Self-tuning of a switching controller for scanning motion systems

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Self-tuning of a switching controller for scanning motion systems

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ABSTRACT

For a class of switching motion control systems, in particular scanning stage systems, a self-tuning method is proposed to find the optimal switching control parameters. In this method, a combined model/data-based approach is used to derive the gradients with respect to these parameters. The gradients are used in an update scheme which subsequently renders an updated set of parameters. Each set is applied to the machine while operating under closed-loop conditions. By repeating the process, the switching control parameters show convergence to an optimized set of values that induce servo performance inaccessible to linear control. This is because high-gain feedback is incidentally switched on to suppress large amplitude oscillations and is otherwise switched off to avoid amplification of small amplitude noises. In time-domain this gives improved low-frequency disturbance rejection properties with minimal deterioration of the sensitivity to high-frequency noises. Stability and convergence of the switching control system and optimization scheme in the face of perturbations is proved using Lyapunov analysis. Servo performance is demonstrated on a commercial and nano-positioning scanning motion system.

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1. Introduction

The classical trade-off between disturbance rejection and measurement noise sensitivity known as the waterbed effect tends to drive the motion control industry towards nonlinear control designs [5,26,23,27]. An example is given by the wafer scanner industry. Dictated by the scanning set-points, switching control is used to improve performance under non-stationary disturbances, see for example [9]. Prior to each scan, large accelerations induce large error responses. These responses can be sufficiently suppressed with high-gain feedback. During the scan, no acceleration is applied to the system such that low-gain feedback induces a preferable noise response. In linear control, e.g. $\mathcal{H}_\infty$ control [24], such properties can generally not be taken into account in the control design. This is because one controller is used for both operating conditions and thus performance is compromised. Other examples of nonlinear control in the given context include optical storage [10] and hard disk drives [16], or hysteresis feedforward control in atomic force microscopes [17].

In this paper, switching control will be implemented by a continuous but non-smooth function with a deadzone interval. This function acts on the magnitude of the input signal which is usually a closed-loop error signal. Large inputs that exceed the deadzone length, induce extra gain, which is in favor of disturbance rejection properties of the closed-loop system. Small inputs that remain

inside the deadzone length induce no extra gain and thus yield a favorable low-gain noise response. See for example [1] for an overview on both magnitude and phase-based switching functions.

Stability for this type of switching control can be guaranteed for a class of Lur'e type systems and irrespective of parameters like the switching (or deadzone) length [9]. The choice for the switching length however strongly affects closed-loop servo performance. It is the aim of this paper to find the switching control parameters like the switching length via a self-tuning method. Using a performance criterion based on measured closed-loop error signals, a machine-in-the-loop optimization scheme is derived. Since this scheme uses machine-specific data obtained from on-line measurement, it is a dedicated machine calibration method which aims at best achievable performance.

For machine-in-the-loop optimization, a combined model/data-based approach is adopted. This approach is novel because of the derivation of the gradients, i.e., the gradients of the closed-loop error signals with respect to the switching parameters, e.g. the switching length. In these gradients, the model part relates to both the (switching) controller, which is exactly known, and the plant, for which an accurate model can be obtained. The data part comes from the closed-loop error signals, which are obtained from on-line measurement. These measured signals capture the effects of machine-specific disturbances and perturbations which are unknown and therefore cannot be modeled properly. In an iterative manner, an updated set of error signals, hence an updated set of model/data-based gradients, is obtained from closed-loop measurement, see also [2,6]. Using a Gauss–Newton-based update scheme this subsequently gives an updated value for the switching length. By repeating the

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process, convergence is demonstrated to an optimized value for this switching length.

In contrast to the model/data-based approach given in this paper, approaches known from literature sometimes use gradients obtained with data-based methods like the perturbation method [18,20,21]. The perturbation method has the advantage that neither model nor model knowledge is needed. The disadvantage is that small perturbations in the switching parameter value yield poor gradient estimates because of noise corruption. Large perturbations induce poor estimates because of lack of linearity in the closed-loop switching system. A well-known data-based approach in closed-loop parameter value optimization that is not using the perturbation method is iterative feedback tuning [7] or IFT. For unbiased gradient estimates, IFT requires three experiments per parameter update [12–14] whilst the method proposed in this paper uses one. Moreover, IFT is based on linear (or at least linearized [8]) closed-loop model relations that are not applicable in the closed-loop switching system. In terms of wafer stage feedforward control a combined model/data-based approach in closed-loop parameter optimization is considered in [22]. Because of the feedforward context in combination with linear dynamics, stability of the optimization scheme then comes naturally. In the nonlinear feedback context as considered in this paper, stability is non-trivial and therefore requires careful analysis [15,29].

Given these observations, the paper has three main contributions. Firstly, a novel method is proposed for the derivation of the model/data-based gradients with respect to the switching (or deadzone) length. Secondly, bounded-input bounded-output stability in the face of perturbations is proved for both the closed-loop switching control system as well as for the machine-in-the-loop optimization scheme which uses these gradients. Thirdly, the main ideas developed in this paper are demonstrated using real experiments on a commercial and nano-positioning scanning motion system.

The rest of the paper is laid out as follows. For a class of Lur'e type systems, Section 2 considers stability of the switching control system in the face of perturbations. Stability and convergence of the optimization scheme used in finding the optimal switching length is addressed in Section 3. In Section 4 the effectiveness of the optimization scheme is assessed both in simulation and experiment by means of a scanning stage example. Conclusions are presented in Section 5.

2. Lur'e type switching control systems

In this paper Lur'e type switching control systems are considered of the following form:

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{b}_1 u + \mathbf{b}_2 v \\ y &= \mathbf{c}^T \mathbf{x} + v \\ u &= -\phi(y)y,\end{aligned}\quad (1)$$

with state vector $\mathbf{x} = \mathbf{x}(t) \in \mathbb{R}^m$, $\mathbf{A} \in \mathbb{R}^{m \times m}$ Hurwitz, $\mathbf{b}_1, \mathbf{b}_2, \mathbf{c} \in \mathbb{R}^m$, the pairs $(\mathbf{A}, \mathbf{b}_1)$ and $(\mathbf{A}, \mathbf{b}_2)$ being state controllable, the pair $(\mathbf{A}, \mathbf{c}^T)$ being state observable, and the switching function $y \mapsto \phi(y)$ defined by:

$$\phi(y) = \begin{cases} 0, & \text{if } |y| \leq \delta \\ \alpha - \frac{\alpha\delta}{|y|}, & \text{otherwise,} \end{cases} \quad (2)$$

with $\alpha \geq 0$ the gain and $\delta \geq 0$ the switching length. In doing so, system (1) is expected to better deal with the non-stationarity contained in $v = r + d$ which consists of the uniformly bounded and recurring reference signals $r = r(t) \in \mathbb{R}$ along with the non-recurring output disturbances $d = d(t) \in \mathbb{R}$. The motivation is shown in Fig. 1. Extra gain $\phi(y) \in \mathbb{R}$ is induced for large amplitude (and generally low-frequency) error signals $y \in \mathbb{R}$ that benefit from this control. Small amplitude (and mostly high-frequency) noises that remain in the switching length δ do not induce extra gain. Consider the example of scanning stage systems [4]. In the acceleration phase, prior to scanning, forced excitation induces large transient error responses, which can be suppressed through extra control gain. During scanning, and in the absence of this excitation, the presence of constant output disturbances induces small but steady-state error responses. The control gain should be kept small as to avoid amplification of these disturbances. In this context, the notion of input-to-state stability (ISS) by Sontag [28] will be explored. Through ISS analysis both transient and steady-state effects in the nonlinear error response will be quantified. Moreover, explicit values for the switching controller parameters (the gain α and the switching length δ) will be derived.

