Partition effects in transverse electron-beam waves

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PARTITION EFFECTS IN TRANSVERSE ELECTRON-BEAM WAVES

PROEFSCHRIFT

TER VERKRIJGING VAN DE GRAAD VAN DOCTOR
IN DE TECHNISCHE WETENSCHAPPEN AAN DE
TECHNISCHE HOGESCHOOL TE EINDHOVEN,
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PAUL ANTON HERMAN HART
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PROMOTOR: PROF. DR. H. GROENDIJK

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1. INTRODUCTION

The innovation of the beam-type parametric amplifier, the so-called Adler-Wade low noise tube ¹⁾²⁾³⁾, has strongly stimulated the interest in transverse waves on a beam focussed by an axial magnetic field. Quite a number of devices using these waves have since been proposed. It must be admitted, however, that the rather intricate nature of the devices has prevented any large scale application up to now. The object of this thesis is to study theoretically and verify experimentally what happens to a transverse wave signal and what noise is generated if the electron beam is partly intercepted. In practice interception takes place in an electron gun that employs one or more narrow-aperture electrodes to avoid the use of a small diameter cathode or to improve the performance of the gun by some kind of collimation. Also unwanted interception in the amplifier electrode system may occur, which can have a deleterious effect upon the noise figure of the tube.

The propagation of signals along a beam with a finite non-zero diameter can be described by means of the "disc" model of Gordon ⁴⁾. According to this model the beam may be thought of as being made up of thin discs, each contained within two transverse cross-sections and having no coupling. The signal propagation can be described in terms of four transverse waves ⁴⁾⁵⁾. The amplitude of the waves is associated with the motion of the centre of mass of the discs rather than directly with the motion of the individual electrons. Inside the discs the electrons execute motions with respect to this centre of mass. According to the laws of classical mechanics this "internal" motion and the centre-of-mass motion are independent provided the same electrons remain in the disc and no inhomogeneous external forces are exerted on the beam.

Interception, scattering of electrons, and spread in axial velocity destroy this complete separation, resulting in a loss of signal power and a generation of noise in every mode of propagation. In the case of interception the number of electrons in each disc is reduced. If scattering occurs then the momentum and displacement of some of the electrons in the disc are changed in a random way. The effect of an axial velocity spread is that after the beam has been allowed to drift over some distance, some electrons in a given disc have been replaced by others from neighbouring discs.

In all cases a displacement of the centre of mass results and its velocity is changed. The motion of the new centre of mass can be conveniently described in terms of two components. One component is simply the motion that corresponds to the undisturbed disc motion, the other describes the perturbation. In all three cases the number of electrons contributing to the first component of the motion is reduced. Consequently the amplitude of the signals is reduced. The second component, describing the perturbation, will show a fluctuation due to the internal motion of the electrons in the beam and the occurrence of a

partition noise current in case of interception or scattering. This fluctuation finds expression in a set of new noise waves.

The effect of spread in axial velocity on a single wave has been investigated by Lea Wilson ⁷⁾, Kompfner ⁹⁾, and in greater detail by Gordon ⁸⁾. A theory of interception and scattering has been given by the present author ⁶⁾. Here we shall extend the latter theory and apply the same mathematical technique for the case of axial velocity spread, thereby considering all the waves simultaneously and also their possible cross-correlation.

2. Characterisation of the waves

In this chapter we shall discuss the transverse waves on the beam by means of the "disc" model of Gordon ⁴⁾. The wave amplitude matrix and the power flow matrix necessary as tools in this thesis will be defined. Also we shall recall some physical properties of the waves. The motion of the individual electrons will be discussed in later chapters.

2.1 The transverse waves

Provided the external fields acting on the beam are constant over the beam's cross-section a given disc moves as a whole under the influence of these fields. The motion of the centre of mass can be found by the force equation:

$$\ddot{\mathbf{r}}_t = -\eta [\mathbf{E} + \dot{\mathbf{r}}_t \times \mathbf{B}], \quad (1)$$

