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Energy contents and vortex dynamics in Mode-C transition of wired-cylinder wake

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The 3D transition of the flow behind a circular cylinder with a near-wake wire disturbance has been investigated experimentally. The flow is oriented horizontally and the wire is positioned in the upper half of the wake. We performed flow visualization and particle image velocimetry experiments to investigate the influence of the wire on various properties of the flow, such as the dynamics of the spanwise structures. Experiments were performed in the Reynolds number range of $Re = 165\text{--}300$. It is shown that in Mode-C transition of the wired cylinder wake, some part of the stream-wise vorticity content of the upper von Kármán vortices located at the perturbed side, is transferred to the secondary vortices. This vorticity transfer results in upper von Kármán vortices which are weaker than the lower ones. The analysis of the discrete energy content of the wake supports this analysis by showing that the energy intensity at von Kármán vortex shedding frequency f_0 at the perturbed side of the wake is less than the energy intensity in the lower half. This leads to conclusion that the excess energy is transferred to the subharmonic frequency $f_1 \approx f_0/2$. © 2013 AIP Publishing LLC. [<http://dx.doi.org/10.1063/1.4801849>]

I. INTRODUCTION

Due to its simple geometry, the flow around circular cylinders has become a central topic in many research fields. A lot of effort was made to explain the physics of circular cylinder wake dynamics.^{1–3} They showed that vortex shedding for low Reynolds numbers can be characterized by vortex splitting and high shear stress occurring in the near-wake. Unal and Rockwell⁴ investigated the near wake vortex formation from the context of absolute instability and showed that the shear layer separating from the cylinder shows an exponential variation of fluctuating kinetic energy with distance downstream of the cylinder.

However, the aforementioned studies were limited to the two-dimensional aspects of the flow. The three-dimensional aspects of wake flows behind circular cylinders are thoroughly reviewed and discussed by Williamson.^{5,6} The laminar vortex shedding regime extends from a Reynolds number of approximately 49 to 140–194 and the flow regime between $Re \approx 190$ and 260 is denoted as the three-dimensional wake transition regime. This regime is associated with two modes of shedding, Mode-A and Mode-B.

The first evidence of Mode-C appeared in the computations and experiments of Zhang *et al.*⁷ when a tiny wire was placed close to a circular cylinder. In their flow visualization experiments, they observed that Mode-C transition takes place over a Reynolds number range of $170 < Re < 270$. Numerical simulations at $Re = 210$ displayed a spanwise periodicity of 1.8 cylinder diameters.

Later on, Mode-C studies were extended to flow behind circular rings by Sheard *et al.*^{8–11} mainly using direct numerical simulations. They demonstrated that the Mode-C instability arises through a subharmonic bifurcation and has a period-doubling nature. Sheard *et al.*⁸ identified this mode as the primary transition mode in the aspect ratio (AR) range of $3.9 \lesssim AR \lesssim 8$, with a spanwise wavelength

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of approximately 1.7 ring cross-section diameters. They suggested that a subharmonic Mode-C instability occurs as a result of the asymmetry between vortices shed from the inner and outer surfaces of the ring due to its curvature. Further nonlinear characterization of Mode-C was made by Sheard *et al.*⁹ They showed that for the flow past a ring with $AR = 5$, Mode-C instability produces a period-doubling in the wake through supercritical and non-hysteretic transition. Subsequently, Sheard *et al.*¹⁰ provided additional computational results and the first experimental observations of the subharmonic mode Mode-C. Later, Sheard *et al.*¹¹ showed that the period-doubling nature of the wake is maintained by a cycle of convection of the perturbation vorticity from the near-wake. A unified description of a time-periodic 2D flow with space-time reflection symmetry (such as the von Karman vortex street) bifurcating to a non-symmetric 3D flow, like the Mode-C transition, using a Floquet stability analysis is presented in Blackburn *et al.*¹² and Blackburn and Sheard.¹³ It is shown that the symmetry-breaking 3D transition of time-periodic wakes is not correlated with the physical characteristics such as the spanwise wavelength of the flows.

Carmo *et al.*¹⁴ also found Mode-C in the wake transition of the flow around staggered arrangements of equi-diameter circular cylinders for different relative positions. They showed that Mode-C appears in the near-wake of the downstream cylinder with an intermediate spanwise wavelength between Mode-A and Mode-B with a period-doubling character. The prevailing feature of the subharmonic Mode-C instability is that it can only be produced in flows which exhibit a breaking of the Z_2 spatio-temporal symmetry, Blackburn *et al.*¹² The placement of a trip-wire near to a circular cylinder but offset from the wake centerline in our experiments serves to break this symmetry and permit the emergence of a subharmonic instability mode.