To show this, consider the following result. System (1) with stable linear part and switching function (2) is input-to-state stable if:

$$\Re\{\mathbf{c}^T(j\omega\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}_1\} \geq -\frac{1}{\alpha}. \quad (3)$$

This inequality stems from the circle criterion and relates the passivity properties of the linear system given by $\mathbf{A} \in \mathbb{R}^{m \times m}$, $\mathbf{b}_1, \mathbf{c} \in \mathbb{R}^m$ in (1) to the upper bound $\alpha > 0$ of the memoryless switching function ϕ in (2). This gives an explicit value for the switching gain α . Moreover, with (3), stability can be assessed for a class of switching functions satisfying the so-called $[0, \alpha]$ sector rather than assessing stability for one specific switching function realization. Through the circle criterion, (3) implies the existence of a positive definite $\mathbf{P} \in \mathbb{R}^{m \times m}$ that satisfies the positive real condition:

$$\begin{aligned}\mathbf{A}^T \mathbf{P} + \mathbf{P} \mathbf{A} &= -\mathbf{q}\mathbf{q}^T - \epsilon \mathbf{P} \\ \mathbf{P} \mathbf{b}_1 &= \mathbf{c} - \sqrt{\frac{2}{\alpha}} \mathbf{q},\end{aligned}\quad (4)$$

with $\epsilon > 0$ and $\mathbf{q} \in \mathbb{R}^m$, see [25]. Choosing the Lyapunov function candidate,

$$c_1 \|\mathbf{x}\|^2 \leq L(\mathbf{x}) = \mathbf{x}^T \mathbf{P} \mathbf{x} \leq c_2 \|\mathbf{x}\|^2, \quad (5)$$

Fig. 1. Motivation for a deadzone switching gain $y \mapsto \phi(y)$ in view of non-stationary disturbances: low-frequency signals in the first time-interval t induce extra gain; high-frequency noises in the second time-interval do not.

for the singular values $c_1 = \sigma_{\min}(\mathbf{P})$ and $c_2 = \sigma_{\max}(\mathbf{P})$ this gives:

$$\begin{aligned} \dot{L}(\mathbf{x}) &= \mathbf{x}^T(\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A})\mathbf{x} + 2\mathbf{x}^T\mathbf{P}\mathbf{b}_1u + 2\mathbf{x}^T\mathbf{P}\mathbf{b}_2v \\ &= -\epsilon L(\mathbf{x}) - \mathbf{x}^T\mathbf{q}\mathbf{q}^T\mathbf{x} + 2\mathbf{x}^T\mathbf{P}\mathbf{b}_1u + 2\mathbf{x}^T\mathbf{P}\mathbf{b}_2v \leq -\epsilon L(\mathbf{x}) \\ &\quad - (\mathbf{x}^T\mathbf{q} + \sqrt{\frac{2}{\alpha}})^2 + 2(\mathbf{x}^T\mathbf{P}\mathbf{b}_2 - u)v \leq -\epsilon L(\mathbf{x}) \\ &\quad + 2(\mathbf{x}^T\mathbf{P}\mathbf{b}_2 - u)v, \end{aligned} \quad (6)$$

where explicit use is made of the sector condition: $\alpha yu \leq -u^2$. Furthermore,

$$\begin{aligned} 2(\mathbf{x}^T\mathbf{P}\mathbf{b}_2 - u)v &\leq 2\|\mathbf{b}_2v\|\|\mathbf{x}^T\mathbf{P}\| + 2\|uv\| \\ &\leq 2c_2\|v\|_\infty\|\mathbf{b}_2\|\|\mathbf{x}\| + 2\alpha\|v\|_\infty\|\mathbf{c}^T\|\|\mathbf{x}\| + 2\alpha\|v\|_\infty^2 \\ &\leq 2c_3\|v\|_\infty\|\mathbf{x}\| + 2\alpha\|v\|_\infty^2, \end{aligned} \quad (7)$$

with $c_3 = c_2\|\mathbf{b}_2\| + \alpha\|\mathbf{c}^T\| > 0$ and $\|v\|_\infty = \sup_{\tau \in [0, t]} |v(\tau)| > 0$ denoting the L_∞ norm of a signal $v: \mathbb{R}_+ \rightarrow \mathbb{R}^n$ on the interval $[0, t]$. Because

$$2c_3\|v\|_\infty\|\mathbf{x}\| + 2\alpha\|v\|_\infty^2 \leq c_3^2\|v\|_\infty^2\lambda + 2\alpha\|v\|_\infty^2 + \frac{1}{\lambda}\|\mathbf{x}\|^2, \quad (8)$$

for any $\lambda > 0$, it follows for:

$$\|\mathbf{x}\| \geq \|v\|_\infty \sqrt{\frac{\lambda(2\alpha + \lambda c_3^2)}{(1 - \sigma)\epsilon c_1\lambda - 1}}, \quad (9)$$

with $\lambda > ((1 - \sigma)\epsilon c_1)^{-1}$ and $0 < \sigma < 1$, that the following inequality (substitution of (8) and (7) in (6)) is satisfied:

$$\begin{aligned} \dot{L}(\mathbf{x}) &\leq -\epsilon L(\mathbf{x}) + c_3^2\|v\|_\infty^2\lambda + 2\alpha\|v\|_\infty^2 + \frac{1}{\lambda}\|\mathbf{x}\|^2, \\ &\leq -\epsilon\sigma L(\mathbf{x}) + c_3^2\|v\|_\infty^2\lambda + 2\alpha\|v\|_\infty^2 \\ &\quad - \left((1 - \sigma)\epsilon c_1 - \frac{1}{\lambda}\right)\|\mathbf{x}\|^2 \leq -\epsilon\sigma L(\mathbf{x}). \end{aligned} \quad (10)$$