where $\mathbf{E}$ and $\mathbf{B}$ represent the electric field and the magnetic induction vectors; η is the ratio of electron charge e to mass m and is a positive number; $\mathbf{r}_t = \mathbf{r}_1 x + \mathbf{r}_2 y + \mathbf{r}_3 z$; $\mathbf{r}_1$, $\mathbf{r}_2$ and $\mathbf{r}_3$ are the unit vectors in the x , y and z directions of the Cartesian system. In the following we shall choose the beam axis to coincide with the z axis of the system. The underlined quantities $\underline{x}$, $\underline{y}$ and $\underline{z}$ denote real values of the complex quantities x , y and z we shall introduce in the course of this section. If we assume that, with the exception of the axially homogeneous magnetic induction B_0 , the influence of magnetic fields can be neglected — which implies that the electron velocity is small compared to the velocity of light — eq. 1 reduces to

$$\ddot{x} + \eta E_x + \omega_c \dot{y} = 0, \quad (2)$$

$$\ddot{y} + \eta E_y - \omega_c \dot{x} = 0, \quad (3)$$

$$\ddot{z} + \eta E_z = 0, \quad (4)$$

and $\omega_c = \eta B_0$, ω_c is called the cyclotron angular frequency. Eqs. (2) and (3) describe interactions with transverse fields. Eq. (4) describes a longitudinal interaction. We will confine ourselves to a small-signal analysis with very nearly transverse fields. Therefore we can say that the longitudinal interaction will be

of second order. Hence $z \approx u_0 t$, where u_0 is the longitudinal drift velocity of the beam.

To solve eq. (2) and (3) we introduce complex functions of time and substitute, in accordance with Siegman ⁵⁾ $E_+ = \frac{1}{2}(E_x + jE_y)$, $E_- = \frac{1}{2}(E_x - jE_y)$, $A_1 = \frac{1}{2}jK(\dot{x} + j\dot{y})$, $A_2 = \frac{1}{2}jK(\dot{x} - j\dot{y})$, $A_3 = \frac{1}{2}jK[\dot{x} + j\dot{y} - j\omega_c(x + jy)]$ and $A_4 = \frac{1}{2}jK[\dot{x} - j\dot{y} + j\omega_c(x - jy)]$. The real constant K will be defined in sec. 2.3 in such a way that $A_i A_i^*$ represents the absolute value of the power flow. A_i^* denotes the complex conjugate of the i th mode amplitude A_i , $i = 1, 2, 3$ or 4 . Since $\dot{z} \approx u_0$ we can write $dA_i/dt \approx \partial A_i/\partial t + u_0 \partial A_i/\partial z$. Assuming complex solutions like $A_i = A_{i0} \exp[j(\omega t - \beta_i z)]$ we obtain

$$A_{10} \left[\beta_1 - \frac{\omega}{u_0} \left(1 - \frac{\omega_c}{\omega} \right) \right] = -\eta K E_+, \quad (5)$$

$$A_{20} \left[\beta_2 - \frac{\omega}{u_0} \left(1 + \frac{\omega_c}{\omega} \right) \right] = -\eta K E_-, \quad (6)$$

$$A_{30} \left[\beta_3 - \frac{\omega}{u_0} \right] = -\eta K E_+, \quad (7)$$

$$A_{40} \left[\beta_4 - \frac{\omega}{u_0} \right] = -\eta K E_-. \quad (8)$$

If the beam travels in free space, i.e. $E_+ = E_- = 0$, these equations are decoupled and the values found for β_i are real so that the solutions represent a set of four independent unattenuated waves. The phase velocity of the i th mode is given by $v_{pi} = \omega/\beta_i$, the group velocity by $v_{gi} = d\omega/d\beta_i = u_0$. The mode A_1 has a phase velocity larger than u_0 if $\omega > \omega_c$, and a negative one if $\omega < \omega_c$. In the literature this wave is called the "fast wave". The phase velocity of A_2 is always smaller than u_0 , this wave is the "slow wave". A_3 and A_4 are synchronous waves, since their phase velocities are equal to the beam velocity u_0 .

