Lagrangian single-particle turbulent statistics through the Hilbert-Huang transform

Citation for published version (APA):

DOI:
10.1103/PhysRevE.87.041003

Document status and date:
Published: 01/01/2013

Document Version:
Publisher’s PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:
• A submitted manuscript is the author's version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher’s website.
• The final author version and the galley proof are versions of the publication after peer review.
• The final published version features the final layout of the paper including the volume, issue and page numbers.

Link to publication

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

• Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
• You may not further distribute the material or use it for any profit-making activity or commercial gain
• You may freely distribute the URL identifying the publication in the public portal.

Take down policy
If you believe that this document breaches copyright please contact us:
openaccess@tue.nl
providing details. We will immediately remove access to the work pending the investigation of your claim.

Download date: 04. Feb. 2019
Lagrangian single-particle turbulent statistics through the Hilbert-Huang transform

Yongxiang Huang (黄永祥),¹,₂ Luca Biferale,² Enrico Calzavarini,¹ Chao Sun (孙超),³ and Federico Toschi⁴
¹Shanghai Institute of Applied Mathematics and Mechanics, Shanghai Key Laboratory of Mechanics in Energy Engineering, Shanghai University, Shanghai 200072, People’s Republic of China
²Department of Physics and INFN, University of Tor Vergata, Via della Ricerca Scientifica 1, I-00133 Roma, Italy
³Laboratoire de Mécanique de Lille, CNRS/UMR 8107, Université Lille I, F-59650 Villeneuve d’Ascq, France
⁴Physics of Fluids Group, Faculty of Science and Technology, J. M. Burgers Centre for Fluid Dynamics, University of Twente, P.O. Box 217, 7500 AE Enschede, The Netherlands
⁵Department of Physics, and Department of Mathematics and Computer Science and J.M. Burgerscentrum, Eindhoven University of Technology, NL-5600 MB Eindhoven, The Netherlands

(Received 21 December 2012; published 22 April 2013)

The Hilbert-Huang transform is applied to analyze single-particle Lagrangian velocity data from numerical simulations of hydrodynamic turbulence. The velocity trajectory is described in terms of a set of intrinsic mode functions $C_i(t)$ and of their instantaneous frequency $\omega_i(t)$. On the basis of this decomposition we define the $\omega$-conditioned statistical moments of the $C_i$ modes, named $q$-order Hilbert spectra (HS). We show that such quantities have enhanced scaling properties as compared to traditional Fourier transform- or correlation-based (structure functions) statistical indicators, thus providing better insights into the turbulent energy transfer process.

The statistical description of a tracer trajectory in turbulent flows still lacks a sound theoretical and phenomenological understanding [1,2]. Presently, analytical results linking the Navier-Stokes equations to the statistics of the velocity increments $v(t + \tau) - v(t)$ along the particle evolution are missing.

The temporal evolution of the velocity field along a Lagrangian trajectory in turbulent flows is strongly influenced by the presence of small-scale vortex filaments inducing visible high-frequency oscillations even on the single-particle velocity signal (see Fig. 1 and Ref. [19]).

HHT has been recently applied to analyze Eulerian turbulent data [26–28], showing an unexpected ability to disentangle multiscale contributions. The main interest in HHT lies in its frequency-amplitude adaptive nature, being based on the decomposition of the original signal on a set of quasieigenmodes.
that are not defined a priori [29,30]. The idea is to not introduce in the analysis any systematic predefined structures as it always happens using Fourier-based methodologies (e.g., Fourier decomposition or wavelet transforms).

In this Rapid Communication, we apply and generalize the HHT methodology to extract the hierarchy of the Lagrangian scaling exponent $\zeta_L(q)$. The method is applied to the fluid tracer trajectories, each composed of $N = 4720$ time samplings of $v_j(t)$ (where $j = 1, 2, 3$ denotes the three velocity components) saved every $0.1\tau_\eta$ time units. Therefore, we can access the time scale from $0 < \tau / \tau_\eta < 236$, corresponding to the frequency range $0.004 < \omega \tau_\eta < 10$.

