Transfer reactions on $^{58}\text{Ni}$ and $^{56}\text{Fe}$ induced by polarized protons of 25 MeV

Citation for published version (APA):

DOI:
10.6100/IR157606

Document status and date:
Published: 01/01/1981

Document Version:
Publisher’s PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

• A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher’s website.
• The final author version and the galley proof are versions of the publication after peer review.
• The final published version features the final layout of the paper including the volume, issue and page numbers.

Link to publication

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

• Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
• You may not further distribute the material or use it for any profit-making activity or commercial gain.
• You may freely distribute the URL identifying the publication in the public portal.

If the publication is distributed under the terms of Article 25fa of the Dutch Copyright Act, indicated by the “Taverne” license above, please follow below link for the End User Agreement:
www.tue.nl/taverne

Take down policy
If you believe that this document breaches copyright please contact us at:
openaccess@tue.nl
providing details and we will investigate your claim.
TRANSFER REACTIONS ON $^{58}$Ni AND $^{56}$Fe
INDUCED BY
POLARIZED PROTONS OF 25 MeV

PROEFSCHRIFT

TER VERKRIJGING VAN DE GRAAD VAN DOCTOR IN DE
TECHNISCHE WETENSCHAPPEN AAN DE TECHNISCHE
HOGESCHOOL EINDHOVEN, OP GEZAG VAN DE
RECTOR MAGNIFICUS, PROF. IR. J. ERKELENS, VOOR
EEN COMMISSIE AANGEWEZEN DOOR HET COLLEGE
VAN DEKANEN IN HET OPENBAAR TE VERDEGEN OP
VRIJDAG 5 JUNI 1981 TE 16.00 UUR

DOOR

JOHANNES HENDRIKUS POLANE

GEBOREN TE AMSTERDAM
DIT PROEFSCHRIFT IS GOEDGEKEURD
DOOR DE PROMOTOREN

PROF. DR. O.J. POPPEMA
IN
PROF. DR. B.J. VERHAAR

-----------------------------

CO-PROMOTOR DR. P.J. VAN HALL
CONTENTS

CHAPTER 1 INTRODUCTION

CHAPTER 2 EXPERIMENTAL PROCEDURE AND DATA REDUCTION
  2.1 Beam production and scattering chamber
  2.2 The telescope system
  2.3 The electronic equipment
  References

CHAPTER 3 SOME ASPECTS OF THE THEORY OF TRANSFER REACTIONS
  3.1 DWBA theory
  3.2 One-neutron transfer
  3.3 Two-neutron transfer
  3.4 The optical-model potential
  3.5 Finite-range and non-locality corrections
  3.6 Adiabatic deuteron potential
  References

CHAPTER 4 THE \(^{1+}(p,d)\) REACTION ON \(^{58}\text{Ni}\) AT 24.6 MeV
  4.1 Introduction
  4.2 The experiment
  4.3 The reaction model and the calculational procedures
    4.3.1 Reaction model
    4.3.2 The neutron form factor
    4.3.3 Optical potentials
  4.4 Discussion
    4.4.1 The particle-states \(3/2^- (0.00), 5/2^- (0.77)\)
      and \(1/2^- (1.11)\)
    4.4.2 The \(7/2^- \) states
    4.4.3 The optical deuteron-potential
    4.4.4 The \(3/2^- \) states
    4.4.5 The states at 3.71 and 5.78 MeV
    4.4.6 The \(1=0\) and \(1=2\) states
    4.4.7 Sum rules
  4.5 Conclusions
  References

CHAPTER 5 THE \(^{(2+)}(p,d)\) REACTION ON \(^{56}\text{Fe}\) AT 24.6 MeV
  5.1 Introduction
  5.2 The experiment
The calculations  55
  5.3.1 Reaction model  55
  5.3.2 Optical-model and other DWBA parameters  57
5.4 Discussion  59
  5.4.1 The negative-parity states  59
  5.4.2 The positive-parity states  64
  5.4.3 Sum-rule results and comparison with $^{58}$Ni$(p,d)$  65
  5.4.4 The deuteron potential  67
5.5 Conclusions  68
  References  70

CHAPTER 6 THE $(p,t)$ REACTION ON $^{58}$Ni AND $^{56}$Fe AT 24.6 MeV  71
6.1 Introduction  71
6.2 Experimental procedures  72
6.3 Calculational procedures  73
  6.3.1 Reaction mechanisms  73
  6.3.2 Simultaneous two-neutron transfer  74
  6.3.3 Sequential transfer  76
  6.3.4 Inelastic scattering  78
  6.3.5 Optical potentials  79
  6.3.6 Normalization parameters  80
6.4 Discussion  81
  6.4.1 Determination of $\lambda$ and $\mu$  81
  6.4.2 The $2_1^+$ and $2_2^+$ states of $^{54}$Fe  83
  6.4.3 The $0_2^+$ state of $^{56}$Fe  92
  6.4.4 DWBA analysis  93
6.5 Conclusions  96
  References  97

CHAPTER 7 THE $(p,\alpha)$ REACTION ON $^{58}$Ni AT 24.6 MeV  99
7.1 Introduction  99
7.2 Reaction model  99
7.3 Discussion  101
7.4 Conclusions  104
  References  105

SUMMARY  107
SAMENVATTING  109
NAWOORD  111
LITERATUUR  112
Nuclear transfer-reactions are characterized by the transfer of a small number of nucleons (protons or neutrons) from one atomic nucleus to another. In this thesis we present the results of various types of transfer reactions induced by bombarding foils consisting of $^{58}$Ni and $^{56}$Fe nuclei, respectively, with a beam of polarized protons having an energy of 24.6 MeV.

The target foils are so thin that the probability of meeting more than one nucleus is very small for the protons. Consequently most of the protons go through the target foil without feeling any nuclear interaction at all. Of the protons that do encounter a nucleus, most are scattered (in)elastically and leave the target as a proton again. In our experiments we selected those reactions, in which the proton "changes" into a deuteron, a triton or an alpha particle by picking up from the target nucleus a neutron, a pair of neutrons or two neutrons plus one proton, respectively. The final nucleus is often left in an excited state by such reactions.

Counting the number of deuterons, tritons and alpha particles emitted in a certain direction and measuring their energy yields so-called energy-spectra. These spectra reveal peaks (i.e. the energies are centered around discrete values), which correspond to the energy levels of the final nuclei. The number of counts in these peaks represent (after proper normalization) the various differential cross sections. In fact, because the protons are polarized, we have two differential cross sections for each level: one for polarization up and one for polarization down. It is convenient to replace these two cross sections by two quantities which are independent of the degree of polarization P. These quantities are the unpolarized cross section $\frac{d\sigma}{d\Omega}$ and the analyzing power $A$ given by

\[
\frac{d\sigma(\theta)}{d\Omega_{\text{unpol}}} = \frac{1}{2} \left( \frac{d\sigma}{d\Omega^+} + \frac{d\sigma}{d\Omega^-} \right) \tag{1.1}
\]

\[
A(\theta) = \frac{1}{P} \times \frac{\left( \frac{d\sigma}{d\Omega^+} - \frac{d\sigma}{d\Omega^-} \right)}{\left( \frac{d\sigma}{d\Omega^+} + \frac{d\sigma}{d\Omega^-} \right)} \tag{1.2}
\]

where $\theta$ is the angle between the emitted particle and the proton beam. Relation (1.2) is only valid for particles emitted in the horizontal plane and to the left of the proton beam direction. To be more precise
\[ A(0), \frac{d\sigma(0)}{d\Omega_{\text{unpol}}} \text{ and the differential cross section in the direction } (0,\phi) \text{ are related by} \]
\[ \frac{d\sigma(0,\phi)}{d\Omega} = (1 + \hat{\mathbf{n}} \cdot \mathbf{A}(0)) \frac{d\sigma(0)}{d\Omega_{\text{unpol}}} \]

where \( \hat{\mathbf{n}} \) is the unit vector corresponding to the vector product of the initial and final light-particle momentum (Basel convention).

Such a study of nuclear reactions yields information on the reaction mechanisms and on the internal structure of the nuclei involved. Both aspects are investigated extensively in the present work. For instance, one- and two-neutron transfer reactions probe the single-neutron and neutron-pairing properties, respectively, of the initial and final nuclei. Concerning the reaction mechanism of e.g. two-neutron transfer reactions one may investigate the relative importance of two successive transfers of one neutron and the simultaneous transfer of a neutron pair. The polarization of the protons often proves to be crucial in answering questions about both nuclear structure and reaction mechanisms.

The choice of \(^{58}\text{Ni}\) and \(^{56}\text{Fe}\) as target nuclei was motivated by two facts. Firstly, \(^{58}\text{Ni}\) and \(^{56}\text{Fe}\) are close to the doubly-magic nucleus \(^{56}\text{Ni}\), which means that their internal structure is relatively simple within the context of the shell-model. Secondly, we expected similarities of the neutron-transfer data since \(^{58}\text{Ni}\) and \(^{56}\text{Fe}\) contain the same number of neutrons.

The experimental set up is described in the following chapter, while we give an outline of the theory in chapter 3. The experimental results are presented and discussed in the remaining chapters. The references are given at the end of each chapter.
The experiments described in this thesis have been induced by means of a polarized proton beam accelerated to 24.6 MeV by the AVF cyclotron of the Eindhoven University of Technology \(^1\). In section 2.1 we give an outline of the production of this polarized proton beam and we describe the arrangement of the scattering chamber, where the nuclear reactions take place. In section 2.2 we discuss how the outgoing particles are identified as protons, deuterons, tritons and alpha particles by the telescope device. Finally we sketch the main features of the electronic circuitry. More details on the experimental set up and the spectrum analysis can be found in the refs. 2 and 3.

2.1 Beam production and scattering chamber

The polarized protons were supplied by a source of the atomic beam type \(^4,5\), which has been developed by Van der Heide \(^6\). The intensity, energy and degree of polarization of the proton beam were 2 to 4 \(\mu\)A, 5 keV and about 80\%, respectively. This proton beam was injected radially into the center of the cyclotron with a trochoidal injection system, which is a copy of the one used in Saclay \(^7\). The Lorentz force acting on the protons during injection was balanced by an electrostatical force generated by a system of electrodes. The shape of the electrical field was such that in combination with the magnetic field a strong focussing was achieved resulting in a transmission efficiency of about 70\%.

Only a small fraction of the injected current is actually accelerated by the cyclotron. During our experiment the protons gained an energy of about 80 keV per revolution, so the total number of revolutions was about 300. The intensity of the extracted beam was 10 to 20 nA with an energy spread of 60 to 90 keV.

After extraction the beam was transported to the scattering chamber by a system composed of five bending magnets, twenty quadrupoles and five steering magnets \(^2,8\) covering a distance of 40 m. In order to get as much intensity as possible on the target we used the doubly achromatic mode of the beam-guiding system, which means conservation of the energy profile of the beam during transport.
We were able to focus the proton beam in a spot of less than two mm diameter at the target. The direction of the beam was checked with probes at the center of the Faraday cup, which was situated two and a half meters after the target.

The outgoing protons, deuterons, tritons and alpha particles were detected by an array of four telescopes T1-T4 (fig. 2.1; see also section 2.2). Two different configurations have been used: in the $^{58}$Ni experiment the telescopes were $10^\circ$ apart, while in the $^{56}$Fe experiment this was $6^\circ$. The angular acceptance was in both cases about $1^\circ$.

Two silicon detectors (D1, D2) in the vertical plane through the beam at constant scattering angles of 45 and $-45$ degrees, respectively, measured elastically scattered protons in order to provide a relative normalization of the various runs and a clock signal for reversing the polarization direction. The vertical plane was chosen because the scattering in that plane is independent of the degree of polarization (the polarization axis of the proton beam being vertical too, cf. formula 1.3).

The degree of polarization of the proton beam was monitored continuously in a second smaller scattering chamber downstream (fig. 2.1) in the following way: first the beam energy was degraded to a mean energy of 16 MeV by an aluminium foil; next the number of protons elastically scattered by the carbon in a thin polyethylene foil was measured by two silicon detectors (D3, D4) placed in the horizontal plane at 52.5 and $-52.5$ degrees. Since the analysing power of $^{12}$C is

\[ A_{\text{el}} \]

\[ A_{\text{el}} = \text{constant} \]

\[ \text{for } \theta = 52.5^\circ \]

\[ \text{for } \theta = -52.5^\circ \]

\[ \text{Fig. 2.1 Schematic drawing of the scattering chamber and polarization monitor.} \]
rather energy-independent at this angle, one can determine in this way a reliable value for the polarization degree of the beam.

2.2 The telescope system

In fig. 2.2 we give a cross-sectional view of the detector block used in the $^{56}$Fe experiment. Each of the four telescopes consists of two silicon surface-barrier detectors: a 200 μm ΔE-detector (Philips) and a 2 mm E-detector (ORTEC). Permanent magnets in front of the telescopes sweep away the secondary electrons arriving from the target. During the $^{58}$Ni experiment we did not yet have this detector block at our disposal and we had to mount each telescope on a separate holder. Both experiments were performed using the same detectors. The telescopes were normalized to each other by comparing measurements done at the same angle.

The particles are identified by their specific energy loss dE/dx in the ΔE-detector, which is approximated by the semiempirical relation 9)

Fig. 2.2 Cross-sectional view of the four-telescope detector block used in the $^{56}$Fe experiment.
\[
- \frac{dE}{dx} = c M^{0.73} Z^2 E^{-0.73}
\] (2.1)

where \( M, Z \) and \( E \) are the mass number, charge number and energy of the particles, respectively. The constants \( c \) and 0.73 depend upon the detector material (silicon) and the energy range (5 to 100 MeV). For silicon is \( c = 0.045 \) if \( E \) is in MeV and \( x \) in \( \mu m \).

In order to determine the particle type irrespective of the energy a quantity PIO (particle identifier output) is calculated according to the algorithm of Goulding et al. 9):

\[ \text{PIO} = (E + \Delta E)^{1.73} - E^{1.73} \] (2.2)

where \( \Delta E \) and \( E \) are the energies deposited by the particles in the \( \Delta E \)- and E-detector, respectively. Assuming that the particle is stopped in the E-detector and using eq. (2.1) one can derive

\[ \text{PIO} = 1.73 c M^{0.73} Z^2 \Delta E \] (2.3)

where \( \Delta E \) is the thickness of the \( \Delta E \)-detector. The relation (2.3) shows that PIO is energy-independent and allows to discriminate between the various particle types. Actually the quantity PIO defined by (2.2)

![Graph showing particle identification spectrum](image)

**Fig. 2.3** Mass spectrum obtained at \( \theta_{\text{lab}} = 30^\circ \) with a 200 \( \mu m \) \( \Delta E \)- and a 2 \( \mu m \) E-detector.
gives rise to a so-called mass spectrum (fig. 2.3), where the peaks are centered around the values given by the relation (2.3). The width of the peaks is caused by several effects, of which electronic noise, inhomogeneities in the thickness of the $\Delta E$-detector and straggling in the $\Delta E$-detector appear to be the most important ones $^{10}$. We will discuss the straggling effect in some detail because it sets an intrinsic lower limit on the peak widths. Since we only want to make here a rough estimate of the importance of the straggling effect we will not use the Vavilov-theory $^{11}$ but we will assume a Gaussian energy-distribution $^{12}$ of the straggled particles, where the standard deviation $\sigma$ is given (in units of keV) by

$$\sigma = 4.3 \ Z \ \sqrt{d} \quad (2.4)$$

with $Z$ the charge number of the particles and with $d$ the thickness of the (silicon) $\Delta E$-detector in $\mu$m. The charge $\Delta P_{\text{IO}}$ induced by the deviation $\sigma$ is approximately given by

$$|\Delta P_{\text{IO}}| = 1.73 \ \sigma \ \mu \text{m}^{0.73} \quad (2.3)$$

Combining (2.3), (2.4) and (2.5) we find

$$\left| \frac{\Delta P_{\text{IO}}}{P_{\text{IO}}} \right| = \frac{0.095 \ \sigma}{2 \ \mu \text{m}^{0.73} \ \sqrt{d}} \ \mu \text{m}^{0.73} \quad (2.6)$$

where $d$ is given in $\mu$m and $E$ in MeV. For instance, eq. (2.6) gives the values 0.07 (protons), 0.03 (deuterons), 0.02 (tritons) and 0.01 ($\alpha$-particles) for the ratio of $\Delta P_{\text{IO}}$ and $P_{\text{IO}}$, when $d = 200$ $\mu$m and the energy of the proton beam is 25 MeV. In order to obtain the FWHM (full-width-half-maximum) one has to multiply the right-hand-side of eq. (2.6) with 2.35. A comparison with the experimental mass-spectra showed that in our case 50 to 70% of the widths could be attributed to straggling. We also tested the relation (2.6) with mass-spectra obtained with different thicknesses of the $\Delta E$-detector (table 2.1) and found a semiquantitative agreement.

From (2.6) it is clear that the resolution of the mass spectrum gets better when the thickness $d$ increases. On the other hand particles that are stopped in the $\Delta E$-detector are not analyzed because the particle type cannot be determined in that case. So the $\Delta E$-detector must not be too thick in order to allow the detection of low-energy deuterons and tritons. To meet the two criteria mentioned above we selected 200 $\mu$m as the appropriate thickness of the $\Delta E$-detectors in our experiments.
Table 2.1
Comparison of calculated and experimental relative peak-widths in the P10 spectrum

<table>
<thead>
<tr>
<th>Thickness of ΔE-det</th>
<th>Particle Type</th>
<th>p</th>
<th>d&lt;sup&gt;a)&lt;/sup&gt;</th>
<th>t</th>
<th>α</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 μm</td>
<td>calc.</td>
<td>1.38</td>
<td>1.00</td>
<td>0.69</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td>exp.</td>
<td>1.11</td>
<td>1.00</td>
<td>0.54</td>
<td>0.49</td>
</tr>
<tr>
<td>200 μm</td>
<td>calc.</td>
<td>1.44</td>
<td>1.00</td>
<td>0.675</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td>exp.</td>
<td>1.43</td>
<td>1.00</td>
<td>0.60</td>
<td>0.29</td>
</tr>
<tr>
<td>500 μm</td>
<td>calc.</td>
<td>1.74</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>exp.</td>
<td>1.46</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<sup>a)</sup> The relative peak-width of the deuteron peak was used to normalize the other results.

The proton peak threatens to swamp the deuteron peak especially at forward angles because of the large number of elastically scattered protons. Therefore - since we were not primarily interested in proton scattering - we shifted the elastic part of the proton peak to lower values by choosing an E-detector which was too thin to stop elastically scattered protons but thick enough to stop all the other particles. One can easily check that the expression (2.2) yields lower values for P10 when calculated for particles that are not stopped by the telescope.

In order to avoid broadening of the mass peaks by unequal amplification of the ΔE- and E-signals we adjusted the relative amplification by comparing the signals induced by the 5.8 MeV alpha particles emitted by <sup>244</sup>Cm.

2.3 The electronic equipment

The main components of the electronic circuitry used for each of the telescopes are displayed in fig. 2.4. Both the ΔE- and the E-signals are fed into a pulse-shaping circuit and a fast-logic circuit. The fast discriminator produces a logical pulse whenever the input signal exceeds a preset level (about 0.1 mV). After conversion from
Fig. 9.4 Electronic circuitry for the telescope system.

PA : preamplifier (CANBERRA 970D)
FA : fast amplifier (LRS 335L)
FIA : filter amplifier (THE)
FD : fast discriminator (LRS 621AL)
LA : level adapter (LHS 688L)
RU : routing unit (THE)
MA : main amplifier (ORTEC 48b)
LGS : linear gate stretcher (ORTEC 442)
PI : particle identifier (THE)
DU : discriminator unit (THE)
PL : particle logic (THE)
MI : mixer (ORTEC 456A)
ADC : analog digital converter (NUCL. CHIC. 27850)
NIM to TTL logic this signal is passed on to the routing unit. Whenever the routing unit receives simultaneously (resolving time 150 ns) a ΔE- and E-signal the gates of the stretchers (LGS) are opened. The two telescope bits identify the telescope in that case. Then the E- and ΔE-outputs of the stretchers are fed into an analogue particle identifier developed by Sluijters et al. 13). This particle identifier generates a PIO (particle identifier output, see formula 2.2) and a total energy signal ΔE+E. The mass peaks of the PIO spectrum are separated by four discriminator levels a, b, c and d in the discriminator unit. Whenever an event in one of the windows of this unit is recorded, the gate of the ADC is opened and two bits are generated to label the particle type.

The ADC bits together with the telescope bits, the particle bits and the spin bit are sent via an ADC controller and a CAMAC system to a MOS-memory (38 k, 24 bits). After each run the contents of the MOS-memory are written to floppy disks by a PDP11 computer. For details on the various control units and the data acquisition we refer to the refs. 3 and 14.
REFERENCES

Nuclear transfer reactions are generally grazing collisions between nuclei, that is the nuclei do not interpenetrate much and the effective interaction is more or less localized at the nuclear surfaces. The relative weakness of this "surface-interaction" allows the use of the so-called distorted-wave Born-approximation (DWBA). In DWBA the surface-interaction is treated in first order only, whereas the elastic scattering before and after this interaction is described to all orders by distorted waves.

In this chapter we will give a small review of DWBA-theory in order to provide some background to the following chapters. More details can be found in e.g. the textbook of Austern 1). The general features of the DWBA are treated in section 3.1, while the applications to one- and two-neutron transfer are presented in the sections 3.2 and 3.3, respectively. For details of the triton transfer formalism, which is applied to the analysis of the $^{58}$Ni(p,α) reaction (chapter 7), we refer to the thesis of Smits 2). Finally, the optical-model potentials, which generate the distorted waves, are discussed in the sections 3.4, 3.5 and 3.6.

3.1 DWBA theory

To describe the reaction $A+a \rightarrow B+b$ we introduce the Hamiltonian

$$ H = H_A + H_a + H_B + T_a + V_a = H_B + H_b + T_B + V_B $$

(3.1)

where $H_A$, $H_a$, $H_B$, and $H_b$ represent the internal motion of the target nucleus $A$, the projectile $a$, the residual nucleus $B$ and the ejectile $b$. The relative motion is governed by the kinetic energy $T_a$ ($T_b$) and the potential $V_a$ ($V_b$).

The state vector $\Psi(E)$ satisfies $H\Psi = E\Psi$ and has in our model the following form

$$ \Psi = \sum_{\gamma} \chi_{\gamma}\psi_c^{\gamma}\psi_c $$

(3.2)

where $\psi_c$ and $\psi_a$ are the internal wave functions of the heavy (C) and light (c) component of the system and consequently are eigenfunctions of $H_C$ and $H_c$. The function $\chi_\gamma$ is the distorted wave for the reaction
channel $\gamma$ and represents the relative motion. The sum in (3.2) generally is limited to a small number of reaction channels, while the remaining processes are hidden in the imaginary (absorption) part of the optical-model potentials mentioned below.

It is in fact the asymptotic behaviour (large separation between $C$ and $c$) of the functions $\chi_y$ that is measured by the detectors. The transition amplitude $T_{a\beta}$ for the reaction $(A,a) \rightarrow (B,b)$ is extracted from $\chi_y^{(+)}(x_{y\rightarrow\infty})$, where the $+$ indicates "outgoing" boundary conditions. All the distorted waves in the expansion (3.2) are of the $\chi^{(+)}$ type except for $\chi_\alpha$ (elastic channel), which also contains incoming waves (the proton beam for instance).