2.2 The wave matrix

In order to be able to treat all the waves simultaneously we use matrix notation and reformulate the relationship between the mode amplitudes A_i and the coördinates of section 2.1. Inspection of the elements shows that the transformation of the coördinates into the wave amplitudes can be given by:

$$\mathbf{A} = j\mathbf{K}\mathbf{N}\mathbf{X}, \quad (9)$$

$\mathbf{A}$ represents the column matrix with the elements A_i , $i = 1, 2, 3$ and 4 re-

spectively. $\mathbf{X}$ is the column matrix of the coördinates $\dot{x}$, $\dot{y}$, x and y , and $\mathbf{N}$ is given by

$$\mathbf{N} = \frac{1}{2} \begin{bmatrix} 1 & j & 0 & 0 \\ 1 & -j & 0 & 0 \\ 1 & j & -j\omega_c & \omega_c \\ 1 & -j & j\omega_c & \omega_c \end{bmatrix}.$$

Let us consider the shape assumed by the beam if a wave travels on it. The locus of the centre of mass of the discs is found by consideration of the transverse vector $\mathbf{r} = \mathbf{r}_1 x + \mathbf{r}_2 y$. The coördinates $\underline{x} = \text{Re}.x$ and $\underline{y} = \text{Re}.y$ follow from the inverse transformation of eq. (9);

$$\mathbf{X} = -jK^{-1}\mathbf{N}^{-1}\mathbf{A}, \quad (10)$$

and

$$\mathbf{N}^{-1} = 1/\omega_c \begin{bmatrix} \omega_c & \omega_c & 0 & 0 \\ -j\omega_c & j\omega_c & 0 & 0 \\ -j & +j & +j & -j \\ -1 & -1 & 1 & 1 \end{bmatrix}.$$

Examination of $\mathbf{X}$ as a function of z at a given instant $t = t_0$ shows that the beam assumes a helical shape for any single wave travelling on it. The pitch of the helix is $2\pi v_{pt}/\omega$ and the radius is

$$(\underline{x}_t^2 + \underline{y}_t^2) = \left(\frac{A_i A_i^*}{4\omega_c^2 K^2} \right)^{\frac{1}{2}}. \quad (11)$$

2.3 The power flow of the waves

The electric field in a practical transverse wave coupler having an axis of symmetry, possesses a longitudinal component except on the axis. For instance a field given by $E_x = E_0 \exp j(\omega t - \beta z)$ and $E_y = 0$ possesses a longitudinal field $E_z = -j\beta x E_0 \exp j(\omega t - \beta z)$ as a first approximation for small values of x as follows from Maxwell's equations. Under the influence of the axial electric field the axial velocity is modified. However, analysis shows that this is only a second order effect. In calculations of the phase constant of the waves it can be neglected if the signal amplitude is small. Even then, however, it has a profound influence on the power flow of the waves. The power flow has been calculated on this basis by a number of authors ⁴⁾⁵⁾. They find

$$P_i = p_i \eta I_0 \omega \omega_c (x_i^2 + y_i^2), \quad (12)$$

with $p_i = 1, -1, -1$ and 1 , for $i = 1, 2, 3$ or 4 respectively. The waves A_1 and A_4 carry a positive energy, which implies that a positive amount of power is required to set up these waves. The waves A_2 and A_3 carry a negative energy, which means that energy must be extracted from the beam to generate a wave

of this kind. Upon substitution of eq. (11) in eq. (12) and choosing

$$K = \left(\frac{I_0 \omega}{2 \eta \omega_c} \right)^{\frac{1}{2}}, \quad (13)$$

we find for the total power flow

$$P = \sum_{i=1}^4 p_i A_i A_i^*.$$

2.4. The power flow matrix

The total power flow can be rewritten in matrix form,

$$P = \text{trace } \mathbf{A} \mathbf{A}^\dagger \mathbf{p} = \mathbf{A}^\dagger \mathbf{p} \mathbf{A},$$

$\mathbf{A}^\dagger$ represents the hermitian conjugate of $\mathbf{A}$, $\mathbf{p} = \text{diag. } (1, -1, -1, 1)$. Haus¹⁰ calls $\mathbf{p}$ the parity matrix. To describe the noise behaviour of the beam we shall use the matrix $\mathbf{A} \mathbf{A}^\dagger \mathbf{p}$ instead of $\mathbf{A}^\dagger \mathbf{p} \mathbf{A}$; $\mathbf{A} \mathbf{A}^\dagger \mathbf{p}$ includes also the elements $A_i A_j^*$ and hence describes also the cross-correlation of the waves¹⁰⁾¹¹⁾¹²⁾,

$$\mathbf{A} \mathbf{A}^\dagger \mathbf{p} = \begin{bmatrix} A_1 A_1^* & -A_1 A_2^* & -A_1 A_3^* & A_1 A_4^* \\ A_2 A_1^* & -A_2 A_2^* & -A_2 A_3^* & A_2 A_4^* \\ A_3 A_1^* & -A_3 A_2^* & -A_3 A_3^* & A_3 A_4^* \\ A_4 A_1^* & -A_4 A_2^* & -A_4 A_3^* & A_4 A_4^* \end{bmatrix}. \quad (14)$$