So system (1), if satisfying the stability conditions underlying the circle criterion, is input-to-state stable because,

$$\|\mathbf{x}\| \leq \rho(\|\mathbf{x}(0)\|, t) + \gamma(\|v\|_\infty), \quad (11)$$

with

$$\rho(\|\mathbf{x}(0)\|, t) = \sqrt{\frac{c_2}{c_1}}\|\mathbf{x}(0)\|\exp^{-\frac{\epsilon\sigma t}{2}}, \quad \text{and} \quad (12a)$$

$$\gamma(\|v\|_\infty) = \|v\|_\infty \sqrt{\frac{c_2\lambda(2\alpha + \lambda c_3^2)}{(1 - \sigma)\epsilon c_1^2\lambda - c_1}}, \quad (12b)$$

holds for all solutions of (1). In (12a) ρ is a $\mathcal{KL}$ -function, see [28] for the details, and relates to the transient part of the solutions of (1). The ISS gain γ in (12b) is a $\mathcal{K}$ -function and addresses the steady-state part of these solutions. That is, when the effect of the initial conditions $\mathbf{x}(0)$ is damped out. The ISS gain γ also provides an explicit value for the switching length δ . Namely for $\delta = \gamma\|\mathbf{c}^T\| + \|v\|_\infty$ all solutions of (1), which are ultimately inside the deadzone length, do not induce extra gain, i.e., no noise amplification in steady-state. Since the ISS gain γ in (12b) is based on stability arguments, which contain conservatism, its practical value in deriving an upper bound on the switching length δ is of limited use. This is why δ will be sought with the aid of a self-tuning method using machine-in-the-loop measurements. Before presenting this method, firstly the switching control design will be considered.

For switching control design, the feedback connection in Fig. 2 is adopted from [9]. Given the linear feedback design with plant $\mathcal{P}$ and nominal (linear) controller $\mathcal{C}_{fb}$, the nonlinear (switching) controller consists of loop-shaping filters $\mathcal{F}_1$ with input $\tilde{u}$, weighting filters $\mathcal{F}_2$ with input $\tilde{y}$, and switching function $y \mapsto \phi(y)$ with input y and output $u(y) = -\phi(y)y$, see also (1). The plant output z , disturbances d , and reference r form the input $\tilde{y}$ via $\tilde{y} = r + d - z$. A feedforward controller (though present in the measurements considered in this paper) is omitted for clarity of presentation. Its effect on the closed-loop output z can be accounted for in reference r .

The switching controller structure in Fig. 2 aims at improved discrimination between low-frequency signals and high-frequency noise. This is motivated in Fig. 3. Assume there exist low-frequency signals in the first (pre-scanning) time-interval t that associate with the earlier-mentioned transient responses. Assume, moreover, that these signals do not discriminate in terms of amplitude from high-frequency noises in the second (scanning) time-interval, the latter coming from steady-state noises. This is shown in the left part of the figure. Then after proper filtering at the frequencies f with weighting filter

Fig. 2. Simplified block diagram of the switching control design.

Fig. 3. Motivation for the weighting filter structure in Fig. 2.

$\mathcal{F}_2$, which is expressed in the middle part of the figure, both intervals (pre-scanning and scanning) become discriminative in terms of amplitude. This is shown in the right part of the figure. By proper choice of the switching function $y \mapsto \phi(y)$, high-frequency noises in the second (scanning) time-interval can thus remain non-amplified whilst the low-frequency response in the first (pre-scanning) interval induces extra feedback, hence extra suppression. The weighting filter operation $\mathcal{F}_2$ is inversely applied after switching as to maintain stability invariance of the closed-loop dynamics [9].

Given this controller structure and ISS stability properties, the problem considered in this paper is the following. Design the switching length δ in (2) such that on the one hand best (transient) disturbance rejection in the face of set-point excitation r is obtained in the pre-scanning interval whereas on the other hand the (steady-state) noise amplification in the face of disturbances d during scanning is minimized. In ISS terminology, fast decay of ρ in (11) is sought in combination with a small ISS gain γ . It is the aim of this paper to find the switching length δ by a self-tuning method. This method is based on machine-in-the-loop optimization. With this method, optimized controller parameters (in particular the switching length) are found that induce dedicated (and improved) machine performance.

3. Switching length optimization

In presenting the self-tuning method, firstly, a lifted system description will be derived for the closed-loop dynamics of the switching control system in (1). Secondly, an update scheme will be proposed which incorporates model/data-based gradient signals derived from this lifted system description. Thirdly, bounded-input bounded-output stability of the update scheme in the face of perturbations will be proved using Lyapunov analysis.

To obtain a switching length update scheme, consider the linear part of system (1) in discrete-time representation:

$$\begin{aligned} \mathbf{x}_d(n+1) &= \mathbf{A}_d \mathbf{x}_d(n) + \mathbf{b}_{d,1} u(n) + \mathbf{b}_{d,2} v(n) \\ \tilde{\mathbf{y}}(n) &= \mathbf{c}_d^T \mathbf{x}_d(n) + v(n), \end{aligned} \quad (13)$$

at sampling instances $n > 0$ with state vector $\mathbf{x}_d = \mathbf{x}_d(n) \in \mathbb{R}^m$, $\mathbf{A}_d \in \mathbb{R}^{m \times m}$ Hurwitz, $\mathbf{b}_{d,1}, \mathbf{b}_{d,2}, \mathbf{c}_d \in \mathbb{R}^m$, the pairs $(\mathbf{A}_d, \mathbf{b}_{d,1})$ and $(\mathbf{A}_d, \mathbf{b}_{d,2})$ being state controllable, and the pair $(\mathbf{A}_d, \mathbf{c}_d^T)$ being state observable. Under the conditions imposing input-to-state stability in (11), (13) can be put in lifted form:

$$\begin{aligned} \begin{bmatrix} \tilde{\mathbf{y}}(1) \\ \vdots \\ \tilde{\mathbf{y}}(n) \end{bmatrix} &= \begin{bmatrix} \overbrace{\mathbf{S}_c \mathbf{F}_2^{-1}}^{\mathbf{S}_c \mathbf{F}_2^{-1}} & & \\ \mathbf{c}_d^T \mathbf{b}_{d,1} & \ddots & 0 \\ \vdots & \ddots & \vdots \\ \mathbf{c}_d^T \mathbf{A}_d^{n-2} \mathbf{b}_{d,1} & \cdots & \mathbf{c}_d^T \mathbf{b}_{d,1} & 0 \end{bmatrix} \begin{bmatrix} u(1) \\ \vdots \\ u(n) \end{bmatrix} \\ &+ \begin{bmatrix} \overbrace{\mathbf{S}_c \mathbf{F}_2^{-1}}^{\mathbf{S}_c \mathbf{F}_2^{-1}} & & \\ \mathbf{c}_d^T \mathbf{b}_{d,2} & \ddots & 0 \\ \vdots & \ddots & \vdots \\ \mathbf{c}_d^T \mathbf{A}_d^{n-2} \mathbf{b}_{d,2} & \cdots & \mathbf{c}_d^T \mathbf{b}_{d,2} & 1 \end{bmatrix} \begin{bmatrix} v(1) \\ \vdots \\ v(n) \end{bmatrix} + \begin{bmatrix} \mathbf{c}_d^T \\ \mathbf{c}_d^T \mathbf{A}_d \\ \vdots \\ \mathbf{c}_d^T \mathbf{A}_d^{n-1} \end{bmatrix} \mathbf{x}_d(1), \end{aligned} \quad (14)$$

with $\mathbf{S}, \mathbf{S}_c \mathbf{F}_2^{-1} \in \mathbb{R}^{n \times n}$ Toeplitz matrices representing the sensitivity $\mathbf{S} \in \mathbb{R}^{n \times n}$, scaled complementary sensitivity $\mathbf{S}_c \in \mathbb{R}^{n \times n}$, and weighting filter matrix $\mathbf{F}_2 \in \mathbb{R}^{n \times n}$ (see Figs. 2 and 3), respectively. For $\mathbf{x}_d(1) = \mathbf{0} \in \mathbb{R}^m$, (14) can be represented by the following set of algebraic equations:

$$\begin{aligned} \tilde{\mathbf{y}}_k &= \mathbf{S}_c \mathbf{F}_2^{-1} \mathbf{u}_k + \mathbf{S} \mathbf{v}_k \\ \mathbf{y}_k &= \mathbf{F}_2 \tilde{\mathbf{y}}_k. \end{aligned} \quad (15)$$