Now, supposing that $<\psi_B^* \psi_B^* | \psi_B^* \psi_B^* > = \delta_{B \gamma}$ (which is only rigorously true for inelastic scattering) and starting from

$$<\psi_B^* \psi_B^* | H - E | \psi_B^* \psi_B^* > = 0$$  (3.3)

we can derive

$$\left( T_{a\beta} + U_{a\beta} - E_{a\beta} \right) \chi_{a\beta} = - \sum_{\gamma} <\psi_B^* \psi_B^* | V | C\gamma | \chi_{a\gamma}$$  (3.4)

where we have replaced $<\psi_B^* \psi_B^* | V | C\gamma | \chi_{a\gamma}$ by the optical potential $U_{a\beta}(r_{a\beta})$ and where the matrix element on the right-hand side is defined by

$$<\psi_B^* \psi_B^* | C\gamma | C\gamma > = <\psi_B^* \psi_B^* | V | C\gamma | \chi_{a\gamma} > = <\psi_B^* \psi_B^* | V | C\gamma | \chi_{a\gamma} >$$  (3.5)

Loosely speaking we can say that $V_{\gamma} - U_{\gamma}$ represents the "surface-interaction" mentioned above.

The equation (3.4) really is a system of coupled differential equations and by solving this system not only the elastic scattering but also the surface-interactions are treated to all orders. In the DWBA some of the terms at the r.h.s. of eq. (3.5) are set equal to zero, which is a reasonable procedure if elastic scattering is the dominant process. We will clarify this procedure by the following example, which includes the outgoing channels $\beta$ and $\gamma$ besides the elastic one (fig. 3.1). The system of equations corresponding to the coupling scheme of fig. 3.1 is given by

$$\left( T_{a\alpha} + U_{a\alpha} - E_{a\alpha} \right) \chi_{a\alpha} = 0$$  (3.6a)

$$\left( T_{a\beta} + U_{a\beta} - E_{a\beta} \right) \chi_{a\beta} = - <\psi_B^* \psi_B^* | C\gamma | \chi_{a\gamma}$$  (3.6b)

$$\left( T_{a\gamma} + U_{a\gamma} - E_{a\gamma} \right) \chi_{a\gamma} = - <\psi_B^* \psi_B^* | C\gamma | \chi_{a\gamma} = <\psi_B^* \psi_B^* | V | \psi_B^* \psi_B^* > - <\psi_B^* \psi_B^* | V | \psi_B^* \psi_B^* >$$  (3.6c)
The transitions (α+β), (β+γ) and (α+γ) are now described by a first-order approximation ("one-way coupling"), whereas the elastic scattering is still treated to all orders. The sequence (α+β)(β+γ) is called a two-step DWBA transition, while (α+γ) is a one-step DWBA transition.

If we are not interested in channel γ then the system (3.6) reduces to (3.6a) and (3.6b) and in that case the transition amplitude \( T_{αβ} \) can also be calculated from the expression

\[
T_{αβ} = (\bar{\chi}_β, <Bb|V|Aa>) \chi_α (3.7)
\]

where \( \bar{\chi}_β \) is the solution of the homogeneous equation corresponding to eq. (3.6b) with incoming boundary conditions. Displaying the integration over relative coordinates explicitly eq. (3.7) becomes

\[
T_{αβ} = J \int d^3x_α \int d^3x_β \chi_β^{(-)}* (\chi_β, \chi_β) <Bb|V|Aa> \chi_α (3.8)
\]

with \( J \) the Jacobian for the transformation to the relative coordinates and with \( k_β \) and \( k_α \) the relative momenta.

Before discussing the matrix element \( <Bb|V|Aa> \) we will first connect \( T_{αβ} \) with the analysing power \( A \) and the unpolarized cross section \( dσ/dΩ \). In expression (3.8) we have not explicitly shown the angular momenta. In fact \( T_{αβ} \) depends upon the spin projections of \( A, \ a, \ B \) and \( b: M_A, m_a, M_B \) and \( m_B \) and the unpolarized cross section is given by...
\[
\frac{d\sigma}{d\Omega} \sim \sum_{E_A^B, m_A^B, m_B^B} |T_{AB}|^2 \tag{3.9}
\]

The analysing power is a measure for the difference between the cross sections obtained with proton spin up \((m_a = +1/2)\) and proton spin down \((m_a = -1/2)\):

\[
A = \frac{\sum_{E_A^B, m_A^B, m_B^B} \left\{ |T_{E_A^B m_a}\rangle \langle m_b| - |T_{E_A^B m_b}\rangle \langle m_a| \right\}^2}{\sum_{E_A^B, m_A^B, m_B^B} |T_{E_A^B m_b}\rangle \langle m_b|} \tag{3.10}
\]

It is clear from (3.10) that \(A\) lies between -1 and 1.

In order to have a quantitative indication of the agreement between calculated and measured (see chapter 1) analysing power and cross section we used the following chi-square expressions:

\[
\chi^2_0 = \frac{1}{N} \sum_i \left\{ \frac{\sigma_{\exp}(i) - \sigma_{\text{cal}}(i)}{\Delta \sigma_{\exp}(i)} \right\}^2 \tag{3.11a}
\]
\[
\chi^2_A = \frac{1}{N} \sum_i \left\{ \frac{A_{\exp}(i) - A_{\text{cal}}(i)}{\Delta A_{\exp}(i)} \right\}^2 \tag{3.11b}
\]

where \(N\) is the total number of experimental data taken at different scattering angles \(\theta_i\). The quantities \(\Delta \sigma\) and \(\Delta A\) are the errors of the experimental cross section and analysing power.

### 3.2 One-neutron transfer

Working out \(\langle \text{Bb} | V | \text{Aa} \rangle\) for a \((p,d)\) transition one arrives at:

\[
\langle \text{Bb} | V | \text{Aa} \rangle = \sqrt{n} \langle \phi_d^p (r - r_n) \psi_B^p (\xi) | V_{np}^p (r_p - r_n) \psi_{A}^p (\xi, r_n) \rangle \tag{3.12}
\]

with \(\xi\) being the internal coordinates of nucleus B and \(n\) the number of "active" (i.e. taken explicitly into account in the wave function \(\psi\)) neutrons in nucleus A. The deuteron internal wave function is indicated by \(\phi_d^p\) and is generally limited to the \(L=0\) component (S-state deuterons), which has the important corollary that the transferred angular momentum is equal to that of the neutron picked up from nucleus A. The "surface interaction" \(V_{np}^p\) is restricted to the potential between the incoming proton and the transferred neutron. In the so-called zero-range approximation one takes \(V_{np}^p\) proportional to the delta function \(\delta(\xi, r_n)\).
i.e. the proton and the neutron only interact when they coincide. In
fact one replaces \( \Phi_d V_{np} \) by \( D_0 \delta(\vec{r}_p - \vec{r}_n) \), where the zero-range constant
\( D_0 \) is calculated from
\[
D_0 = \int d\vec{r} \, \Phi_d(\vec{r}) V_{np}(\vec{r}) \tag{3.13}
\]
with \( V_{np} \) a more or less realistic potential.

The nuclear overlap ("form factor") \( \langle \psi_B | \psi_A \rangle \) is expanded as
follows:
\[
\sqrt{n} \langle \psi_B(\vec{r}) | \psi_A(\vec{r}, \vec{r}_n) \rangle = \sum_j S^{1/2}(j) \langle j \rangle B_{AB} M_A \langle j \rangle A \times \phi_{jm}(\vec{r}_n) \tag{3.14}
\]
One usually takes for \( \phi_{jm} \) a single-particle shell-model wave function
calculated with a real Woods-Saxon potential (eq. 3.29), of which the
well-depth is adjusted to comply with the condition that the binding
energy of the neutron has to be equal to the experimental separation
energy \( E_B - E_A \) in order to guarantee a correct asymptotic behaviour of
\( \langle \psi_B | \psi_A \rangle \). This is called the well-depth procedure (WDP).

The nuclear structure information is contained in the spectroscopic amplitudes \( S^{1/2} \) and they measure the amplitudes with which the various single-particle states appear in the nuclear wave functions.
For example, the \( 0^+ \) ground state of \( ^{58}\text{Ni} \) may be described by the
following two-neutron wave function:
\[
|0^+\rangle = a|\Phi_{1/2^+}\rangle + b|\Phi_{3/2^+}\rangle + c|\Phi_{5/2^+}\rangle \tag{3.15}
\]
In this model the \( ^{56}\text{Ni} \) core is considered to be inert (closed shells)
and the two "valence" neutrons are occupying \( 2p_{1/2}, 2p_{3/2} \) and \( 1f_{5/2} \)
orbits. The configuration mixing is caused by the residual interaction
(not present in the shell-model potential) between the neutrons. The
spectroscopic amplitude \( S^{1/2}(2p_{3/2}) \) for the transition of this \( 0^+ \)
state to the \( 3/2^- \) ground state of \( ^{58}\text{Ni} \) is then (using eq. 3.14) \( b\sqrt{2} \).
So by measuring the spectroscopic amplitudes one thus determines the
magnitude of the coefficients of the various components of the \( 0^+ \) wave function.

Since we want to investigate the interference between one-step
and two-step DWBA transitions, knowledge of the relative signs of the
spectroscopic amplitudes is crucial. Therefore one must be sure that
the phase conventions used in calculating these amplitudes from shell-
model wave functions are the same as those of the computer code that
calculates the cross sections and the analysing powers.
For the coupled-channels code CHUCK2, with which the cross sections and analysing powers in this thesis have been calculated, one has to use spectroscopic amplitudes defined by

$$S^{1/2}(\rho) = \sqrt{\alpha} \langle \phi_{B A}^{as} \rangle J_A ^{1/2} \psi_{A}^{as} \rangle$$  \hspace{1cm} (3.16)

where $\rho$ stands for the quantum numbers $(nlj)$ and "as" means: anti-symmetrized. In addition the radial part of the single-particle wave functions is positive at infinity and the angular part is defined without the factor $\gamma$.

Summarizing we get the following expression for the one-step DWBA transition amplitude

$$T_{AB} = \frac{\alpha}{\rho_{pd}} \sum_j S^{1/2}(nlj) \int dr \chi_{B}^{(-)}(r) \phi_{nlj}^{(+)}(r) \chi_{A}^{(+)}(r) \chi_{A}^{(A-1)}(r)$$  \hspace{1cm} (3.17)

which is also valid for other one-neutron transfers, but with a different zero-range constant $\alpha$. The mass number of the target nucleus is indicated by $A$. The angular momentum transfer $j$ has to satisfy the triangle condition implied by the Clebsch-Gordan coefficient of eq. 3.14 and if $J_A = 0$ then only one term survives: $j = J_B$.

French and Macfarlane have derived sum rules, which connect neutron and proton occupation numbers with the spectroscopic amplitudes. Since we apply these sum rules to our experimental results (chapters 4 and 5) we give them here. If the target nucleus has isospin $T_A$ then one can reach via one-neutron transfer $(t = 1/2, t_2 = 1/2)$ final states with $T_A = T_A - 1/2$ and $T_A = T_A + 1/2$. The sum rules then are:

$$\frac{1}{2} C^2 S_{T_A}^{1/2} (lj) = v^{lj} - \frac{1^{lj}}{2T_A + 1}$$  \hspace{1cm} (3.18)

$$\frac{1}{2} C^2 S_{T_A}^{1/2} (lj) = v^{lj} - \frac{2^{lj}}{2T_A + 1}$$

where $v^{lj}$ is the number of neutrons occupying the $lj$ orbit and correspondingly $y^{lj}$ for the protons. $C$ stands for the isospin Clebsch-Gordan coefficient $\langle T_B T_A 1/2 1/2 | T_A T_A \rangle$ with either $T_B = T_A - 1/2$ or $T_B = T_A + 1/2$. 

18
3.3 Two-neutron transfer

In case of a one-step \((p,t)\) transition the explicit form of the transition density is \(^5\):

\[
\langle bb | v | AA \rangle = \sqrt{\frac{n(n-1)}{2}} \langle \phi_B | v_1 \rho (\vec{r}_1 - \vec{r}_2) + v_2 \rho (\vec{r}_2 - \vec{r}_1) | \psi_A (\vec{r}_1, \vec{r}_2) \rangle
\]

**(3.19)**

In the zero-range approximation one takes:

\[
\phi_B (\rho v_{p1} + v_{p2}) \sim d_0 (\vec{r}) \delta (\rho)
\]

**(3.20)**

where \( \vec{r} = \vec{r}_1 - \vec{r}_2 \) and \( \vec{s} = \vec{r}_p - \frac{\vec{r}_1 + \vec{r}_2}{2} \)

**(3.21)**

In this approximation the interaction takes place when the proton coincides with the center of mass of the two neutrons. The function \( d_0 \) can be determined analytically from:

\[
d_0 (\vec{r}) = \int d^3 \phi (\vec{r}, \vec{s}) (\rho v_{p1} + v_{p2})
\]

**(3.22)**

if one takes Gaussian shapes for \( \phi \) and \( V \). The important parameter entering this evaluation is the root-mean-square radius of the triton, which usually is chosen equal to 1.7 fm. In addition a pure \( L=0 \) triton internal wave function is assumed.

The nuclear overlap \( \langle \psi_B (\xi) | \psi_A (\xi, \vec{r}_1, \vec{r}_2) \rangle \) is related to the spectroscopic amplitudes in the following way

\[
\sqrt{\frac{n(n-1)}{2}} \langle \psi_B | \psi_A \rangle = \sum_{j_1 j_2} s^{1/2} (j_1 j_2; J) \langle JM | N_B | J A A \rangle \langle \phi \rho (\vec{r}_1) \times \phi \rho (\vec{r}_2) \rangle
\]

**(3.23)**

The one-particle wave functions in this equation are determined in the same way as in the case of one-neutron transfer, while the binding energy is usually taken equal to half the separation energy \( E_B - E_A \) of the two neutrons.

The inverse of eq. 3.23 is given by

\[
s^{1/2} (\delta \lambda, J) = \sqrt{\frac{n(n-1)}{2}} \langle \left( \phi \rho \times \phi \rho \right)_J | \psi_B \rangle | \psi_A \rangle
\]

**(3.24)**

where \( \delta \) and \( \lambda \) stand for \((n_1j_1)\) and \((n_2j_2)\), respectively.

In order to calculate the transition amplitude \( T_{0B} \) one then proceeds as follows. First \( \phi \rho (\vec{r}_1) \times \phi \rho (\vec{r}_2) \) is transformed into a sum of products of radial and angular functions expressed in the relative
and center-of-mass coordinates $\mathbf{\hat{r}} = \mathbf{\hat{r}}_2 - \mathbf{\hat{r}}_1$ and $\mathbf{\hat{R}} = 1/2 (\mathbf{\hat{r}}_1 + \mathbf{\hat{r}}_2)$, respectively. Next one restricts this transformed function to the part with relative $1-0$ (i.e. the two neutrons being in an $S$-state relative to each other). This means that the $S$-transfer is zero and that $J = L$. Then the integration over $\mathbf{\hat{r}}$ and $\mathbf{\hat{p}}$ is performed resulting in the form factor $F^J_{\rho\lambda}(A/2, R)$, where $A$ is the mass number of the target nucleus.

The final result then is

$$T_{\alpha B}(J) \sim D_0(\lambda t) \int \frac{d^4\mathbf{\hat{R}}}{(2\pi)^4} \mathcal{S}^{1/2}(\lambda t; J) \int d^4\mathbf{\hat{x}} \chi^{(-\alpha)}_n(\mathbf{\hat{x}}) F^J_{\rho\lambda}(\mathbf{\hat{R}}) \chi^{(+\lambda)}_p(\mathbf{\hat{R}})$$

(3.25)

### 3.4 The optical-model potential

The distorted waves are calculated from equations like (3.6a) using an optical-model potential of the following form:

$$U(r) = U_C(r) - Vf(x_0) - iWvF(x_1) - 4iWv'F'(x_1) - \left(\frac{\hbar}{m_\pi c}\right)^2 v \frac{d^2}{dr^2} - \frac{1}{r} \frac{df(x_2)}{dx}$$

(3.26)

where $f(x_1)$ has the Woods-Saxon shape

$$(x^2 + 1)^{-1} \quad \text{with} \quad x_1 = \frac{r - r_A A^{1/3}}{a}$$

(3.27)

and where $U_C$ is the Coulomb potential generated by a uniformly charged sphere of radius $r_C A^{1/3}$. For protons and tritons $\mathcal{S} = 2s$ and for deuterons $\mathcal{S} = \frac{1}{2}$, where $\mathcal{S}$ is the spin operator. The vector $\mathcal{S}$ is the orbital angular momentum of the scattered particle. The two imaginary terms of $U(r)$ represent the volume absorption ($W_v$) and the surface absorption ($W_S$).

### 3.5 Finite-range and non-locality corrections

In section 3.2 we have discussed the zero-range approximation for one-neutron transfer. It is, however, possible to make a first-order correction to this approximation, which is called the local-energy approximation (LEA). The LEA consists of multiplying the form factor (eq. 3.14) with the Bultenb form

$$\Lambda(x) = \left\{1 + \frac{2}{\hbar^2} \frac{a m}{m_\pi^2} R^2 \left(\frac{x}{a} - V_\alpha \left(\frac{a - 1}{A} x - E_n - V_n(x) - E_S + V_S(x)\right)\right)\right\}^{-1}$$

(3.28)
with \( V_n \) the bound-state potential and \( R \) the finite-range parameter.

The optical potential should really be a non-local potential. As shown by Perey \(^7\) a non-local potential reduces the amplitude of the distorted waves in the nuclear interior. This Perey-effect can be simulated approximately by multiplying the form factor (eq. 3.14) with the factors

\[
N_i(r) = \exp \left( \frac{\mu_i \beta_i^2}{4h^2} U_i(r) \right)
\]

where \( \beta_i \) is the non-locality parameter of projectile, neutron and ejectile, respectively, and \( \mu_i \) the corresponding reduced mass.

### 3.6 Adiabatic deuteron potential

In the reaction model implied by eq. 3.2 three-body effects are not taken into account. Since the deuteron is a loosely-bound particle, the nuclear field of the final nucleus can easily split the deuteron in a proton and a neutron and thus give rise to the channel \((3,p,n)\).

The adiabatic model of Johnson and Soper \(^8\) provides an approximate treatment of these effects by replacing the optical deuteron potential by a kind of folding potential:

\[
\bar{U}(r) = D_0^{-1} \int ds \left( U_n \left( r + \frac{s}{2} \right) + U_p \left( r - \frac{s}{2} \right) \right) V_{pn}(s) \phi_s(s) \quad (3.30)
\]

where

\[
D_0 = \int ds V_{pn}(s) \phi_s(s)
\]

and where the proton and the neutron each get half of the deuteron energy. The distorted waves generated by the "adiabatic" potential \( \bar{U} \) also include unbound \((p,n)\) states, in which the proton and the neutron continue to move together in an \( S \)-state with little relative momentum.

An approximate expression for \( \bar{U}(r) \) has been derived by Satchler \(^9\) for the case that \( U_n \) and \( U_p \) are Woods-Saxon potentials (or first derivatives thereof) with the same geometry parameters but different well-depths. We will give here the formulas leading to this approximate expression for \( \bar{U}(r) \) and refer to the study of Satchler for details.

If \( U_n \) and \( U_p \) have a Woods-Saxon shape with parameters \((V_n,r,a)\) and \((V_p,r,a)\), respectively, then \( \bar{U} \) has \( \bar{r} = r \) and

\[
a = a + \frac{0.0217}{a} \quad (3.32a)
\]

\[
\bar{V} = (1 - 0.0217 \frac{2\pi^2}{h^2}) (V_p + V_n) \quad (3.32b)
\]
where \( R = r A^{1/3} \). We remark that eq. 3.32b differs from the prescription of Satchler by the factor 2 in front of \( r^2 \). We think, however, that our approximation is better. We will not go into details, since the final results are hardly influenced by this alternative prescription.

If \( U_n \) and \( U_p \) have the shape of the derivative of a Woods-Saxon potential with parameters \( (W_n, r, a) \) and \( (W_p, r, a) \), respectively, then the same is true for \( U \) with \( r = r \) and

\[
\bar{a} = a + \frac{0.0182}{a} \tag{3.33a}
\]

\[
\bar{W} = \left( 1 - 0.0182 \left( \frac{2}{\bar{a}^2} + \frac{2}{3} \frac{\pi^2}{R^2} \right) \right) (W_n + W_p) \tag{3.33b}
\]

We also had to construct potentials \( \bar{W} \) starting from different geometry parameters for the neutron and the proton potentials. This was done by separately calculating \( \bar{W}_n \) and \( \bar{W}_p \) from eq. 3.33b putting \( W_p = 0 \) and \( W_n = 0 \), respectively and then adding them.
REFERENCES

3. P.D. Kunz, University of Colorado, unpublished.
CHAPTER 4  THE \( ^{\,}^p_d \) REACTION ON \( ^{\,}^{58}Ni \) AT 24.6 MeV

4.1 Introduction

The \( ^{\,}^p_d \) reaction is a well-known instrument for obtaining spectroscopic information like \( l \)-values and spectroscopic factors. The usefulness of this reaction is greatly enhanced by using polarized protons, because the analysing powers show a clear \( j \)-dependence \(^1\). As a general rule one can say that the sign of the analysing power at the angle where the cross section reaches its main maximum, is positive or negative depending on whether \( j = 1 + \frac{1}{2} \) or \( j = 1 - \frac{1}{2} \). By comparing the measured analysing powers with DWBA calculations or with the experimental results for levels with well-established \( j \)-values ("pattern recognition") one thus obtains a reliable spin assignment of the final state in the case of spin-zero target nuclei.

Some \( ^{\,}^p_d \) cross sections do not exhibit the characteristic \( l \)-dependent diffraction pattern predicted by the one-step DWBA, but a rather flat and featureless shape \(^2,^3\). A common property of the levels showing such a behaviour is that they are weakly excited, wherefore in these cases two-step processes are generally thought to be competing with the one-step pick-up. Such transitions are present in the \( ^{\,}^{58}Ni(p,d) \) reaction at 27.5 MeV \(^3\).

Mayer et al. \(^1\) already performed the \( ^{\,}^{58}Ni(p,d) \) experiment with polarized protons at the same energy as we did. These authors, however, only analysed the strongly excited levels. So we decided to extend this study to the weaker levels especially in view of possible two-step contributions. In addition we wanted to investigate the importance of two-step processes for strongly excited levels.

All cross sections and analysing powers have been calculated with the computer code \textsc{Chuck2} \(^4\).

Our results are discussed in section 4.4. An outline of the reaction model and the ingredients for the calculations is presented in section 4.3. In section 4.2 we have brought together the details of the experiment and finally in section 4.5 we have summarized the main conclusions of this chapter.
4.2 The experiment

The (p,d) reaction was measured simultaneously with the (p,p), (p,t) and (p,α) reactions. The outgoing particles were identified using the telescope system described in chapter 2. In order to determine absolute cross sections we compared the elastic proton-scattering data with the optical-model analysis performed before by our group \(^5\) for the reaction \(^{58}\)Ni(p,p) at 24.6 MeV. The estimated accuracy of the normalization is 20%.

