (SISO) controller for each diagonal element of the transfer function matrix neglecting cross-coupling. The combination of these SISO controllers is called a decentralized controller. The disadvantage is that the cross-coupling is (most likely) a limiting factor to further improving closed-loop performance [113]. Further more, the cross-coupling could reduce the designed closed-loop performance or even destabilize the closed-loop system.

Since the advanced techniques on identification and control design of MIMO systems were developed, previous research [126, 16] shows that direct MIMO control design has the potential of further performance improvement over the decentralized control. However, deriving a complete MIMO model and direct MIMO control design require high implementation cost and are usually complex. This complexity is increased with the number of variables that need to be controlled.

By applying decoupling prior to the decentralized control, the decentralized control performance can be improved without significantly increasing the design and implementation cost. Therefore, the decoupling-based decentralized control is often applied in the industry. As the decentralized control design does not take the cross-coupling into account, the performance of the decentralized control highly relies on the quality of decoupling.

There are two issues concerned by decoupling:

- How to quantify the cross-coupling of a MIMO system;
- How to derive the decoupling matrices.

Both issues have been extensively studied in the past. The criteria used to quantify the cross-coupling of a MIMO system is referred to as the interaction measure and the process to derive the decoupling matrices is referred to as the decoupling procedure. After necessary notations defined in subsection 2.1.2, subsection 2.1.3 and subsection 2.1.4 provide a brief review on the interaction measures and the decoupling procedures developed in the past.

2.1.2 Notations of decoupling

Static decoupling decouples a MIMO system using constant matrices. The general block diagram of the decentralized control based on static decoupling is illustrated in Fig. 2.1. The block $C$ denotes the diagonal transfer-function-matrix of the decentralized controller and its diagonal entries are $C_i, \forall i \in \{1, 2, ..., n\}$. The block $T_u$ is the input decoupling matrix and the block $T_y$ is the output decoupling matrix. The blocks $P$ and $P_d$ denote the transfer function matrices of the original plant and the decoupled plant, respectively. They are related by

$$P_d = T_y P T_u. \quad (2.1)$$

The signals $u$ and $y$ are the input and output vectors of $P$. The signals $u_t$ and $y_t$ are the input and output vectors of $P_d$. They are transformed by $u_t = T_u^{-1} u$ and $y_t = T_y y$. The decoupling is perfect if $P_j$ is diagonal. The measured Frequency Response Function (FRF) of the original plant and the decoupled plant are denoted by $\tilde{P}$ and $\tilde{P}_d$, respectively. The $i^{th}$ row and $j^{th}$ column entry of $P$, $P_d$, $\tilde{P}$, and $\tilde{P}_d$ are denoted by $P_{ij}$, $P_{dj}$, $\tilde{P}_{ij}$, and $\tilde{P}_{dij}$, respectively. The complementary sensitivity function for the $i^{th}$ SISO loop is $H_i = P_{d,ii} C_i (1 + P_{d,ii} C_i)^{-1}$. 

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As a transfer function matrix model of a MIMO system is hard to derive, the following definitions are based on the measured FRF. The FRF matrix which takes only the diagonal entries of $\tilde{P}(j\omega)$ is denoted by $\text{diag}\{\tilde{P}(j\omega)\}$:

$$\text{diag}\{P(j\omega)\} = \begin{bmatrix} \tilde{P}_{11}(j\omega) & 0 & \cdots & 0 \\ 0 & \tilde{P}_{22}(j\omega) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \tilde{P}_{nn}(j\omega) \end{bmatrix}. \quad (2.2)$$

2.1.3 Interaction measure

This subsection reviews four interaction measures developed in the past. Among the four interaction measures, the $\mu$-interaction measure is recommended to quantify the cross-coupling of a practical MIMO system. The reasons to make this recommendation are explained.

Many criteria have been proposed to quantify the cross-coupling of a MIMO system: the Relative Gain Array (RGA) [11], the diagonal dominance [24], the $\bar{\sigma}$-interaction measure [111], and the $\mu$-interaction measure [34]. As the interaction measure will be used to compare the decoupling performance of many different pairs of decoupling matrices, it has to be a single-valued function of frequency to make this comparison a straightforward process. Furthermore, the interaction measure is preferred to be related to stability and performance of the decentralized control loop to facilitate the control design.

In mathematics, a matrix is diagonally dominant if for every row of the matrix, the amplitude of the diagonal entry in a row is larger than or equal to the sum of the amplitudes of all the other (non-diagonal) entries in that row. Similarly, the diagonal dominance [24] for row $i$ of an $n \times n$ system $P$ is defined as:

$$\frac{|P_{ii}|}{\sum_{j=1, j\neq i}^{n} |P_{ij}|}. \quad (2.3)$$

According to this definition, the diagonal dominance is different from row to row. Therefore, the diagonal dominance is not a single-valued function of frequency.

The RGA [11] of an $n \times n$ system $P(s)$ is defined as $P(s)P^{-T}(s)$. It can be calculated based on the measured FRF by $\tilde{P}(j\omega)\tilde{P}^{-T}(j\omega)$ if a MIMO model is not available. For each frequency, the RGA is an $n \times n$ constant symmetric matrix and it is invariant to the input
Theorem 2.1.1. The decentralized closed-loop system is stable if all $H_i$ are stable and

$$|H_i(j\omega)| \leq \mu^{-1}_\Delta(E(P_d(j\omega))), \quad \forall \omega$$

is satisfied for all SISO loops.

The proof is found in [34].

Theorem 2.1.1 provides a sufficient but not necessary condition for the stability of the decentralized control loop. However, it provides the tightest bound on $|H_i|$ to evaluate the stability of the decentralized control loop. Experimental work in [108] shows that the improved $\mu$-interaction measure corresponds to an improved available control performance. In Chapter 10 of [98], the $\mu$-interaction measure is even used to define the diagonal dominance instead of the definition (2.3). Therefore, the $\mu$-interaction measure is recommended as the criterion to quantify the strength of cross-coupling for a MIMO system.

As accurate $H_i(j\omega)$ and $P_d(j\omega)$ are difficult to obtain, the measured FRF of $P_d(j\omega)$ can be used to calculate the FRF of $H_i(j\omega)$. As such, this theory can be used to evaluate the stability of the decentralized control in practice. Although the structured singular value can
not be exactly calculated, methods to calculate its tight upper and lower bounds are already available in Matlab [9]. These close lower and upper bounds give an accurate estimate of the true structured singular value. The calculated upper bound of the $\mu$-interaction measure is used as the practical $\mu$-interaction measure in this thesis.

### 2.1.4 Decoupling procedure

There are two categories of decoupling procedures: dynamic decoupling [40] is to decouple a MIMO system using transfer function matrices and static decoupling is to decouple a MIMO system using constant real matrices. Dynamic decoupling is theoretically able to achieve better performance but the complexity and the implementation cost could be comparable to the direct MIMO control. On the other hand, the static decoupling has a simpler structure and requires less implementation cost. Therefore, only static decoupling is discussed.

Many different decoupling procedures have been developed in the past. Classic methods are the ALIGN-procedure [73], the method to achieve diagonal dominance [77], the method uses only the input-decoupling matrix [38], model decomposition [43, 26, 74], the Owens method [88], etc.

More recent achievement has been done by D.Veas [109] who developed the optimal static decoupling. In [109], the minimal number of parameters needed in the decoupling matrices are found. Subsequently, the input and output decoupling matrices are parameterized for numerical optimization which is intended to minimize the $\mu$-interaction measure.

### 2.2 Decoupling of an LTI mechanical system with proportional damping

Modal decomposition [43, 26, 74] is a decoupling procedure specifically developed for a Linear, Time-Invariant (LTI) mechanical system with the property of proportional damping (the damping matrix is linearly dependent on the inertia matrix or the stiffness matrix). It has been widely applied in the industry [41, 42]. The Owens method is based on the Dyadic Transfer function Matrix (DTM) theory [87, 70, 5]. A system which can be described by a real-coefficient transfer function matrix is called dyadic if and only if there exist two constant matrices which could diagonalize this transfer function matrix. A more complete definition follows later. D.Owens [87] developed a method to derive the two decoupling matrices based on eigenvalue decomposition but its application is limited to dyadic systems.

It will be shown in this section that an LTI mechanical system with proportional damping is dyadic. To decouple such a system, the Owens method and the modal decomposition are both applicable and are both based on the eigenvalue decomposition. However, the connection between these two methods is not found in literature.

This section starts with necessary definitions and terminologies. Subsequently, the two methods are both described and applied to an LTI mechanical system with proportional damping. It will be proved that the input and output decoupling matrices derived by both methods can be normalized to the same pair of input and output decoupling matrices. As such, the performance of the two methods is identical.
2.2. Decoupling of an LTI mechanical system with proportional damping

2.2.1 Nomenclature and definitions

In this section, all matrices are considered as constant, square, and complex unless they are clearly defined otherwise. A matrix $\mathbf{A}$ is called symmetric if and only if it is equal to its direct transpose: $\mathbf{A} = \mathbf{A}^T$. For a matrix $\mathbf{A}$, $\mathbf{A}^{-T}$ denotes the inverse of $\mathbf{A}^T$ and $\mathbf{A}^H$ denotes the conjugate transpose of $\mathbf{A}$. $\mathbf{I}$ and $\mathbf{O}$ denote the identity matrix and the zero matrix with proper dimensions. Two matrices, $\mathbf{A}$ and $\mathbf{B}$, commute if and only if $\mathbf{AB} = \mathbf{BA}$. For a row vector $\vec{v}$, $\vec{v}^H$ denote the conjugate column vector of $\vec{v}$. $\vec{v}(i)$ denote the $i^{th}$ element of vector $\vec{v}$. A real vector $\vec{v}$ is normalized if and only if its absolute value is one and its first component, $\vec{v}(1)$, is non-negative. A complex column vector $\vec{v}$ can be normalized by the following process:

1. Calculate $\vec{v}_1 = \vec{v}(1)^H \vec{v}$;
2. The normalized vector is given by $\frac{\vec{v}_1}{\sqrt{\vec{v}_1^H \vec{v}_1}}$.

The transfer function of a Linear, Time-Invariant (LTI) lumped mechanical systems has the form of

$$P(s) = \left[ M s^2 + D s + K \right]^{-1}, \quad (2.7)$$

in which

$\mathbf{M}$: $n \times n$ real positive-definite inertia matrix.
$\mathbf{D}$: $n \times n$ real damping matrix.
$\mathbf{K}$: $n \times n$ real stiffness matrix.

An LTI mechanical system possesses the property of proportional damping if

$$\mathbf{D} = \alpha \mathbf{M} + \beta \mathbf{K}, \quad (2.8)$$

where $\alpha$ and $\beta$ are real constant scalars.

The definition of dyadic transfer function matrix [5] is described as follows.

**Definition 2.2.1.** An $n \times n$ transfer function matrix $P(s)$, belonging to the field of real rational functions of $s$, is said to be dyadic if and only if, $\det(P(s)) \neq 0$ and there exist constant $n \times n$ matrices $\mathbf{T}_u$ and $\mathbf{T}_y$ and rational transfer functions with coefficients $p_1(s), ..., p_n(s)$ such that

$$\mathbf{T}_y P(s) \mathbf{T}_u = \text{diag}\{p_1(s), ..., p_n(s)\}. \quad (2.9)$$

**Remark 2.2.1.** The two constant matrices, $\mathbf{T}_u$ and $\mathbf{T}_y$, in Definition 2.2.1 are allowed to be complex. It is important to mention that complex matrices can not be implemented in a practical system. Therefore, a dyadic system can not be perfectly diagonalized in practice unless $\mathbf{T}_u$ and $\mathbf{T}_y$ are both real matrices.

**Lemma 2.2.1.** If one of the two matrices, $\mathbf{A}$ and $\mathbf{B}$, is invertible, $\mathbf{AB}$ and $\mathbf{BA}$ have the same eigenvalues.

**Proof.** Without losing generality, we assume that $\mathbf{A}$ is invertible. According to the properties of the eigenvalue decomposition, $\mathbf{BA}$ and $\mathbf{A}^{-1}(\mathbf{BA})\mathbf{A}^{-1}$ have the same eigenvalues and $\mathbf{AB} = \mathbf{A}(\mathbf{BA})\mathbf{A}^{-1}$. Therefore, $\mathbf{AB}$ and $\mathbf{BA}$ have the same eigenvalues. \qed
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Remark 2.2.2. The eigenvalues of $AB$ and $BA$ are the same but the eigenvalue matrices for $AB$ and $BA$ can be different because the sequence of eigenvalues in the matrix can be different. In this work, we assume that the sequence of the eigenvalues is properly organized such that $AB$ and $BA$ have the same eigenvalue matrix.

Lemma 2.2.2. If an $n \times n$ matrix $A$ fulfills the following two assumptions,

- $A$ is invertible and diagonal.
- $A$ has no repeating diagonal entries.

any constant matrix which commutes with $A$ is diagonal.

Proof. Without losing generality, we assume an $n \times n$ matrix $B$ commutes with $A$. The entry at $i^{th}$ row and $j^{th}$ column in matrix $B$ is denoted by $B_{ij}$. The $i^{th}$ diagonal entry of the matrix $A$ is denoted by $A_{ii}$. The two matrices $AB$ and $BA$ are calculated as follows.

\[
AB = \begin{bmatrix}
A_{11}B_{11} & A_{11}B_{12} & \cdots & A_{11}B_{1n} \\
A_{22}B_{21} & A_{22}B_{22} & \cdots & A_{22}B_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
A_{nn}B_{n1} & A_{nn}B_{n2} & \cdots & A_{nn}B_{nn}
\end{bmatrix}, \quad (2.10)
\]

\[
BA = \begin{bmatrix}
A_{11}B_{11} & A_{12}B_{12} & \cdots & A_{1n}B_{1n} \\
A_{11}B_{21} & A_{12}B_{22} & \cdots & A_{1n}B_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
A_{11}B_{n1} & A_{12}B_{n2} & \cdots & A_{1n}B_{nn}
\end{bmatrix}. \quad (2.11)
\]

Since $AB = BA$, the corresponding entries of these two matrices are equals: $A_{ii}B_{ij} = A_{jj}B_{ji}$. Since $A_{ii} \neq A_{jj}$ with $i \neq j$, $B_{ij} = B_{ji} = 0$. Therefore, the matrix $B$ is diagonal. \hfill \square

2.2.2 Modal decomposition

Modal decomposition [43, 26, 74] is a procedure to derive two constant matrices to decouple an LTI mechanical system $P(s)$ which possesses the property of proportional damping. It is described as follows.

The inertia matrix $M$ is positive definite so it is always invertible. Substitute (2.8) into (2.7), (2.7) can be reformed to

\[
P(s) = [Is^2 + (\alpha I + \beta M^{-1}K)s + M^{-1}K]^{-1}M^{-1}, \quad (2.12a)
\]

or

\[
P(s) = M^{-1} [Is^2 + (\alpha I + \beta KM^{-1})s + KM^{-1}]^{-1}. \quad (2.12b)
\]

According to Lemma 2.2.1 and Remark 2.2.2, $M^{-1}K$ and $KM^{-1}$ have the same eigenvalue matrix. Apply the eigenvalue decomposition to two real matrices $M^{-1}K$ and $KM^{-1}$, we have

\[
M^{-1}K = T_1 \Lambda T_1^{-1}, \quad (2.13a)
\]

and

\[
KM^{-1} = T_2 \Lambda T_2^{-1}, \quad (2.13b)
\]
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where \( \Lambda \) is a diagonal matrix which contains the eigenvalues and \( T_1, T_2 \) are the eigenvector matrices. Substitute (2.13) into (2.12), we have

\[
P(s) = T_1 [I s^2 + (\alpha I + \beta \Lambda) s + \Lambda]^{-1} T_1^{-1} M^{-1}, \tag{2.14a}
\]
and

\[
P(s) = M^{-1} T_2 [I s^2 + (\alpha I + \beta \Lambda) s + \Lambda]^{-1} T_2^{-1}. \tag{2.14b}
\]

Since \([I s^2 + (\alpha I + \beta \Lambda) s + \Lambda]^{-1}\) is diagonal, the considered system \( P(s) \) can be decoupled by two pairs of matrices:

1. \( T_1^{-1} \) and \( MT_1 \);
2. \( T_2^{-1} M \) and \( T_2 \).

Remark 2.2.3. In literature, the modal decomposition is derived in the differential equation model of the LTI mechanical system and only the first matrix pair is applied for decoupling. Using \( T_2^{-1} M \) and \( T_2 \) for decoupling is not found in literature.

Remark 2.2.4. If the eigenvector matrix of \( M^{-1} K \) (or \( KM^{-1} \)) is complex, practical implementation of the decoupling matrices becomes a problem because they are complex. In practice, the complex decoupling matrices are usually implemented by their real approximation (the real parts or absolute values). Therefore, decoupling would not be perfect any more. To avoid this problem, system design has to make sure that the resulting decoupling matrices are real. For example, \( M \) and \( K \) by modeling the system using differential equations or Finite Element Method (FEM) during system-design evaluation; subsequently, the eigenvalues of \( M^{-1} K \) are numerically calculated to check whether or not they are all real.

Remark 2.2.5. According to (2.14) and Definition 2.2.1, an LTI mechanical system which has the property of proportional damping is a dyadic system. Therefore, the Owens method is also applicable to derive the decoupling matrices.

2.2.3 Introduction to Owens method

The Owens method [88] is a procedure to find an input decoupling matrix \( T_u \) and an output decoupling matrix \( T_y \) for an arbitrary dyadic system \( P(s) \) such that the decoupled system \( T_y P(s) T_u \) is diagonal. This procedure is described as follows.

- Select two constant numbers \( c_1 \) and \( c_2 \).
- \( T_y^{-1} \) is the eigenvector matrix of \( P(c_1) P(c_2)^{-1} \).
- \( T_u \) is the eigenvector matrix of \( P(c_2)^{-1} P(c_1) \).

This procedure can be easily understood. Since

\[
P(c_1) = T_y^{-1} P_d(c_1) T_u^{-1}, \quad P(c_2) = T_y^{-1} P_d(c_2) T_u^{-1}, \tag{2.15}
\]

where \( P_d(c_1) \) and \( P_d(c_2) \) are both diagonal. Subsequently, we have

\[
P(c_1) P(c_2)^{-1} = T_y^{-1} P_d(c_1) P_d(c_2)^{-1} T_y, \tag{2.16a}
\]

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and
\[ P(c_2)^{-1}P(c_1) = T_uP_d(c_2)^{-1}P_d(c_1)T_u^{-1}. \]  
(2.16b)

As both \( P_d(c_1) \) and \( P_d(c_2)^{-1} \) are diagonal, \( T_y^{-1} \) and \( T_u \) are eigenvector matrices of \( P(c_1)P(c_2)^{-1} \) and \( P(c_2)^{-1}P(c_1) \), respectively.

The Owens method is applicable to all dyadic systems. D.Vaes [110] proposes to choose the two constants, \( c_1 \) and \( c_2 \), as two frequencies: \( c_1 = j\omega_1 \) and \( c_2 = j\omega_2 \). Then \( P(j\omega_1) = P_1 \) and \( P(j\omega_2) = P_2 \) are FRF matrices of the system \( P(s) \).

### 2.2.4 Application of Owens method to LTI mechanical system

As mentioned in Remark 2.2.5, an LTI mechanical system which has the property of proportional damping is a dyadic system. In this subsection, the Owens method is applied to the same LTI mechanical system \( P(s) \) which has the property of proportional damping. The two frequencies are chosen as \( \omega_1 \) and \( \omega_2 \). From (2.14), the matrices \( P_1P_2^{-1} \) and \( P_2^{-1}P_1 \) are calculated as

\[ P_1P_2^{-1} = T_1[\Lambda - I\omega_1^2 + (\alpha I + \beta \Lambda)j\omega_1]^{-1} \]
\[ T_1[\Lambda - I\omega_2^2 + (\alpha I + \beta \Lambda)j\omega_2]T_1^{-1}. \]  
(2.17a)

and

\[ P_2^{-1}P_1 = T_2[\Lambda - I\omega_1^2 + (\alpha I + \beta \Lambda)j\omega_1]^{-1} \]
\[ T_2[\Lambda - I\omega_2^2 + (\alpha I + \beta \Lambda)j\omega_2]T_2^{-1}. \]  
(2.17b)

The two equations, (2.17a) and (2.13a), show that the eigenvector matrix of \( M^{-1}K \) (real) is also the eigenvector matrix of \( P_1P_2^{-1} \) (complex). The two equations, (2.17b) and (2.13b), show that the eigenvector matrix of \( KM^{-1} \) (real) is also the eigenvector matrix of \( P_2^{-1}P_1 \) (complex).

Substitute \( c_1 = j\omega_1 \) and \( c_2 = j\omega_2 \) into (2.16) and compare it with (2.17), it can be concluded that \( T_1^{-1}P(s)T_2 \) is diagonal.

### 2.2.5 Decoupling matrix normalization

A short summary is provided for the results of subsection 2.2.2 and subsection 2.2.4: for an LTI mechanical system \( P(s) \) with the property of proportional damping,

1. \( T_1^{-1}P(s)MT_1 \) is diagonal;
2. \( T_2^{-1}MP(s)T_2 \) is diagonal;
3. \( T_1^{-1}P(s)T_2 \) is diagonal.

To study the connection of the three pairs of decoupling matrices, the following theorem is provided.

**Theorem 2.2.1.** \( M \) and \( K \) are the inertia matrix and the stiffness matrix as (2.7), respectively. \( T_1, T_2, \) and \( \Lambda \) are derived by (2.13). Assuming that the matrix \( \Lambda \) fulfill the assumptions of Lemma 2.2.2, the two output decoupling matrices, \( T_1^{-1} \) and \( T_2^{-1}M \), will converge to the same matrix by normalizing all of their row vectors; the two input decoupling matrices, \( MT_1 \) and \( T_2 \), will converge to the same matrix by normalizing all of their column vectors.
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Proof. From (2.13), we have

\[ K = MT_1 \Delta T_1^{-1} = T_2 \Delta T_2^{-1} M. \]  
(2.18)

Reforming (2.18), we have

\[ T_2^{-1} MT_1 \Delta = \Delta T_2^{-1} MT_1. \]  
(2.19)

According to Lemma 2.2.2, \( T_2^{-1} MT_1 \) is diagonal. Let \( T_2^{-1} MT_1 = A \), we have

\[ T_2 A = MT_1. \]  
(2.20)

So, \( \tilde{c}_i [T_2] a_i = \tilde{c}_i [MT_1] \), where \( a_i \) is the \( i \)th diagonal entry of the diagonal matrix \( A \). Therefore, the normalized \( \tilde{c}_i [T_2] \) and \( \tilde{c}_i [MT_1] \) would be identical. It can be concluded that \( MT_1 \) and \( T_2 \) will converge to the same matrix by normalizing all of their column vectors.

As \( T_2^{-1} M = AT_1^{-1} \), we have \( \tilde{r}_i [T_1^{-1}] a_i = \tilde{r}_i [T_2^{-1} M] \). Therefore, the normalized \( \tilde{r}_i [T_1^{-1}] \) and \( \tilde{r}_i [T_2^{-1} M] \) would be identical. It can be concluded that \( T_1^{-1} \) and \( T_2^{-1} M \) will converge to the same matrix by normalizing all of their row vectors.

It has been proved by D.Vaes [109] that scaling of any row vector of the output decoupling matrix or any column vector of the input decoupling matrix does not influence the \( \mu \)-interaction measure, or the decoupling performance. Therefore, all of the three pair decoupling matrices derived by modal decomposition and the Owens method are equivalent in decoupling the LTI mechanical system \( P(s) \) with the property of proportional damping, thereby making the system dyadic.

A numerical example is given to show the performances of the three pairs of decoupling matrices.

2.2.6 Numerical example

The inertia matrix and the stiffness matrix are taken from a practical mechanical system [21] that will be presented in Chapter 4 and will be used for the validation of the proposed control.

\[ M = \begin{bmatrix}
3.2531 & -0.0189 & 0.0047 \\
-0.0159 & 0.0258 & -0.0003 \\
0.0060 & -0.0000 & 0.0284
\end{bmatrix} \]  
(2.21)

\[ K = \begin{bmatrix}
567.6356 & 15.4369 & 23.9811 \\
53.4371 & 80.9241 & -3.1536 \\
18.8218 & -2.1851 & 135.7891
\end{bmatrix} \]  
(2.22)

However, this system does not have the property of proportional damping. So the damping matrix is assumed to be \( D = 2M + 0.02K \) in order to make the system dyadic. The
eigenvector matrices of $M^{-1}K$ and $KM^{-1}$ are calculated in Matlab:

$$T_1 = \begin{bmatrix} -0.7985 & 0.0078 & -0.0002 \\ 0.5902 & 0.9988 & -0.0410 \\ 0.1184 & 0.0475 & 0.9992 \end{bmatrix}, \quad (2.23)$$

$$T_2 = \begin{bmatrix} -0.9999 & 0.2500 & 0.1684 \\ 0.0107 & 0.9668 & -0.0470 \\ -0.0005 & 0.0527 & 0.9846 \end{bmatrix}. \quad (2.24)$$

Although a few choices of two frequencies $\omega_1$ and $\omega_2$ are tried, the $T_1$ and $T_2$ calculated by (2.17) are all the same as above values.

The two input decoupling matrices ($MT_1$ and $T_2$) can be normalized and the resultant matrix is exactly $T_2$. The two output decoupling matrices ($T_1^{-1}$ and $T_2^{-1}M$) can be normalized and the resultant matrix is derived as

$$T_1^{-1} = \begin{bmatrix} -1.0000 & 0.0078 & 0.0001 \\ 0.5972 & 0.8014 & 0.0330 \\ 0.1116 & -0.0481 & 0.9926 \end{bmatrix}. \quad (2.25)$$

The LTI mechanical system $P(s)$ is calculated by (2.7) and its magnitude is plotted in Fig. 2.3. The decoupled systems using the four different pairs of decoupling matrices are also compared with $P(s)$ in Fig. 2.3. This comparison gives an impression of the decoupling performance. To quantify the decoupling performance, the $\mu$-interaction measures of the original system $P(s)$ and the four decoupled systems are plotted in Fig. 2.2. It shows that all the three pairs of decoupling matrices have the same decoupling performances when the inertia matrix and the stiffness matrix are exactly known.

In theory, the off-diagonal entries of the decoupled systems are all zero. The $\mu$-interaction measures of the four decoupled systems are also zero. However, the calculated off-diagonal entries are in the order of $10^{-20}$ (-400dB) and the calculated $\mu$-interaction measures of the four decoupled systems are in the order of $10^{-15}$ (-300 dB). They can be supposed to be numerical errors in Matlab calculation.

### 2.2.7 Comparison

The above theoretical and numerical study shows that the modal decomposition and the Owens method have equivalent performance in decoupling. The difference between the two methods would be the conditions to apply. This subsection compares the pre-conditions to apply the two methods.

To apply the modal decomposition to an LTI mechanical system, knowledge of the inertia matrix and the stiffness matrix is a must. In practice, there are two approaches to acquire this knowledge:

1. System modeling: Modeling using motion equations based on the mechanical design.
2. MIMO identification: Parameter estimation based on experimental data acquired in identification tests.

The first approach does not need experiments but the error of derived parameters ($M$ and $K$) is usually up to 10%. It is possible to derive more accurate parameters using the second
2.2. Decoupling of an LTI mechanical system with proportional damping

Figure 2.2: The $\mu$-interaction measure of the original system $P(s)$ and the decoupled systems $T_1^{-1}P(s)MT_1$, $T_2^{-1}MP(s)T_2$, $T_1^{-1}P(s)T_2$, and $T_1^{-1}P(s)MT_1$.

Figure 2.3: Comparison of the original system $P(s)$ with the decoupled systems $T_1^{-1}P(s)MT_1$, $T_2^{-1}MP(s)T_2$, $T_1^{-1}P(s)T_2$, and $T_1^{-1}P(s)MT_1$. The legend is exactly the same as Fig. 2.2.
approach provided with necessary equipments but the cost in both time and effort would be relatively much higher.

The Owens method can be applied based on two types of system parameters:

1. The FRF matrices of $P(s)$ at only two different frequencies;
2. The inertia matrix $M$ and the stiffness matrix $K$ used for modal decomposition.

The FRF measurement at two different frequencies allows the Owens method to achieve comparable decoupling performance to the second approach of modal decomposition with less cost in both time and effort. Therefore, the Owens method is a good alternative to the two approaches of applying modal decomposition.

If the inertia matrix, damping matrix, and stiffness matrix are all known with reasonable accuracy by identification or modeling, direct Multi-Input-Multi-Output (MIMO) control design could be more advantageous than the decentralized control. In that case, decoupling of the multi-DOF system might not be necessary any more.

2.3 Optimal static decoupling

The modal decomposition and the Owens method are both limited to dyadic systems. However, perfect dyadic system hardly exists in practice. If the Owens method is extended to a non-dyadic system, perfect decoupling can be achieved only at the two chosen frequencies. With different pairs of chosen frequencies, the derived input and output decoupling matrices ($T_y$ and $T_u$) would be different. As the FRF can only be derived at limited number of frequencies, the choices of the frequency-pair to apply the Owens method form are limited. In other words, the pairs of decoupling matrices ($T_y$ and $T_u$) that can be derived by the Owens method has limited number. D. Vaes developed a numerical process to search for the two frequencies to apply the Owens method, which leads to a pair of $T_y$ and $T_u$ corresponding to the minimal $\mu$-interaction measure. Even though, it is still possible that there exist another pair of $T_y$ and $T_u$ which can not be derived by the Owens method. D. Vaes has further developed a numerical process [109] to directly search for the optimal $T_y$ and $T_u$. This process is referred to as the Vaes-procedure. It is referred to as "optimal" in the sense that the $\mu$-interaction measure is minimized.

This section describes and analyzes the Vaes-procedure. Subsequently, two modifications are developed to improve the Vaes-procedure.

2.3.1 Vaes-procedure description

For an $n \times n$ MIMO system $P(s)$, two $n \times n$ real decoupling matrices $T_y$ and $T_u$ are needed. The Vaes-procedure takes two steps to derive them.

First, the two matrices $T_y$ and $T_u^{-1}$ are parameterized in such a way that the number of parameters are minimized without losing decoupling accuracy. It has been proved in [109] that scaling of a row vector in matrices $T_y$ and $T_u^{-1}$ does not affect the $\mu$-interaction measure. According to this conclusion, each row of the two matrices $T_y$ and $T_u^{-1}$ are parameterized by an $n$-dimensional unit vector without losing degrees of freedom. In this way, the number of variables that are needed to parameterize $T_y$ and $T_u^{-1}$ would be $2n(n - 1)$. Trigonometric functions are used for parameterizations.
In Vaes-procedure, $T_y$ is parameterized as

$$
\begin{bmatrix}
\Pi_{i=1}^{n-1} \cos(\alpha_{i,1}) & \Pi_{i=1}^{n-1} \cos(\alpha_{i,2}) & \ldots & \Pi_{i=1}^{n-1} \cos(\alpha_{i,n}) \\
\sin(\alpha_{n,1}) \Pi_{i=1}^{n-2} \cos(\alpha_{i,1}) & \sin(\alpha_{n,2}) \Pi_{i=1}^{n-2} \cos(\alpha_{i,2}) & \ldots & \sin(\alpha_{n,n}) \Pi_{i=1}^{n-2} \cos(\alpha_{i,n}) \\
\sin(\alpha_{n-1,1}) \Pi_{i=1}^{n-3} \cos(\alpha_{i,1}) & \sin(\alpha_{n-1,2}) \Pi_{i=1}^{n-3} \cos(\alpha_{i,2}) & \ldots & \sin(\alpha_{n-1,n}) \Pi_{i=1}^{n-3} \cos(\alpha_{i,n}) \\
\vdots & \vdots & \ldots & \vdots \\
\sin(\alpha_{2,1}) \cos(\alpha_{1,1}) & \sin(\alpha_{2,2}) \cos(\alpha_{1,2}) & \ldots & \sin(\alpha_{2,n}) \cos(\alpha_{1,n}) \\
\sin(\alpha_{1,1}) & \sin(\alpha_{1,2}) & \ldots & \sin(\alpha_{1,n})
\end{bmatrix}^T
$$

\hspace{1cm}(2.26)

and $T_u^{-1}$ is parameterized as

$$
\begin{bmatrix}
\Pi_{i=1}^{n-1} \cos(\beta_{i,1}) & \Pi_{i=1}^{n-1} \cos(\beta_{i,2}) & \ldots & \Pi_{i=1}^{n-1} \cos(\beta_{i,n}) \\
\sin(\beta_{n,1}) \Pi_{i=1}^{n-2} \cos(\beta_{i,1}) & \sin(\beta_{n,2}) \Pi_{i=1}^{n-2} \cos(\beta_{i,2}) & \ldots & \sin(\beta_{n,n}) \Pi_{i=1}^{n-2} \cos(\beta_{i,n}) \\
\sin(\beta_{n-1,1}) \Pi_{i=1}^{n-3} \cos(\beta_{i,1}) & \sin(\beta_{n-1,2}) \Pi_{i=1}^{n-3} \cos(\beta_{i,2}) & \ldots & \sin(\beta_{n-1,n}) \Pi_{i=1}^{n-3} \cos(\beta_{i,n}) \\
\vdots & \vdots & \ldots & \vdots \\
\sin(\beta_{2,1}) \cos(\beta_{1,1}) & \sin(\beta_{2,2}) \cos(\beta_{1,2}) & \ldots & \sin(\beta_{2,n}) \cos(\beta_{1,n}) \\
\sin(\beta_{1,1}) & \sin(\beta_{1,2}) & \ldots & \sin(\beta_{1,n})
\end{bmatrix}^T
$$

\hspace{1cm}(2.27)

such that the row vectors of $T_y$ and $T_u^{-1}$ are all unit vectors. All the variables $\alpha_{i,j}$ and $\beta_{i,j}$ are bounded within the range of $[-\pi, \pi]$. As such, each parameterized unit vector could point to all possible points on the $n$-dimensional unit sphere by proper combinations of the variables.

Second, the cost function

$$
\max_{\omega} \mu_{\Delta} \left( E(P_d(\omega))W(\omega) \right)
$$

\hspace{1cm}(2.28)

is numerically minimized. $P_d$ is calculated by (2.1) with parameterized $T_y$ and the inverse of the parameterized $T_u^{-1}$. $W(\omega)$ is a weighting filter to enhance a certain frequency range. The decoupling matrices derived from the Owens method or modal decomposition are recommended as the initial parameter values.

### 2.3.2 Vaes-procedure modification

The allowed variation range for each variable ($\alpha_{i,j}$ or $\beta_{i,j}$) in the Vaes-procedure may be reduced. It is expected that reducing the variable bounds could speed up the optimization process.

In the Vaes-procedure, each row vector of $T_y$ and $T_u^{-1}$ is able to point to all the points on the $n$-dimensional unit sphere when variables ($\alpha_{i,j}$ and $\beta_{i,j}$) vary within their bounds $[-\pi, \pi]$. However, this is conservative because it would be sufficient that each unit vector may point to all the points on half of the unit sphere. Assume that an $n$-dimensional unit vector $\vec{v}$ is parameterized as a row vector of $T_y$ or $T_u^{-1}$, using the vector $-\vec{v}$ to replace $\vec{v}$ in $T_y$ or $T_u^{-1}$ would not affect the $\mu$-interaction measure. Besides, the variable bounds proposed by D. Vaes is so loose that more than one combinations of $\alpha_{i,j}$ and $\beta_{i,j}$ are corresponding to a single unit vector.
Chapter 2. Methodologies of static decoupling for a general MIMO system

It is proposed in this thesis that the bounds for all of the variables $\alpha_i, j$ and $\beta_i, j$ can be tightened to $(-\frac{\pi}{2}, \frac{\pi}{2})$. In this way, each unit vector may point to all the points on half of the unit sphere. This can be explained as follows.

Assume that the elements of the $n$-dimensional unit vector $\vec{v}$ are denoted by $v_i$, $\forall i \in \{1, 2, ..., n\}$ and $\vec{v}$ is parameterized as

$$ v_1 = \sin \theta_1, $$
$$ v_2 = \sin \theta_2 \cos \theta_1, $$
$$ \vdots $$
$$ v_{n-1} = \sin \theta_{n-1} \prod_{i=1}^{n-2} \cos \theta_i, $$
$$ v_n = \prod_{i=1}^{n-1} \cos \theta_i. $$

If $\theta_i \in (-\frac{\pi}{2}, \frac{\pi}{2})$, we have $\sin \theta_i \in (-1, 1]$ and $\cos \theta_i \in [0, 1]$. Therefore, $v_i \in (-1, 1]$, $\forall i \in \{1, 2, ..., n\}$ and $v_n \in [0, 1]$, which indicate that $\vec{v}$ can only access half of the unit sphere.

Assume that $\theta_1 \in (-\frac{\pi}{2}, \frac{\pi}{2})$ and $\vec{v}$ can access all points on the half of the unit sphere ($v_n \in [0, 1]$ and $v_i \in (-1, 1]$, $\forall i \in \{1, 2, ..., n-1\}$), we have $v_1^2 + v_2^2 + \ldots + v_n^2 = 1$. Subsequently, $v_1 = \sin \theta_1 \leq \sqrt{1 - v_2^2} \Rightarrow v_2^2 \leq \cos^2 \theta_1$. As $\theta_1 \in (-\frac{\pi}{2}, \frac{\pi}{2})$, $\cos \theta_1 \in [0, 1]$. Therefore, we have $v_2 \cos \theta_1 \in (-1, 1]$. Similarly, we have $\frac{v_{n-1}}{\prod_{i=1}^{n-2} \cos \theta_i} \in (-1, 1]$, $\forall n$ and $\frac{v_n}{\prod_{i=1}^{n-1} \cos \theta_i} \in [0, 1]$. Considering that,

$$ \theta_1 = \arcsin v_1, $$
$$ \theta_2 = \arcsin \frac{v_2}{\cos \theta_1}, $$
$$ \vdots $$
$$ \theta_{n-1} = \arcsin \frac{v_{n-1}}{\prod_{i=1}^{n-2} \cos \theta_i}, $$
$$ \theta_n = \arccos \frac{v_n}{\prod_{i=1}^{n-1} \cos \theta_i}. $$

it can be concluded that $\theta_i \in (-\frac{\pi}{2}, \frac{\pi}{2})$.

The calculation of inverting $T_u^{-1}$ is not preferred in the optimization process. The numerical process in Matlab sometimes comes up with a $T_u^{-1}$ which is nearly singular. Inverse of this matrix induces calculation error which would interrupt the optimization process. Besides, inverse of a matrix costs extra calculation time. Therefore, instead of $T_u^{-1}$, $T_u$ is
2.4. Conclusions

Parameterized as

\[
\begin{bmatrix}
\prod_{i=1}^{n-1} \cos(\beta_{i,1}) & \sin(\beta_{1,2}) & \ldots & \sin(\beta_{n-1,n}) \prod_{i=1}^{n-2} \cos(\beta_{i,n}) \\
\sin(\beta_{n-1,1}) \prod_{i=1}^{n-2} \cos(\beta_{i,1}) & \prod_{i=1}^{n-1} \cos(\beta_{i,2}) & \ldots & \sin(\beta_{n-1,n}) \prod_{i=1}^{n-3} \cos(\beta_{i,n}) \\
\sin(\beta_{n-2,1}) \prod_{i=1}^{n-3} \cos(\beta_{i,1}) & \sin(\beta_{n-1,2}) \prod_{i=1}^{n-2} \cos(\beta_{i,2}) & \ldots & \sin(\beta_{n-3,n}) \prod_{i=1}^{n-4} \cos(\beta_{i,n}) \\
\vdots & \vdots & \ddots & \vdots \\
\sin(\beta_{2,1}) \cos(\beta_{1,1}) & \sin(\beta_{2,2}) \cos(\beta_{2,1}) & \ldots & \sin(\beta_{1,n}) \prod_{i=1}^{n-1} \cos(\beta_{i,n}) \\
\sin(\beta_{1,1}) & \sin(\beta_{2,2}) \cos(\beta_{1,2}) & \ldots & \sin(\beta_{1,n}) \prod_{i=1}^{n-1} \cos(\beta_{i,n}) 
\end{bmatrix}
\]

so that the calculation of inverting \( T_u^{-1} \) is no longer necessary. It has been explained in [109] that scaling the column vector of \( T_u \) does not affect the \( \mu \)-interaction measure. Therefore, this modification does not affect the decoupling accuracy.

Accordingly, \( T_y \) is parameterized as

\[
\begin{bmatrix}
\prod_{i=1}^{n-1} \cos(\alpha_{i,1}) & \sin(\alpha_{1,2}) & \ldots & \sin(\alpha_{n-1,n}) \prod_{i=1}^{n-2} \cos(\alpha_{i,n}) \\
\sin(\alpha_{n-1,1}) \prod_{i=1}^{n-2} \cos(\alpha_{i,1}) & \prod_{i=1}^{n-1} \cos(\alpha_{i,2}) & \ldots & \sin(\alpha_{n-1,n}) \prod_{i=1}^{n-3} \cos(\alpha_{i,n}) \\
\sin(\alpha_{n-2,1}) \prod_{i=1}^{n-3} \cos(\alpha_{i,1}) & \sin(\alpha_{n-1,2}) \prod_{i=1}^{n-2} \cos(\alpha_{i,2}) & \ldots & \sin(\alpha_{n-3,n}) \prod_{i=1}^{n-4} \cos(\alpha_{i,n}) \\
\vdots & \vdots & \ddots & \vdots \\
\sin(\alpha_{2,1}) \cos(\alpha_{1,1}) & \sin(\alpha_{2,2}) \cos(\alpha_{1,2}) \cos(\alpha_{2,2}) & \ldots & \sin(\alpha_{1,n}) \prod_{i=1}^{n-1} \cos(\alpha_{i,n}) \\
\sin(\alpha_{1,1}) & \sin(\alpha_{2,2}) \cos(\alpha_{1,2}) & \ldots & \sin(\alpha_{1,n}) \prod_{i=1}^{n-1} \cos(\alpha_{i,n}) 
\end{bmatrix}^T
\]

(2.29)

2.4 Conclusions

This section concludes the two parts that have been studied in this chapter.

2.4.1 Modal decomposition and Owens method

An LTI mechanical system which has the property of proportional damping has been proved to be a dyadic system. Both modal decomposition and Owens method have been applied to derive the input- and output- matrices for static decoupling. The two methods lead to different pairs of decoupling matrices but they can be normalized to the same pair of decoupling matrix. Therefore, the two methods have equivalent decoupling performance. As the Owens method requires FRF matrices measured at two frequencies, it requires less time and effort compared with the modal decomposition which requires the inertia matrix and stiffness matrix.

2.4.2 Vaes-procedure modification

Two modifications have been proposed to improve the Vaes-procedure in terms of computational complexity and numerical stability. The bound of each variable to parameterize the two decoupling matrices are minimized from \([-\pi, \pi]\) to \([-\frac{\pi}{2}, \frac{\pi}{2}]\). As such, the computational complexity is reduced. The input decoupling matrix \( T_u \) is directly parameterized instead of parameterization of its inverse. As a result, it avoids the possibility of inversion a nearly singular matrix during the numerical optimization process so that the numerical stability is improved.
Chapter 3

Vibration isolation control using a sliding surface control approach

As explained in Chapter 1, the Single Electro-Magnetic Isolator System (SEMIS) concerned in this thesis is a Multi-Input-Multi-Output (MIMO) contactless suspension system. The six Degrees-of-Freedom (DoF) motions, the three translations and three rotations, are cross-coupled. This thesis explores a control strategy which combines static decoupling and single-DoF vibration isolation control. The static decoupling, which is used to decouple the MIMO system using constant real matrices, is studied in Chapter 2. In this chapter, vibration isolation control is studied for a single-DoF (or 1-DoF) suspension system to simultaneously achieve stabilization, vibration isolation, and disturbance rejection. As explained in Chapter 1, the Permanent Magnet (PM) based gravity compensator is only weakly nonlinear. Therefore, a linear model of the 1-DoF suspension system is used to study the vibration isolation control.

As defined in Chapter 1, the terminology vibration isolation control in this thesis specifically refers to as the control design for an active suspension system which is aiming to simultaneously achieve stabilization, isolation of floor vibrations (transmissibility), and rejection of direct-acting disturbance forces (compliance). As the vibration isolation control is highly dependent on the measurement scheme, the possible measurement schemes are discussed in this chapter. First, three different types of sensors, displacement sensor, geophone (absolute velocity sensor), and accelerometer are modeled. The accelerometer and the geophone are referred to as absolute motion sensors because the motion being measured is with respect to an inertially fixed reference. General control diagrams for three possible measurement schemes are given. Second, the influence of the controller on the four performance criteria are calculated. Finally, a measurement scheme combining the relative displacement and the payload absolute motion is chosen for SEMIS.