On the other hand, several experimental studies have been performed to characterize the secondary vortices in the unperturbed circular cylinder wake,^{5,15–18} which preserves $Z_2 \times O(2)$ spatio-temporal symmetry. Williamson⁵ reported results obtained from hot-wire measurements in the transition regime $Re > 180$, while Lin *et al.*¹⁵ and Chyu and Rockwell¹⁶ measured the secondary vortices in a cross-stream plane using Particle Image Velocimetry (PIV) at a Reynolds number of $Re = 10\,000$ (in the turbulent flow regime). Huang *et al.*¹⁷ also performed PIV experiments to measure secondary vortices in all three perpendicular Cartesian planes. In recent work, Scarano and Poelma¹⁸ performed time-resolved tomographic PIV experiments in the Reynolds number range of $Re = 180$ –5540. All these experiments focused on non-wired cylinders.

Regarding the perturbed cylinder case, it is shown by Zhang *et al.*⁷ that Mode-C transition can be characterized by the formation of secondary vortices with a period-doubling nature and a spanwise wavelength of approximately 2.2 cylinder diameters. It is also shown that the shedding frequency in Mode-C transition is lower than the Mode-A and Mode-B frequencies for the non-wired cylinder in the same Reynolds number range. Yildirim *et al.*¹⁹ investigated the laminar two-dimensional wake flow for the wired-cylinder case, investigating its vortex dynamics and vortex shedding process. Furthermore, Parezanovic and Cadot^{20,21} characterized the modification of a two-dimensional wake by a smaller cylinder using sensitivity maps. This paper mainly focuses on further experimental characterization of Mode-C transition regarding the fluctuating flow features, energy characteristics, and vortex dynamics using PIV velocimetry measurements.

The outline of the paper is as follows. Experimental details are given in Sec. II. In Sec. III the fluctuating velocity characteristics are examined to characterize the effect of the wire on the temporal behavior of the wake. The energy spectrum analysis showed that there is an energy transfer from the primary vortices to the secondary vortices in the upper half of the wired cylinder wake. This observation is further evaluated by calculating discrete energy components at different frequency components of the energy spectrum in Sec. IV. The vortex dynamics in Mode-C transition is investigated in Sec. V. Section VI gives the concluding remarks.

II. EXPERIMENTAL METHODOLOGY

The experiments are performed in a towing tank with dimensions of $L \times W \times H = 5\text{ m} \times 0.5\text{ m} \times 0.75\text{ m}$.²² The experimental model is a circular cylinder with a diameter of $D = 15\text{ mm}$ and length of $L = 480\text{ mm}$. It is placed between circular end plates in order to force parallel shedding. In order to elucidate the effect of the wire, both non-wired and wired experiments

FIG. 1. Measurement configurations. (a) Side-view configuration and (b) upstream-view configuration.

are performed. Mode-C instability is triggered using a wire with a diameter of $d = 0.15$ mm which is placed in the near wake. The ratio of diameter of the cylinder and the wire is $D/d = 100$. The position of the wire is chosen in accordance with the work of Zhang *et al.*,⁷ which is $(x/D, y/D) = (0.75, 0.75)$, as shown in Figure 1.

Figure 1(a) illustrates the configuration, which is used for the investigation of the evolution of the primary (i.e., spanwise) vortices. The images are recorded synchronously with two side-by-side cameras to cover a larger field-of-view and are later combined in the post-processing stage. Some details of the merging procedure are described here.

The second configuration is used for the investigation of the evolution of the secondary (i.e., streamwise) vortices. The layout of this configuration is shown in Figure 1(b). The laser plane illuminates the $Y-Z$ plane where the images are recorded through a mirror placed in the downstream of the wake. The size of the mirror is $100 \text{ mm} \times 100 \text{ mm}$ and is placed at an angle of 45° with respect to the free-stream direction. The effect of the mirror on vortex shedding is evaluated using top-view flow visualization experiments (not shown here). The results showed that when the mirror center is located $x_m = 16D$ downstream of the cylinder, a clear parallel shedding is seen and at this position the generation of secondary vortices is not affected by the mirror.

Regarding the PIV experiments, the water in the towing tank was seeded with PSP particles having a diameter of $20 \mu\text{m}$. The flow was illuminated by using an Nd-Yag laser and the images were captured with a 12-bit camera with $1600 \text{ px} \times 1200 \text{ px}$ pixel resolution and 30 Hz frame rate. A summary of the experimental details for the two types of experiments is given in Table I. One may notice the camera field of view in both side-view and upstream-view experiments is similar, and close to each other and corresponds to approximately $(S_x, S_y) \approx (7.3D, 5.5D)$ and $(S_y, S_z) \approx (5.2D, 6.9D)$, respectively. However, in the side-view experiments, two cameras in a side-by-side arrangement are used to cover a larger field of view which resulted in field of view of approximately $(S_x, S_y) \approx (14.1D, 5.5D)$.

TABLE I. Summary of PIV experiment details for side-view (XY -plane) and upstream-view (YZ -plane) wired cylinder measurements [$Re = 180$].