The target consisted of a self-supporting foil of isotopically enriched (99.9%) \(^{58}\)Ni with a thickness of 1 mg/cm\(^2\). The intensity and degree of polarization of the proton beam were about 10 nA and 80%, respectively. Data were taken from 10\(^\circ\) to 80\(^\circ\) in steps of 5\(^\circ\). The energy resolution was about 100 keV FWHM, which imposed a limit to the number of levels that could be analysed.

We measured the analysing powers and cross sections of eighteen levels (table 4.8). These can be seen in the spectrum of fig. 4.1

![Energy spectrum of \(^{58}\)Ni(p,d)\(^{57}\)Ni at \(E_p = 24.6\) MeV for proton-spin down; \(\theta_{lab} = 20^\circ\)](image)
together with levels of contaminants and other levels of $^{57}$Ni which were excited too weakly to yield reliable angular distributions. The excitation energies were taken from the survey of Auble 6) and from the study of the $^{58}$Ni($\tau$,a) reaction at 25 MeV by Fortier and Galès 7).

The $^{58}$Ni(p,d) reaction has already been measured several times at different energies and with unpolarized protons 3,8-10). A polarized proton beam has been used by Mayer et al. 1) (24.5 MeV) and by Hosono et al. 11) (65 MeV).

4.3 The reaction model and the calculational procedures

4.3.1 Reaction model

Seven out of the measured eighteen transitions have been analysed according to the reaction model that is implied by the coupling scheme given in fig. 4.2. The remaining transitions were described by one-step DWBA only. In the two-step process inelastic proton-scattering to the first $2^+$ (1.45 MeV) of $^{58}$Ni is followed by the pick-up of a $p3/2$, $p1/2$, $f5/2$ or $f7/2$ neutron. Pick-up followed by inelastic deuteron-scattering turned out to yield negligible contributions and therefore was ignored. Replacing DWBA by CCBA in the proton channel (i.e. a two-way coupling between the $0^+$ and $2^+$ states) did not change the results significantly and so CCBA was also left out of consideration.

The one- plus two-step DWBA calculation (fig. 4.2) is complicated by the fact that five spectroscopic amplitudes may be involved against one in case of a simple one-step DWBA analysis. Therefore we made use of shell-model calculations done by Koops and Glaudemans 12) and Van Hees et al. 13). The latter calculations allow for one $1f7/2$.

![Fig. 4.2 Coupling scheme for the coupled-channels calculations](image_url)

$J^\pi = 1/2^-,3/2^-,5/2^-,7/2^-$
nucleon-hole in the $^{56}$Ni core with the particles occupying 2p$_3/2$, 2p$_1/2$ and 1f$_5/2$ orbits and they are performed with the surface-delta (SD1) or with a renormalized Kuo-Brown (KB1) residual interaction. The calculations contained in ref. 12 were restricted to 2p$_3/2$, 2p$_1/2$ and 1f$_5/2$ particles (i.e. an inert $^{56}$Ni core) and the surface-delta interaction (SDO). From the results of these nuclear structure studies three sets of spectroscopic amplitudes, also indicated by SD1, KB1 and SDO, have been computed 14). The set SDO obviously describes transitions to one 3/2$^-$, 5/2$^-$ and 1/2$^-$ state only, while SD1 and KB1 lead to a whole spectrum of negative parity states ranging from 1/2$^-$ to 11/2$^-$. Of these the 3/2$^-$ and 11/2$^-$ states can not be reached in one step from the 0$^+$ ground state of $^{58}$Ni.

The 3/2$^-$, 5/2$^-$ and 1/2$^-$ states are identified with the $^{57}$Ni levels at 0.00, 0.77 and 1.11 MeV, respectively. These three states are the only ones which turn out to have an appreciable 1/2$^-$, 3/2$^-$ and 5/2$^-$ transfer strength from the 2$^+$ state of $^{58}$Ni (table 4.1). The remaining states are populated by 7/2$^-$ transfer from either the 0$^+$ only or the 2$^+$ only. Calculations show that the states which can only be reached

<table>
<thead>
<tr>
<th>Process</th>
<th>$2J(2j)^{(a)}$</th>
<th>SDO</th>
<th>SD1</th>
<th>KB1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0$^+$, J</td>
<td>5(5)</td>
<td>-0.78</td>
<td>-0.94</td>
<td>-0.53</td>
</tr>
<tr>
<td></td>
<td>3(3)</td>
<td>1.11</td>
<td>0.89</td>
<td>1.25</td>
</tr>
<tr>
<td></td>
<td>1(1)</td>
<td>0.40</td>
<td>0.41</td>
<td>0.47</td>
</tr>
<tr>
<td>2$^+\to$ J</td>
<td>5(5)</td>
<td>-0.42</td>
<td>-0.57</td>
<td>-0.22</td>
</tr>
<tr>
<td></td>
<td>3(3)</td>
<td>0.32</td>
<td>0.35</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>3(1)</td>
<td>0.35</td>
<td>0.44</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>5(7)</td>
<td>0.00</td>
<td>0.10</td>
<td>&lt;0.10</td>
</tr>
<tr>
<td></td>
<td>3(5)</td>
<td>0.32</td>
<td>0.36</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>3(3)</td>
<td>1.01</td>
<td>0.72</td>
<td>1.12</td>
</tr>
<tr>
<td></td>
<td>3(1)</td>
<td>-0.42</td>
<td>-0.39</td>
<td>-0.41</td>
</tr>
<tr>
<td></td>
<td>3(7)</td>
<td>0.00</td>
<td>0.22</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>1(5)</td>
<td>-0.35</td>
<td>-0.44</td>
<td>-0.14</td>
</tr>
<tr>
<td></td>
<td>1(3)</td>
<td>0.42</td>
<td>0.37</td>
<td>0.40</td>
</tr>
</tbody>
</table>

(a) the transferred angular momentum is indicated by j.
via the $2^+$ are too weak to be observed in our experiment. Some of these states probably have been found by Portier and Galès and may be identified with a weak-coupling multiplet $2^+ \times 7/2^-$ somewhere around $E_X(2^+) + E_X(7/2^-) = 4.0$ MeV excitation energy.

The cross sections and analysing powers of the particle-states at 0.00, 0.77 and 1.11 MeV have been calculated using the spectroscopic amplitudes from table 4.1. We applied the usual one-step DWBA analysis to the other states except for four cases, where we added a two-step $7/2^-$ transfer to the one-step transition.

The inelastic scattering has been described macroscopically using the following form factor

$$F_2(r) = \frac{8}{\pi} \left\{ R_0 \frac{dU_0}{dr} + R_1 \frac{dU_1}{dr} - \frac{3}{5} z e^2 \left( \frac{R_C^2}{r^3} \theta(r-R_C) + \frac{1}{r^2} \theta(R_C-r) \right) \right\}$$

where $U_0$ and $U_1$ are the real (volume W-S) and the imaginary parts, respectively, of the optical proton-potential (eq. 3.26). The parameters $R_i$ (i=0,1,C) are defined by $R_i = r_i 1^{1/3}$, while $\theta$ is the usual step function. For the deformation parameter $\beta_2$ we used the value -0.22 found before by our group from the $^{58}$Ni($p,p'$) reaction at 24.6 MeV.

Actually the sign of the deformation parameter cannot be determined by the $(p,p')$ experiment since it depends on the relative phase of the wave functions of the $0^+$ and $2^+$ states. The sign of $\beta_2$ has to be found by trial and error. It was possible to make a definite choice for this sign by making use of the interference between the one-step and the two-step transitions. This is illustrated in fig. 4.3 (3/2$, solid and dashed-dotted lines).

4.3.2 The neutron form factor

The code CHUCK2 does not account for the D-state of the deuteron and the finite range of the interaction. So our calculations are limited to deuterons in the S-state and the finite range is described in the local-energy approximation (LEA). We used the Hulthen form (3.29) with the value of 0.69 fm for the finite-range parameter. The LEA reduces the cross sections by about 3% and has no significant effect on the shapes. The cross sections were normalized with the empirical constant $D_0(pd) = -122.5$ MeV fm$^{3/2}$.

The need for a non-locality correction of the neutron wave function has been demonstrated recently for neutron- and proton-
density calculations $^{15-17}$. So we apply this correction in our case too. Following the authors of refs. $^{15-17}$ we used a Gaussian form (3.30) with the value of 0.85 fm for the non-locality parameter. The cross sections increase with about 35% by the non-locality correction, whereas the shapes remain virtually unchanged.

Usually the neutron form factor is calculated by the well-depth procedure with binding energy of the neutron equal to the experimental separation energy (SE): $S_n = S_n + E_x$ with $S_n$ the one-neutron separation energy of $^{58}$Ni (g.s.) and $E_x$ the excitation energy. In this way the correct asymptotic behaviour of the form factor is assured.

In order to explain the observed $J$-dependence of certain (p,d) cross sections Sherr et. al. $^{18}$ proposed to replace the separation energies of the particle-states by effective energies, which are more in line with the single-particle shell-model energies and are supposed to give a better description in the nuclear interior.

In fact both approaches can be reconciled by a more sophisticated treatment of the form factor, which explicitly accounts for the residual interaction of the valence neutrons by considering the form factor as a solution of an inhomogeneous Schrödinger equation $^{19-24}$. We tried to construct an approximate solution to such an equation in the following way: first by choosing the residual interaction to be a delta interaction we simplified the inhomogeneous Schrödinger equation to

$$ (E - T - U) F_j(x) = \left( \sum_{j' = 1}^{2J+1} a_{jj'} F_{j'}^2 \right) F_j(x) \quad (4.2) $$

with $E$ the separation energy, $T$ the kinetic energy operator, $U$ the shell-model potential of the $^{56}$Ni core and $a_{jj'}$ certain coefficients determined by the spectroscopic amplitudes; secondly, we substituted the solutions $F_j^{(0)}$ of the homogeneous equations

$$ (E - T - U) F_j^{(0)}(x) = 0 \quad (4.3) $$

in the right-hand side of the equations (4.2) and solved these numerically; finally we repeated this procedure by substituting the newly-found solutions in the right-hand side of (4.2). The solutions did not change any more after two or three iterations. The cross sections and analysing powers calculated with these form factors, however, turned out to be virtually the same as those obtained following the separation-energy prescription.
Table 4.2

Effective energies (EE) for the bound-neutron wave function

<table>
<thead>
<tr>
<th>$j^n$</th>
<th>EE - $S_n$</th>
<th>EE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3/2^-$</td>
<td>0.00</td>
<td>12.20</td>
</tr>
<tr>
<td>$5/2^-$</td>
<td>-0.77</td>
<td>11.43</td>
</tr>
<tr>
<td>$1/2^-$</td>
<td>-1.11</td>
<td>11.09</td>
</tr>
<tr>
<td>$7/2^-$</td>
<td>2.58$^b)$</td>
<td>14.78</td>
</tr>
<tr>
<td>$7/2^-$</td>
<td>5.22$^c)$</td>
<td>17.43</td>
</tr>
<tr>
<td>$1/2^+$</td>
<td>5.50</td>
<td>17.78</td>
</tr>
<tr>
<td>$3/2^+$</td>
<td>6.01</td>
<td>18.21</td>
</tr>
</tbody>
</table>

$a)$ $S_n$ is the separation energy of a neutron from the ground state of $^{58}$Ni; $S_n = 12.20$ MeV

$b)$ if $E_x < 5.23$ MeV

$c)$ if $E_x > 5.23$ MeV

Since more carefully constructed solutions of the inhomogeneous Schrödinger equation yield results$^{21,22)}$ which deviate significantly from that prescription, we decided to compare the separation-energy (SE) with the following effective-energy (EE) prescription:

$$EE = S_n \pm E_x, \quad S_n$$ is the one-neutron separation energy of $^{58}$Ni in the ground state and $E_x$ is the excitation energy of the final $^{57}$Ni state. The minus and the plus signs are chosen for particle (i.e. $1/2^-$, $3/2^-$, $5/2^-$) and hole ($7/2^-$) states, respectively. The effective binding-energies are also used in a well-depth procedure, which implies a homogeneous Schrödinger equation. They are given in table 4.2.

We took the usual geometry parameters for the Woods-Saxon well:

$r = 1.25$ fm, $a = 0.65$ fm and the factor multiplying the Thomas term $\lambda = 25$ (i.e. $V_{SO}/V = 0.138$).

4.3.3 Optical potentials

The optical-model parameters are given in table 4.3. The proton potential has been determined before by our group$^{51)}$. It was extracted from a global fit to elastic scattering data of polarized protons on several iron and nickel isotopes at energies ranging from 15 to 25 MeV.
Table 4.3

Optical-model parameters

<table>
<thead>
<tr>
<th>Name</th>
<th>$V$</th>
<th>$r_o$</th>
<th>$a_o$</th>
<th>$W_v$</th>
<th>$W_d$</th>
<th>$r_I$</th>
<th>$a_I$</th>
<th>$v_{so}$</th>
<th>$r_{so}$</th>
<th>$a_{so}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>52.0</td>
<td>1.15</td>
<td>0.76</td>
<td>2.45</td>
<td>6.7</td>
<td>1.35</td>
<td>0.47</td>
<td>5.6</td>
<td>1.04</td>
<td>0.54</td>
</tr>
<tr>
<td>D1</td>
<td>107.1</td>
<td>1.05</td>
<td>0.86</td>
<td></td>
<td>14.7</td>
<td>1.43</td>
<td>0.69</td>
<td>7.0</td>
<td>0.75</td>
<td>0.50</td>
</tr>
<tr>
<td>D2</td>
<td>100.4</td>
<td>1.17</td>
<td>0.78</td>
<td></td>
<td>10.1</td>
<td>1.26</td>
<td>0.61</td>
<td>11.8</td>
<td>1.04</td>
<td>0.56</td>
</tr>
<tr>
<td>D4</td>
<td>98.4</td>
<td>1.17</td>
<td>0.78</td>
<td></td>
<td>15.2</td>
<td>1.32</td>
<td>0.57</td>
<td>11.8</td>
<td>1.04</td>
<td>0.56</td>
</tr>
</tbody>
</table>

Energy dependence of the deuteron potentials

\[ \Delta W/\Delta E_x \quad \Delta W_d/\Delta E_x \]

| D2 | 0.313 | 0.118 |
| D4 | 0.290 | -0.626 |

---

\[ a) \quad v, w_v, w_d \] and $v_{so}$ in MeV, the other parameters in fm; $r_c = 1.25$ fm for the proton potential and $r_c = 1.30$ fm for the deuteron potentials

\[ b) \quad \] The value of this parameter is reported to be 0.80 in ref. 26 but was later corrected by the authors to be 0.86.

\[ c) \quad \] For the ground state of $^{58}$Ni

\[ d) \quad \] Potential D3 is the same as D2 except for a 4.5\% reduction of $V$

This potential is rather close to the Becchetti-Greenlees 25\) potential except for the diffuseness of the spin-orbit part.

We tried several deuteron potentials. The first one (D1) is the global Lohr-Haeberli 26\) potential obtained from elastic scattering of polarized and unpolarized deuterons on nuclides with $A > 40$ at energies from 8 to 13 MeV, which is just the region of our experiment.

The adiabatic deuteron potential (D2) has been constructed from the Becchetti-Greenlees 25\) potentials for protons and neutrons according to the prescription of Satchler 27\) (Chapter 3, section 3.6). In constructing the spin-orbit potential we replaced the geometry parameters of the BG potential by those of our proton potential, since the latter was determined by a polarization experiment.

Out of the present study arose the need for two more deuteron potentials. One of these (D3) is the same as D2 but for a reduction
of 4.5% of the real well-depth and is left out of table 4.3. The other one (D4), which gave the best results, differs from D2 by a 7.5% reduction of the real well-depth and by an increase of the imaginary potential. This increase is 50% for the ground state and about 10% at 5 MeV excitation energy. The curves displayed in the figs. 4.3-4.11 have been calculated using deuteron potential D4, unless otherwise mentioned.

Non-locality corrections have been applied using the Gaussian form of the correction factor with $\beta(p) = 0.85$ fm and $\beta(d) = 0.54$ fm. The cross sections rise about 3% by these corrections and show slightly more pronounced diffraction patterns. The finite-range and non-locality corrections for the proton and the deuteron together raise the cross sections by about 5%.

4.4 Discussion

4.4.1 The particle-states $3/2^-(0.00)$, $5/2^-(0.77)$ and $1/2^-(1.11)$

One-neutron pick-up preceded by inelastic scattering (fig. 4.2) yields cross sections which are at least one order of magnitude smaller than those obtained by pick-up only. Therefore all levels which are strongly populated by the $(p,d)$ reaction, are mainly reached by a one-step transition, while the two-step transition may dominate a weakly-populated level.

For the strong particle-states at 0.00, 0.77 and 1.11 MeV the addition of the two-step transitions introduces a small but significant change of the calculated cross sections and analysing powers (fig. 4.3, solid and dashed lines). All cross section curves but one in fig. 4.3 were normalized to the first maximum in order to facilitate the comparison of the shapes.

Calculations with different sets of spectroscopic amplitudes (SD0, SD1, KB1) yield almost identical results except for the magnitudes of the cross sections. Only for the $1/2^-$ state there is a slight difference in shape when KB1 is used instead of SD0 or SD1 (fig. 4.3).

The experimental data are described rather well by the calculations apart from the cross section of the $1/2^-$ state which is falling off too slowly. This does not appear to be a general problem, because in the corresponding $^{56}\text{Fe}(p,d)^{55}\text{Fe}(1/2^-)$ reaction (chapter 5) the discrepancy
Fig. 1.3 Cross sections and analyzing powers for the transitions to the particle states of $^{57}$Ni. The solid lines represent the complete calculations, normalization factors are displayed only for the 5/2\(^{-}\) state. The dashed-dotted curves for the 3/2\(^{-}\) state have been calculated using the opposite sign for the deformation parameter $\beta$. 

34
between the calculated and measured cross sections is considerably smaller.

The inclusion of the two-step processes does not always improve the fit to the data as can be seen from the cross section of the $\frac{1}{2}^-$ state and the analysing power of the $\frac{3}{2}^-$ state, which are better described by the one-step calculations (fig. 4.3).

The first maximum of the $\frac{3}{2}^-$ and $\frac{1}{2}^-$ states is decreased by about 10% after adding the two-step processes, which means that the spectroscopic factors extracted from a DWBA-analysis have to be increased by about 10%. The magnitude of this correction is rather insensitive to the optical deuteron-potential and to the set of spectroscopic amplitudes (SD0, SD1, KB1). There is no need for a correction of the spectroscopic factor of the $\frac{5}{2}^-$ state. So the interference between the one-step and the two-step processes at the first maximum is destructive for all three states. The interference angle is about 120° for the $\frac{3}{2}^-$ and $\frac{1}{2}^-$ states and 100° for the $\frac{5}{2}^-$ state.

The separation-energy (SE) prescription for the bound neutron (section 4.3.2) generally gives minor differences in the results compared to the effective-energy (EE) prescription. Yet we preferred the latter, because in that case the cross sections of the $\frac{5}{2}^-$ and $\frac{1}{2}^-$ states are falling off more rapidly, which can be understood from the fact that the neutron is less bound when using EE. All curves displayed in the figures of this chapter are calculated with the effective-energy prescription.

Table 4.4

Spectroscopic factors of the one-step transitions from the ground state of $^{58}$Ni to the particle states of $^{57}$Ni

<table>
<thead>
<tr>
<th>$J^\pi(R_x)$</th>
<th>theoretical</th>
<th>experimental$^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SDO</td>
<td>SD1</td>
</tr>
<tr>
<td>$\frac{3}{2}^-$ (0.00)</td>
<td>1.23</td>
<td>0.79</td>
</tr>
<tr>
<td>$\frac{5}{2}^-$ (0.77)</td>
<td>0.62</td>
<td>0.88</td>
</tr>
<tr>
<td>$\frac{1}{2}^-$ (1.11)</td>
<td>0.16</td>
<td>0.17</td>
</tr>
<tr>
<td>$\Sigma c^2$</td>
<td>2.01</td>
<td>1.64</td>
</tr>
</tbody>
</table>

$^a$ determined by using potential D4
In table 4.4 we give the experimental spectroscopic factors, which are corrected as described above, together with the predictions of the nuclear-structure calculations SDO, SDI and KB1. None of these calculations gives the correct magnitude of all the experimental C^2S-factors. Yet we can say that the surface-delta interaction (SDO and SDI) is generally closer to the mark than the Kuo-Brown interaction. The summed strength is best described by the calculations which allow for one-hole states (SDI and KB1).

4.4.2 The 7/2⁻ states

The cross sections of the 5/2⁻ state (fig. 4.3) and the 7/2⁻ states (figs. 4.4 and 4.5) show an unmistakable j-dependence which is only slightly reproduced by the calculations. This j-dependence, which has already been noticed by Sherr et al. 18), cannot be explained by two-step processes, because all investigated 7/2⁻ states show this effect to the same extent. Johnson and Santos 28) found the inclusion of the deuteron D-state to be largely responsible for this effect, although Delic and Robson 29) claim that taking into account "exact finite-range" cancels the D-state effects almost completely. It may also be that a more careful treatment of the neutron form factor 19-24), as discussed in section 4.3.2, will shed light on the problem.

The structure calculations SDI and KB1 do not predict a sizable two-step contribution for the 7/2⁻ states, so we only applied a one-step DWBA-analysis in these cases and this resulted in a reasonable fit to the analysing powers. It is obvious from the j-effect of the analysing powers that the levels at 4.23, 4.58 and 7.12 MeV are 7/2⁻ states.

The spectroscopic factors are given in table 4.5. They show that the structure calculations do not reproduce the fragmentation of the T=1/2 states.

Following the study of Fortier and Gaëtes 7) we assume that the strength of the T=3/2 transitions is mainly concentrated in the states at 5.23 and 7.12 MeV. Then according to the following sum rule 30)

\[
\sum_{T=3/2} C^2S(7/2^-) = \frac{\pi(7/2^-)}{2T_A + 1} (4.4)
\]

where \(\pi(7/2^-)\) is the average number of f7/2 protons and \(T_A=1\) is the
Fig. 4.4 Cross sections and analysing powers for the transitions to the $7/2^-$ states at 2.58, 5.12 and 7.12 MeV. The dashed curves have been calculated assuming a $5/2^-$ final state.
Fig. 4.6 Cross sections and analysing powers for the transitions to the $7/2^-$ states at 8.76, 4.78 and 4.86 MeV. The dashed curves have been calculated assuming a $5/2^-$ final state.
Table 4.5
C²S-factors of the one-step transitions from the ground state of $^{58}\text{Ni}$ to the $7/2^-$ states of $^{57}\text{Ni}$

<table>
<thead>
<tr>
<th>$l^b)$</th>
<th>$E_x$</th>
<th>$C²S$</th>
<th>$C²S(T=3/2)$</th>
<th>$C²S(T=1/2)$</th>
<th>$E_x$</th>
<th>$E_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/2</td>
<td>5.23</td>
<td>1.68</td>
<td>1.68</td>
<td>1.98</td>
<td>2.25</td>
<td>2.25</td>
</tr>
<tr>
<td></td>
<td>7.12</td>
<td>0.10</td>
<td>0.53</td>
<td>2.11</td>
<td>0.56</td>
<td>0.67</td>
</tr>
</tbody>
</table>

Theoretical

<table>
<thead>
<tr>
<th>$l^b)$</th>
<th>$E_x$</th>
<th>$E_x$</th>
<th>$E_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2</td>
<td>2.58</td>
<td>3.60</td>
<td>3.00</td>
</tr>
<tr>
<td></td>
<td>3.23</td>
<td>0.56</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>3.37</td>
<td>0.11</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>4.23</td>
<td>0.21</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>4.58</td>
<td>0.22</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Experimental$^a)$

<table>
<thead>
<tr>
<th>$l^b)$</th>
<th>$E_x$</th>
<th>$E_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4.16</td>
<td>3.88</td>
</tr>
</tbody>
</table>

$^a)$ determined by using potential D4

$^b)$ isospin of the final state

isospin of the target nucleus, the summed strength should not exceed the value 2.67. We used this limit in determining the potential D4 (section 4.4.3).