The sliding surface control proposed in this chapter is based on the frequency-shaped sliding surface approach in [128]. It is generalized as a two-step vibration isolation control design method, which is applicable to two possible types of measurement schemes. Numerical examples are provided to further illustrate the design process. The numerical results are compared. The accelerometer is concluded to be preferred rather than the absolute velocity sensor.
3.1 Nomenclature and definition

3.1.1 Model of a 1-DoF suspension

A 1-DoF suspension system is introduced as an example plant to study the vibration control. The schematic of the 1-DoF plant is shown in Fig. 3.1. The payload represents the isolated object or the metrology frame in the lithography machine. The spring and damper represent the pneumatic isolator or the PM-based gravity compensator. The payload mass, spring stiffness, and damping coefficient are denoted by \( m \), \( k \), and \( c \), respectively. For pneumatic isolators, \( c > 0 \) and \( k > 0 \). For 1-DoF contactless electromagnetic isolators, \( c = 0 \) and \( k < 0 \) (For multi-DoF contactless electromagnetic isolators, there is at least one DoF that has negative stiffness). The payload absolute displacement, payload absolute velocity, and base absolute displacement are denoted by \( x_a \), \( v_a \), and \( x_b \), respectively. The term \textit{absolute} indicates that the referred physical variable is with respect to an inertially fixed reference. The actuator force is denoted by \( f_a \). The equation of motion for the payload is given by

\[
mx_a'' + cx_r + kx_r = f_a + f_d,
\]  

where \( x_r = x_a - x_b \) is the relative displacement. The passive transmissibility and compliance are calculated as:

\[
T_p = \frac{cs + k}{ms^2 + cs + k}, \quad C_p = \frac{1}{ms^2 + cs + k}.
\]

Table 3.1 lists the symbols of the variables used in this chapter. The Laplace transform of a signal is denoted by the capital letter with the same subscript. Take \( x_a \) for example, its Laplace transform is denoted by \( X_a \).

The transmissibility \( T \) is used to evaluate the vibration isolation performance. It is defined by the transfer function from the base displacement \( x_b \) to the payload absolute displacement \( x_a \):

\[
T = \frac{X_a}{X_b}.
\]
Chapter 3. Vibration isolation control using a sliding surface control approach

<table>
<thead>
<tr>
<th>symbol</th>
<th>explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_a$</td>
<td>payload absolute displacement (m).</td>
</tr>
<tr>
<td>$x_r$</td>
<td>payload relative displacement (m).</td>
</tr>
<tr>
<td>$x_b$</td>
<td>base absolute displacement (m).</td>
</tr>
<tr>
<td>$v_a$</td>
<td>payload absolute velocity (m/s).</td>
</tr>
<tr>
<td>$a_a$</td>
<td>payload absolute acceleration (m/s²).</td>
</tr>
<tr>
<td>$v_b$</td>
<td>base absolute velocity (m/s).</td>
</tr>
<tr>
<td>$a_b$</td>
<td>base absolute acceleration (m/s²).</td>
</tr>
<tr>
<td>$\tilde{x}_r$</td>
<td>measured payload relative displacement (m).</td>
</tr>
<tr>
<td>$\tilde{v}_a$</td>
<td>measured payload absolute velocity (m/s).</td>
</tr>
<tr>
<td>$\tilde{a}_a$</td>
<td>measured payload absolute acceleration (m/s²).</td>
</tr>
<tr>
<td>$\tilde{v}_b$</td>
<td>measured base absolute velocity (m/s).</td>
</tr>
<tr>
<td>$\tilde{a}_b$</td>
<td>measured base absolute acceleration (m/s²).</td>
</tr>
<tr>
<td>$n_x$</td>
<td>displacement sensor noise (m).</td>
</tr>
<tr>
<td>$n_a$</td>
<td>absolute sensor (accelerometer (m/s²) or geophone (m/s)) noise.</td>
</tr>
<tr>
<td>$r$</td>
<td>the reference, which is assumed to be constant (m).</td>
</tr>
<tr>
<td>$f_d$</td>
<td>disturbance force directly applied to the payload (N).</td>
</tr>
<tr>
<td>$f_a$</td>
<td>control force applied to the payload (N).</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>sliding surface variable.</td>
</tr>
</tbody>
</table>

Table 3.1: The notations of variables.

The compliance $C$ is used to evaluate the disturbance rejection performance. It is defined by the transfer function from the disturbance force $f_d$ to the payload absolute displacement $x_a$:

$$C = \frac{X_a}{F_d}.$$ (3.4)

To simultaneously improve the transmissibility and compliance, only measuring the relative displacement $x_r$ for feedback control is not sufficient. Another absolute motion signal has to be measured. This point will be explained later in this chapter. Therefore, the vibration isolation controller is usually a Double-Input-Single-Output (DISO) controller. Some exceptions and the corresponding limitations will be explained later.

In closed-loop control, the influences of the sensor noises cannot be ignored. They are evaluated by two sensitivity functions. The relative sensitivity $\mathcal{R}$ is defined by the transfer function from the relative displacement sensor noise $n_x$ to the payload absolute displacement $x_a$:

$$\mathcal{R} = \frac{X_a}{N_x}.$$ (3.5)

The absolute sensitivity $\mathcal{S}$ is defined by the transfer function from the absolute motion sensor noise $n_a$ to the payload absolute displacement $x_a$:

$$\mathcal{S} = \frac{X_a}{N_a}.$$ (3.6)
3.2 Introduction

3.2.1 Sensor models

This subsection describes dynamic models for possible sensors that could be used in an active suspension system. There are three different types of sensors available for a suspension system:

- relative displacement sensor;
- absolute velocity sensor;
- absolute acceleration sensor.

By introducing their dynamic models, their limitations and the corresponding influences to the control design will be analyzed.

To measure the relative displacement, many commercial contactless sensors are applicable, including eddy current sensor, fiber-optical sensor, capacitive sensor, and laser interferometer. The displacement sensor usually has a very high bandwidth (in the order of $10^4$ Hz) so that the sensor dynamics are negligible at the interested frequency range. The true and measured relative displacement signals are denoted by $x_r$ and $\tilde{x}_r$, respectively. The sensor noise, denoted by $n_x$, is assumed to be Gaussian white noise. As such, the measured relative displacement $\tilde{x}_r$ can be derived by

$$\tilde{x}_r = x_r - n_x. \quad (3.7)$$

Two types of absolute motion sensors are commercially available: the geophone and the accelerometer. The geophone is a type of absolute velocity sensor. The dynamic model [1] for the geophone has the form of

$$G_v(s) = \frac{s^2}{s^2 + 2\omega_v \xi_v s + \omega_v^2}, \quad (3.8)$$

where $\omega_v$ is the natural frequency and $\xi_v$ is the damping ratio. The true and measured velocity signals are denoted by $v$ and $\tilde{v}$, respectively. The sensor noise $n_a$ is assumed to be Gaussian white noise so that $\tilde{v}$ is modeled by

$$\tilde{v} = G_v(s)v - n_a. \quad (3.9)$$

Because of the geophone dynamics, the DC velocity is not measurable. Furthermore, the signal to noise ratio at frequencies lower than its natural frequency is not good due to its second-order dynamics. However, to further reduce the geophone natural frequency is difficult. The recent-developed geophone which uses passive magnetic spring [85, 19] achieves 0.9 Hz natural frequency.

The considered accelerometer also has much higher bandwidth than the interested frequency range. The true and measured acceleration signals are denoted by $a$ and $\tilde{a}$, respectively. The sensor noise, denoted by $n_a$, is assumed to be independent of $a$ so that $\tilde{a}$ is modeled by

$$\tilde{a} = a - n_a. \quad (3.10)$$
Different from the other sensor noises, the $n_a$ is significantly biased at zero frequency especially when the sensor measures the vertical acceleration. For this reason, the acceleration sensor noise is assumed to be the sum of Gaussian white noise and a DC component. As this DC component is difficult to be compensated, the DC acceleration is also not measurable. Since the acceleration signal at low frequencies is weak in nature, the signal to noise ratio of the measured acceleration is also not good at low frequencies.

### 3.2.2 Performance requirements

As explained in Chapter 1, the objective of the vibration isolation control is to minimize the payload absolute displacement $x_a$ despite of the disturbance force $f_d$ and base displacement $x_b$. This is why transmissibility and compliance are used as performance criteria. The base displacement $x_b$ is typically seismic vibrations which have a broad band spectrum and unpredictable waveform. The disturbance forces $f_d$ are mostly acoustic noises and other self-generated forces.

**Remark 3.2.1.** Base vibrations lower than a threshold $\omega_c$ ($\omega_c$ is mostly less than 1 Hz) may be transmitted to the payload. Collision between the payload and the base frame has to be avoided, as described in Chapter 1. The influence of these low-frequency vibrations on the high precision applications on the payload can be eliminated by feedback control.

For both transmissibility and compliance, the general requirement is to minimize their magnitudes by active control. The phases of these two criteria, however, do not matter. Therefore, the two terms, transmissibility and compliance, in this thesis are referred to as the magnitudes of these two criterion by default.

Beside the minimized magnitude, an extra requirement for advanced suspension system is to have zero compliance ($-\infty$ dB) at zero frequency. This can be easily achieved by adding an integral control action in the displacement loop.

The challenging requirements are posted on the transmissibility. As the mass of the passive system acts as a passive second-order low-pass filter, the transmissibility is better at higher frequencies. For this reason, the vibration isolation control focuses on low-frequency transmissibility. The actuators in advanced suspension systems perform reasonably close to the ideal actuator at low frequencies but the two types of absolute sensors both have limited performances at low frequencies, as described in Section 3.2.1. Therefore, the low-frequency vibration isolation is limited mainly by the absolute sensor performances.

The ideal transmissibility is shown in Fig. 3.2. At low frequencies ($< \omega_c$), the ideal transmissibility is 1 (0 dB) to follow the base vibration. At high frequencies ($> \omega_c$), the ideal transmissibility is 0 ($-\infty$ dB) which indicates perfect isolation. The ideal transition ratio of the transmissibility from low frequencies to high frequencies is $-\infty$ dB/dec, which is not possible in practice. In industry, the transition ratio of $-40$ dB/dec is considered good enough. A resonance peak around the cut-off frequency $\omega_c$ is quite common in practice but this resonance peak is not desired and has to be minimized. The practical transmissibility shown in Fig. 3.2 is also referred to as skyhook transmissibility in literature.

For the two sensitivity functions, the general requirement of control design is to reduce their magnitudes. The absolute sensitivity $\mathcal{S}$ is further constrained by $\mathcal{S}(0) = 0$ ($-\infty$ dB) as both types of absolute sensors are useless at zero frequency.
3.3. Measurement schemes for vibration isolation control

The objective of vibration control is to minimize the payload absolute displacement which is the payload displacement with respect to an inertially fixed reference. Unfortunately, such an inertially fixed reference is actually a perfectly isolated object, which does not exist in practice. To build a nearly-perfect isolated object is feasible but the complexity and cost would be very high. Therefore, direct measurement of the payload absolute displacement or the base-frame absolute displacement is not feasible. In theory, the absolute displacement signal can be derived by integration of the absolute velocity or acceleration. However, the DC absolute displacement is still impossible to derive due to the limitation of the corresponding sensors. As a result, it is impossible to stabilize an inherent unstable suspension system using only absolute sensors. On the other hand, the relative displacement can be measured and used for stabilization. All of these possibilities and their limitations are discussed in this section.

3.3.1 Only relative displacement feedback

To stabilize an inherent unstable suspension system, measurement scheme of the relative displacement $x_r$ is applicable. However, it is impossible to simultaneously improve transmissibility and compliance using only relative displacement feedback. This can be explained as follows.

Assume that only the relative displacement is measured, the control diagram is shown in Fig. 3.3. The closed-loop transmissibility $T_c$ and compliance $C_c$ are calculated as:

$$T_c = \frac{C + cs + k}{ms^2 + cs + k + C} = \frac{CC_p + T_p}{1 + CC_p}, \quad C_c = \frac{1}{ms^2 + cs + k + C} = \frac{C_p}{1 + CC_p}. \quad (3.11)$$

As $T_c + C_c ms^2 = 1$, it is impossible to simultaneously reduce the magnitudes of $T_c$ and $C_c$. 
Chapter 3. Vibration isolation control using a sliding surface control approach

Figure 3.3: Control diagram for a 1-DoF suspension system using only relative displacement feedback.

(∞ frequency is an exception).

To simultaneously achieve stabilization, vibration isolation, and disturbance rejection, the measurement scheme must combine the relative displacement and one of the absolute motion signals.

### 3.3.2 Relative displacement & payload absolute motion

In addition to the relative displacement, measurement of the payload absolute motions, the payload absolute velocity [44, 128, 115] or the payload absolute acceleration [93], is commonly seen in literature. As two variables are measured, the controller in this case is a Double-Input-Single-Output (DISO) controller which can be separated into two SISO controllers. Assume that the relative displacement and the payload absolute motion are both measured, the general control diagram is shown in Fig. 3.4. The two blocks, $C_1$ and $C_2$, are the SISO controllers of the position loop and acceleration loop, respectively. The block $S$ represents the dynamics of the absolute sensor. For geophone, $S = sG_v$, where $G_v$ is defined in (3.8). For acceleration, $S = s^2$. The closed-loop transmissibility $T_c$ and compliance $C_c$ are calculated accordingly.

$$ T_c = \frac{C_1 + cs + k}{ms^2 + cs + k + C_1 + C_2 S} = \frac{C_1 C_P + T_p}{1 + C_P(C_1 + C_2 S)}. \quad (3.12) $$

$$ C_c = \frac{1}{ms^2 + cs + k + C_1 + C_2 S} = \frac{C_P}{1 + C_P(C_1 + C_2 S)}. \quad (3.13) $$

To reduce both $T_c$ and $C_c$, an obvious way is to increase the magnitude of $C_2$ and to reduce the magnitude of $C_1$. The two sensitivity functions are calculated as:

$$ R_c = \frac{C_1}{ms^2 + cs + k + C_1 + C_2 S} = \frac{C_1 C_P}{1 + C_P(C_1 + C_2 S)}. \quad (3.14) $$

$$ S_c = \frac{C_2}{ms^2 + cs + k + C_1 + C_2 S} = \frac{C_2 C_P}{1 + C_P(C_1 + C_2 S)}. \quad (3.15) $$
Combining (3.12), (3.13), and (3.15), we have
\[ T_c + Ccms^2 + ScS = 1. \] (3.16)

It indicates that it is impossible to simultaneously improve, or reduce the magnitudes, of all the three criteria \( T_c, Cc, \) and \( Sc \) by control design (\( \infty \) frequency is an exception). According to (3.12) and (3.14), \( Rc \) and \( Tc \) are highly correlated. Given any two DISO controllers, if the resultant \( Tc \) from the first controller has lower magnitude than that of the second controller, the resultant \( Rc \) from the first controller also has lower magnitude than that of the second controller. In other words, it is impossible to simultaneously increase \( Tc \) magnitude and reduce \( Rc \) magnitude by control design. Due to this correlation and the constraint (3.16), it is impossible to simultaneously improve all the four criteria \( T_c, C_c, R_c, \) and \( S_c \) by control design. The control design has to make a trade-off between the two groups of criteria:

- \( T_c, C_c, \) and \( R_c \); 
- \( S_c \).

For this measurement scheme, the open-loop transfer function, or, the loop-gain, \( L_c \) is defined as
\[ L_c = C_p(C_1 + C_2S). \] (3.17)

The control bandwidth is defined as the frequency that \( |L_c| = 1 \) (0 dB), which is also referred to as the cross-over frequency of \( L_c \). The loop-gain \( L_c \) can be separated into two parts: \( L_r = C_pC_1 \) and \( L_a = C_pC_2S \), which are referred to as the relative loop-gain and the absolute loop-gain, respectively. They are useful for loop-gain analysis.

**Remark 3.3.1.** In literature, most suspension systems are inherently stable and only the payload absolute motion is measured for control. For these systems, the relative displacement measurement is not a hard requirement because the combination of a spring and
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![Figure 3.5: General control diagram for a 1-DoF suspension system assuming that relative displacement and base absolute motion are both measured.](image)

A damper acts as a passive PD controller applied to the relative displacement control loop. Such a measurement scheme can be considered as a special form of the measurement scheme of relative displacement and payload absolute motion.

### 3.3.3 Relative displacement & base absolute motion

The absolute motion sensors which are specialized in low-frequency measurement usually have large dimensions (in the order of 0.1 m) and comparably large mass (in the order of 1 kg). In some practical situations, mounting the sensors on the payload is difficult for the mechanical design. An alternative to the payload absolute motion measurement is to measure the base absolute motion. Assume that the relative displacement and the base absolute motion are both measured, the general control diagram is shown in Fig. 3.5. This measurement scheme also requires a DISO controller, represented by the two SISO controllers $C_1$ and $C_2$ in Fig. 3.5.

The closed-loop transmissibility $T_c$ and compliance $C_c$ are calculated as:

$$T_c = \frac{C_1 - C_2S + cs + k}{ms^2 + cs + k + C_1} = \frac{(C_1 - C_2S)C_p + T_p}{1 + C_pC_1}. \quad (3.18)$$

$$C_c = \frac{1}{ms^2 + cs + k + C_1} = \frac{C_p}{1 + C_pC_1}. \quad (3.19)$$

To reduce $C_c$, we have to increase the magnitude of $C_1$. To reduce $T_c$ with a high-gain $C_1$, $C_2$ has to be properly designed such that the increase of $C_1$ is compensated by $C_2S$. Therefore, it is possible to simultaneously reduce both $T_c$ and $C_c$ using this measurement scheme. The two sensitivity functions are calculated as:

$$R_c = \frac{C_1}{ms^2 + cs + k + C_1} = \frac{C_1C_p}{1 + C_pC_1}. \quad (3.20)$$

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3.3. Measurement schemes for vibration isolation control

\[
S_c = \frac{C_2}{ms^2 + cs + k + C_1} = \frac{C_2C_p}{1 + C_pC_1}.
\]

(3.21)

Combining (3.18), (3.19), (3.20), and (3.21), we have

\[
T_c + C_cms^2 - S_cS = 1, \quad R_c + C_cC_p^{-1} = 1.
\]

(3.22)

It indicates that it is impossible to simultaneously improve, or reduce the magnitudes of, all the three criteria \(T_c\), \(C_c\), and \(S_c\) by control design (zero frequency is an exception). It is also not possible to simultaneously improve both \(C_c\) and \(R_c\) by control design. The control design has to make a trade-off between the two groups of criteria:

- \(T_c\) and \(C_c\);
- \(R_c\) and \(S_c\).

For this measurement scheme, the loop-gain \(L_c\) is defined as

\[
L_c = C_pC_1.
\]

(3.23)

The control bandwidth is defined as the frequency that \(|L_c| = 1\) (0 dB), which is also referred to as the cross-over frequency of \(L_c\).

3.3.4 Summary and comparison

Three measurement schemes are studied in the section:

1. Only relative displacement feedback;
2. Relative displacement and payload absolute motion;
3. Relative displacement and base absolute motion.

The first measurement scheme is not feasible due to the fact that it is impossible to simultaneously improve both transmissibility and compliance by control design. The other two measurement schemes are both feasible to simultaneously improve both transmissibility and compliance by control design. These two measurement schemes are compared in terms of requirements on the sensors.

For the second measurement scheme, the control design has to make a trade-off between the two groups of criteria:

- \(R_c\), \(T_c\) and \(C_c\);
- \(S_c\).

To improve transmissibility \(T_c\) and compliance \(C_c\), the relative sensitivity \(R_c\) is also improved and only the absolute sensitivity \(S_c\) is sacrificed. To achieve higher closed-loop performances \((T_c\) and \(C_c)\), a better absolute motion sensor is needed. However, the requirements on the displacement sensor are not increased.

For the third measurement scheme, the control design has to make a trade-off between the two groups of criteria:

- \(T_c\) and \(C_c\);
- \(R_c\) and \(S_c\).
To reduce transmissibility $T_c$ and compliance $C_c$, the relative sensitivity $R_c$ and the absolute sensitivity $S_c$ have to be both sacrificed. To achieve higher closed-loop performances ($T_c$ and $C_c$), a better absolute motion sensor and a better displacement sensor are both needed.

Comparing the last two measurement schemes, the third measurement scheme has higher demanding on the relative displacement sensor than the second measurement scheme. It indicates that the cost of the third measurement scheme is higher. The advantage of the third measurement scheme is that the accelerometers do not need to be mounted on the isolated platform such that there is no disturbance induced by the power/signal wires of the accelerometer. Furthermore, mechanical design for the isolated platform is easier if no accelerometer has to be mounted.

According to this comparison, the second measurement scheme is chosen for the SEMIS because disturbances induced by the wires of the chosen accelerometers are expected to be sufficiently weak.

### 3.4 Sliding surface control review

The terminology *sliding mode control* used in this thesis is specifically referred to as the conventional sliding mode control [99, 122]. The sliding mode control was first elaborated upon in the Soviet Union in the early 1960’s [124]. The idea only appeared outside Russia when a book by Itkis [48] and a survey paper by Utkins [106] were published in English in the mid 1970’s. It has become a well-know nonlinear control technique in academy as well as in industry. The design of a sliding mode controller involves:

1. Constructing a sliding surface such that the system dynamics exhibit desirable behavior when confined to this sliding surface;
2. Applying the switching control such that system dynamics intersect and stay on the sliding surface.

In the field of vibration isolation control, it is commonly seen in literature [123, 120, 72] that the sliding mode controller is applied to control the payload absolute motion loop while the relative displacement loop is not included in the design.

L. Zuo and J.J.E. Slotine [128] exploit the frequency-shaped sliding surface approach in a way that relative displacement and payload absolute velocity are both used to construct the sliding surface. In this way, the constructed sliding surface is physically connected to the transmissibility such that the transmissibility can be shaped in frequency domain by sliding surface design toward the skyhook performance. The work in [128] is reviewed based on the 1-DoF suspension model (3.1).

In [128], the sliding surface variable $\sigma$ is defined as

$$\sigma = v_a + \frac{b_1 s + b_0}{s + a_0} x_r.$$  \hspace{1cm} (3.24)

The final control force $f_a$ is derived by applying the switching control to the sliding surface variable according to the conventional sliding mode control.

$$f_a = -f_{\text{max}} \text{sgn}(\sigma),$$  \hspace{1cm} (3.25)
where $f_{\text{max}}$ is the maximal control effort that can be provided by the system and the function $\text{sgn}(\sigma)$ indicates the sign of $\sigma$.

$$\text{sgn}(\sigma) = \begin{cases} 
1 & \sigma > 0, \\
0 & \sigma = 0, \\
-1 & \sigma < 0.
\end{cases}$$  \hspace{1cm} (3.26)

By applying the switching control together with the boundary layer control which helps to remove the 'chatter', the system dynamics can be kept on the sliding surface, which indicates $\sigma = 0$. Substitute the sliding surface design (3.24), $x_r = x_a - x_b$, and $v_a = \dot{x}_a$ into $\sigma = 0$, we have

$$\dot{x}_a + \frac{b_1 s + b_0}{s + a_0} (x_a - x_b) = 0.$$  \hspace{1cm} (3.27)

Apply the Laplace transform, we have

$$\frac{X_a}{X_b} = \frac{b_1 s + b_0}{s^2 + (a_0 + b_1) s + b_0},$$  \hspace{1cm} (3.28)

where $X_a$ and $X_b$ are the Laplace transforms of $x_a$ and $x_b$, respectively. Note that $\frac{X_a}{X_b}$ is indeed the transmissibility. In this way, the sliding surface design (3.24) is physically interpreted to the vibration isolation performance (3.28). Let $b_1 = 0$, (3.28) yields

$$\frac{X_a}{X_b} = \frac{b_0}{s^2 + a_0 s + b_0},$$  \hspace{1cm} (3.29)

which is the exactly the skyhook transmissibility (the practical transmissibility shown in Fig. 3.2).

However, the ideal skyhook performance can only be realized at the frequency range where both sensors and actuators function close to the ideal devices. Assume that the payload absolute velocity is measured by a geophone. Due to the geophone dynamics (3.8), the achieved transmissibility on the sliding surface becomes

$$\frac{X_a}{X_b} = \frac{b_0(s^2 + 2\omega_v \xi_v + \omega_v^2)}{s^4 + a_0 s^3 + b_0(s^2 + 2\omega_v \xi_v + \omega_v^2)}.$$  \hspace{1cm} (3.30)

It is derived in [128] that the achieved transmissibility (3.30) is stable if

$$\frac{\omega_d}{\omega_v} > \frac{\xi_d}{\xi_v} + \frac{\xi_v}{\xi_d},$$  \hspace{1cm} (3.31)

where $\omega_d$ and $\xi_d$ are the resonant frequency and damping ratio of the ideal skyhook transmissibility ($\omega_d = \sqrt{b_0}$ and $\xi_d = a_0/(2\sqrt{b_0})$). A necessary condition to keep $T_d$ stable is $\omega_d > 2\omega_v$. Ideal skyhook transmissibility can be achieved only if $\omega_d \gg \omega_v$. This leads to the conclusion that the vibration isolation performance at low frequencies is limited by the geophone dynamics using this sliding surface design.

The sliding surface control in [128] provides a physical connection between the sliding surface and the transmissibility. However, the assumption of ideal sensors and ideal actuators is not always fulfilled in practice. Geophone dynamics induce unnecessary limitations.
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on the transmissibility using this sliding surface design. The switching control inherited from the conventional sliding mode control is not desired by precision machines because it demands extremely high actuator bandwidth and it is nonlinear which is usually more complex for performance prediction.

3.5 Sliding surface control with payload absolute motion

3.5.1 Introduction

In [128] the sliding surface, constructed by both relative displacement and payload absolute motion, is physically connected to the transmissibility. In theory, the performance limitation induced by sensor dynamics as described in [128] can be removed as the sensor dynamics can always be compensated actively by control design. Existence of the sensor noises, however, is the limiting factor to further improve the transmissibility and compliance as the absolute sensitivity can not be improved simultaneously.

By including the sensor dynamics modeled in Section 3.2.1 in the sliding surface design, the sliding surface control introduced in [128] can be extended and generalized as a two-step vibration isolation control design approach:

1. Construct a sliding surface using both relative displacement and one of the absolute motion signals which is physically connected to desired transmissibility and sensitivities;
2. Design a linear regulator such that system dynamics converge to and stay on the sliding surface, to realize the desired performance criteria. Such a linear regulator will also improve the compliance, which will be explained later in this section.

This vibration isolation control approach is referred to as sliding surface control to distinguish it from the conventional sliding mode control and the work described in [128].

The performance criteria determined by the sliding surface are referred to as the designed performance criteria, which are the designed transmissibility $T_d$ and the two designed sensitivity functions $R_d$ and $S_d$. The sliding surface design can be customized according to the corresponding measurement scheme and the corresponding sensor dynamics. Since the boundary layer control, which is commonly used to avoid the ’chatter’, is within the linear control framework [122], a linear regulator is applied instead of the switching control. In this way, the four closed-loop performance criteria can be theoretically calculated.

3.5.2 Sliding surface control methodology

The sliding surface control is described in this subsection based on the 1-DoF suspension system with the measurement scheme of relative displacement and payload absolute motion.

If the payload absolute motion of the 1-DoF suspension system is measured, the block diagram of sliding surface control is provided in Fig. 3.6. The dashed rectangular shows the block diagram for the passive system. The two transfer functions $\Lambda_1$ and $\Lambda_2$ are used to shape the sliding surface in frequency domain. The block $R$ is the regulator designed based on $\Lambda_1$, $\Lambda_2$ and the passive system to keep the system dynamics on the sliding surface. Same
3.5. Sliding surface control with payload absolute motion

Figure 3.6: Control diagram of a 1-DoF suspension system with the measurement scheme of relative displacement and payload absolute motion using the generalized sliding surface control.

as Fig. 3.4, \( S \) represents the sensor dynamics. For payload absolute velocity measurement, \( S = sG_v \). For payload absolute acceleration measurement, \( S = s^2 \).

The sliding surface equation \( \sigma = 0 \) is equivalent to

\[
\Lambda_1 (x_r - n_x) + \Lambda_2 (Sx_a - n_a) = 0. \tag{3.32}
\]

The reference \( r \) is not taken into account because it is a constant and it does not influence the system dynamics. Substitute \( x_r = x_a - x_b \) and apply the Laplace transform, (3.32) yields

\[
\frac{X_a}{X_b} = \frac{\Lambda_1}{\Lambda_1 + \Lambda_2 S} + \frac{\Lambda_1 N_x}{\Lambda_1 + \Lambda_2 S X_b} + \frac{\Lambda_2 N_a}{\Lambda_1 + \Lambda_2 S X_b}, \tag{3.33}
\]

where \( X_a, X_b, N_x, \) and \( N_a \) are the Laplace transforms of the signals \( x_a, x_b, n_x, \) and \( n_a, \) respectively. Therefore, the designed transmissibility \( T_d \), the two designed sensitivity functions \( R_d \) and \( S_d \) are derived as

\[
T_d = R_d = \frac{\Lambda_1}{\Lambda_1 + \Lambda_2 S}, \quad S_d = \frac{\Lambda_2}{\Lambda_1 + \Lambda_2 S}. \tag{3.34}
\]

According to (3.34), \( S_d \) and \( T_d \) are related by

\[
T_d + S S_d = 1. \tag{3.35}
\]

Therefore, to simultaneously improve, or reduce the magnitudes of, both \( S_d \) and \( T_d \), it is impossible with predefined absolute-motion-sensor dynamics. The sliding surface design has to make a trade-off between \( S_d \) and \( T_d \). The upper bound of \( T_d \) magnitude, \(|T_d|\), can be derived as

\[
|T_d| \leq |T_d| = |T_d| + |R_d| \left| \frac{N_x}{X_b} \right| + |S_d| \left| \frac{N_a}{X_b} \right|. \tag{3.36}
\]
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The sliding surface, the sensor models, and the original plant model can be regarded as a new SISO system, illustrated as the gray area in Fig. 3.6. The regulator is designed to regulate this new SISO system. In other words, the objective of regulator design is to confine the system dynamics to the sliding surface \((\sigma = 0)\). A simple PID controller could be sufficient. More advanced control techniques, like \(H_\infty\) control or LQG can also apply. As long as the regulator is linear, the closed-loop performances can be calculated according to Fig. 3.6. The closed-loop transmissibility and compliance are calculated as

\[
T_c = \frac{R\Lambda_1 C_p + T_p}{1 + R C_p (\Lambda_1 + \Lambda_2 S)} = \frac{\Lambda_1 + T_p (RC_p)^{-1}}{(RC_p)^{-1} + (\Lambda_1 + \Lambda_2 S)},
\]

(3.37)

\[
C_c = \frac{1 + R C_p (\Lambda_1 + \Lambda_2 S)}{R} = \frac{1}{(RC_p)^{-1} + (\Lambda_1 + \Lambda_2 S)}.
\]

(3.38)

The two closed-loop sensitivity functions are calculated as

\[
S_c = \frac{R\Lambda_2 C_p}{1 + R C_p (\Lambda_1 + \Lambda_2 S)} = \frac{\Lambda_2}{(RC_p)^{-1} + (\Lambda_1 + \Lambda_2 S)},
\]

(3.39)

\[
R_c = \frac{R\Lambda_1 C_p}{1 + R C_p (\Lambda_1 + \Lambda_2 S)} = \frac{\Lambda_1}{(RC_p)^{-1} + (\Lambda_1 + \Lambda_2 S)}.
\]

(3.40)

The upper bound of the closed-loop transmissibility, \(|T_c|\), is calculated as

\[
|T_c| \leq |T_d| = |T_c| + |R_c| \left| \frac{N_a}{X_b} \right| + |S_c| \left| \frac{N_d}{X_b} \right| + |C_c| \left| \frac{F_d}{X_b} \right|.
\]

(3.41)

If the loop-gain is so high that the approximations

\[
\Lambda_1 + T_p (RC_p)^{-1} \approx \Lambda_1,
\]

(3.42a)

\[
(RC_p)^{-1} + (\Lambda_1 + \Lambda_2 S) \approx \Lambda_1 + \Lambda_2 S,
\]

(3.42b)

may be assumed, we have \(T_c = T_d\), \(R_c = R_d\), and \(S_c = S_d\). Meanwhile, the higher the loop-gain, the lower \(|C_c|\), which is exactly the goal of control design. Another benefit of increasing loop-gain is that \(|T_c|\) converges to \(|T_d|\). Therefore, a high-gain \(R\) is desired. In fact, the regulator design has no difference from a common SISO design, wherein, the difference between the reference and the plant output is suppressed by high control gain at frequencies lower than control bandwidth. As control bandwidth is always limited, the approximation (3.42) does not hold for frequencies above or close to the bandwidth. Therefore, the higher the control bandwidth, the better achievable transmissibility and compliance.

According to (3.32), the sliding surface design is dependent on the sensor dynamics. There are two types of payload absolute motion signal that can be measured by commercially available sensors: velocity and acceleration. The sliding surface design is studied separately for these two possibilities in the following two subsections.

3.5.3 Sliding surface design with geophone

The only commercially available absolute velocity sensor is geophone [1]. The model of such a sensor is described in Section 3.2.1. The sliding surface design is studied in this
subsection assuming that the geophone is used: $S = sG_v$. In this case, the measurement
scheme is exactly the same as the work of Zuo and Slotine [128]. It will be shown that the
performance limit in [128] can be eliminated by the improved sliding surface design.

Let $N_i$ and $D_i$ denote the numerators and denominators of $\Lambda_i, \forall i \in \{1, 2\}$, respectively.
Substituting $\Lambda = \frac{N_1}{D_1} \Lambda_2 = \frac{N_2}{D_2}$, and $D_1 = (s^2 + 2\omega_0 \xi_0 s + \omega_0^2)D_2$ into (3.34) yields

$$T_d = \frac{N_1}{N_1 + N_2 s^3}, \quad S_d = \frac{N_2(s^2 + 2\omega_0 \xi_0 s + \omega_0^2)}{N_1 + N_2 s^3}. \quad (3.43)$$

To achieve $S_d(0) = 0$, the constant term of the polynomial $N_2$ should be zero. Let $N_2 = N'_2 s$, (3.43) becomes

$$T_d = \frac{N_1}{N_1 + N'_2 s^4}, \quad S_d = \frac{N'_2 s(s^2 + 2\omega_0 \xi_0 s + \omega_0^2)}{N_1 + N'_2 s^4}. \quad (3.44)$$

$T_d$ can be designed by the choice of $N_1$ and $N'_2$. To achieve the -40 dB/dec decreasing rate
of $|T_d|$ at high frequencies, the denominator order should be the numerator order plus two.
If the order of $T_d$ is four (this is the lowest), $N'_2$ has to be a constant and the order of $N_1$ has
three possibilities: zero, one or two. In this case, $T_d$ has the possible forms of

$$T_d = \frac{a_0}{a_4 s^4 + a_0}, \text{ or } T_d = \frac{a_1 s + a_0}{a_4 s^4 + a_1 s + a_0},$$

or $T_d = \frac{a_2 s^2 + a_1 s + a_0}{a_4 s^4 + a_2 s^2 + a_1 s + a_0}$.

To make $T_d$ stable, proper sets of constants $\{a_0, a_4\}$ or $\{a_0, a_1, a_4\}$ or $\{a_0, a_1, a_2, a_4\}$ have
to be found, which are all impossible according to the Routh-Hurwitz criterion.

If the order of $T_d$ is five, the two numerators can be designed as $N_1 = a_3 s^3 + a_2 s^2 + a_1 s + a_0$ and $N'_2 = a_5 s + a_4$ so that $T_d$ has the form of

$$T_d = \frac{a_3 s^3 + a_2 s^2 + a_1 s + a_0}{a_5 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}. \quad (3.45)$$

The five poles of $T_d$ can be selected according to criteria of stability and low cut-off fre-
quency. Assuming that $a_5 = 1$, the five constants $a_i, \forall i \in \{0, 1, 2, 3, 4\}$ are uniquely deter-
dined by the five poles without losing generality. There are also no theoretical performance
limit induced by the geophone natural frequency. Substitute $N_1$ and $N'_2$ into (3.44), $S_d$ has the form of

$$S_d = \frac{(s^2 + a_4 s)(s^2 + 2\omega_0 \xi_0 s + \omega_0^2)}{s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}. \quad (3.46)$$

In this way, both $T_d$ and $S_d$ fulfill the general requirements. The design of $D_1$ and $D_2$ is
to make $\Lambda_1$ and $\Lambda_2$ stable and to simplify the regulator design. If we continue increasing
the order of $T_d$, higher decreasing rate of $|T_d|$ or lower $|S_d|$ could be achieved. The price
would be the increased order of the controller.

Although the geophone dynamics could be compensated in the improved sliding sur-
face design, a geophone with higher natural frequency increases the magnitude of $S_d$ at
low frequencies. Therefore, a geophone with low natural frequency is preferred for low-frequency vibration isolation. For commercially available geophones [1, 19], the natural frequency is as low as 0.9 Hz - 1 Hz. The cost of further reducing the natural frequency of a geophone would be greatly increased.

To further illustrate the effectiveness of the improved sliding surface design with respect to the sliding surface design in [128], numerical examples are provided to make a comparison. The parameters of geophone dynamics are assumed to be $\omega_v = 2\pi \text{ rad/s (1 Hz)}$ and $\xi_v = 0.7$ so that geophone dynamics are calculated as

$$G_v = \frac{s^2}{s^2 + 8.796s + 39.48}.$$  

(3.47)

We also assume that $|\frac{N_v(\omega)}{X_v(\omega)}| = 0.2$ and $|\frac{\dot{N}_v(\omega)}{\ddot{X}_v(\omega)}| = 0.1, \forall \omega \geq 0$.

According to (3.31), $\omega_d$ and $\xi_d$ are chosen as $\omega_d = 8\pi \text{ rad/s (4 Hz)}$ and $\xi_d = 0.7$. The designed sliding surface ignoring the geophone dynamics according to (3.24) is

$$b_1 = 0, \quad b_0 = 64\pi^2, \quad a_0 = 11.2\pi.$$  

(3.48)

The desired transmissibility as in (3.29) is

$$T_d = \frac{631.7}{s^2 + 35.19s + 631.7},$$  

(3.49)

which is the ideal skyhook transmissibility. The desired transmissibility calculated by (3.29) and the real designed transmissibility calculated with the geophone dynamics by (3.30) are compared in Fig. 3.7. Under assumption $|\frac{N_v(\omega)}{X_v(\omega)}| = 0.2$ and $|\frac{\dot{N}_v(\omega)}{\ddot{X}_v(\omega)}| = 0.1, \forall \omega \geq 0$, the upper bound of the designed transmissibility is also plotted. It shows that the desired transmissibility which ignores geophone dynamics is quite different from the real designed transmissibility which has a resonance peak of 6.3 dB at about 2 Hz. The designed transmissibility upper bound is reasonably close to the real designed transmissibility which indicates that the influence of the sensor noises is not significant.

Including the geophone dynamics (3.8) into the sliding surface design, the designed transmissibility $T_d$ would have higher order. If the order of $T_d$ is five, the five poles of the designed transmissibility are chosen as $[-2 + 2j, -2 - 2j, -2 + 2j, -2 - 2j, -4]$. The designed sliding surface is

$$\Lambda_1 = \frac{64(s + 1.453)}{s^2 + 8.796s + 39.48}, \quad \Lambda_2 = \frac{s^2 + 12s}{s^2 + 1.547s + 2.752}.$$  

(3.50)

The designed transmissibility is

$$T_d = \frac{64(s + 1.453)(s^2 + 1.547s + 2.752)}{(s + 4)(s^2 + 4s + 8)^2}.$$  

(3.51)

The designed transmissibility and its upper bound under the assumption $|\frac{N_v(\omega)}{X_v(\omega)}| = 0.2$ and $|\frac{\dot{N}_v(\omega)}{\ddot{X}_v(\omega)}| = 0.1, \forall \omega \geq 0$ are plotted in Fig. 3.8. It shows that the designed transmissibility
3.5. Sliding surface control with payload absolute motion

Figure 3.7: Designed transmissibility and its upper bound using previous sliding surface design assuming that payload absolute velocity is measured.

has a resonance peak of 7.4 dB at about 0.5 Hz. The difference between the designed transmissibility and its upper bound around the resonant frequency is much larger than that of higher frequencies. It indicates that the influence of the sensor noises is quite significant around the resonant frequency. Comparing with the real designed transmissibility in Fig. 3.7, the designed transmissibility has much less cut-off frequency and the resonance peak is slightly larger. We can conclude that the sliding surface design taking the geophone dynamics into account eliminates the performance limit induced by the stability condition (3.31).

3.5.4 Sliding surface design with accelerometer

If an accelerometer is used as the absolute sensor instead of a geophone, the block $S$ in Fig. 3.6 is $S = s^2$. Using the similar procedure as in the last subsection, design of $T_d$ with the lowest order has the following form

$$T_d = \frac{a_2s^2 + a_1s + a_0}{a_4s^4 + a_3s^3 + a_2s^2 + a_1s + a_0},$$

(3.52)

where the four constants $a_i, \forall i \in \{0, 1, 2, 3\}$ are uniquely determined by selecting four stable poles for $T_d$ assuming that $a_4 = 1$. The corresponding $S_d$ has the form of

$$S_d = \frac{a_4s^2 + a_3s}{a_4s^4 + a_3s^3 + a_2s^2 + a_1s + a_0},$$

(3.53)

In this way, both $T_d$ and $S_d$ fulfill the general requirements. The main concerns of the denominator design for $\Lambda_1$ and $\Lambda_2$ are stability and simplification of the regulator design.
Example designs are

\[
\begin{align*}
\Lambda_1 &= \frac{a_2 s^2 + a_1 s + a_0}{a_4 s^2 + a_3 s}, \quad \text{or} \\
\Lambda_2 &= 1
\end{align*}
\]

The same trade-off between the controller order and the performance as the last subsection holds here.

A numerical example is also provided. The same sensor noise assumption is made: \( |N_a(\omega)| = 0.2 \) and \( |N_x(\omega)| = 0.1, \forall \omega \geq 0 \). If the order of \( T_d \) is four, the four poles of the designed transmissibility are chosen as \([-2 + 2j, -2 - 2j, -2.5 + 2.5j, -2.5 - 2.5j]\). The designed sliding surface is

\[
\Lambda_1 = 1, \quad \Lambda_2 = \frac{0.02469(s + 9)}{s^2 + 2.222s + 2.469}.
\]

The designed transmissibility is

\[
T_d = \frac{40.5(s^2 + 2.222s + 2.469)}{(s^2 + 4s + 8)(s^2 + 5s + 12.5)}.
\]

The designed transmissibility and its upper bound under the assumption \( |N_a(\omega)| = 0.2 \) and \( |N_x(\omega)| = 0.1, \forall \omega \geq 0 \) are plotted in Fig. 3.9. It shows that the designed transmissibility has a resonance peak of 6.2 dB at about 0.5 Hz. The designed transmissibility and its upper bound are reasonably close, which indicates that the influence of the sensor noises is not significant.

Comparing Fig. 3.9 and Fig. 3.8, the resonance peak and cut-off frequency of the designed transmissibility using accelerometer are both lower than that of the geophone. The dynamics of the geophone play a part in the higher resonance peak and higher cut-off frequency. Furthermore, the order of the designed transmissibility using accelerometer is lower than that of using geophone. For these reasons, the accelerometer is preferred rather than the geophone.

### 3.6 Sliding surface control with base absolute motion

The sliding surface control described in the last section is generalized from the work in [128]. It is also applicable to the measurement scheme of relative displacement and base absolute motion. This measurement scheme has an obvious disadvantage, proved in Section 3.3. However, it could be the only choice because it is possible that the measurement scheme with payload absolute motion is not applicable. The absolute sensors which are suitable for low-frequency measurement usually have large mass and large dimensions, which could cause problems in mechanical design. Although it is unlikely that the base absolute motion is applied to SEMIS, the sliding surface control is still studied for the measurement scheme of relative displacement and base absolute motion in this section to make a complete description of sliding surface control. Numerical examples will be provided as a support to the conclusions of Section 3.3.
3.6. Sliding surface control with base absolute motion

Figure 3.8: Designed transmissibility and its upper bound using the improved sliding surface design assuming that payload absolute velocity is measured.

Figure 3.9: Designed transmissibility and its upper bound using the payload acceleration measurement scheme.
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3.6.1 Sliding surface control methodology

If the absolute motion of the base structure is measured, the sliding surface control diagram is shown in Fig. 3.10.