At each iteration k (or so-called trial), which involves a new set of experiments, the data-sampled error signals are given by $\tilde{\mathbf{y}}_k = [\tilde{\mathbf{y}}(1) \dots \tilde{\mathbf{y}}(n)]^T \in \mathbb{R}^n$, the data-sampled error signals after filtering

with $\mathbf{F}_2 \in \mathbb{R}^{n \times n}$ are represented by $\mathbf{y}_k = [y(1) \dots y(n)]^T \in \mathbb{R}^n$, and the inputs are represented by $\mathbf{u}_k = [u(1) \dots u(n)]^T \in \mathbb{R}^n$; see also Fig. 2. The vector $\mathbf{v}_k = \mathbf{r} + \mathbf{d}_k = [v(1) \dots v(n)]^T \in \mathbb{R}^n$ contains the recurring (trial-invariant) reference commands $\mathbf{r} = [r(1) \dots r(n)]^T \in \mathbb{R}^n$ and the non-recurring (trial varying) disturbances $\mathbf{d}_k = [d(1) \dots d(n)]^T \in \mathbb{R}^n$.

For the nonlinear part of system (1), the inputs $\mathbf{u}_k$ result (by definition) from:

$$\mathbf{u}_k = -\varphi(\mathbf{y}_k), \quad (16)$$

where

$$\varphi(\mathbf{y}_k) = \varphi_1(\mathbf{y}_k) \mathbf{y}_k + \delta_k \varphi_2(\mathbf{y}_k), \quad (17)$$

with $\varphi_1(\mathbf{y}_k) \in \mathbb{R}^{n \times n}$ a positive semi-definite diagonal matrix:

$$\varphi_1(\mathbf{y}_k)[i, i] = \begin{cases} 0, & \text{if } |\mathbf{y}_k[i]| < \delta_k \\ \alpha, & \text{otherwise,} \end{cases} \quad (18)$$

and $\varphi_2(\mathbf{y}_k) \in \mathbb{R}^n$ given by:

$$\varphi_2(\mathbf{y}_k)[i] = \begin{cases} 0, & \text{if } |\mathbf{y}_k[i]| < \delta_k \\ -\alpha \text{sign}(\mathbf{y}_k[i]), & \text{otherwise,} \end{cases} \quad (19)$$

with δ_k denoting the switching length at iteration k . This value is kept constant at each iteration separately, but generally varies over subsequent iterations.

Now consider the objective function:

$$V(\tilde{\mathbf{y}}_k) = \tilde{\mathbf{y}}_k^T(\delta_k) \tilde{\mathbf{y}}_k(\delta_k). \quad (20)$$

For system (15) and objective function (20) the aim is to find the optimal switching length δ_{opt} such that:

$$\delta_{\text{opt}} := \arg \min_{\delta_k} V(\tilde{\mathbf{y}}_k). \quad (21)$$

Hereto the following Gauss–Newton scheme is used:

$$\delta_{k+1} = \delta_k + \beta \frac{\mathbf{p}_k^T(\mathbf{y}_k)}{\|\mathbf{p}_k(\mathbf{y}_k)\|^2} \tilde{\mathbf{y}}_k, \quad 0 \leq \delta_k < \delta_{\text{max}} \leq \|\mathbf{y}_k\|_{\infty}, \quad (22)$$

with convergence parameter $0 < \beta < 1$ and gradients:

$$\begin{aligned} \mathbf{p}_k(\mathbf{y}_k) &= -\frac{\partial \tilde{\mathbf{y}}_k}{\partial \delta} = \mathbf{S}_c \mathbf{F}_2^{-1} \varphi_1(\mathbf{y}_k) \mathbf{F}_2 \frac{\partial \tilde{\mathbf{y}}_k}{\partial \delta} + \mathbf{S}_c \mathbf{F}_2^{-1} \varphi_2(\mathbf{y}_k) \\ &= \mathbf{A}_k^{-1}(\mathbf{y}_k) \mathbf{S}_c \mathbf{F}_2^{-1} \varphi_2(\mathbf{y}_k), \end{aligned} \quad (23)$$

with

$$\mathbf{A}_k(\mathbf{y}_k) = \mathbf{I} + \mathbf{S}_c \mathbf{F}_2^{-1} \varphi_1(\mathbf{y}_k) \mathbf{F}_2, \quad (24)$$

being invertible and $0 < \|\mathbf{p}_k(\mathbf{y}_k)\| \leq \|\mathbf{p}_0\|$ uniformly bounded. The uniform bound follows from the fact that $\mathbf{p}_k(\mathbf{y}_k) \in \mathbb{R}^n$ only contains the stable linear time-invariant filter operations $\mathbf{S}_c \mathbf{F}_2^{-1} \in \mathbb{R}^{n \times n}$ and $\mathbf{F}_2 \in \mathbb{R}^{n \times n}$, which have bounds on their frequency response functions, and nonlinear operations bounded by $\|\varphi_1(\mathbf{y}_k)\|, \|\sqrt{\varphi_2(\mathbf{y}_k) \varphi_2^T(\mathbf{y}_k)}\| \leq \alpha$. Moreover, (23) is derived given the fact that:

$$\frac{\partial \varphi_1(\mathbf{y}_k)}{\partial \delta} \mathbf{y}_k + \delta_k \frac{\partial \varphi_2(\mathbf{y}_k)}{\partial \delta} = \mathbf{0} \in \mathbb{R}^n. \quad (25)$$

This is because the partial derivatives $\partial \varphi_1(\mathbf{y}_k)/\partial \delta$ and $\partial \varphi_2(\mathbf{y}_k)/\partial \delta$, which result from step functions, see (50) and (51) in Appendix A, are given by Dirac delta functions. These delta functions, see (52) in Appendix A, are non-zero valued at $y_k = \delta_k$ and $y_k = -\delta_k$ and are canceled in (25) and therefore do not appear in (23); see Appendix A for the proof.