4.4.3 The optical deuteron-potential

We took some trouble in finding an acceptable deuteron potential. In this section we describe how this was done.

The Lohr-Haeberli potential (D1) yields cross sections which fall off too slowly except for the $7/2^-$ states (fig. 4.6). This is remedied by using the adiabatic potential (D2), but then the fit to the analysing power of the $7/2^-$ state at 2.58 MeV is quite bad (fig. 4.6). Starting from the adiabatic potential one has to reduce the strength of the real part with 5 to 10% in order to obtain an acceptable fit to this analysing power (fig. 4.7). Such a reduction has also been proposed for other reasons by Satchler 27) and Blankert 31) and is
possibly due to the neglect of exchange effects in the construction of the adiabatic potential.

It seems that this reduction has to be accompanied by a simultaneous increase of the absorption potential, because a reduction of the real potential alone causes a fall-off of the cross section of the $5/2^-$ state, which is too slow. This is demonstrated by the calculations using the D3 and D4 potentials. The applied reduction is 4.5% and 7.5% for D3 and D4, respectively, while in addition for the D4 potential the absorption potential is increased by about 35% (fig. 4.7).

An increase of that size, however, causes the spectroscopic factors of the $7/2^-$ states at 5.23 and 7.12 MeV to be so large that the sum-rule limit of 2.67 is largely exceeded. We solved this problem
by assuming a linear energy-dependence for the percentage of increase (the higher $E_x$, the lower the percentage; see table 4.3).

Eventually we fixed the parameters of D4 by taking the sum of the spectroscopic strengths of the states at 5.23 and 7.12 MeV approximately equal to the sum-rule limit and by fitting as well as possible the shapes of the cross sections and the analysing powers of the particle states at 0.00, 0.77 and 1.11 MeV and the hole states at 2.58 and 5.12 MeV.

The increase of the surface absorption turned out to be almost equivalent to an increase of the volume absorption with slightly better fits in the former case.

Generally there is not so much difference between the results obtained with D3 and D4 (see for instance the $7/2^-$ state in fig. 4.7).
Table 4.6

Potential dependence of the $C^2S$-factors $a)$

<table>
<thead>
<tr>
<th>$J^\pi(E_x)$</th>
<th>D2</th>
<th>D3</th>
<th>D4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3/2^- (0.00)$</td>
<td>0.82</td>
<td>0.80</td>
<td>0.91</td>
</tr>
<tr>
<td>$5/2^- (0.77)$</td>
<td>0.51</td>
<td>0.43</td>
<td>0.53</td>
</tr>
<tr>
<td>$1/2^- (1.11)$</td>
<td>0.12</td>
<td>0.11</td>
<td>0.12</td>
</tr>
<tr>
<td>$E C^2S$</td>
<td>1.45</td>
<td>1.34</td>
<td>1.56</td>
</tr>
<tr>
<td>$7/2^- (2.58)$</td>
<td>3.1</td>
<td>2.8</td>
<td>3.0</td>
</tr>
<tr>
<td>$7/2^- (5.23)$</td>
<td>2.75</td>
<td>2.3</td>
<td>2.25</td>
</tr>
</tbody>
</table>

$a)$ determined using the effective-energy prescription (EE)

This is also true for the spectroscopic factors which vary at most 20\% (table 4.6). Unless otherwise stated, all curves displayed in the figures have been calculated using D4.

Finally we like to remark that the finite-range (LRH) and non-locality corrections (for proton and deuteron) mimic to a certain extent an increase of the absorption, so that the need for such an increase is more urgent if one does not apply these corrections.

4.4.4 The $3/2^-$ states

Apart from the ground state we found four more $3/2^-$ states. Although their cross sections show the characteristic forward peak, they fall off much more slowly than the one-step DWBA curves (figs. 4.8 and 4.9, dashed lines) for which reason Edwards et al. $^3$ label these as unassigned levels.

Fortier and Galès $^7$, however, do not meet this problem in their analysis of the $^{58}$Ni($t,\alpha$) reaction leading to these states. Since the resolution of this experiment is much better (20 keV) than ours (100 keV) or the one of Edwards et al. (75 keV), we checked for possible unresolved multiplets. According to the results of ref. $^7$ only one of the four states is an unresolved doublet, namely the one at 4.95 MeV which is not resolved from the $7/2^-$ state at 4.89 MeV. The fit to the cross section indeed improves somewhat by including the $7/2^-$ state, while the two calculated analysing powers are very similar and
describe the data very well (fig. 4.9).

Cross sections originating from a two-step transition tend to be rather flat and featureless (e.g. 5/2^- state, fig. 4.3). The cross sections of the remaining three states are too large, however, to be explained by two-step processes exclusively and we are forced to allow for a one-step component too.

The only sizable two-step contributions arise from inelastic excitation to the 2^+_1 state of $^{58}$Ni followed by the pick-up of a 7/2^- neutron. Moreover, leaving aside the three particle-states, the structure calculations SD1 and KB1 generate only spectroscopic strength from the 2^+_1 state to states of $^{57}$Ni through 7/2^- transfer. Therefore we performed calculations involving such a two-step transition for the states at 3.85, 4.46 and 5.95 MeV. The C^2S-factors
Fig. 4.9  One-step calculations for the transitions to the 3/2^- states at 4.85 and 6.95 MeV compared with a one- plus two-step calculation (8.95 MeV) and a one-step calculation for the transition to an unresolved doublet (4.85 MeV).

are given in table 4.7. The C^2S_2 strength was chosen to be 18 times larger than the C^2S_0 strength.

The fits to the cross sections are considerably improved by the inclusion of the two-step processes (figs. 4.8 and 4.9, dashed lines), while the C^2S_0-factors are unaffected, because the cross sections are completely dominated by the one-step processes at forward angles. So it seems that in spite of the large C^2S_2-factors a pure one-step analysis is sufficient to determine C^2S_0.

Finally, although the analysing powers are not well described or lack statistics, we can tentatively assign a J=3/2 value from the behaviour around 15°.
4.4.5 The states at 3.71 and 5.78 MeV

From the very flat structure of the cross section at 3.71 MeV we get no indication about the $l$-value, a result already reported by Edwards et al. 3). The $^{58}\text{Ni}(p,d)$ reaction at 121 MeV 9), however, shows a clear $l=3$ behaviour for this state, to which Portier and Galés 7) and Anderson et al. 9) tentatively assign a $J$-value of 7/2. We think, on the contrary, that this value has to be 5/2. In analogy with the 3/2$^-$ states discussed above we added a strong two-step process (table 4.7) and this improved the fit to in particular the analysing power (fig. 4.10). We were not able to obtain an equally good fit assuming a 7/2$^-$ final state. The one-step DWBA calculations of the analysing power already seem to indicate a 5/2$^-$ value (fig. 4.10).

The state at 5.78 MeV is, according to ref. 7), a multiplet of 7/2$^-$ states. This is confirmed by the shape of the analysing power (fig. 4.10). The shape of the cross section, however, deviates strongly from a 7/2$^-$ state, a fact already noticed before 3,9). We investigated the possibility of the presence of an unresolved 3/2$^-$ state (fig. 4.10, solid line), but found no real improvement.

Table 4.7

<table>
<thead>
<tr>
<th>$J^-$</th>
<th>$E_x$</th>
<th>$C^2S(0^+\rightarrow J^-)$</th>
<th>$C^2S(2^+\rightarrow J^-)$ b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5/2$^-$</td>
<td>3.71</td>
<td>0.05</td>
<td>3.2</td>
</tr>
<tr>
<td>3/2$^-$</td>
<td>3.85</td>
<td>0.04</td>
<td>0.77</td>
</tr>
<tr>
<td>3/2$^-$</td>
<td>4.46</td>
<td>0.02</td>
<td>0.32</td>
</tr>
<tr>
<td>3/2$^-$</td>
<td>6.95</td>
<td>0.06</td>
<td>1.10</td>
</tr>
</tbody>
</table>

a) determined using potential D4 and the EE prescription
b) via 7/2$^-$ transfer; relative sign of $CS^2(0^+)$ and $CS^2(2^+)$ negative in all cases

45
Fig. 4.10 Cross sections and analysing powers for the transitions to the states at 3.71 MeV (comparison of $5/2^-$ and $7/2^-$ states) and at 5.78 MeV, for which the results for an unresolved ($7/2^-$, $5/2^-$) doublet are also shown.

4.4.6 The $l=0$ and $l=2$ states

The $1/2^+$ state at 5.50 MeV is reasonably well described by a one-step DWBA calculation and taking account of the unresolved $3/2^-$ state at 5.67 MeV brings even more agreement with the experimental data (fig. 4.11).

The behaviour at forward angles of the analysing power of the $1=2$ state at 6.01 MeV favours a j-value of $3/2$ as can be seen from a comparison with the one-step DWBA calculations (fig. 4.11). The overall fit to both the cross section and the analysing power, though, is rather bad. Including a two-step process via the $3^-$ of
Fig. 4.11 One-step calculations for the transitions to the $1/2^+$ state at 5.58 MeV and the $3/2^+$ state at 6.01 MeV. The results for an unresolved $(1/2^+, 3/2^-)$ doublet are also shown.

$^{58}$Ni or assuming an unresolved $7/2^-$ or $5/2^+$ state did not improve this situation.

4.4.7 Sum rules

In table 4.8 we have summarized the $C^2 S$-factors obtained with the effective-energy (EE) and separation-energy (SE) prescription for the bound neutron. It should be remembered that the "previous results" are not corrected for two-step contributions.

The summed $C^2 S$-factors are related to the average numbers of protons and neutrons in the various shell-model orbits through the
Table 4.8  
Results of the $^{58}\text{Ni}(p,d)^{57}\text{Ni}$ reaction

<table>
<thead>
<tr>
<th>$E_k$</th>
<th>1</th>
<th>$J^P$</th>
<th>$E_x$</th>
<th>EE</th>
<th>SE</th>
<th>FG</th>
<th>NDS</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>1</td>
<td>$3/2^-$</td>
<td>12.20</td>
<td>0.91</td>
<td>0.91</td>
<td>1.13</td>
<td>0.83</td>
</tr>
<tr>
<td>0.77</td>
<td>3</td>
<td>$5/2^-$</td>
<td>11.43</td>
<td>0.53</td>
<td>0.68</td>
<td>1.14</td>
<td>0.54</td>
</tr>
<tr>
<td>1.11</td>
<td>1</td>
<td>$1/2^-$</td>
<td>11.09</td>
<td>0.12</td>
<td>0.19</td>
<td>0.22</td>
<td>0.16</td>
</tr>
<tr>
<td>2.58</td>
<td>3</td>
<td>$7/2^-$</td>
<td>14.78</td>
<td>3.0</td>
<td>3.0</td>
<td>3.3</td>
<td>2.8</td>
</tr>
<tr>
<td>3.23</td>
<td>3</td>
<td>$7/2^-$</td>
<td>14.78</td>
<td>0.50</td>
<td>0.54</td>
<td>0.61</td>
<td>0.39</td>
</tr>
<tr>
<td>3.37</td>
<td>3</td>
<td>$7/2^-$</td>
<td>14.78</td>
<td>0.11</td>
<td>0.12</td>
<td>0.11</td>
<td>0.15</td>
</tr>
<tr>
<td>3.71</td>
<td>3</td>
<td>$(5/2^+)$</td>
<td>11.43</td>
<td>0.05</td>
<td>0.09</td>
<td>0.16</td>
<td>0.16</td>
</tr>
<tr>
<td>3.85</td>
<td>1</td>
<td>$(3/2^-)$</td>
<td>12.20</td>
<td>0.04</td>
<td>0.07</td>
<td>0.09</td>
<td>0.09</td>
</tr>
<tr>
<td>4.22</td>
<td>3</td>
<td>$7/2^-$</td>
<td>14.78</td>
<td>0.21</td>
<td>0.26</td>
<td>0.34</td>
<td>0.22</td>
</tr>
<tr>
<td>4.46</td>
<td>1</td>
<td>$(1/2^-)$</td>
<td>12.20</td>
<td>0.02</td>
<td>0.04</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>4.56</td>
<td>3</td>
<td>$7/2^-$</td>
<td>14.78</td>
<td>0.22</td>
<td>0.23</td>
<td>0.25</td>
<td>0.23</td>
</tr>
<tr>
<td>4.96</td>
<td>1</td>
<td>$3/2^-$</td>
<td>12.20</td>
<td>0.04</td>
<td>0.04</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>5.23</td>
<td>3</td>
<td>$7/2^-$</td>
<td>17.43</td>
<td>2.25</td>
<td>2.25</td>
<td>2.29</td>
<td>2.0</td>
</tr>
<tr>
<td>5.58</td>
<td>0</td>
<td>$1/2^+$</td>
<td>17.70</td>
<td>1.0</td>
<td>1.0</td>
<td>0.70</td>
<td>0.60</td>
</tr>
<tr>
<td>5.79</td>
<td>3</td>
<td>$(7/2^-)$</td>
<td>17.43</td>
<td>0.32</td>
<td>0.34</td>
<td>0.36</td>
<td>0.26</td>
</tr>
<tr>
<td>6.01</td>
<td>2</td>
<td>$(3/2^+)$</td>
<td>18.21</td>
<td>0.96</td>
<td>0.96</td>
<td>0.98</td>
<td>0.92</td>
</tr>
<tr>
<td>6.95</td>
<td>1</td>
<td>$(3/2^-)$</td>
<td>12.20</td>
<td>0.04</td>
<td>0.06</td>
<td>0.11</td>
<td>0.11</td>
</tr>
<tr>
<td>7.12</td>
<td>3</td>
<td>$7/2^-$</td>
<td>17.43</td>
<td>0.56</td>
<td>0.57</td>
<td>0.72</td>
<td>0.44</td>
</tr>
</tbody>
</table>

a) from the study of Portier and Caldas 7)
b) from the survey of Auble 6)
c) based on $J^P=7/2^-$
d) multiplets

sum rules of Fronch and Macfarlane 30), which are also given in chapter 3.

In Table 4.9 we compare our results with the predictions of the SDI and KBI calculations 14) for the ground state of $^{59}\text{Ni}$. The BE results are somewhat lower than the theoretical limits 2.0 and 8.0, but thus is not surprising, since we have not found all levels. If we take the remaining strength from ref. 7) the BE results become 1.9 and 7.7, respectively, which is very close to the limits indeed.
The average number of neutrons in a $\frac{3}{2}^-$ orbit seems to be somewhat underestimated by the structure calculations. The determination of $C^2S$-factors is hampered by many uncertainties and although the sum-rule results give us some confidence, we estimate the error of these factors for the strongly-excited states to be 20%.

Throughout this chapter we applied a non-locality correction to the neutron form-factor. One has to raise all $C^2S$-factors with about 35% if this correction is left out. It is obvious that the sum-rule limits are violated in that case. This remains true if one uses the D2 and D3 potentials. So the only alternative then is to change these potentials (for instance the energy dependence).

Finally we like to remark that the summed strength for $7/2^-$ transfer from the $2_1^+$ state to $^{57}\text{Ni}$ (table 4.7), which is equal to 5.4, does not exceed the theoretical limit of 7.96 given by the structure calculations.

4.5 Conclusions

In this chapter we studied the reaction $^{58}\text{Ni}(p,d)$ leading to eighteen levels of $^{57}\text{Ni}$. Four of these levels showed a featureless shape of the cross section, which could be interpreted to a large extent by a strongly competing two-step process. This process consists of inelastic proton-scattering to the $2_1^+$ state of $^{58}\text{Ni}$ followed by the pick-up of a $7/2^-$ neutron.

The description of the analysing powers, which were also rather featureless, was generally not improved, though, by the inclusion of
the two-step processes.

It turned out that the usual determination of the S-factors of the ground state and the $1/2^-$ state at 1.11 MeV by a one-step DWBA analysis yield values which are 10% too low. The sum rules of the spectroscopic factors appeared to be satisfied rather well, while the structure calculations that we used, failed to describe the fragmentation of the $7/2^-$ levels.

We also determined some new spin assignments using the $j$-effect of the analysing power. The problem of the $j$-effect of the $l=3$ cross sections remained unsolved and was tentatively ascribed to the neglect of the D-state of the deuterons.

Finally, we were forced to reduce the strength of the real part of the adiabatic deuteron potential in order to obtain satisfying fits to the data and we got some indications that this reduction has to be accompanied by an increase of the absorption potential.
REFERENCES

4 P.D. Kunz, Univ. of Colorado, unpublished.
14 A.G.M. van Hees, private communication.


5.1 Introduction

Since the nuclide $^{56}\text{Fe}$ contains the same number of neutrons as $^{58}\text{Ni}$, one can expect a certain analogy between the one-neutron pick-up reactions $^{56}\text{Fe}(p,d)$ and $^{58}\text{Ni}(p,d)$. Consequently our treatment of the reaction $^{56}\text{Fe}(p,d)$ will be quite similar to that of $^{58}\text{Ni}(p,d)$ as described in the foregoing chapter.

Only for the most prominent transitions to $^{55}\text{Fe}$ there are spectroscopic factors available from the literature. The values given for these spectroscopic factors are plainly conflicting in some cases or largely exceed the sum-rule limits. Using the experience acquired through analysing the reaction $^{58}\text{Ni}(p,d)$ we hope to remove some of these inconsistencies, the more so as the sum rules generally seemed to be fulfilled quite well in that analysis.

Until now there has been reported only one $^{56}\text{Fe}(p,d)$ experiment, in which a polarized-proton beam was used. This experiment was performed by Hosono et al. at a much higher proton energy (65 MeV) than ours and the analysis was restricted to the transitions to the three lowest levels of $^{55}\text{Fe}$. So we may be able to make some new spin assignments for states of $^{55}\text{Fe}$ by exploiting the $j$-effect of the analysing power.

The results of our investigations are discussed in section 5.4, while the experimental and calculational details are given in the sections 5.2 and 5.3, respectively.

5.2 The experiment

We analysed the transitions to nine levels of $^{55}\text{Fe}$ (table 5.4). These are shown in the energy spectrum of fig. 5.1 together with some other levels, which were excited too weakly to give reliable angular distributions.

As can be seen from this spectrum the peak shape is slightly asymmetrical: there is a small hump on the right flank of the peaks. Most of the spectra showed this feature but sometimes the hump appeared on the left flank or had about the same magnitude as the
main peak. We did not investigate the origin of these peculiar peak shapes, since they hardly affected the results of the analyses of the spectra.

A good definition of the peak shape is important when resolving doublets. Actually we met this situation for the states at 1.32 and 1.41 MeV, since the energy resolution was about 100 keV FWHM. The peak-fitting computer code (see chapter 2) that we used provides for peak shapes such as described above. We determined the peak parameters from the low-lying single peaks and used these parameters together with precise excitation-energies from the literature in order to resolve the states at 1.32 and 1.41 MeV.

The target consisted of a self-supporting foil of isotopically enriched (99.9%) $^{56}$Fe with a thickness of about 1 mg/cm$^2$. The intensity and degree of polarization of the proton beam were about 10 nA and 80%, respectively. The data were taken at 11°, from 15° to 33° in steps of 3° and from 33° to 81° in steps of 6°.

We did not determine the absolute normalization of the cross sections independently but interpolated linearly between data obtained
at 17.0 \(^4\), 22.3 \(^5\) and 28 MeV \(^6\), we felt confident in using such an interpolation because these data virtually were lying on a straight line.

The reaction \(^{56}\text{Fe}(p,d)\) has been measured before \(^3,4,7\) but except for the experiment of Hosono et al. \(^3\) (65 MeV proton energy) in all cases with an unpolarized proton beam.

5.3 The calculations

5.3.1 Reaction model

We applied the same kind of coupling scheme to the reaction \(^{56}\text{Fe}(p,d)\) as we did in the case of \(^{58}\text{Ni}(p,d)\), i.e. inelastic proton-scattering followed by neutron pick-up (two-step DWBA) in addition to neutron pick-up from the ground state of the target nucleus (one-step DWBA). We included two \(^2^+\) states in the proton channel: the \(^2_1^+\) at 0.85 MeV with \(\beta_{21} = 0.24\) and the \(^2_2^+\) at 2.66 MeV with \(\beta_{22} = 0.10\) (scheme A, fig. 5.2). The inelastic scattering was described macroscopically (see formula 4.1). The magnitudes of the deformation

![Diagram](image_url)

Fig. 5.2 Coupling schemes for the coupled-channels calculations
\(J^m = 1/2^-, 3/2^-, 5/2^-, 7/2^-\)
parameters have been determined before by our group \(^8\) from scattering of polarized protons by the nucleus \(^{56}\)Fe.

We were able to reduce the computing time with a factor of two by combining both \(^2^+\) states in an effective \(^2^+\) state at 1.12 MeV (scheme B, fig. 5.2). The excitation energy of this state was calculated according to the following formula:

\[
x(2^+) = (E_{2_1}^2 + E_{2_2}^2)^{-1} (E_{2_1}^2 B_{2_1}^2 + E_{2_2}^2 B_{2_2}^2)
\]  

(5.1)

The deformation parameter \(h_2\) was chosen to be -0.24. The spectroscopic amplitudes for neutron pick-up from the second \(^2^+\) state were multiplied

<table>
<thead>
<tr>
<th>Process</th>
<th>(2J(2j)^a)</th>
<th>SD1 (^\text{c)})</th>
<th>KB1</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0^+\rightarrow j)</td>
<td>(5 ,(5))</td>
<td>-0.91</td>
<td>-0.42</td>
</tr>
<tr>
<td></td>
<td>(3 ,(3))</td>
<td>0.65</td>
<td>1.06</td>
</tr>
<tr>
<td></td>
<td>(1 ,(1))</td>
<td>0.66</td>
<td>0.64</td>
</tr>
<tr>
<td></td>
<td>(7_1 ,(7)^b)</td>
<td>0.60</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(7_3 ,(7)^b)</td>
<td>0.73</td>
<td></td>
</tr>
<tr>
<td>(2^+\rightarrow j)</td>
<td>(5 ,(5))</td>
<td>-0.37</td>
<td>-0.18</td>
</tr>
<tr>
<td></td>
<td>(5 ,(3))</td>
<td>0.16</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>(5 ,(1))</td>
<td>0.46</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>(5 ,(7))</td>
<td>0.14</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>(3 ,(5))</td>
<td>0.14</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(3 ,(3))</td>
<td>0.76</td>
<td>0.54</td>
</tr>
<tr>
<td></td>
<td>(3 ,(1))</td>
<td>-0.38</td>
<td>-0.45</td>
</tr>
<tr>
<td></td>
<td>(3 ,(7))</td>
<td>0.17</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>(1 ,(5))</td>
<td>-0.35</td>
<td>-0.07</td>
</tr>
<tr>
<td></td>
<td>(1 ,(3))</td>
<td>0.33</td>
<td>0.56</td>
</tr>
<tr>
<td></td>
<td>(7_1 ,(3))</td>
<td>0.72</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(7_1 ,(7))</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(7_3 ,(3))</td>
<td>-0.24</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(7_3 ,(7))</td>
<td>0.30</td>
<td></td>
</tr>
</tbody>
</table>

\(^a\) The transferred angular momentum is indicated by \(j\)

\(^b\) The states at 1.32 MeV and 2.94 MeV: \(7_1\) and \(7_3\), respectively

\(^c\) Transitions to \(7/2^-\) states not calculated with SD1
by -0.10, divided by -0.24 and then added to the corresponding spectroscopic amplitudes for the first $2^+$ state.