The HHT is a procedure composed of two steps. The first step is the decomposition of the signal into its intrinsic mode functions (IMFs) followed by the Hilbert transform on such modes. In the first step, through a procedure called empirical mode decomposition (EMD), we decompose each velocity time series into the sum

$$v(t) = \sum_{i=1}^{n} C_i(t) + r_n(t),$$

where $C_i(t)$ are the IMFs and $r_n(t)$ is a small residual, an almost constant function characterized by having at most one extreme along the whole trajectory (which will therefore be neglected in the following analysis) [29,30]. In Eq. (2) $n$ may depend on the trajectory, with a maximum value which is limited to its length as $n_{\text{max}} = \log_2(N) \approx 12$. Given the actual length of our trajectories, with $n \approx 6–7$, we are typically able to reconstruct the full behaviors (see Fig. 1).

To be an IMF, each $C_i(t)$ must satisfy the following two conditions: (1) The difference between the number of local extrema and the number of zero crossings must be zero or one; and (2) the running mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero. Indeed, the IMF is an approximation of the so-called monocomponent signal, which possesses a well defined instantaneous frequency [29,32]. The physical meaning of such decomposition is clear: We want to decompose the original trajectory into quasieigenmodes with locally homogeneous oscillating properties [29,33]. In the second step, one performs a Hilbert transform for each of the IMFs,

$$\bar{C}_i(t) = \frac{1}{\pi} P \int_0^\infty \frac{C_i(t')}{t - t'} dt',$$

where P stands for the Cauchy principal value. This allows to retrieve the instantaneous frequency associated to each $C_i$ via

$$\bar{\omega}_i(t) = \frac{1}{2\pi} \frac{d}{dt} \arctan \left( \frac{\bar{C}_i(t)}{C_i(t)} \right),$$

[29]. Therefore, we construct the pair of functions $[C_i(t), \bar{\omega}_i(t)]$ for all IMF modes, and this concludes the standard HHT procedure. Let us stress again the fully adaptive nature of the HHT: The IMFs are not defined a priori, and they accommodate the oscillatory degree of the analyzed signal without postulating systematic “structures” [29,30]. The most important consequence is that the HHT is typically free of subharmonics [23,27,28]. Here, in order to investigate the amplitude of turbulent velocity fluctuations versus their characteristic frequency, we define the $q$-dependent $\omega$-statistical moment $L_q(\omega)$ by computing the moments of each IMF conditioned on those instants of time where the corresponding instantaneous frequency has a given value $\bar{\omega}_i(t) = \omega$,

$$L_q(\omega) \equiv \sum_{i=1}^{n} |\bar{C}_i|^q |\bar{\omega}_i|,$$

where $q \geq 0$ is a real number, and with $\{ \cdot \}$ we denote time and ensemble averaging over different trajectory realizations. We call it the Hilbert spectrum (HS) of order $q$. Let us notice that each HS can be seen as a superimposition of spectra obtained from different IMFs.

From a dimensional point of view the simplest link between the instantaneous frequency $\omega$ and the coherence time of
In Fig. 2 we show the second-order HS, as a function of its coherence time or characteristic frequency.

0 and a lognormal signal with an intermittent parameter validated by using both fractional Brownian motion with compensated behavior of the Fourier spectrum, \( S_2(\omega) \). Contributions from each IMF, \( \langle \xi_i \rangle \), are superimposed with the contributions from each different IMF. In order to better compare the HS to order. As one can see, only the whole reconstructed HS shows superimposed with the contributions from each different IMF.

An eddy \( \tau \) is the reciprocal relation \( \omega \sim \tau^{-1} \). Therefore, we postulate for the general HS of order \( q \) a scaling relation of the form

\[
\mathcal{L}_q(\omega) \sim \omega^{-\xi_L(q)},
\]