Measurement plane	X - Y plane	Y - Z plane ($x_l/D = 4$)
Number of cameras	2 (side-by-side)	1
Magnification	$M = 14.53$ px/mm	$M = 15.49$ px/mm
Field of view (single camera)	$S_x = 110.12$ mm $S_y = 82.59$ mm	$S_y = 77.47$ mm $S_z = 103.29$ mm
Field of view (after merging)	$S_x = 211.29$ mm $S_y = 82.59$ mm	
Lens focal length	$f = 50$ mm	$f = 50$ mm
Lens aperture	$f_\# = 2.8$	$f_\# = 2.8$
Exposure time	$t_{\text{exposure}} = 25$ ms	$t_{\text{exposure}} = 25$ ms
Pulse delay	$\Delta t = 1/15$ s	$\Delta t = 1/15$ s
Number data files	$N_f = 3500$	$N_f = 2700$

A sample merging procedure is demonstrated in Figure 2 showing an instantaneous spanwise vorticity field for a wired case experiment at $Re = 180$. The overlapping region between the two frames is 6 data points (≈ 130 px) long in the x -direction. Hence, the final merged data field has a size of $N_x \times N_y = 190 \times 73$ data points. Figure 2 represents a challenging example where a vortex is present in the overlapping region. Part of the vortex is measured by two cameras and the velocity field needs to be continuous through the frame boundaries.

The merging approach of two independently measured vector fields is similar to the one used in Herpin *et al.*²³ where they have used a simple averaging in the overlapping region. It is assumed that a flow variable φ in the overlapping region can be simply calculated by taking a linear combination of the data from frame 1 and frame 2 as follows:

$$\varphi = a_1 \varphi_1 + a_2 \varphi_2, \quad (1)$$

with a_1 and a_2 being the corresponding weighting coefficients.

The method of Herpin *et al.*²³ is improved by using a linear weighted averaging method to minimize the image boundary effects. The weighting coefficients a_1 and a_2 are taken as linear functions of the horizontal distance within the overlapping region, as shown in Figure 2 and formulated as

$$a_1 = \frac{-x}{x_2 - x_1} + \frac{x_2}{x_2 - x_1},$$

$$a_2 = 1 - a_1. \quad (2)$$

The results of the merging are presented in Figure 3 which shows that the image boundary effects are minimized in the merged profiles.

FIG. 2. Instantaneous spanwise vorticity contours after merging, [$Re = 180$]. Two different frames are labeled as frame 1 and frame 2 and indicated with arrows. To assess the averaging methods which are given on the right hand side velocity and vorticity data are extracted along the indicated line.

FIG. 3. Extracted spanwise vorticity, horizontal and vertical velocity profiles after weighted averaging in the overlapping region. Each symbol in the figures corresponds to a PIV grid point.

The Reynolds number range investigated is between $185 \leq Re \leq 230$ for the non-wired experiments and between $180 \leq Re \leq 225$ for the wired experiments. The difference in the Reynolds number ranges stems from an “unfortunate” correction of the kinematic viscosity due to differences in temperature of the water for the wired and non-wired cases.

III. FLUCTUATING VELOCITY CHARACTERISTICS

The vortex shedding frequency behind a circular cylinder is closely related to its fluctuating velocity characteristics, such that, if the fluctuations are reduced below some level, even a vortex shedding suppression can be achieved. The importance of velocity fluctuations comes from the fact that the vortex shedding frequency depends on the enhancement or damping of velocity fluctuations in the wake.²⁴ The fluctuating wake flow is characterized by the standard deviation from the mean value (u_{rms}) of its velocity components. For the analysis only the streamwise component (u) of the velocity field is used, since the dominating flow is in that direction.

Figure 4 shows the non-dimensional rms-velocity profiles for the wired case at two different Reynolds numbers and at three different downstream positions. The flow is actually three-dimensional at these Reynolds numbers for the wired case. The non-wired case is also plotted for comparison. The rms-velocity at $Re = 180$ for the wired case is presented in Figure 4(a). For $x/D = 1.5$, both wired and non-wired cases exhibit similar curves with two peaks which are almost symmetric with respect to the wake centerline ($y/D = 0$). These two peaks are associated with the upper and lower vortex rows of the von Kármán vortex street. The locations of maximum u_{rms} values are aligned with the upper and lower shoulder of the cylinder, indicating that the highest fluctuation values are achieved in the separating shear layer. At this downstream position, the effect of the wire is evident in reducing the rms-velocity levels. On the other hand, at $x/D = 4.5$ the symmetry breaking effect of the wire is more apparent which is evident from the vertical locations of the peaks.

At a higher Reynolds number $Re = 225$, the near-wake velocity fluctuation characteristics of Mode-C transition do not change considerably from the $Re = 180$ case. The shape of the u_{rms}/U_∞ profile of the wired case has the same curve pattern as the profile of the non-wired cylinder. However, at $x/D = 1.5$, the reduction in the velocity fluctuation level is higher than in the $Re = 180$ case.