The results of using the scheme A or B were compared in several cases and were found to yield negligible differences. Apart from the saving of computer time there is another advantage of the simplified coupling scheme B, namely that it can be transferred unchanged to the multistep calculations for the $^{56}\text{Fe}(p,t)$ reaction (chapter 6), where the maximum number of channel couplings allowed by the computer code CHUCK2 would be largely exceeded when using scheme A.

The states at 1.41, 4.51, 4.28 and 7.79 MeV were analysed with one-step DWBA calculations only. The spectroscopic amplitudes for the one- plus two-step transitions to the five remaining states are given in table 5.1. These amplitudes are based upon the same kind of shell-model calculations as described in section 4.3.1 of the foregoing chapter.

Two sets of spectroscopic amplitudes, indicated by SDI (Surface delta) and KBI (renormalized Kuo-Brown) were computed by Van Hees \cite{9} from the work of Vennink and Glaudemans \cite{10}. The configuration space used for these calculations is described in the following sentences. The nuclide $^{56}\text{Fe}$ has two protons less than $^{58}\text{Ni}$ and these are represented by two $f7/2$ holes in the $^{56}\text{Ni}$ core. The configuration space is constructed with $p3/2$, $p1/2$ and $f5/2$ particles and $f7/2$ holes with the only restriction that the maximum number of $f7/2$ holes is three for both $^{56}\text{Fe}$ and $^{55}\text{Fe}$. Since two of the holes are proton holes this means that one additional proton or neutron hole is allowed.

5.3.2 Optical-model and other DWBA parameters

The neutron form-factor was calculated using the well-depth procedure with effective binding-energies (section 4.3.2), which can be found in table 5.4 in the column headed by $\Delta E$. The zero-range normalization constant $D_0(pd)$ was taken equal to $-122.5$ MeV fm$^{3/2}$. We applied the same non-locality and finite-range corrections (table 5.2) as we did for the reaction $^{58}\text{Ni}(p,d)$ (chapter 4).

The optical-model parameters are given in table 5.2. The proton potential has been determined before by our group \cite{8} in the way described in section 4.3.3. We tried the same optical deuteron-potentials as we did for the reaction $^{58}\text{Ni}(p,d)$ with essentially the same results.
In this chapter we therefore will only present results obtained with the adiabatic potentials D2 and D4 as described in chapter 4. The potential D4 generally gave the best description of the cross sections and analysing powers and was used in all calculations unless stated otherwise.

Table 5.2
Optical-model parameters

<table>
<thead>
<tr>
<th>Name</th>
<th>$V$</th>
<th>$r_C$</th>
<th>$a_0$</th>
<th>$W_V$</th>
<th>$W_d$</th>
<th>$r_I$</th>
<th>$a_I$</th>
<th>$V_{so}$</th>
<th>$r_{so}$</th>
<th>$a_{so}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>51.8</td>
<td>1.15</td>
<td>0.76</td>
<td>2.50</td>
<td>6.17</td>
<td>1.34</td>
<td>0.55</td>
<td>5.5</td>
<td>1.04</td>
<td>0.54</td>
</tr>
<tr>
<td>D2</td>
<td>105.0b</td>
<td>1.17</td>
<td>0.78</td>
<td></td>
<td>10.0 b</td>
<td>1.26</td>
<td>0.61</td>
<td>11.8</td>
<td>1.04</td>
<td>0.56</td>
</tr>
<tr>
<td>D4</td>
<td>97.9b</td>
<td>1.17</td>
<td>0.78</td>
<td></td>
<td>9.6  b</td>
<td>1.32</td>
<td>0.57</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n</td>
<td></td>
<td>1.25</td>
<td>0.65</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Energy dependence of the deuteron potentials

$\Delta V/\Delta E_x$  $\Delta W_d/\Delta E_x$

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>D2</td>
<td>0.311</td>
<td>0.118</td>
</tr>
<tr>
<td>D4</td>
<td>0.290</td>
<td>-0.626</td>
</tr>
</tbody>
</table>

Non-locality parameter $\delta$

<table>
<thead>
<tr>
<th>$p$</th>
<th>$n$</th>
<th>$d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>0.85</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Finito-range parameter $R$

<table>
<thead>
<tr>
<th>$(p,d)$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.69</td>
</tr>
</tbody>
</table>

---

a) $V$, $V_{so}$, $W_V$ and $W_d$ in MeV, the other parameters in fm; $r_C = 1.25$ fm for the proton potential and $r_C = 1.30$ fm for the deuteron potentials

b) For the ground state of $^{55}$Fe

c) $V$ is found from the well-depth procedure

d) Factor multiplying the Thomas-term is 25
5.4 Discussion

5.4.1 The negative-parity states

The transitions to the particle-states at 0.00, 0.41 and 0.93 MeV yield cross-section shapes and analysing powers (fig. 5.3) which are very similar to those obtained for the corresponding transitions to $^{57}$Ni. This result can be understood from the fact that in both cases the particle-states are strongly excited and consequently are reached dominantly by the one-step transfer.

The analysing powers of the transitions to the $1/2^-$ states, though, are somewhat different beyond 45°. This difference is mainly due to the presence of a larger two-step contribution to the transition to the $1/2^-$ state of $^{57}$Ni as the DWBA curves show (figs. 4.3 and 5.3).

Although the experimental cross sections of the transitions to both $1/2^-$ states are almost identical in shape, the best fit is obtained in the case of $^{55}$Fe. This must be ascribed to the sensitivity of the calculation to the deuteron energy (or the Q-value), which differs about 1.7 MeV (for the $3/2^-$ and $5/2^-$ states the differences are 1.0 and 0.85 MeV, respectively). It is not clear if this implies a weaker energy dependence of the deuteron potential, since we do not know yet the exact influence of neglecting exact finite-range and the deuteron D-state.

The unrealistic sharp peaks of the calculated analysing powers of the $3/2^-$ state (at 90°) and the $1/2^-$ state (at 30°) coincide with minima of the cross sections. It is obvious that instabilities of the calculations will show up first at such places (see also section 5.4.4).

The inclusion of the two-step transitions generally improves the fits to the cross sections and analysing powers of the particle-states, although the difference with the one-step calculation remains small (fig. 5.3).

The analysing power of the first $7/2^-$ state (1.32 MeV) clearly shows a phase shift with respect to that of the second $7/2^-$ state (1.41 MeV) and since the latter state, being strongly excited, is mainly reached by a one-step transition, it is evident that the transition to the first $7/2^-$ state receives a considerable two-step contribution (fig. 5.4). This is confirmed by the shell-model calculations (table 5.1) and is related to the presence of a large
Fig. 5.3 Cross sections and analysing powers for the transitions to the particle states of $^{56}$Fe. The calculated cross sections have been normalized to the forward maximum.
two-hole component in the first 7/2$^-$ state which can of course only be reached from the 2$^+$ state of $^{56}$Fe, whereas the second 7/2$^-$ state is dominated by a three-hole component (two protons and one neutron).

The direction of the phase shift is correctly predicted by the structure calculations (fig. 5.4, 1.32 MeV), although the comparison with the experimental data is obscured by an additional shift between the experimental and calculated analysing powers of the one-step transition (fig. 5.4, 1.41 MeV). The instability of the calculations of the analysing powers near 0° will be discussed in section 5.4.4.

The analysing powers of the transitions to the 7/2$^-$ states at 2.94 and 1.41 MeV are almost identical, so we conclude that the state at 2.94 MeV is mainly reached by a one-step transfer, contrary to the predictions of the shell-model calculations (fig. 5.5).

The state at 7.7 MeV probably is the isobaric analog (IAS) of the 7/2$^-$ (0.126 MeV) state of $^{55}$Mn. The analysing power indicates that

---

**Fig. 5.4** Cross sections and analysing powers for the transitions to the 7/2$^-$ states at 1.32 and 1.41 MeV. The one- plus two-step calculation is based on the amplitudes KBI.
the spin of this state is indeed $7/2$ (fig. 5.5).

The two sets of spectroscopic amplitudes SD1 and KB1 generated identical shapes of the cross sections and analysing powers like they did for the $^{58}$Ni(p,d) transitions. So their relative merits have to be judged from the spectroscopic factors they predict (table 5.3). The Kuo-Brown interaction clearly gives the best results. Therefore all curves displayed in this chapter are calculated using the set of spectroscopic amplitudes KB1.

The relative strength of the second and first $7/2^-$ state when analysed with a one-step transition only (1ST), is about 4.0. This lies closely to the value of 4.4 found by Majumder et al. 11) from the reaction $^{56}$Fe($d$,t) at 12 MeV.

In table 5.4 we compare our results with those obtained by other authors. In all cases we found (sometimes considerably) smaller
Table 5.3

Comparison of experimental and theoretical $C^2 S$-factors

<table>
<thead>
<tr>
<th>$E_x$</th>
<th>$J^\pi$</th>
<th>theoretical</th>
<th>experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>SDL</td>
<td>KBI</td>
</tr>
<tr>
<td>0.00</td>
<td>3/2$^-$</td>
<td>0.42</td>
<td>1.13</td>
</tr>
<tr>
<td>0.41</td>
<td>1/2$^-$</td>
<td>0.44</td>
<td>0.41</td>
</tr>
<tr>
<td>0.93</td>
<td>5/2$^-$</td>
<td>0.84</td>
<td>0.18</td>
</tr>
<tr>
<td>1.32</td>
<td>7/2$^-$</td>
<td>0.04</td>
<td>0.35</td>
</tr>
<tr>
<td>1.41</td>
<td>7/2$^-$</td>
<td>3.72</td>
<td>2.66</td>
</tr>
<tr>
<td>2.94</td>
<td>7/2$^-$</td>
<td>0.41</td>
<td>0.54</td>
</tr>
<tr>
<td>7.79</td>
<td>7/2$^-$ IAS</td>
<td>&lt;1.17</td>
<td>&lt;1.17</td>
</tr>
</tbody>
</table>

$^a$) $C^2 S$ extracted from a one- plus two-step DWBA-analysis.

Table 5.4

Results of the $^{56}$Fe($^3$H,$d$)$^{55}$Fe reaction

<table>
<thead>
<tr>
<th>$E_x$$^a)$</th>
<th>l</th>
<th>$J^\pi$</th>
<th>$E_0$$^a)$</th>
<th>$C^2 S$($0^+$,$J^\pi$)</th>
<th>THS$^b)$</th>
<th>$pd^c)$</th>
<th>$dt^c)$</th>
<th>$^3$He,$\alpha$$^c)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>1</td>
<td>3/2$^-$</td>
<td>11.20</td>
<td>0.69</td>
<td>1.0</td>
<td>1.02</td>
<td>1.12</td>
<td></td>
</tr>
<tr>
<td>0.41</td>
<td>1</td>
<td>1/2$^-$</td>
<td>10.79</td>
<td>0.28</td>
<td>0.42</td>
<td>0.48</td>
<td>0.42</td>
<td></td>
</tr>
<tr>
<td>0.93</td>
<td>3</td>
<td>5/2$^-$</td>
<td>10.27</td>
<td>0.33</td>
<td>0.97</td>
<td>0.54</td>
<td>1.71</td>
<td></td>
</tr>
<tr>
<td>1.32</td>
<td>3</td>
<td>7/2$^-$</td>
<td>12.60</td>
<td>0.41</td>
<td>0.69</td>
<td>0.69</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.41</td>
<td>3</td>
<td>7/2$^-$</td>
<td>12.60</td>
<td>2.41</td>
<td>6.1</td>
<td>3.02</td>
<td>4.65</td>
<td></td>
</tr>
<tr>
<td>2.94</td>
<td>3</td>
<td>7/2$^-$</td>
<td>12.60</td>
<td>0.87</td>
<td></td>
<td></td>
<td>5.04</td>
<td></td>
</tr>
<tr>
<td>4.51$^d)$</td>
<td>0</td>
<td>1/2$^+$</td>
<td>15.70</td>
<td>0.31</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.88$^d)$</td>
<td>2</td>
<td>3/2$^+$</td>
<td>16.07</td>
<td>0.97</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.79$^d)$</td>
<td>3</td>
<td>7/2$^-$</td>
<td>18.99</td>
<td>0.82</td>
<td></td>
<td></td>
<td>4.88</td>
<td></td>
</tr>
</tbody>
</table>

$^a$) $E_x$ and $E_0$ in MeV

$^b$) This experiment

$^c$) From the survey of Kocher $^1$)

$^d$) Given as 4.45, 4.82 and 7.73 MeV, respectively, by Kocher $^1$)
spectroscopic factors and we think these to be more realistic because of their consistency with the sum rules (see section 5.4.3).

5.4.2 The positive-parity states

The angular distributions of the analysing power and the cross section for the state at 4.51 MeV (fig. 5.6) closely resemble those for the $1/2^+$ (5.58 MeV) state of $^{57}$Ni. Therefore a spin-parity assignment of $1/2^+$ is obvious. The DWBA calculation gives a rather poor fit to the data. This is partly caused by the calculation being too sensitive to the Q-value or the deuteron energy, since the transition to the $1/2^+$ state of $^{57}$Ni is described considerably better by the one-step DWBA analysis (fig. 4.11, dashed line). Notice that

![Graphs showing cross sections and analysing powers for transitions](image)

*Fig. 5.6 Cross sections and analysing powers for the transitions to the $1/2^+$ (4.51) and the $3/2^+$ (4.08) states of $^{56}$Fe. The dashed curves have been calculated assuming a $3/2^+$ (4.08) final state.*
the unrealistic peaks of the calculated analysing power coincide again with minima of the cross section.

The shape of the cross section and the analysing power of the transition to the state at 4.88 MeV (fig. 5.6) clearly excludes an \( l=0 \) or \( l=1 \) transfer. We have to choose between \( l=2 \) and \( l=3 \). The analysing power then limits the choice of the \( J^\pi \) values to \( 3/2^+ \) and \( 5/2^- \).

Assuming a \( 5/2^- \) state results in a spectroscopic factor of 0.47 which is improbably large. Moreover Fulmer and McCarthy\(^{12}\) find an \( l=2 \) transfer for the \( ^{54}\text{Fe}(d,p) \) transition to this state. So we feel confident in assigning a \( J^\pi \) value of \( 3/2^+ \) to the state at 4.88 MeV. The one-step DWBA-analysis gives a fair description of both the cross section and the analysing power.

5.4.3 Sum-rule results and comparison with \( ^{58}\text{Ni}(p,d) \)

In table 5.5 we compare the summed \( C^2S \)-strength with the results of the shell-model calculations (KB1, SD1). The total strength for the particle-states is 32\% lower than given by KB1 and SD1. This may be an indication that our values are too low. The more so, since the total strength, which has to be larger than 2.0, seems to be mainly concentrated in the states analysed by us.

The strength of the transitions to the \( 7/2^- \) states is given rather well by the structure calculations. We found about 60\% of the

<table>
<thead>
<tr>
<th>( J^\pi )</th>
<th>( C^2S(0^+\rightarrow J^\pi) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1/2^-+3/2^-+5/2^- )</td>
<td>( 3/2 )</td>
</tr>
<tr>
<td>( 7/2^- )</td>
<td>( 3/2 )</td>
</tr>
<tr>
<td>( 7/2^- )</td>
<td>( 5/2 )</td>
</tr>
<tr>
<td>( 7/2^- )</td>
<td>( 3/2,5/2 )</td>
</tr>
</tbody>
</table>

a) isospin of the final state  
b) this experiment  
c) restricted to the states analysed in this chapter  
d) total \( C^2S \)-strength for \( J^\pi \)
total 7/2⁻ strength. The remaining strength is probably spread over a large number of weakly-excited states.

The ratios of the spectroscopic factors of the transitions to the particle states of ⁵⁵Fe and ⁵⁷Ni are reproduced remarkably well by the KB1 calculations (table 5.6) although the experimental spectroscopic factors are smaller than the calculated ones for the ¹/²⁻ and ³/²⁻ states and larger for the ⁵/²⁻ state.

The total calculated strength for transitions to the particle states is smaller for ⁵⁵Fe than for ⁵⁷Ni. This is confirmed by the experimental results (table 5.6).

Table 5.6
Comparison between C²S-factors of the reactions ⁵⁶Ni(⁵⁺,d) and ⁵⁶Fe(⁵⁺,d)

<table>
<thead>
<tr>
<th>J⁻</th>
<th>Nucleus</th>
<th>C²S</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>exp.</td>
</tr>
<tr>
<td>¹/²⁻</td>
<td>⁵⁵Fe</td>
<td>0.28</td>
</tr>
<tr>
<td></td>
<td>⁵⁷Ni</td>
<td>0.12</td>
</tr>
<tr>
<td>³/²⁻</td>
<td>⁵⁵Fe</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td>⁵⁷Ni</td>
<td>0.91</td>
</tr>
<tr>
<td>⁵/²⁻</td>
<td>⁵⁵Fe</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>⁵⁷Ni</td>
<td>0.53</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>C²S(⁵⁵Fe)/C²S(⁵⁷Ni)</th>
</tr>
</thead>
<tbody>
<tr>
<td>¹/²⁻</td>
</tr>
<tr>
<td>³/²⁻</td>
</tr>
<tr>
<td>⁵/²⁻</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>¹ C²S</th>
</tr>
</thead>
<tbody>
<tr>
<td>⁵⁵Fe</td>
</tr>
<tr>
<td>⁵⁷Ni</td>
</tr>
</tbody>
</table>
5.4.4 The deuteron potential

Fig. 5.7 Comparison of various deuteron potentials for the transitions to the 7/2^- states at 1.11 and 2.94 MeV. The calculated cross sections have been normalized to the forward maximum except for those of the lowest part of the figure.
In fig. 5.7 we show the effect of using the adiabatic deuteron potential D2 instead of the adjusted deuteron potential D4 for the transitions to two 7/2^- states of 55Fe. It is clear from these examples that D4 is to be preferred to D2 for the reaction 56Fe(p,d) too.

It is also clear from this figure that there are three regions of instability for the calculated analysing powers: around 5, 50 and 80 degrees. They coincide approximately with minima of the cross sections. Notice that in the stable regions the fit to the experimental data is quite reasonable.

The instability around 50° depends upon the deuteron energy outside the nucleus (or the Q-value) and is relatively insensitive to the deuteron potential used. The energy of the deuteron inside the nucleus \( E_{\text{in}} \approx E_{\text{out}} - V(d) \), so a change of \( E_{\text{out}} \) (or the Q-value) is more or less equivalent to a change of the real well-depth \( V(d) \) of the deuteron potential.

The influence of \( E_{\text{in}} \) on the unstable regions is shown in fig. 5.7 by three calculations performed for \( E_x = 2.94 \text{ MeV} \) using potential D2 with 0%, 8% and 16% reduction of \( V(d) \).

We feel that the introduction of exact finite-range and D-state deuterons may stabilize the calculated analysing powers and at the same time fill up the minima of the \( l=1 \) cross sections.

The use of the analysing power in assigning the j-value is not seriously affected by the instability problem, because its j-dependence is most evident at the angle where the cross section reaches its main maximum (a stable region).

Finally, we like to remark that because of the mentioned instabilities the magnitudes of the adjustments of the real and imaginary well-depths of the deuteron potential D4 are quite uncertain.

5.5 Conclusions

We investigated the one-neutron pick-up transitions to nine levels of 55Fe by the reaction 56Fe(p,d). All of these transitions were dominated by one-step transfer. The results were in general quite similar to those found for the reaction 58Ni(p,d).
The shell-model calculations based on the renormalized Kuo-Brown interaction yielded a better description of the spectroscopic factors than those based on the surface-delta interaction.

We added some new spectroscopic factors and spin-parity assignments. Our spectroscopic factors turned out to be (sometimes considerably) smaller than those obtained by other authors.

Finally, we found the calculated analysing powers of especially the transitions to $7/2^-$ states to be unstable against variations of the deuteron energy inside and outside the nucleus. Since these instabilities occur at angles where the cross sections have minima, we think that "second-order" effects like exact finite range or the $D$-state of the deuteron may remove this problem.
REFERENCES

1 D.C. Kocher, Nucl. Data Sheets 18 (1976) 463.
9 A.G.M. van Hees, private communication.
CHAPTER 6  \( \text{(p,t) REACTION ON } ^{58}\text{Ni and } ^{56}\text{Fe AT 24.6 MeV} \)

6.1 Introduction

In the analysis of one-neutron transfer reactions interference between different possible processes arises only when going beyond one-step calculations. So interference generally is a small effect as we have seen in the two foregoing chapters.

In two-neutron transfer reactions, however, interference already shows up in one-step calculations. The importance of interference in two-neutron transfer reactions makes the \((p,t)\) process a valuable tool for testing calculated nuclear wave functions. For the Ni and Fe isotopes sophisticated wave functions are available \(^1,^2\) and these were used in our calculations.

The significance of two-step processes like for instance sequential transfer (two successive one-neutron transfers) for "allowed" \(^3-^6\) and even for "forbidden" \(^7-^9\) \((p,t)\) transitions is still a matter of discussion. In our case we included the following processes in the reaction mechanism: simultaneous transfer (one-step), sequential transfer, inelastic scattering following or preceding either simultaneous transfer (two-step) or sequential transfer (three-step).

Since there are indications \(^9,^{10}\) that especially the analysing powers are sensitive to interference between one-step and multistep processes, we focussed our attention on obtaining a good fit to these.

Several authors \(^11,^{12}\) found that if they used the "wrong" relative sign of the sequential and the simultaneous transfers the agreement with the experimental data was improved in particular at forward angles. Therefore we considered this possibility in our analyses too. Because we made the zero-range approximation and neglected non-orthogonality effects \(^13\) our results will probably give no final answer to this problem.

The calculations of the cross sections and analysing powers have been performed with the coupled-reactions code CHUCK2 \(^14\), which had been modified to raise the maximum number of couplings to twenty one.

The results of our investigations are discussed in section 6.4, while the main conclusions are summarized in section 6.5. The reaction model and all numbers relevant to the calculations can be found in section 6.3. The experimental details are given in the next section.
6.2 Experimental Procedures

The \((p,t)\) reactions investigated in this chapter have been measured simultaneously with the \((p,d)\) reactions on \(^{58}\)Ni and \(^{56}\)Fe, which have been dealt with in the chapters 4 and 5. We refer to these chapters for more information on the experimental details.

In the \(^{58}\)Ni experiment the data were taken from \(15^\circ\) to \(80^\circ\) in steps of \(5^\circ\), while for the \(^{56}\)Fe experiment the size of the steps was \(4^\circ\) from \(11^\circ\) to \(15^\circ\), \(2^\circ\) from \(15^\circ\) to \(33^\circ\) and \(6^\circ\) from \(33^\circ\) to \(81^\circ\). The energy resolution was about \(100\) keV FWHM, of which most was due to the energy spread of the proton beam.