3. Noise and internal motion

In general the electrons of the beam will have transverse velocities due to the thermal emission by the cathode, and due to electric and magnetic fields acting as lenses in the gun. The beam will also show shot noise. As a result the centre of mass of a disc has a fluctuating transverse displacement vector $\mathbf{r}$ and a fluctuating velocity vector $\dot{\mathbf{r}}$ even in the absence of signals on the beam. By means of well-known techniques these fluctuating quantities can be expanded in Fourier density spectra¹³⁾¹⁴⁾. By considering only a small frequency range $d\mathbf{f}$ at a time we can describe this noise in terms of a complex signal having a slowly varying random amplitude and phase and hence by means of the waves and the power flow matrix $\mathbf{A} \mathbf{A}^\dagger \mathbf{p}$ of the preceding section.

3.1 The noise waves

Let the transverse displacement and the velocity of the j th electron be denoted by $\mathbf{r}_j$ and $\dot{\mathbf{r}}_j$. When there are N electrons in a disc which is assumed to be very thin, the centre-of-mass vectors are given by

$$\mathbf{r} = \frac{1}{N} \sum_j^N \mathbf{r}_j, \quad (15)$$

and

$$\dot{\mathbf{r}} = \frac{1}{N} \sum_I^N \dot{\mathbf{r}}_j. \quad (16)$$

According to the central limit theorem ¹⁵⁾ the vectors $\mathbf{r}$ and $\dot{\mathbf{r}}$ follow a normal distribution, provided the motions of the electrons are uncorrelated and N is a large number. This applies regardless of the distribution functions of $\mathbf{r}_j$ and $\dot{\mathbf{r}}_j$. Further it follows from this theorem that:

$$\overline{r^2}^\tau = \frac{1}{N} \sum_I^N \overline{r_j^2}^\tau, \quad (17)$$

and

$$\overline{\dot{r}^2}^\tau = \frac{1}{N} \sum_I^N \overline{\dot{r}_j^2}^\tau, \quad (18)$$

r_j and $\dot{r}_j$ are the root mean square amplitudes of $\mathbf{r}_j$ and $\dot{\mathbf{r}}_j$.

The averages are taken over a long time τ . The Fourier density functions ¹⁴⁾ $\overline{r^2}$ and $\overline{\dot{r}^2}$ in the frequency range df are given by

$$\overline{r^2} = \frac{2 e df}{I_0} \overline{r_j^2}^\tau, \quad (19)$$

and

$$\overline{\dot{r}^2} = \frac{2 e df}{I_0} \overline{\dot{r}_j^2}^\tau, \quad (20)$$

I_0 being the beam current and e the electron charge. In accordance with $\mathbf{r} = \mathbf{r}_1 x + \mathbf{r}_2 y$ we have $r^2 = x x^* \cos^2(\arg x) + y y^* \cos^2(\arg y)$ and since the arguments of x and y are evenly distributed we find

$$\overline{r^2} = \frac{1}{2} (\overline{x x^*} + \overline{y y^*}), \quad (21)$$

and similarly

$$\overline{\dot{r}^2} = \frac{1}{2} (\overline{\dot{x} \dot{x}^*} + \overline{\dot{y} \dot{y}^*}). \quad (22)$$