As seen in Figure 4, one of the major effects of the wire on the fluctuating velocity characteristics is a reduction of the rms-velocity levels in the shear layers in the vortex formation region. Another major effect is the introduction of an asymmetry to the u_{rms}/U_∞ profiles. This effect is more pronounced at further downstream, i.e., outside of the formation region. This asymmetry exhibits itself as a broadening of the u_{rms}/U_∞ profile in the upper half of the wake. This broadening of the profile might be an indication of a spread in the fluctuating velocity, possibly due to the vorticity braids originating from the upper vortices, see Figure 5. This figure also confirms the existence of three-dimensional Mode-C flow structures in the wake at $Re = 180$ for the wired case.

FIG. 4. u_{rms}/U_∞ profiles in the near-wake at two different Reynolds numbers ($Re = 185, 230$) and three downstream positions ($x/D = 1.5, 4.5, 12$). (a) Wire: $Re = 180$, no wire: $Re = 185$; (b) wire: $Re = 225$, no wire: $Re = 230$.

The damping effect of the wire on u_{rms}/U_∞ values can be quantified by comparing the maximum values for the wired and non-wired cylinder wake at different Reynolds numbers. Figure 6 shows the downstream variation of maximum rms-velocity values $u_{rms}|_{max}/U_\infty$ in the upper separating shear layer of both the wired and non-wired cylinder wakes in the transition regime.

The data in Figure 6 primarily demonstrate that $u_{rms}|_{max}/U_\infty$ levels do not change significantly in the wired cases. They stay almost constant throughout the Reynolds number range $Re = 180$ – 225 . As summarized in Table II, the wire reduces the $u_{rms}|_{max}/U_\infty$ levels by more than 10% and this amount of reduction increases with Reynolds number, providing an almost constant fluctuation level in the upper shear layer during the Mode-C transition.

FIG. 5. Instantaneous pattern of non-dimensional spanwise $\omega_z D/U_\infty$ (left figure) and streamwise $\omega_x D/U_\infty$ (right figure) vorticity. Streamwise vorticity is measured at cross-stream plane $x/D = 4.5$ at the corresponding von Kármán vortex shedding phase in the left figure. Contour levels are $|\omega_z| D/U_\infty = 0.4, 0.8, \dots, 4$ and $|\omega_x| D/U_\infty = 0.2, 0.4, \dots, 2$. Solid line (blue online) and dashed line (red online) contours indicate positive and negative vorticity, respectively, at $Re = 180$.

FIG. 6. Downstream variation of maximum levels of $u_{rms}|_{max}/U_\infty$ in the upper half of the wake. (a) No wire; (b) wire.

IV. ENERGY CONTENT OF THE MODE-C WAKE

Time history of velocity data is used to calculate the frequency and energy content of the Mode-C wake. The velocity time history is obtained from the time resolved PIV measurements. Figure 7 shows that the energy spectrum in the Mode-C transition has predominant peaks at von Kármán vortex shedding frequency f_0 and at a subharmonic frequency f_1 . The subharmonic frequency f_1 is associated with the formation of secondary vortices and points to the period-doubling character of the Mode-C transition since $f_1 \approx f_0/2$.²⁵ It is possible to calculate the energy content of Mode-C wake at particular frequencies by integrating the energy spectrum curve.^{26–28} For that purpose the energy intensity e at a point in the flow field is defined as²⁶

$$e = \frac{\int_{-\infty}^{+\infty} G(f)df}{U_\infty^2} = \frac{u_{rms}^2}{U_\infty^2}, \quad (3)$$

with $G(f)$ defining the energy spectrum curve. This shows that $G(f)df$ is the energy in a frequency interval df centered at f .²⁸ If the boundaries of the integral cover a specific frequency interval in the spectrum then the integration is the discrete energy intensity in that frequency interval. Furthermore, the integration of the energy intensity across the wake gives the total integrated wake energy,²⁷

$$E = \int_{-\infty}^{+\infty} e \, d\left(\frac{y}{D}\right) = \int_{-\infty}^{+\infty} \left(\frac{u_{rms}}{U_\infty}\right)^2 d\left(\frac{y}{D}\right). \quad (4)$$

There are two ways of calculating the energy intensity and integrated energy. The first is to use the rms-velocity calculations and the second is to use the spectrum calculations. Figure 8 shows the comparison of the wake energy intensity e and the integrated energy E obtained from rms-velocity and spectrum calculations of side-view PIV experiments. The energy intensity plots represent cross-stream data profiles for wired and non-wired cases at $x/D = 4.5$ for $Re = 180$ and

TABLE II. Maximum u_{rms}/U_∞ values of the upper shear layer at different Reynolds numbers and percentage difference between wired and non-wired experiments.

Non-wired experiments		Wired experiments		Difference (%)
Re	$u_{rms} _{max}/U_\infty$	Re	$u_{rms} _{max}/U_\infty$	
185	0.446	180	0.400	−10.3
200	0.462	195	0.394	−14.8
215	0.472	210	0.408	−13.6
230	0.490	225	0.407	−16.9

FIG. 7. Example of an energy spectrum and corresponding discrete energy intensities. Energy spectrum is obtained from side-view PIV experiment for $Re = 180$ at data point $(x/D, y/D) = (4.5, 1.0)$.