The sliding surface equation \( \sigma = 0 \) is equivalent to

\[
\Lambda_1 (x_r - n_x) + \Lambda_2 (Sx_b - n_a) = 0. \tag{3.57}
\]

Substitute \( x_r = x_a - x_b \) and apply the Laplace transform, (3.57) yields

\[
\frac{X_a}{X_b} = \frac{\Lambda_1 - \Lambda_2 S}{\Lambda_1} + \frac{N_x}{X_b} + \frac{\Lambda_2 N_a}{\Lambda_1 X_b}, \tag{3.58}
\]

where \( X_a, X_b, N_x, \) and \( N_a \) are the Laplace transforms of the signals \( x_a, x_b, n_x, \) and \( n_a, \) respectively. Therefore, the designed transmissibility \( T_d \) and the two designed sensitivity functions \( R_d \) and \( S_d \) are derived as

\[
T_d = \frac{\Lambda_1 - \Lambda_2 S}{\Lambda_1}, \quad R_d = 1, \quad S_d = \frac{\Lambda_2}{\Lambda_1}. \tag{3.59}
\]

According to (3.59), \( S_d \) and \( T_d \) are related by

\[
T_d + S_d = 1. \tag{3.60}
\]

Therefore, the performance conflict in (3.35) still holds. The upper bound of the designed transmissibility, \( |T_d| \), has the same expression as in (3.36):

\[
|T_d| \leq |T_d| = |T_d| + |R_d| \left| \frac{N_x}{X_b} \right| + |S_d| \left| \frac{N_a}{X_b} \right|. \tag{3.61}
\]
The closed-loop transmissibility and compliance are calculated according to the control diagram in Fig. 3.10.

\[
T_c = \frac{RC_p(\Lambda_1 - \Lambda_2S) + T_p}{1 + RC_p\Lambda_1} = \frac{\Lambda_1 - \Lambda_2S + T_p(RC_p)^{-1}}{(RC_p)^{-1} + \Lambda_1},
\]

(3.62)

\[
C_c = \frac{C_p}{1 + RC_p\Lambda_1} = \frac{R^{-1}}{(RC_p)^{-1} + \Lambda_1}.
\]

(3.63)

The two closed-loop sensitivity functions are calculated as

\[
S_c = \frac{RA_2C_p}{1 + RC_p\Lambda_1} = \frac{\Lambda_2}{(RC_p)^{-1} + \Lambda_1},
\]

(3.64)

\[
R_c = \frac{RA_1C_p}{1 + RC_p\Lambda_1} = \frac{\Lambda_1}{(RC_p)^{-1} + \Lambda_1}.
\]

(3.65)

The upper bound of the closed-loop transmissibility, \(|T_c|\), is calculated as

\[
|T_c| \leq |\overline{T}_c| = |T_c| + |R_c| \left| \frac{N_x}{X_b} \right| + |S_c| \left| \frac{N_d}{X_b} \right| + |C_c| \left| \frac{F_d}{X_b} \right|.
\]

(3.66)

If the open loop gain is so high that the approximations

\[
\Lambda_1 - \Lambda_2S + T_p(RC_p)^{-1} \approx \Lambda_1 - \Lambda_2S,
\]

(3.67a)

\[
(RC_p)^{-1} + \Lambda_1 \approx \Lambda_1,
\]

(3.67b)

are feasible, we have \(T_c = T_d\), \(R_c = R_d\), and \(S_c = S_d\). Meanwhile, the higher loop-gain, the lower \(|C_c|\), which is exactly the goal of control design. Another benefit of reducing \(|C_c|\) is that \(|T_c|\) converges to \(|\overline{T}_d|\). Therefore, a high-gain \(R\) is desired. The principle of regulator design in this case is the same as the regulator design described in Section 3.5.2.

### 3.6.2 Sliding surface control with geophone

Same as Section 3.5.2, the sliding surface design is studied separately for the two sensor choices: geophone and accelerometer. This subsection analyzes the sliding surface design assuming a geophone is used as the absolute sensor. The model of this geophone is described in Section 3.2.1. The block diagram in control diagram Fig. 3.10 is given by \(S = sG_y\).

The lowest order of \(T_d\) is still five. The reasoning is the similar to that in Section 3.5.2.

\[
T_d = \frac{a_3s^3 + a_2s^2 + a_1s + a_0}{a_5s^5 + a_4s^4 + a_3s^3 + a_2s^2 + a_1s + a_0}.
\]

(3.68)

Assuming that \(a_5 = 1\), the four constants \(a_i, \forall i \in \{0, 1, 2, 3, 4\}\) are uniquely determined by selecting five stable poles for \(T_d\). The corresponding \(S_d\) has the form of

\[
S_d = \frac{(a_5s^2 + a_4s)(s^2 + 2\omega_\xi s + \omega_0^2)}{a_5s^5 + a_4s^4 + a_3s^3 + a_2s^2 + a_1s + a_0}.
\]

(3.69)

In this way, both \(T_d\) and \(S_d\) fulfill the general requirements. The main concerns of the denominator design for \(\Lambda_1\) and \(\Lambda_2\) are stability and simplification of the regulator design.
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Same as the Section 3.5.2, there is also a trade-off between the controller order and the performance.

The parameters of the geophone dynamics are also assumed to be \( \omega_r = 2\pi \text{ rad/s} \) (1 Hz) and \( \xi_v = 0.7 \) so that the geophone dynamics is calculated as

\[
G_v = \frac{s^2}{s^2 + 8.796s + 39.48}.
\]  

(3.70)

We assume that \( \left| \frac{N_v(\omega)}{X_v(\omega)} \right| = 0.2, \forall \omega \geq 0 \). Since the displacement sensor noise is critical to the transmissibility magnitude at high frequencies, the designed transmissibility upper bound is calculated based on both assumptions on the displacement sensor noises:

\[
\left| \frac{N_v(\omega)}{X_v(\omega)} \right| = 0.1, \forall \omega \geq 0 \quad \text{and} \quad \left| \frac{N_v(\omega)}{X_v(\omega)} \right| = 0.01, \forall \omega \geq 0.
\]

If the order of \( T_d \) is five, the five poles of the designed transmissibility are chosen as \([-2 + 2j, -2 - 2j, -2 + 2j, -2 - 2j, -4]\). The designed sliding surface is

\[
\Lambda_1 = \frac{s^2 + 4s + 8}{s^2 + 8.796s + 39.48}, \quad \Lambda_2 = \frac{s^2 + 12s}{(s + 4)(s^2 + 4s + 8)}. \]  

(3.71)

The designed transmissibility is

\[
T_d = \frac{64(s + 1.453)(s^2 + 1.547s + 2.752)}{(s + 4)(s^2 + 4s + 8)^2},
\]  

(3.72)

which is exactly the same with the one with payload velocity feedback. The designed transmissibility and its upper bounds under two different assumptions are plotted in Fig. 3.11.

It shows that the designed transmissibility has a resonance peak of 7.4 dB at about 0.5 Hz. The magnitude of \( |T_d| \) assuming \( \left| \frac{N_v(\omega)}{X_v(\omega)} \right| = 0.1 \), converges to -20 dB at high frequencies and the magnitude of \( |T_d| \) assuming \( \left| \frac{N_v(\omega)}{X_v(\omega)} \right| = 0.01 \), converges to -40 dB at high frequencies.

It is consistent to the conclusion of Section 3.3 that the realizable transmissibility at high frequencies is highly dependent on the displacement sensor resolution.

3.6.3 Sliding surface design with accelerometer

If an accelerometer is used as the absolute sensor instead of a geophone, the block \( S \) in Fig. 3.10 is \( S = s^2 \). Using the similar procedure as in Section 3.5.2, design of \( T_d \) with the lowest order has the following form

\[
T_d = \frac{a_2s^2 + a_1s + a_0}{a_4s^4 + a_3s^3 + a_2s^2 + a_1s + a_0},
\]  

(3.73)

where the four constants \( a_i, \forall i \in \{0, 1, 2, 3\} \) are uniquely determined by selecting four stable poles for \( T_d \) assuming that \( a_4 = 1 \). The corresponding \( S_d \) has the form of

\[
S_d = \frac{a_4s^2 + a_3s}{a_4s^4 + a_3s^3 + a_2s^2 + a_1s + a_0}.
\]  

(3.74)

In this way, both \( T_d \) and \( S_d \) fulfill the general requirements. The main concerns of the denominator design for \( \Lambda_1 \) and \( \Lambda_2 \) are stability and simplification of the regulator design.

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3.7. Sliding surface optimization

The same trade-off between the controller order and the performance as the last subsection holds here.

A numerical example is provided to further illustrate the sliding surface design. We assume that \( |N_x(\omega)| = 0.2, \forall \omega \geq 0 \). Since the displacement sensor noise is critical to the transmissibility magnitude at high frequencies, the designed transmissibility upper bound is calculated based on both assumptions on the displacement sensor noises:

\[
|N_x(\omega)| = 0.1, \forall \omega \geq 0 \quad \text{and} \quad |N_x(\omega)| = 0.01, \forall \omega \geq 0.
\]

If the order of \( T_d \) is four, the four poles of the designed transmissibility are chosen as \([-2+2j, -2-2j, -2.5+2.5j, -2.5-2.5j]\). The designed sliding surface is

\[
\Lambda_1 = \frac{s^2 + 4s + 8}{s^2 + 8.796s + 39.48}, \quad \Lambda_2 = \frac{s^2 + 12s}{(s+4)(s^2 + 4s + 8)}.
\]

(3.75)

The designed transmissibility is

\[
T_d = \frac{40.5(s^2 + 2.222s + 2.469)}{(s^2 + 4s + 8)(s^2 + 5s + 12.5)},
\]

(3.76)

which is exactly the same as the one with payload acceleration feedback. The designed transmissibility and its upper bounds under two different assumptions are plotted in Fig. 3.12. It shows that the designed transmissibility has a resonance peak of 6.2 dB at about 0.5 Hz. The magnitude of \(|T_d|\) assuming \( |N_x(\omega)| = 0.1 \), converges to -20 dB at high frequencies and the magnitude of \(|T_d|\) assuming \( |N_x(\omega)| = 0.01 \), converges to -40 dB at high frequencies. It is consistent to the conclusion of Section 3.3 that the realizable transmissibility at high frequencies is highly dependent on the displacement sensor resolution.

3.7 Sliding surface optimization

The sliding surface design described in the previous sections searches for a good compromise between the designed transmissibility and the designed sensitivity by heuristic pole placement. The final result could be good but the process could be very cumbersome and experience-demanding as it is still heuristic. This section defines the optimized sliding surface and proposes a numerical process to find the optimal parameters of the designed performances. The measurement scheme is assumed to be the combination of relative displacement and payload absolute motion.

3.7.1 Introduction

We assume that the base absolute motion and the absolute sensor noise spectrum are all measurable, the upper bound of the designed transmissibility can be calculated for a set of chosen poles. Assume that the cut-off frequency of \(|T_d|\), denoted by \( \omega_c \), has a required upper-bound, \( \omega_1 \). Assume that \( \omega_i \) and \( \epsilon_i, \forall i \in \{0,1,2,...,n\} \) are predefined constants that satisfy:

- \( \omega_0 < \omega_c \);
- \( \omega_1 = \omega_c \);
- \( \omega_i > \omega_c \ \forall \ i \in \{2,3,...,n\} \);
Figure 3.11: Designed transmissibility and its upper bound using the base velocity measurement scheme.

Figure 3.12: Designed transmissibility and its upper bound using the base acceleration measurement scheme.
3.7. Sliding surface optimization

- $\varepsilon_0 > 1$;
- $\varepsilon_1 = 1$;
- $\varepsilon_i < 1 \forall i \in \{2, ..., n\}$.

Let $\hat{a}$ denote a set of controller parameters to be designed. The transmissibility optimization is to find a set $\hat{a}$ which minimizes the resonance peak of the transmissibility upper bound under constraints.

$$\hat{a} = \min_a \sup_{\omega} |T_d(\omega)|,$$

under the constraints of

- $|T_d(\omega)| \leq \varepsilon_0, \forall \omega \leq \omega_0$;
- $|T_d(\omega)| \leq \varepsilon_i, \forall i \in \{1, 2, ..., n\}$.

Note that the above constraints are common industrial requirements for a particular suspension system. The detailed optimization processes are described for each measurement scheme.

3.7.2 Payload acceleration feedback

The Power Spectrum Density (PSD) of the sensor noises ($n_x$ and $n_a$) is usually provided in the specifications. If not, the PSD can also be experimentally measured [10]. Similarly, the PSD of the base displacement can also be experimentally measured. The PSD ratios of the sensor noises over the base-frame displacement vary with the frequency. These variations can be described by two functions.

$$G_x(\omega) = \frac{N_x(\omega)}{X_G(\omega)}, \quad G_a(\omega) = \frac{N_a(\omega)}{X_G(\omega)}.$$

(3.78)

Note that both $G_x(\omega)$ and $G_a(\omega)$ can be either continuous functions of $\omega$ or discontinuous look-up tables of $\omega$. (3.36) can be reformed to

$$|T_d(\omega)| = |T_d(\omega)| (1 + |G_x(\omega)|) + |S_d(\omega)| |G_a(\omega)|.$$

(3.79)

There are two ways to parameterize the cost function $|T_d(\omega)|$:

1. Parameterize $|T_d(\omega)|$ using the denominator poles;
2. Parameterize $|T_d(\omega)|$ using the denominator coefficients.

Since the numerical optimization program in Matlab can not guarantee the global optimum, both parameterization are applied to double confirm the results of the optimization processes.

Pole parameterization

Assume that $T_d$ takes the form of (3.52), there are three possibilities of the four poles. Assume that $r_i < 0, \forall i \in \{1, 2, 3, 4\}$ are independent real variables, the three possible combinations of the four stable poles are:

- Four real poles ($r_i, \forall i \in \{1, 2, 3, 4\}$);
- Two real poles ($r_1$ & $r_2$) and a conjugate pair ($r_3 \pm r_4 j$);
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- Two conjugate pairs \((r_1 \pm r_2 j)\) and \((r_3 \pm r_4 j)\).

In each case, \(|T_d(\omega)|\) can be numerically calculated according to (3.79) and (3.34). The transmissibility optimization is formulated as follows.

To find the set of four negative variables \(r_i, \forall i \in \{1, 2, 3, 4\}\) which minimizes \(\sup |T_d(\omega)|\) under constraints of

- \(|T_d(\omega)| \leq \varepsilon_0, \forall \omega \leq \omega_0\);
- \(|T_d(\omega_i)| \leq \varepsilon_i, \forall i \in \{1, 2, ..., n\}\).

The above optimization problem can be solved numerically using the Matlab Optimization Toolbox [75] for each case of pole combinations. The final optimal solution is the one with lowest \(\sup |T_d(\omega)|\).

Coefficient parameterization

Assume that \(T_d\) takes the form of (3.52), the constants \(a_i, \forall i \in \{0, 1, 2, 3\}\) are used as parameters and the constant \(a_4\) is fixed as \(a_4 = 1\). The transmissibility optimization is formulated as follows.

To find the set of four positive variables \(a_i, \forall i \in \{0, 1, 2, 3\}\) which minimizes \(\sup |T_d(\omega)|\) under constraints of

- \(|T_d(\omega)| \leq \varepsilon_0, \forall \omega \leq \omega_0\);
- \(|T_d(\omega_i)| \leq \varepsilon_i, \forall i \in \{1, 2, ..., n\}\);
- \(a_2 - a_1/a_3 > 0\);
- \(a_1 - a_3 a_0/(a_2 - a_1/a_3) > 0\).

The last three constraints are used to keep \(T_d\) stable. They are derived using the Routh-Hurwitz criterion.

Numerical example

A simple numerical example of the optimization process is given. Assume that

- For mode \(i\), \(|G_x(\omega)| = 0.1\) and \(|G_a(\omega)| = 0.2\);
- \(\omega_0 = 0.01\) Hz, \(\omega_1 = 0.5\) Hz, \(\omega_2 = 10\) Hz;
- \(\varepsilon_0 = 1.1885\) (1.5 dB), \(\varepsilon_1 = 1\) (0 dB), \(\varepsilon_2 = 3.162 \times 10^{-3}\) (-50 dB).

Using the pole parameterization, the initial values are set as \(r_i = -1, \forall k \in \{1, 2, 3, 4\}\).

Three results are obtained for each combination of the four poles:

- Four real poles \((r_i = -1.3648, \forall i \in \{1, 2, 3, 4\})\);
- Two real poles \((r_1 = r_2 = -0.2609)\) and a conjugate pair \((-2.0004 \pm 2.2374 j)\);
- Two conjugate pairs \((r_1 = r_3 = -1.3648\) and \(r_2 = r_4 = 0)\).

Since the results of four real poles and two conjugate pairs converge, there are only two different results left. The corresponding \(|T_d|\) curves are plotted in Fig. 3.13. The second pole combination (two real poles and one conjugate pair) gives the lowest peak of \(|T_d|\) (3.1797 dB) so that it is the optimized solution.
3.7. Sliding surface optimization

Figure 3.13: Optimized $|\mathcal{T}_d|$ using the pole parameterization if an acceleration sensor is applied.

Figure 3.14: Optimized $|\mathcal{T}_d|$ using the coefficient parameterization if an acceleration sensor is applied.
Using the coefficient parameterization, the initial values are set as $a_3 = 4$, $a_2 = 6$, $a_1 = 4$, and $a_0 = 1$. This set of initial values places four real poles at $-1$. The optimized parameters are $a_3 = 4.4836$, $a_2 = 11.1627$, $a_1 = 4.6461$, and $a_0 = 0.6173$. The corresponding $|T_d|$ curve is plotted in Fig. 3.14. The peak value is $3.1522$ dB, which is lower than the pole parameterization method.

In principle, the two parameterization methods should lead to the same result. However, the optimization process in Matlab does not guarantee global optimum [75]. Nevertheless, the result of the optimization process gives much better result than the heuristic pole placement. Remark that it is a coincidence that the coefficient parameterization gives better result because the derived results could be both local optimum. In practice, it is recommended to compare the results from both parameterization methods.

3.7.3 Payload velocity feedback

The frequency response of a geophone is usually plotted in the sensor specification documents. The parameters of $G_v$ can be estimated. The Power Spectrum Density (PSD) of the sensor noises ($n_x$ and $n_a$) can be experimentally measured [10]. Similarly, the PSD of the base acceleration can also be experimentally measured. The PSD ratios of the sensor noises over the base-frame displacement vary with the frequency. These variations can be described by two functions.

$$G_x(\omega) = \frac{N_v(\omega)}{X_G(\omega)}, \quad G_a(\omega) = \frac{N_v(\omega)}{X_G(\omega)}.$$  (3.80)

Note that both $G_x(\omega)$ and $G_v(\omega)$ can be either continuous functions or look-up tables. (3.36) can be reformed to

$$|T_d(\omega)| = |T_d(\omega)|(1 + |G_x(\omega)|) + |S_d(\omega)||G_a(\omega)|.$$  (3.81)

There are also two ways to parameterize the cost function $|T_d(\omega)|$:

1. Parameterize $|T_d(\omega)|$ using the denominator poles;
2. Parameterize $|T_d(\omega)|$ using the denominator coefficients.

Since the numerical optimization program in Matlab can not guarantee the global optimum, both parameterization are applied to double confirm the results of the optimization processes.

**Pole parameterization**

Assume that $T_d$ takes the form of (3.45), there are four possibilities of the five poles. Assume that $r_i < 0, \forall i \in \{1, 2, 3, 4, 5\}$ are independent negative variables, the three possible combinations of the five stable poles are:

- Five real poles ($r_i, \forall i \in \{1, 2, 3, 4, 5\}$);
- Three real poles ($r_i, \forall i \in \{1, 2, 3\}$) and a conjugate pair ($r_4 \pm r_5 j$);
- One real pole ($r_1$) and two conjugate pairs ($r_2 \pm r_3 j$ and $r_4 \pm r_5 j$).

In each case, $|T_d(\omega)|$ can be numerically calculated according to (3.81) and (3.34). The transmissibility optimization is formulated as follows.
3.7. Sliding surface optimization

To find the set of four negative variables \( r_i, \forall i \in \{1, 2, 3, 4, 5\} \) which minimizes \( \sup |T_d(\omega)| \) under constraints of

\[
\begin{align*}
    &|T_d(\omega)| \leq \varepsilon_0, \forall \omega \leq \omega_0; \\
    &|T_d(\omega_i)| \leq \varepsilon_i, \forall i \in \{1, 2, \ldots, n\}.
\end{align*}
\]

The above optimization problem can be solved numerically in Matlab for each case of pole combinations. The final optimal solution is the one with lowest \( \sup |T_d(\omega)| \).

Coefficient parameterization

Assume that \( T_d \) takes the form of (3.45), the constants \( a_i, \forall i \in \{0, 1, 2, 3, 4\} \) are used as parameters and the constant is defined as \( a_5 = 1 \). The transmissibility optimization is formulated as follows.

To find the set of four positive variables \( a_i, \forall i \in \{0, 1, 2, 3, 4\} \) which minimizes \( \sup |T_d(\omega)| \) under constraints of

\[
\begin{align*}
    &|T_d(\omega)| \leq \varepsilon_0, \forall \omega \leq \omega_0; \\
    &|T_d(\omega_i)| \leq \varepsilon_i, \forall i \in \{1, 2, \ldots, n\}; \\
    &a_i > 0, \forall i \in \{0, 1, 2, 3, 4\}; \\
    &b_1 = a_3 - a_2/a_4 > 0; \\
    &c_1 = a_2 - b_2a_4/b_1 > 0, \text{ where } b_2 = a_1 - a_0/a_4; \\
    &b_2 - b_1a_0/c_1 > 0, \text{ where } b_2 = a_1 - a_0/a_4.
\end{align*}
\]

The last four constraints are used to keep \( T_d \) stable. They are derived using the Routh-Hurwitz criterion.

Numerical example

A simple numerical example of the optimization process is given. Assume that

\[
\begin{align*}
    &\text{For mode } i, |G_x(\omega)| = 0.1 \text{ and } |G_d(\omega)| = 0.2; \\
    &\omega_0 = 0.001 \text{ Hz, } \omega_1 = 1 \text{ Hz, } \omega_2 = 10 \text{ Hz}; \\
    &\varepsilon_0 = 1.4125 (3 \text{ dB}), \varepsilon_1 = 1 (0 \text{ dB}), \varepsilon_2 = 0.01 (-40 \text{ dB}).
\end{align*}
\]

Using the pole parameterization, the initial values are set as \( r_i = -1, \forall i \in \{1, 2, 3, 4, 5\} \). Three results are obtained for each combination of the five poles:

\[
\begin{align*}
    &\text{Five real poles } (r_i = -1.5632, \forall i \in \{1, 2, 3, 4, 5\}); \\
    &\text{Three real poles } (r_1 = -0.7670, r_2 = -0.8573, r_3 = -0.7327) \text{ and a conjugate pair } (-1.4248 \pm 3.7058 j); \\
    &\text{One real pole and two conjugate pairs } (-0.0345, -1.7862 \pm 1.5838 j \text{ and } -1.7861 \pm 1.5836 j).
\end{align*}
\]

The resultant three \( |T_d| \) curves are plotted in Fig. 3.15. The third pole combination (one real pole and two conjugate pairs) gives the lowest peak of \( |T_d(\omega)| \) (8.3899 dB). It will be compared with the result of the coefficient parameterization.

Using the coefficient parameterization, the initial values are set as \( a_4 = 5, a_3 = 10, a_2 = 10, a_1 = 5, \text{ and } a_0 = 1 \). Note that this set of initial values places five real poles at \(-1\). But the result is not as good as that of pole parameterization. Therefore, the results of pole
Chapter 3. Vibration isolation control using a sliding surface control approach

Figure 3.15: Optimized $|T_d|$ using the pole parameterization if a geophone is applied.

Figure 3.16: Optimized $|T_d|$ using the coefficient parameterization if a geophone is applied.
parameterization are used as initial values. The optimized parameters are $a_4 = 4.7503$, $a_3 = 24.3178$, $a_2 = 35.3461$, $a_1 = 31.5798$, and $a_0 = 3.4184$. The corresponding $|T_d|$ curve is plotted in Fig. 3.16. The peak value is 8.3797 dB, which is lower than the pole parameterization method. This is the optimal solution.

### 3.7.4 Comparison

Both geophone and accelerometer can be used to measure the payload absolute motion. The numerical examples in the section are used to compare the two sensors. The magnitude peak of $|T_d|$ in Fig. 3.14 is much lower than that in Fig. 3.16. Considering that the spectrum assumptions of sensor noises and base vibrations are the same, it can be concluded that using accelerometer may achieve lower magnitude peak and lower cut-off frequency than using geophone. The drawback of the geophone is induced by the second-order dynamics $G_v$, as described in Section 3.2.1. The measurement scheme of SEMIS is finally chosen as a combination of relative displacement and payload absolute acceleration.

### 3.8 Conclusions

In this chapter, vibration isolation control design and possible measurement schemes have been studied for a 1-DoF active suspension system. To simultaneously improve all the four performance criteria (transmissibility, compliance, and the two sensitivity functions) has been proved to be impossible. The measurement scheme of SEMIS has been chosen as a combination of relative displacement and payload absolute acceleration. The terms, open-loop gain, control bandwidth, and the two stability margins, which are conventionally defined for SISO control design, have been extended to vibration isolation control design (DISO).

A new vibration isolation control approach, the sliding surface control, has been developed for a 1-DoF active suspension system based on the frequency-shaped sliding surface approach in literature. This sliding surface control is applicable to all four possible measurement schemes. The four concerned performance criteria are designed in two separated steps. The sliding surface design which is constructed by relative displacement and absolute motion is physically interpreted to the transmissibility and the two sensitivity functions. The regulator is designed to realize the designed performances and to reduce the closed-loop compliance. The regulator design is a typical SISO design problem which is solvable by many classic SISO control design tools, for example, SISO loop-shaping.

The sliding surface design problem has been converted to a heuristic pole placement problem. Subsequently, a transmissibility optimization problem has been formulated to optimize the sliding surface design and it is numerically solved using the Matlab Optimization Toolbox [75]. It provides an automatic method to determine a set of parameters which are corresponding to the optimal transmissibility and the two sensitivity functions.

Compared with the conventional DISO control design approach (iterative tuning of the two SISO controllers), the advantages of the sliding surface control are:

1. the sliding surface design is independent of the regulator design and it has negligible influence on the compliance;
2. transmissibility and the two sensitivity functions can be optimized by sliding surface design;
3. the regulator design has negligible influence on the transmissibility and the two sensitivity functions if the regulator gain is sufficiently high;
4. the original DISO control design problem is reduced to SISO control design problem.

The sliding surface control will be compared with $H_\infty$-optimization in Chapter 5. Examples will be given to further illustrate the control design process.
Part II

Validation of vibration isolation control on a 3-DoF demonstrator
Chapter 4

Validation of static optimal decoupling on a 3-DoF demonstrator

4.1 Introduction

The optimal static decoupling described in Section 2.3 derives the pair of input and output decoupling matrices which minimizes the $\mu$-interaction measure numerically using the Matlab Optimization Toolbox [75]. However, this numerical process does not guarantee the global minimum. The derived "optimal" pair of decoupling matrices could be just a local minimum. Furthermore, this numerical optimization process could be negatively influenced by modeling errors induced by Frequency Response Function (FRF) measurement. Therefore, experimental results are necessary to evaluate the practical effectiveness of this decoupling procedure.

A 3-DoF system, described in Section 4.2, is used as an experimental demonstrator to validate the optical static decoupling instead of the Single Electro-Magnetic-Isolator System (SEMIS) for safety. The payload of this system is subjected to 3-DoF motions: $q_z$ (translation along z-axis), $q_\phi$ (rotation about x-axis), and $q_\theta$ (rotation about y-axis). These 3-DoF motions are cross-coupled. The reasons are explained in Section 4.2. The other DoF's are fixed.

In Section 4.3, the modified Vaes-procedure as described in Section 2.3.2 is applied to derive the optimal pair of real decoupling matrices based on the measured Frequency Response Function (FRF). To show the effectiveness of the optimal static decoupling, the $\mu$-interaction measure, as defined in Section 2.1.3, is calculated to compare the cross-coupling strength of the original plant and the decoupled plant.

The decoupling-based decentralized control is also compared with direct MIMO control design in terms of closed-loop cross-coupling. In Section 4.4, the asymptotic identification approach [127] is applied to derive a complete MIMO model of the original 3-DoF system and three SISO models of the decoupled system. Subsequently, the inertia matrix, stiffness matrix, and damping matrix of the 3-DoF system are calculated from the derived MIMO model using a state transformation approach. Finally, the identified MIMO models are validated.

In Section 4.5, $H_\infty$-optimization is used to design a MIMO controller for the original
system and a decentralized controller for the decoupled system toward reference tracking performance. This performance is tested instead of the vibration isolation control performance because vibration isolation control can not be implemented at the moment due to unavailability of the absolute motion sensors. The necessity of the absolute motion sensors for vibration isolation control has been explained in Section 3.3. The reference stepwise response is used to compare the cross-coupling of the closed-loop system. It shows that the direct MIMO control design has only slightly less closed-loop cross-coupling than the proposed decentralized control with static optimal decoupling.

4.2 The 3-DoF setup

The experimental setup was originally designed to validate the control design for a contactless planar actuator with manipulator [29]. With some of the unnecessary components (the H-bridge and the manipulator) removed, the drawings of the setup are shown in Fig. 4.1. The payload (approximately 3 kg) is subjected to 3-DoF motions: the translation along the vertical axis ($q_z$) and the two rotations about the two horizontal axes (roll $q_\phi$ and pitch $q_\theta$). The rest of the DoFs are limited by the three leaf-springs connecting the payload and the base-frame. The mass of this base-frame is approximately 70 kg and the solid iron table that is supporting this base-frame is approximately 200 kg. The leaf-springs can easily bend at their two ends but they can not shrink or extend. These leaf-springs are designed such that the three free DoFs have very low stiffness. Four mechanical hard-stops are installed at the four corners of the payload for safety. Each hard-stop limits the vertical displacement of the corresponding payload corner within the limit of $\pm 1$ mm. There are nine positions to install the Voice Coil Actuators (VCA), labeled by numbers 1-9, show in Fig. 4.1(b). There are only five of the VCAs (from H2W Technologies, Inc.) installed: two of them (VCA-4 and VCA-6) provide DC force for gravity compensation and three of them (VCA-1, VCA-3, and VCA-8) are mainly used for control. All the translators (coils) of the VCAs are rigidly fixed to the payload and all the stators (magnets) are rigidly fixed to the base-frame. Fig. 4.2 shows the photo of the setup. The control force provided by each VCA is proportional to the coil current and is virtually independent of the translator-stator relative position. Each VCA is powered by a linear current amplifier module (from Quanser Consulting Inc.), which provides current output proportional to the control signal. Each VCA also provides damping force due to the trapped air between the translator and the stator. The three inductive sensors (S-1, S-2, and S-3) installed underneath are used to measure the vertical translator displacements of VCA-1, VCA-3, and VCA-8, respectively. A dSpace system is used to calculate the control signals based on the three displacement feedback signals.

The origin of the coordinate system is set as the geometric center of the payload. The directions of the three axes are shown in Fig. 4.1(a). The output signals of the three sensors S-1, S-2, and S-3 are denoted by $q_1$, $q_2$, and $q_3$, respectively. The force produced by the three VCAs, VCA-1, VCA-3, and VCA-8 are denoted by $f_1$, $f_2$, and $f_3$, respectively. The transformation matrix $qT_s$ is used to transform the direct sensor output vector $\vec{q}_s = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$ to the vector $\vec{q} = \begin{bmatrix} q_z \\ q_\phi \\ q_\theta \end{bmatrix}$, which is used to describe the 3-DoF motions.

$$\vec{q} = qT_s \vec{q}_s.$$  

(4.1)

In this chapter, we assume $\vec{q}_r = \vec{q}_a = \vec{q}$ as zero floor/base vibrations are assumed. The
control wrench vector is \( \vec{w}_a = [f_z, t_x, t_y]^T \), where \( f_z \), \( t_x \), and \( t_y \) are the vertical control force, the control torque about \( x \)-axis, and the control torque about \( y \)-axis, respectively. The transformation matrix \( \mathbf{T}_w \) is used to transform the control wrench \( \vec{w}_a \) to the VCA force vector \( \vec{f}_a = [f_1, f_2, f_3]^T \).

\[
\vec{f}_a = \mathbf{T}_w \vec{w}_a .
\] (4.2)

4.2. The 3-DoF setup
assumption that \(\sin \phi \approx \phi, \sin \theta \approx \theta, \cos \phi \approx 1,\) and \(\cos \theta \approx 1.\) The plant has the wrench vector \(\vec{w}_a\) as its input and the physical variable vector \(\vec{q}\) as its output.

There are a few reasons for the cross-coupling of the plant. First of all, both the spring forces and the damping forces are unsymmetrical about the origin. For the three leaf-springs, the two leaf-springs in parallel are symmetrical about the \(y\)-axis but the third leaf-spring is unsymmetrical. It induces cross-coupling between the vertical translation \(q_z\) and the horizontal rotation \(q_\phi.\) The allocation of the five VCAs is unsymmetrical about the origin. It induces cross-coupling among all of the three DoFs because each of the five VCAs provides damping forces. Second, the coordinate system origin and the mass center of the payload do not exactly coincide. Obviously, the mass distribution of the leaf-springs and the VCA translators are unsymmetrical about the origin. Third, the imperfect mechanical/electrical components and subsystems could also induce the cross-coupling. For example, the noise and drift outputs of the current amplifiers provide inaccurate control effort.

### 4.3 Static decoupling performance

To derive the optimal pair of decoupling matrices, the Frequency Response Function (FRF) of the 3-DoF system has to be measured. Subsequently, the static optimal decoupling matrices are derived using the modified Veas-procedure, as described in Section 2.3.2. For decoupling-based decentralized control, the upper bound of the complementary sensitivity of each SISO loop is the inverse of the \(\mu\)-interaction measure [34]. For this reason, the inverse of the \(\mu\)-interaction measures of the original system and the decoupled system are calculated and compared. It shows that the static optimal decoupling is effective in increasing the inverse of the \(\mu\)-interaction measure, or in reducing the \(\mu\)-interaction measure.

#### 4.3.1 FRF measurement

As the 3-DoF system has nearly double-integrator properties, the FRF is measured in closed-loop, shown in Fig. 4.3. \(P\) represents the 3-DoF plant. \(C\) is the controller used for stabilization during the FRF measurement. It is given by

\[
C = \begin{bmatrix}
200 + \frac{100}{s} & 0 & 0 \\
0 & 20 + \frac{5}{s} & 0 \\
0 & 0 & 20 + \frac{5}{s}
\end{bmatrix}.
\]

(4.3)

The vector \(\vec{r}\) is the reference signal. The vector \(\vec{w}_d\) denotes the identification excitation signals (Gaussian white noise). The strength of \(\vec{w}_d\) is tuned in such a way that the response \(\vec{q}\) is large but the payload has no danger of touching the hard-stops. During the FRF measurement, \(\vec{r}\) is set to zero which indicates that each VCA is in the center of its translational range. Using the system analyzer ‘Siglab’ (from Spectral Dynamics, Inc.), the closed-loop FRF \(\tilde{G}(\vec{w}_d \rightarrow \vec{q})\) is averaged from results of 200 independent experiments. Subsequently, \(\tilde{P}\) is calculated by

\[
\tilde{P}(\omega) = [\tilde{G}(\omega)^{-1} - C(\omega)]^{-1},
\]

(4.4)

where \(C(\omega)\) is the FRF of the controller \(C\) at frequency \(\omega.\) Remark that the controller is very weak so that the calculated \(\tilde{P}\) describes system dynamics well. The data collection
4.4. MIMO identification

takes about one hour. The magnitude of the FRF \( \tilde{P} \) is plotted in Fig. 4.4. The inverse of the \( \mu \)-interaction measure, \( \mu^{-1}_\Delta(E(\tilde{P}(\omega))) \), is plotted in Fig. 4.5. The interested frequency range is \([0.5, 500]\) Hz and the sampling rate is set to 1280 Hz.

4.3.2 Optimal static decoupling

The modified Vaes algorithm is applied to find the optimal static decoupling matrices. The initial values are taken from the results of the Owens method. The optimization process takes about four hours to complete. The two decoupling matrices are derived as

\[
T_y = \begin{bmatrix}
1.0000 & -0.0093 & -0.0014 \\
0.1308 & 0.9892 & 0.0668 \\
0.1098 & -0.0489 & 0.9928 \\
\end{bmatrix}, \quad (4.5)
\]

\[
T_u = \begin{bmatrix}
0.9999 & 0.2402 & 0.0676 \\
-0.0113 & 0.9688 & -0.0455 \\
0.0001 & 0.0604 & 0.9967 \\
\end{bmatrix}. \quad (4.6)
\]

Since both decoupling matrices are close to the identity matrix, the input and output vectors of the decoupled plant \( \tilde{P}_d \) are denoted by \( w_{\omega_0} = [f_{\omega_0}, l_{\omega_0}, f_{\omega_0}]^T \) and \( w_{\omega_0} = [q_{\omega_0}, q_{\theta_0}, q_{\phi_0}]^T \), respectively. The FRF \( \tilde{P}_d \) of the decoupled plant \( P_d = T_yPT_u \) is measured according to the diagram in Fig. 4.6 using similar procedure described in the last subsection. The magnitude plot of \( \tilde{P}_d \) is compared with that of \( \tilde{P} \) in Fig. 4.7. The inverse of the \( \mu \)-interaction measure calculated based on (2.4), \( \mu^{-1}_\Delta(E(P_d(\omega))) \), is compared with \( \mu^{-1}_\Delta(E(\tilde{P}(\omega))) \) in Fig. 4.8. \( \mu^{-1}_\Delta(E(\tilde{P}(\omega))) \) is calculated based on \( E(\tilde{P}(\omega)) \), the relative error of the measured FRF of the decoupled plant \( \tilde{P}_d \). \( \mu^{-1}_\Delta(E(P_d(\omega))) \) and \( \mu^{-1}_\Delta(E(\tilde{P}_d(\omega))) \) are reasonably closed to each other which validates the performance of the decoupling matrices.

The FRF comparison in Fig. 4.7 shows that the three diagonal entries of \( \tilde{P}_d \) are very close to that of \( \tilde{P} \) while the off-diagonal entries of \( \tilde{P}_d \) are lower than that of \( \tilde{P} \), especially at low frequencies (<10 Hz). This proves the effectiveness of the decoupling. Compare Fig. 4.8 with Fig. 4.5, \( \mu^{-1}_\Delta(E(\tilde{P}_d(\omega))) \) is higher than \( \mu^{-1}_\Delta(E(\tilde{P}(\omega))) \) at frequencies lower than 70 Hz. For frequencies lower than 6 Hz, \( \mu^{-1}_\Delta(E(\tilde{P}(\omega))) \) is increased from \( \mu^{-1}(E(\tilde{P}(\omega))) \) by 10 dB. The notch-like spike of \( \mu^{-1}_\Delta(E(\tilde{P}_d(\omega))) \) at frequencies around 9 Hz, as shown in Fig. 4.5, is removed in \( \mu^{-1}_\Delta(E(\tilde{P}_d(\omega))) \). This is another proof of the decoupling effectiveness. According to (2.6), the complementary sensitivity of each SISO loop should have lower magnitude than \( \mu^{-1}_\Delta(E(\tilde{P}_d(\omega))) \). The cross-over frequency of \( \mu^{-1}_\Delta(E(\tilde{P}_d(\omega))) \) is the same as the bandwidth-limiting resonance at 287 Hz. It indicates that the stability limit to the closed-loop bandwidth induced by the imperfect decoupling is the same as that induced by the high-frequency parasitic resonances.

4.4 MIMO identification

As the direct MIMO control design requires a dynamic MIMO model, this section describes the identification of the 3-DoF system as well as the decoupled system using the derived matrices in subsection 4.3.2. The asymptotic identification method [125] is introduced and applied. A state-transformation approach is proposed to derive the physical parameters.
Chapter 4. Validation of static optimal decoupling on a 3-DoF demonstrator

Figure 4.3: Diagram of closed-loop identification test on the 3-DoF system.

Figure 4.4: Magnitude plot of the measured FRF $\tilde{P}$.

Figure 4.5: Magnitude plot of $\mu^{-1}_\Delta(E(\tilde{P}(\omega)))$ from the measured FRF $\tilde{P}$. 
of the 3-DoF system, including inertia matrix, damping matrix, and stiffness matrix. The derived models are also validated.

4.4.1 Model identification

As the 3-DoF system has nearly double-integrator properties, the input-output data used for identification are collected in closed-loop tests. The asymptotic identification method [125] is used to identify a dynamic MIMO model. This procedure is as follows.

1. Collect the input output data set by closed-loop identification tests.
2. Prepare the data set and estimate a high-order ARX (equation error) model.
3. Derive a low-order model by applying implicit balancing techniques [117] to the high-order ARX model. The order of the low-order model is chosen according to Hankel singular values.
4. Model validation.
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Figure 4.8: The $\mu$-interaction measure validation. The solid curve is $\mu_{\Delta}^{-1}(E(P_d(\omega)))$, which is calculated by the decoupling matrices and the plant FRF $\tilde{P}$. The dashed curve is $\mu_{\Delta}^{-1}(E(\tilde{P}_d(\omega)))$, which is calculated from the FRF of the decoupled plant, $\tilde{P}_d$.

5. Physical parameters derivation.

This asymptotic identification method is chosen because of its simple procedure and its effectiveness to derive a parametric linear model.

The diagram of the closed-loop identification test is the same as the FRF measurement, shown in Fig. 4.3. During the identification test, the reference vector $\vec{r}$ is set to zero and the identification excitation signal $\vec{w}_d$ is Gaussian white noise with appropriate power. The plant input and output vector, $\vec{w}_a$ and $\vec{q}$, are recorded as the input-output data set. The time span of this data set is about 34 s. The sampling frequency is 1280 Hz. With the DC offset removed, the data set is subsequently used to estimate a high-order ARX model (240 states).

For control design, a low-order model is preferred. If the high frequency resonances may be ignored, the 3-DoF system can be described by the following motion equation.

$$\mathbf{M}_p \ddot{\vec{q}} + \mathbf{D}_p \dot{\vec{q}} + \mathbf{K}_p \vec{q} = \vec{w}_a,$$

where $\mathbf{M}_p$, $\mathbf{D}_p$, and $\mathbf{K}_p$ are the inertia matrix, damping coefficient matrix, and stiffness matrix, respectively. They are all physical parameters of the 3-DoF system. This motion equation indicates that we need at least six states to model the 3-DoF system well. The order of the low-order model is chosen according to the Hankel singular values. The selected order should be larger than or equal to six. Model order reduction [117] can be applied to derive a low-order model.
4.4. MIMO identification

4.4.2 Derivation of the physical parameters

The physical parameters are important in mechanical engineering. For example, they can be used to validate the mechanical design. The inertia matrix and the stiffness matrix determines the natural frequencies of the system. If the derived low-order model has exactly six states, the three $3 \times 3$ matrices $M_p, D_p, \text{ and } K_p$ can be derived from the identified 6-state model using the following state transformation.

Assume that the state space representation of the derived 6-state model is

\[
\begin{align*}
\dot{\vec{x}} &= A_o \vec{x} + B_o \vec{w}_a \\
\vec{q} &= C_o \vec{x}
\end{align*}
\]  

(4.8)

Define the new states by physical meanings as

\[
\vec{x}_p = \begin{bmatrix} \vec{q} \\ \dot{\vec{q}} \end{bmatrix}. 
\]  

(4.9)

As $\vec{q} = C_o \vec{x}$, $\dot{\vec{q}} = C_o \dot{\vec{x}} = C_o A_o \vec{x} + C_o B_o \vec{w}_a$. Therefore, $\vec{x}_p = T_s \vec{x}$, where $T_s$ is the state transformation matrix, derived as

\[
T_s = \begin{bmatrix} C_o \\ C_o A_o \end{bmatrix}. 
\]  

(4.10)

The new state space representation of the 6-state model is

\[
\begin{align*}
\dot{\vec{x}}_p &= A_n \vec{x}_p + B_n \vec{w}_a \\
\vec{q} &= C_n \vec{x}_p
\end{align*}
\]  

(4.11)

where $A_n = T_s A_o T_s^{-1}$, $B_n = T_s B_o$, and $C_n = C_o T_s^{-1}$. According to the physical equation (4.7), the theoretical forms of three matrices are

\[
A_n = \begin{bmatrix} 0_{3 \times 3} & I_{3 \times 3} \\ -M_p^{-1} K_p & -M_p^{-1} D_p \end{bmatrix}, \quad B_n = \begin{bmatrix} 0_{3 \times 3} \\ M_p^{-1} \end{bmatrix}, \quad C_n = \begin{bmatrix} I_{3 \times 3} \\ 0_{3 \times 3} \end{bmatrix}. 
\]  

(4.12)

By comparing the derived state space matrices $(A_n, B_n, C_n)$ with their theoretical forms in (4.12), the three matrices, $M_p, D_p, \text{ and } K_p$ can be derived. By default, the 6-state model mentioned in the following text indicates the one in the theoretical form of (4.11) and (4.12). Note that this 6-state model is accurate only if the high frequency resonances are not significant. Otherwise, the derived low-order model would have more than six states.