The noise power flow expressed by the noise matrix $\mathbf{A} \mathbf{A}^\dagger \mathbf{p}$ can be found if the values of $\overline{r_j^2}$ and $\overline{\dot{r}_j^2}$ are known, and if $\mathbf{r}_j$ and $\dot{\mathbf{r}}_j$ of the electrons are uncorrelated, i.e. if $\overline{\dot{x} y^*}$ and similar terms are zero. Hence we obtain with the aid of eq. (9)

$$\mathbf{A} \mathbf{A}^\dagger \mathbf{p} = K^2 \mathbf{N} \mathbf{X} \mathbf{X}^\dagger \mathbf{N}^\dagger \mathbf{p} = \frac{1}{2} K^2 \mathbf{Q} \overline{\dot{r}^2} =$$

$$K^2 \frac{e}{I_0} \mathbf{Q} \overline{\dot{r}_j^2}^\tau df, \quad (23)$$

with

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \\ 1 & 0 & -(1+\delta^2) & 0 \\ 0 & -1 & 0 & (1+\delta^2) \end{bmatrix} \quad \text{and } \delta^2 = \omega_c^2 \cdot \overline{r_j^2}^\tau / \overline{\dot{r}_j^2}^\tau.$$

In case of a homogeneous beam with a circular cross-section having a diameter D $\overline{r_j^2}$ is given by ¹¹⁾

$$\overline{r_j^2}^\tau = \frac{1}{8} \Gamma^2 D^2, \quad (24)$$

Γ^2 represents the longitudinal shot noise suppression factor that is brought about by the space charge. If the electron density cannot be assumed constant over the beam's cross-section but can be approximated by a parabolical distribution, so that $dn(r) = 3N(1-4r^2D^{-2})/D$ represents the number of electrons in the interval between $r + dr$ and r we find, by using the same technique as has been used to derive eq. (24),

$$\overline{r_j^2}^\tau = \frac{1}{20} \Gamma^2 D^2. \quad (25)$$

Similarly for a hollow beam of circular cross-section having a constant electron density between an inner diameter D_1 and an outer diameter D_2 we find

$$\overline{r_j^2}^\tau = \frac{1}{8} \Gamma^2 (D_1^2 + D_2^2). \quad (26)$$

In a beam having only transverse thermal velocities characterised by the transverse electron temperature $T_\perp$, $\overline{r_j^2}^\tau$ is given by:

$$\overline{r_j^2}^\tau = \frac{2kT_\perp}{m}. \quad (27)$$

For a smooth homogeneously emitting cathode $T_\perp$ equals the cathode temperature T_c . In other cases the velocity distribution is not Maxwellian and this results in an increase of $T_\perp$ ¹⁷⁾.

The requirement that the electron motions are fully uncorrelated is met only in the region just beyond the cathode surface, so that eqs (19) and (20) do not apply to any other cross-section of the beam. In order to find $\overline{r^2}$ and $\overline{\dot{r}^2}$ we must have recourse to the wave picture of the beam. First we calculate the waves on the beam excited by the cathode ¹⁶⁾. Then we must calculate the amplitude of the waves in the region where we want to determine $\overline{r^2}$ and $\overline{\dot{r}^2}$. Having obtained these waves the quantities sought are obtained with the aid of eqs (10), (22) and (23).

A calculation of the transformation of the waves along the beam is possible in a limited number of cases only. In contrast with the foregoing, such an analysis is not required for an understanding of the interception phenomena we wish to study. Therefore we shall assume the waves to be given, at least in the region that is of interest to us.

For the sake of completeness it should be mentioned that the transformation of the waves along a drifting beam follows from the solutions of eqs (5), (6), (7) and (8), all the waves being multiplied by their propagation factor $\exp[-j\beta_z z]$. The effect of single electric or magnetic lenses has been described by Gordon ⁴⁾ and by Gould and Johnson ¹⁸⁾; combinations of lenses and drift sections pro-

viding linear noise transformers by Wessel-Berg and Blötekjaer ¹¹⁾¹²⁾, deflection plate couplers by Smith, Suhlman and Cuccia ¹⁹⁾²⁰⁾, slow-wave structures by Siegmán ⁵⁾ and Hart and Weber ²¹⁾, and quadrupolesections by Gordon, Gould, Johnson, Ashkin, Blötekjaer and Wessel-Berg ⁴⁾¹⁸⁾²²⁾²³⁾.