The update scheme (22) is clearly model/data based. The Toeplitz matrices $\mathbf{S}_c \in \mathbb{R}^{n \times n}$ and $\mathbf{F}_2 \in \mathbb{R}^{n \times n}$ are obtained through models. The model relations φ_1 and φ_2 are known. The signals $\tilde{\mathbf{y}}_k \in \mathbb{R}^n$ (and subsequently $\mathbf{y}_k \in \mathbb{R}^n$) are obtained from closed-loop experiment. In fact, each iteration k requires a new experiment from which

$\tilde{\mathbf{y}}_k \in \mathbb{R}^n$ and the gradients $\mathbf{p}_k(\mathbf{y}_k) \in \mathbb{R}^n$ are obtained. As such a machine-in-the-loop optimization approach is derived that optimizes for best performance.

Stability of scheme (22) applied to system (15) follows from the error update scheme. Substitution of (17) and (16) in (15) gives:

$$\begin{aligned}\tilde{\mathbf{y}}_k &= -\mathbf{S}_c \mathbf{F}_2^{-1}(\varphi_1(\mathbf{y}_k)\mathbf{y}_k + \delta_k \varphi_2(\mathbf{y}_k)) + \mathbf{S}\mathbf{v}_k \\ &= -\mathbf{A}_k^{-1}(\mathbf{y}_k) \mathbf{S}_c \mathbf{F}_2^{-1} \varphi_2(\mathbf{y}_k) \delta_k + \mathbf{A}_k^{-1}(\mathbf{y}_k) \mathbf{S}\mathbf{v}_k \\ &= -\mathbf{p}_k(\mathbf{y}_k) \delta_k + \mathbf{A}_k^{-1}(\mathbf{y}_k) \mathbf{S}\mathbf{v}_k,\end{aligned}\quad (26)$$

and similarly

$$\tilde{\mathbf{y}}_{k+1} = -\mathbf{p}_{k+1}(\mathbf{y}_{k+1}) \delta_{k+1} + \mathbf{A}_{k+1}^{-1}(\mathbf{y}_{k+1}) \mathbf{S}\mathbf{v}_{k+1}. \quad (27)$$

Subtracting both equations and using (22) gives the error update law:

$$\tilde{\mathbf{y}}_{k+1} = \tilde{\mathbf{y}}_k - \beta \mathbf{p}_k \tilde{\mathbf{y}}_k + \mathcal{O}_k(\mathbf{y}_{k+1}, \mathbf{y}_k), \quad (28)$$

with positive semi-definite matrix:

$$\mathbf{P}_k = \frac{\mathbf{p}_k(\mathbf{y}_k) \mathbf{p}_k^T(\mathbf{y}_k)}{\|\mathbf{p}_k(\mathbf{y}_k)\|^2}, \quad (29)$$

and remainder terms:

$$\begin{aligned}\mathcal{O}_k(\mathbf{y}_{k+1}, \mathbf{y}_k) &= -\delta_{k+1}(\mathbf{p}_{k+1}(\mathbf{y}_{k+1}) - \mathbf{p}_k(\mathbf{y}_k)) + \mathbf{A}_{k+1}^{-1}(\mathbf{y}_{k+1}) \mathbf{S}\mathbf{v}_{k+1} \\ &\quad - \mathbf{A}_k^{-1}(\mathbf{y}_k) \mathbf{S}\mathbf{v}_k,\end{aligned}\quad (30)$$

for which it holds that $\|\mathcal{O}_k(\mathbf{y}_{k+1}, \mathbf{y}_k)\| \leq \eta$ with uniform bound $\eta > 0$. Let (20) be a candidate Lyapunov function which satisfies:

$$c_1 \|\tilde{\mathbf{y}}_k\|^2 \leq \tilde{\mathbf{y}}_k^T \mathbf{P}_k \tilde{\mathbf{y}}_k \leq \|\tilde{\mathbf{y}}_k\|^2 = V(\tilde{\mathbf{y}}_k). \quad (31)$$

Since $\mathbf{P}_k$ is positive semi-definite, choosing $c_1 = \sigma_{\min}(\mathbf{P}_k) = 0$ does not prove stability of scheme (22) applied to system (15) in the way used earlier to obtain (11). To avoid this problem, define the finite sequence $(\delta_1, \dots, \delta_o)$ of length o that satisfies (22) and let this sequence have the property that $\tilde{\mathbf{y}}_k \neq \tilde{\mathbf{y}}_{k+1}$ for any index k with $\tilde{\mathbf{y}}_k$ and $\tilde{\mathbf{y}}_{k+1}$ satisfying (28). Under the assumption that $\delta_1 \neq \delta_{\text{opt}}$, $c_1 \|\tilde{\mathbf{y}}_k\|^2 \leq \tilde{\mathbf{y}}_k^T \mathbf{P}_k \tilde{\mathbf{y}}_k$ with $0 < c_1 \leq 1$. Namely for any $\delta_k \neq \delta_{\text{opt}}$ it follows by definition that $\mathbf{p}_k^T \tilde{\mathbf{y}}_k \neq 0$, or $\tilde{\mathbf{y}}_k^T \frac{\mathbf{p}_k \mathbf{p}_k^T}{\|\mathbf{p}_k\|^2} \tilde{\mathbf{y}}_k = \tilde{\mathbf{y}}_k^T \mathbf{P}_k \tilde{\mathbf{y}}_k \neq 0$, which gives the non-zero lower bound on c_1 . The upper bound follows from $\sigma_{\max}(\mathbf{P}_k) = 1$. For the sequence defined above, the difference $V(\tilde{\mathbf{y}}_{k+1}) - V(\tilde{\mathbf{y}}_k)$ becomes:

$$\begin{aligned}V(\tilde{\mathbf{y}}_{k+1}) - V(\tilde{\mathbf{y}}_k) &= -\beta(2 - \beta) \tilde{\mathbf{y}}_k^T \mathbf{P}_k \tilde{\mathbf{y}}_k + 2\mathcal{O}_k^T(\mathbf{y}_{k+1}, \mathbf{y}_k)(\mathbf{I} \\ &\quad - \beta \mathbf{P}_k) \tilde{\mathbf{y}}_k + \mathcal{O}_k^T(\mathbf{y}_{k+1}, \mathbf{y}_k) \mathcal{O}_k(\mathbf{y}_{k+1}, \mathbf{y}_k) \\ &\leq -c_1 \beta(2 - \beta) V(\tilde{\mathbf{y}}_k) + 2\eta(1 + \beta) \|\tilde{\mathbf{y}}_k\| + \eta^2.\end{aligned}\quad (32)$$

Here the fact is used that $\mathbf{P}_k^T \mathbf{P}_k = \mathbf{P}_k \in \mathbb{R}^{n \times n}$. Since

$$2\eta(1 + \beta) \|\tilde{\mathbf{y}}_k\| \leq \lambda \eta^2 (1 + \beta)^2 + \frac{1}{\lambda} \|\tilde{\mathbf{y}}_k\|^2, \quad (33)$$

for any $\lambda > 0$, it now follows for:

$$\|\tilde{\mathbf{y}}_k\| \geq \eta \sqrt{\frac{\lambda(1 + \lambda(1 + \beta)^2)}{\beta c_1 \lambda - 1}}, \quad (34)$$

with $\beta c_1 \lambda > 1$ that

$$V(\tilde{\mathbf{y}}_{k+1}) - V(\tilde{\mathbf{y}}_k) \leq -c_1 \beta(1 - \beta) V(\tilde{\mathbf{y}}_k). \quad (35)$$