If the derived low-order model has more than six states, the six states which represents the natural frequencies can be selected by the following steps.

1. Derive the Jordan Canonical Form of the low-order model.
2. Select the six states which fit the natural frequencies.
3. Collect the corresponding state space matrices for these six states.
4. Apply the above procedure to derive the physical parameters.
4.4.3 Model validation

The identified model is validated in both the frequency domain and the time domain. Fig. 4.9 plots the magnitudes of the 240-states model and the measured FRF. Both magnitude curves are reasonably close to each other, even for some of the high frequency resonances. The measured FRF is taken from Fig. 4.4.

The 12 largest Hankel singular values of the 240-state model are plotted in Fig. 4.10. There is a significant drop from state 8 to state 9. Therefore, the 240-state model is reduced to an 8-state model. The 8-state model is compared with the 240-state model in Fig. 4.11. It shows that the 8-state model describes the 287 Hz resonance well but higher-frequency resonances are not modeled.

The physical parameters derived from the 8-state model are

$$ M_p = \begin{bmatrix} 3.2531 & -0.0189 & 0.0047 \\ -0.0159 & 0.0258 & -0.0003 \\ 0.0060 & -0.0000 & 0.0284 \end{bmatrix}, $$

$$ D_p = \begin{bmatrix} 64.1733 & -0.9402 & -0.1257 \\ -0.9595 & 0.2922 & 0.0059 \\ -0.1925 & 0.0094 & 0.3773 \end{bmatrix}, $$

$$ K_p = \begin{bmatrix} 567.6357 & 15.4369 & 23.9848 \\ 53.4371 & 80.9241 & -3.1536 \\ 18.8206 & -2.1851 & 135.7941 \end{bmatrix}. $$

A 6-state model is derived using the derived physical parameters according to (4.7). Fig. 4.12 compares the magnitude plots of the 240-state model and the 6-state model. It shows that the two magnitudes are reasonably close at lower frequencies. It can be concluded that the 6-state model describes the system frequency response well at low frequencies but it neglects all high frequency resonances. Comparing Fig. 4.12 and Fig. 4.11, the 8-state model describes the resonance at 287 Hz but the 6-state model does not. The Hankel singular values for state 7 and 8 is only slightly smaller than that of state 6. It indicates that the 287 Hz resonance is the most significant resonance.

The time response of the identified model to the input vectors of the identification data set is simulated and subsequently compared with the output vectors of the identification data set. To eliminate the effect of wrong initial state value in the simulation, only the last 30 s data are compared. The fitting percentage is calculated for each output vector by

$$ fit = 100\% \times \left(1 - \frac{\sqrt{\sum_{i=1}^{n}(\hat{q}_i - \tilde{q}_i)^2}}{\sqrt{\sum_{i=1}^{n}(\tilde{q}_i - \tilde{q}_{im})^2}}\right), $$

where $\hat{q}_i$ is the elements of the simulated response and $\tilde{q}_i$ is the elements of the output vector of the measured identification data. This fitting percentage is used to evaluate the accuracy of the identified model. $\tilde{q}_{im}$ is the mean value of the corresponding measured output vector. Fig. 4.13 compares the measured output data and the simulated time response of the 240-state model. It confirms that the 240-state model is very accurate. Note that the fitting
4.4. MIMO identification

Figure 4.9: Magnitude plot comparison of the averaged FRF (solid) and the 240-state model (dashed).

Figure 4.10: The 12 largest Hankel singular values of the 240-state model.
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Figure 4.11: Magnitude plot comparison of the 240-state model (solid) and the 8-state model (dashed).

Figure 4.12: Magnitude plot comparison of the 240-state model (solid) and the 6-state model (dashed).
percentage for the vertical translation $q_z$ is lower than the two rotations. The reason is that the output signals of the two rotations, $q_\theta$ and $q_\phi$, have much larger values than that of the vertical translation $q_z$. Fig. 4.14 compares the measured output data and the simulated time response of the 6-state model. By comparing the fitting percentage calculated in Fig. 4.13 and Fig. 4.14, it can be concluded that the influence of the model reduction on the time response simulation is ignorable small. In other words, the influences of the high frequency resonances on the time domain simulation are ignorable small.

To further validate the 6-state model, another data set (31 s) is measured and is processed to remove the linear trends. The time response of the 6-state model to the input vector of the new data set is simulated and is subsequently compared with the output vector of the new data set. Similarly, only the last 30 s of the output vector is used for comparison and fitting percentage calculation. Fig. 4.15 plots the comparison results. The fitting percentage of the three output vectors are almost the same as that in Fig. 4.14.

The high frequency resonances are probably caused by the flexible modes of the leaf-springs. Removing the high frequency resonances from the high-order model almost has no influence on the fitting percentage of the simulated time responses. It can be concluded that the influence of the 287 Hz resonance on the time responses is ignorable. The data collection takes less than one minute to complete in total. The 240-state model estimation, model order reduction, and model validation take less than 10 minutes.

Remark 4.4.1. Shown in Fig. 4.13, Fig. 4.14, and Fig. 4.15, the fitting percentage of the simulated time response with respect to the measured time response for the vertical translation $q_z$ is lower than $q_\theta$ and $q_\phi$. The reason is explained as follows. The asymptotic identification estimates a parametric MIMO model by minimizing the prediction error. Shown in the aforementioned figures, the signal values of $q_z$ is approximately 10 times lower than that of $q_\theta$ and $q_\phi$. For this reason, the influence of $q_z$ on the prediction error is weaker than that of $q_\theta$ and $q_\phi$. Therefore, the estimated transfer functions from all inputs to the output $q_z$ have larger error than the other models.

In conventional decentralized control, SISO identification is usually applied to identify each decoupled SISO system or PID tuning is applied to avoid the identification. Since the above MIMO identification process is not complicated, the decoupled system is also identified using above identification procedure. The three diagonal input-output models ($f_{zo} \rightarrow q_{zo}$, $t_{xo} \rightarrow q_{\theta o}$, and $t_{yo} \rightarrow q_{\theta o}$) are further reduced to second-order models. Usually, the derived second-order models do not perfectly fit the physical form of $\omega_n^2/(s^2 + 2\zeta_n\omega_n s + \omega_n^2)$. The high order terms of the numerator are manually set to zero to fit the physical form. The magnitudes of the derived second-order models and the measured FRF are plotted in Fig. 4.16 - Fig. 4.18. Note that the measured FRF is an average of 200 measured frequency response curves, taken from Fig. 4.7. The comparison shows that the identified models are reasonably close to the measured FRF up to 100 Hz. The three second-order models will be used for the subsequent SISO control design.

4.5 MIMO control

In this section, a MIMO controller and a decentralized controller are designed using $H_\infty$-optimization based on the models identified in Section 4.4. The low-order models are used
Figure 4.13: Time domain validation. Comparison of the measured output vectors (solid) and the simulated time response of the high-order (240 states) ARX model (dashed). The fitting percentage is calculated for each output vector.

Figure 4.14: Time domain validation. Comparison of the measured output vectors (solid) and the simulated time response of the 6-state model (dashed). The fitting percentage is calculated for each output vector.
4.5. MIMO control

Figure 4.15: Time domain validation. Comparison of the output vectors from another measured data set (solid) and the simulated time response of the 6-state model (dashed). The fitting percentage is calculated for each output vector.

Figure 4.16: Magnitude comparison of the measured FRF and the identified model for \( f_{zo} \rightarrow q_{zo} \)
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Figure 4.17: Magnitude comparison of the measured FRF and the identified model for $t_{xo} \rightarrow q_{\phi o}$

Figure 4.18: Magnitude comparison of the measured FRF and the identified model for $t_{yo} \rightarrow q_{\theta o}$
for control design. As a result, a low-order controller is derived. The high-order models and the measured FRF are used to predict the closed-loop stability. Both MIMO control and decentralized control are aiming at reference-tracking performance. A reference stepwise response is used to compare the closed-loop cross-coupling of the two control strategies. The step size for $\vec{q}$ is $[0.2, 1, 1]^T$ mm or mrad. The step sizes for each DoF motion are selected as approximately one fifth of the total stroke length.

### 4.5.1 $H_\infty$ control design

The diagram of $H_\infty$ control design towards reference-tracking is shown in Fig. 4.19. The block $G$ denotes the identified model and the block $K$ denotes the $H_\infty$ controller to be designed. The signal vectors $\vec{w}_d$, $\vec{n}_x$, $\vec{r}$, $\vec{u}$, and $\vec{e}$ denote the disturbance force/torque, displacement sensor noise, the reference, the control signal, and the error signal, respectively. The blocks $V_d$, $V_n$, $V_r$, $W_u$, and $W_e$ are the weighting filters for the signal vectors $\vec{w}_d$, $\vec{n}_x$, $\vec{r}$, $\vec{u}$, and $\vec{e}$, respectively. The signal vectors $\vec{w}_1$, $\vec{w}_2$, and $\vec{w}_3$ are exogenous input of the augmented plant. The signal vectors $\vec{z}_1$ and $\vec{z}_2$ are the augmented plant outputs that have to be minimized. The signal vectors $\vec{u}$ and $\vec{e}$ are the control effort and the measured output of the augmented plant. The dimensions of all signal vectors depend on the model $G$. The controller $K$ can be synthesized in the Matlab Robust Control Toolbox [8].

For the 6-state model ($\vec{w}_a \rightarrow \vec{q}$), the weighting filters are designed as follows. $V_r$ is designed according to the step sizes for the corresponding motion. The step sizes is 0.2 mm for the vertical translation and 1 mrad for the two horizontal rotations.

$$V_r = 10^{-3} \times \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$ 

$V_n$ is designed according to the sensor resolutions for the corresponding motion. The sensor
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resolution is 4 \( \mu \text{m} \) for the vertical translation and 20 \( \mu \text{rad} \) for the two horizontal rotations.

\[
V_n = 10^{-5} \times \begin{bmatrix} 0.4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}.
\]

\( V_d \) is designed according to the estimated disturbance induced by the amplifier noise current for the corresponding motion. The vertical disturbance force is estimated as 0.5 mN and the horizontal disturbance torques are estimated as 12 mN\( \cdot \)m.

\[
V_d = 10^{-3} \times \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 12 \end{bmatrix}.
\]

The penalty filter \( W_e \) is designed to be first-order to result in a low-order controller. The penalty filter \( W_e \) is designed to have high-gain at low frequencies to suppress the tracking error. For vertical translation, the DC-gain of \( W_e \) is \( 1 \times 10^3 \), which is corresponding to 10\( \mu \text{m} \) tracking error if \( \gamma = 1 \). For the two horizontal rotations, the DC-gain of \( W_e \) is \( 5 \times 10^4 \), which is corresponding to 20\( \mu \text{rad} \) tracking error if \( \gamma = 1 \). The zero of \( W_e \) is set to 125.7 \( \text{rad/s} \), which is corresponding to a cut-off frequency of 20 Hz.

\[
W_e = \frac{s + 125.7}{s + 0.006283} \times \begin{bmatrix} 5 & 0 & 0 \\ 0 & 2.5 & 0 \\ 0 & 0 & 2.5 \end{bmatrix}.
\]

The penalty filter \( W_u \) is designed as a diagonal constant matrix. The diagonal values are tuned iteratively such that the synthesized \( \gamma \) is close to one.

\[
W_u = 10^{-4} \times \begin{bmatrix} 0.15 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}.
\]

The resultant \( \gamma \) of the \( H_{\infty} \)-optimization is 1.0007. The synthesized controller is a 9\( ^{th} \)-order \( 3 \times 3 \) transfer function matrix.

For the design of the decentralized controller, or the three SISO controllers, the weighting filter design for each of them is exactly the same as the corresponding diagonal entries of the MIMO case. For the second-order model \( f_{zo} \rightarrow q_{zo} \), the weighting filters are designed as

\[
V_r = 2 \times 10^{-4}, \ V_n = 4 \times 10^{-6}, \ V_d = 5 \times 10^{-4},
\]

\[
W_e = \frac{5(s + 125.7)}{s + 0.006283}, \ W_u = 1.5 \times 10^{-5}.
\]

For the second-order model \( t_{xo} \rightarrow q_{\phi o} \) and \( t_{yo} \rightarrow q_{\theta o} \), the weighting filters are designed as

\[
V_r = 10^{-3}, \ V_n = 2 \times 10^{-5}, \ V_d = 0.012,
\]

\[
W_e = \frac{2.5(s + 125.7)}{s + 0.006283}, \ W_u = 2 \times 10^{-4}.
\]
4.5. MIMO control

The resultant $\gamma$ of the $H_\infty$-optimization are 0.4003, 1.0007, and 1.0007. The decentralized controller is three 3rd-order SISO controllers.

Remark that the control bandwidth (the frequency at which the corresponding open-loop transfer function crosses over 0 dB) of the two controllers are the same.

4.5.2 Closed-loop stability

The robust stability of the $H_\infty$ control is analyzed using the small gain theory. The true plant is denoted by $P$ and the 6-state model is denoted by $G_L$. The difference between $P$ and $G_L$ is modeled by the additive uncertainty, denoted by $\Delta = P - G_L$. The control diagram of the additive uncertain model is shown in Fig. 4.20. The grey rectangular denote the true plant $P$. Compare Fig. 4.20 with the basic perturbation model in Fig. 4.21, the corresponding interconnection matrix $H$ is calculated as

$$H = \left(CG_L + I_{3 \times 3}\right)^{-1}C,$$

where $C$ is the designed $H_\infty$ controller. According to the small gain theory, the closed-loop system is stable with stable $H$ and $\Delta$ if $\|H\Delta\|_\infty \leq 1$. As the true plant is unknown, the measured FRF and the identified 240-state model are used to calculate $\Delta$. Subsequently, the interconnection matrix $H$ is calculated by (4.14). Finally, the singular values of $H\Delta$ are calculated and plotted in Fig. 4.22. The maximum of the singular values is more than 4 dB below 1 (0 dB), which indicates that the perturbed closed-loop system remains stable.

![Control diagram of the additive uncertain model.](image)

![Diagram of the basic perturbation model.](image)
the controller. The $\mu$-interaction measure inverse of the decoupled system and the complementary sensitivity function of the three SISO loop are plotted in Fig. 4.23. All complementary sensitivities of the three SISO loop are below the $\mu$-interaction measure inverse, which indicates that the closed-loop system remains stable for the assumed uncertainty $\Delta$.

4.5.3 Step response

The designed MIMO controller is implemented to the 3-DoF system by replacing the PI controller designed for identification by the designed $H_\infty$ controller in Fig. 4.3. The response of the closed-loop system to the stepwise reference vectors $\vec{r} = [2 \times 10^{-4}, 0, 0]^T$, $\vec{r} = [0, 1 \times 10^{-3}, 0]^T$, and $\vec{r} = [0, 0, 1 \times 10^{-3}]^T$ are plotted in Fig. 4.24(a) and zoomed in Fig. 4.24(b). The corresponding control wrench is plotted in Fig. 4.26(a).

The three designed SISO controllers are implemented to the 3-DoF system with the static optimal decoupling matrices according to Fig. 4.6. The response of the closed-loop system to the stepwise reference vectors $\vec{r} = [2 \times 10^{-4}, 0, 0]^T$, $\vec{r} = [0, 1 \times 10^{-3}, 0]^T$, and $\vec{r} = [0, 0, 1 \times 10^{-3}]^T$ are plotted in Fig. 4.25(a) and zoomed in Fig. 4.25(b). The corresponding control wrench is plotted in Fig. 4.26(b). Comparing the step responses of the two control strategies, the peak transient responses of the control wrench are similar.

Shown in Fig. 4.24(b), transient response of $q_\theta$ is up to $\pm 3 \mu$rad (1.5 times of the tracking error at steady state) when $q_z$ is stepped up (at 1 s) and stepped down (at 3 s). This is the highest transient responses of the cross-coupling for the direct MIMO control. Shown in Fig. 4.25(b), transient response of $q_\phi$ is up to $\pm 8 \mu$rad (four times of the tracking error at steady state) when $q_z$ is stepped up (at 1 s) and stepped down (at 3 s). This is the highest transient responses of the cross-coupling for the decentralized control. Therefore, the closed-loop cross-coupling of the decentralized control is only slightly stronger than that of the direct MIMO control.

4.6 Conclusions

The effectiveness of the optimal static decoupling has been demonstrated on a 3-DoF experimental setup in this chapter. The two control strategies, decoupling-based decentralized control and direct MIMO control, are compared in experiments in terms of closed-loop cross-coupling and complexity of design and implementation.

4.6.1 Optimal static decoupling

The Frequency Response Function (FRF) of the 3-DoF system, $\tilde{P}$, has been measured indirectly in a stabilized closed-loop. Subsequently, the inverse of the $\mu$-interaction measure, $\mu_\Delta^{-1}(E(P(\omega)))$, has been calculated and plotted in Fig. 4.5. The modified Vea-procedure has been applied to obtain the optimal pair of decoupling matrices in terms of minimizing the $\mu$-interaction measure. The FRF of the decoupled system $\tilde{P}_d$, has been measured using the same procedure. The corresponding inverse of the $\mu$-interaction measure, $\mu_\Delta^{-1}(E(\tilde{P}_d(\omega)))$, has been calculated and plotted in Fig. 4.8.

Comparing the two $\mu$-interaction measure inverse, $\mu_\Delta^{-1}(E(\tilde{P}_d(\omega)))$ is higher than $\mu_\Delta^{-1}(E(P(\omega)))$ at frequencies lower than 70 Hz. For frequencies lower than 6 Hz, $\mu_\Delta^{-1}(E(\tilde{P}_d(\omega)))$ is increased from $\mu_\Delta^{-1}(E(P(\omega)))$ by 10 dB. At frequencies around 9 Hz,
Figure 4.22: Singular values of $H\Delta$ from the direct MIMO control. The solid curves are the singular values calculated from the identified high-order ARX model. The dashed curves are the singular values calculated from the measured FRF.

Figure 4.23: Comparison of the $\mu$-interaction measure inverse and the complementary sensitivity functions of the three SISO loops.
Chapter 4. Validation of static optimal decoupling on a 3-DoF demonstrator

Figure 4.24: The response of the MIMO control loop output to the stepwise reference.

Figure 4.25: The response of the decentralized control output to the stepwise reference.

Figure 4.26: The response of the control wrench to the stepwise reference.
the notch-like spike of $\mu^{-1}_{\Delta}(E(\tilde{P}(\omega)))$ is removed in $\mu^{-1}_{\Delta}(E(\tilde{P}_d(\omega)))$. This is the effectiveness proof of the decoupling matrices.

The cross-over frequency of $\mu^{-1}_{\Delta}(E(\tilde{P}_d(\omega)))$ is the same as the bandwidth-limiting resonance at 287 Hz. It indicates that the stability limit to the closed-loop bandwidth induced by the imperfect decoupling is the same as that induced by the high-frequency parasitic resonances.

4.6.2 MIMO identification and MIMO control

Asymptotic identification [125] has been applied to derive a dynamic MIMO model of the 3-DoF system based on input-output data acquired in closed-loop test. The derived MIMO model has been validated in both time-domain and frequency-domain. Using the derived MIMO model, a state-transformation approach has been developed to derive the inertia matrix, damping matrix, and stiffness matrix. As the asymptotic identification is not complex and is effective for this 3-DoF system, it is also used to derive the three SISO models for the decoupled system.

Based on the identified models, $H_\infty$-optimization has been applied to design a MIMO controller and a decentralized controller toward reference-tracking performance. The time-domain response to the stepwise reference has been measured and compared in terms of closed-loop cross-coupling.

4.6.3 Comparison

Shown in Fig. 4.24(b), transient response of $q_\theta$ is up to $\pm 3 \, \mu\text{rad}$ (1.5 times of the tracking error at steady state) when $q_z$ is stepped up (at 1 s) and stepped down (at 3 s). This is the highest transient response of the cross-coupling for the direct MIMO control. Shown in Fig. 4.25(b), transient response of $q_\phi$ is up to $\pm 8 \, \mu\text{rad}$ (four times of the tracking error at steady state) when $q_z$ is stepped up (at 1 s) and stepped down (at 3 s). This is the highest transient response of the cross-coupling for the decentralized control. Therefore, the closed-loop cross-coupling of the decentralized control is only slightly stronger than that of the direct MIMO control. Considering that the direct MIMO control using $H_\infty$-optimization results in a $9^{th}$-order $3 \times 3$ transfer function matrix and the decentralized controller is three $3^{rd}$-order transfer functions, the decentralized control achieves slightly stronger closed-loop cross-coupling using much simpler controllers.

For this 3-DoF system, the dynamic MIMO model required by direct MIMO control is identified using the asymptotic approach [125] and this process takes less time than the FRF measurement. However, MIMO identification requires higher theoretical demanding than FRF measurement or SISO identification. For this reason, the direct MIMO control design is less straightforward and requires higher cost than the decentralized control design.
Chapter 5

Design process of vibration isolation control

As explained in Chapter 1, the objective of vibration isolation control is to simultaneously achieve stabilization, isolation of floor vibrations (transmissibility), and rejection of directly acting disturbance forces (compliance). This chapter applies vibration isolation control design to the identified models of the 3-DoF system described in Chapter 4. The measurement scheme is assumed to combine the absolute acceleration of the isolated platform (payload) and its relative displacement to its supporting base frame. The reason to apply this measurement scheme is explained in Section 3.3. Two design methods of vibration isolation control, direct MIMO control using $H_{\infty}$-optimization and the sliding surface control combined with optimal static decoupling, are both studied. The closed-loop performance criteria are compared based on the calculated results. The design process as well as the controller complexity are compared. Closed-loop stability is evaluated taking un-modeled system dynamics into account.

Vibration isolation control of a 1-DoF suspension system is described in Section 5.1. This is a preliminary study in order to explain vibration isolation control of a multi-DoF suspension system easier. The aforementioned two design methods of vibration isolation control are both applied and compared. All the four closed-loop performance criteria are calculated. In Section 5.2, these two design methods of vibration isolation control are applied to the identified model of the 3-DoF system. The design process as well as the controller complexity are also compared. Only the transmissibility and compliance are calculated.

5.1 1-DoF vibration isolation control

The motion $f_{zo} \rightarrow q_{zo}$ of the 3-DoF system is used as an example 1-DoF plant to design the vibration isolation controllers using $H_{\infty}$-optimization and the sliding surface control. The designed controllers as well as the corresponding open-loop transfer function are compared. The four closed-loop performance criteria (transmissibility, compliance, and two sensitivity functions) are also compared.
5.1. 1-DoF vibration isolation control

5.1.1 $H_\infty$-optimization

To perform the $H_\infty$-optimization design, the structure of the augmented plant is defined. Subsequently, the weighting filters are designed to synthesize the controller using the Matlab Robust Control Toolbox [8]. Remark that $H_\infty$-optimization is also possible by solving Linear Matrix Inequalities (LMIs) as this problem is convex. However, this method often leads to a controller with unstable poles. Such a controller could produce infinite control effort which cannot be realized in practice. Therefore, this approach is not used for $H_\infty$-optimization.

Augmented plant

The motion equation (3.1) of the mass-spring-damper system (Fig. 5.1) can be reformed as

\[ m \ddot{x}_r + c \dot{x}_r + k x_r = f_a + f_d - m a_b. \]

(5.1)

The symbols $m$, $c$, and $k$ denote system physical parameters: payload inertia, passive damping coefficient, and stiffness, respectively. The two forces, $f_a$ and $f_d$, denote the control force and the disturbance force directly acting on the payload. The two motion signals, $x_r$ and $a_b$, denote the payload relative displacement to its supporting base frame and absolute acceleration of the base frame. Define the state vector $\vec{x} = [x_r, \dot{x}_r]^T$, the input vector $\vec{u} = [a_b, f_d, f_a]^T$, and the output vector $\vec{y} = [x_r, a_a]^T$, the state space representation is derived as

\[
\begin{align*}
\dot{\vec{x}} &= A \vec{x} + B \vec{u}, \\
\vec{y} &= C \vec{x} + D \vec{u},
\end{align*}
\]

(5.2)

where

\[
A = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 \\ -1 & -\frac{1}{m} & -\frac{1}{m} \end{bmatrix},
\]

\[
C = \begin{bmatrix} 1 & 0 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\frac{1}{m} & -\frac{1}{m} \end{bmatrix}.
\]

The diagram of this state space representation is shown in Fig. 5.1.

The diagram of the augmented plant for vibration isolation control using $H_\infty$-optimization is illustrated in Fig. 5.2. The block $G_L$ denotes the mass-spring-damper model described by the state space representation (5.2). The block $K$ denotes the $H_\infty$ controller to be synthesized. The two signals $n_x$ and $n_a$ denote the position sensor noise and the accelerometer noise, respectively. The four signals $w_1$, $w_2$, $w_3$, and $w_4$ denote the exogenous inputs. The three signals $z_1$, $z_2$, and $z_3$ are to be minimized. $y_1$ and $y_2$ are the measured signals. $f_a$ is the control effort. The blocks $V_b$, $V_d$, $V_x$, and $V_a$ are characterizing filters. The blocks $W_a$, $W_r$, and $W_u$ are penalizing filters. They are all weighting filters to be designed.

Weighting filters design

As the order of the designed $H_\infty$ controller equals to the sum of the orders for the weighting filters and the model, the order of the weighting filters should be as low as possible. The characterizing filters for the exogenous inputs should describe the maximal signal strength
with respect to the frequency. $V_x$ and $V_a$ can be designed as constants that represent the corresponding sensor resolution. The position sensor resolution is $4 \mu m$ such that $V_x$ is designed to be $4 \times 10^{-6}$. The accelerometer sensor resolution is $1.3 \mu g \ (g$ is the gravitational acceleration) at frequencies less than 10 Hz. For this reason, $V_a$ is is designed to be $1.3 \times 10^{-5}$. The disturbance force is assumed to be white so that $V_d$ is chosen as a constant ($0.05 \text{ N}$). According to [4, 33], the velocity of the base vibration is more close to white noise compared with the base acceleration. For this reason, $V_b$ should be a first-order high pass filter. The gain of this filter is adjusted such that the base velocity is 0.02 m/s within the frequency range of $[0.01, 100]$ rad/s.

The payload should follow the motion of the base at low frequencies to avoid drifting away and the payload absolute motion should be minimized at high frequencies. For this reason, the penalty on relative displacement $x_r$, $W_r$, should be high at low frequencies and low at high frequencies. The penalty on payload absolute acceleration $a_a$, $W_a$, should be the opposite of $W_r$. Therefore, $W_r$ should be a low pass filter and $W_a$ should be a high pass filter. The actuator has limited bandwidth so that the control force $f_a$ should be penalized at high frequencies. Therefore, $W_u$ is designed to be a high-pass filter. The poles, zeros, and gains of these three penalty filters are tuned such that

- the transmissibility has a desired shape.
- the infinity norm of the compliance is minimized.
- $\gamma$ approaches one.

The final design of the filters are

\[
V_x = 4 \times 10^{-6}, V_a = 1.3 \times 10^{-5}, V_d = 0.05,
\]
\[
V_b = \frac{2(s+0.01)}{s+100}, W_a = \frac{12(s+40)}{s+100},
\]
\[
W_r = \frac{0.16(s+10^4)}{s+0.1}, W_a = \frac{50(s+10)}{s+10^4}.
\]

The derived $\gamma$ is 0.9216. The $H_\infty$ controller is two $6^{th}$-order transfer functions.

![Figure 5.1: The diagram of the state space representation.](image-url)
5.1. 1-DoF vibration isolation control

![Diagram of vibration isolation control](image)

**Figure 5.2:** The augmented plant diagram of vibration isolation control using $H_\infty$-optimization.

Closed-loop performance

The closed-loop performances and the corresponding upper bounds are plotted in Fig. 5.3. This upper bound is calculated by the derived $\gamma$ over the corresponding characterizing filter and the penalizing filter. At frequencies lower than 1 Hz, the bottleneck of further improving the performance is the transfer function $a_b \to x_r$. Within the frequency range [1,100] Hz, this bottleneck is the transmissibility $(a_b \to a_a)$. For higher frequencies (> 100 Hz), this bottleneck is the transfer function $f_d \to a_a$.

The closed-loop performance criterion are calculated and compared with the passive performance criterion in Fig. 5.4(a) - Fig. 5.5(b). As defined in Chapter 3, the relative sensitivity is defined as the transfer function from position sensor noise $n_x$ to payload absolute displacement $x_a$. The absolute sensitivity is defined as the transfer function from accelerometer noise $n_a$ to payload absolute displacement $x_a$. The infinity norm of the closed-loop compliance is -73 dB. For the closed-loop transmissibility, the cut-off frequency is about 1.5 Hz and the magnitude peak is 1.9 dB at 0.7 Hz. Its magnitude is -18.8 dB at 10 Hz. From 1.5 Hz to 15 Hz, the magnitude slope is -20 dB/dec. From 15 Hz to 100 Hz, the closed-loop transmissibility has lower magnitude than the passive transmissibility. The closed-loop transmissibility converges to the passive transmissibility at frequencies higher than 100 Hz.

5.1.2 Sliding surface control

The Power Spectral Density (PSD) ratios of the sensor noises over the base displacement are estimated as $|G_x(\omega)| = 0.1$ and $|G_a(\omega)| = 1$. The constraint constants are defined as

- $\omega_0 = 0.01$ Hz, $\omega_1 = 2$ Hz, $\omega_2 = 20$ Hz.
- $\varepsilon_0 = 1.4125$ (3 dB), $\varepsilon_1 = 1$ (0 dB), $\varepsilon_2 = 0.01$ (-40 dB).

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Figure 5.3: The closed-loop performances (solid) and the corresponding upper bound (dashed). This upper bound is calculated by the derived $\gamma$ over the corresponding characterizing filter and the penalizing filter.

Apply the optimization procedure described in Section 3.7.2, then the designed transmissibility is obtained as

$$T_d = \frac{111.2s^2 + 77.84s + 20.95}{s^2 + 12.07s + 111.2s^2 + 77.84s + 20.95}.$$  \hspace{1cm} (5.3)

The two transfer functions used to shape the sliding surface are designed as

$$\Lambda_1 = 1, \quad \Lambda_2 = \frac{s^2 + 12.07s}{111.2s^2 + 77.84s + 20.95}.$$  \hspace{1cm} (5.4)

The regulator is designed as

$$R = \frac{2.3 \times 10^8}{s(s+2000)} \frac{s^2 + 19.75s + 167.9}{s^2 + 75s + 2500}.$$  \hspace{1cm} (5.5)

The sliding surface controller is therefore a 5th-order transfer function ($R\Lambda_2$) and a 4th-order transfer function ($R\Lambda_1$).

The closed-loop performance criterion are calculated and compared with the passive performance criterion in Fig. 5.6(a) - Fig. 5.7(b). The infinity norm of the closed-loop compliance is -63.5 dB. The closed-loop compliance is improved from the passive compliance up to 100 Hz. At low frequencies ($< 1$ Hz), the closed-loop compliance magnitude
increasing rate is 20 dB/dec which is due to the integrator of the controller. The closed-loop compliance converges to the passive compliance at high frequencies (> 100 Hz) due to the decreasing regulator gain. At low frequencies (< 5Hz), the closed-loop compliance of the sliding surface control has higher magnitude than that of the $H_{\infty}$-optimization. Around 10 Hz, the closed-loop compliance of the sliding surface control has lower magnitude than that of the $H_{\infty}$-optimization. At high frequencies (> 100Hz), they both converge to the passive compliance.

For the closed-loop transmissibility, the cut-off frequency is about 0.7 Hz and the magnitude peak is 0.6 dB at 0.2 Hz. Its magnitude is -25 dB at 10 Hz. From 2 Hz to 10 Hz, the magnitude slope is approximately -40 dB/dec. From 10 Hz to 100 Hz, the closed-loop transmissibility flattens. The closed-loop transmissibility converges to the passive transmissibility at frequencies higher than 100 Hz. The closed-loop transmissibility has lower magnitude than the passive transmissibility within the frequency range 1 ~ 100 Hz. The closed-loop transmissibility of the sliding surface control has lower magnitude than that of
Chapter 5. Design process of vibration isolation control

5.1.3 Comparison of the two controllers

The magnitudes of the two designed controllers are plotted in Fig. 5.8. Both curves have similar shape. For $C_1$, the sliding surface controller has lower gain compared with the $H_\infty$ controller. It is consistent to the fact that the closed-loop compliance of the sliding surface control is not as good as that of the $H_\infty$ controller. For $C_2$, the sliding surface controller has higher gain from 6 Hz to 40 Hz compared with the $H_\infty$ controller. It is consistent to the fact that the closed-loop transmissibility of the sliding surface control is better than that of the $H_\infty$ controller. Both transmissibility and the compliance are improved from the passive system at the frequency range $1 \sim 100$ Hz.

The open-loop gain, as defined in (3.17), of the $H_\infty$ controller and the sliding surface controller are calculated according to (3.17). They are plotted in Fig. 5.9 and Fig. 5.10, respectively. The magnitude of the relative open-loop and the absolute open-loop, as defined
5.1. 1-DoF vibration isolation control

Figure 5.8: Magnitude comparison of the $H_\infty$ controller and the sliding surface controller. $C_1$ takes the relative displacement feedback and $C_2$ takes the payload absolute acceleration feedback.

in Section 3.3 are also plotted. For both controllers, the relative open-loop has higher gain at low frequencies ($< 1.5$ Hz) and the absolute open-loop has higher gain at high frequencies ($> 1.5$ Hz). Both open-loop gains start to decrease at about 10 Hz, which is also the frequency that the closed-loop transmissibility starts flattening. The cross-over frequency of the open-loop gain, or the control bandwidth, is about 53 Hz for the $H_\infty$ controller and about 50 Hz for the sliding surface controller. The phase margins of the $H_\infty$ controller and the sliding surface controller are $188^\circ$ and $93^\circ$, respectively. The gain margins of the $H_\infty$ controller and the sliding surface controller are both infinity. The sliding surface control does not optimize the compliance. The control gain at frequencies lower than the control bandwidth can be further increased which would further reduce the compliance and also the phase margin.

Remark that the relative open-loop itself is stable for both controllers. This property is good to have because the acceleration control loop can be shut off without compromising the closed-loop stability. However, it is not compulsory. This feature will be further explained in Chapter 9.

The open-loop gain of the sliding surface controller is calculated as $C_p R (\Lambda_1 + \Lambda_2 s^2)$ and plotted in Fig. 5.10. The open-loop magnitude with only the relative displacement loop ($\Lambda_2 = 0$) and the open-loop magnitude with only the absolute acceleration loop ($\Lambda_1 = 0$) are also plotted. The relative displacement loop has higher gain at low frequencies ($< 1.5$ Hz) and the absolute acceleration loop has higher gain at high frequencies ($> 1.5$ Hz). The open-loop gain starts to decrease at about 10 Hz, which is also the frequency that the closed-loop transmissibility starts flattening. The cross-over frequency of the total open-loop gain is about 50 Hz.
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Figure 5.9: Open-loop gain using the $H_\infty$ controller.

Figure 5.10: Open-loop gain using the sliding surface controller.
5.1. 1-DoF vibration isolation control

The robust stability is analyzed using the small gain theory, which is a similar method as in subsection 4.5.2. During the calculation of the interconnection matrix $H$, the identified high-order ARX model is assumed as the true plant. The magnitude of $H\Delta$ for the $H_\infty$ controller is plotted in Fig. 5.11(a). The magnitude of $H\Delta$ for the sliding surface controller is plotted in Fig. 5.11(b). It shows that each of the two controllers would stabilize the perturbed closed-loop. The sliding surface controller has less stability margin than the $H_\infty$ controller. Note that the two curve calculated from the 240-state model and the one calculated from the measured FRF has significant difference at low frequencies. The reason is that the derived second-order model has a relatively large difference compared with the measured FRF at low frequencies.

![Figure 5.11: Magnitude plot of the $H\Delta$.](image)

5.1.4 Conclusion

In general, the performances of the two control design methods are comparable. Compared with the sliding surface control, the $H_\infty$ control has a higher stability margin and a slightly higher bandwidth. It is consistent to the theory that the $H_\infty$ control is robust. The $H_\infty$ controller has only slightly higher order than the sliding surface controller. Compared with the $H_\infty$-optimization, the sliding surface control achieves much better transmissibility but the compliance at low frequencies has higher magnitude. This is consistent to the fact that the transmissibility is optimized with relatively lower conservativeness but the compliance is not optimized. The limiting factor of further increasing the bandwidth is the 380 Hz resonance.

In $H_\infty$-optimization, each set of the weighting filters results in one combination of the four performance criteria. Any weighting filter tuning will influence all four performance criteria. On the other hand, the sliding surface control design takes two steps. The first step designs the transmissibility and the two sensitivities by a numerical optimization process. The second step designs the compliance using classic SISO loop shaping. Using such a design process, the transmissibility and the compliance can be improved in two separated steps. Therefore, the sliding surface control is more straightforward and easier than $H_\infty$-optimization. Furthermore, the sliding surface control has less theoretical demanding on...
5.2 Multi-DoF vibration isolation control

This section takes the identified 3-DoF model in Chapter 4 as an example plant to study the vibration control of a multi-DoF system using both $H_{\infty}$-optimization and the sliding surface control with static optimal decoupling.

5.2.1 $H_{\infty}$-optimization

Like in the 1-DoF design, the structure of the augmented plant has to be prepared for the $H_{\infty}$-optimization. Subsequently, the weighting filters are designed to optimize the closed-loop performance. This process is similar to the 1-DoF case except that all signals become 3 dimensional vectors and all weighting filters become $3 \times 3$ transfer function matrices.

Augmented plant

The augmented plant that is required for $H_{\infty}$-optimization has two input vectors ($\vec{w}_a$ and $\vec{a}_b$) and two output vectors ($\vec{q}_r$ and $\vec{a}_d$). The transfer function matrix from $\vec{w}_a$ to the two output vectors can be experimentally identified. Identification of the transfer function matrix from $\vec{a}_b$ to the six outputs is practically difficult because practical active suspension systems are built on a solid base structure and measurement of its absolute displacement $\vec{a}_b$ is difficult. In this thesis, this augmented plant is constructed using the identified inertia matrix, damping matrix, and the stiffness matrix.

The inertia matrix, damping matrix, and the stiffness matrix are denoted by $M_p$, $D_p$, and $K_p$, respectively. The physical vectors are the payload absolute displacement $\vec{q}_a$, the relative displacement $\vec{q}_r$, the payload absolute acceleration $\vec{a}_a$, the base acceleration $\vec{a}_b$, the control wrench $\vec{w}_a$, and the disturbance wrench $\vec{w}_d$. The motion equation of the 3-DoF system is

$$M_p \ddot{\vec{q}}_a + D_p \dot{\vec{q}}_r + K_p \vec{q}_r = \vec{w}_a - \vec{w}_d. \quad (5.6)$$

It can be reformulated as

$$M_p \ddot{\vec{q}}_a + D_p \dot{\vec{q}}_r + K_p \vec{q}_r = \vec{w}_a - \vec{w}_d - M_p \dot{\vec{a}}_b. \quad (5.7)$$

Define the state vector $\vec{x} = [\vec{q}_r^T, \vec{a}_a^T]^T$, the input vector $\vec{u} = [\vec{a}_b^T, \vec{w}_d^T, \vec{w}_a^T]^T$, and the output vector $\vec{y} = [\vec{q}_r, \vec{a}_a]^T$, the state space representation is derived as

$$\begin{cases} \dot{\vec{x}} = A \vec{x} + B \vec{u} \\ \vec{y} = C \vec{x} + D \vec{u} \end{cases}, \quad (5.8)$$

where

$$A = \begin{bmatrix} 0 & 1 \\ -M_p^{-1} K_p & -M_p^{-1} D_p \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 0 \\ -1 & -M_p^{-1} & M_p^{-1} \end{bmatrix},$$

$$C = \begin{bmatrix} 1 \\ -M_p^{-1} K_p & -M_p^{-1} D_p \end{bmatrix}, D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -M_p^{-1} & M_p^{-1} \end{bmatrix}. $$

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5.2. Multi-DoF vibration isolation control

The diagram of the augmented plant for vibration isolation control using $H_{\infty}$-optimization is shown in Fig. 5.12. It is very similar to the 1-DoF case except that the control effort is filtered by a notch filter $F_N$ with a notch at 287 Hz.

$$F_N = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{s^2 + 27.77s + 3.249 \times 10^6}{s^2 + 1000s + 3.249 \times 10^6} \end{bmatrix}.$$  

In this way, the 287 Hz resonance can be filtered and higher control bandwidth can be achieved. The block $G_L$ denotes the mass-spring-damper model described by the state space representation (5.8). The block $K$ denotes the $H_{\infty}$ controller to be synthesized. The four signal vectors $\vec{w}_1$, $\vec{w}_2$, $\vec{w}_3$, and $\vec{w}_4$ denote the external inputs. The three signal vectors $\vec{z}_1$, $\vec{z}_2$, and $\vec{z}_3$ are to be minimized. $\vec{y}_1$ and $\vec{y}_2$ are the measured signal vectors. $\vec{w}_a$ is the control effort. The blocks $V_b$, $V_d$, $V_x$, $V_a$, $W_r$, and $W_u$ are weighting filters to be designed.

Figure 5.12: The augmented plant diagram of vibration isolation control using $H_{\infty}$-optimization.

Weighting filters design

The weighting filters are all diagonal $3 \times 3$ transfer function matrices. The design criterion is similar to that of the 1-DoF system. The designed weighting filters are

$$V_x = 10^{-5} \times \begin{bmatrix} 0.4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}, V_a = 10^{-5} \times \begin{bmatrix} 1.3 & 0 & 0 \\ 0 & 13 & 0 \\ 0 & 0 & 13 \end{bmatrix},$$

$$V_b = 2 \times \begin{bmatrix} \frac{s + 0.01}{s + 100} & 0 & 0 \\ 0 & \frac{s + 0.1}{s + 100} & 0 \\ 0 & 0 & \frac{s + 0.1}{s + 100} \end{bmatrix}, V_d = 10^{-3} \times \begin{bmatrix} 50 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix},$$
Chapter 5. Design process of vibration isolation control

\[ W_a = \begin{bmatrix} 4.2(s+40) & 0 & 0 \\ 0 & 3(s+20) & 0 \\ 0 & 0 & 1.5(s+20) \end{bmatrix} \frac{s+80}{s+100}, \quad W_u = \frac{10^3(s+20)^2}{(s+10^4)^2} \begin{bmatrix} 2.25 & 0 & 0 \\ 0 & 30 & 0 \\ 0 & 0 & 60 \end{bmatrix}, \]

\[ W_r = \begin{bmatrix} 0.45(s+10^4) & 0 & 0 \\ 0 & 1.8(s+10^4) & 0 \\ 0 & 0 & 3(s+10^4) \end{bmatrix} \frac{s+0.1}{s+1}. \]

The order of the entries of the penalizing filter \( W_a \) is increased to two. This is because the control effort requires higher-order low pass property at high frequencies to deal with the high frequency resonances. The derived \( \gamma \) is 0.8192. The designed \( H_\infty \) controller is a \( 3 \times 6 \) 23\textsuperscript{rd}-order transfer function matrix, which has 23 states.

Closed-loop performance

The control diagram of implementing the \( H_\infty \) controller is shown in Fig. 5.13. The block diagram is as follows:

Figure 5.13: The diagram of \( H_\infty \) controller implementation for vibration isolation.

\( C_1 \) is the negative of the designed \( H_\infty \) controller for the relative displacement feedback and \( C_2 \) is the negative of the designed \( H_\infty \) controller for the absolute acceleration feedback. The vectors \( \vec{a}_b \) and \( \vec{a}_a \) denote the base acceleration and payload acceleration, respectively. They are defined as

\[ \vec{a}_b = [a_{b_z}, a_{b\phi}, a_{b\theta}]^T, \quad \vec{a}_a = [a_z, a_{\phi}, a_{\theta}]^T. \]  

(5.9)

The passive compliance can be calculated by

\[ C_p = [M_p s^2 + D_p s + K_p]^{-1}, \]  

(5.10)

where \( M_p, D_p, \) and \( K_p \) are the inertia matrix, the damping matrix, and the stiffness matrix, respectively. They are identified in Section 4.4.