here, \( \xi_L(q) \) must be compared with the scaling exponent provided by the LSF [28]. The above scaling relation was validated by using both fractional Brownian motion with various Hurst numbers \( 0 < H < 1 \) for monofractal processes and a lognormal signal with an intermittent parameter \( \mu = 0.15 \) as an example of a multifractal process. For all cases, the scaling exponents provided by the HHT agree with the ones derived by the standard SMF method and with the theoretical ones [28]. To begin with, we focus on the case \( q = 2 \), that, as mentioned, is related to the amplitude of energy fluctuations as a function of its coherence time or characteristic frequency. In Fig. 2 we show the second-order HS, \( \mathcal{L}_2(\omega) \) vs \( \omega \) in log-log, superimposed with the contributions from each different IMF order. As one can see, only the whole reconstructed HS shows a good scaling behavior. In order to better compare the HS to LSF curves we plot them in Fig. 3 in compensated form in such a way that the expected behavior in the inertial range would be given by a constant, respectively, \( S_2(\tau)/(\epsilon \tau) \sim \tau \) and \( \mathcal{L}_2(\omega)e^{-\omega \tau} \) vs \( 1/\omega \). For completeness, in the same figure, the compensated behavior of the Fourier spectrum, \( E(f)e^{-f^2/\epsilon^2} \) vs \( 1/f \), is also provided. The first striking difference between HS and LSF or Fourier is the enhanced scaling property of the new quantity. We also note that the shape of the LSF curve is consistent with the one in Refs. [8,9], where no plateau was observed in the inertial range. On the compensated scale the Fourier spectrum behaves better than the LSF, but the range of scaling is about half of that of the Hilbert spectrum. Such a difference is even more evident when the logarithmic local slopes are compared (see the inset of Fig. 3). A clear inertial scaling range, \( 0.01 < \tau \eta \sim 0.2 \), corresponding to an interval of time scales \( 5 < \tau \eta \sim 100 \), is observed for the compensated \( \mathcal{L}_2 \). The reason why LSF fails in displaying scaling is that it mixes low (infrared, IR)/high (ultraviolet, UV) frequency fluctuations to the ones in the inertial range. On the compensated scale the low-frequency contributions, and \( \mathcal{L}_2(\omega) \sim \omega^{-\xi_L(2)} \), with \( \xi_L(2) = 1 \).

UV) frequency fluctuations to the ones in the inertial range \(-[10^{-2}, 10^{-1}] \epsilon \). This becomes explicit when considering the relation \( S_2(\tau) \sim \epsilon \epsilon/\tau \), \( E(f) \sim f^{-1} \), \( E(f) \sim f^{-1} \epsilon(\tau^{2}+\epsilon) \), and \( \mathcal{L}_2(\omega) \sim \omega^{-\xi_L(2)} \), with \( \xi_L(2) = 1 \).

\[
R_{\tau \eta}^{f}(f) \equiv S_2(\tau^{-1}) \int_{f_{\eta}}^{f_{\epsilon}} E(f')(1 - \cos(2\pi f' \tau))df',
\]

(7)

which measures the relative contributions to \( S_2(\tau) \) from the frequency range \([f_{\eta}, f_{\epsilon}] \). When such an interval is set to \([0, 10^{-2}] \epsilon \) we get the low-frequency contributions, and with \([10^{-1}, +\infty] \epsilon \) the high ones. In Fig. 4, we show that such spurious nonlocal contributions can be as high as 80%.
First, let us notice the evident departure from the dimensional same analysis on random subsets with $1/2$ (in log scale) of the fitted frequency range. Note that the indicated errors are larger than the estimated statistical errors. The numerical values for the exponents are estimated as the average of the logarithmic local slope empirically measured with the multifractal prediction $\xi_{L}(q)$ without applying ESS. Our measurements provide a solid confirmation to the predictions of the multifractal model. The Hilbert method we applied here is general and can be applied to other systems with multiscale dynamics, e.g., Rayleigh-Bénard convection [35], two-dimensional turbulence [36,37].