In fig. 6.1 we display a typical energy spectrum for the reaction \(^{56}\)Fe\((p,t)\). We have chosen the angle so as to give a good picture of the dependence on the polarization for the ground state of \(^{56}\)Fe. We analysed the transitions to the four lowest states, to wit the \(0^+_1\) (ground state), the \(2^+_1\) (1.41 MeV), the \(0^+_2\) (2.56 MeV) and the \(2^+_2\) (2.96 MeV). For the \(^{58}\)Ni\((p,t)\) reaction we obtained data only for the ground-state to ground-state transition.

To obtain absolute cross sections we used the same normalization factors as deduced in the analyses of the \((p,d)\) reactions. The estimated accuracy of the normalization is \(20\%\) in both cases.

![Energy spectrum of \(^{56}\)Fe\((p,t)\) at \(E_p = 24.6\) MeV for both spin conditions; \(\theta_{lab} = 61^\circ\)](image-url)
There have been other \((p,t)\) experiments on \(^{58}\text{Ni}\) \cite{15-18} and \(^{56}\text{Fe}\) \cite{18,19} before, but at higher energies and not with a polarized proton beam.

6.3 Calculational procedures

6.3.1 Reaction mechanisms

In order to limit computing time and to comply with the maximum number of channels and of channel-couplings in the code CHUCK2 we had to restrict our coupling schemes somewhat. These restrictions depend upon the final state and will be justified in section 6.4. We described the ground-state transitions with the coupling scheme A (fig. 6.2; the \(2^+\) state of \(^{54}\text{Fe}\) is the effective state at 1.12 MeV, see chapter 5), which consists of the one-step transition \((pt)\), the two-step transitions \((pdt)\) and \((pp't)\) and the three-step transition \((pp'dt)\).

\[\begin{align*}
58\text{Ni} & \rightarrow 2^+ \text{ (ground state)} \\
57\text{Ni} & \rightarrow 56\text{Ni} \\
56\text{Fe} & \rightarrow 55\text{Fe} \\
\end{align*}\]

A

\[\begin{align*}
58\text{Fe} & \rightarrow 59\text{Fe} \\
57\text{Fe} & \rightarrow 56\text{Fe} \\
54\text{Fe} & \rightarrow 53\text{Fe} \\
\end{align*}\]

B

\[\begin{align*}
58\text{Fe} & \rightarrow 57\text{Fe} \\
57\text{Fe} & \rightarrow 56\text{Fe} \\
54\text{Fe} & \rightarrow 53\text{Fe} \\
\end{align*}\]

C

Fig. 6.2 Coupling schemes for the coupled-channels calculations

\(J^\pi = 1/2^+, 3/2^-, 5/2^-, 7/2^-\)
For the analysis of the transitions to the $2^+$ states of $^{56}$Fe we replaced the inelastic proton scattering by inelastic triton scattering. Then we have the following transitions: $(pt')$, $(pdt')$, $(pt'')$ and $(pdt'')$. These are summarized in scheme B (fig. 6.2). We applied a two-way coupling to the triton scattering, but we found that in our case the difference with the calculation using a one-way coupling was not significant (less than 3% in magnitude of the cross section). In the case of the $0^+_2$ state we only investigated the $(pt')$ and the $(pdt')$ transitions (scheme C, fig. 6.2).

6.3.2 Simultaneous two-neutron transfer

Since we used the code CHUCK2 we had to make the zero-range approximation, which means that the interaction is non-zero only, when the proton coincides with the center of mass of the two transferred neutrons. After this simplification the transition amplitude for the transfer of two neutrons with a given angular momentum $J$ becomes:

$$T(J) \propto N_0(pt) \sum_{\lambda} S^{1/2}(\rho \lambda; J) \int \frac{d^3r}{r} \chi_{\rho \lambda}^{(+)}(r) F^J_{\rho \lambda}(r) \chi_{\rho}^{(+)}\left(\frac{\Lambda-2}{\Lambda}r\right)$$

$F^J_{\rho \lambda}$ is the normalized form factor of the neutron pair picked-up from the shell-model orbitals $\rho$ and $\lambda$ and coupled to an angular momentum $J$. The spectroscopic amplitudes $S^{1/2}(\rho \lambda; J)$ follow from nuclear wave functions according to formula (3.24). The coherent summation over the configuration $(\rho \lambda)$ in eq. (6.1) is the origin of the interference effects in the one-step calculation as mentioned in section 6.1.

The zero-range normalization constant $N_0(pt)$ has been taken equal to $-1560$ MeV fm$^3$ which corresponds to $D_0(pt) = -500$ MeV fm$^{3/2}$.

The form factors $F^J_{\rho \lambda}$ have been generated by the Bayman-Kallio 20) method with the root-mean-square radius of the triton equal to 1.7 fm. The neutron bound-state wave functions were constructed according to the well-depth procedure using half the two-neutron separation energy of the target nucleus as the binding energy.

We assumed an inert $^{56}\text{Ni}$ core in the $^{58}\text{Ni}$ experiment and derived the spectroscopic amplitudes $S^{1/2}(\rho \lambda; J)$ from the wave functions of Koops and Glaudemans (given in ref. 11), which are based on a residual interaction of the surface-delta form. The two transfer-neutrons are occupying the $2p3/2, 1f5/2$ and $2p1/2$ shells.
Since the amount of \((1f7/2)^{-2}\) configurations in the ground state of \(^{56}\text{Ni}\) is probably small, we think that neglecting the \(1f7/2\) shell will not change very much.

The \(^{56}\text{Fe}\) and \(^{54}\text{Fe}\) wave functions have been taken from the work of Vennink and Glaudemans\(^2\). They allowed \(1p-1h\) excitations of neutrons and protons with the hole being in the \(1f7/2\) shell. Together with two \(1f7/2\) proton holes this results e.g. for \(^{54}\text{Fe}\) in \((f7/2)^{-n-2}(p3/2,f5/2,p1/2)^n\) configurations with \(n = 0\) and \(n = 1\). We have chosen the wave functions computed with the modified Kuo-Brown.

**Table 6.1**

<table>
<thead>
<tr>
<th>Process</th>
<th>((2j_1,2j_2))</th>
<th>(s^b)</th>
<th>Process</th>
<th>((2j_1,2j_2))</th>
<th>(s^b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{56}\text{Fe})</td>
<td>(0^+\rightarrow 0^+_1)</td>
<td>(0)</td>
<td>(^{56}\text{Ni})</td>
<td>(0^+\rightarrow 0^+_0)</td>
<td>(0)</td>
</tr>
<tr>
<td>((5,5))</td>
<td>-0.21</td>
<td>((3,3))</td>
<td>-0.58</td>
<td>((3,3))</td>
<td>-0.78</td>
</tr>
<tr>
<td>((1,1))</td>
<td>-0.29</td>
<td>((5,5))</td>
<td>-0.30</td>
<td>((5,5))</td>
<td>-0.32</td>
</tr>
<tr>
<td>(^{54}\text{Fe})</td>
<td>(2^+\rightarrow 0^+_1)</td>
<td>(2)</td>
<td>(^{56}\text{Ni})</td>
<td>(2^+\rightarrow 0^+_2)</td>
<td>(2)</td>
</tr>
<tr>
<td>((5,5))</td>
<td>0.16</td>
<td>((5,5))</td>
<td>-0.23</td>
<td>((5,1))</td>
<td>0.36</td>
</tr>
<tr>
<td>((5,3))</td>
<td>-0.09</td>
<td>((3,3))</td>
<td>-0.71</td>
<td>((3,1))</td>
<td>-0.71</td>
</tr>
<tr>
<td>((3,3))</td>
<td>0.13</td>
<td>(3,1)</td>
<td>-0.42</td>
<td>(3,1)</td>
<td>-0.42</td>
</tr>
<tr>
<td>((3,7))</td>
<td>-0.16</td>
<td>(3,7)</td>
<td>-0.16</td>
<td>(3,7)</td>
<td>-0.16</td>
</tr>
<tr>
<td>(^{56}\text{Fe})</td>
<td>(0^+\rightarrow 2^+_1)</td>
<td>(2)</td>
<td>(^{56}\text{Fe})</td>
<td>(0^+\rightarrow 2^+_2)</td>
<td>(2)</td>
</tr>
<tr>
<td>((5,5))</td>
<td>0.09</td>
<td>((5,5))</td>
<td>-0.02</td>
<td>((5,5))</td>
<td>-0.02</td>
</tr>
<tr>
<td>((5,3))</td>
<td>-0.01</td>
<td>((5,1))</td>
<td>0.09</td>
<td>((5,1))</td>
<td>0.09</td>
</tr>
<tr>
<td>((3,3))</td>
<td>-0.07</td>
<td>((3,3))</td>
<td>-0.15</td>
<td>((3,3))</td>
<td>-0.15</td>
</tr>
<tr>
<td>((3,1))</td>
<td>0.21</td>
<td>((3,1))</td>
<td>-0.17</td>
<td>((3,1))</td>
<td>-0.17</td>
</tr>
<tr>
<td>((3,7))</td>
<td>0.25</td>
<td>((3,7))</td>
<td>0.43</td>
<td>((3,7))</td>
<td>0.43</td>
</tr>
<tr>
<td>(^{56}\text{Fe})</td>
<td>(2^+\rightarrow 2^+_1)</td>
<td>(0)</td>
<td>(^{56}\text{Fe})</td>
<td>(2^+\rightarrow 2^+_2)</td>
<td>(0)</td>
</tr>
<tr>
<td>((3,3))</td>
<td>0.38</td>
<td>((3,7))</td>
<td>0.10</td>
<td>((3,1))</td>
<td>0.20</td>
</tr>
<tr>
<td>((1,1))</td>
<td>0.21</td>
<td>(3,7)</td>
<td>0.11</td>
<td>(3,7)</td>
<td>0.11</td>
</tr>
<tr>
<td>((1,1))</td>
<td>0.21</td>
<td>(3,7)</td>
<td>0.11</td>
<td>(3,7)</td>
<td>0.11</td>
</tr>
</tbody>
</table>

\(a\) we took the effective \(2^+\) state of \(^{56}\text{Fe}\) at 1.12 MeV (chapter 5)

\(b\) we only considered the amplitudes originating from the first \(2^+\) state of \(^{56}\text{Fe}\) at 0.85 MeV

75
residual interaction (KB1), because this interaction generally gives better results \(^2\) for the Fe isotopes than the surface-delta interaction (SD1). We have calculated the spectroscopic amplitudes according to formula 15.48 of ref. \(^1\).

Care has been taken to bring the resulting spectroscopic amplitudes into line with the phase conventions of CHUCK2. We refer to chapter 3 for details. The spectroscopic amplitudes are given in table 6.1 as they entered CHUCK2 (apart from the normalization constant \(N_0\)).

6.3.3 Sequential transfer

In our calculation the sequential transfer proceeds through the following intermediate states: \(1/2^+, 3/2^+, 5/2^-\) for \(57\text{Ni}\) and in addition for \(55\text{Fe}\) also the three lowest \(7/2^-\) states. The transfer strength is almost exhausted by these states, so we feel confident in neglecting higher states.

The one-neutron form factors were computed with the well-depth procedure. The binding energy of a neutron in orbit \(p\) is given by

\[
E_b(p) = S_n(A) + E(p) = S_n(A-1) + E_p(A-1) + E_x(A-1)
\]

for the \((pd)\) step and

\[
E_b(p) = S_n(A-1) + E_p(A-1) + E_x(A-1)
\]

for the \((dt)\) step. \(S_n\) is the one-neutron separation energy, \(A\) the mass number of the target nucleus and \(E_x\) the excitation energy. The minus and the plus sign are chosen for particle and hole states, respectively.

We applied the finite-range correction of the interaction in the local-energy approximation in all one-neutron transfer-channels. For this we used the Hulthén form with the value of 0.69 fm for the finite-range parameter.

The one-neutron transfer amplitudes have to be normalized with empirical constants, because the finite range of the interaction has not been treated in an exact way. We took the widely used values

\[
N_0(pd) = +122.5 \text{ MeV fm}^{3/2} \quad \text{and} \quad N_0(dt) = -225.0 \text{ MeV fm}^{3/2}
\]

These correspond to \(D_0(pd) = -122.5 \text{ MeV fm}^{3/2}\) and \(D_0(dt) = -184.0 \text{ MeV fm}^{3/2}\). For convenience' sake we have gathered all normalization constants in table 6.31.

The spectroscopic amplitudes for the one-neutron transfers have been computed \(^2\) with the nuclear wave functions of section 6.3.2 extended to the intermediate nuclides \(57\text{Ni}\) and \(55\text{Fe}\). We renormalized the spectroscopic amplitudes of the \((pd)\)-step in order to reproduce the experimental cross sections of the \((p,d)\) reactions (see chapters 4 and 5). The relative magnitude of the \((p',d)\) and \((pd)\) amplitudes

76
Table 6.2
Spectroscopic amplitudes for sequential two-neutron transfer

<table>
<thead>
<tr>
<th>Process</th>
<th>$2J(2j)^a$</th>
<th>$S^b$</th>
<th>Process</th>
<th>$2J(2j)$</th>
<th>$S^b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fe $^{0+}$</td>
<td>5 (5)</td>
<td>-0.58</td>
<td>Ni $^{0+}$</td>
<td>5 (5)</td>
<td>-0.73</td>
</tr>
<tr>
<td></td>
<td>3 (3)</td>
<td>0.83</td>
<td></td>
<td>3 (3)</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>1 (1)</td>
<td>0.53</td>
<td></td>
<td>1 (1)</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>7 (7)</td>
<td>1.53</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fe $^{2+}$</td>
<td>5 (5)</td>
<td>-0.24</td>
<td>Ni $^{2+}$</td>
<td>5 (5)</td>
<td>-0.39</td>
</tr>
<tr>
<td></td>
<td>5 (3)</td>
<td>0.16</td>
<td></td>
<td>5 (3)</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>5 (1)</td>
<td>0.36</td>
<td></td>
<td>5 (1)</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td>5 (7)</td>
<td>0.13</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 (5)</td>
<td>0.01</td>
<td></td>
<td>3 (5)</td>
<td>0.28</td>
</tr>
<tr>
<td></td>
<td>3 (3)</td>
<td>0.42</td>
<td></td>
<td>3 (3)</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td>3 (1)</td>
<td>-0.35</td>
<td></td>
<td>3 (1)</td>
<td>-0.36</td>
</tr>
<tr>
<td></td>
<td>3 (7)</td>
<td>0.10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 (5)</td>
<td>-0.06</td>
<td></td>
<td>1 (5)</td>
<td>-0.31</td>
</tr>
<tr>
<td></td>
<td>1 (3)</td>
<td>0.48</td>
<td></td>
<td>1 (3)</td>
<td>0.37</td>
</tr>
<tr>
<td>Fe $^{3+}$</td>
<td>5 (5)</td>
<td>0.70</td>
<td>Ni $^{3+}$</td>
<td>5 (5)</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>3 (3)</td>
<td>-0.74</td>
<td></td>
<td>3 (3)</td>
<td>-1.00</td>
</tr>
<tr>
<td></td>
<td>1 (1)</td>
<td>-0.61</td>
<td></td>
<td>1 (1)</td>
<td>-1.00</td>
</tr>
<tr>
<td>Fe $^{1+}$</td>
<td>5 (5)</td>
<td>-0.28</td>
<td>Fe $^{1+}$</td>
<td>5 (5)</td>
<td>-0.14</td>
</tr>
<tr>
<td></td>
<td>5 (3)</td>
<td>-0.27</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5 (1)</td>
<td>0.37</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 (5)</td>
<td>-0.14</td>
<td></td>
<td>3 (5)</td>
<td>-0.20</td>
</tr>
<tr>
<td></td>
<td>3 (3)</td>
<td>0.41</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 (1)</td>
<td>0.35</td>
<td></td>
<td>3 (1)</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>3 (7)</td>
<td>0.17</td>
<td></td>
<td>3 (7)</td>
<td>0.56</td>
</tr>
<tr>
<td></td>
<td>1 (5)</td>
<td>-0.30</td>
<td></td>
<td>1 (5)</td>
<td>-0.14</td>
</tr>
<tr>
<td></td>
<td>1 (3)</td>
<td>-0.64</td>
<td></td>
<td>1 (3)</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>7 (3)</td>
<td>0.40</td>
<td></td>
<td>7 (3)</td>
<td>0.28</td>
</tr>
</tbody>
</table>

$a)$ the transferred angular momentum is indicated by j

$b)$ we took the effective $2^+$ state of $^{56}$Fe at 1.12 MeV (chapter 5)
Table 6.3

Zero-range normalization constants

<table>
<thead>
<tr>
<th>Reaction</th>
<th>$D_0$</th>
<th>$N_0$ (^a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p,d)</td>
<td>-122.5</td>
<td>+122.5</td>
</tr>
<tr>
<td>(d,t)</td>
<td>-184.0</td>
<td>-225.0</td>
</tr>
<tr>
<td>(p,t)</td>
<td>-500.0</td>
<td>-1560.0</td>
</tr>
</tbody>
</table>

\(^a\) used in the code CHUCK2; \(N_0\) includes light-ion spectroscopic amplitudes

leading to the same final state in the deuteron channel remained unchanged. Since we performed calculations with and without inclusion of inelastic proton scattering, we employed two slightly different renormalizations. The corresponding spectroscopic amplitudes are given in Table 6.2.

We remark that for the ground-state to ground-state transitions, the relation $\sqrt{2} \frac{\Gamma_{1/2}(p\alpha;0)}{\Gamma_{1/2}(p)} = \frac{\Gamma_{1/2}(d)}{\Gamma_{1/2}(t)}$ is approximately valid. This relation enables one to check the signs of the spectroscopic amplitudes.

6.3.4 Inelastic Scattering

We described the inelastic proton and triton scattering using the form factor given by eq. 4.1. We found no determination of the deformation parameters $\beta_2$ from $^{54}$Fe(t,t') experiments in the literature. Therefore we used the values obtained in our own laboratory from $^{54}$Fe(p,p') experiments \(^{22}\). The $\beta_2$ parameters for inelastic proton scattering were taken also from experiments \(^{22}\) done before by our group. For the proton channel the actual values are $\beta_2 = -0.22$ and $\beta_2 = -0.24$ for $^{58}$Ni (1.45 MeV) and $^{56}$Fe (1.12 MeV), respectively (see chapter 4 and 5). For the triton channel the values $\beta_2 = +0.15$ and $\beta_2 = +0.13$ have been used. The inelastic triton scattering is not applicable to the Ni experiment, because we measured only the transition to the ground state of $^{56}$Ni.

The signs of the deformation parameters depend upon the phases of the nuclear wave functions and can only be determined by a microscopic calculation of the form factor. Because of the interference between the various processes a reversal of the sign of $\beta_2$ considerably
changes the results of a multistep calculation. Our procedure was simply to try both signs and then to compare the results of the calculations with the experimental data. In this way we were always able to make an unambiguous choice.

6.3.5 Optical potentials

The optical-model parameters for the triton potentials are listed in Table 6.4. The proton and deuteron potentials have been taken from the chapters 4 and 5. We chose the modified adiabatic deuteron potential D4, because this potential describes the (p,d) experiments best.

We tried three different triton potentials indicated by TF, TE and T6. TF is the result of a fixed-geometry fit \(^{23}\) to elastic-scattering data of 20 MeV tritons over a broad range of the mass number A. Since there is no TF potential for \(^{56}\)Ni, we have extrapolated the TF potential of \(^{54}\)Fe to \(^{56}\)Ni. We have also applied an energy extrapolation to the TF potentials. Both extrapolations were done according to the energy and symmetry dependence of the global triton potential of Becchetti and Greenlees \(^{24}\). The TB potential was obtained from a global fit of (t,p) reaction data with angular momentum transfers \(l=0\) and \(l=2\) using a triton energy of 12 MeV \(^{25}\), which is close to the experimental data.

Table 6.4

<table>
<thead>
<tr>
<th>Name</th>
<th>Nucleus</th>
<th>(V)</th>
<th>(r_C)</th>
<th>(a_C)</th>
<th>(V_r)</th>
<th>(W_d)</th>
<th>(x_I)</th>
<th>(a_I)</th>
<th>(r_C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TB</td>
<td>(^{56})Ni, (^{54})Fe</td>
<td>155.0</td>
<td>1.24</td>
<td>0.70</td>
<td>30.0</td>
<td>0</td>
<td>1.37</td>
<td>0.70</td>
<td>1.25</td>
</tr>
<tr>
<td>TF</td>
<td>(^{56})Ni</td>
<td>144.2</td>
<td>1.24</td>
<td>0.69</td>
<td>34.1</td>
<td>0</td>
<td>1.43</td>
<td>0.87</td>
<td>1.25</td>
</tr>
<tr>
<td></td>
<td>(^{54})Fe</td>
<td>143.6</td>
<td>1.24</td>
<td>0.69</td>
<td>29.4</td>
<td>0</td>
<td>1.43</td>
<td>0.87</td>
<td>1.25</td>
</tr>
<tr>
<td>T6</td>
<td>(^{56})Ni</td>
<td>137.6</td>
<td>1.10</td>
<td>0.85</td>
<td>32.3</td>
<td>1.31</td>
<td>0.75</td>
<td>1.25</td>
<td></td>
</tr>
</tbody>
</table>

Non-locality parameter \(\beta\) | p | d | t | n
---|---|---|---|---
\(\beta\) | 0.85 | 0.54 | 0.25 | 0.85

Finite-range parameter \(R\) | (p,d) | (d,t)
---|---|---
\(R\) | 0.69 | 0.69

a) \(V, W_v\) and \(W_d\) in MeV; the other parameters in fm.
The potential was tested only in the $^{56}$Ni experiment. This potential has been applied with some success to Ti($p,t$) reactions $^{26}$ and originated from fits of elastic triton and $^3$He scattering data for energies of 20 MeV to 80 MeV. Since TF generally gave the best results we used this potential in most of our calculations. We did not account for differences in triton energy in the final channel. In other words we used the same triton potential for all final states.

In the calculations where a two-way coupling is present in the $(t,t')$ process one must change the imaginary triton potential somewhat $^{27}$. We applied an estimated reduction of 5% to the imaginary potential and we found that the change of cross sections and analysing powers were negligible.

The merits of the non-locality corrections have already been discussed in chapter 4. All calculations described in this chapter were done with the non-locality corrections. The corresponding parameters are given in table 6.4.

### 6.1.6 Normalization parameters

A comparison of exact finite-range and zero-range calculations of one-step $(pt)$ cross sections $^{3,28}$ shows that the shapes of the angular distributions are very similar and that the main difference lies in the magnitude of the cross section. The exact finite-range calculations often underestimate the experimental cross sections considerably. Although the effect of finite range on the magnitude of $(pd)$ and $(dt)$ cross sections is negligible, the same is not true for the two-step process $(pd_t)$ as a recent study by Burch et al. $^3$ showed. Again the only difference between the finite-range and zero-range calculations is the magnitude of the cross section. It was also found $^3$ that the relative phase angle between the $(pt)$ and $(pd_t)$ transition does not change significantly by the introduction of exact finite range. So if we renormalize the zero-range $(pt)$ amplitude with a factor $\lambda$ and the zero-range $(pd_t)$ amplitude with a factor $\mu$, we can simulate an exact finite-range calculation.