The passive transmissibility can be calculated by

\[ T_p = C_p(D_p s + K_p). \]  

(5.11)
5.2. Multi-DoF vibration isolation control

The closed-loop compliance is calculated by

\[ C_c = [I + C_p (C_1 + C_{2s^2})]^{-1} C_p, \]  

(5.12)

The closed-loop transmissibility is calculated by

\[ T_c = [I + C_p (C_1 + C_{2s^2})]^{-1} [C_p C_1 + T_p]. \]  

(5.13)

The closed-loop transmissibility is calculated and compared with the passive transmissibility in Fig. 5.14. For the diagonal entries, the closed-loop transmissibility is slightly improved from the passive transmissibility between 4 Hz and 30 Hz. For the off-diagonal entries, the closed-loop transmissibility has much lower magnitude up to 100 Hz.

The closed-loop compliance is calculated based on the 6-state model and compared with the passive compliance in Fig. 5.15. The resonant peak around 10 Hz is removed by the closed-loop control. The magnitude slope of the closed-loop compliance is not 20 dB/dec for extremely low frequencies. This is because the \( H_\infty \) controller does not have a perfect integrator. But the integrator can be added manually by moving some of the near-zero poles to zero. Similar to the transmissibility, the effectiveness of the closed-loop control is more significant for off-diagonal entries.

Fig. 5.16 plots the singular values of \( H_\Delta \). All singular values are below 0 dB which indicates that the closed-loop system is stable.

5.2.2 Sliding surface control

This thesis explores a design strategy of vibration isolation control which combines the sliding surface control as described in Chapter 3 and the optimal static decoupling as described in Chapter 2. This approach can be classified into the category of decentralized control. To apply this approach, optimal static decoupling is used to derive a pair of constant decoupling matrices to minimize the \( \mu \)-interaction measure. On top of that, each diagonal entry of the decoupled system is treated as a single-DoF suspension system to apply the sliding surface control. For the 3-DoF system described in Chapter 4, optimal static decoupling matrices have been derived and the diagonal entries of the decoupled system have been identified. Three sliding surface controllers are designed for the three identified second-order models. For the model \( f_{zo} \rightarrow q_{zo} \), the sliding surface design is described in subsection 5.1.2.

\[ \Lambda_1,z = 1, \quad \Lambda_2,z = \frac{s^2 + 12.07s}{111.2s^2 + 77.84s + 20.95}. \]  

(5.14)

The regulator is designed as

\[ R_z = \frac{2 \times 10^8}{s(s+2000)} \frac{s^2 + 19.75s + 167.9}{s^2 + 75s + 2500} \frac{(600\pi)^2}{(s+600\pi)^2}. \]  

(5.15)

The difference is that the new design includes a second-order low-pass filter to deal with the high frequency resonances. For the model \( t_{xo} \rightarrow q_{\phi o} \) and \( t_{yo} \rightarrow q_{\theta o} \), the control design is described as follows.

The Power Spectral Density (PSD) ratios of the sensor noises over the base displacement are estimated as \( |G_x(\omega)| = 0.3 \) and \( |G_{\phi}(\omega)| = 3 \). The constraint constants are defined as
Figure 5.14: Closed-loop transmissibility of $H_{\infty}$ control (dashed) and the passive transmissibility (solid).

Figure 5.15: Closed-loop compliance of $H_{\infty}$ control (dashed) and the passive compliance (solid).
5.2. Multi-DoF vibration isolation control

- \( \omega_0 = 0.01 \) Hz, \( \omega_1 = 5 \) Hz, \( \omega_2 = 50 \) Hz.
- \( \varepsilon_0 = 1.4125 \) (3 dB), \( \varepsilon_1 = 1 \) (0 dB), \( \varepsilon_2 = 0.01 \) (-40 dB).

Apply the optimization procedure described in subsection 3.7.2, the designed transmissibility is obtained as

\[
T_d = \frac{755.5s^2 + 1272s + 940.3}{s^4 + 34.58s^3 + 755.5s^2 + 1272s + 940.3}. \tag{5.16}
\]

The two transfer functions used to shape the sliding surface are designed as

\[
\Lambda_{1,\phi} = \Lambda_{1,\theta} = 1, \quad \Lambda_{2,\phi} = \Lambda_{2,\theta} = \frac{s^2 + 34.58s}{755.5s^2 + 1272s + 940.3}. \tag{5.17}
\]

The regulator design for the second model\( t_{xo} \rightarrow q_{\theta_o} \) is

\[
R_\phi = \frac{5 \times 10^3(s+1)}{s^2}. \tag{5.18}
\]

The regulator design for the third model\( t_{yo} \rightarrow q_{\theta_o} \) is

\[
R_\theta = \frac{5 \times 10^3(s+1)}{s^2} \frac{s^2 + 30.14s + 3.393 \times 10^6}{s^2 + 700s + 3.393 \times 10^6} \frac{(400\pi)^2}{(s + 400\pi)^2}. \tag{5.19}
\]

The difference between \( R_2 \) and \( R_3 \) is that \( R_3 \) has a second-order notch filter and a second-order low-pass filter to deal with the 287 Hz resonance. It is explained in Chapter 4 that the transfer \( t_{yo} \rightarrow q_{\theta_o} \) has significant high frequency resonances.
With respect to Fig. 5.13, the equivalent $C_1$ and $C_2$ for the sliding surface control with static optimal decoupling are

$$C_1 = T_u R \Lambda_1 T_y, \quad C_2 = T_u R \Lambda_2 T_y,$$

(5.20)

where

$$R = \begin{bmatrix} R_z & 0 & 0 \\ 0 & R_\phi & 0 \\ 0 & 0 & R_\theta \end{bmatrix}, \Lambda_1 = \begin{bmatrix} \Lambda_{1,z} & 0 & 0 \\ 0 & \Lambda_{1,\phi} & 0 \\ 0 & 0 & \Lambda_{1,\theta} \end{bmatrix}, \Lambda_2 = \begin{bmatrix} \Lambda_{2,z} & 0 & 0 \\ 0 & \Lambda_{2,\phi} & 0 \\ 0 & 0 & \Lambda_{2,\theta} \end{bmatrix}.$$

$T_u$ and $T_y$ are the two decoupling matrices derived in subsection 4.3.2. The sliding surface controller has 6 transfer functions and the order of these transfer functions range from 2 to 8.

The closed-loop performances are calculated based on the 6-state model and compared with the passive performances in Fig. 5.17 and Fig. 5.18. The closed-loop transmissibility is improved within the frequency range $1 - 100$ Hz and converges to the passive transmissibility above 100 Hz. The diagonal entries of the closed-loop transmissibility are 1 (0 dB) at low frequencies. For the off-diagonal entries, the closed-loop transmissibility is not as good as the passive transmissibility at low frequencies but the 10 Hz resonance is removed.

The closed-loop compliance, including the off-diagonal entries, is improved up to 100 Hz. At low frequencies ($< 1$ Hz), the closed-loop compliance increasing rate is 20 dB/dec which is due to the integrator of the controller. The closed-loop compliance is improved from the passive compliance up to 50 Hz. The closed-loop compliance converges to the passive compliance at high frequencies ($> 100$ Hz) due to the decreasing open-loop gain.

The robust stability of the decentralized control is analyzed using the $\mu$-interaction measure inverse based on the theory in [97]. The complementary sensitivity function is calculated using the measured FRF of the decoupled system and the frequency response of the controller. The inverse of the $\mu$-interaction measure of the decoupled system and the equivalent complementary sensitivity function of the three SISO loops are plotted in Fig. 5.19 and zoomed in Fig. 5.20. All equivalent complementary sensitivities of the three SISO loops are below the $\mu$-interaction measure, inverse which indicates that the closed-loop system is stable.

5.2.3 Comparison of the two controllers

The $H_\infty$ controller is a $3 \times 6$ 23$^{rd}$-order transfer function matrix. On the other hand, the sliding surface controller is a combination of 6 transfer functions and their order is less than or equal to eight. Therefore, the $H_\infty$ controller is more complex than the sliding surface controller. The closed-loop transmissibility designed by the $H_\infty$-optimization and the sliding surface control with static optimal decoupling are compared in Fig. 5.21. For the diagonal entries, the sliding surface control with static optimal decoupling gives better transmissibility. The reason is that the $H_\infty$-optimization is conservative. For the off-diagonal entries, the $H_\infty$-optimization gives generally better transmissibility. The reason is that the sliding surface control ignores the off-diagonal entries of the decoupled system while the $H_\infty$-optimization takes all entries of the original plant into account.
5.2. Multi-DoF vibration isolation control

Figure 5.17: Transmissibility of the sliding surface control with decoupling matrices for the 3-DoF suspension system. The solid curve is the passive transmissibility and the dashed curve is the closed-loop transmissibility.

Figure 5.18: Compliance of the sliding surface control with decoupling matrices for the 3-DoF suspension system. The solid curve is the passive compliance and the dashed curve is the closed-loop compliance.
Chapter 5. Design process of vibration isolation control

Figure 5.19: Comparison of the $\mu$-interaction measure inverse and the equivalent complementary sensitivity functions of the three SISO loops.

Figure 5.20: Zoomed Fig. 5.19 for the frequency range of 350~400 Hz.
5.3. Conclusions

The closed-loop compliance designed by the $H_\infty$-optimization and the sliding surface control with static optimal decoupling are compared in Fig. 5.22. It shows that the sliding surface control with static optimal decoupling gives generally better compliance.

5.2.4 Conclusion

The $H_\infty$-optimization and the sliding surface control with static optimal decoupling have been both applied to the 3-DoF system. The two strategies are compared in terms of controller complexity and closed-loop performance. Compared with the $H_\infty$-optimization, the sliding surface control with static optimal decoupling has an easier design process and results in much lower order controller. The residue cross-coupling of the decoupled system has been ignored during the sliding surface control. Nevertheless, it gives generally better performance compared with the $H_\infty$-optimization. The main reason is the conservativeness of the $H_\infty$ control theory. According to experience, the numerical solution given by the Matlab Robust Control Toolbox [8] is not always reliable for complicated augmented plants.

5.3 Conclusions

Vibration isolation control design has been studied using a 1-DoF model and a 3-DoF model as example plants. Two control strategies, $H_\infty$-optimization and sliding surface control with static optimal decoupling, have been studied. They have been compared in terms of design process, controller complexity, as well as the achieved performance.

For the 1-DoF vibration isolation control, the comparison of the two control design methods is summarized in Table 5.1. The performances of the two control design methods are comparable. Compared with the sliding surface control, the $H_\infty$ control has a higher stability margin and a slightly higher bandwidth. It is consistent to the theory that the $H_\infty$ control is robust. The $H_\infty$ controller has only slightly higher order than the sliding surface controller. Compared with the $H_\infty$-optimization, the sliding surface control achieves much better transmissibility but slightly worse compliance at low frequencies. This is consistent to the fact that the transmissibility is optimized with relatively lower conservativeness but the compliance is not optimized. The limiting factor of further increasing the bandwidth is the 380 Hz resonance.

In $H_\infty$-optimization, each set of the weighting filters results in one combination of the four performance criteria. Any weighting filter tuning will influence all four performance criteria. On the other hand, the sliding surface control design takes two steps. The first step designs the transmissibility and the two sensitivities by a numerical optimization process. The second step designs the compliance using classic SISO loop shaping. Using such a design process, the transmissibility and the compliance can be improved in two separated steps. Therefore, the sliding surface control is more straightforward and easier than $H_\infty$-optimization. Furthermore, the sliding surface control has less theoretical demanding on the control engineer than the $H_\infty$-optimization.

For the 3-DoF vibration isolation control, the comparison of the two control design methods is summarized in Table 5.2. The $H_\infty$-optimization requires an augmented plant which is constructed from the identified inertia matrix, damping matrix, and stiffness matrix in this thesis. It depends on weighting filters design to optimize the closed-loop performance. The complexity of the weighting filter tuning is increased quadratically from
Chapter 5. Design process of vibration isolation control

Figure 5.21: Transmissibility comparison of the two control design. The solid curve is the transmissibility of the $H_{\infty}$ controller. The dashed curve is the transmissibility of the sliding surface controller with static optimal decoupling.

Figure 5.22: Compliance comparison of the two control design. The solid curve is the compliance of the $H_{\infty}$ controller. The dashed curve is the compliance of the sliding surface controller with static optimal decoupling.
the 1-DoF $H_\infty$-optimization because tuning of a single filter would affect the closed-loop performance of all the 3-DoF. On the other hand, the sliding surface control for the 3-DoF design has three times more work than the 1-DoF design but the complexity is not increased. Furthermore, the designed $H_\infty$ controller is a $3 \times 6$ 23rd-order transfer function matrix, which is much more complex than the sliding surface controller (six transfer functions with their order equal to or less than eight). Above all, the sliding surface control with static optimal decoupling requires less cost and is less complex than the direct MIMO control design using $H_\infty$-optimization.

For the 3-DoF vibration isolation design, the general performance of the $H_\infty$-optimization is no better than the sliding surface control with static optimal decoupling. The $H_\infty$-optimization gives better transmissibility for only the off-diagonal entries. On the other hand, the sliding surface control with static optimal decoupling gives better transmissibility for the diagonal entries. The reason is that the sliding surface control ignores the off-diagonal entries of the decoupled system while the $H_\infty$-optimization takes all entries of the original plant into account. Furthermore, the sliding surface control with static optimal decoupling gives generally better compliance. It can be concluded that sliding surface control with static optimal decoupling has higher performance-cost ratio comparing with direct MIMO control design using $H_\infty$-optimization.

<table>
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Table 5.1: Comparison of control design for a 1-DoF suspension system.

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Table 5.2: Comparison of control design for a 3-DoF suspension system.
Chapter 6

Implementation of vibration isolation control on the 3-DoF demonstrator

A 3-DoF suspension system, which is modified from the 3-DoF system introduced in Chapter 4, is used to demonstrate multi-DoF vibration isolation control. Both control strategies, direct MIMO control design ($H_{\infty}$-optimization) and decoupling-based decentralized control (sliding surface control with static optimal decoupling), are practically implemented and demonstrated. Both performance criteria, compliance and transmissibility, are measured and compared with the performance criteria of passive vibration isolation.

6.1 The 3-DoF suspension system

6.1.1 System description

The 3-DoF system introduced in Chapter 4 is modified to validate the control design for multi-DoF vibration isolation. The mass of the payload is less than 4 kg and the mass of the supporting base-frame is approximately 300 kg. Three acceleration sensors (Kistler 8330) are rigidly fixed to the payload to measure the 3-DoF payload absolute acceleration. As the acceleration sensors have their own mass, this addition will change the payload mass and moment of inertias. Also, the distribution of the payload mass is changed. The analog current amplifiers are replaced by digital current amplifiers (from Prodrive). The dSpace system used to implement the controller is replaced by a set of xPC system (from Speedgoat). The control signals generated by the xPC target machine are sent to the digital current amplifiers via serial communication (RS422). The sampling frequency is 3 kHz.

Because of so many hardware changes, the 3-DoF system is identified again using exactly the same procedure introduced in Chapter 4. The Frequency Response Function (FRF) is measured again and the static optimal decoupling matrices are also recalculated. The FRF is measured manually instead of using Siglab, which is an automatic FRF measurement device. First, input and output signals are recorded in experiments. Second, the FRF is estimated by calculating the Cross Power Spectral Density (CPSD) of these input and output signals using the Welch’s averaged periodogram method which is commercially available in Matlab Signal Processing Toolbox [76].

The re-identified 240-state model is compared with the measured FRF in Fig. 6.1. They
have a good match at most of the frequency region except at the very low frequencies (< 0.5 Hz). This might be due to insufficient excitation around these frequencies during the FRF measurement.

The re-identified inertia matrix, damping matrix, and the stiffness matrix are

$$\mathbf{M}_p = \begin{bmatrix} 3.1484 & -0.0226 & -0.0024 \\ -0.0210 & 0.0253 & -0.0001 \\ -0.0015 & 0.0001 & 0.0266 \end{bmatrix}, \quad (6.1a)$$

$$\mathbf{D}_p = \begin{bmatrix} 57.1607 & -0.7418 & -0.0164 \\ -0.6667 & 0.2682 & 0.0045 \\ 0.0956 & 0.0062 & 0.3438 \end{bmatrix}, \quad (6.1b)$$

$$\mathbf{K}_p = \begin{bmatrix} 424.9948 & 29.9514 & 21.6203 \\ 59.0395 & 74.6115 & -3.6705 \\ 13.0490 & -2.2771 & 118.0839 \end{bmatrix}. \quad (6.1c)$$

The re-identified physical parameters are slightly smaller than the previously identified values in Chapter 4. This is probably because the gains of the current amplifiers are different. According to these re-identified physical parameters, the mass of the payload is 3.15 kg.
6.1.2 3-DoF shaker table

To validate the vibration isolation performance, the 3-DoF suspension system is fixed on a 3-DoF shaker table (table mass is approximately 300 kg), shown in Fig. 6.2. Fig. 6.3 shows the front view of this shaker table.

Figure 6.2: The photo of the total setup.

Figure 6.3: The photo of the shaker table (front view from the endpoint 3).

The drawings of the shaker table in Fig. 6.4 shows the top view. The three endpoints of the table are labeled by numbers one to three. The origin of the coordinate system is defined as the geometric center. The two horizontal axes are labeled in Fig. 6.4. The shaker table is supported by three mechanical springs and excited by three Shaker Voice Coil Actuators.
6.1. The 3-DoF suspension system

(SVCAs). Fig 6.5 shows the top view of the drawings with the table invisible. Each SVCA has force output capability of 500 N continuously up to 1 kHz. The shaker table is designed to be symmetrical. The three springs and the three SVCAs both forms a regular triangle. The table is actuated to provide 3-DoF vibrations: the vertical translation and the rotations about the two horizontal axes. Three acceleration sensors are used to measure the 3-DoF table accelerations.

6.1.3 Data acquisition of acceleration

The base-frame (70 kg) of the 3-DoF suspension system is clumped to the shaker table (300 kg). Additional acceleration sensors are rigidly fixed on the top of the three Voice Coil Actuators (VCA) for control purpose (VCA-1, VCA-3, and VCA-8). They are used to measure the vertical acceleration of the three VCA translators. The sensitivity of the acceleration sensors is $1.2 \, \text{V/g}$ ($\text{g}$ denotes the gravitational acceleration) and the resolution is $1.3 \, \mu\text{g}$ which indicates that the sensor noise is about $15 \, \mu\text{V}$. The measurement range of the 16-bit ADC is $\pm 10 \, \text{V}$ so that the lowest significant bit is corresponding to about $0.3$ mV, which is much higher than the sensor noise. If the sensor output is directly connected to the ADC, the measured data would be mostly the ADC noises. Therefore, analog signal filters (Krohn-Hite 3360 series) are used to amplify the acceleration signal by a factor of 50. This signal amplification factor will be compensated digitally in the xPC target machine. According to the specifications, the accelerometer output signal has a DC bias up to 0.25 V when it is aligned to measure horizontal accelerations. This DC bias would be increased to a level of 1.4 V if the accelerometer is aligned to measure the vertical acceleration. The reason is that the accelerometer measures the gravity force of its own components. An RC circuit is used to filter the DC bias before acceleration signal is amplified. The equivalent transfer function of the RC circuit is

$$F_{RC} = \frac{s}{s + (RC)^{-1}}.$$  \hfill (6.2)

The advantage of low $(RC)^{-1}$ is that the acceleration measurement can be achieved at lower frequencies. The disadvantage is that the filter needs longer time to reach its steady state when the system is powered up. The total resistance $R$, taking the input impedance of the signal amplifiers into account, is about $0.8 \, \text{M}\Omega$ and the capacitance $C$ is about $6 \, \mu\text{F}$. So the value of $(RC)^{-1}$ is about 0.21. The time constant is about 5 s.

The drawback of amplifying the acceleration signal is that the measurement range of each channel is reduced to about $\pm 1.6 \, \text{m/s}^2$. The acceleration of the payload has very high amplitude around the resonance frequencies ($> 100 \, \text{Hz}$). The measured acceleration signal can be easily clipped because of the limited range of the acceleration measurement. This nonlinear behavior could affect the closed-loop stability. To avoid this problem, the analog signal filters are set to low-pass filters. The cut-off frequency is set to 300 Hz. The equivalent transfer function of the signal filter is

$$F_L = \frac{1.2624 \times 10^{13}}{(s^2 + 3483s + 3.553 \times 10^6)(s^2 + 1443s + 3.553 \times 10^6)}.$$  \hfill (6.3)
Chapter 6. Implementation of vibration isolation control on the 3-DoF demonstrator

Figure 6.4: The drawings of the shaker table (top view).

Figure 6.5: The drawings of the support of the shaker table (top view). The table is invisible.
6.1. The 3-DoF Suspension System

6.1.4 Vibration Isolation Control Implementation

Fig. 6.6 shows the block diagram of the vibration isolation control implementation on the 3-DoF suspension system. $C_1$ and $C_2$ denote the two designed controllers for each loop. $q_{Ts}$ and $f_{Tw}$ are coordinate transformation matrices. $\vec{a}_s$ and $\vec{q}_s$ are the direct output signal vectors directly from the acceleration sensors and the displacement sensors. $\vec{n}_a$ and $\vec{n}_x$ are the corresponding sensor noises. The block $RC$ denotes the RC circuits used to filter the acceleration sensor DC bias. The block $F_L$ denotes the signal amplifiers. The blocks inside the dashed rectangular are all implemented on the xPC target machine. The digital current amplifiers are used to produce electric current which flows through the VCAs on the 3-DoF system according to the command signal from the xPC target machine. As the bandwidth of these current amplifiers are in the order of kHz, they may be assumed to be ideal devices at frequencies lower than 1 kHz. The current amplifiers for the SVCAs are also from Prodrive but they are capable of higher power output and are also commanded by the xPC target machine via serial communication (RS422). The sampling rate of the performance validation experiments is 3 kHz.

Remark 6.1.1. The 3-DoF suspension system has three inputs (represented by the vector $\vec{w}_a$) and six outputs, represented by the relative displacement vector $\vec{q}_r$, and the payload absolute acceleration vector $\vec{a}_a$. For this reason, the 3-DoF suspension system is treated as two square systems ($\vec{w}_a \rightarrow \vec{q}_r$ and $\vec{w}_a \rightarrow \vec{a}_a$) during static optimal decoupling. In theory, these two square systems are different and should be decoupled by different pairs of decoupling matrices. In practice, it may be assumed that the two pairs of decoupling matrices are the same. The reasons are explained as follows.

The relative displacement $\vec{q}_r$ is defined as payload absolute displacement $\vec{q}_a$ minus the base absolute displacement $\vec{q}_b$. In practice, the active suspension systems are built on a base frame which is solid and rigid. As a result, the control force from the actuators can hardly induce any base displacements. In other words, the transfer function matrix $\vec{w}_a \rightarrow \vec{q}_b$ may be ignored because its magnitude is much lower than $\vec{w}_a \rightarrow \vec{a}_a$. Therefore, $\vec{w}_a \rightarrow \vec{q}_r$ may be assumed to be equal to $\vec{w}_a \rightarrow \vec{q}_a$ in practice. As $\vec{w}_a \rightarrow \vec{q}_a = \vec{w}_a \rightarrow \vec{a}_a \vec{s}_a^2$, the two transfer function matrices can be decoupled by the same pair of matrices. To conclude, the two
square systems $\vec{w}_a \rightarrow \vec{q}_r$ and $\vec{w}_a \rightarrow \vec{a}_a$ can be decoupled by the same pair of matrices.

For this 3-DoF suspension system, the shaker table and the base frame of the 3-DoF system on its top have a total mass of 370 kg. They are supported by three mechanical springs and the stiffness of each spring is $9 \times 10^2$ N/mm. On the other hand, the payload is only 3 kg. The magnitude of $\vec{w}_a \rightarrow \vec{q}_b$ is less than 1% of $\vec{w}_a \rightarrow \vec{q}_a$. Therefore, $\vec{w}_a \rightarrow \vec{q}_r = \vec{w}_a \rightarrow \vec{q}_a$ may be assumed in this case.

6.1.5 Performance measurement process

The two performance criteria, compliance and transmissibility, are measured separately. This procedure is described more extensively in the following sections. It is briefly introduced here. During the compliance measurement, the shaker table is rigidly fixed to the floor. White noise force disturbance is injected to each DoF of the closed-loop system separately. This disturbance injection is large enough to ignore the floor vibrations and the environment disturbance forces. All the relative displacement signals are recorded as the output response. The FRF of the compliance is calculated taking the disturbance injection as input and the relative displacement response as output. The measured FRF matrix of the 3-DoF suspension system is the passive compliance.

During the transmissibility measurement, the disturbance excitation is set to zero and the shaker table is fixed to the floor only by the three mechanical springs. The three SVCAs are used to excite vibrations on the shaker table for each DoF. The FRF of the transmissibility is calculated taking the table acceleration as input and the payload acceleration as output. It will be explained in the following sections that vibration excitation on only one DoF is not possible due to the cross-coupling of the shaker table. Therefore, the transmissibility measurement is not as accurate as the compliance measurement.

6.2 Compliance validation

The measured FRF and the identified model of the 3-DoF suspension system shown in Fig. 6.1 are both representation of the passive compliance. The closed-loop compliance is measured using the same method as the FRF measurement described in Chapter 4. To validate the two designed controllers, $H_\infty$ controller and the sliding surface controller with static optimal decoupling, the FRF of the closed-loop compliance is measured and compared with the theoretical calculations.

6.2.1 Sliding surface control

The sliding surface design for each DoF is exactly the same as described in Chapter 5.

$$\Lambda_1 = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \Lambda_2 = \begin{bmatrix} \frac{s^2+12.07s}{111.2s^2+77.84s+20.95} & 0 & 0 \\ 0 & \frac{s^2+34.58s}{755.5s^2+1272s+940.3} & 0 \\ 0 & 0 & \frac{s^2+34.58s}{755.5s^2+1272s+940.3} \end{bmatrix}.$$
6.2. Compliance validation

The regulator is also similar except that the low-pass filters are removed because all measured acceleration signals are filtered by the analog low-pass filter $F_L$ (6.3).

$$R = \begin{bmatrix}
\frac{2 \times 10^8}{s(s+2000)} & \frac{s^2 + 19.75s + 167.9}{s^2 + 75s + 2500} & 0 & 0 \\
0 & \frac{5 \times 10^4(s+1)}{s^2} & 0 & 0 \\
0 & 0 & \frac{5 \times 10^3(s+1)(s^2 + 30.14s + 3.393 \times 10^6)}{s^2 + 1000s + 3.393 \times 10^6}
\end{bmatrix}.$$

Note that the regulator still includes a notch filter $F_{\text{notch}}$. It is used to filter the 287 Hz resonance of the transfer $t_{yo} \rightarrow q_{\theta o}$.

$$F_{\text{notch}} = \frac{s^2 + 30.14s + 3.393 \times 10^6}{s^2 + 1000s + 3.393 \times 10^6}.$$  (6.4)

The theoretical closed-loop compliance is calculated by

$$C_c = [I + C_pC_{ss}]^{-1}C_p,$$  (6.5)

where $C_p$ is the identified passive compliance and $C_{ss}$ is the equivalent controller, given by

$$C_{ss} = C_1 + C_2F_Ls^2.$$

The $F_L$ is the transfer function for the analog low-pass filter given in (6.3). The $C_1$ and $C_2$ are the two controllers for the two loops, indicated in Fig. 6.6. For the sliding surface control with static optimal decoupling, they are calculated by

$$C_1 = T_uR\Lambda_1T_y.$$  (6.6a)

$$C_2 = T_uR\Lambda_2T_y.$$  (6.6b)

$T_u$ and $T_y$ are the constant matrices derived by static optimal decoupling.

The theoretical closed-loop compliance is compared with the passive compliance in Fig. 6.7. The closed-loop compliance has lower magnitude than the passive compliance up to 30 Hz and it converges to the passive compliance at high frequencies ($> 80$ Hz). With the frequency range $30 \sim 80$ Hz, the closed-loop compliance has a little higher magnitude than the passive compliance due to the analog fourth-order low-pass filters.

The FRF matrix of the closed-loop compliance is measured and compared with that of the theoretically calculated closed-loop compliance in Fig. 6.8. All entries are reasonably close except that two off-diagonal entries ($f_z \rightarrow q_\theta$ and $t_x \rightarrow q_\theta$) have some difference at the mid frequency range. It will be shown in the following subsection that the similar inconsistency also occurs at the $H_\infty$ control. It is probably due to lack of excitation at these frequencies during FRF measurement.

6.2.2 $H_\infty$ control

The weighting filters design for the $H_\infty$-optimization has the same augmented plant with the one described in Chapter 5. Only some gains of the penalty filters are adjusted. To increase the bandwidth, a notch filter is applied to filter the 287 Hz resonance. The designed controller is converted to the Jordan Canonical Form. Subsequently, three of the diagonal
Chapter 6. Implementation of vibration isolation control on the 3-DoF demonstrator

Figure 6.7: The passive compliance (solid) and the theoretical closed-loop compliance (dashed) using sliding surface control with static optimal decoupling.

Figure 6.8: Compliance validation of the sliding surface control with static optimal decoupling. The solid curve is the measured FRF of the closed-loop compliance. The dashed curve is the theoretical calculated closed-loop compliance based on the 240-state model.
entries in the A-matrix are manually set to zero. This is to derive the integration action to achieve the zero compliance at zero frequency. The transfer function representation of the designed controller has the form of $C_1$ and $C_2$ as in Fig. 6.6. The $H_\infty$ controller is implemented together with a notch filter, which is used to filter the measured acceleration signal $q_\theta$.

The theoretical closed-loop compliance is compared with the passive compliance in Fig. 6.9. Compared with the passive compliance, the closed-loop compliance removes the 8 Hz resonance and the magnitude-frequency ratio is 20 dB/dec at low frequencies. For two of the diagonal entries, $t_x \rightarrow q_\phi$ and $t_y \rightarrow q_\theta$, the closed-loop compliance has higher magnitude than the passive compliance within $2 \sim 6$ Hz. The improvement is also significant for most of the off-diagonal entries (except $t_z \rightarrow q_z$). However, the closed-loop compliance is not as good as that of the sliding surface control with static optimal decoupling.

The FRF matrix of the closed-loop compliance is measured and compared with that of the theoretically calculated closed-loop compliance in Fig. 6.10. All entries are reasonably close except that some off-diagonal entries ($f_z \rightarrow q_\theta$ and $t_x \rightarrow q_\theta$) have some difference at the mid frequency range. Comparing with Fig. 6.8, this inconsistency occurs at similar frequency range. It is probably due to insufficient excitation at these frequencies during FRF measurement.

6.2.3 Conclusive remarks

The closed-loop compliance has been validated in experiments for both designed controllers. Generally, the predicted compliance using controller and model matches the measured compliance well. There are only slightly mismatch for the two off-diagonal entries ($f_z \rightarrow q_\theta$ and $t_x \rightarrow q_\theta$). They can not be accurately measured because their magnitudes are more than 30 dB lower than the diagonal entries.

The order of the sliding surface controller is less than or equal to six. The $H_\infty$ controller is a $3 \times 6$ 23rd-order transfer function matrix. The sliding surface control with static optimal decoupling provides much better closed-loop compliance than the $H_\infty$-optimization with relatively lower order. It is consistent with the theory that $H_\infty$-optimization is robust but conservative. Theoretically, the $\mu$ synthesis is able to reduce the conservativeness but the designed controller would have even higher order.

6.3 Transmissibility validation

6.3.1 Shaker table dynamics

The transmissibility is measured by exciting vibrations on the shaker table where the base frame of the 3-DoF suspension system is fixed. The photo of the shaker table is shown in Fig. 6.3. It consists of one rigid table, three mechanical springs, and three SVCAs. Three acceleration sensors are used to measure the 3-DoF acceleration of the shaker table. The allocation of the springs and actuators is symmetric about the geometric center of the table. The table mass is approximately 320 kg and the spring stiffness is approximately $9 \times 10^2$ N/mm. The excitation wrench vector for the shaker table is denoted by $\vec{w}_e = [f_{bz}, t_{bx}, t_{by}]^T$. The acceleration output vector of the shaker table is denoted by $\vec{a}_b = [a_{bz}, a_{b\phi}, a_{b\theta}]^T$.

The 3-DoF FRF from the control wrench to the acceleration is measured and plotted in
Figure 6.9: The passive compliance (solid) and the theoretical closed-loop compliance (dashed) using $H_\infty$ control.

Figure 6.10: Compliance validation of the $H_\infty$ control. The solid curve is the measured FRF of the closed-loop compliance. The dashed curve is the theoretical calculated closed-loop compliance based on the 240-state model.
6.3. Transmissibility validation

Fig. 6.11. The damping induced by the three SVCAs is very weak. As a result, the FRF have very high peak at the first resonant frequency (natural frequency). To measure the transmissibility of the 3-DoF suspension system, the table acceleration is preferred to be white. For this reason, the classic skyhook control is applied to reduce the resonance peak of the shaker table. The skyhook control is the proportional control of the absolute velocity, which is derived by integration the measured acceleration signal. The FRF of the skyhook-controlled shaker table is measured and compared with the passive FRF in Fig. 6.11. It shows that the skyhook control is quite effective on reducing the resonance peak around 10-20 Hz. Nevertheless, the shaker table has significant cross-coupling.

The measured FRF shows significant nonlinear behavior around the natural frequency. The reason is that the coils and the magnets of the SVCAs are not aligned to the center position of the vertical linear range. The shaker table is designed for the suspension system with 700 kg payload. The gravity force of the total suspension system and the table would compress the springs. The equilibrium table-position determined by the total gravity force and the spring force is designed such that the coils and the magnets of the SVCAs are aligned to the center position of the linear range. The mass of the current 3-DoF suspension system is only 70 kg. It is not heavy enough to compress the springs so that the coils and magnets of the SVCAs are aligned to the boundary of the linear range. When the table is excited at the resonant frequency, the table displacement at the vertical direction is very large. The translators of the SVCAs can be easily positioned out of the linear range of the SVCAs. As a result, the force output of the SVCAs would not be linear with the current input. It induces significant nonlinear behavior for the shaker table.

6.3.2 Passive transmissibility

The passive transmissibility of the 3-DoF suspension system can be calculated by

$$ T_p = C_p [D_p s + K_p] $$

(6.7)

where $C_p$ is the passive compliance. $D_p$ and $K_p$ are the damping matrix given in (6.1b) and the stiffness matrix given in (6.1c), respectively.

The passive transmissibility can be measured in experiments. The constant forces provided by the five VCAs are adjusted to levitate the payload. Subsequently, the shaker table is separately excited for each DoF to provide low-pass filtered white noise acceleration. The FRF of the passive transmissibility is calculated using table acceleration as input and the payload acceleration as output. The measured passive transmissibility is compared with the theoretical calculations in Fig. 6.12. The diagonal entries are reasonably close at low frequencies. For the off-diagonal entries, the measured FRF has significant higher magnitude than the theoretical calculations. The measurement errors are mainly due to the cross-coupling and the nonlinearity of the shaker table.

Due to this nonlinear behavior, sinusoidal shaker table vibration is difficult to create. Fortunately, Gaussian white noise can be used to excite the shaker vibration for transmissibility measurement instead of sinusoidal excitation. It is possible that vibrations at certain frequency range can not be sufficiently excited. As a consequence, transmissibility measurement has relatively larger error at these frequencies.
Chapter 6. Implementation of vibration isolation control on the 3-DoF demonstrator

Figure 6.11: The measured FRF of the 3-DoF shaker table (wrench to acceleration). The solid curve is the FRF of the passive system and the dashed curve is the FRF of the skyhook-controlled system.

Figure 6.12: The passive transmissibility of the 3-DoF suspension system. The solid curve is measured in experiments and the dashed curve is calculated based on the identified compliance.
### 6.3. Transmissibility validation

#### 6.3.3 Sliding surface control

The theoretical closed-loop transmissibility is calculated by

\[ T_c = \left[ I + C_p (C_1 + C_2 F_L s^2) \right]^{-1} \left[ C_p C_1 + T_p \right], \tag{6.8} \]

where \( C_p \) and \( T_p \) are the passive compliance and transmissibility, respectively. \( C_1 \) and \( C_2 \) are the two controllers as indicated in Fig. 6.6 and they are given in (6.6).

The theoretical closed-loop transmissibility is compared with the passive transmissibility in Fig. 6.13. At high frequencies \( > 100 \) Hz, the closed-loop transmissibility converges to the passive transmissibility. The diagonal entries of the closed-loop transmissibility are 1 (0 dB) at low frequencies and are improved at the mid frequencies. For the off-diagonal entries, the closed-loop transmissibility eliminates the 10 Hz resonance from the passive transmissibility.

The measured FRF of the closed-loop transmissibility and the passive transmissibility are compared in Fig. 6.14. The closed-loop transmissibility has lower magnitude from 1 Hz up to 30 Hz. The improvement is the most significant around 10 Hz.

#### 6.3.4 \( H_\infty \) optimization

The theoretical closed-loop transmissibility is compared with the passive transmissibility in Fig. 6.15. At high frequencies \( > 100 \) Hz, the closed-loop transmissibility converges to the passive transmissibility. The diagonal entries of the closed-loop transmissibility are 1 (0 dB) at low frequencies and are improved at the mid frequencies. For the off-diagonal entries, the closed-loop transmissibility eliminates the 10 Hz resonance from the passive transmissibility. The diagonal entries are not as good as that of the sliding surface control with static optimal decoupling but most of the off-diagonal entries have lower magnitude. This is because the static optimal decoupling is still not perfect decoupling.

The measured FRF of the closed-loop transmissibility and the passive transmissibility are compared in Fig. 6.16. The closed-loop transmissibility has lower magnitude from 1 Hz up to 30 Hz. The improvement is the most significant around 10 Hz. Based on the measured FRF, the \( H_\infty \) control does not perform as good as the sliding surface control with static optimal decoupling on vibration isolation.

#### 6.3.5 Conclusive remarks

The closed-loop transmissibility has been validated in experiments for both controllers. Due to the cross-coupling and the nonlinearity of the shaker table, the off-diagonal entries of the transmissibility can not be accurately measured. Therefore, the measured FRF of the closed-loop transmissibility is compared with the measured FRF of the passive transmissibility. This comparison is reasonable because the excited table acceleration has consistent power spectrum in all measurement.

The sliding surface control with static optimal decoupling provides better transmissibility on the diagonal entries. The most likely reason is that the \( H_\infty \) control is conservative. For off-diagonal entries, \( H_\infty \) control provides lower transmissibility magnitude according to theoretical calculations. Unfortunately, this could not be validated by measurements due to the cross-coupling and the nonlinearity of the shaker table.
Chapter 6. Implementation of vibration isolation control on the 3-DoF demonstrator

Figure 6.13: The theoretically calculated passive transmissibility (solid) and closed-loop transmissibility (dashed) using the sliding surface control with static optimal decoupling.

Figure 6.14: The measured FRF of the passive transmissibility (solid) and the closed-loop transmissibility (dashed) using the sliding surface control with static optimal decoupling.
6.3. Transmissibility validation

Figure 6.15: The theoretically calculated passive transmissibility (solid) and closed-loop transmissibility (dashed) using the $H_\infty$ control.

Figure 6.16: The measured FRF of the passive transmissibility (solid) and the closed-loop transmissibility (dashed) using the $H_\infty$ control.
Chapter 6. Implementation of vibration isolation control on the 3-DoF demonstrator

6.4 Conclusions

The closed-loop performance of the two control strategies, sliding surface control with static optimal decoupling and the $H_\infty$-optimization, are measured in experiments and are compared with their theoretical curves. According to the theoretical calculations, the former control strategy provides better compliance and diagonal entries of the transmissibility although the static optimal decoupling is not perfect decoupling. The most likely reason is that the $H_\infty$-optimization is conservative. $H_\infty$ control provides lower transmissibility on off-diagonal entries according to theoretical calculations. Unfortunately, this could not be validated by measurements due to the cross-coupling and the nonlinearity of the shaker table.

$H_\infty$-optimization using the Matlab Robust Control Toolbox [8] is not an analytical solution but a numerical process. This numerical process sometimes can not converge to a reasonable solution in the 3-DoF vibration isolation control design. It can be expected that this convergence problem of the numerical process would become more serious for the 6-DoF vibration isolation control design. Therefore, the sliding surface control with static optimal decoupling is a good alternative for the 6-DoF vibration isolation control design.
Part III

System design, behavior, and performance of the contactless electromagnetic suspension system
Chapter 7

System-level design and static measurements of the contactless electromagnetic suspension system

7.1 Introduction

As introduced in Chapter 1, the Single Electro-Magnetic Isolator System (SEMIS) is designed to serve two purposes:

1. Verification of the predicted static and dynamic properties of the realized PM-based gravity compensator, including high vertical passive force, low stiffness, low passive damping, contactless design, and suitability for high-bandwidth control;
2. Establishing a feasibility proof of a contactless electromagnetic suspension system which explores the full performance potential of Permanent Magnet (PM) based gravity compensator.

To complete these two tasks, the single-gravity-compensator concept is proposed as the system-level design for SEMIS. This chapter describes the single-gravity-compensator concept and verifies three properties of the PM-based gravity compensator: high vertical passive force, low stiffness, and contactless design.

The single-gravity-compensator concept is proposed as the system-level design of SEMIS because only one electromagnetic isolator is realized. In fact, this single-gravity-compensator concept is also applicable to the pneumatic suspension systems by employing a pneumatic isolator [41] instead of the electromagnetic isolator. Therefore, this single-gravity-compensator concept is compared with conventional multi-gravity-compensator concepts [42, 41, 89] based on the assumption that the same type of isolators are applied. This comparison, which shows advantages and disadvantages of the corresponding concepts, is also presented in this chapter.

SEMIS is designed as a contactless electromagnetic suspension system. The heart of this system is the recent-realized electromagnetic isolator, which combines:

1. Vertical and horizontal Lorentz actuators, which provide virtually position-independent force output;
2. Fully-passive PM-based gravity compensator which acts as a contactless magnetic spring.

The gravity force of a floating metrology frame (730 kg) is compensated by the passive interactions between the PMs in the gravity compensator. As a rigid body, the floating metrology frame is subjected to 6-DoF motions: translations ($x$, $y$, and $z$) along the three Cartesian axes and the rotations ($\phi$, $\theta$, and $\psi$) about the three Cartesian axes. As fully-passive PM devices are inherently unstable [23], six additional Lorentz actuators are used to provide stabilizing active force for all 6-DoF motions.

The gravity compensator consists of two parts with PMs that interact with each other. It is hysteresis-free due to the absence of soft-magnetic materials. The PM structure is designed such, that it combines a high vertical force density to compensate the gravity force with minimized stiffness. These properties have been predicted by means of the analytical surface charge model [60, 53, 57], which is a dedicated modeling method for ironless PM-based devices. The translational range of this gravity compensator is $\pm 1$ mm along all the three Cartesian axes.

The two properties, high vertical passive force and low stiffness, are predicted in the companion research [54] to this thesis using the aforementioned modeling method for ironless PM-based devices. Experimental verification of these properties is necessary as explained as follows:

- It is one of the goals of the companion research [54].
- The high vertical passive force is a critical criterion of the gravity compensator. If this vertical passive force is not high enough, the metrology frame can not be lifted by the Lorentz actuators due to their limited force output. It would lead to a system failure of SEMIS.
- The stiffness is an important dynamic property of SEMIS for control design.

For these reasons, the relation between the passive wrench (a vector of forces and torques) and displacement (the three translations and three rotations) is measured statically. Contactless design indicates that the two parts of the gravity compensator do not have any mechanical contact during operation. Experimental demonstration is necessary to verify this concept.

The magnetic flux density produced by the PMs (NdFeB) has a negative temperature coefficient of approximately 1 $\%/K$ [107]. Although low in relative terms, this effect becomes quite significant in the vertical passive offset force of 7.5 kN (according to the prediction in [54]). Its magnitude is comparable to the force variations that result from a displacement. These ambient temperature variations have a very large time constant and therefore they are regarded as static effect here. Nevertheless, their influence must be taken into account to reconstruct a wrench-versus-displacement characteristic.

This chapter is organized as follows. The single-gravity-compensator concept of the Single Electro-Magnetic Isolator System (SEMIS) is described and compared with conventional multi-gravity-compensator concepts in Section 7.2. Subsequently, section 7.3 models SEMIS by means of Newton’s equations and develops the necessary geometric transformations for the sensors and actuators. The static measurements and their results are described in Section 7.4. Finally, the conclusions are presented in Section 7.5.
7.2 SEMIS system-level design

As mentioned, the SEMIS has been realized for verification purposes and as a proof-of-concept. Further, the setup is suitable for dynamic testing of floor vibration isolation by means of a shaker table. The use of advanced control schemes would simultaneously improve the rejection of direct force disturbances and the isolation of floor vibrations. However, these last functionalities are not part of this chapter as it focuses on static measurements. This section describes the system-level design of SEMIS and subsequently introduces all of its subsystems. A more detailed description of the various subsystems (gravity compensator, Lorentz actuators, and the mechanical components) can be found in [54]. The single-gravity-compensator concept is also compared with conventional multi-gravity-compensator concepts based on the assumption that the same type of isolator is applied. The results revealed by this comparison show many advantages of this novel single-gravity-compensator concept.