So the finite sequence obtained from system (15) with nonlinear feedback (16) and optimization scheme (22) converges under the conditions stated earlier, with

$$\|\tilde{\mathbf{y}}_k\| \leq \rho(\|\mathbf{y}_0\|, k) + \gamma(\eta), \quad (36)$$

where

$$\rho(\|\mathbf{y}_0\|, k) = \|\mathbf{y}_0\| (1 - c_1 \beta (1 - \beta))^{k/2}, \quad \text{and} \quad (37a)$$

$$\gamma(\eta) = \eta \sqrt{\frac{\lambda(1 + \lambda(1 + \beta)^2)}{\beta c_1 \lambda - 1}}. \quad (37b)$$

Eq. (36) implies that after the transient part in (37a) has damped out, all closed-loop solutions $\tilde{\mathbf{y}}_k \in \mathbb{R}^n$ converge to an invariant set dictated by (37b). This has similarity with the ISS analysis from the previous section. By comparing (11) with (36) it is clear that the solutions of both dynamic systems converge to an invariant set dictated, among others, by the set-point r and the disturbances d .

To show the effectiveness of the self-tuning method in finding the optimized switching length δ in (2), the update scheme (22) and the control scheme in Fig. 2 are studied regarding their application to scanning motion systems.

4. Scanning motion systems, modeling and results

In presenting the results of the self-tuning method, this section is divided in two parts. In the first part, a wafer scanning system will be presented. This includes modeling and model validation. In the second part, the results obtained with this system will be presented both in simulation as well as in experiment. This includes parameter convergence of the optimization scheme and time-domain performance assessment on an industrial system.

4.1. Scanning stage example

As an example of scanning motion systems consider the wafer scanning process in Fig. 4. Light from a laser passes a reticle which contains a blueprint of the chip to be processed. The reticle is mounted atop the reticle stage, the latter performing scanning motion. The resulting image is scaled down by a lens system and projected onto the light sensitive layers of a wafer. The wafer which is mounted on the wafer stage performs a combined (and synchronized) scanning motion with the reticle stage [19].

In terms of stage modeling consider a simplified short-stroke wafer stage. In scanning direction, the short stroke is modeled by a fourth-order model:

Fig. 4. Schematics of the wafer scanning principle.

$$\mathcal{P}(s) = \frac{m_2 s^2 + b_{12} s + k_{12}}{m_1 m_2 s^4 + b_{12} (m_1 + m_2) s^3 + k_{12} (m_1 + m_2) s^2}, \quad (38)$$

with $m_1 = 17.55$ kg the mass at the motor side, $m_2 = 4.95$ kg the mass at the load side, $k_{12} = 7.510^7$ N m⁻¹ the internal stiffness, and $b_{12} = 910^1$ Ns m⁻¹ the internal damping. Because control is collocated, both actuation and measurement is done at the motor side. Eq. (38) expresses double integrator behavior. At the low-frequency interval $\mathcal{P}$ is characterized by the overall mass $m_1 + m_2$. At the high-frequency interval it is characterized by the motor mass m_1 only. The corresponding controller is defined by:

$$C_{fb}(s) = \mathcal{F}_{pid}(s) \mathcal{F}_{lp,i}(s) \mathcal{F}_{n,1}(s) \mathcal{F}_{n,2}(s), \quad (39)$$

with proportional–integrator–derivative (PID) filter:

$$\mathcal{F}_{pid}(s) = \frac{k_p (s^2 + 2\pi f_d s + 4\pi^2 f_d^2)}{2\pi f_d s}, \quad (40)$$

second-order low-pass filter:

$$\mathcal{F}_{lp,i}(s) = \frac{4\pi^2 f_{lp,i}^2}{s^2 + 4\pi \beta_{lp,i} f_{lp,i} s + 4\pi^2 f_{lp,i}^2}, \quad i \in \{1, 2\}, \quad (41)$$

and notch filters:

$$\mathcal{F}_{n,i}(s) = \left(\frac{f_{p,i}}{f_{z,i}} \right)^2 \frac{s^2 + 4\pi \beta_{z,i} f_{z,i} s + 4\pi^2 f_{z,i}^2}{s^2 + 4\pi \beta_{p,i} f_{p,i} s + 4\pi^2 f_{p,i}^2}, \quad i \in \{1, 2, 3, 4\}. \quad (42)$$

The controller parameters are given in Table 1. The PID filter aims at achieving performance under high-gain tracking control with zero offset error. The low-pass filter is added to avoid

high-frequency noise amplification. The notch filters are used to suppress the influence of structural flexibilities of the stage on the attainable bandwidth.

To check stability of the nominal (and linear) control design, Fig. 5 shows in Bode representation the open-loop frequency response functions: $C_{fb}(j\omega)\mathcal{P}(j\omega)$, with C_{fb} from (39) and $\mathcal{P}$ from (38) either modeled as well as obtained by closed-loop measurement. Given a sampling frequency of 5 kHz, robust stability of the linear feedback loop is shown to be guaranteed. From the figure, it follows that a bandwidth of 161 Hz is obtained with a phase margin of 28.8° and a gain margin of 4.8 dB. Lowering the sampling frequency, though viable, induces smaller stability margins, which is undesirable in dealing with machine-to-machine variations.

To check stability of the switching control design the fact is used that (3) in frequency-domain reads:

$$\Re\{S_c(j\omega)\} = \Re\left\{ \frac{\mathcal{F}_1(j\omega)C_{fb}(j\omega)\mathcal{P}(j\omega)}{1 + C_{fb}(j\omega)\mathcal{P}(j\omega)} \right\} \geq -\frac{1}{\alpha}, \quad (43)$$

where the loop shaping filter $\mathcal{F}_1$ is given by:

$$\mathcal{F}_1(s) = \mathcal{F}_{lp,2}(s) \mathcal{F}_{n,3}(s), \quad (44)$$

hence a second-order low-pass filter as in (41) in series connection with a second-order notch filter as in (42). The control parameters are given in Table 2. Tuning of $\mathcal{F}_1$ aims at obtaining a significant amount of extra (low-frequency) gain α in (2) without causing the switching closed-loop system to become unstable. The importance of (43) lies in the graphical interpretation which is given by the Nyquist plot of Fig. 6. Robust stability of the switching closed-loop system is guaranteed for $\alpha = 3$ because the frequency response functions obtained from either modeling or stage measurement remain sufficiently to the right of a straight line through the point $(-1/3, 0)$. In fact an equivalent gain margin of 4.43 dB is obtained given the distance between the points $(-1/3, 0)$ and $(-1/5, 0)$ and which follows from the measured frequency response function.

The choice for the weighting filter $\mathcal{F}_2$ (see also Fig. 3) which is closed-loop stability invariant (but performance relevant) reads:

Table 1
Controller parameter values of C_{fb} as in (39).