However, we do not know the magnitudes of $\lambda$ and $\mu$. Burch et al. $^3$ obtained $\lambda = 0.69$ and $\mu = 0.65$ for the reaction $^{62}$Ni($p,t$)$^{60}$Ni($0^+_1$) at 27 MeV and $\lambda = 0.60$ and $\mu = 0.39$ for the same reaction at 40 MeV. It is important to realize that these results depend much upon the details of the assumed internal deuteron and triton wave functions.
We decided to determine the ratio of the normalization parameters \( \lambda \) and \( \mu \) from a best fit to the analysing power of the \((p,t)\) reaction to the ground state. Next a normalization of the calculated cross section to the experimental one fixed the magnitudes of \( \lambda \) and \( \mu \). Then these values have also been used in the calculations for the excited states of the final nucleus.

6.4 Discussion

6.4.1 Determination of \( \lambda \) and \( \mu \)

The analysing powers and the cross sections of the \((p,t)\) reactions leading to the ground states of \( ^{56}\text{Ni} \) and \( ^{54}\text{Fe} \) have been computed from transition amplitudes of the following form (scheme A, fig. 6.2):

\[
T = \lambda \{T(p^t) + T(p^p't)\} + \mu \{T(p^d) + T(p^p'dt)\}
\]

The results of the calculations with \((\lambda,\mu) = (1,0)\) and \((\lambda,\mu) = (0,1)\) for \( ^{56}\text{Fe} \) and \( ^{58}\text{Ni} \) using the potential TF are shown in fig. 6.3. First we like to remark that the analysing powers do not behave like the derivatives of the cross sections at forward angles as they do for instance in the reaction \( ^{116}\text{Sn}(p,t) \). A conspicuous difference between the experimental analysing powers of \( ^{55}\text{Fe} \) and \( ^{58}\text{Ni} \) is the deepness of the minimum at 25°. This is reproduced qualitatively by the calculations. We see that the simultaneous transfer gives larger cross sections than the sequential transfer and that for both transfer modes the cross sections are of the same order of magnitude as the experimental results.

One now can determine the ratio of \( \lambda \) and \( \mu \) from fitting the angular distribution of the cross section or from applying the same procedure to the analysing power. We examined both ways. Since we were interested in the relative sign \(^{1)} \) of the simultaneous- and sequential-transfer amplitudes, we also looked into negative values of \( \lambda/\mu \). Using the chi-square values \( \chi^2_c \) and \( \chi^2_A \) defined by (3.11a) and (3.11b) to indicate the quality of the fits we found in general two minima for both \( \chi^2_c \) and \( \chi^2_A \), one for \( \lambda/\mu \) positive and one for \( \lambda/\mu \) negative. In a consistent calculation the positions of the minima of \( \chi^2_c \) and \( \chi^2_A \) should coincide. This is more or less the case for \( ^{58}\text{Ni} \), but not for \( ^{56}\text{Fe} \). The sensitivity to variations of \( \lambda/\mu \) is of the same order of magnitude for the cross section and the analysing power, but the minima are more
pronounced for $\chi^2_A$ than for $\chi^2_0$. Moreover the value of $\lambda/\mu$ which gives a minimum $\chi^2_0$ depends upon the normalization procedure used for the cross section. So we decided to choose the best fit of the analysing power as the criterion for determining $\lambda/\mu$.

The magnitudes of $\lambda$ and $\mu$ have to be found from the absolute cross sections. As a rule one normalizes the cross section to the first maximum. For an $I=0$ transfer this maximum is situated at $0^\circ$. Consequently, one has to extrapolate the experimental data to this angle. For $^{58}$Ni, however, the experimental points at forward angles are too scarce for a reliable determination of the maximum at $0^\circ$. Since we wanted to compare the results for $^{58}$Ni and $^{56}$Fe, we decided to normalize the cross sections to the second maximum in both cases.

Actually for $^{56}$Fe we chose the experimental point at $51^\circ$ as a reference, while for $^{58}$Ni we applied a best-fit procedure from $30^\circ$ to
80°, because in the latter case the maximum at 50° is less prominent. We will return to this normalization problem later on in this section.

The resulting values of $\lambda$ and $\mu$ are given in Table 6.5. Since there is no minimum of $\chi^2_A$ for $\lambda/\mu$ positive when the triton potential T6 is used, we give the results obtained from the minimum of $\chi^2_0$ instead.

The ratio of $\lambda$ and $\mu$ tends to be independent of the triton potential. This property is a consequence of the two following facts. Firstly, the main difference between the various triton potentials lies in the magnitudes of the cross sections with about the same relative normalization factors for simultaneous and sequential transfer, whereas the analysing powers and the shapes of the cross sections are quite similar. Secondly, the interference angle between the simultaneous- and sequential-transfer amplitudes is rather insensitive to the triton potential. This angle is generally about 90° when both amplitudes are large.

If we look at the chi-square values in Table 6.5 it is clear that the best fits are obtained with TF. But the fact that T6 consistently gives cross sections which are too large is even more important for

<table>
<thead>
<tr>
<th>Nucleus Potential</th>
<th>$\lambda/\mu$</th>
<th>$\lambda$</th>
<th>$\mu$</th>
<th>$\chi^2_A/N$</th>
<th>$\chi^2_0/N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{54}_{Fe}$ TF</td>
<td>0.76</td>
<td>0.81</td>
<td>1.065</td>
<td>46</td>
<td>440</td>
</tr>
<tr>
<td></td>
<td>-0.11</td>
<td>-0.17</td>
<td>1.59</td>
<td>41</td>
<td>718</td>
</tr>
<tr>
<td></td>
<td>0.69</td>
<td>0.64</td>
<td>0.93</td>
<td>39</td>
<td>609</td>
</tr>
<tr>
<td></td>
<td>-0.10</td>
<td>-0.13</td>
<td>1.30</td>
<td>49</td>
<td>771</td>
</tr>
<tr>
<td>$^{56}_{Ni}$ TF</td>
<td>0.67</td>
<td>0.53</td>
<td>0.78</td>
<td>25</td>
<td>85</td>
</tr>
<tr>
<td></td>
<td>-0.22</td>
<td>-0.22</td>
<td>0.97</td>
<td>13</td>
<td>154</td>
</tr>
<tr>
<td></td>
<td>0.67</td>
<td>0.40</td>
<td>0.59</td>
<td>29</td>
<td>99</td>
</tr>
<tr>
<td></td>
<td>-0.21</td>
<td>-0.15</td>
<td>0.72</td>
<td>18</td>
<td>187</td>
</tr>
<tr>
<td></td>
<td>0.81</td>
<td>0.69</td>
<td>0.85</td>
<td>48</td>
<td>85</td>
</tr>
<tr>
<td></td>
<td>-0.20</td>
<td>-0.20</td>
<td>1.04</td>
<td>16</td>
<td>196</td>
</tr>
</tbody>
</table>

a) N is the number of experimental points
b) obtained from the absolute minimum of $\chi^2_0$
Table 6.6

Ratio of calculated and experimental cross sections
for the ground-state transitions

<table>
<thead>
<tr>
<th>Solution</th>
<th>Process (^{a)} )</th>
<th>(54 \text{Fe} )</th>
<th>(56 \text{Ni} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C_+ )</td>
<td>( p + p' d t )</td>
<td>0.19</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>( p + p' t )</td>
<td>0.91</td>
<td>0.39</td>
</tr>
<tr>
<td></td>
<td>( p t )</td>
<td>0.91</td>
<td>0.30</td>
</tr>
<tr>
<td>(C_- )</td>
<td>( p + p' d t )</td>
<td>0.86</td>
<td>0.82</td>
</tr>
<tr>
<td></td>
<td>( p + p' t )</td>
<td>0.04</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>( p t )</td>
<td>0.04</td>
<td>0.06</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>TF</th>
<th>TR</th>
<th>TF</th>
<th>TB</th>
</tr>
</thead>
<tbody>
<tr>
<td>(54 \text{Fe} )</td>
<td>0.45</td>
<td>0.45</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(56 \text{Ni} )</td>
<td>0.54</td>
<td>0.36</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^{a)}\) the complete \(C_+\) and \(C_-\) calculations have 1.0 as ratio

the choice of the triton potential. For example, the \(58 \text{Ni} \) cross section calculated from the \((pt)\) amplitude only \((\lambda=1)\) is 2.4 times larger than the experimental cross section (for TF this factor is 1.2). Therefore all curves presented in the figures have been computed with the TF potential.

In table 6.6 we have listed the magnitudes of the cross sections of the separate processes relative to the experimental cross sections at \(50^\circ\) (Fe) and \(80^\circ\) (Ni). The sequential transfer is the dominant process for the \(C_- (\lambda/\mu < 0) \) solutions, while the simultaneous transfer contributes less than 10\%. In contrast, the sequential- and simultaneous-transfer cross sections for the \(C_+ (\lambda/\mu > 0) \) solutions are of the same order of magnitude. Comparing the results of \(56 \text{Fe} \) and \(58 \text{Ni} \) in this table, the constancy of the sequential contribution is noteworthy.

From fig. 6.4 one can see that the analysing power of \(58 \text{Ni} \) is fitted better by the solution with \(\lambda/\mu < 0 \) \((C_-)\), because the dip at \(25^\circ\) is described well by \(C_-\). Concerning the analysing power of \(56 \text{Fe} \) (fig. 6.4), it is clear that the main difference between \(C_-\) and \(C_+\) is again around \(25^\circ\), but now both fits are equally good. The cross section of \(58 \text{Ni} \) clearly favours the \(C_+\) solution, which is related to the fact that the absolute minimum of \(\chi^2\) is reached for \(\lambda/\mu\) positive. The \(C_-\) cross section of \(56 \text{Fe} \) is too low at forward angles. This introduces
Fig. 6.4 The complete calculations (coupling scheme \( \lambda \)) with \( \lambda/\mu \) positive (\( C_+ \)) and \( \lambda/\mu \) negative (\( C_- \)) for the ground state transitions.

an ambiguity in the normalization and hence in the values of \( \lambda \) and \( \mu \).
The extrapolated value of the cross section of \(^{56}\text{Fe} \) at 0° is about 6.0 mb/sr and if one normalizes to this value then the \( C_+ \) cross section has to be multiplied by the factor 1.7. As mentioned before we can not give such a reliable estimate of the cross section at 0° in the case of \(^{58}\text{Ni} \).

We will discuss two reasons why one usually prefers the first maximum when normalizing the calculated cross sections. Firstly, second-order effects are thought to be less important for this maximum than for the remaining part of the angular distribution, which is nicely illustrated in fig. 6.5 by the \((pt)\) and \((pt)+(pp't)\) processes. Now, the relative contributions to the cross sections at 0° are rather similar to the results at 50° and 80° (table 6.6) and since these
results show that for the C_+ solutions all processes give comparable contributions there is no clear-cut first-order process in these cases.

Secondly, the shape of the angular distribution beyond the first maximum is often sensitive to the optical potentials. For the C_+ solutions, however, these shapes are remarkably similar for all three triton potentials.

Finally, Hashimoto and Kawai 10) showed that a non-orthogonality correction applied to a two-step calculation of $^{40}$Ca(p,t)$^{46}$Ca(0^+_1) at 40 MeV considerably reduced the magnitude of the cross section at 0°, although at 27 MeV this correction is much smaller. Anyway, we will quote the consequences of normalizing to the first maximum at the appropriate places.

Comparing the magnitudes of $\mu$ (table 6.5) calculated with the same potential and sign of $\lambda/\mu$ we find considerably larger values for $^{56}$Fe than for $^{58}$Ni. This is connected to the fact that for $^{58}$Ni the (dt) amplitudes (table 6.2) are about a factor 1.5 larger than for $^{56}$Fe. Hence the sequential transfer with $\mu=1$ gives larger cross sections for $^{58}$Ni than for $^{56}$Fe (fig. 6.3), whereas the magnitudes of the experimental cross sections are quite close to one another. Since the amplitudes for $^{58}$Ni have been obtained from a simpler configuration space (no 7/2^- holes) than the one used for $^{56}$Fe (up till one 7/2^- hole) we think that the values of $\mu$ extracted from the $^{56}$Fe analysis are more realistic.

According to Burch et al. 3) finite-range corrections result in a reduction of the cross section for both types of transfer, so we expect $\lambda$ and $\mu$ to be smaller than one. The C_+ solution for $^{56}$Fe (TP) gives $\mu=1.59$ and the C_+ solution $\mu=1.065$. This is another point in favour of the latter possibility.

The relative C_+ cross section of the (pt+pp't) transfer is about two times larger for $^{56}$Fe than for $^{58}$Ni (table 6.6). This has been caused by the different values of $\lambda$. Because the C_+ solution seems more reliable for $^{58}$Ni than for $^{56}$Fe (better fits, see table 6.5, and coinciding minima of $x_0^2$ and $x_A^2$) we think that the (pt+pp't) cross section is closer to 0.39 than to 0.91.

Summarizing we can say that $\lambda/\mu$ is 0.7±0.2 and 0.5±0.2 for $^{56}$Fe and $^{58}$Ni (after correction for the (dt) amplitudes), respectively. The error of 0.2 was obtained from comparing $\chi^2_{\text{min}}$ with 1.5 $\chi^2_{\text{min}}$. 

56
These results are rather independent of the triton potential and they are not affected by the uncertainties in the normalization since they follow from the analysing power. They should be compared with $\lambda/\mu = 1.1$ obtained from finite-range calculations $^3$ of the cross section of $^6\text{Li}(p,t)^4\text{He}(^1_p)$ at 47 MeV. Keeping $\nu$ fixed at 1.0 Feix et al. $^6$ found $\lambda/\mu = 0.6$ for $^{116}\text{Sn}(p,t)^{114}\text{Sn}(^1_p)$ at 25 MeV.

We wondered if the fact that the $^{56}\text{Fe}$ data are not described as well as the Ni data (table 6.5 and fig. 6.4) could be connected with the presence of a four-hole component in the wave function of the ground state of $^{56}\text{Fe}$ (because of mixing with the $0^+_2$ state). Therefore we added the transfer of two $7/2^-$ neutrons with an adjustable strength to the original calculation. A new search was made with $\lambda$ and $\mu$ free.

![Diagram](image)

Fig. 6.5 Calculations with and without inelastic proton scattering for the ground-state transition of $^{56}\text{Ni}$; $\lambda/\mu$ is positive in all cases; see caption of fig. 6.4 for $C_\mu$. 
Yet the best fit of the analysing power was obtained with no \((7/2)^{-}\) transfer at all.

We want to finish this subsection with some remarks about the coupling scheme A (fig. 6.2) which we used for the ground-state transitions. Leaving out the inelastic triton scattering only changed the magnitude of the cross section with about \(5\%\), which we verified with the limited calculation \((\text{pt})+(\text{pdt})\). In coupling scheme B, which is used for the \(2^+\) states, we have dropped the inelastic proton scattering. Therefore it is of some importance to look at the effect of this simplification on the ground-state transition. We performed a \((\text{pt})+(\text{pdt})\) calculation for Ni with the same values of \(\lambda\) and \(\mu\) as for the C solution (fig. 6.5). Although the fits have become worse, the change of the cross section and the analysing power is not drastic. So we expect that leaving out the inelastic proton scattering is a reasonable approximation for the calculation of the \(2^+\) states. Of course one could also optimize \(\chi_A^2\) for the ground state within this truncated scheme, but in that case the fit of the cross section gets worse \((\chi_g^2 = 190)\) and \(\lambda\) changes from 0.53 to 0.17. Apparently the determination of \(\lambda\) is very sensitive to the inclusion of \((\text{pp}'\text{dt})\) and \((\text{pp}'\text{t})\). We found that the change of \(\lambda\) is chiefly due to the omission of the three-step process \((\text{pp}'\text{dt})\). That the importance of the \((\text{pp}'\text{t})\) process should not be underestimated is clear from fig. 6.5 where we compare the \((\text{pt})\) and the \((\text{pt}+\text{pp}'\text{t})\) calculations using \(\lambda = 1\). The description of the analysing power is substantially improved after inclusion of the two-step process \((\text{pp}'\text{t})\).

6.4.2 The \(2^+\) and \(2^\pi\) states of \(54\text{Fe}\)

The \(2^+\) states have been calculated according to coupling scheme B with the normalization parameter values from table 6.5. Both triton potentials \((\text{TF}, \text{TB})\) give very similar analysing powers and (absolute) cross sections. Therefore we only present the results obtained with TF. We already mentioned the fact that replacing the two-way coupling in the triton channel by a one-way coupling to the \(2^+\) state leads to virtually the same results. Consequently, the calculations are really based on amplitudes of the following form:

\[
T = \lambda(T(\text{pt}') + T(\text{ptt}')) + \mu(T(\text{pdt}') + T(\text{pdtt}'))
\]  \hspace{1cm} (6.3)

In our discussion we will discern the simultaneous transfer \((\text{pt}')\), the
Fig. 6.6 Calculations of simultaneous ($\lambda$=1) and sequential ($\nu$=1) transfer to the $2^+$ states of $^{56}$Fe. The cross sections are normalized with the factors displayed in the figure.

sequential transfer (pdt') and the inelastic transition (ptt'+pdtt') as the basic processes.

Fig. 6.6 shows the cross sections and analysing powers of two of these processes calculated with $\lambda = 1$ and $\nu = 1$. The cross section curves are normalized to the first maximum to allow a better judgement of the shapes. The experimental analysing powers of both states are quite different (almost opposite phase), which is connected with their difference in structure, the $2_1^+$ being predominantly a two-hole state and the $2_2^+$ a three-hole state. The (pt') transfer by itself describes the transition to the $2_2^+$ state very well. The first maximum of the $2_1^+$ cross section, on the contrary, is better fitted by the (pdt') process. But neither the (pt') nor the (pdt') transfer produces a
Fig. 6.7 The complete calculations (coupling scheme B) with λ/μ positive (C_+) and λ/μ negative (C_-) for the \( 2^+ \) state transitions. The cross sections are normalised with the factors displayed in the figure.

<table>
<thead>
<tr>
<th>Table 6.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio of calculated(^{a)}) and experimental cross sections for the ( 2^+ ) transitions of ( ^{54})Fe</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Solution</th>
<th>Final state</th>
<th>pt'</th>
<th>p(\text{dt}')</th>
<th>pt'</th>
<th>Complete p(\text{dt}') calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>C_+</td>
<td>( 2^+_1 )</td>
<td>0.80</td>
<td>1.29</td>
<td>2.0</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td>( 2^-_2 )</td>
<td>0.91</td>
<td>0.13</td>
<td>0.84</td>
<td>2.0</td>
</tr>
<tr>
<td>C_-</td>
<td>( 2^+_1 )</td>
<td>0.035</td>
<td>2.9</td>
<td>2.9</td>
<td>1.4</td>
</tr>
<tr>
<td></td>
<td>( 2^+_2 )</td>
<td>0.04</td>
<td>0.29</td>
<td>0.38</td>
<td>0.7</td>
</tr>
</tbody>
</table>

\(^{a)}\) the C_+ results have to be multiplied by 1.7 if one normalizes the ground-state transition to the max. at \( 0^\circ \)
reasonable analysing power for the $2^+_1$ state. Note the smallness of the $(pdt')$ cross section for the $2^+_2$ state. There is no simple reason for this.

The angular distribution and analysing powers of the $2^+_1$ states clearly select the $C_+$ solutions as the better ones (fig. 6.7), although the $C_-$ solutions give cross section normalizations closer to experiment. However, one should not worry too much about these normalizations since they turned out to be rather sensitive to the magnitude and the sign of the spectroscopic amplitudes.

In table 6.7 we have brought together the cross sections of various processes normalized by the experimental cross section at $25^\circ$. The

![Diagram](image_url)

**Fig. 6.8** The calculation including inelastic scattering in the proton channel (+pp't') and the one with $\beta_2$ negative compared with the $C_+$ result; $\lambda/\mu$ is positive in all cases; the cross sections have been multiplied with the factors indicated in the figure.
multistep processes give considerable contributions to the total
cross sections. Notice the drastic change of magnitude when the in-
elastic transition is included. The inelastic transition interferes
destructively for the first and constructively for the second $2^+$
state.

We also performed calculations based on coupling scheme B ex-
tended with an $L=0$ (pp't') transfer (table 6.1). The change introduced
by this additional two-step process is negligible for the $2^+_2$ state and
disastrous for the $2^+_1$ state (fig. 6.8). The situation for the $2^+_1$ state
might be improved by also including the L=2 and L=4 (pp't') transfer
and the (pp'ttt') and (pp'dtt') processes. Anyhow, this teaches us to
be cautious when neglecting inelastic proton scattering for transitions
to excited states in the triton channel. The C$_0$ solution for the $2^+_1$
state is not changed significantly by the addition of the (pp't')
process because of the small value of $\lambda$.

Finally, we show how to distinguish the sign of the deformation
parameter $\beta_2$ (fig. 6.8). Observe the normalization of the cross
section when $\beta_2$ is negative.

6.4.3 The $0^+_2$ state of $^{81}$Rb

Since the shell-model calculations of Vennink et al. 2) do not
contain four-hole states and the $0^+_2$ state is thought to be pre-
dominantly a four-hole state, we do not have calculated spectroscopic
amplitudes at our disposal. Therefore we computed the analysing power
and the cross section with adjustable spectroscopic amplitudes. In
fact we combined the one-step simultaneous transfer and the two-step
sequential transfer of two $\gamma$-neutrons (scheme C, fig. 6.2). By
minimizing the chi-square of the analysing power and next normalizing
the cross section to the maximum at $0^\circ$ we determined both spectro-
scopic amplitudes. The result is given in fig. 6.9.

Although the fit of the complete calculation is far from being
perfect, it is certainly better than the one-step calculation. The
(pp't') and (pd't') processes separately give 49% and 85% of the cross
section, respectively. The strength of both processes was found to be

$$\lambda \frac{g_{1/2}(7/2)}{g_{2/2}(1/2; 0)} = 0.61$$

and

$$\mu \frac{g_{1/2}(7/2)}{S_{1/2}(7/2)} = 2.50$$

92
If we substitute the values of \( \lambda \) from table 6.5 (TP) we find for the (pt') amplitude 0.78 and -3.6 for \( \lambda > 0 \) and \( \lambda < 0 \), respectively. Apparently \( \lambda \) has to be positive because the theoretical limit of \( |S_{1/2}(7/2; 7/2; 0)\) is 1.

Finally, for a pure one-step calculation the spectroscopic amplitude is 0.90 if one takes \( \lambda = 1 \).

### 6.4.4 DWBA analysis

The multistep calculations fit the data of the 0\(^+\) states and the 2\(^+\) state significantly better than the one-step DWBA calculations (figs. 6.9, 6.10 and 6.11). The 2\(^+\) state is also well described by a pure one-step transition (fig. 6.10). However, the magnitudes of the cross sections are surprisingly close to the experimental values in the DWBA analysis (table 6.8).

The wave function of the 2\(^+\) state \(^2\) consists predominantly of two proton holes coupled to angular momentum 2. The reason that the one-step two-neutron transfer has a large cross section stems from the fact that there is a considerable component in the wave function of the ground state of \(^{56}\text{Fe}\) with two proton holes coupled to angular momentum 2. The magnitude of the (pt') cross section of the 2\(^+\) state (table 6.8) should be compared with the results obtained at 52 MeV \(^19\) where the calculated cross section is an order of magnitude too small.
Fig. 6.18  The one-step calculations (normalized) and the $C_\gamma$ calculations for the ground-state transitions.