3.2. The internal motion

A knowledge of the noise waves, i.e. of the centre-of-mass motion, is sufficient to describe the noise associated with interaction of the beam and homogeneous external fields. However, if there is an inconstancy in the number of electrons in a disc, or if there is exchange of the electrons between different discs, we need more information about the motion of the electrons, in fact their motion relative to the centre of mass. The transverse vectors $\mathbf{r}_j$ and $\dot{\mathbf{r}}_j$ can be split up into two components, so that

$$\mathbf{r}_j = \mathbf{r} + \mathbf{r}_{j \text{ in}}, \quad (28)$$

$$\dot{\mathbf{r}}_j = \dot{\mathbf{r}} + \dot{\mathbf{r}}_{j \text{ in}}. \quad (29)$$

The vectors $\mathbf{r}_{j \text{ in}}$ and $\dot{\mathbf{r}}_{j \text{ in}}$ represent the displacement and velocity of the electrons with respect to the centre of mass, the motion of the latter is represented by $\mathbf{r}$ and $\dot{\mathbf{r}}$. By definition it follows that

$$\frac{1}{N} \sum_I^N \mathbf{r}_{j \text{ in}} = 0 \text{ and } \frac{1}{N} \sum_I^N \dot{\mathbf{r}}_{j \text{ in}} = 0.$$

In other words, that part of the motion of the individual electrons given by $\mathbf{r}_{j \text{ in}}$ and $\dot{\mathbf{r}}_{j \text{ in}}$ is cross-correlated in such a way that the centre of mass of the disc remains always at rest. This part of the electron motion we will call the "internal" motion. If electron changes take place or if there are heterogeneous forces exerted on the disc this correlation is partly or in whole destroyed. As a consequence also the centre-of-mass motion is disturbed.

Let us here consider the case that the correlation is completely destroyed; the partly uncorrelated case will be considered in the next chapters. The fluctuation of the new centre-of-mass can be found on basis of the central limit theorem as has been outlined in the preceding chapter. However, unless the process disturbs the distributions of $\mathbf{r}_{j \text{ in}}$ and $\dot{\mathbf{r}}_{j \text{ in}}$ to such an extent that $\overline{r_{j \text{ in}}^2}$ and $\overline{\dot{r}_{j \text{ in}}^2}$ are modified, eq. (19) and (20) apply as they stand. Hence, if we substitute $\overline{r_{j \text{ in}}^2}$ and $\overline{\dot{r}_{j \text{ in}}^2}$ instead of respectively $\overline{r_j^2}$ and $\overline{\dot{r}_j^2}$, then $\overline{r^2}$ and $\overline{\dot{r}^2}$ will correspond to the fluctuations of the new centre of mass.

4. Attenuation of the original waves

In this chapter we will discuss what happens to the original waves on the beam if the number of electrons in each disc is changed, or if some electrons are replaced by others having a different velocity and displacement.

The case of interception of electrons by electrodes or by positive ions is the simplest one to consider, because the captured electrons do not remain in the system and need no further consideration.

Scattering of electrons occurs for instance if electrons of the beam are reflected against the electrode causing the interception. Secondary emission and strongly heterogeneous fields also cause scattering. As we shall see from the measurements, some scattering will always accompany interception. The cases of scattering and spread in axial velocity are more complicated, since the electrons all remain in the system and have to be accounted for.

4.1 Interception

Let us consider a beam carrying the waves **A**; the beam current is I_0 (fig. 1a and 1b). This beam we shall call the incident beam. According to eq. (10) the coördinates **X** of the centre of mass of this beam can be found from the waves in any region of the beam. Let us suppose that in a given region interception occurs; then what will remain of the original waves beyond this region, i.e. on the transmitted beam?

Fig. 1a). The beam is partly intercepted by an electrode. The transmitted beam can be considered to consist of two partial beams, one containing scattered electrons only.

Fig. 1b). The signal waves **A** give rise to attenuated signal waves **A'** on the transmitted beam. Simultaneously the noise waves **A''** are generated.

Fig. 1c). The signal waves **B'** on the unscattered partial beam are caused by the signal waves **B** on the beam prior to scattering. The noise waves **B''** and **B'''** are generated by the scattering. **C** and **C'** represent respectively the signal and the noise waves of the resulting beam.