$\mathcal{F}_{pid}$ as in (40)	$k_p = 10^7$ N/m	$f_i = 70$ Hz	$f_d = 72$ Hz
$\mathcal{F}_{lp,1}$ as in (41)	$f_{lp,1} = 570$ Hz	$\beta_{lp,1} = 0.05$	
$\mathcal{F}_{n,1}$ as in (42)	$f_{z,1} = 730$ Hz	$\beta_{z,1} = 1.510^{-6}$	$f_{p,1} = 810$ Hz
$\mathcal{F}_{n,2}$ as in (42)	$f_{z,2} = 1100$ Hz	$\beta_{z,2} = 0.19$	$f_{p,2} = 730$ Hz
			$\beta_{p,1} = 0.88$
			$\beta_{p,2} = 0.18$

Fig. 5. Bode diagram of the open-loop characteristics $C_{fb}(j\omega)\mathcal{P}(j\omega)$ showing robust stability of the linear motion system given a bandwidth of 161 Hz, phase margin of 28.8°, and gain margin of 4.8 dB.

Table 2Controller parameter values of $\mathcal{F}_1$ as in (44).

$\mathcal{F}_{lp,2}$ as in (41)	$f_{lp,2} = 700$ Hz	$\beta_{lp,2} = 0.7$		
$\mathcal{F}_{n,3}$ as in (42)	$f_{z,3} = 250$ Hz	$\beta_{z,3} = 0.6$	$f_{p,3} = 250$ Hz	$\beta_{p,3} = 4.8$

Fig. 6. Nyquist plot of the circle criterion condition in (3) showing robust stability of the switching motion system (1) with $\alpha = 3$: $\Re\{S_c(j\omega)\}$ lies sufficiently to the right of a straight line through the point $(-1/3, 0)$.**Table 3**Controller parameter values of $\mathcal{F}_2$ as in (45).

$\mathcal{F}_{n,4}$ as in (42)	$f_{z,4} = 10$ Hz	$\beta_{z,4} = 0.2$	$f_{p,4} = 10$ Hz	$\beta_{p,4} = 2.0$
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$$\mathcal{F}_2(s) = \mathcal{F}_{n,4}(s), \quad (45)$$

i.e., a second-order notch filter as in (42). The notch frequencies are chosen at 10 Hz (see Table 3) and relate to the fundamental excitation frequency of the set-point r in the scanning process.

At this point in the analysis, stability is sufficiently checked for both the linear closed-loop system without switching function as well as the switching closed-loop system. As a result, the switching closed-loop system is ISS, see (11), and all solutions converge to a steady-state response which is bounded by the ISS gain γ in (12b). Thus, a stable lifted system description is obtained in (14) that guarantees stability (given the further conditions posed) of the update scheme in (22) and its closed-loop solutions in (36). Apart from stability, it remains to be checked how the self-tuning method helps in finding the switching length parameter δ that associates with improved closed-loop performance. This is done in the next part.

4.2. Results

For the stage characteristics given in the previous part, the results of switching length optimization are depicted in Fig. 7. In both simulation and measurement, Fig. 7 shows that given two initial values (upper part) and fixed convergence rate $\beta = 0.25$, convergence to an optimized switching length δ_{opt} is achieved in less than ten iterations k . Increasing β generally fastens this convergence but at the cost of an increased noise response; recall (37b). It is important to mention that in simulation the reference signal r is chosen as:

$$r(t) = r_1 \psi(t) \sin(2\pi 10t) + r_2 \psi(t - \tau) \sin(2\pi 358t), \quad (46)$$

with $r_1 = 4 \mu\text{m}$, $r_2 = 3.5 \text{ nm}$, $\tau = 0.05 \text{ s}$, and Hanning function:

$$\psi(t) = \begin{cases} 1 - \cos^2\left(\frac{\pi t}{\tau}\right), & \text{if } 0 \leq t \leq \tau \\ 0, & \text{otherwise.} \end{cases} \quad (47)$$

The reason for this is twofold. On the one hand, a simplified excitation signal is sought that clearly illustrates the features of the switching controller. On the other hand, a signal is required that induces a relevant closed-loop error response from the simulation model in the absence of any feedforward control design or disturbance modeling; recall the discussion related to Fig. 2 on disregarding the feedforward controller. The signal in (46) accommodates for both requirements. Namely, the combined effect of the set-point and the feedforward controller on the simulated error signals during pre-scanning is accounted for by a single harmonic contribution at 10 Hz. Additionally, the effect of the disturbances during scanning is lumped into an harmonic contribution at 358 Hz. The experimental results (right part) are obtained under realistic scanning set-points, feedforward control design, and disturbances and yield error responses in the same order of magnitude as in the simulations.¹ This is in favor of the choice in (46). Nevertheless, the results obtained from simulation cannot be compared directly with the results obtained from measurement. The goal of the simulation is to show convergence towards a single optimized value in a finite number of steps. This optimized value corresponds to the minimum of a non-convex objective function (lower left part). The goal of the experiment is to demonstrate that convergence still applies to the more complex excitation spectrum during real scanning. In the experiments, it can be seen (lower right part) that the objective function evaluation itself is of poor quality. In obtaining the gradients, the proposed model/data-based method is therefore preferred over the proposed data-based method in [18]. Namely if the data-based objective function evaluation is of poor quality, then the data-based gradients are of poor quality too.

The results from time-domain simulation are shown in Fig. 8. Compared to a low-gain linear design (upper part) where $\alpha = 0$ and a high-gain linear design (lower part) where $\alpha = 3$ and $\delta = 0 \text{ nm}$, it can be observed that an optimized switching controller with $\alpha = 3$ but $\delta = 4.4 \text{ nm}$ combines low-frequency disturbance rejection properties resulting from high-gain feedback (in the first time-interval) with a preferable high-frequency noise response resulting from low-gain feedback (in the second time-interval).

Fig. 9 shows measured time-domain error signals after post-processing. Post-processing refers to the moving average filter operation:

$$\mathcal{M}_a(\tilde{\mathbf{y}}_k[i]) = \frac{1}{T_p} \sum_{j=i-T_p/2}^{i+T_p/2-1} \tilde{\mathbf{y}}_k[j], \quad \forall i \in \{1, \dots, n\}, \quad (48)$$

with $T_p = 8$, which provides a measure for machine overlay [3] and typically resembles a low-pass filter. Additionally the moving standard deviation filter operation:

$$\mathcal{M}_{sd}(\tilde{\mathbf{y}}_k[i]) = \sqrt{\frac{1}{T_p} \sum_{j=i-T_p/2}^{i+T_p/2-1} (\tilde{\mathbf{y}}_k[j] - \mathcal{M}_a(\tilde{\mathbf{y}}_k[j]))^2}, \quad \forall i \in \{1, \dots, n\}, \quad (49)$$

provides a measure for imaging and resembles a high-pass filter [4]. In terms of $\mathcal{M}_a$ -filtering the upper part of Fig. 9 shows that optimized switching control with $\alpha = 3$ and $\delta = 10.9 \text{ nm}$ performs equally effective as high-gain feedback with $\alpha = 3$ and $\delta = 0 \text{ nm}$ (upper right part) but more effective than low-gain feedback with $\alpha = 0$ (upper left part); all measurements are conducted twice as to distinguish between structural improvements and noises. Fig. 9 also shows the scaled third-order (acceleration) set-point profile.