7.2.1 SEMIS schematic

A 2D schematic of the total setup (SEMIS and the shaker table) is shown in Fig. 7.1. The electromagnetic suspension system, SEMIS, incorporates a heavy metrology frame supported by a single gravity compensator under its center of mass and six Lorentz actuators that stabilize it. For such a PM-based system, 6-DoF stabilization is a necessity as it is inherently unstable according to Earnshaw’s law [23].

The SEMIS is mounted on a shaker table, formed by a rigid table that is supported by coil springs and excited by Shaker Voice Coil Actuators (SVCAs). The forcing coils of the six Lorentz actuators and one part of the gravity compensator are mounted to this shaker table. The mechanical design of this shaker table is described in Section 6.1.2 and the dynamics of this shaker table are described in Section 6.3.1.

The rigid body that is formed by the granite, the floating part of the gravity compensator, and the translators of the six Lorentz actuators, is referred to as the metrology frame. All interactions between this metrology frame and the shaker table are contactless, electromagnetically induced forces. As control of a flexible body is not within the scope of this research, the metrology frame and the shaker table have been designed to have their flexible modes above 1 kHz. At lower frequencies, both platforms can be seen as a rigid body to facilitate the control design. The relative position between both bodies is measured with position sensors and their absolute acceleration with accelerometers.

Fig. 7.2 shows a schematic top view of the system-level design of the SEMIS. The square-shaped electromagnetic isolator is placed in the geometric center of the triangular metrology frame near its center of mass (CoM). This center of mass is defined as the origin of the Cartesian coordinate frame in Fig. 7.2. The center of actuation (CoA) of the gravity compensator is located slightly below this CoM and is defined as the geometrical center of its PM structure. Fig. 7.3 shows the realized SEMIS.

7.2.2 Subsystems

Electromagnetic isolator

As the core device of SEMIS, the electromagnetic isolator is designed and realized in the companion research [54] of this thesis. The schematic of this electromagnetic isolator is
Chapter 7. System-level design and static measurements of the contactless electromagnetic suspension system

presented in Fig. 7.4 and is briefly explained in this section. The design details are described in [54]. A single electromagnetic isolator consists:

1. Two vertical and two horizontal Lorentz actuators, which virtually provide position-independent force output;
2. A cross-shaped fully-passive PM-based gravity compensator which acts as a contactless magnetic spring.

Figure 7.1: The system-level design of the total setup (SEMI S and the shaker table): the front-view.

Figure 7.2: The system-level design of SEMIS: the top-view.

The outer dimensions of a single electromagnetic isolator are $300 \times 300 \times 200$ mm and its stroke length along all the three Cartesian axes is $\pm 0.5$ mm, limited by removable guidance. The gravity compensator has two cross-shaped parts that interact with each other. The inner-cross is fixed to the supporting shaker table and the outer-cross is fixed to the floating metrology frame. The passive interaction between the integrated PMs provides
a vertical force of 7.2 kN that matches the gravitational force of the floating metrology frame. The mass of this metrology frame is calculated as 730 kg by a CAD software used to make the mechanical drawings. This will be validated experimentally in Chapter 8. The 7.2 kN vertical passive force provided by the PM-based gravity compensator is 4% less than the 7.5 kN predicted by the aforementioned analytical modeling tools. The design of this gravity compensator is aiming at combining this high vertical force with low stiffness along all degrees of freedom. By design, the stiffness values have been reduced to values below 7.5 kN/m, thus resulting in a sub-Hertz mass-spring system [54], which is preferred for good vibration isolation. As the (unadjustable) vertical force of the gravity compensator is a given property and prone to manufacturing tolerances, the mass of the isolated platform has been made adjustable by means of the steel disks that are shown in Fig. 7.3. The close match that is achieved by tuning the resulting gravity force to the vertical magnetically induced force results in reduction of the additional vertical offset forces that must be delivered by the six Lorentz actuators. This mass-tuning, in combination with the very low stiffness values, reduces the energy consumption significantly.

At four corners of the gravity compensator, two vertical and two horizontal Lorentz actuators are integrated. These four Lorentz actuators are referred to as isolator-actuators, to distinguish them from the six Lorentz actuators at the three corners of SEMIS. The design of the vertical and horizontal isolator-actuators are exactly the same as the vertical and horizontal Lorentz actuators at the three corners of SEMIS, respectively.

Each Lorentz actuator has two parts: a coil and an array of PMs. The coil is sealed in resin and integrated into a holding frame which is made of stainless 316 steel. These isolator actuators are not used for active control in any experiments described in this thesis. However, their existence influences dynamic behaviors of SEMIS due to interactions between the PMs and the holding frame. The effects of these interactions will be explained later. The coordinate system defined in Fig. 7.4 is exactly the same as the one defined in Fig. 7.2. This determines the orientation of the electromagnetic isolator on SEMIS.

**Actuators**

As the 6-DoF PM-based gravity compensator is unstable by nature [23], six Lorentz actuators are used to execute the control action that is necessary to stabilize the SEMIS. These actuators have nearly linear relation between the input current and the Lorentz force which is virtually independent of position [54]. Fig. 7.2 shows that these Lorentz actuators are grouped in pairs at the three corners of the triangular metrology frame. Each of these groups contains one vertically oriented actuator and one horizontally oriented actuator; their respective forces are represented by \( f_{iv} \) and \( f_{ih} \), respectively, with \( i \in \{1, 2, 3\} \) corresponding to the number of the three corners shown in Fig. 7.2. During the static measurements to be described in this chapter, the metrology frame is stabilized by six PID controllers using feedback signals from the six position sensors.

**Sensors**

The shaker table is rigidly connected to the laboratory floor. The displacement between the floating metrology frame and the supporting shaker table is measured with six optical position sensors (Philtec RC100) which are placed in orthogonal pairs near the actuator units at the three corners. The floating metrology frame and the supporting shaker table each have
Chapter 7. System-level design and static measurements of the contactless electromagnetic suspension system

Figure 7.3: Photo of SEMIS.

Figure 7.4: Schematic of the electromagnetic isolator: top view. The arrows indicate the magnetization direction of the corresponding PM.
six inertial accelerometers (Kistler 8330) which are used for advanced vibration isolation control purposes. The necessity of this measurement scheme has been explained in Chapter 3. Although the measurements described in this chapter do not use these accelerometers, they are still introduced to give a complete description of the setup.

*Shaker table*

The shaker table consists of a 300 kg aluminum rigid table supported by coil springs with natural frequencies at 15-25 Hz in all DoF motions. Three shaker voice coil actuators which are placed parallel to these coils are used to excite artificial floor vibrations for future vibration isolation tests. For the measurements that are described in this chapter, steel plates, as shown in Fig. 7.2 and Fig. 7.3, are installed in parallel to the coil springs to effectively couple the shaker table to the lab floor. More detailed description of this shaker table, including its dynamic behavior, can be found in Chapter 6.

### 7.2.3 Concept comparison

The single-gravity-compensator concept is a dedicated design for SEMIS because only one electromagnetic isolator is realized. Nevertheless, this single-gravity-compensator concept is also applicable to the pneumatic suspension systems by employing a pneumatic isolator [41] instead of the electromagnetic isolator. Therefore, this single-gravity-compensator concept is a system-level design of a general active suspension system. It is compared with conventional multi-gravity-compensator concepts [42, 41, 89] based on the assumption that the same type of isolators (regardless which type) is applied. The advantages of the single-gravity-compensator concept are:

- Lower stiffness and lower passive damping;
- Lower cross-coupling;
- Lower cost.

The single-gravity-compensator concept is compared with a multi-gravity-compensator concept using a simplified model of the isolators. A double-gravity-compensator concept is used as an example of the multi-gravity-compensator concepts in this comparison. The purpose of this comparison is to show the disadvantages of using more than one gravity-compensator in an active suspension system. These disadvantages are also possessed by other multi-gravity-compensator concepts.

Only dynamics along the vertical translation are modeled for each isolator. More complicated dynamic models, in which more DoF motions are modeled, will lead to the same conclusion. We assume the same type of gravity-compensator (stiffness $k$ and damping coefficient $c$) is used. The two concepts are compared using simple inertia-spring-damper models, as shown in Fig. 7.5 and Fig. 7.6. Only 3-DoF motions are modeled: the translations along $z$-axis and $x$-axis; the rotation around $y$-axis.

For translations along $z$-axis and $x$-axis, the stiffness and damping coefficient of the single-gravity-compensator concept are only half of that of the double-gravity-compensator concept. The more employed isolators, the higher stiffness and damping coefficient of the system. For the rotation around $y$, the stiffness and damping coefficient of the single-gravity-compensator concept are zero while the stiffness and damping coefficient the double-gravity-compensator concept are $\frac{1}{2}kd^2$ and $\frac{1}{2}cd^2$, respectively. This proves that
the single-gravity-compensator concept has lower stiffness and lower passive damping. As explained in Section 1.2.1 in Chapter 1, low stiffness and low passive damping are both preferred properties of suspension systems.

In practice, it is impossible to have two identical gravity-compensators. If the two gravity-compensators have different stiffness or different damping coefficient, the two motions, $z$-translation and the $y$-rotation, are cross-coupled for the double-gravity-compensator design. On the other hand, the single-gravity-compensator design does not have this problem. Although the cross-coupling could be possibly minimized by decoupling techniques, reducing the cross-coupling by system-level design is still more advantageous.

The single-gravity-compensator concept requires less isolators than the double-gravity-compensator concept. As a result, the cost of the single-gravity-compensator concept is lower.

There are also challenges for this single-gravity-compensator concept. The first challenge is compensation of the metrology frame gravitational force. The allowed mass of the suspended metrology frame for the single-gravity-compensator concept is only half of the double-gravity-compensator concept, one third of that for the three-gravity-compensator concept, and one fourth of that for the four-gravity-compensator concept. Further increasing the metrology frame mass demands more on the gravity compensation capability of the gravity-compensator. This challenge is applicable to both electromagnetic suspension systems and pneumatic suspension systems. Second, if the mass center of the suspended
7.3. Motion equation model

object is above the supporting point of the gravity-compensator, this concept is inherently unstable even if the gravity-compensator itself is stable. As the electromagnetic isolator itself is already inherent unstable, this property is not an additional challenge for electromagnetic suspension systems. Third, this single-gravity-compensator concept requires higher accuracy on mechanical manufacturing and assembling. If the mass center of the suspended object has a horizontal offset from the supporting gravity-compensator, the horizontal torque produced by this offset has to be compensated by the stabilizing actuators which demands higher power consumption than necessity.

7.3 Motion equation model

Before proceeding with measurement results, necessary coordinates and notations are defined.

7.3.1 Coordinate system definition

The centers of actuation of the six stabilizing Lorentz actuators are denoted by \( L_{iv} \) and \( L_{ih} \), with \( i \in \{1, 2, 3\} \) indicating the corner number and the subscripts \( v \) or \( h \) the vertical or horizontal direction, respectively. Similarly, the respective locations of the position sensor are defined by \( P_{iv} \) and \( P_{ih} \) and those of the accelerometers by \( A_{iv} \) and \( A_{ih} \). The triangles that are formed by the actuators (\( \triangle L_{1v}L_{2v}L_{3v} \) and \( \triangle L_{1h}L_{2h}L_{3h} \)), the relative position sensors (\( \triangle P_{1v}P_{2v}P_{3v} \) and \( \triangle P_{1h}P_{2h}P_{3h} \)) and the inertial accelerometers (\( \triangle A_{1v}A_{2v}A_{3v} \) and \( \triangle A_{1h}A_{2h}A_{3h} \)) are all parallel regular triangles. Furthermore, the geometric centers of these triangles are on the same vertical line as the metrology frame’s CoM and the gravity compensator’s CoA.

7.3.2 Denotations and vector definitions

The position of a point, take \( P_{iv} \) for example, with respect to the coordinate system defined in Fig. 7.2 is denoted by \( \vec{p}_{iv} = [\vec{p}_{iv}.x, \vec{p}_{iv}.y, \vec{p}_{iv}.z]^T \).

Table 7.1 defines the input and output vectors of the SEMIS. The subscripts \( x, y, \) and \( z \) denote the three axes of the Cartesian coordinate system. The subscripts \( \phi, \theta, \psi \) denote the Euler angles (roll-pitch-yaw rotational angles). The vectors \( \vec{f} \) and \( \vec{t} \) denote the force and torque vector, respectively, and are combined in a wrench vector. The abbreviation MF denotes the metrology frame. The passive wrench \( \vec{w}_p \) is the summation of the wrench induced by the metrology frame gravity force and the passive wrench exhibited by the gravity compensator.

7.3.3 Motion equation

The motion equation of the metrology frame is derived by Newton’s second Law assuming that the disturbance forces caused by the sensor wires and eddy currents may be ignored.

\[
M_{\hat{q}a} = \vec{w}_a + w_p (q_r),
\]

where \( M \) is the inertia matrix. A linearization of \( \vec{w}_p (q_r) \) around the middle-of-stroke position \( q_{r0} \) using a first-order Taylor’s expansion gives

\[
\vec{w}_p (q_r) = \vec{w}_p (q_{r0}) - K_p(q_{r0}) \vec{q}_r,
\]

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<table>
<thead>
<tr>
<th>name</th>
<th>definition</th>
<th>description</th>
</tr>
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<tr>
<td>$\mathbf{w}_a$</td>
<td>$[f_x, f_y, t_z, t_x, t_y, f_z]^T$</td>
<td>active control wrench (N or N·m)</td>
</tr>
<tr>
<td>$\mathbf{w}_p$</td>
<td>$[f_{px}, f_{py}, t_{pz}, t_{px}, t_{py}, f_{pz}]^T$</td>
<td>total passive wrench (N or N·m)</td>
</tr>
<tr>
<td>$i_a$</td>
<td>$[i_{1h}, i_{2h}, i_{3h}, i_{1v}, i_{2v}, i_{3v}]^T$</td>
<td>current of Lorentz actuator (A)</td>
</tr>
<tr>
<td>$f_a$</td>
<td>$[f_{1h}, f_{2h}, f_{3h}, f_{1v}, f_{2v}, f_{3v}]^T$</td>
<td>Lorentz actuator force (N)</td>
</tr>
<tr>
<td>$\mathbf{q}_s$</td>
<td>$[q_{1h}, q_{2h}, q_{3h}, q_{1v}, q_{2v}, q_{3v}]^T$</td>
<td>position sensor output (m)</td>
</tr>
<tr>
<td>$\mathbf{a}_s$</td>
<td>$[a_{1h}, a_{2h}, a_{3h}, a_{1v}, a_{2v}, a_{3v}]^T$</td>
<td>accelerometer output (m/s²)</td>
</tr>
<tr>
<td>$\mathbf{q}_a$</td>
<td>$[q_x, q_y, q_\psi, q_\theta, q_\phi, q_z]^T$</td>
<td>MF absolute displacement (m)</td>
</tr>
<tr>
<td>$\mathbf{q}_r$</td>
<td>$[q_{rx}, q_{ry}, q_{r\psi}, q_{r\theta}, q_{r\phi}, q_{rz}]^T$</td>
<td>MF relative displacement to shaker (m)</td>
</tr>
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<td>$\mathbf{q}_b$</td>
<td>$[q_{bx}, q_{by}, q_{b\psi}, q_{b\theta}, q_{b\phi}, q_{bz}]^T$</td>
<td>shaker absolute displacement (m)</td>
</tr>
<tr>
<td>$\mathbf{a}_a$</td>
<td>$[a_x, a_y, a_\psi, a_\phi, a_\theta, a_z]^T$</td>
<td>MF absolute acceleration (m/s²)</td>
</tr>
<tr>
<td>$\mathbf{a}_b$</td>
<td>$[a_{bx}, a_{by}, a_{b\psi}, a_{b\phi}, a_{b\theta}, a_{bz}]^T$</td>
<td>shaker absolute acceleration (m/s²)</td>
</tr>
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</table>

Table 7.1: Vector Definitions. MF denotes metrology frame.

where $\mathbf{w}_p (\mathbf{q}_r)$ is a constant force and $\mathbf{K}_p$ is the stiffness matrix, defined as

$$\mathbf{K}_p(\mathbf{q}_r) = -\frac{\partial \mathbf{w}_p}{\partial \mathbf{q}_r}. \quad (7.3)$$

Substitute (7.2) into (7.1), we have

$$\mathbf{M} \mathbf{q}_a + \mathbf{K}_p(\mathbf{q}_r) \mathbf{q}_r = \mathbf{w}_a + \mathbf{w}_p (\mathbf{q}_r). \quad (7.4)$$

It is important to mention that $\mathbf{w}_p (\mathbf{q}_r)$ is constant and it will not affect the system dynamics. Theoretically, the stiffness matrix $\mathbf{K}_p(\mathbf{q}_r)$ is dependent on $\mathbf{q}_r$. However, it will be shown in Section 7.4 that the dependency of $\mathbf{K}_p$ on $\mathbf{q}_r$ is ignoingly weak. As a result, the force depends almost linearly on displacement.

The direct input vector to SEMIS is $\mathbf{f}_a$ (actuator force) and the direct output vectors are $\mathbf{q}_s$ and $\mathbf{a}_s$ (position sensors and accelerometers, respectively). The generalized input/output vectors $\mathbf{w}_a$, $\mathbf{q}_r$, and $\mathbf{a}_a$ are calculated by linear transformation from the direct input/output vectors. This linear transformation is calculated based on geometry in the following subsection.

### 7.3.4 Geometric transformation

Transformation matrices between the sensor/actuator space and the generalized vector space are calculated around the center position $\mathbf{q}_{r0}$ of the metrology frame based on geometry. The calculated matrices are needed for a full description of the setup. Table 7.2 lists the coordinates of the most important points on the metrology frames. The variables in Table 7.2 are defined in Section 7.3.2.
7.4. Static measurements

The actuator force transformation matrix \( \mathbf{w} \mathbf{T}_f \), defined according to \( \mathbf{w}_a = \mathbf{w} \mathbf{T}_f \mathbf{f}_a \), is given by

\[
\mathbf{w} \mathbf{T}_f = \begin{bmatrix}
\frac{1}{2} & -\frac{\sqrt{3}}{2} & -1 & 0 & 0 & 0 \\
\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\
l_{lh} & l_{lh} & l_{lh} & 0 & 0 & 0 \\
\frac{1}{2} h_l & \frac{1}{2} h_l & -h_l & \frac{\sqrt{3}}{2} l_{lv} & \frac{\sqrt{3}}{2} l_{lv} & 0 \\
0 & 0 & 0 & 1 & 1 & 1 \\
\end{bmatrix},
\]

(7.5)

where \( l_{lh} = l_{3h} \cdot \mathbf{y} \), \( l_{lv} = l_{3v} \cdot \mathbf{y} \), and \( h_l = l_{3h} \cdot \mathbf{z} \). The position signal transformation matrix \( \mathbf{s} \mathbf{T}_q \), defined according to \( \mathbf{q}_s = \mathbf{s} \mathbf{T}_q \mathbf{q}_r \), is given by

\[
\mathbf{s} \mathbf{T}_q = \begin{bmatrix}
\frac{1}{2} & -\frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} & p_{3h} \cdot \mathbf{y} & -\frac{\sqrt{3}}{2} p_{3h} \cdot \mathbf{y} & -p_{3h} \cdot \mathbf{z} & 0 \\
\frac{1}{2} & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & p_{3h} \cdot \mathbf{y} & \frac{\sqrt{3}}{2} p_{3h} \cdot \mathbf{y} & -p_{3h} \cdot \mathbf{z} & 0 \\
-1 & 0 & 0 & p_{3h} \cdot \mathbf{y} & 0 & -p_{3h} \cdot \mathbf{z} & 0 \\
0 & 0 & 0 & p_{1y} \cdot \mathbf{y} & -p_{1y} \cdot \mathbf{x} & 1 \\
0 & 0 & 0 & p_{2y} \cdot \mathbf{y} & -p_{2y} \cdot \mathbf{x} & 1 \\
0 & 0 & 0 & p_{3y} \cdot \mathbf{y} & -p_{3y} \cdot \mathbf{x} & 1 \\
\end{bmatrix},
\]

(7.6)

The acceleration signal transformation matrix \( \mathbf{s} \mathbf{T}_a \), defined according to \( \mathbf{a}_s = \mathbf{s} \mathbf{T}_a \mathbf{a}_r \), is given by

\[
\mathbf{s} \mathbf{T}_a = \begin{bmatrix}
\frac{1}{2} & -\frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} & a_{3h} \cdot \mathbf{y} & -\frac{\sqrt{3}}{2} a_{3h} \cdot \mathbf{y} & -a_{3h} \cdot \mathbf{z} & 0 \\
\frac{1}{2} & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & a_{3h} \cdot \mathbf{y} & \frac{\sqrt{3}}{2} a_{3h} \cdot \mathbf{y} & -a_{3h} \cdot \mathbf{z} & 0 \\
-1 & 0 & 0 & a_{3h} \cdot \mathbf{y} & 0 & -a_{3h} \cdot \mathbf{z} & 0 \\
0 & 0 & 0 & a_{1y} \cdot \mathbf{y} & -a_{1y} \cdot \mathbf{x} & 1 \\
0 & 0 & 0 & a_{2y} \cdot \mathbf{y} & -a_{2y} \cdot \mathbf{x} & 1 \\
0 & 0 & 0 & a_{3y} \cdot \mathbf{y} & -a_{3y} \cdot \mathbf{x} & 1 \\
\end{bmatrix},
\]

(7.7)

7.4 Static measurements

As introduced in Section 7.1, the static measurements of the SEMIS consist of

1. the vertical passive force with respect to the ambient temperature;
2. the relations between the passive wrench \( \mathbf{w}_p \) and the metrology frame relative displacement \( \mathbf{q}_r \) with respect to the shaker table.

These two measurements are conducted in two consecutive steps, described in the following two subsections. Without knowing the linearity of SEMIS, a safe way to calculate the stiffness matrix is to measure the relation between \( \mathbf{w}_p \) and \( \mathbf{q}_r \) and subsequently calculate the stiffness matrix according to (7.4). Position-dependent stiffness matrix indicates system nonlinear behavior.

The SEMIS is inherently unstable and for this reason the static measurements have been performed in a stabilized closed-loop system. The control diagram is given in Fig. 7.7.
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<td>p₁h</td>
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<td>-290</td>
<td>-238</td>
<td>a₁v</td>
<td>476</td>
<td>-275</td>
<td>70</td>
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<tr>
<td>p₂h</td>
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<td>-238</td>
<td>a₂v</td>
<td>-476</td>
<td>-275</td>
<td>70</td>
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<tr>
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<td>-238</td>
<td>a₃v</td>
<td>0</td>
<td>550</td>
<td>70</td>
</tr>
</tbody>
</table>

Table 7.2: Coordinates of important points on the metrology frame. All quantities are in millimeters.

The block $C(s)$ denotes six PID controllers used for stabilization. The matrices $T_w^f = w^{-f}T_f$ and $T_s^q = s^{-q}T_q$ are the transformation matrices. The signal vectors $\vec{r}$ and $\vec{e}$ are the reference and error, respectively. Because of the integrator in the PID controller, the reference-tracking error $\vec{e}$ at steady state is accurate up to the sensor accuracy ($\pm 1 \mu m$). As the shaker table is fixed to the floor, it is assumed that $\vec{q}_r = \vec{q}_a$ for these static measurements.

7.4.1 Temperature sensitivity of the vertical passive force

When the metrology frame is positioned at a fixed equilibrium, the vertical active force $f_z$ is equal and opposite to the difference between the MF’s gravity force $f_{MF}$ and the vertical passive force of the gravity compensator $f_{gc}$

\[
(f_{gc} - f_{MF}) + f_z = f_{pz} + f_z = 0.
\]

Therefore, recording the vertical control force $f_z$ while the system is at steady state yields $f_{pz}$. A temperature sensor is mounted in the gravity compensator and is used to record the gravity compensator temperature, which varies sufficiently slow to be considered equal to the PM temperature. The $f_z$ as function of the ambient temperature is shown in Fig. 7.8. Each $f_z$-temperature data point in Fig. 7.8 is averaged from a 10 s recording of $f_z$ (3 kHz sampling rate). The temperature variation during this 10 s is assumed to be ignorable. All data points are measured while the MF position $\vec{q}_r$ is kept constant. The total measurement process takes a few days. A first-order least-square fitting is sufficient in this
7.4. Static measurements

case and illustrates that $f_z$ has a linear temperature dependency of 12.1 N/K within the range [21.5,25]$^\circ$C. In relative terms this corresponds to 1.7 $\%$/K. Although this temperature dependency is clearly measurable on the vertical force, its influence on the other force components and the torque is negligible due to their low absolute values.

![Figure 7.8: The vertical control force $f_z$ versus the temperature. Different markers indicate they are measured in different days. The straight line is the first-order least-square fitting.](image)

7.4.2 Position dependency of the passive wrench

Analogous to (7.8), a balance in the wrenches is found when the system is in equilibrium

$$\vec{w}_p + \vec{w}_a = 0.$$ 

(7.9)

Similar to the measurement of the vertical passive force $f_{pz}$, $\vec{w}_p$ has been measured by averaging the measured wrench $\vec{w}_a$ over 5 seconds (1 kHz sampling rate). The aforementioned temperature reading has been recorded in parallel and used to correct the measured $f_z$ for the ambient temperature. With this method the wrench $\vec{w}_p$ has been measured at selected positions within the stroke range. The number of measurement positions and rotations may grow very large as a result of the 6-DoF behavior. To reduce this number, the passive wrench $\vec{w}_p$ with respect to the translations and rotations have been measured separately.

**Translation**

The passive wrench $\vec{w}_p$ was measured while rotational displacements were maintained zero. The control wrench $\vec{w}_a$ was recorded on three evenly distributed horizontal planes at $z = -0.5$ mm, $z = 0$ mm, and $z = 0.5$ mm, illustrated in Fig. 7.9(a). The averaged reading of $f_z$ were corrected to a reference temperature of 23$^\circ$C as mentioned above. The resulting measured wrench values $\vec{w}_a$ are shown in Fig. 7.10 - Fig. 7.12 and illustrate the near-linear relations between the wrench and displacement. These figures show that the components
Chapter 7. System-level design and static measurements of the contactless electromagnetic suspension system

Figure 7.9: Positions where the control wrench is recorded. (a): The three horizontal planes (gray) where the control wrench is measured while rotational displacements are zero. (b): The seven points (black) where the control wrench is measured with respect to different rotation angles.

$f_x$ and $t_y$ are mainly a function of a displacement $\vec{q}_{rx}$ and that $f_y$ and $t_x$ depend on $\vec{q}_{ry}$. The vertical force $f_z$ and torque $t_z$ are significantly less sensitive to these translations.

**Rotation**

The passive wrench $\vec{w}_p$ was measured at seven different positions, shown in Fig. 7.9(b). At each position, it was measured as function of the three rotations ($q_\phi, q_\theta, q_\psi$) while the three translations are kept zero. The results are not graphically shown here, but have been used to obtain a stiffness matrix.

**Stiffness matrix**

From the aforementioned static measurements a stiffness matrix has been derived that describes the behavior of the SEMIS around it’s middle-of-stroke. Combining (7.3) and (7.9), we have

$$K_p = \frac{\partial \vec{w}_a}{\partial \vec{q}_r}.$$  \hspace{1cm} (7.10)

$K_p$ is calculated from the static measurements and presented in Table. 7.3. This stiffness matrix validates the low-stiffness property of the gravity compensator.

The two entries, $\frac{\partial f_x}{\partial q_x}$ and $\frac{\partial f_y}{\partial q_y}$, are negative, which indicates instability, and are significantly higher than the other entries. However, the difference between the two entries, $\frac{\partial f_x}{\partial q_x}$ and $\frac{\partial f_y}{\partial q_y}$, is not consistent to the symmetric design of the gravity compensator. The explanation of this phenomena is described as follows.

The interactions between the PM array and the holding frame (stainless 316 steel) for the coil of all Lorentz actuators introduce stiffness to the system [27, 50, 58], in addition to the gravity compensator. When the PM array moves perpendicular to the surface of the holding frame, the interaction force is position-dependent which introduces a negative
7.4. Static measurements

Figure 7.10: Horizontal control force versus translational displacements.

Figure 7.11: Horizontal control torque versus translational displacements.

Figure 7.12: Vertical control force/torque versus translational displacements.
stiffness. To explain the influence of this interaction to system dynamics, the schematic of all Lorentz actuators is presented in Fig. 7.13.

<table>
<thead>
<tr>
<th>$q_x$ (mm)</th>
<th>$f_{px}$ (N)</th>
<th>$f_{py}$ (N)</th>
<th>$t_{pz}$ (N m)</th>
<th>$t_{px}$ (N m)</th>
<th>$t_{py}$ (N m)</th>
<th>$f_{pc}$ (N)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.2</td>
<td>-3.1</td>
<td>1.9</td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>-75</td>
<td>0.1 ± 0.1</td>
<td>2.1</td>
<td>-0.3</td>
<td>-0.6</td>
<td></td>
</tr>
<tr>
<td>2 ± 1</td>
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<td>-0.8 ± 0.1</td>
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</tr>
<tr>
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<td>1 ± 1</td>
<td>0.1 ± 0.1</td>
<td>0.4</td>
<td>0 ± 0.04</td>
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<tr>
<td>-1.5</td>
<td>1 ± 2</td>
<td>-0.3 ± 0.2</td>
<td>0 ± 0.1</td>
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<td>-0.2 ± 0.1</td>
<td></td>
</tr>
<tr>
<td>1.3</td>
<td>-0.5 ± 1.5</td>
<td>-0.9</td>
<td>-1.0</td>
<td>-0.3</td>
<td>1.0</td>
<td></td>
</tr>
</tbody>
</table>

Table 7.3: The stiffness matrix derived from the static measurements.

The six Lorentz actuators used for stabilization are grouped as three pairs: each pair has a vertical-oriented and a horizontal-oriented Lorentz actuators. These three actuator-pairs are at the three corners of SEMIS. The four isolator-actuators can also be treated as two pairs. The same design is applied to all vertical Lorentz actuators and the same design is applied to all horizontal Lorentz actuators, described in [54]. As such, it is reasonable to assume that all five actuator-pairs have the same stiffness along the direction that perpendicular to the surface of its coil-holding-frame. This stiffness is denoted by $k_a$.

According to geometry shown in Fig. 7.13, the total stiffness induced by all the five actuator-pairs along the $y$-axis is $4k_a$. On the other hand, the total stiffness induced by all the five actuator-pairs along the $x$-axis is only $\sqrt{3} k_a$. The difference $\sqrt{3} k_a - 4k_a$ is exactly the difference between the two stiffness entries: $\frac{\partial f_{px}}{\partial q_x} - \frac{\partial f_{py}}{\partial q_y} = -61 - (-75)$. Therefore, $k_a$ is estimated as -0.62 N/mm. The stiffness of the gravity compensator along $x$-axis is calculated as $-61 - \sqrt{3} k_a = -50.3$ N/mm, which is a perfect match to the stiffness of the gravity compensator along $y$-axis, calculated as $-75 - 4k_a = -50.3$ N/mm.

The calculated stiffness of the gravity compensator for the two horizontal translations coincide, which is consistent to the symmetric design of the gravity compensator.

Still, the calculated horizontal stiffness (-50.3 N/mm) of the gravity compensator by removing the effect of the Lorentz actuators is much higher than the predicted values (up to -5 N/mm) using the analytical modeling tools. Possible reasons are assembly and manufacturing tolerances.

The $\mu$-interaction measure of the stiffness matrix $K_p$ is 10 dB, which indicates significant cross-coupling. A few possible reasons for these cross-couplings are:

1. Cross-coupling behavior of the gravity compensator;
2. Interactions between the PMs and the stainless steel plate of the Lorentz actuators;
3. External disturbances induced by the sensor cables, for example, the entry $\frac{\partial f_{pc}}{\partial q_x}$;
4. Mis-calculation of the geometric transformation matrix, for example, the entry $\frac{\partial t_{py}}{\partial q_x}$;
7.4. Static measurements

Figure 7.13: Schematic of all Lorentz actuators of SEMIS: top view.

Figure 7.14: The power consumption while the metrology frame is stabilized at different positions.
7. System-level design and static measurements of the contactless electromagnetic suspension system

Power consumption

The power consumption for stabilizing the metrology frame is calculated as the total ohmic loss of the six Lorentz actuators.

\[ P_w = \left| \vec{i}_a \right|^2 R = \left| \frac{1}{k} \vec{T}_w \vec{w}_a \right|^2 R, \]  

(7.11)

where \( R = 1.5 \Omega \) is the resistance of a single Lorentz actuator and \( k = 31.75 N/A \) is the current to force ratio of the Lorentz actuators. In this calculation, \( \vec{w}_a \) is taken as the data plotted in Fig. 7.10 – Fig. 7.12. As \( \vec{w}_a \) is position-dependent, the power consumption is also position-dependent. The calculated results with respect to the translational positions are plotted in Fig. 7.14. It shows that the power consumption used for stabilization is very low (0.3~6 W). The coordinates of the minimum-power-point are \([x, y, z]^T = [0.2, 0.1, -0.5]^T\). This probably suggests that the PM-based gravity compensator is not in its geometric center when the metrology frame is positioned in its center working point \( \vec{q}_0 = [0, 0, 0]^T \) due to mechanical assembly errors.

7.5 Conclusions

This chapter has presented the single-gravity-compensator design, modeling, and static measurements of the Single Electro-Magnetic Isolator System (SEMIS), which is a 6-DoF contactless electromagnetic suspension system. A prototype of the PM-based gravity compensator serves as a passive magnetic spring to support a metrology frame (730 kg). The gravity force of the metrology frame is compensated by the vertical passive interaction of the PMs in the gravity compensator and the system is stabilized by position-controlled Lorentz actuators. An integrated shaker table enables to excite artificial floor vibrations. The translational stroke length along all the three Cartesian axes is ±0.5 mm, limited by removable guidance.

Two properties of SEMIS, high passive vertical force and low stiffness, have been obtained in closed-loop position-controlled experiments. Contactless design has been demonstrated. The passive vertical force produced by the gravity compensator is about 7.2 kN and has a temperature sensitivity of -12.1 N/K which corresponds to 1.7 %/K of the total 7.2 kN. The total passive wrench with respect to the translational and rotational displacements is measured at selected positions. It shows that the position-dependency of the passive wrench is nearly constant. The stiffness matrix has been calculated and the low-stiffness design of the gravity compensator has been validated. The stiffness of vertical translation is much lower than that of pneumatic isolators. The power consumption of SEMIS at steady state has been found to be between 0.3 W and 6 W, which demonstrates the system’s low power consumption. The results presented in this chapter provide a solid base for the dynamic measurements and vibration isolation control which will be described in Chapter 8 and Chapter 9, respectively.
Chapter 8

Dynamic behavior of the contactless electromagnetic suspension system

8.1 Introduction

As introduced in Chapter 1, the Single Electro-Magnetic Isolator System (SEMIS) is designed to serve two purposes:

1. Verification of the predicted static and dynamic properties of the realized PM-based gravity compensator, including high vertical passive force, low stiffness, low passive damping, contactless design, and suitability for high-bandwidth control;
2. Establishing the feasibility of a contactless electromagnetic suspension system which explores the full performance potential of the Permanent Magnet (PM) based gravity compensator.

In Chapter 7, the property of low stiffness has been verified by deriving stiffness matrix from static measurements. Contactless design has been demonstrated. The property of high vertical passive force of 7.2 kN is verified based on the assumption that the mass of the floating metrology frame is 730 kg. This value is derived using a CAD software.

This chapter focuses on the dynamics of SEMIS. The Frequency Response Function (FRF) of SEMIS is measured for the following purposes:

1. To measure the mass and the moments of inertia of the floating metrology frame;
2. To identify the flexible modes of the floating metrology frame;
3. To evaluate the cross-coupling of SEMIS;
4. To measure damping coefficients (velocity-derivative of the passive wrench).

The mass and the moments of inertia can be quantified by curve fitting on the measured FRF. This is an experimental proof of the 730 kg floating mass and is therefore an experimental proof of the 7.2 kN vertical passive force provided by the gravity compensator.

The flexible modes of the metrology frame are limiting factors to increase the control bandwidth. Knowledge of these flexible modes is necessary to evaluate the suitability of high-bandwidth control using the PM-based gravity compensator. It is also important for the control design that is described in Chapter 9.
Chapter 8. Dynamic behavior of the contactless electromagnetic suspension system

The \( \mu \)-interaction measure, as defined in Chapter 2, is an index to quantify the cross-coupling of a Multi-Input-Multi-Output (MIMO) system. It is calculated based on the measured FRF of SEMIS in this chapter. It shows that the cross-coupling of SEMIS is sufficiently small around the intended control bandwidth.

As explained in Chapter 1, stiffness and passive damping are preferred to be low for an active suspension system. The low-stiffness property of SEMIS has been verified in static measurements described in Chapter 7. The passive damping is evaluated by the damping coefficient, which can be estimated by curve-fitting on the measured FRF for a Linear Time-Invariant (LTI) mechanical system. As the stiffness of this system is as low as 1 N/mm (the vertical translation), the natural frequency is about 0.2 Hz. FRF measurement at these low frequencies is difficult. For this reason, the damping coefficient is identified by curve-fitting on position-controlled closed-loop transmissibility measured using the shaker table.

This chapter is organized as follows. Section 8.2 describes the experiments to measure the FRF of SEMIS from 1 Hz to 1000 Hz. The mass and moments of inertia of the metrology frame are quantified in Section 8.3. Section 8.4 analyzes the nonlinear behavior of SEMIS introduced by interaction forces between the two parts of each Lorentz actuator and its effects on future control design. In Section 8.5, the \( \mu \)-interaction measure is calculated from the measured FRF of SEMIS. It is also explained in this section that to further decouple SEMIS by applying static decoupling matrices other than the geometric transformation matrices that are described in Chapter 7 is neither necessary nor possible. Section 8.6 identifies the damping coefficients of 3-DoF motions. For the other 3-DoF motions, damping coefficient cannot be measured due to the limitation of the shaker table. It is estimated based on the assumption that all damping coefficients are of the same order of magnitude. This chapter is concluded in Section 8.7.

8.2 FRF measurement

8.2.1 Experiment description

As SEMIS is inherently unstable, the Frequency Response Function (FRF) of SEMIS is measured indirectly in a stabilized closed-loop using a similar procedure as described in Chapter 4. Fig. 8.1 shows the control diagram of the closed-loop experiments. The input vector of SEMIS is defined as the wrench vector \( \vec{w}_a = [f_x, f_y, t_z, t_x, t_y, f_z]^T \) and the output vectors of SEMIS are the relative displacement vector \( \vec{q}_r = [q_{rx}, q_{ry}, q_{r\phi}, q_{r\theta}, q_{rz}]^T \) and the absolute acceleration of the metrology frame \( \vec{a}_r = [a_x, a_y, a_{\phi}, a_{\theta}, a_z]^T \). The absolute acceleration of the shaker table is denoted by the vector \( \vec{a}_b = [a_{bx}, a_{by}, a_{b\phi}, a_{b\theta}, a_{bz}]^T \). The subscripts \( x, y, z, \phi, \theta, \) and \( \psi \) denote the six Degrees-of-Freedom (DoF) motions.

The block Plant denotes the physical plant of SEMIS, which takes the six Lorentz forces as input and the twelve sensor signals as output. The block \( C(s) \) denotes six PID controllers used for stabilization. The geometric transformation matrices \( T_w = w T_f^{-1}, q T_s = q T_q^{-1}, \) and \( a T_s = a T_a^{-1} \) are calculated and presented in (7.5), (7.6), and (7.7), respectively. The signal vectors \( \vec{r} \) and \( \vec{e} \) are the reference and error, respectively. The acceleration signals are acquired using a similar procedure as described in Chapter 6. The same RC circuit and the same analog low-pass filter with different settings are applied. The
8.2. FRF measurement

The cut-off frequency of the analog low-pass filter is set to 500 Hz and the signal amplification gain is set to 100.

During the FRF measurement, the shaker table is coupled to the floor by three steel plates. The design intention is to make this coupling as rigid as possible. An assumption is made in the beginning of the experiments that the two vectors, \( \vec{q}_r \) and \( \vec{q}_a = \vec{a}_a s^{-2} \), are the same. For this reason, they are both used to calculate the FRF of the SEMIS. However, it will be shown later in the plotted FRF that this assumption is not true. This will be further analyzed later in Remark 8.2.2.

Low-pass filtered Gaussian white noise is used as the excitation signal and each component of the disturbance wrench \( \vec{w}_d \) is excited in separated experiments. The response of relative displacement \( \vec{q}_r \) and absolute acceleration \( \vec{a}_a \) are both measured and recorded. The excitation signal and the recorded responses are used as input and output signals to calculate the corresponding FRF, \( \tilde{G} \). Remark that this measured FRF \( \tilde{G} \) is actually the closed-loop compliance of this position-controlled closed-loop. Its parametric form is a 6 \times 6 transfer function matrix and each column is measured in an independent experiment. Subsequently, the FRF of SEMIS, \( \tilde{P} \), is calculated by

\[
\tilde{P}(\omega) = [\tilde{G}(\omega)^{-1} - C(\omega)]^{-1}.
\]  

The dynamics of the analog filters are considered as a part of the controller \( C(\omega) \).

**Remark 8.2.1.** The calculated FRF \( \tilde{P} \) denotes the system which includes the geometric decoupling matrices and the physical plant of SEMIS. The input to this system is a 6-dimensional wrench vector, which contains three forces and three torques and can be calculated as \( "T_f \vec{f}_a \). The output of this system is a 6-dimensional displacement vector \( \vec{q}_a = \vec{a}_a s^{-2} \). If the displacement of the shaker table may be ignored, \( \vec{q}_a = \vec{q}_r \).

The FRF is measured within the frequency range of 1 - 1000 Hz. These flexible modes are designed to be above 1000 Hz. It is necessary to prove it by measurement because these flexible modes are expected to be the bandwidth-limiting factor for closed-loop control. The measured FRF will also be used for control design and analysis which will be described in Chapter 9.

### 8.2.2 Measured FRF

The FRF of SEMIS is measured using a sampling rate of 6000 Hz and each input-output data collection takes 50 s. The responses are measured using the position sensors and the accelerometers. The calculated FRF is plotted separately in Fig. 8.2 and Fig. 8.3. The FRF
measured by position sensors (blue) are very noisy at high frequencies (> 100 Hz for diagonal entries and > 10 Hz for off-diagonal entries) due to bad signal-to-noise ratio at these frequencies. The FRF measured by accelerometers is much less noisy for diagonal entries than the off-diagonal entries. The reason is that the off-diagonal entries have much lower magnitude than the diagonal entries so that their responses are too low. Excitation can not be increased because the responses of the diagonal entries are close to their physical limits. For diagonal entries, the FRF measured by accelerometers shows that the flexible modes of the metrology frame are above 700 Hz. It is possible that this 700 Hz flexible mode is introduced by the cube-shaped mechanical components which connect the accelerometers and the metrology frame because these components are not taken into account during flexible mode prediction using FEM.

Remark 8.2.2. In Fig. 8.2 and Fig. 8.3, there is a pair of anti-resonance and resonance found on each diagonal entry of SEMIS FRF measured by position sensors. For the two horizontal translations ($f_x \rightarrow q_x$ and $f_y \rightarrow q_y$), there is an anti-resonance (40 Hz) followed by a resonance (80 Hz). The 80 Hz resonance is also visible (a small spike on magnitude and phase) on the FRF measured by accelerometers, see Fig. 8.4 and Fig. 8.5. For the remaining four DoFs, the anti-resonances are not clearly seen in the FRF measured by position sensors but the resonances are visible: 180 Hz for $t_z \rightarrow q_w$ and $f_z \rightarrow q_z$; 300 Hz for $t_x \rightarrow q_\phi$ and $t_y \rightarrow q_\theta$. However, these resonances or anti-resonances are not seen on the FRF measured by accelerometers except the small spikes on the FRF of $f_x \rightarrow q_x$ and $f_y \rightarrow q_y$.

These resonances and anti-resonances are induced by the shaker table vibrations. During FRF measurement, the white noise disturbances ($\vec{w}_d$) excite vibrations not only on the metrology frame but also on the shaker table due to the flexibility of the steel plates. Our intention is to design the steel plates such that the shaker table can be coupled to the floor as rigid as possible. However, these steel plates act as leaf springs. Because of the geometry of the steel plates, the coupling between shaker table and the floor for the two horizontal translations is weaker than that of the other four DoFs. This explains that, comparing the other four DoFs, the anti-resonance and resonance are more significant on the two FRF $f_x \rightarrow q_x$ and $f_y \rightarrow q_y$ measured by position sensors (blue curve in Fig. 8.2).