Table 6.8

Ratio of DWBA and experimental cross sections for the ground state and the $2^+$ states of $^{54}$Fe

<table>
<thead>
<tr>
<th>Potential</th>
<th>$\lambda^{a)}$</th>
<th>$\sigma_1^+$</th>
<th>$\sigma_2^+$</th>
<th>$\sigma_3^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TF</td>
<td>0.84</td>
<td>1.00</td>
<td>0.82</td>
<td>0.96</td>
</tr>
<tr>
<td>TB</td>
<td>0.66</td>
<td>1.00</td>
<td>0.87</td>
<td>1.02</td>
</tr>
</tbody>
</table>

$a)$ for Ni: $\lambda = 0.92$ (TF) and $\lambda = 0.84$ (TB)
Fig. 6.11 The one-step and the $C_1$ calculations for the $2^+$ state transitions. The normalisation factors are given in the figure.
6.5 Conclusions

In this chapter we studied five natural parity (i.e. "allowed") \((p,\gamma)\) transitions. In all cases multistep processes compete strongly with the one-step process. The shape of the cross sections and analysing powers improve after addition of the multistep processes. Therefore the shell-model wave functions \(^1\),\(^2\) which were used to generate the spectroscopic amplitudes seem to be reliable.

The calculations have been performed with the zero-range code CHUCK\(^2\). We have tried to make up for the omission of effects such as finite range by renormalizing the simultaneous- and sequential-transfer amplitudes. By minimizing the chi-square of the analysing power of the ground-state transitions we found two solutions for the normalization parameters, of which one yielded a reverse relative phase between the simultaneous and the sequential amplitudes. But, considering all results, the solution with the natural phase clearly is the best one.

The ratio of the normalization parameters \(\lambda\) and \(\mu\) was found to be approximately equal to 0.7 for \(^{68}\)Se and 0.5 for \(^{68}\)Ni. So we had to reduce the simultaneous transfer two to four times more than the sequential transfer.
REFERENCES

9 M. Igarashi and K.-I. Kubo, Fifth International Symposium on Polarization Phenomena in Nuclear Physics, Santa Fe, August 1980, contribution 2.112.
12 P. Guazzoni, M. Pignanelli, E. Colombo and F. Crescentini, preprint.
14 P.D. Kunz, Univ. of Colorado, unpublished.
21 A.C.W. van Hees, private communication.
30 N. Hashimoto and M. Kawai, preprint.
CHAPTER 7 THE (p,a) REACTION ON $^{58}$Ni AT 24.6 MeV

7.1 Introduction

Although direct (p,a) reactions have been studied for quite a long while $^1$, it is only during the last few years that polarized proton beams have been used $^2$-$^7$ to induce them. It was found by van Hall et al. $^2$ that the analyzing powers generally show much more structure than the cross sections and that they discriminate clearly between positive and negative parity states, whereas the cross sections usually display a j-dependence $^8$ only.

A $^{58}$Ni(p,a) experiment has already been performed by Tagishi et al. $^6$ at the proton energy of 22 MeV, which is close to our value of 24.6 MeV. The data of our experiment were taken simultaneously with those of the $^{58}$Ni(p,d) (chapter 4) and $^{58}$Ni(p,t) (chapter 6) reactions and we thought it worthwhile to extend the multistep calculations applied to these cases to the (p,a) reaction.

We only observed the transition to the $\frac{7}{2}^-$ ground state of $^{55}$Co, because the alpha particles, that leave the $^{55}$Co nucleus in an excited state, are too slow to pass through the 200 µm ΔE-detectors of the telescopes. The experimental results are in close agreement with those obtained by Tagishi et al. $^6$.

7.2 Reaction model

The $\frac{7}{2}^-$ ground state of $^{55}$Co is assumed to be a pure $\frac{7}{2}^+$ proton hole in the $^{56}$Ni core and the (p,a) reaction is thought to proceed by the pick-up of the two valence neutrons of $^{58}$Ni and one $\frac{7}{2}^+$ proton. The reaction model is summarized in the coupling scheme of fig. 7.1, which is scheme A of fig. 6.2 supplemented with a (t,a) and a (p,a) step. We did not take into account transitions via excited states of $^{58}$Ni.

The one-step (p,a) transition was treated in a cluster approximation, wherein the transition was described by the pick-up of a triton bound in a Woods-Saxon well with the binding energy equal to the experimental separation energy of the triton. The number of nodes N of the form factor is equal to 3 and follows from the harmonic-
oscillator rule: \( I(2n_i + 1_J) = 2N + L \), where the \( n_i \) and \( l_i \) are quantum numbers of the three constituent nucleons. The geometry parameters of the potential well are listed in table 7.1. The small size of the diffuseness parameter was found to be necessary for one-step calculations in order to get good fits to the cross sections and is widely accepted now. The well-depth procedure resulted in a value of 120 MeV for the triton binding potential.

The \((t,\alpha)\) step was calculated like the other one-neutron transfers and using the zero-range normalization constant \( D_0(t\alpha) = -479 \text{ MeV fm}^{3/2} \) corresponding to the CHUCK normalization constant \( N_0(t\alpha) = -678 \text{ MeV fm}^{3/2} \).

The proton and deuteron \((D4)\) potentials are discussed in chapter 4 (section 4.3.3) and the triton potentials \(TV\) and \(TB\) in chapter 6 (section 6.3.3). We tried two alpha-potentials \(A1\) and \(A2\) (table 7.1). The first one \((A1)\) originates from a study by Brodowska et al. \(^{11}\) and was applied successfully to the one-step cluster analysis of the reaction \(^{58}\text{Ni}(p,\alpha)\) at 30 MeV by Smits and Siemssen \(^{10}\). The potential \(A2\) was taken from a global fit to data obtained from elastic scattering of alpha particles of 24.7 MeV on nuclei in the region of \(A=60\) by Budzanowski et al. \(^{12}\).

The non-locality parameters for the neutron, triton and the alpha particle are given in table 7.1 together with the finite-range parameters \((LEA)\) for the \((t,\alpha)\) transition. We applied no finite-range correction to the triton pick-up.

We introduced normalization parameters \(\lambda\) and \(\mu\) for the \((p,\alpha)\) and \((t,\alpha)\) step, respectively, because of the uncertainty of the normalization of the \((p,\alpha)\) transition and the neglect of effects such
Table 7.1
Optical-model and other DWBA parameters a)

<table>
<thead>
<tr>
<th>Name</th>
<th>V</th>
<th>r_o</th>
<th>a_o</th>
<th>W_v</th>
<th>r_x</th>
<th>a_x</th>
<th>V_so</th>
<th>r_so</th>
<th>a_so</th>
<th>r_c</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>175.3</td>
<td>1.46</td>
<td>0.48</td>
<td>26.0</td>
<td>1.46</td>
<td>0.48</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A2</td>
<td>204.7</td>
<td>1.42</td>
<td>0.52</td>
<td>24.0</td>
<td>1.42</td>
<td>0.52</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>b)</td>
<td>1.25</td>
<td>0.65</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>t</td>
<td>b)</td>
<td>1.25</td>
<td>0.45</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>c)</td>
<td>1.25</td>
<td>0.65</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Non-locality parameter θ

<table>
<thead>
<tr>
<th>n</th>
<th>t</th>
<th>α</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.85</td>
<td>0.25</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Finite-range parameter R

<table>
<thead>
<tr>
<th>(t,α)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.70</td>
<td></td>
</tr>
</tbody>
</table>

a) V, W_v and V_so in MeV, the other parameters in fm
b) V is found from the well-depth procedure
c) Factor multiplying the Thomas-term is 25

as exact finite-range. The normalization parameters were determined analogously to the procedure of section 6.3.6, namely by minimizing the chi-square value of the fit to the analysing power of the (p,α) reaction and then normalizing the calculated cross section to the experimental data.

7.3 Discussion

The results of the procedure for determining λ and μ are displayed in fig. 7.2 for the C_+ and C_- solutions (see chapter 6) using triton potential TF and alpha potential A2. It is clear that the difference with the one-step calculation is small but significant (especially for the analysing power).

The contribution of the (t,α) path to the cross section (at 35°) is about 6% for both the C_+ and the C_- solution. Since we found the C_+ solution to give the correct description of the (p,t) reaction, we will from now on only discuss calculations based on this solution. The (p,α) step alone gives 130% of the cross section (at 35°), so the interference between the (t,α) and (p,α) paths is destructive (the
Fig. 7.2 Cross sections and analyzing powers for the transition to the $7/2^-$ ground state of $^{55}\text{Co}$. The symbols are explained in the text.
interference angle is $130^\circ$). The $6\%$ contribution of the $(t,\alpha)$ path can be translated in a spectroscopic factor $S(t,\alpha) = 0.04$ (disregarding exact finite-range effects), which is much smaller than the expected value of $8$ based on a simple shell-model. Even if we only try to obtain the correct magnitude of the cross section, not caring about the shape of the cross section and the analysing power, we find that $S(t,\alpha)$ cannot exceed the value $0.9$ (assuming an interference angle of $130^\circ$). In section 7.4 we give some possible explanations of this problem.

In fact if we look at the chi-square values only then the result of fitting the analysing power by varying $\lambda$ and $\mu$ is consistent with a pure one-step description ($\mu = 0$) of the $(p,\alpha)$ reaction, namely $\mu/\lambda = -0.1 \pm 0.15$. The corresponding result from minimizing the chi-square value of the cross-section shape sustains this finding: $\mu/\lambda = 0.0 \pm 0.1$.

That the $(p,\alpha)$ path has to be dominant, is also clear from the pure $(p,\alpha)$ and $(\ldots) (t,\alpha)$ calculations (fig. 7.2), which show a much closer agreement with especially the cross-section data in the former case.

The choice of the triton potential has some influence on the shape of the cross section and the analysing power of the $(\ldots) (t,\alpha)$ transition, while the magnitude of the cross section calculated with potential TB is about twice as large at that obtained with TF, which is quite similar to the dependence upon the triton potential found for the $(p,t)$ reaction (chapter 6).

Whereas the alpha potential $A_1$ is quite successful in describing the cross section calculated in the one-step cluster approximation (fig. 7.2), it obviously is deficient in reproducing the analysing power. The global potential $A_2$, on the other hand, gives here (also one-step) a reasonable fit to both the cross section and the analysing power. For this reason we selected this potential for the calculations including the multistep transitions.

The ratio of the experimental cross section and the one given by the code DWUCK4 $^{13}$ is 40.5 and 16.7 using the potentials $A_2$ and $A_1$, respectively. These values correspond to $\beta = 2600$ and $\beta = 1600$ where $\beta$ is the coupling parameter of the code CHUCK2 $^{13}$. These numbers contain normalization constants as well as spectroscopic factors.
One-step \((p,\alpha)\) calculations based on a microscopic form factor generally yield fits to the experimental data which are inferior to those obtained with a cluster form factor \(^{14}\). In our analysis we mixed the "microscopic approach" of the \((...t)(t\alpha)\) path with the "cluster treatment" of the \((p\alpha)\) path and found that the contribution of the former process was much smaller than expected. One obvious reason for this result is the fact that the geometry parameters of the potential that binds the triton cluster have been determined from one-step analyses of \((p,\alpha)\) reactions. Therefore it may be that multi-step effects are hidden in the cluster approximation and that the results obtained with microscopic form factors will be improved by the inclusion of e.g. \((pt)(t\alpha)\) transitions, in which case our results seem to indicate that the interference has to be destructive. Another explanation may be that the \((p,^3\text{He})(^3\text{He},\alpha)\) process interferes destructively with the \((p,t)(t,\alpha)\) process resulting in a seeming dominance of the one-step \((p,\alpha)\) process.
REFERENCES

13 P.D. Kunz, Univ. of Colorado, unpublished.
SUMMARY

Nuclear transfer-reactions are characterized by the transfer of a small number of nucleons from one atomic nucleus to another. This thesis deals with several types of transfer reactions which were induced by bombarding $^{58}\text{Ni}$ and $^{56}\text{Fe}$ foils, respectively, with a beam of polarized protons having an energy of 24.6 MeV. Three types of reactions have been studied: the transfer of one neutron (p,d), the transfer of two neutrons (p,t) and the transfer of two neutrons and one proton (p,q).

The purpose of this study was twofold, namely the extension of the spectroscopic knowledge of the nuclei $^{57}\text{Ni}$ and $^{55}\text{Fe}$ - by means of the (p,d) reactions - and the determination of the importance of multistep processes for all reactions investigated in this thesis. For this the analysing powers, which were available thanks to the polarization of the proton beam, turned out to be very helpful.

The description of the (p,d) reactions generally improved by taking account of the two-step process $(pp')(p'd)$, i.e. inelastic proton scattering (to the $2^+$ states) followed by one-neutron transfer, in addition to the direct (pd) process. The resulting corrections of the experimental spectroscopic factors amounted to at most 10%.

In most cases the multistep calculations were based on transition amplitudes obtained from shell-model calculations of Glaudemans, Koops and Vennink. The latter calculations have been performed using two different types of residual nucleon-nucleon interaction: surface-delta (SD) and Kuo-Brown (KB). The shapes of the differential cross sections and the analysing powers were insensitive to the type of residual interaction, but not, however, the magnitudes of the cross sections. The KB interaction generally reproduced these magnitudes better than the SD interaction in the case of the $^{56}\text{Fe}(p,d)$ reaction, while it was not possible to make a definite choice for the $^{58}\text{Ni}(p,d)$ reaction.

The analysis of the (p,d) reactions yielded some problems with the adiabatic deuteron potential: we were forced to reduce the real well-depth with about 7% in order to obtain an acceptable description of both the cross sections and the analysing powers. In addition there were indications that the imaginary well-depth has to be increased (i.e. more deuteron absorption) in that case. We guess that these
results are consequences of the fact that exchange effects were not taken into account in the construction of the adiabatic deuteron potential. Not even the adjusted deuteron potential D4 could sufficiently explain the difference observed between the experimental cross sections with j=7/2 and j=5/2. We think that this shortcoming is due to the neglect of the D-state of the deuterons.

The sequential transfer of two neutrons (pd)(dt) generally contributed as much to the (p,t) cross sections as the direct two-neutron transfer (pt). Because of uncertainties about the normalization of both the (pd)(dt) and the (pt) process due to - among other things - the zero-range approximation, we introduced two normalization parameters λ and μ. The ratio of λ and μ was determined by fitting the calculated analyzing powers to the experimental ones. Next the magnitude of λ and μ followed from the magnitudes of the experimental cross sections.

In the case of the $^{38}$Ni(p,a) reaction we used such parameters too. From this we found the one-step process (pd) to be dominant. The (...,t)(ta) processes contributed much less than expected possibly because of destructive interference with (...,$^3$He)($^3$Hea) processes, which were left out of the calculations.

In view of the accumulation of uncertainties about the various ingredients of the calculations for the (p,t) and (p,a) reactions, we think it wise to concentrate - for the time being - on gaining more insight into the details of the (p,d) reactions.
**SAMENVATTING**

Kernreacties, waarbij een klein aantal nucleonen (protonen of neutronen) van de ene atoomkern naar de andere overgaat, worden overdrachtreacties genoemd. Dit proefschrift handelt over de resultaten van enige overdrachtreacties, die werden opgewekt door folies van bijna zuiver $^{56}\text{Ni}$ respectievelijk $^{56}\text{Fe}$ materiaal te beschieten met gepolariseerde protonen, waarvan de energie 24,6 MeV was ten gevolge van versnelling door het A.V.F. cyclotron van de T.H. Eindhoven. Er werden drie types reacties bestudeerd: overdracht van één neutron $(p,d)$, overdracht van twee neutronen $(p,t)$ en overdracht van twee neutronen en één proton $(p,\alpha)$.

Het doel van dit onderzoek was tweeledig, n.l. uitbreiding van de spectroscopische informatie over de kernen $^{57}\text{Ni}$ en $^{55}\text{Fe}$ - via de $(p,d)$ reacties - en bepaling van het belang van meestapsprocessen voor al de bestudeerde overdrachtreacties. Hierbij werd met vrije gebruik gemaakt van de extra gegevens die experimenten met gepolariseerde protonen opleveren: de analyserende vermogens.

Het in rekening brengen van het tweestapsproces $(pp')(p'd)$, d.w.z. inelastische protonverstrooiing (naar $2^+$ niveau's) gevolgd door de overdracht van een neutron, naast het direkte $(pd)$ proces bleek in het algemeen de beschrijving van de $(p,d)$ reacties te verbeteren. De hierdoor noodzakelijk geworden correctie van de experimentele spectroscopische factoren bedroeg maximaal 10%.

In de meeste gevallen waren bovengenoemde meestapsberekeningen gebaseerd op overgangsamplitudes verkregen uit de schijfmodelberekeningen van Glaudemans, Koops en Vennink. Deze laatste berekeningen zijn uitgevoerd voor twee verschillende vormen van de residue nucleon-nucleon wisselwerking: oppervlakte-delta (SD) en Kuo-Brown (KB). De vorm van de differentiële werkzame doorsnaden en de analyserende vermogens bleek ongevoelig voor de keus tussen SD en KB, echter niet de grootte van de werkzame doorsnades. Daarbij was KB over het algemeen beter dan SD voor de $^{56}\text{Fe}(p,d)$ reactie, terwijl het niet mogelijk was een voorkeur te bepalen bij de $^{58}\text{Ni}(p,d)$ reactie.

Bij de analyse van de $(p,d)$ reacties stuitten we op problemen met de adiabatische deuterongpotentialia: we waren gedwongen de punderie van het reële deel met ongeveer 7% te verkleinen om een redelijke beschrijving van zowel de werkzame doorsnades als de analyserende vor-
mogens te verkrijgen. Verder waren er aanwijzingen dat de putdiepte van het imaginaire deel dan tegelijk vergroot moet worden (meer deuteronabsorptie). We vermoeden dat deze resultaten te maken hebben met het feit dat "exchange" effecten niet in rekening zijn gebracht bij de constructie van de adiabatische deuteronpotentiaal. Ook de aangepaste deuteronpotentiaal D4 kon het experimentele verschil tussen werkzame doorsneden met j=7/2 en die met j=5/2 niet afdoende verklaren. We denken dat dit te wijten is aan het verwaarlozen van de D-toestand van het deuteron.

Bij de (p,t) reacties bleek de sequentiële overdracht van twee neutrons (pd) (dt) in het algemeen ongeveer evenveel bij te dragen aan de werkzame doorsnede als de directe overdracht (pt). Gezien de onzekerheid over de normering van zowel het (pd) (dt) als het (pt) proces in verband met o.a. de dracht-nul benadering, voerden we twee normeringsparameters λ en μ in. De verhouding van λ en μ werd bepaald door de berekende analyserende vermogens zo goed mogelijk aan te passen aan de experimentele analyserende vermogens. De grootte van λ en μ volgde dan uit de grootte van de experimentele werkzame doorsnede.

Ook voor de \(^{58}\text{Ni}(p,a)\) reactie voerden we dergelijke parameters λ en μ in. Daarbij bleek het eenstapproces (pa) dominant te zijn. De bijdrage van de (...,t)(ta) processen was veel kleiner dan verwacht, mogelijk ten gevolge van destructieve interferentie met (...,\(^{3}\text{He})(^{3}\text{He}a) processen, die niet expliciet in rekening werden gebracht.

Het lijkt ons verstandig voorlopig meer aandacht te schenken aan het begrijpen van de details van de (p,d) reacties, gezien de openstaapel van onzekerheden over de verscheidene onderdelen van de berekeningen bij de (p,t) en (p,a) reacties.
Het onderzoek dat in dit proefschrift beschreven is, werd verricht binnen de onderwerpgroep "experimentele kernfysica" en met de daadwerkelijke steun van alle leden van die groep.


Van essentieel belang was ook de ontwikkeling van het computersysteem door Adri de Raa.

Ik denk met plezier terug aan de nauwe samenwerking met Hans van Oosten gedurende de ontwikkeling en het testen van het telescoop-systeem.

Heel erkentelijk ben ik prof. Glaudemans en zijn medewerkers — in het bijzonder Fons van Hees — voor het verschaffen van golffuncties en spectroscopische amplitudes van de ijzer- en nikkelkern.

Van Wolfgang Peix heb ik — tijdens zijn verblijf hier als "research fellow" — veel geleerd over de meertijdsanalyse van (p,t) reacties.

Veel dank ook jegens de cyclotronsbedrijfsgroep voor het leveren van dopolizeerde protonenbundel.

De uiteindelijke vormgeving van dit proefschrift, tenslotte, kwam tot stand dank zij de grote inzet van Ruth Gruyters (tekeningen), Rian Teurlings en Aafje Smit (typewerk).
**Levensloop**

10 februari 1946 geboren te Amsterdam

mei 1964 eindexamen gymnasium-II aan het ir. Lely-lyceum te Amsterdam

september 1972 doctoraal examen theoretische natuurkunde (hoge-energie-fysica) aan de Universiteit van Amsterdam

september 1972 - augustus 1974 wetenschappelijk medewerker aan de Vrije Universiteit van Brussel

augustus 1974 - augustus 1976 leraar natuurkunde aan het Michaelcollege te Zaandam

STELLINGEN

behorend bij het proefschrift van

J.R. Polane

Eindhoven, 5 juni 1981
In de uitdrukking die Donner 1) geeft voor de aanslag-energie van tweedeeljesexcitatie in het RGS-model, dient de term 
\[ C(u_{k_1}^* u_{k_2}^* + v_{k_1}^* v_{k_2}^*)^2 \]
vervangen te worden door 
\[ -2C u_{k_1}^* u_{k_2}^* v_{k_1} v_{k_2}^* \].

1) W. Donner, Einführung in die Theorie der Kernspektren 
(M. I., Mannheim 1971), p. 236

In hun bespreking van de oppervlakte-delta-wisselwerking maken Brussaard en Glaudemans 1) ten onrechte onderscheid tussen de parameters B' en C' enerzijds en de parameters B en C anderzijds.

1) P.J. Brussaard en P.W.M. Glaudemans, Shell-model applications in nuclear spectroscopy (North-Holland, Amsterdam 1977), p. 112

Effecten van de tweede orde kunnen ook bij voorwaartse hoeken een aanzienlijke bijdrage geven aan de differentiële werkzame doornsneede van een deeltjesoverdrachtrecht. Het is raadzaam hiermee rekening te houden bij het normeren van berekende differentiële werkzame doornsneeden.

Dit proefschrift

Voor het bepalen van de relatieve bijdragen van de verschillende processen aan de overgangsamplitudes voor \((p,t)\) reacties verdient een goede aanpassing van het analyserende vermogen de voorkeur boven die van de differentiële werkzame doornsneede.

Dit proefschrift

In een aantal recente populaire artikelen 1, 2) over de unificatie van de elektromagnetische, de sterke en de zwakke wisselwerking wordt ten onrechte de mogelijkheid van een neutrino met rustmassa ongelijk nul genegeerd.

1) P. Wilczek, Sci. American, december 1980, p. 60
Een kerncentrale die getroffen wordt door een kernhorm, verandert voor lange tijd in een verwoestend stalingswapen \textsuperscript{1). Dit feit zou een afdoende reden moeten zijn om af te zien van kernenergie.}

\textsuperscript{1) S.A. Fetter en K. Teipie, So. American, april 1981, p. 33}

Het verplichte eindexamenprogramma natuurkunde voor het HAVO laat te weinig ruimte over voor projectonderwijs.

Ten onrechte wordt de aanleg van een afvoerputje in de badkamer van veel huizen als overbodige luxe beschouwd.