$Re = 185$, respectively, while the integrated energy plot is for the wired case only. It is evident from Figure 8(b) that both approaches of calculating wake energy produce very similar results. Therefore, it is concluded that spectrum analysis of PIV velocity data can be used for further characterization of the wake energy at discrete frequency intervals.

As demonstrated in Figure 7, the total energy intensity e can be written as the sum of energy intensities at discrete frequencies,²⁶

$$e = e_0 + e_1 + e_h + e_n, \quad (5)$$

where e_0 and e_1 are the energy intensities at vortex shedding frequency f_0 and subharmonic secondary frequency f_1 , respectively. e_h represents the contributions of first harmonics of f_0 and f_1 . e_n includes high-frequency noise, higher harmonics of f_0 and f_1 , and the remaining fluctuating energies in the wake.

Figure 9(a) shows each discrete component that contributes to the energy of the wired cylinder wake during Mode-C transition. It demonstrates the cross-stream variation of e_0 , e_1 , e_h , and e_n at the downstream position $x/D = 4.5$ for the wired case with $Re = 180$. The wake is dominated by the von Kármán vortex street which is reflected as two almost symmetrical peaks in the e_0 plot. The upper

FIG. 8. Comparison of wake energy intensity and integrated energy obtained from rms and spectrum calculation of side-view PIV experiments. The energy intensity plots show cross-stream data profiles at $x/D = 4.5$. (a) Energy intensity, e ; (b) integrated energy, E .

FIG. 9. (a) and (b) Discrete energy components in Mode-C wake which are calculated at specific frequencies of the energy spectrum on the cross-stream line at $x/D = 4.5$.

half of the e_0 curve has a wider shape with a lower peak value than the lower half of the curve. The effect of the wire is evident in the plot of e_1 , which shows a peak in the upper wake at position of $y/D = 1.25$. The position of this peak corresponds to the vortex centers of the secondary vortices as depicted in Figure 5. e_h contains the harmonics of the frequencies f_0 and f_1 . It has a maximum value on the wake centerline since the wake centerline faces the influence of the von Kármán vortices from both sides of the wake, causing a harmonic frequency of $2f_0$. On the other hand, as seen from the plot of e_n , the high frequency noise is constant across the wake. e_n is calculated by taking the integral of Eq. (3) for the frequencies $f > 2f_0$.

Having determined the energy intensities at specific frequencies, it is worth considering the energy contents of the primary and secondary vortices, namely, e_0 and e_1 , and comparing them with the total energy intensity for a better understanding of the wake.

Figure 9(b) shows a comparison of the total energy intensity e and the sum of energies at the primary and secondary frequencies $e_0 + e_1$. It is clear that $e_0 + e_1$ follows the e curve with an almost constant deviation due to the noise and higher harmonic contributions. The energy intensity e_0 at the von Kármán vortex shedding frequency in the upper half of the wake is less than the energy intensity in the lower half. The excess energy is stored in e_1 in the upper half of the wake. Thus, some portion of the energy at the von Kármán vortex shedding frequency f_0 is transferred to the subharmonic frequency f_1 in the upper half of the wake. This is due to the fact that the secondary vortices are originating from the upper von Kármán vortices only. Hence, there is no energy content at the secondary frequency f_1 , i.e., $e_1 \approx 0$, for $y/D < 0$. Figure 10 demonstrates the variation of the integrated wake energy E in the wake of a wired cylinder for Reynolds numbers $Re = 180$ and $Re = 225$ and for a non-wired cylinder at $Re = 185$.

FIG. 10. Downstream variation of total wake energy E for wired and non-wired cylinders.

FIG. 11. Downstream variation of total $(E_0 + E_1)/E$ and discrete energy ratios E_0/E and E_1/E in the wake of a wired cylinder at $Re = 180$ and $Re = 225$.

The discrete energy components exhibit different behaviors, as shown in Figure 11. The downstream variation of the ratio of the sum of discrete energies to the total wake energy, $(E_0 + E_1)/E$, does not change with Reynolds number in the Mode-C transition regime. E_0/E and E_1/E are measures of the contribution of von Kármán vortices and secondary vortices to the total integrated energy. More than 50%–60% of the wake energy is coming from the von Kármán vortices but the percentage in the total energy is decreasing with increasing Reynolds number and downstream position. On the other hand, the percentage of the contribution of secondary vortices in the total energy is increasing for $Re = 225$ and almost constant for $Re = 180$. The energy trade between primary and secondary vortices can be further investigated by looking into their vortex strengths and trajectory deviations due to this relationship.