¹ The integrator in the controller (39) together with the double integrator of the plant (38) induce 60 dB/dec closed-loop suppression at low-frequencies, hence the difference in amplitude between r_1 and r_2 of a factor 1000 which is needed to get closed-loop error responses in the same order of magnitude.

Fig. 7. Convergence and convexity properties in simulation and experiment when optimizing δ with $\beta = 0.25$.

Fig. 8. Time-series simulation of the error signals $\tilde{y}_k(i)$ with $i \in \{1, \dots, 500\}$; low gain refers to the case with $\alpha = 0$ and high gain to the case with $\alpha = 3$ and $\delta = 0$ nm; optimal switching control is conducted at $\alpha = 3$ but $\delta = 4.4$ nm.

As the main source of excitation this profile clearly associates with a pre-scanning interval with non-zero accelerations and a scanning interval with zero accelerations, thus explaining for the transient

error responses. In terms of $\mathcal{M}_{sd}$ -filtering, the lower part in Fig. 9 shows that optimized switching control performs equally effective as low-gain feedback (lower left part) but more effective than

Fig. 9. Time series experiment of the filtered moving average ($\mathcal{M}_a$) and moving standard deviation ($\mathcal{M}_{sd}$) error signals $\tilde{\mathbf{y}}_k(i)$ with $i \in \{1, \dots, 500\}$; low gain refers to the case with $\alpha = 0$ and high gain to the case with $\alpha = 3$ and $\delta = 0$ nm; optimal switching control is conducted at $\alpha = 3$ but $\delta = 10.9$ nm; the figure includes a scaled representation of the acceleration scanning set-points.

high-gain feedback (lower right part). The amplification of noises in the scanning interval under linear high-gain feedback is avoided whereas low-frequency disturbances are rejected. Both observations are in correspondence with the simulation results of Fig. 8. Different, however, is the fact that performance as shown in Fig. 9 is obtained from a state-of-the-art commercial machine in the million dollar range and thus is hard to compete with. Using the self-tuned switching controller, the $\mathcal{M}_a$ -filtered peak values during scanning are reduced with 42% with respect to the nominal (low-gain) control design. At the same time, the noise amplification in terms of $\mathcal{M}_{sd}$ -filtered peak values is reduced with 18% with respect to the high-gain control design.

5. Conclusions

In terms of switching control, the optimal switching length is found using a combined model/data-based approach towards its gradients. Within a self-tuning and iterative machine-in-the-loop approach, this gives rise to machine-dedicated calibrations in which improved scanning stage performance is demonstrated. More specifically, improved low-frequency disturbance rejection properties are obtained that compete with the properties of linear high-gain feedback. At the same time, the noise response is kept at the lower (preferred) level of linear low-gain feedback. The switching controller therefore gives access to performance improvements inaccessible to any linear controller within the considered control configuration. The optimization method is strictly performance driven. This is because of the stability invariance in tuning the switching length parameter. Stability of the switching system and optimization scheme is proved by Lyapunov theory and extensions thereof. All solutions converge to an invariant set related to the level of noise. Multi parameter optimization in which both the parameters: switching length and gain, are optimized, makes

a logical extension towards the presented single-parameter optimization scheme, see for example [11].

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Appendix A

To show the validity of (25), let the matrix $\varphi_1(\mathbf{y}_k)[i, i]$ and vector $\varphi_2(\mathbf{y}_k)[i]$ in (18) and (19), respectively, be defined for arbitrary index i as:

$$\varphi_1(y) = \lim_{z \rightarrow 0} \left\{ \alpha - \left(\frac{\alpha}{\pi} \arctan \left(\frac{2\pi(y + \delta)}{z} \right) - \frac{\alpha}{\pi} \arctan \left(\frac{2\pi(y - \delta)}{z} \right) \right) \right\}, \quad (50)$$

and

$$\varphi_2(y) = \lim_{z \rightarrow 0} \left\{ -\frac{\alpha}{\pi} \arctan \left(\frac{2\pi(y + \delta)}{z} \right) - \frac{\alpha}{\pi} \arctan \left(\frac{2\pi(y - \delta)}{z} \right) \right\}, \quad (51)$$

with input $y \in \mathbb{R}$ and switching length $\delta > 0$; the case $\delta = 0$ is excluded from this part of the analysis because it renders the switching controller linear. The partial derivatives are given by:

$$\begin{aligned} \frac{\partial \varphi_1(y)}{\partial y} &= \lim_{z \rightarrow 0} \left\{ -\frac{2\alpha z}{z^2 + 4\pi^2(y + \delta)^2} + \frac{2\alpha z}{z^2 + 4\pi^2(y - \delta)^2} \right\}, \\ \frac{\partial \varphi_2(y)}{\partial y} &= \lim_{z \rightarrow 0} \left\{ -\frac{2\alpha z}{z^2 + 4\pi^2(y + \delta)^2} - \frac{2\alpha z}{z^2 + 4\pi^2(y - \delta)^2} \right\}. \end{aligned} \quad (52)$$

For $y \neq \pm \delta$, the partial derivatives in (52) are zero such that:

$$\begin{aligned} \frac{\partial \varphi_1(y)}{\partial y} = \frac{\partial \varphi_2(y)}{\partial y} = 0 &\Rightarrow \frac{\partial \varphi_1(y)}{\partial y} y + \frac{\partial \varphi_2(y)}{\partial y} \delta \\ &= 0 \text{ (for } y \neq \pm \delta \text{)}. \end{aligned} \quad (53)$$

For $y = \delta$, the partial derivatives in (52) are non-zero, but it follows that:

$$\begin{aligned} \frac{\partial \phi_1(y)}{\partial y} \delta + \frac{\partial \phi_2(y)}{\partial y} \delta &= \lim_{z \rightarrow 0} \left\{ -\frac{2\alpha\delta z}{z^2 + 16\pi^2\delta^2} + \frac{2\alpha\delta z}{z^2} - \frac{2\alpha\delta z}{z^2 + 16\pi^2\delta^2} - \frac{2\alpha\delta z}{z^2} \right\} \\ &= \lim_{z \rightarrow 0} \left\{ -\frac{4\alpha\delta z}{z^2 + 16\pi^2\delta^2} \right\} = 0. \end{aligned} \quad (54)$$

The same result applies to $y = -\delta$. It is therefore concluded that:

$$\frac{\partial \phi_1(y)}{\partial \delta} y + \frac{\partial \phi_2(y)}{\partial \delta} \delta = \frac{\partial y}{\partial \delta} \left(\frac{\partial \phi_1(y)}{\partial y} y + \frac{\partial \phi_2(y)}{\partial y} \delta \right) = 0, \quad (55)$$

which applies to each index i in (18) and (19), hence the validity of (25).

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