This work is sponsored by the National Natural Science Foundation of China (Grants No. 11072139, No. 11032007, and No. 11202122), “Pu Jiang” project of Shanghai (No. 12PJ1403500), the Shanghai Program for Innovative Research Team in Universities, COST Action MP0806 “Particles in turbulence,” and in part by the Foundation for Fundamental Research on Matter (FOM), which is part of the Netherlands Organization for Scientific Research (NWO). The DNS data used in this study are freely available from the iCFD database [38]. We thank Dr. G. Rilling and Prof. P. Flandrin for sharing their EMD code, which is available at [39].

\[ \xi_{L}(q) = d \log L_{q}(\omega)/d \log \omega \text{ with the multifractal prediction } \xi_{L}^{MF}(q). \]

The HS functions $L_{q}(\omega)$ have good scaling properties also for other $q$ orders. We calculated $L_{q}(\omega)$ for the orders $q = 1, 2, 3, 4$, and empirically found a good power law behavior in the range $0.01 < \omega \tau_{q} < 0.1$ (respectively $10 < \tau / \tau_{q} < 100$), as shown in Fig. 5. This allows to extract the scaling exponents directly in the instantaneous frequency space, without resorting to the above mentioned ESS procedure. The numerical values for the $\xi_{L}(q)$ extracted from the fit in the range $0.01 < \omega \tau_{q} < 0.1$ are reported in Table I. The values of the scaling exponents are estimated as the average of the logarithmic local slope $\xi_{L}(q, \omega) = d \log L_{q}(\omega)/d \log \omega$ on the above interval and the error bars as the difference between the averages taken on only the first or the second half (in log scale) of the fitted frequency range. Note that the indicated errors are larger than the estimated statistical errors. Here, statistical convergence was checked by performing the same analysis on random subsets with $1/64$ of the total data. First, let us notice the evident departure from the dimensional estimate (named K41 [34]), $\xi_{L}^{K41}(q) = q/2$. Second, the measured values are in good agreement with the prediction given by the multifractal model $\xi_{L}^{MF}[12]$. In order to better appreciate the quality of our scaling, we show in the inset of Fig. 5 the logarithmic local slope empirically measured with the HHT, $\xi_{L}(q, \omega)$, compensated with the predicted value from the multifractal phenomenology, such that a plateau around the value 1 is the indication of the existence of an intermittent multifractal power law behavior.

In summary, we have presented a Hilbert-Huang transform-based methodology to capture the intermittent nature of the turbulent Lagrangian velocity fluctuations. Our test bench has been a numerical database of homogeneous isotropic turbulence at $Re_{\lambda} = 400$. The first result is that for the second-order statistical moment $L_{q}(\omega)$, an energy-like quantity, we observe a clear inertial range versus time defined as $\tau = \omega^{-1}$ for at least one decade, in the range $0.01 < \omega \tau_{q} < 0.2$. Such clean scaling can not be highlighted using more standard methods. Second, we extracted the hierarchy of the scaling exponent $\xi_{L}(q)$ without applying ESS. Our measurements provide a solid confirmation to the predictions of the multifractal model. The Hilbert method we applied here is general and can be applied to other systems with multiscale dynamics, e.g., Rayleigh-Bénard convection [35], two-dimensional turbulence [36,37].

This work is sponsored by the National Natural Science Foundation of China (Grants No. 11072139, No. 11032007, and No. 11202122), “Pu Jiang” project of Shanghai (No. 12PJ1403500), the Shanghai Program for Innovative Research Team in Universities, COST Action MP0806 “Particles in turbulence,” and in part by the Foundation for Fundamental Research on Matter (FOM), which is part of the Netherlands Organization for Scientific Research (NWO). The DNS data used in this study are freely available from the iCFD database [38]. We thank Dr. G. Rilling and Prof. P. Flandrin for sharing their EMD code, which is available at [39].

\[ \begin{array}{cccc}
q = 1 & q = 2 & q = 3 & q = 4 \\
\xi_{L}^{K41}(q) & 0.5 & 1.0 & 1.5 & 2.0 \\
\xi_{L}^{MF}(q) & 0.55 & 1.0 & 1.38 & 1.71 \\
\xi_{L}^{HS}(q) & 0.59 \pm 0.06 & 1.03 \pm 0.03 & 1.39 \pm 0.10 & 1.70 \pm 0.14 \\
\end{array} \]

\[ \begin{array}{c}
\text{Table I. Lagrangian scaling exponents } \xi_{L}(q) \text{ for orders } q = 1.4 \text{ as estimated from dimensional analysis } q/2 \text{ (K41), from the multifractal model (MF) [12], and as obtained here from Hilbert spectra (HS).} \\
\end{array} \]
[38] http://cfd.cineca.it.