V. VORTEX STRENGTHS AND TRAJECTORIES

A. Spanwise vortices

The instantaneous spanwise vorticity $\omega_z D/U_\infty$ iso-contours obtained from side-view PIV experiments at different Reynolds numbers are shown in Figure 12. It demonstrates the vorticity structure at different stages of Mode-C transition. The images are chosen at approximately the same phase in the shedding cycle. The figure shows that the overall layout of the flow structure in Mode-C transition does not change with increasing Reynolds number in the range $180 \leq Re \leq 244$. The vorticity field for $Re = 244$ is not shown, though it showed the same flow features in a slightly more disordered manner.

The discussion about the vortex trajectories is further extended in Figure 14 which demonstrates how the shapes of the different trajectories are associated with the von Kármán vortices at $Re = 180$ in the wake of a wired cylinder. The vortices are identified using the λ_2 method.^{29,30} The boundary of a vortex is represented by a closed contour line of $\lambda_2 = -0.1$. The vortices are tracked in time for 6–10 shedding periods. The incomplete vortex trajectories in Figure 13 correspond either to the beginning or the end of the data.

The vortex trajectories are superposed on the instantaneous spanwise vorticity fields at two instants which are T_{shed} apart from each other, where T_{shed} is the shedding period of the von Kármán vortices. The corresponding trajectories of each von Kármán vortex are marked with an arrow. The snapshots are chosen such that they show the wake just after the formation of an upper von Kármán vortex. The subscripts 1 and 2 refer to the two distinct vortices of upper and lower trajectories UT_1 , UT_2 , LT_1 , and LT_2 . Trajectory UT_1 is associated with the upper vortex which has a compact center region and two elongated braids, as shown in Figure 14(a). The concentrated vorticity in the vortex center can be noted from the color contrast in the vorticity contours. The succeeding lower vortex follows the trajectory LT_1 . At the phase $t = t_0 + T_{shed}$, the upper vortex which follows the trajectory UT_2 is formed. This vortex has a different shape as the former one. It does not have long braids and has a more diffused vorticity concentration in the center. This upper vortex is the one from which

FIG. 12. Instantaneous non-dimensional spanwise vorticity patterns $\omega_z D/U_\infty$ in the wake of the wired cylinder. Contour levels are $|\omega_z| D/U_\infty = 0.4, 0.8, \dots, 4$. Solid line (blue online) and dashed line (red online) contours indicate positive and negative vorticity, respectively.

the secondary Mode-C vortices are originating and is succeeded by a lower vortex which follows the curved trajectory of LT_2 .

The effect of the wire on the trajectories and strengths of von Kármán vortices in the transition regime $Re \geq 180$ is shown in Figure 13. As illustrated in Figure 13, there are two different trajectories taken by the upper and lower vortices. They are denoted in the uppermost left plot of Figure 13. UT_1 and UT_2 point out the trajectories of upper vortices and LT_1 and LT_2 the trajectories of the lower vortices. Despite some deviations, the same trajectory patterns appear for all Reynolds numbers considered and UT_1 , UT_2 , LT_1 , and LT_2 almost preserve their shapes.

It is noted that all vortices except the ones that follow UT_2 are formed almost at the same downstream position $x/D \approx 2$. The vortices of UT_1 are formed at a position approximately $0.6D$

FIG. 13. Vortex trajectories and strengths of von Kármán vortices in the wake of a wired cylinder as a function of Reynolds number. (UT_1 , UT_2 : trajectories of upper vortices and LT_1 , LT_2 : trajectories of lower vortices.) Note that the upper vortices have a negative circulation value.

further upstream and $0.25D$ higher as compared to those of UT_2 . They go upwards until $x/D \approx 4$ and then downwards until they reach the trajectory UT_2 . So, the vortex trajectory UT_1 has a curved shape with a high peak in the near-wake region. On the other hand, the trajectory UT_2 has a rather linear shape. The vortices following the UT_2 trajectory have a steady upward motion for $Re = 180$ and for $x/D > 7$ the UT_2 trajectory almost coincides with the UT_1 trajectory. For the other Reynolds numbers presented in Figure 13, UT_2 has a slightly curved shape but not as pronounced as UT_1 .

In the lower half of the wake, LT_1 has a similar tendency as UT_2 but in the opposite direction. It has a quasi-linear shape with a steady downwards deflection. LT_2 has the same curved shape

FIG. 14. Vortex trajectories of von Kármán vortices at $Re = 180$ in the wake of a wired cylinder. Vortex trajectories are superposed on the instantaneous spanwise vorticity field at two instants which are T_{shed} apart from each other. The corresponding trajectory of each vortex is shown with an arrow. Contour levels are $|\omega_z|D/U_\infty = 0.4, 0.8, \dots, 4$. Darker gray and lighter gray contours (blue and red, respectively, online) indicate positive and negative vorticity, respectively. The solid lines represent the contour line of $\lambda_2 = -0.1$. (a) $t = t_0$; (b) $t = t_0 + T_{shed}$.

like UT_1 but for the lower half of the wake. Vortices of LT_2 follow a downwards trajectory until $x/D \approx 5.5$ – 6.0 and a slightly upwards trajectory further downstream.