There are small spikes on the FRF $f_x \rightarrow q_x$ and $f_y \rightarrow q_y$ measured by accelerometers at the corresponding resonance frequencies. For the other four DoFs, no spikes are visible. This can be explained by a combination of two factors. First, under the same excitation from the control wrench, the horizontal translational displacements of the shaker table have higher responses than the other four DoF motions due to weaker coupling to the floor. Second, the SEMIS stiffness induced by the PM-based gravity compensator for the two horizontal translations is significantly higher than that of the other four DoFs.

Each of the 6-DoF motions of SEMIS can be modeled by an inertia-spring-damper model. The parameters (inertia, stiffness, and damping coefficient) of the models are provided in Table 8.1. The stiffness of each DoF is taken from the stiffness matrix derived from the static measurements in Chapter 7. The inertia of each DoF is derived by curve-fitting on the measured FRF. The curve-fittings are shown later in Fig. 8.4 - Fig. 8.9. The damping coefficients will be quantified later in Section 8.6.
8.2. FRF measurement

Figure 8.2: The FRF of SEMIS: the first half. The blue curves are measured using position sensors and the red curves are measured using accelerometers. The unit of the wrench input is \( N \) or \( N \cdot m \) and the unit of the displacement output is \( m \) or \( \text{rad} \).
Figure 8.3: The FRF of SEMIS: the second half. The blue curves are measured using position sensors and the red curves are measured using accelerometers. The unit of the wrench input is $N$ or $N \cdot m$ and the unit of the displacement output is $m$ or $rad$. 

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8.3 Inertia validation

According to the measured FRF by accelerometers in Fig. 8.2 and Fig. 8.3, SEMIS exhibits perfect double-integrator behavior at frequencies from 5 Hz to 200 Hz. Therefore, the diagonal entries of the FRF are compared to their corresponding inertia-spring-damper model to validate the corresponding inertias. The parameters of these inertia-spring-damper models are described in Table 8.1. This comparison is shown in Fig. 8.4 - Fig. 8.9.

For all diagonal entries, the magnitude of these models match their corresponding FRF reasonably well from 3 Hz to 500 Hz. The magnitude mismatch at higher frequencies is due to the 700 Hz resonance. The phase of the models match their corresponding FRF reasonably well from 2 Hz to 100 Hz. The phase mismatch at higher frequencies is due to the 700 Hz resonance and the time-delay in the closed-loop system used for FRF measurement. The phase lag of the FRF up to 700 Hz is corresponding to a time-delay of 0.2 ms. This time-delay is slightly higher than the unit time-delay (0.17 ms) induced by sampling. The most probable reason is other time-delay factors in the closed-loop, for example, the control effort calculation inside the xPC target machine and the communication (RS422) between the xPC target machine and the digital current amplifiers.

A good match between the inertia-spring-damper model and the measured FRF also proves the mass of the metrology frame is indeed 730 kg. A straightforward conclusion is that the gravity force of the metrology frame is 7.2 kN, which is an experimental validation of the passive vertical force produced by the PM-based gravity compensator.

### Table 8.1: Parameters of the inertia-spring-damper model for each DoF.

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<thead>
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<th>DoF</th>
<th>inertia</th>
<th>stiffness</th>
<th>damping coefficient</th>
</tr>
</thead>
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<tr>
<td>$f_x \rightarrow q_z$</td>
<td>730 kg</td>
<td>-61 N/mm</td>
<td>600 N·s/m</td>
</tr>
<tr>
<td>$f_y \rightarrow q_z$</td>
<td>730 kg</td>
<td>-75 N/mm</td>
<td>600 N·s/m</td>
</tr>
<tr>
<td>$t_z \rightarrow q_{\psi}$</td>
<td>90 kg·m²</td>
<td>0.4 N·m/mrad</td>
<td>50 N·m/s/rad</td>
</tr>
<tr>
<td>$t_x \rightarrow q_{\phi}$</td>
<td>57 kg·m²</td>
<td>0.4 N·m/mrad</td>
<td>50 N·m/s/rad</td>
</tr>
<tr>
<td>$l_y \rightarrow q_{\theta}$</td>
<td>57 kg·m²</td>
<td>0.2 N·m/mrad</td>
<td>50 N·m/s/rad</td>
</tr>
<tr>
<td>$f_z \rightarrow q_z$</td>
<td>730 kg</td>
<td>1 N/mm</td>
<td>600 N·s/m</td>
</tr>
</tbody>
</table>

8.4 Nonlinear damping analysis

SEMIS exhibits certain nonlinear behavior. The vertical translation is taken as an example to show these behaviors. Vertical sinusoidal force (0.5 Hz and 8 Hz) is applied to excite the position-controlled closed-loop system and the vertical relative displacement responses $q_{rz}$ are recorded (200 s for 0.5 Hz and 50 s for 8 Hz). Remark that the shaker table is coupled to the floor during these measurements. Fast Fourier Transform (FFT) magnitudes of the responses $q_{rz}$ are calculated and plotted in Fig. 8.10 and Fig. 8.11. The existence of harmonics other than the fundamental harmonics in Fig. 8.10 proves the nonlinear behavior of SEMIS. The ratio of higher order harmonics over the fundamental harmonic is used to quantify the nonlinear behavior. This ratio for the response of the 8 Hz excitation is
Chapter 8. Dynamic behavior of the contactless electromagnetic suspension system

![Graph](image)

**Figure 8.4:** The FRF $f_x \rightarrow q_x$ (solid) compared to the corresponding model (dashed). The mass of the model is 730 kg.

![Graph](image)

**Figure 8.5:** The FRF $f_y \rightarrow q_y$ (solid) compared to the corresponding model (dashed). The mass of the model is 730 kg.
8.4. Nonlinear damping analysis

Figure 8.6: The FRF $t_z \rightarrow q_\psi$ (solid) compared to the corresponding model (dashed). The moment of inertia of the model is 90 kg $\cdot$ m$^2$.

Figure 8.7: The FRF $t_x \rightarrow q_\phi$ (solid) compared to the corresponding model (dashed). The moment of inertia of the model is 57 kg $\cdot$ m$^2$. 
Figure 8.8: The FRF $t_y \rightarrow q_{\theta}$ (solid) compared to the corresponding model (dashed). The moment of inertia of the model is $57 \text{ kg} \cdot \text{m}^2$.

Figure 8.9: The FRF $f_z \rightarrow q_z$ (solid) compared to the corresponding model (dashed). The mass of the model is $730 \text{ kg}$.
much smaller than that of the 0.5 Hz excitation, which can be clearly seen in Fig. 8.10 and Fig. 8.11. It indicates that the nonlinear behavior at 8 Hz is weaker than that of 0.5 Hz.

These nonlinear behaviors are most probably introduced by the interactions between the two parts of the Lorentz actuators. The coil of each Lorentz actuator is sealed in resin and integrated to a holding frame which is made of stainless 316 steel [54]. The stainless 316 steel can be very weakly magnetized by a Permanent Magnet (PM). The interaction between a PM and a piece of stainless 316 steel is much weaker than that between a PM and a piece of iron but it is strong enough to be felt. The effects of this interaction on dynamic behaviors of the Lorentz actuators are briefly described as follows.

We assume the following scenario to explain this interaction: a cuboidal PM is positioned with its magnetized direction perpendicular to the surface of a stainless 316 steel block. When the PM is moving perpendicular to the surface of the stainless 316 steel, the interaction force between these two objects is position-dependent. This position-dependent force introduces stiffness and this effect has been explained in Chapter 7. When the PM is moving parallel to the surface of the stainless 316 steel block, the force acting on the PM introduced by the interaction between the two objects is against the motion and is velocity-dependent [27, 50, 58]. This velocity-dependent force is a type of damping force with a decreasing coefficient (velocity-derivative of the damping force) with increasing frequency. This frequency-dependent coefficient indicates that it is a nonlinear behavior.

It is shown in Fig. 8.4 - Fig. 8.9 that the FRF at frequencies close to 1 Hz is not smooth. It is possibly due to bad signal to noise ratio. It is also possible that these nonlinear behaviors play a part. Fig. 8.4 - Fig. 8.9 also indicate that these nonlinear behaviors are not affecting high-bandwidth (>100 Hz) closed-loop control which will be implemented later.

8.5 Cross-coupling

8.5.1 Cross-coupling quantification

To quantify the cross-coupling of SEMIS, the \( \mu \)-interaction measure is plotted in Fig. 8.12. The \( \mu \)-interaction measure (solid) is calculated from the FRF measured by accelerometers. The FRF measured by position sensors is compromised by position sensor noises and the shaker table vibrations at frequencies above 10 Hz so that it does not represent the true SEMIS dynamics.

The magnitude of the \( \mu \)-interaction measure can be trusted at frequencies higher than 3 Hz. Within the frequency range of \([3.5, 200]\) Hz, its magnitude is a little bit noisy and is mostly within the range of \([-30, -20]\) dB. The small spike at 80 Hz is induced by the shaker table vibrations due to the flexible coupling to the floor. At 600 Hz, the high-frequency \( \mu \)-interaction measure crosses over the 0 dB.

The noisy magnitude between 3.5 and 200 Hz is induced by the noisy off-diagonal FRF measurement. To prove this point, the FRF of SEMIS is re-measured at selected frequencies using sine excitations instead of white noise excitations. The calculated \( \mu \)-interaction measure is plotted as star-markers in Fig. 8.12. The advantage of sine excitation is that the excitation energy is focused at a single frequency so that the FRF, especially the off-diagonal entries which have lower magnitude, is calculated using output vectors with better signal-to-noise ratio. The star-markers in Fig. 8.12 are almost flat with magnitude of -30 dB. They all have lower magnitude than the solid curve, which proves that the noisy \( \mu \)-
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Figure 8.10: FFT magnitude calculated from the response of the vertical relative displacement $q_{rz}$ to a 0.5 Hz vertical sine force excitation. Existence of harmonics proves the nonlinear behavior of SEMIS.

Figure 8.11: FFT magnitude calculated from the response of the vertical relative displacement $q_{rz}$ to an 8 Hz vertical sine force excitation.
interaction measure at corresponding frequencies is due to lack of excitation power during FRF measurement.

![Graph showing frequency (Hz) vs. magnitude (dB) for FRF measurements.]

**Figure 8.12:** The $\mu$-interaction measure of SEMIS.

### 8.5.2 Static decoupling

Beside the geometric transformation matrices, additional static decoupling matrices will not be applied to this 6-DoF system. There are two reasons explained as follows.

The first reason is the low-mass design of the shaker table which is intended to save cost. SEMIS has six inputs and twelve outputs. It can be treated as two square systems ($\vec{w}_a \rightarrow \vec{q}_r$ and $\vec{w}_a \rightarrow \vec{a}_a$) but these two square systems cannot be decoupled using the same pair of static matrices. The reason is that the equality $\vec{w}_a \rightarrow \vec{q}_r = \vec{w}_a \rightarrow \vec{a}_a s^{-2}$ is not true for this particular system. The shaker table (400 kg), which serves as a test rig, is designed to have less mass than the metrology frame (700 kg) to save cost. As such, the transfer function from control wrench to shaker table vibration ($\vec{w}_a \rightarrow \vec{q}_b$) may not be ignored.

**Remark 8.5.1.** In practice, an active suspension system is usually built on a solid base frame, which has much more mass than the metrology frame. As such, the transfer function from control wrench to the base frame displacement ($\vec{w}_a \rightarrow \vec{q}_b$) may be ignored compared to the transfer function from the control wrench to metrology frame displacement ($\vec{w}_a \rightarrow \vec{q}_a$). Therefore, the static decoupling is still feasible for practical active suspension systems.

The second reason is the limitation of SEMIS itself. The cross-coupling between 3 Hz and 200 Hz cannot be further reduced because the off-diagonal entries of FRF cannot be measured with higher accuracy due to their low magnitudes and limited actuator...
capacity. The cross-coupling at high frequencies (> 200 Hz) are induced by flexible modes of the metrology frame. Further reducing of these cross-couplings is not possible by static decoupling.

Due to these two reasons, it is not possible to apply static decoupling matrices in addition to the geometric transformation matrices. Fortunately, it is not necessary to apply these static decoupling matrices. This will be proved by experimental results presented in Chapter 9. The reason is briefly explained as follows.

Further reducing the cross-coupling at frequencies less than 200 Hz is not necessary because the $\mu$-interaction measure is already lower than that of the 3-DoF system described in Chapter 4, wherein, static optimal decoupling is successfully implemented. This frequency range is of the most important because it includes the intended control bandwidth.

Remark 8.5.2. The low cross-coupling of SEMIS proves the success in gravity compensator design and system-level design. The PM-based gravity compensator is designed to minimize the stiffness which also minimizes the cross-coupling. The single-isolator concept proposed in this thesis eliminates unnecessary cross-coupling that exists in conventional multi-isolator concepts, as explained in Chapter 7.

8.6 Passive damping identification

8.6.1 Motivation

The passive damping is one of the most important system characteristics of SEMIS. It influences the passive transmissibility as well as the closed-loop transmissibility beyond the control-bandwidth, as explained in Chapter 1. The damping coefficient is defined as the velocity-derivative of the passive wrench and it is used to evaluate the influence of the damping force to system dynamics. For a linear mechanical system, the damping coefficient is a constant. For SEMIS, the damping coefficients for all 6-DoF motions are not constants due to the nonlinear behaviors of SEMIS, as described in Section 8.4. There are three factors that induce passive damping of SEMIS:

1. The nonlinear damping introduced by interactions between the two parts of the Lorentz actuators (stainless 316 steel plate and the PMs), as described in Section 8.4;
2. Trapped air in the electromagnetic isolator;
3. Dynamic interactions between the PMs that are integrated in the two parts of the gravity compensator.

The first factor has been analyzed in Section 8.4. The damping coefficient of this factor decreases with increasing frequency. The influence of the nonlinear effect introduced by this factor is not visible on FRF at frequencies above 2 Hz. However, it is still possible that the damping coefficient is not zero at higher frequencies. In future research or applications, the passive damping introduced by this factor can be removed by employing another type of material instead of stainless 316 steel. This type of material should possess the following properties:

1. the electricity-conductivity should be as low as possible to avoid eddy currents;
2. good heat-conductivity for heat dissipation;
Due to the mechanical design of the electromagnetic isolator, the air trapped between the two parts of the electromagnetic isolator acts as a passive damper. As this gravity compensator is intended to be implemented in vacuum, as required by the future EUV lithography, practical elimination of the passive damping introduced by this factor is feasible.

The PMs integrated in the gravity compensator is a type of steel-alloy using rare-earth elements. Such material is a conductor to electricity. Relative motion between a conductor and a PM produces eddy currents in the conductor. For this reason, the PMs in the gravity compensator all have eddy currents flowing in themselves. The interaction forces introduced by these eddy currents are velocity dependent and are therefore damping forces. These damping forces are hypothetically assumed to be weak in [54]. Validation of this hypothesis is necessary as it remains with the gravity compensator in future applications.

As explained, the first two factors can be removed in future practical applications. Due to the difficulty of creating a vacuum environment in our lab, the passive damping introduced by the trapped air in electromagnetic isolator can not be removed here. As SEMIS has been built, it is not feasible to change the stainless-steel plate in the Lorentz actuators. For these reasons, it is not possible to quantify the damping coefficient introduced by each factor separately using this setup. The total damping coefficient introduced by all three factors, which is also the damping coefficient of SEMIS, is to be measured as an upper-bound of the damping coefficient introduced by the third factor. Nevertheless, the influence of the first factor can be reduced as its influence to system dynamics is frequency-dependent.

It has been described in Section 8.4 that the influence of the first factor is only significant at frequencies lower than 5 Hz. To reduce the influence of the first factor, the damping coefficient at frequencies around 10 - 50 Hz is estimated by curve-fitting on position-controlled closed-loop transmissibility. At higher frequencies, accurate measurement of the transmissibility is difficult due to the limitations of the shaker table.

8.6.2 Experiment description

As passive damping affects on passive transmissibility, the FRF of passive transmissibility can be measured to identify the passive damping coefficient. Since SEMIS is inherently unstable, the transmissibility of a weakly position-controlled closed-loop is measured instead of the passive transmissibility to identify the passive damping coefficient by curve fitting in frequency domain.

The FRF of the position-controlled closed-loop transmissibility is measured using the same procedure as described in Chapter 6. The three Shaker Voice Coil Actuators (SVCAs) are used to excite vibrations on the shaker table. Low-pass filtered white noise signal is used as excitation force or torque. The shaker table acceleration and the metrology frame acceleration are recorded as input and output signals, respectively. The transmissibility FRF is estimated by calculating the Cross Power Spectral Density (CPSD) of the input and output signals using the Welch’s averaged periodogram method which is commercially available in Matlab Signal Processing Toolbox [76]. As explained in Chapter 6, only 3-DoF vibrations can be excited on the shaker table: the vertical translation \(a_{b_{z}}\) and the two horizontal rotations \(a_{b_{\phi}}\) and \(a_{b_{\theta}}\). For this reason, transmissibility measurement is only possible for a \(3 \times 3\) transfer function matrix. However, due to the cross-coupling of the
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shaker table, the off-diagonal entries of the transmissibility can not be accurately measured, as described in Chapter 6. Nevertheless, accurate measurement of the diagonal entries is sufficient to identify the corresponding damping coefficients. These three diagonal entries are measured in separated experiments. In each experiment, a single-DoF motion of the shaker table is excited using the SVCAs and the cross-coupling is simply ignored. The acceleration of this DoF motion of the shaker table is recorded as the input signal. The same DoF motion of the metrology frame is measured by accelerometers as the output signal. These input-output signals are used to calculate the transmissibility. The sampling frequency for these experiments is 2 kHz.

In Fig. 8.13 - Fig. 8.15, the diagonal entries of the position-controlled closed-loop transmissibility are compared to the calculated curves based on the corresponding controller and model. The model parameters are presented in Table 8.1. As the values of stiffness and inertia have been validated, the damping coefficient is the only variable to validate. To further illustrate the accuracy of the values of the damping coefficient, another set of models with higher damping coefficient than the values in Table 8.1 is used to calculate the corresponding transmissibility curves, which are also plotted in Fig. 8.13 - Fig. 8.15.

For the vertical translational transmissibility (see Fig. 8.13), the measured FRF magnitude matches the magnitude calculated from the identified damping coefficient listed in Table 8.1 (600 \( N \cdot s/m \)) from 2 Hz to 30 Hz. From 30 Hz to 80 Hz, the measured FRF magnitude is slightly lower than the calculated curve. It is consistent to the theory that the damping coefficient introduced by the Lorentz actuators decreases with increasing frequency. For higher frequencies, the measured FRF magnitude is very noisy, which is due to weak excitation at these frequencies. The transmissibility magnitude calculated using higher damping coefficient (800 \( N \cdot s/m \)) is slightly higher than the other two curves. This gives an estimation on relative error of the identified damping coefficient (600 \( N \cdot s/m \)).

For the two rotational transmissibility (see Fig. 8.14 and Fig. 8.15), the measured FRF magnitude matches the magnitude calculated from the identified damping coefficient listed in Table 8.1 (50 \( N \cdot m \cdot s/\text{rad} \)) from 20 Hz to 100 Hz. The transmissibility magnitude calculated using higher damping coefficient (100 \( N \cdot m \cdot s/\text{rad} \)) is slightly higher than the other two curves. This gives an estimation error on the identified damping coefficient (50 \( N \cdot m \cdot s/\text{rad} \)). For frequencies lower than 20 Hz, the measured FRF’s do not match the corresponding calculations. This is possibly due to the effects of the nonlinear damping force introduced by Lorentz actuators.

These comparison results validate the three damping coefficients: \( c_z = 600 \ N \cdot s/m \), \( c_\phi = c_\theta = 50 \ N \cdot m \cdot s/\text{rad} \). For the damping coefficient of the other 3-DoF motions, validation using transmissibility measurement is not possible due to limitation of the shaker table. The damping coefficients of the two motions, \( f_x \rightarrow q_x \) and \( f_y \rightarrow q_y \), are expected to have the same order as that of \( f_z \rightarrow q_z \) and they are assumed to be the same as \( c_z \) in Table 8.1. The damping coefficient of the motion \( t_z \rightarrow q_\psi \) is expected to be of the same order of \( c_\phi \) and \( c_\theta \) and it is assumed to be the same as \( c_\phi \) and \( c_\theta \) in Table 8.1.

8.7 Conclusions

Two types of experiments have been described in this chapter. First, the FRF of SEMIS has been measured within the frequency range of 1 Hz - 1000 Hz. Second, the transmis-
8.7. Conclusions

Figure 8.13: Transmissibility of position-controlled SEMIS: vertical translation $a_b \rightarrow a_z$. $c_z$ denotes the corresponding damping coefficient.

Figure 8.14: Transmissibility of position-controlled SEMIS: rotation around $x$-axis $a_b \rightarrow a_\phi$. $c_\phi$ denotes the corresponding damping coefficient.
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![Graph of Transmissibility](image)

Figure 8.15: Transmissibility of position-controlled SEMIS: rotation around y-axis $a_{\theta} \rightarrow a_{\theta}$. $c_\theta$ denotes the corresponding damping coefficient.

measured FRF of the position-controlled closed-loop system has been measured to identify the damping coefficients. Results of these two types of experiments are concluded in two separated subsections.

8.7.1 Conclusions on the measured FRF of SEMIS

The mass and moments of inertia of the floating metrology frame have been quantified, listed in Table 8.1. The identified mass (730 kg) provides an experimental proof of the vertical passive force (7.2 kN) produced by the PM-based gravity compensator.

The flexible modes of the metrology frame have been identified to be above 700 Hz. This is a support to the suitability of high-bandwidth control of the PM-based gravity compensator. The experimental validation will be given in Chapter 9, in which, high-bandwidth vibration isolation control is implemented. The causes of these flexible modes are hypothetically assumed to be the cube-shaped mechanical components which connect the accelerometers and the metrology frame.

Weak cross-coupling from 3 Hz to 200 Hz has been proved by the $\mu$-interaction measure calculated from the measured FRF. The $\mu$-interaction measure increases to 0 dB at 600 Hz. This is because of the resonances introduced by flexible modes of the metrology frame.

It has been concluded that further decoupling SEMIS using static matrices in addition to the geometric transformation matrices is neither possible nor necessary. It is not possible because

- SEMIS can be treated as two six-input-six-output systems and these two systems can
not be decoupled using the same pair of static matrices, as explained in Section 8.5.2.

- Within frequency range 3 Hz to 200 Hz, off-diagonal entries of the FRF can not be accurately measured because their magnitudes are too low compared to the diagonal entries.
- At higher frequencies, static decoupling is not possible due to the flexible modes of the metrology frame.

It is not necessary because the cross-coupling is already minimized at a frequency range which includes the intended control-bandwidth. The $\mu$-interaction measure around these frequencies is comparable to that of the 3-DoF system described in Chapter 4, wherein, static optimal decoupling has been successfully implemented.

### 8.7.2 Conclusions on passive damping quantification

The damping coefficients of 3-DoF motions $t_x \rightarrow q_\phi$, $t_y \rightarrow q_\theta$, and $f_z \rightarrow q_z$ have been experimentally identified by curve fitting on position-controlled closed-loop transmissibility using the 3-DoF shaker table. The values are summarized in in Table 8.1. For the other 3-DoF motions, the damping coefficient can not be directly measured using this method. The values listed in Table 8.1 are based on reasonable estimation.

Experiments elucidate the low passive damping property of SEMIS. This also proves the low passive damping property of the PM-based gravity compensator, because the passive damping introduced by interactions between PMs inside the gravity compensator is always smaller than that of the total system (SEMIS).
Chapter 9

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As explained in Chapter 8, the six Degrees-of-Freedom (DoFs) motions of SEMIS are already well decoupled around the intended control bandwidth. Further static decoupling is neither possible nor necessary. Therefore, each DoF motion is treated as a single-DoF system during the vibration isolation control design. The sliding surface control as described in Chapter 3 and Chapter 5 is applied to design a vibration isolation controller for each DoF motion. The measured FRF presented in Chapter 8 for each DoF motion is used to calculate the open-loop bode plot in order to predict closed-loop stability and to indicate the stability margins.

9.1 System description

An xPC target machine is used to implement the designed controllers for fast prototyping. The relative displacement between the metrology frame and the shaker table is measured using six fiber-optical sensors (Philtec RC100). The output signals of these six displacement sensors are acquired by a 16-bit Analog-to-Digital Converter (ADC), which is an integrated module of the xPC target machine. The absolute acceleration of the metrology frame is measured using six accelerometers (Kistler 8330). The details of the acceleration signal acquisition are described in Section 6.1.3. It is briefly described here. The acceleration signals measured by accelerometers are filtered by a first-order high-pass filter (an analog passive RC circuit) to remove DC bias and subsequently filtered by a 4\textsuperscript{th}-order analog filter (Krohn-Hite 3360 series). Finally, the filtered signals are acquired to the xPC target machine using the same 16-bit ADC. This analog filter is set to butterworth low-pass filter with 500 Hz cut-off frequency. Using this analog filter, the acceleration signals are amplified by a factor of 100. The purpose of this amplification is to increase the total voltage-to-acceleration sensitivity of the acceleration measurement loop. As a result, the digitalized acceleration signals have better resolution. This amplification gain will be compensated in the digital controller (the xPC target machine). The output signal of the RC circuit has a small DC offset (around 10 mV), due to the imperfect electrical components (resistors and capacitors). This DC offset will be also amplified by a factor of 100. Due to this amplified DC offset, SEMIS will produce high transient response as soon as high-
9.1. System description

The xPC target machine calculates the control effort and sends the control command to digital current amplifiers by serial communication (RS422). The digital current amplifiers control the electrical current of the six Lorentz actuators so that desired control forces can be applied to the metrology frame. As these current amplifiers have their bandwidth in the order of kHz, their dynamics at frequencies up to the order of 100 Hz are ignored.

The block diagram of vibration isolation control implementation on SEMIS is provided in Fig. 9.1. It is similar to the control diagram for the 3-DoF suspension system described in Chapter 6. The blocks in the dashed rectangular are digitally implemented in the xPC target machine. Fig. 9.2 provides the diagram of SEMIS itself.

For both figures, the signal vectors \( \vec{q}_s \), \( \vec{a}_s \), \( \vec{q}_r \), \( \vec{q}_b \), \( \vec{a}_a \), \( \vec{w}_a \), \( \vec{w}_p \), \( \vec{i}_L \), and \( \vec{f}_L \) are defined in Table 7.1. The signal vector \( \vec{w}_d \) is the disturbance wrench, which can be artificially excited for purposes of SEMIS FRF measurement or closed-loop compliance measurement. The signal vectors \( \vec{n}_v \) and \( \vec{n}_a \) denote the noises for position sensors and accelerometers, respectively. The blocks \( RC \) and \( F_L \) denote the RC circuit and the \( 4^{th} \)-order analog filter, respectively. The dynamics of these two blocks can be described by the following transfer functions.
functions:

\[ F_{RC} = \frac{s}{s+0.21}, \quad (9.1) \]

\[ F_L = \frac{9.741 \times 10^{13}}{(s^2 + 5803s + 9.87 \times 10^6)(s^2 + 2406s + 9.87 \times 10^6)}. \quad (9.2) \]

The block \( C_1 \) denotes the controller for the position loop and the block \( C_2 \) denotes the controller for the acceleration loop. Each of them is a \( 6 \times 6 \) diagonal transfer function matrix. As such, each of them can be treated as six Single-Input-Single-Output (SISO) controllers. The geometric transformation matrices \( fT_w = w^{-1}T_{fw}, qT_s = q^{-1}T_{qs}, \) and \( aT_s = a^{-1}T_{sa} \) are calculated and presented in (7.5), (7.6), and (7.7), respectively. In Fig. 9.2, the matrix \( M \) denotes the inertia matrix of the metrology frame. The block PM denotes the PM-based gravity compensator.

### 9.2 Vibration isolation control design

#### 9.2.1 Controller structure

As no further static decoupling is applied, vibration isolation control is designed for each DoF motion of SEMIS using the sliding surface control approach described in Chapter 3 and 5. The two controllers, \( C_1 \) and \( C_2 \) in Fig. 9.1, have the following forms:

\[ C_1 = RA_1, \quad C_2 = A_2, \quad (9.3) \]

where \( R \) denotes the transfer function matrix of the regulator and \( A_i, \forall i \in \{1, 2\} \) denote the transfer function matrices used to shape the sliding surface in frequency domain. The regulator and \( A_i, \forall i \in \{1, 2\} \) have the form of

\[
R = \begin{bmatrix}
R_x & 0 & 0 & 0 & 0 & 0 \\
0 & R_y & 0 & 0 & 0 & 0 \\
0 & 0 & R_\psi & 0 & 0 & 0 \\
0 & 0 & 0 & R_\phi & 0 & 0 \\
0 & 0 & 0 & 0 & R_\theta & 0 \\
0 & 0 & 0 & 0 & 0 & R_z
\end{bmatrix}, \quad A_i = \begin{bmatrix}
A_{ix} & 0 & 0 & 0 & 0 & 0 \\
0 & A_{iy} & 0 & 0 & 0 & 0 \\
0 & 0 & A_{i\psi} & 0 & 0 & 0 \\
0 & 0 & 0 & A_{i\phi} & 0 & 0 \\
0 & 0 & 0 & 0 & A_{i\theta} & 0 \\
0 & 0 & 0 & 0 & 0 & A_{iz}
\end{bmatrix}. \quad (9.4)
\]

The high-pass filter, \( F_h \), has a form of

\[
F_h = \begin{bmatrix}
F_{hx} & 0 & 0 & 0 & 0 & 0 \\
0 & F_{hy} & 0 & 0 & 0 & 0 \\
0 & 0 & F_{h\psi} & 0 & 0 & 0 \\
0 & 0 & 0 & F_{h\phi} & 0 & 0 \\
0 & 0 & 0 & 0 & F_{h\theta} & 0 \\
0 & 0 & 0 & 0 & 0 & F_{hz}
\end{bmatrix}. \quad (9.5)
\]

The subscripts \( x, y, z \) denote the three translations along the three Cartesian axes. The subscripts \( \phi, \theta, \psi \) denote the three rotations about the three Cartesian axes \( x, y, z \), respectively.
9.2. Vibration isolation control design

9.2.2 Performance criteria

The loop-gain for each DoF $L_i, \forall i \in \{x, y, \psi, \phi, \theta, z\}$ can be calculated by

$$L_i = R_i (\Lambda_{1i} + \Lambda_{2i} F_{RL} F_{RC} F_{h} s^2) P_i, \forall i \in \{x, y, \psi, \phi, \theta, z\}. \tag{9.6}$$

$P_i$ denotes the diagonal entries of the transfer function matrix of SEMIS. To calculate the loop-gain, $P_i$ is represented by the inertia-spring-damper model provided in Table 8.1 and the measured FRF plotted in Fig. 8.4 - Fig. 8.9. As defined in Chapter 3, the loop-gain $L_i$ can be separated into two parts:

1. The relative loop-gain is the open-loop transfer function of the relative displacement loop:

$$L_{ri} = R_i \Lambda_{1i} P_i, \forall i \in \{x, y, \psi, \phi, \theta, z\}.$$ 

2. The absolute loop-gain is the open-loop transfer function of the absolute acceleration loop:

$$L_{ai} = R_i F_{RL} \Lambda_{2i} F_{RC} F_{h} s^2 P_i, \forall i \in \{x, y, \psi, \phi, \theta, z\}.$$ 

Let $m_i$, $c_i$, and $k_i$ denote the inertia, damping coefficient, and stiffness of each of 6-DoF motions $i \in \{x, y, \psi, \phi, \theta, z\}$. The values of these parameters are provided in Table 8.1 in Chapter 8. The passive transmissibility $T_p$ and the passive compliance $C_p$ have the form of

$$T_{pi} = \frac{c_i s + k_i}{m_i s^2 + c_i s + k_i}, \quad C_{pi} = \frac{1}{m_i s^2 + c_i s + k_i}. \tag{9.7}$$

The transmissibility determined by the sliding surface, or the designed transmissibility referred in Chapter 3, is defined as

$$T_{di} = \frac{\Lambda_{1i}}{\Lambda_{1i} + \Lambda_{2i} F_{RL} F_{RC} F_{h} s^2}, \forall i \in \{x, y, \psi, \phi, \theta, z\}. \tag{9.8}$$

According to the formulas given in Chapter 3, the closed-loop performance criteria are calculated. The closed-loop transmissibility is

$$T_{ci} = \frac{C_1 C_{pi} + T_{pi}}{1 + C_{pi} (C_1 + C_2 s^2)}, \tag{9.9}$$

The closed-loop compliance is

$$C_{ci} = \frac{C_{pi}}{1 + C_{pi} (C_1 + C_2 s^2)}. \tag{9.10}$$

As explained in Chapter 3, there are two sensitivity functions concerned in vibration isolation control because there are two types of sensors employed. The relative sensitivity is the transfer function from the position sensor noise to the payload absolute displacement. The absolute sensitivity is the transfer function from the accelerometer noise to the payload absolute displacement. The closed-loop relative sensitivity is

$$R_{ci} = \frac{C_1 C_{pi}}{1 + C_{pi} (C_1 + C_2 s^2)}, \tag{9.11}$$
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The closed-loop absolute sensitivity is
\[
S_{ci} = \frac{C_2 C_{pi}}{1 + C_{pi}(C_1 + C_2 s^2)}.
\] (9.12)

The relative sensitivity determined by the sliding surface, or the designed relative sensitivity, is the same as the designed transmissibility: \( R_{di} = T_{di}, \forall i \in \{x, y, \psi, \phi, \theta, z\} \). The absolute sensitivity determined by the sliding surface, or the designed absolute sensitivity, is
\[
S_{di} = \frac{\Lambda_{2i}}{\Lambda_{1i} + \Lambda_{2i} F_{RC} F_{hi} s^2}, \forall i \in \{x, y, \psi, \phi, \theta, z\}.
\] (9.13)

The details of the control design are described in the following subsection.

9.2.3 Controller parameters

As described in Chapter 3, the sliding surface control design for a 1-DoF suspension system takes two steps. In the first step, the sliding surface (consisting two transfer functions \( \Lambda_{1i} \) and \( \Lambda_{2i} \)) is designed to optimize the three designed performance criteria (transmissibility and two sensitivity functions). In the second step, a regulator \( R_i \) is designed to realize the designed performance criteria and to suppress the compliance. For the sliding surface design, the Power Spectrum Density (PSD) ratios of the sensor noises over the base displacement are estimated as \( |G_x(\omega)| = 0.1 \) and \( |G_y(\omega)| = 1 \).

For the two horizontal translations, the constraint constants for sliding surface optimization are defined as
- \( \omega_0 = 0.01 \text{ Hz}, \omega_1 = 1 \text{ Hz}, \omega_2 = 10 \text{ Hz} \).
- \( \epsilon_0 = 1.2589 \text{ (2 dB)}, \epsilon_1 = 1 \text{ (0 dB)}, \epsilon_2 = 0.01 \text{ (-40 dB)} \).

Applying the optimization procedure described in Section 3.7.2 yields the following sliding surface design:
\[
\Lambda_{1x} = \Lambda_{1y} = \frac{31.29 s^2 + 25.12 s + 7.781}{s^2 + 6.712 s}, \quad \Lambda_{2x} = \Lambda_{2y} = 1.
\] (9.14)

The regulator is designed as
\[
R_x = R_y = \frac{7 \times 10^5 (s^2 + 197.4 s + 2 \times 10^4)}{(s + 5)(s^2 + 0.8026 s + 0.2487)}.
\] (9.15)

For the three rotations, the constraint constants for sliding surface optimization are defined as
- \( \omega_0 = 0.01 \text{ Hz}, \omega_1 = 2 \text{ Hz}, \omega_2 = 20 \text{ Hz} \).
- \( \epsilon_0 = 1.4125 \text{ (3 dB)}, \epsilon_1 = 1 \text{ (0 dB)}, \epsilon_2 = 0.01 \text{ (-40 dB)} \).

Applying the optimization procedure described in Section 3.7.2 yields the following sliding surface design:
\[
\Lambda_{1\psi} = \Lambda_{1\phi} = \Lambda_{1\theta} = \frac{111.2 s^2 + 77.84 s + 20.95}{s^2 + 12.07 s}, \quad \Lambda_{2\psi} = \Lambda_{2\phi} = \Lambda_{2\theta} = 1.
\] (9.16)
Due to the difference in inertia, the regulator design is different for the three rotational motions. The regulator for the vertical rotational motion is designed as

\[
R_\psi = \frac{7.5 \times 10^4(s^2 + 242.2s + 3 \times 10^4)}{(s + 5)(s^2 + 0.7003s + 0.1885)}.
\]  
(9.17)

The regulator for the two horizontal rotational motions is designed as

\[
R_\phi = R_\theta = \frac{5 \times 10^4(s^2 + 242.2s + 3 \times 10^4)}{(s + 1)(s^2 + 0.7003s + 0.1885)}.
\]  
(9.18)

For the vertical translation, the constraint constants for sliding surface optimization are defined as

- \( \omega_0 = 0.01 \text{ Hz}, \omega_1 = 0.7 \text{ Hz}, \omega_2 = 7 \text{ Hz}. \)
- \( \varepsilon_0 = 1.2589 \text{ (2 dB)}, \varepsilon_1 = 1 \text{ (0 dB)}, \varepsilon_2 = 0.01 \text{ (-40 dB)}. \)

Applying the optimization procedure described in Section 3.7.2 yields the following sliding surface design:

\[
\Lambda_1 z = \frac{15.25s^2 + 10.24s + 1.382}{s^2 + 4.962s}, \quad \Lambda_2 z = 1.
\]  
(9.19)

The regulator is designed as

\[
R_z = \frac{7 \times 10^5(s^2 + 197.4s + 2 \times 10^4)}{(s + 1)(s + 0.4845)(s + 0.1871)}.
\]  
(9.20)

The limit of further lowering the transmissibility cross-over frequency for the vertical translation (0.7 Hz) and the three rotations (2 Hz) is noises in the acquired acceleration signals. These noises are partly from the accelerometer themselves and are partly caused by the RC circuits. For the two horizontal translations, the limit of further lowering the transmissibility cross-over frequency (1 Hz) is the horizontal rotation-translation cross-talk caused by false signals of the accelerometers. This will be further analyzed in Section 9.4.1.

The regulators are designed to make a trade-off between the control bandwidth and stability margins. The control bandwidth and stability margins for each DoF motion are summarized in Table 9.1. It will be shown in Section 9.3.1 that the control bandwidth and the stability margins are pushed to their system limits by the corresponding regulator design.

To calculate the designed transmissibility, the dynamics of the RC circuit and the first-order high-pass filter \( (F_{hi}) \) have to be considered as well. The pole of the RC circuit is 0.21 rad/s which is corresponding to 0.03 Hz. In theory, this low frequency RC circuit has virtually no influence on the sliding surface dynamics. However, significant noises (much higher than specified noise spectrum by accelerometer supplier) with main frequency component of 0.01 Hz are observed on the acquired acceleration signals. This noise is from the RC circuit and amplified by the 4\(^{th}\)-order analog filter by a factor of 100. For capacitors whose capacitance are in the order of 1 \( \mu \text{F} \), their capacitance are usually time-variant with small amplitudes and large time constants. The first-order high-pass filter \( (F_{hi}) \) has a second
purpose, which is to filter these acceleration signal noises. For this purpose, it would be better for \( F_{hi} \) to have larger pole. On the other hand, \( F_{hi} \) would compromise the optimized sliding surface dynamics if the pole is too large. For the vertical translation, the first-order high-pass filter is designed as
\[
F_{hz} = \frac{s}{s + 0.8}. \tag{9.21}
\]
For other five DoF motions,
\[
F_{hx} = F_{hy} = F_{h\psi} = F_{h\phi} = F_{h\theta} = \frac{s}{s + 1}. \tag{9.22}
\]
The reason to choose a smaller pole for \( F_{hz} \) is that the designed transmissibility for the vertical translation crosses over 0 dB at a frequency less than 0.7 Hz, which is relatively lower than the other 5-DoF motions.

9.3 Performance prediction

9.3.1 Open-loop gain
To evaluate the closed-loop stability, the loop-gains for all 6-DoF motions are calculated separately using the measured FRF and the inertia-spring-damper model. They are plotted in the figure on the left of Fig. 9.3 - Fig. 9.8. The magnitude curves calculated using the two methods match each other except at the high frequencies (> 500 Hz) due to the resonances higher than 700 Hz. The phase lag at high frequencies (> 100 Hz) is caused by the low-pass behavior of the regulator and the analog filter \( F_L \). The difference between the two phase curves at these frequencies is induced by the time-delay during the FRF measurement.

The relative loop-gain, absolute loop-gain, and the loop-gain calculated from the inertia-spring-damper model are compared in the figure on the right of Fig. 9.3 - Fig. 9.8. These figures show that the loop-gain coincides with the relative loop-gain at low frequencies and with the absolute loop-gain at high frequencies. In between, there is a one-decade frequency range, wherein, the loop-gain exchange from the relative loop-gain to the absolute loop-gain. The frequency interval of this one-decade frequency range is determined by the sliding surface design.

The phase margin, gain margin, and the control bandwidth according to the loop-gain calculated from the model are summarized in Table 9.1 for all 6-DoF motions. The stability margins predict the closed-loop stability. The three rotations have slightly lower bandwidth (130 - 140 Hz) but higher stability margins compared with the three translations. The time-delay in the FRF measurement will consume about 10° phase margin. According to the plotted loop-gain, the control bandwidth and stability margins are pushed to a limit by the sliding surface control design.

9.3.2 Transmissibility
The transmissibility determined by the sliding surface, or the designed transmissibility, is compared with the passive transmissibility and the closed-loop transmissibility in Fig. 9.9 - Fig. 9.11 for all 6-DoF motions. The designed transmissibility for each DoF motion is calculated according to (9.8) using the corresponding \( \Lambda_{1i} \) and \( \Lambda_{2i} \), \( \forall i \in \{x, y, \psi, \phi, \theta, z\} \).
9.3. Performance prediction

Figure 9.3: The open-loop bode plot of the motion $f_x \rightarrow q_x$.

Figure 9.4: The open-loop bode plot of the motion $f_y \rightarrow q_y$.

Figure 9.5: The open-loop bode plot of the motion $t_z \rightarrow q_\psi$. 
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**Figure 9.6:** The open-loop bode plot of the motion $t_x \rightarrow q_\phi$.

**Figure 9.7:** The open-loop bode plot of the motion $t_y \rightarrow q_\theta$.

**Figure 9.8:** The open-loop bode plot of the motion $f_z \rightarrow q_z$. 
9.3. Performance prediction

<table>
<thead>
<tr>
<th></th>
<th>Phase margin</th>
<th>Gain margin</th>
<th>Bandwidth</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_x \rightarrow q_x )</td>
<td>32.5°</td>
<td>4.9 dB</td>
<td>152.5 Hz</td>
</tr>
<tr>
<td>( f_y \rightarrow q_y )</td>
<td>32.5°</td>
<td>4.9 dB</td>
<td>152.5 Hz</td>
</tr>
<tr>
<td>( t_z \rightarrow q_{\psi} )</td>
<td>33.5°</td>
<td>5.9 dB</td>
<td>132.6 Hz</td>
</tr>
<tr>
<td>( t_x \rightarrow q_{\phi} )</td>
<td>31.9°</td>
<td>5.4 dB</td>
<td>139.6 Hz</td>
</tr>
<tr>
<td>( t_y \rightarrow q_{\theta} )</td>
<td>32.0°</td>
<td>5.4 dB</td>
<td>139.5 Hz</td>
</tr>
<tr>
<td>( f_z \rightarrow q_z )</td>
<td>32.0°</td>
<td>4.8 dB</td>
<td>152.5 Hz</td>
</tr>
</tbody>
</table>

Table 9.1: Stability margins and the control bandwidth of the designed sliding surface controller.

<table>
<thead>
<tr>
<th></th>
<th>Cross-over</th>
<th>Magnitude peak</th>
<th>-40 dB/dec range</th>
</tr>
</thead>
<tbody>
<tr>
<td>translation along x-axis</td>
<td>1 Hz</td>
<td>5.9 dB</td>
<td>1 - 20 Hz</td>
</tr>
<tr>
<td>translation along y-axis</td>
<td>1 Hz</td>
<td>5.9 dB</td>
<td>1 - 20 Hz</td>
</tr>
<tr>
<td>rotation around z-axis</td>
<td>1.7 Hz</td>
<td>3.0 dB</td>
<td>2 - 40 Hz</td>
</tr>
<tr>
<td>rotation around x-axis</td>
<td>1.7 Hz</td>
<td>3.0 dB</td>
<td>2 - 50 Hz</td>
</tr>
<tr>
<td>rotation around y-axis</td>
<td>1.7 Hz</td>
<td>3.0 dB</td>
<td>2 - 50 Hz</td>
</tr>
<tr>
<td>translation along z-axis</td>
<td>0.7 Hz</td>
<td>7.2 dB</td>
<td>0.7 - 20 Hz</td>
</tr>
</tbody>
</table>

Table 9.2: Characteristics of the closed-loop transmissibility calculated based on the sliding surface controller and the model.