Unlike in the vortex trajectories, the downstream variation of the vortex strength Γ_z of the von Kármán vortices does not show a significant difference between the upper and lower wakes. Comparison of results from different Reynolds numbers indicate that the tendency of the circulation values as a function of downstream position x/D is almost the same. However, there is a clear difference between the strengths of upper and lower von Kármán vortices. The circulation values and the difference between upper and lower vortices at downstream position $x/D = 10$ are given in Table III for different Reynolds numbers. As quantified in the table, the strength difference between the upper and lower von Kármán vortices is in the order of $\mathcal{O}(10\%)$. This suggests that during Mode-C transition the upper vortices are weaker than the lower vortices, since some part of the vorticity in the upper shear layer is transferred into secondary vortices.

B. Streamwise vortices

The strengths of the secondary vortices associated with Mode-C transition are calculated from upstream-view PIV experiments. Figure 15 shows the resulting velocity vectors and streamwise vorticity distribution at the cross-stream plane of $x/D = 4.5$ for a Reynolds number of $Re = 180$. Each vortex is defined with a $\lambda_2 = -0.1$ contour. The circulation Γ_x of the corresponding streamwise

TABLE III. Circulation values of von Kármán vortices at downstream position $x/D = 10$. Circulation values in the table are non-dimensionalized with $1/DU_\infty$.

Re	$ \Gamma_U $	Γ_L	$ \Gamma_U - \Gamma_L$	Difference (%)
180	1.521	2.284	-0.763	-33.4
195	1.270	1.938	-0.668	-34.5
210	1.512	1.773	-0.261	-14.7
225	1.337	1.518	-0.181	-11.9
244	1.372	1.624	-0.252	-15.5

vortical structures are calculated in the same manner as for the spanwise vortices. In Figure 15, Γ_x^+ and Γ_x^- indicate secondary vortices with positive and negative circulation, respectively. Γ_x^+ implies counter-clock-wise and Γ_x^- implies clock-wise rotation. The strength of the vortices is calculated at the instant at which they are at their highest position with respect to the wake centerline $y/D = 0$. This instant corresponds to the vortex shedding phase where the measurement plane is between the upper and lower primary vortices, see Figure 5, for an example. In Figure 15, the spanwise distance between the vortices of the same sign determines the spanwise wavelength λ_z of Mode-C transition.

Figure 16 shows the variation of the strengths of the secondary vortices with Reynolds number and downstream position. The overall strength Γ_x is calculated by taking the average of Γ_x^+ and $|\Gamma_x^-|$ in several snapshots for each upstream-view PIV experiment. The standard deviation of the circulation calculation ($\sigma/DU_\infty = 0.057$) is shown as an error bar for a representative case. The data show a clear outlier for the case $Re = 180$ and $x/D = 3.5$. Nevertheless, it might be concluded that Γ_x is increasing almost linearly with the Reynolds number for $1.5 < x/D < 4.5$ in the near-wake. The average strengths of the Mode-C vortices are $\Gamma_x/DU_\infty = 0.40$ for $Re = 180$ and $\Gamma_x/DU_\infty = 0.56$ for $Re = 244$.

The results of the spanwise wavelength calculation of Mode-C transition are shown in Figure 17. Although the data show a scatter of λ_z values, it appears that λ_z is decreasing slightly with increasing Reynolds number. The standard deviation of the calculation of the spanwise wavelength is $\sigma/D = 0.15$ and is shown as an error bar for a representative case in Figure 17. The average spanwise wavelength values for each corresponding Reynolds number are calculated as $\lambda_z/D = 2.28, 2.20, 2.17, 2.06, 2.10$ for $Re = 180, 195, 210, 225, 244$, respectively. On the other hand, λ_z seems to increase with downstream position x/D . For example, the average λ_z values at each downstream location is $\lambda_z/D = 2.02, 2.13, 2.21, 2.27$ for $x/D = 1.5, 2.5, 3.5, 4.5$, respectively. Most likely, this increase is caused by the end conditions imposed by the end plates. The spanwise wavelength of Mode-C transition for the wired cylinder case is calculated by averaging all the values given in

FIG. 15. Velocity vectors and streamwise vorticity patterns at the cross-stream plane of $x/D = 4.5$ at $Re = 180$. Contour levels of vorticity are $|\omega_x|DU_\infty = 0.2, 0.4, \dots, 2$. Γ_x^+ and Γ_x^- indicate secondary vortices with positive and negative circulation, respectively. λ_z denotes the spanwise wavelength of the secondary vortices and is measured between the centers of vortices of the same sign.

FIG. 16. Variation of strengths of secondary vortices Γ_x / DU_∞ with Reynolds number Re and downstream position x/D in the near-wake.

Figure 17 and is found to be

$$\frac{\lambda_z}{D} = 2.2. \quad (6)$$

The spanwise wavelength of $1.8D$ is reported by Zhang *et al.*⁷ for the Mode-C instability behind circular cylinders. Considering the experimental uncertainty, it can be said that the results presented above matches the previous observations. Furthermore, Sheard *et al.*³¹ mentioned that a spanwise wavelength of 2.6 diameters for the Mode-C in square cylinders. However, in stability calculations of the flow around bluff rings, Sheard *et al.*⁸ found a maximum growth rate for Mode-C instability for spanwise wavelengths between $1.6D$ and $1.7D$.

The quantification of the relationship between secondary and primary vortices is enabled by comparing their strengths. In Figure 18, the variation of the strength ratio Γ_x / Γ_z with Reynolds number is shown for the downstream positions $x/D = 2.5, 3.5, 4.5$. The strength of the upper von Kármán vortex is used in the calculations, since the Mode-C vortices originate from the upper vortices. It can be seen that the circulation of the secondary vortices is significantly smaller than the circulation of the primary von Kármán vortices. The strength of the Mode-C vortices is $\approx 20\%$ of the strength of the von Kármán vortices for $Re = 180$ and is $\approx 30\%$ for $Re = 244$. Discarding the outlier data point at $Re = 180$ and $x/D = 3.5$, one may conclude that Γ_x / Γ_z varies linearly with the

FIG. 17. Variation of spanwise wavelength of secondary vortices λ_z / D with Reynolds number Re and downstream position x/D in the near-wake.

FIG. 18. Variation of strength ratio Γ_x/Γ_z of secondary and primary vortices with Reynolds number Re and downstream position x/D in the near-wake.

Reynolds number. This result indicates that the strength of secondary vortices is increasing faster than the strength of primary vortices during Mode-C transition.

VI. CONCLUDING REMARKS

In this paper, the characterization of Mode-C transition is extended by investigating and quantifying observations such as the energy content of the wake, the vortex trajectories of von Kármán vortices, and the spanwise wavelength of Mode-C transition. These analyses are done mainly by focusing on fluctuating flow features, energy characteristics, and vortex dynamics. The data used in the analysis are obtained from several side-view and upstream-view PIV measurements.

First of all, the effect of the wire on temporal characteristics is investigated by evaluating the variation of rms-velocity characteristics of the wake. It is shown that the u_{rms}/U_∞ profile in the upper half of the wake is broadened. Peak u_{rms} values do not change significantly in the wired cases but they are reduced by approximately 10%–15% compared to the non-wired cases. Moreover, they stay almost constant throughout the Reynolds number range $Re = 180$ – 225 .

Second, discrete energy components are calculated at different frequencies of the energy spectrum. The effect of the wire is evident in the cross-stream plot of energy intensity at subharmonic frequency $f_1 \approx f_0/2$. There is a clear peak in the energy intensity of f_1 in the upper half of the wake around $y/D = 1.25$. The energy intensity at the von Kármán vortex shedding frequency in the upper half of the wake is less than the energy intensity in the lower half, meaning that the excess energy is stored at subharmonic frequency $f_1 \approx f_0/2$. Furthermore, from the downstream variation of the discrete integrated energies, one may conclude that von Kármán vortices decay faster than secondary Mode-C vortices.

Finally, the vortex dynamics in Mode-C transition is investigated. It is shown that the period-doubling nature of the transition also exhibits itself in the vortex trajectories. In non-wired cylinder flows there is one single trajectory describing the path of the von Kármán vortices for each half of the wake. However, for the wired cylinder case (Mode-C transition), there are two different trajectories for both upper and lower vortices. The strength difference between the upper and lower von Kármán vortices is in the order of $\mathcal{O}(10\%)$, which suggests that during Mode-C transition the upper vortices become weaker than the lower vortices, since some part of the vorticity in the upper shear layer is transferred into the secondary vortices. Most probably the distinct trajectories of the lower vortices are highly linked to the distinct trajectories of the upper vortices, which on their turn are linked to the sub-harmonic appearance of the Mode-C structures causing a strength difference between the upper and lower vortices.

The vortex strength calculations of secondary vortices indicate that the strength of the secondary vortices, Γ_x , is increasing almost linearly with Reynolds number Re . The strength of secondary

Mode-C vortices is $\approx 20\%$ of the strength of von Kármán vortices for $Re = 180$ and is $\approx 30\%$ for $Re = 244$. This result suggests that the growth rate of secondary vortices is higher than the growth rate of primary vortices during Mode-C transition. Finally, a quantification of the spanwise wavelength of Mode-C instability is done. The spanwise wavelength of Mode-C transition for the wired cylinder case is calculated as $\lambda_z/D = 2.2$.

The energy transfer occurs through a connection between the spanwise and streamwise vortices. From visualization experiments it was observed that the Mode-C structures are formed as secondary streamwise vortices around the primary von Karman vortices. In fact, the secondary vortices in Mode-C transition are actually the vortex loops that originate from the upper vortex. These vortex loops are further stretched in the braid region and roll-up to form streamwise vortex pairs from the sides of the loops. They are located between the upper and lower vortices and affect the near-wake vortex shedding process. The physics behind the coupling of the spanwise and the streamwise vortices is documented elsewhere Yildirim.²⁵

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