The passive transmissibility for each DoF motion is calculated according to (9.7) using the parameters given in Table 8.1. The closed-loop transmissibility for each DoF motion is calculated according to (9.9).

At low frequencies (< 20 Hz), the open-loop gain is sufficiently high to force the convergence of the closed-loop transmissibility to the designed transmissibility. At high frequencies (> 500 Hz), the closed-loop transmissibility converges to the passive transmissibility because the open-loop gain is rather low. At the transition frequency range in between, the open-loop gain is not sufficiently high to force the convergence of the two curves but high enough to force the closed-loop transmissibility away from the passive transmissibility. In addition, a magnitude peak is found on the closed-loop transmissibility typically around 200 Hz. This peak is because of the phase lag induced by the low-pass analog filters. The measured closed-loop transmissibility is expected to have even higher peak because of the unavoidable time-delay in the control loop.

The characteristics of the calculated closed-loop transmissibility are summarized for the 6-DoF motions in Table 9.2. As defined in Chapter 3, the cross-over frequency is the frequency that the transmissibility crosses over the 0 dB magnitude. Magnitude peak is the maximum magnitude, or the infinity norm, of the transmissibility. The -40 dB/dec range is the frequency range that the transmissibility has a magnitude slope of -40 dB/dec.
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Figure 9.9: Transmissibility of the two horizontal translations.

Figure 9.10: Transmissibility of the two horizontal rotations.

Figure 9.11: Transmissibility of the vertical translation and vertical rotation.
9.3. Performance prediction

The passive transmissibility has much lower cross-over frequency than the closed-loop transmissibility. However, it is impossible to take advantage of the low passive transmissibility in practice because the passive system is unstable. The passive transmissibility curves plotted in Fig. 9.9 - Fig. 9.11 are used to show that the closed-loop transmissibility will converge to the passive transmissibility at frequencies higher than the control bandwidth.

9.3.3 Compliance

The closed-loop compliance and the passive compliance are compared in Fig. 9.12 - Fig. 9.14 for all the 6-DOF motions. The passive compliance for each DoF motion is calculated according to (9.7) using the parameters given in Table 8.1. The closed-loop compliance for each DoF motion is calculated according to (9.10). Similar to the closed-loop transmissibility, the magnitude peak around 200 Hz is caused by the phase lag of the analog filters. Higher magnitude peak is expected in closed-loop control because of the unit time delay induced by sampling. At high frequencies (>400 Hz), the closed-loop compliance converges to the passive compliance. At lower frequencies, the closed-loop compliance is shaped by the designed sliding surface controllers.

The characteristics of the calculated closed-loop compliance are summarized in Table 9.3 for all the 6-DoF motions. The magnitude peak is the maximum magnitude, or the infinity norm, of the compliance magnitude curve. The peak frequency is the frequency where the magnitude peak occurs. The peak reduction is calculated by the passive compliance magnitude at zero frequency (inverse of the corresponding stiffness) minus the closed-loop compliance magnitude peak.

<table>
<thead>
<tr>
<th></th>
<th>Magnitude peak</th>
<th>Peak frequency</th>
<th>Peak reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>translation along x-axis</td>
<td>-161.7 dB</td>
<td>25 Hz</td>
<td>66.0 dB</td>
</tr>
<tr>
<td>translation along y-axis</td>
<td>-161.7 dB</td>
<td>25 Hz</td>
<td>66.0 dB</td>
</tr>
<tr>
<td>rotation around z-axis</td>
<td>-143.4 dB</td>
<td>33 Hz</td>
<td>91.4 dB</td>
</tr>
<tr>
<td>rotation around x-axis</td>
<td>-140.0 dB</td>
<td>33 Hz</td>
<td>88.0 dB</td>
</tr>
<tr>
<td>rotation around y-axis</td>
<td>-140.0 dB</td>
<td>33 Hz</td>
<td>94.0 dB</td>
</tr>
<tr>
<td>translation along z-axis</td>
<td>-161.7 dB</td>
<td>25 Hz</td>
<td>101.7 dB</td>
</tr>
</tbody>
</table>

Table 9.3: Characteristics of the closed-loop compliance calculated based on the sliding surface controller and the model.

The magnitude peak for all the three translations is -161.7 dB, which indicates that the displacement response of a sine disturbance force of 1 N is less than 10 nm regardless the frequency of this disturbance. The magnitude peak for all the three rotations is less than -140 dB, which indicates that the displacement response of a sine disturbance torque of 1 N-m is less than 100 nrad regardless the frequency of this disturbance. The ratio between the magnitude peaks of each DoF motion is approximately the same as the ratio between the inertia values. This is because system dynamics around the control bandwidth are perfect double-integrator behavior and the similarity of the control design (bandwidth, poles, zeros). It is shown in Table 9.3 that the closed-loop compliance is greatly improved.
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Figure 9.12: Compliance of the two horizontal translations.

Figure 9.13: Compliance of the two horizontal rotations.

Figure 9.14: Compliance of the vertical translation and vertical rotation.
from the corresponding passive compliance.

9.3.4 Sensitivity functions

The closed-loop relative sensitivity and the designed relative sensitivity (same as the designed transmissibility) are compared in Fig. 9.15 - Fig. 9.17. They are calculated according to (9.11) and (9.8), respectively. The closed-loop relative sensitivity and the designed relative sensitivity coincide at frequencies lower than the control bandwidth, due to the sufficiently high open-loop gain.

The closed-loop absolute sensitivity and the designed absolute sensitivity are compared in Fig. 9.18 - Fig. 9.20. The closed-loop absolute sensitivity and the designed absolute sensitivity coincide at frequencies lower than the control bandwidth. They are calculated according to (9.12) and (9.13), respectively. The closed-loop absolute sensitivity and the designed absolute sensitivity coincide at frequencies lower than the control bandwidth, due to the sufficiently high open-loop gain.

9.4 Starting-up sequence

9.4.1 Problems in SEMIS starting-up

The designed sliding surface controller has a high control bandwidth (130 - 150 Hz). To start up this high-bandwidth controlled SEMIS, the metrology frame is guided by the reference signal to the center of its translational and rotational range \( \vec{q}_r = [0, 0, 0, 0, 0, 0]^T \) from its off-position (the metrology frame position when SEMIS is power-off). However, the transient response of the metrology frame relative displacement \( \vec{q}_r \) during this process is higher than the allowed translational or rotational range, limited by the mechanical hard-stops. The root-cause to this high transient response will be explained later. As a result, the metrology frame crashes on one of these hard-stops. This collision is forbidden because it will lead to an unstable behavior of SEMIS, which has a high-bandwidth controlled acceleration loop. Any collision between the metrology frame on the hard-stops will produce a very high peak on the accelerometer outputs. The high-bandwidth acceleration loop controller would respond to this peak and generate a control effort that saturates the Lorentz actuators. As a result, the metrology frame will respond to this high control effort and crash on an opposite hard-stop. This collision and Lorentz actuator saturation will repeat and will never stop.

The high transient response of the relative displacement \( \vec{q}_r \) to the reference is produced by the false signal of the accelerometers. An accelerometer output is influenced by the gravity force of the components of the accelerometer itself. As a result, the output signal of an accelerometer statically depends on its horizontal orientation. If an accelerometer rotates around a horizontal axis, its output signal varies statically although there is no acceleration along its measuring direction. Therefore, this output signal is a false signal.

When SEMIS is power-off, the position of the metrology frame is horizontally rotated from its center of translational and rotational range \( \vec{q}_r = [0, 0, 0, 0, 0, 0]^T \). To start up SEMIS, the metrology frame has to be guided to \( \vec{q}_r \) from its off-position. During this process, the metrology frame is horizontally rotated which will lead to a false acceleration signal along the horizontal axes. The high-bandwidth controller will respond to this signal and produce a high control effort. This high control effort will result in a collision and
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Figure 9.15: Relative sensitivity of the two horizontal translations.

Figure 9.16: Relative sensitivity of the two horizontal rotations.

Figure 9.17: Relative sensitivity of the vertical translation and vertical rotation.
9.4. Starting-up sequence

Figure 9.18: Absolute sensitivity of the two horizontal translations.

Figure 9.19: Absolute sensitivity of the two horizontal rotations.

Figure 9.20: Absolute sensitivity of the vertical translation and vertical rotation.
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subsequently the aforementioned undesirable behavior.

This problem is referred to as the horizontal translation-rotation cross-talk. It is a well-known problem in the field of active vibration isolation. It is induced by imperfect sensor (accelerometer or geophone) and there is no solution to eliminate this problem up to now. Some methods are developed in literature to improve the absolute motion measurement of horizontal translations. For example, the absolute motion of horizontal translations can be re-constructed by means of, for instance, the relative displacement of horizontal rotations; dedicated mechanical design [115, 7] can be used to couple the absolute motion sensor to only horizontal translations using leaf-springs. These methods are effective under one condition: there are limited horizontal-rotational vibrations on the floor.

9.4.2 Designed starting-up sequence

To avoid aforementioned undesirable behavior, a dedicated starting-up sequence is designed and successfully implemented on SEMIS.

1. Download the Matlab simulink model to the xPC target machine. In this model, high-bandwidth sliding surface control is implemented for only 4-DOF motions: vertical translation and the three rotations. The two horizontal translations are only position-controlled using two low-bandwidth PID controllers.

2. Measure the off-position of the metrology frame and make the positioning error zero by setting the reference to the measured off-position. As the reference is low-pass filtered, it takes about half a minute for the tracking error to settle.

3. Enable the controller and set the reference to $\mathbf{q}_\mathbf{r} = [0, 0, 0, 0, 0, 0]^T$. For the 4-DoF motions that are controlled by sliding surface controllers, the transient response of the relative displacements ($\mathbf{q}_r z$, $\mathbf{q}_r \phi$, $\mathbf{q}_r \theta$, and $\mathbf{q}_r \psi$) are within the allowed translational or rotational range. The reason is that the transfer function from the reference to the relative displacement is exactly the relative sensitivity, which is designed to have its cross-over frequency (0.7 - 2 Hz) the same as the transmissibility. For the two horizontal translations, the transient responses of the relative displacements ($\mathbf{q}_r x$ and $\mathbf{q}_r y$) are also within the allowed translational range because of the low-bandwidth position control. This process takes 1 - 2 minutes to settle.

4. Measure the DC component of the 6-DoF control wrench when the system is at steady-state. Remark that the DC component of the 6-DoF control wrench has a small offset (a few N or N·m) to compensate the passive wrench produced by the electromagnetic isolator. Set the DC component of the disturbance wrench to the measured DC control wrench and wait a few seconds for settling. At steady-state, the DC component of the 6-DoF control wrench would be zero.

5. Perform a controller switch for the two horizontal translations. The low-bandwidth position controllers are switched to the designed high-bandwidth sliding surface controller.

The controller switch in the last step is theoretically stable because only a single switch is required. The transient response of $\mathbf{q}_r$ caused by the controller switch is within its allowed ranges. This start-up sequence has been tested for a few hundred times and no failure (collision) is observed. This proves its practical reliability.
9.5. Steady-state behavior

The shutting-down process is just to shut off the power. The metrology frame will be safely landed on the mechanical hard-stops because the passive wrench is weak and the translational range of the metrology frame is only $\pm 0.5\,\text{mm}$.

9.5 Steady-state behavior

The steady-state behavior of SEMIS is referred to as system time-domain responses under the following conditions:

- There is no artificial excitation on the shaker table or on the metrology frame;
- The shaker table is coupled to the floor using the three steel plates.

![Figure 9.21: Steady-state response of SEMIS with sliding surface control implemented.](image)

In steady-state, the relative displacement $\vec{q}_r$ is recorded for a period of 200 s and is plotted in Fig. 9.21. It shows that the response is mostly within the range of $\pm 10\,\mu\text{m}$ or $\mu\text{rad}$ except a few peaks. This steady-state response is mainly the closed-loop system response to sensor noises. Shaker table vibrations also play a part.

9.6 Compliance validation

While the closed-loop compliance of the sliding surface control is measured, the shaker table is coupled to the lab floor using the three steel plates.

Fig. 9.22 shows the diagram of SEMIS with sliding surface control implemented. It will also be used to describe the measurement process of the closed-loop compliance. The signal vectors $\vec{n}_s$ and $\vec{n}_a$ denote the corresponding sensor noises. The signal vectors $\vec{q}_s$, $\vec{a}_s$, $\vec{q}_r$, $\vec{a}_a$, $\vec{w}_d$, and $\vec{f}_d$ are defined in Table 7.1. $\vec{w}_d$ denotes the disturbance wrench, which is excited artificially to measure the compliance. Each component of $\vec{w}_d$ is excited in an independent experiment using Gaussian white noise and is recorded as input signal. Both $\vec{q}_r$ and $\vec{a}_a$ are measured as output signals. FRF of the closed-loop compliance is calculated
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by estimating the Cross Power Spectral Density (CPSD) of the input and output signals. The calculated FRF $\vec{w}_d \rightarrow \vec{a}_d$ is multiplied with the FRF of a double-integrator $s^{-2}$ to derive the FRF $\vec{w}_d \rightarrow \vec{q}_a$. The sampling frequency of the compliance measurement is 5 kHz.

The measured FRF of the closed-loop compliance is plotted separately in two figures, Fig. 9.23 and Fig. 9.24. These plots validate the closed-loop compliance characteristics presented in Table 9.3. The diagonal entries of the measured FRF are compared with the corresponding theoretical calculations (red-dashed). A good match between the two magnitude curves is observed at all frequencies except the frequency range 100 - 200 Hz. Each diagonal entry of the measured FRF has a magnitude peak centered at 150 Hz, which is up to 10 dB higher than the corresponding calculated closed-loop compliance. These magnitude difference at 150 Hz are caused by a time-delay of 0.35 ms in the control loop. This time-delay is partially induced by the unit time-delay (0.2 ms) of sampling. The rest 0.15 ms time-delay is induced by other time-delay factors in the control loop, for example, control effort calculation by the xPC target machine and serial communication (RS422) between the xPC target machine and the digital current amplifiers.

The sliding surface controller has been tuned such that the magnitude peak at 30 Hz is lower than or equal to the magnitude peak at 150 Hz. If the stability margins are not reduced, further increasing the control bandwidth will reduce the magnitude peak at 30 Hz but it will increase the magnitude peak at 150 Hz. Further lowering the control bandwidth will reduce the magnitude peak at 150 Hz but it will increase the magnitude peak at 30 Hz. Therefore, it is not possible to achieve lower peak magnitude for closed-loop compliance using current sampling frequency and current analog filter settings while the stability margins are not reduced. In other words, the peak magnitude, or the infinity norm, of the closed-loop compliance for each DoF motions has been minimized by the corresponding control design.

9.7 Transmissibility validation

While the closed-loop transmissibility of the sliding surface control is measured, the three steel plates used to couple the shaker table to the lab floor are removed. The shaker table is supported by three coil mechanical springs and the natural frequency of this system is

Figure 9.22: The diagram of SEMIS with sliding surface control implemented.

The measured FRF of the closed-loop compliance is plotted separately in two figures, Fig. 9.23 and Fig. 9.24. These plots validate the closed-loop compliance characteristics presented in Table 9.3. The diagonal entries of the measured FRF are compared with the corresponding theoretical calculations (red-dashed). A good match between the two magnitude curves is observed at all frequencies except the frequency range 100 - 200 Hz. Each diagonal entry of the measured FRF has a magnitude peak centered at 150 Hz, which is up to 10 dB higher than the corresponding calculated closed-loop compliance. These magnitude difference at 150 Hz are caused by a time-delay of 0.35 ms in the control loop. This time-delay is partially induced by the unit time-delay (0.2 ms) of sampling. The rest 0.15 ms time-delay is induced by other time-delay factors in the control loop, for example, control effort calculation by the xPC target machine and serial communication (RS422) between the xPC target machine and the digital current amplifiers.

The sliding surface controller has been tuned such that the magnitude peak at 30 Hz is lower than or equal to the magnitude peak at 150 Hz. If the stability margins are not reduced, further increasing the control bandwidth will reduce the magnitude peak at 30 Hz but it will increase the magnitude peak at 150 Hz. Further lowering the control bandwidth will reduce the magnitude peak at 150 Hz but it will increase the magnitude peak at 30 Hz. Therefore, it is not possible to achieve lower peak magnitude for closed-loop compliance using current sampling frequency and current analog filter settings while the stability margins are not reduced. In other words, the peak magnitude, or the infinity norm, of the closed-loop compliance for each DoF motions has been minimized by the corresponding control design.

9.7 Transmissibility validation

While the closed-loop transmissibility of the sliding surface control is measured, the three steel plates used to couple the shaker table to the lab floor are removed. The shaker table is supported by three coil mechanical springs and the natural frequency of this system is
Figure 9.23: The closed-loop compliance measured on SEMIS with sliding surface control implemented: the first half. The green curves are measured by position sensors and the blue curves are measured by accelerometers. The red-dashed curves are calculated from the model and the controller.
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Figure 9.24: The closed-loop compliance measured on SEMIS with sliding surface control implemented: the second half. The green curves are measured by position sensors and the blue curves are measured by accelerometers. The red-dashed curves are calculated from the model and the controller.
9.7. Transmissibility validation

around 20 Hz. Three Shaker Voice Coil Actuators (SVCAs) are used to excite 3-DoF artificial vibrations on the shaker table: vertical translation \((z)\) and two horizontal rotations \((\phi \text{ and } \theta)\). For this reason, only transmissibility of these 3-DoF motions can be measured.

Two types of vibration are excited independently on the shaker table: the sine force excitation and Gaussian white noise force excitation. The Gaussian white noise excitation is used to produce Gaussian white noise acceleration on the shaker table such that transmissibility can be measured in a frequency band in a single experiment. The transmissibility FRF is calculated by estimating the Cross Power Spectral Density (CPSD) of the input-output signals. The input signal is the measured 3-DoF acceleration of the shaker table. The output signal is the measured 3-DoF acceleration of the metrology frame. The corresponding coherence of the input-output signals is also calculated to indicate the confidence of the calculated FRF. In Fig. 9.25 - Fig. 9.27, the measured transmissibility (green/solid) is compared with the calculated transmissibility (blue/dashed). They have a good match at a frequency range \((0.5 - 20 \text{ Hz for vertical translation and } 0.7 - 40 \text{ Hz for the two horizontal rotations})\). The measured transmissibility at low frequencies \(< 0.5 \text{ Hz for vertical translation and } < 0.8 \text{ Hz for the two horizontal rotations}\) and high frequencies \(> 40 \text{ Hz for vertical translation and } > 80 \text{ Hz for the two horizontal rotations}\) cannot be trusted because the corresponding coherence is too low \(< 0.9\). At frequencies wherein coherence is less than 0.9, sine force excitation is used to measure closed-loop transmissibility at some selected frequency points. The purpose of this measurement is to focus excitation power on a single frequency to gain better signal-to-noise ratio. The transmissibility magnitude at the excitation frequency is calculated as the Fast Fourier Transform (FFT) magnitude ratio of the metrology frame acceleration and the shaker table acceleration. These results are represented by the red-stars in Fig. 9.25 - Fig. 9.27.

The measured transmissibility matches the calculated transmissibility very well at low frequencies \(< 20 \text{ Hz for vertical translation and } < 40 \text{ Hz for the two horizontal rotations}\). This is consistent to the theory that closed-loop performance is not sensitive to modeling errors in case of high-gain control. At higher frequencies, there are visible differences between the measured (red stars) and calculated (blue/dashed) transmissibility magnitude curves for the two rotations, as shown in Fig. 9.25 and Fig. 9.26 but no magnitude peak is observed. For the vertical translation, the measured transmissibility (red stars in Fig. 9.27) indicates a magnitude peak at 200 Hz and this peak is 15 dB higher than the calculated transmissibility (blue/dashed in Fig. 9.27). The reason of the mismatch between the calculations and measurements at high frequencies is that the closed-loop performance is sensitive to modeling errors if the loop-gain is not high enough. In this case, the closed-loop transmissibility is sensitive to the damping coefficient if the loop-gain is not high enough. In addition, the identified damping coefficient in the model has relatively high uncertainty. However, this does not explain the high magnitude peak at 200 Hz for the transmissibility of vertical translation. This 200-Hz peak is further studied as follows.

To clearly show the 200-Hz peak on the transmissibility to vertical translation, bandpass \((50 \text{ Hz} - 300 \text{ Hz})\) filtered Gaussian white noise is used to excite vertical translational vibrations on the shaker table. The calculated FRF of the transmissibility (green/solid) is compared with the measured transmissibility using sine excitations (red stars) in Fig. 9.28. The two measurements have a good match so they show the true transmissibility.

As this transmissibility resonance differs from the predicted curve very much, it is un-
Chapter 9. Achieved performance on the contactless electromagnetic suspension system

Figure 9.25: The measured closed-loop transmissibility of vertical translation ($a_{b\theta} \rightarrow a_\phi$) with sliding surface control implemented.

Figure 9.26: The measured closed-loop transmissibility of vertical translation ($a_{b\theta} \rightarrow a_\theta$) with sliding surface control implemented.
9.7. Transmissibility validation

Figure 9.27: The measured closed-loop transmissibility of vertical translation \((a_{bz} \rightarrow a_z)\) with sliding surface control implemented.

Figure 9.28: The measured closed-loop transmissibility of vertical translation \((a_{bz} \rightarrow a_z)\) with sliding surface control implemented. The green-solid curve is measured by band-passed Gaussian white noise excitation. The red-star markers are measured by sine excitation.
likely that it is caused only by the closed-loop control. To find it out, the transmissibility for vertical translation of an only position-controlled closed-loop is measured using the band-pass (50 Hz - 300 Hz) filtered Gaussian white noise. It is compared with the sliding-surface-controlled (bandwidth 150 Hz) closed-loop transmissibility in Fig. 9.29. The control bandwidth of the position-controlled closed-loop is less than 2 Hz so that the measured closed-loop transmissibility can be treated as the passive transmissibility at frequencies higher than 100 Hz. It is clearly shown in Fig. 9.29 that the passive transmissibility also has a high resonance at the frequency range 150 Hz - 250 Hz and the peak is at 215 Hz. The comparison of the two magnitudes shows that the high-bandwidth sliding surface controller increases the passive transmissibility by approximately 10 dB within the frequency range of 150 Hz - 250 Hz. This increment is reasonable because this frequency interval is close to the bandwidth (150 Hz) and there is significant phase lag around these frequencies, which is caused by the controller, the $4^{th}$-order low-pass analog filter, and the unit-time delay (0.2 ms).

The 215-Hz resonance peak on the passive transmissibility for vertical translation is not caused by the Lorentz actuators. The reason is that similar resonance peaks are not found on the transmissibility magnitudes of the two horizontal rotations.

One possible reason of this 215-Hz resonance is the dynamics of the air-cushion (approximately 3 cm thick) between the metrology frame and the shaker table. Another pos-
sibility is the relatively flexible connection between the granite and the moving part of the gravity compensator. To experimentally evaluate these assumptions is very difficult with the current setup.

For the three rotations and the vertical translation, the bottle-neck to further lower the cross-over frequency of the closed-loop transmissibility is the accelerometer noise. Under the same accelerometer noise level, further lower the transmissibility cross-over frequency will sacrifice the absolute sensitivity. The consequence is that the time-domain response of the relative displacement at steady-state increases. For the two horizontal translations, the bottle-neck to further lower the cross-over frequency of the closed-loop transmissibility is the aforementioned horizontal translation-rotation cross-talk.

9.8 Conclusions

Sliding surface control described in Chapter 3 has been applied to design a vibration isolation controller for each DoF motion of SEMIS. Based on the calculated open-loop gain, the control bandwidth and stability margins for all 6-DoF motions have been identified and summarized in Table 9.1. According to these results, the control bandwidth of the 6-DoF motions (130 - 150 Hz) have been pushed to system limits. The four closed-loop performance criteria (transmissibility, compliance, and two sensitivity functions) have been calculated and presented. The main characteristics of closed-loop transmissibility and closed-loop compliance have been summarized in Table 9.2 and Table 9.3, respectively.

The designed controllers have been implemented on SEMIS and high-bandwidth (130 - 150 Hz) control has been demonstrated. This proves the suitability of high-bandwidth control of the electromagnetic isolator. In steady-state, the relative displacement is within \( \pm 10 \mu \text{m} \) or \( \mu \text{rad} \).

9.8.1 Compliance validation

The 6-DoF closed-loop compliance has been validated in experiments. The measured closed-loop compliance has a good match with the prediction using the model and controller at all frequencies except the frequency range 100 Hz - 200 Hz. The measured closed-loop compliance for each DoF motion has a peak centered at 150 Hz and its magnitude is up to 10 dB higher than the corresponding calculations. It is caused by a time-delay of 0.35 ms in the control loop.

The peak magnitude, or the infinity norm, of the closed-loop compliance for each DoF motions has been minimized by control design. The magnitude peak for all the three translations is -161.7 dB, which indicates that the displacement response of a sine disturbance force of 1 N is less than 10 nm regardless the frequency of this sine disturbance. The magnitude peak for all the three rotations is less than -140 dB, which indicates that the displacement response of a sine disturbance torque of 1 N·m is less than 100 nrad regardless the frequency of this sine disturbance.

9.8.2 Transmissibility validation

Due to the limitation of the shaker table, the closed-loop transmissibility has been validated for only 3-DoF motions: vertical translation and the two horizontal rotations. For the two horizontal rotations, a good match between the measured transmissibility and the calculated transmissibility has been observed at frequencies lower than 80 Hz.

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For the vertical translation, a good match between the measured transmissibility and the calculated transmissibility has been observed at frequencies lower than 20 Hz. From 20 Hz to 100 Hz, the measured transmissibility is approximately 5 dB lower than the calculated transmissibility. A resonance peak between 150 Hz and 250 Hz has been observed on the measured transmissibility. It is mainly inherited from the parasitic resonance in the passive system, proved by additional measurement of the passive transmissibility. The magnitude of this resonance is increased by the high-bandwidth controller by approximately 10 dB.

The cross-over frequency for the transmissibility of each DoF motion has been minimized. For the vertical translation and the three rotations, to further lower the cross-over frequency, better signal-to-noise ratio of the acquired acceleration signals are required, or, higher steady-state responses may be allowed. For the two horizontal translations, the bottleneck of further lowering the cross-over frequency is the horizontal translation-rotation cross-talk caused by imperfect accelerometers.

For each DoF motion, the bottleneck of further lowering the magnitude peak of the transmissibility is the signal-to-noise ratio of the acquired acceleration signals. For each DoF motion, the frequency range of the transmissibility slope of -40 dB/dec is more than one decade. Further increasing this frequency range requires higher control bandwidth.
Part IV

Closing
Chapter 10

Conclusions and recommendations

This thesis has explored opportunities for improving the performance (transmissibility and compliance) of a 6-DoF contactless electromagnetic suspension system, which is referred to as the Single Electro-Magnetic Isolator System (SEMIS). This chapter summarizes the main conclusions and contributions of this thesis, and provides recommendations for future work.

10.1 Vibration isolation control of a multi-DoF active suspension system

A design strategy for vibration isolation control of a multi-DoF active suspension system, which combines optimal static decoupling and the sliding surface control, has been explored. It has been compared with $H_\infty$-optimization in terms of control design process complexity, closed-loop performance, robustness, requirements to be posted on the MIMO model. The closed-loop performance of both control strategies have been validated on a 3-DoF experimental demonstrator. The main conclusions are summarized as follows.

10.1.1 Optimal static decoupling

Modal decomposition and Owens method

An LTI mechanical system which has the property of proportional damping has been proved to be a dyadic system. Both modal decomposition and Owens method have been applied to derive the input- and output- matrices for static decoupling. The two methods lead to different pairs of decoupling matrices, which can be normalized to the same pair of decoupling matrices. Therefore, the two methods have equivalent decoupling performance. As the Owens method requires FRF matrices measured at two frequencies, it requires less time and effort compared with the modal decomposition which requires the inertia matrix and stiffness matrix to be known.

Vaes-procedure modification

For an LTI mechanical system without the property of proportional damping, the Vaes-procedure has been proposed to derive the a pair of static decoupling matrices which minimizes the $\mu$-interaction measure. Two modifications have been proposed to improve the Vaes-procedure in terms of computational complexity and numerical stability.
The modified Vaes-procedure has been experimentally validated on a 3-DoF demonstrator. The $\mu$-interaction measure of the decoupled system has been improved from that of the original system by 10 dB at frequencies lower than 6 Hz. The notch-like spike around 9 Hz on the inverse of the $\mu$-interaction measure of the original system has been removed.

10.1.2 Vibration isolation control of a 1-DoF suspension system

The simultaneous improvement of all four performance criteria (transmissibility, compliance, and the two measurement noise sensitivity functions) has been proved to be contradictory. So inevitably solutions will suffer from a trade-off and sensor noise levels will bound the ultimate performance. Furthermore, it has been shown, that necessarily two types of measurements are indispensable. As a consequence, the measurement scheme of SEMIS has been chosen as a combination of relative displacement and payload absolute acceleration. The terms, open-loop gain, control bandwidth, and the two stability margins, which are conventionally defined for SISO control design, have been extended to vibration isolation control design (DISO for a SIDO-plant).

A new vibration isolation control approach, the sliding surface control, has been developed for a 1-DoF active suspension system based on the frequency-shaped sliding surface approach. This sliding surface control is applicable to all four possible measurement schemes. The four concerned performance criteria are designed in two separated steps. The sliding surface design which is constructed by relative displacement and absolute motion is physically connected to the transmissibility and the two sensitivity functions. The regulator is designed to realize the designed performances and to reduce the closed-loop compliance. The regulator design problem is solvable by many classic SISO control design tools.

The sliding surface design problem has been converted to a heuristic pole placement problem. Subsequently, a transmissibility optimization problem has been formulated to optimize the sliding surface design and it is numerically solved using the Matlab Optimization Toolbox [75]. It provides an automatic method to determine a set of parameters which correspond to the optimal transmissibility.

Compared with the conventional DISO control design approach (iterative tuning of the two SISO controllers), the advantages of the sliding surface control are:

1. the sliding surface design is independent of the regulator design and it has negligible influence on the compliance;
2. transmissibility can be optimized by sliding surface design;
3. the regulator design has negligible influence on the transmissibility and the two sensitivity functions if the regulator gain is sufficiently high;
4. the original DISO control design problem is reduced to one SISO control design problem.

Comparison of the sliding surface control and the $H_\infty$-optimization has been summarized in Table 5.1. The sliding surface control has comparable performance, less robustness, lower controller order, simpler design process, and less requirements on identification.

10.1.3 Vibration isolation control of a 3-DoF suspension system

The proposed control strategy for a multi-DoF suspension system, a combination of the sliding surface control and optimal static decoupling, has been compared with the $H_\infty$-optimization.
optimization based on an identified 6th-order model of the 3-DoF demonstrator. This comparison has been summarized in Table 5.2.

The $H_\infty$-optimization requires an augmented plant which is constructed from the identified inertia matrix, damping matrix, and stiffness matrix in this thesis. It depends on weighting filters design to optimize the closed-loop performance. The complexity of the weighting filter tuning is increased quadratically from the 1-DoF $H_\infty$-optimization because tuning of a single filter would affect the closed-loop performance of all the 3-DoF. On the other hand, the sliding surface control for the 3-DoF design has three times more work than the 1-DoF design but the complexity is not increased. Furthermore, the designed $H_\infty$ controller is a $3 \times 6$ 23rd-order transfer function matrix, which is much more complex than the sliding surface controller (six transfer functions with their order equal to or less than eight). Above all, the sliding surface control with static optimal decoupling requires less cost and is less complex than the direct MIMO control design using $H_\infty$-optimization.

For the 3-DoF vibration isolation design, the general performance of the $H_\infty$-optimization is not better than the sliding surface control with static optimal decoupling. The $H_\infty$-optimization gives better transmissibility for only the off-diagonal entries. On the other hand, the sliding surface control with static optimal decoupling gives better transmissibility for the diagonal entries. The reason is that the sliding surface control ignores the off-diagonal entries of the decoupled system while the $H_\infty$-optimization takes into account all entries of the original plant. Furthermore, the sliding surface control with static optimal decoupling gives generally better compliance. From the above observation and analysis, it can be concluded that sliding surface control with static optimal decoupling has higher performance-cost ratio comparing with direct MIMO control design using $H_\infty$-optimization.

10.1.4 Demonstration of the developed strategy on a 3-DoF suspension system

The closed-loop performance of the two control strategies, sliding surface control with static optimal decoupling and the $H_\infty$-optimization, are measured in experiments and are compared with their theoretical curves. According to the theoretical calculations, the former control strategy provides better compliance and diagonal entries of the transmissibility although the static optimal decoupling is not perfect. The most likely reason is that the $H_\infty$-optimization is too conservative. $H_\infty$ control provides lower transmissibility on off-diagonal entries according to theoretical calculations. Unfortunately, this could not be validated by measurements due to the cross-coupling and the nonlinearity of the shaker table.

10.2 Behavior and performance of the contactless electromagnetic suspension system (SEMIS)

The Single Electro-Magnetic Isolator System (SEMIS) has been realized according to the proposed single-isolator concept. It is the-first-of-its-kind 6-DoF contactless suspension system, which combines the advantage of passive gravity force (7.2 kN) compensation and low power consumption (0.3∼6 W). The design requirements of SEMIS have been fulfilled:

1. Static and dynamic properties that are predicted for the realized PM-based gravity
compensator, have been validated;
2. A feasibility proof of a contactless electromagnetic suspension system has been established. The performance of SEMIS has been pushed to its hardware limit.

This is a proof of the success in system-level design. The main conclusions regarding behaviors and achieved performance of SEMIS are summarized in the following two subsections.

10.2.1 Static and dynamic behaviors

Static behaviors

The mass of the floating metrology frame has been validated as 730 kg by the measured FRF, which is consistent to the calculated mass using the CAD software. The high vertical passive force (7.2 kN) produced by the realized PM-based gravity compensator is therefore proved experimentally. The variation of the two horizontal passive forces produced by the gravity compensator has been proved to be less than or equal to 71 N when the metrology frame moves within its stroke range of ±0.5 mm.

The temperature dependency of the vertical passive force produced by the PM-based gravity compensator has been measured as -12.1 N/K, which is 1.7% K of the total 7.2 kN.

Dynamic behaviors

The dynamic behaviors of SEMIS, low cross-coupling, low stiffness, low passive damping, and suitability of high-bandwidth control have been experimentally validated. These dynamic behaviors are inherited from the PM-based gravity compensator. They also prove the success of the system-level design.

The low passive damping has been experimentally proved for only 3-DoF motions. The damping coefficients for the vertical translation and the two horizontal rotations are validated by magnitude fitting on the measured position-controlled closed-loop transmissibility. The damping coefficient for the other 3-DoF motions can not be measured using this method.

Cross-coupling

Weak cross-coupling from 3 Hz to 200 Hz has been proved by the $\mu$-interaction measure calculated from the measured FRF. The $\mu$-interaction measure increases to 0 dB at 600 Hz. This is because of the resonances induced by flexible modes of the metrology frame.

It has been concluded that further decoupling SEMIS using static matrices in addition to the geometric transformation matrices is neither possible nor necessary. It is not possible because

- SEMIS can be treated as two six-input-six-output systems and these two systems can not be decoupled using the same pair of static matrices, as explained in Section 8.5.2.
- within frequency range 3 Hz to 200 Hz, off-diagonal entries of the FRF can not be accurately measured because their magnitudes are too low compared to the diagonal entries.
- at higher frequencies, static decoupling is not possible due to the flexible modes of the metrology frame.
10.2. Behavior and performance of the contactless electromagnetic suspension system (SEMIS)

It is not necessary because the cross-coupling is already minimized at a frequency range which includes the intended control-bandwidth. The $\mu$-interaction measure around these frequencies is comparable to that of the 3-DoF system described in Chapter 4, wherein, static optimal decoupling has been successfully implemented.

10.2.2 Control design for SEMIS

The proposed control strategy, a combination of the sliding surface control and optimal static decoupling has been applied to the control design of SEMIS. The optimal input- and output-decoupling matrices are both identity matrix as the $\mu$-interaction measure of SEMIS has been minimized. The sliding surface control has been applied to control design for each DoF motion of SEMIS.

Based on the calculated open-loop gain, the control bandwidth and stability margins for all 6-DoF motions have been identified and summarized in Table 9.1. According to these results, the control bandwidth of the 6-DoF motions (130 - 150 Hz) have been pushed to system limits. The four closed-loop performance criteria (transmissibility, compliance, and two sensitivity functions) have been calculated and presented. The main characteristics of closed-loop transmissibility and closed-loop compliance have been summarized in Table 9.2 and Table 9.3, respectively.

10.2.3 Achieved performance

The designed high-bandwidth (130 - 150 Hz) controller has been implemented on SEMIS. As such, the suitability of high-bandwidth control has been demonstrated. At steady-state, the relative displacement is within $\pm 10 \mu \text{m or } \mu \text{rad}$. The calculated closed-loop compliance and transmissibility are validated in experiments. It shows that the closed-loop compliance and transmissibility have been pushed to hardware limits by control design.

Compliance validation

The 6-DoF closed-loop compliance has been validated in experiments. The measured closed-loop compliance has a good match with the prediction using the model and controller at all frequencies except the frequency range 100 Hz - 200 Hz. The measured closed-loop compliance for each DoF motion has a peak centered at 150 Hz with its magnitude is up to 10 dB higher than the corresponding calculations with time-delay excluded. Further calculation including a time-delay of 0.35 ms has a good match with the measurement results.

The peak magnitude, or the infinity norm, of the closed-loop compliance for each DoF motions has been minimized by control design. The magnitude peak for all the three translations is -161.7 dB, which indicates that the displacement response of a sine disturbance force of 1 N is less than 10 nm regardless the frequency of this sine disturbance. The magnitude peak for all the three rotations is less than -140 dB, which indicates that the displacement response of a sine disturbance torque of 1 N-m is less than 100 nrad regardless the frequency of this sine disturbance.

Transmissibility validation

Due to the limitation of the shaker table, the closed-loop transmissibility has been validated for only 3-DoF motions: vertical translation and the two horizontal rotations. For the two
horizontal rotations, a good match between the measured transmissibility and the calculated transmissibility has been observed at frequencies lower than 80 Hz.

For the vertical translation, a good match between the measured transmissibility and the calculated transmissibility has been observed at frequencies lower than 20 Hz. From 20 Hz to 100 Hz, the measured transmissibility is approximately 5 dB lower than the calculated transmissibility. A resonance peak between 150 Hz and 250 Hz has been observed on the measured transmissibility. It is mainly inherited from the parasitic resonance in the passive system, proved by additional measurement of the passive transmissibility. The magnitude of this resonance is increased by the high-bandwidth controller by approximately 10 dB, which is not preferred but acceptable.

The cross-over frequency for the transmissibility of each DoF motion has been minimized. For the vertical translation and the three rotations, the bottleneck of further lowering the cross-over frequency is the specification of the relative displacement response at steady-state ($\pm 10 \mu m$ or $1 \mu rad$) or the noises of the acquired acceleration signals. For the two horizontal translations, the bottleneck of further lowering the cross-over frequency is the horizontal translation-rotation cross-talk caused by imperfect accelerometers.

For each DoF motion, the bottleneck of further lowering the magnitude peak of the transmissibility is the noises in the acquired acceleration signals. For each DoF motion, the frequency range of the transmissibility slope of -40 dB/dec is more than one decade. Further increasing the upper limit of this frequency range requires higher control bandwidth. Further increasing the lower limit of this frequency range requires better signal-to-noise ratio of the acquired acceleration signals.

10.3 Contributions

10.3.1 Optimal static decoupling

For an LTI mechanical system which has the property of proportional damping, the input- and output- static decoupling matrices derived by the two methods (modal decomposition and Owens method) have been theoretically proved to have equivalent decoupling performance.

For an LTI mechanical system which does not have the property of proportional damping, two modifications have been developed to improve the Vaes-procedure in terms of computational complexity and numerical stability.

10.3.2 Vibration isolation control

A measurement scheme combining relative displacement and payload absolute acceleration has been shown to be necessary and sufficient for SEMIS.

A new vibration isolation control approach, the sliding surface control, has been developed for a 1-DoF active suspension system based on the frequency-shaped sliding surface approach in literature. It has been compared with $H_\infty$-optimization.

A method has been developed to derive the inertia matrix, the damping matrix, and the stiffness matrix from an identified LTI dynamic model of a mechanical system.

10.3.3 Active suspension system-level design

The single-gravity-compensator concept has been developed and realized on SEMIS.
10.3.4 Identification and control of SEMIS

Static and dynamic behaviors of SEMIS have been measured and analyzed. High-bandwidth sliding surface control has been implemented on SEMIS. A dedicated starting-up procedure has been developed for SEMIS. A feasibility proof of a contactless electromagnetic suspension system has been established. The performance of SEMIS has been pushed to its hardware limit.

10.4 Recommendations

10.4.1 Low-order dynamic decoupling

Static decoupling has the advantage of simplicity and low implementation cost. However, the performance of static decoupling is limited. It would be interesting to see how much improvement can be achieved on the $\mu$-interaction measure if low-order (first-order or second-order) dynamic decoupling is applied.

10.4.2 System-level design

The single-gravity-compensator concept is a dedicated system-level design for SEMIS. Nevertheless, this concept can be used as a system-level design of a general active suspension system. The principle of this concept is that the gravity compensator is vertically aligned with the mass center of the metrology frame and subsequently position-independent control force is applied to positions on the metrology frame which are far away from its mass center. It has many advantages with respect to the conventional multi-gravity-compensator concepts: low stiffness, low passive damping, low cost, and low cross-coupling. Therefore, it is more advantageous than the multi-gravity-compensator concepts for an active suspension system.

10.4.3 Subsystem-level design

The interactions between the two parts of the Lorentz actuators cause many problems on measurements of SEMIS. The stainless 316 steel should be replaced by a type of material which is a conductor to heat but not to electricity and have no interactions with permanent magnets.

The cables of the position sensors (fiber-optical sensors) induce disturbances during static measurements and low-frequency FRF measurements. The probes of these sensors should be fixed on the shaker table and the targets on the metrology frame.

The PM-based gravity compensator has higher (a ratio of 60 - 70) stiffness for the two horizontal translations than the vertical translation. In addition, the stiffness for the two horizontal translations are negative which indicates instability. A combination of these two properties make the start-up sequence more challenging. It is recommended to reduce the stiffness of the two horizontal translations in the future design of the PM-based gravity compensator. If necessary, the stiffness of the vertical translation can be made higher as a trade-off.

10.4.4 Acquisition of acceleration signals

The data acquisition system for acceleration signals induces additional noises which are, in some frequencies, even more powerful than the signal. This is mainly because the RC
circuit noises are amplified by a ratio of 100. Another drawback of this amplification is a reduction of the acceleration measurement range.

Instead of using the 16-bit ADC, 24-bit ADC could be more advantages as the high amplification ratio of 100 can be reduced without reducing the signal to noise ratio of the acquired acceleration signals.

10.4.5 Starting-up sequence

The dedicated starting-up sequence designed for SEMIS includes a control switch from a position-controlled (SISO) closed-loop to a sliding surface controlled (DISO) closed-loop. The transient response caused by this control switch is significantly higher than the steady-state response, which is not desired. This control switch is a temporary engineering solution. However, a smooth control switch would be beneficial for a practical active suspension system.

10.4.6 Shaker table

The shaker table is an auxiliary tool used to evaluate the vibration isolation performance of SEMIS. The off-diagonal entries of the transmissibility can not be measured due to the limitations of this shaker table. It indicates that the system-level design of this shaker table is not very successful.

First, the damping coefficient is too low, which results in high magnitude peaks and high cross-couplings at natural frequencies of each DoF motion. In this thesis, closed-loop control using acceleration feedback is applied to create active damping to reduce these magnitude peaks. It works but it is not the preferred solution. Damping elements should be added to increase the passive damping. The preferred scenario is to have critical passive damping for all DoF motions.

Second, the cross-coupling is too high. Actions should be taken to reduce the cross-couplings.

10.4.7 Acceleration correction for horizontal translations

The horizontal rotation-translation cross-talk causes two problems:

- It provides a lower bound for the cross-over frequency of the transmissibility.
- It makes the start-up sequence complicated.

To remove this cross-talk by correcting the accelerations of horizontal translations is very beneficial. This has been studied in literature. However, practical implementation on SEMIS is not achieved yet.
Bibliography


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