According to eqs (28) and (29), and on account of the fact that the displacement and velocity vector of each electron not intercepted remains the same, the new centre-of-mass motion is given by

$$\mathbf{r}' = \frac{1}{N'} \sum_i^{N'} (\mathbf{r} + \mathbf{r}_{f \text{ in}}) = \mathbf{r} + \frac{1}{N'} \sum_i^{N'} \mathbf{r}_{f \text{ in}}, \quad (30)$$

and

$$\dot{\mathbf{r}}' = \frac{1}{N'} \sum_i^{N'} (\dot{\mathbf{r}} + \dot{\mathbf{r}}_{j \text{ in}}) = \dot{\mathbf{r}} + \frac{1}{N'} \sum_i^{N'} \dot{\mathbf{r}}_{j \text{ in}}, \quad (31)$$

N' is the number of electrons remaining in the discs of the transmitted beam of current I_1 .

We observe that the right-hand side of eqs (30) and (31) consists of two parts. The first term equals the centre-of-mass motion vectors $\mathbf{r}$ and $\dot{\mathbf{r}}$ of the incident beam, it represents that part of the new centre-of-mass motion due to the motion of the incident beam. The second terms of eqs. (30) and (31) being sums of $\mathbf{r}_{j \text{ in}}$ and $\dot{\mathbf{r}}_{j \text{ in}}$ describe the shifts of the displacement and velocity that the centre of mass undergoes by the interception and due to internal motion. Consequently the coördinate matrix after interception consists of a sum of the coördinate matrix before interception $\mathbf{X}' = \mathbf{X}$ and of a new matrix $\mathbf{X}''$. Since the centre-of-mass motion and the internal motion of a piece of drifting beam are independent, $\mathbf{r}$ and $\dot{\mathbf{r}}$ are independent of $\mathbf{r}_{j \text{ in}}$ and $\dot{\mathbf{r}}_{j \text{ in}}$ and so are $\mathbf{X}'$ and $\mathbf{X}''$, provided that the interception occurs at random. Because of $\mathbf{X}' = \mathbf{X}$, the waves $\mathbf{A}'$ of the transmitted beam due to the incident beam waves $\mathbf{A}$ can be found from eqs. (9), (10) and (13):

$$\mathbf{A}' = jK' \mathbf{N} \mathbf{X} = K'/K \cdot \mathbf{N} \mathbf{N}^{-1} = \sqrt{I_1/I_0} \cdot \mathbf{A}, \quad (32)$$

and

$$K' = \sqrt{I_1 \omega / 2 \eta \omega_c}.$$

In general the sum of vectors in the right hand side of eqs (30) and (31) will not be zero. This will correspond to a noise component. The noise waves $\mathbf{A}''$ due to this term we shall examine in sec. 5.1. These waves $\mathbf{A}''$ will be noise, since the electrons are intercepted in a random way and often neither $\mathbf{r}_{j \text{ in}}$ nor $\dot{\mathbf{r}}_{j \text{ in}}$ are uniform. Returning to eq. (32) we observe that the original waves $\mathbf{A}$ are attenuated by a factor $(I_1/I_0)^{1/2}$ and that no cross terms occur.

In order to derive eq. (32) it has been assumed that the interception occurs at random. Strictly speaking, random interception takes place only in case of capture either by a cloud of positive ions or by a fine grid. In many cases the interception is caused by some obstacle, like the edge of an electrode. Here, one is tempted to expect that the interception does not occur at random.

To see what happens then, let us assume that really random interception does not take place. Consequently the sums in the right-hand sides of eqs (30) and (31) are not completely independent of $\mathbf{r}$ and $\dot{\mathbf{r}}$. Since we are attempting a small-signal analysis we can assume a linear correlation. Let the completely correlated part of the sums be denoted by $\mathbf{s}$ and $\dot{\mathbf{s}}$, which are linear functions of respectively $\mathbf{r}$ and $\dot{\mathbf{r}}$. According to eqs. (30) and (31) the residual disc's centre-of-mass motion, brought about by the motion of the original disc, can be written as $\mathbf{r}' = \mathbf{r} + \mathbf{s} = a_1 \mathbf{r}$ and $\dot{\mathbf{r}}' = \dot{\mathbf{r}} + \dot{\mathbf{s}} = a_2 \dot{\mathbf{r}}$. In matrix notation this becomes $\mathbf{X}' = \text{diag}(a_2, a_2, a_1, a_1) \cdot \mathbf{X}$. Using eqs. (9), (10) and (13) we obtain:

$$\mathbf{A}' = \sqrt{I_1/I_0} \cdot \begin{bmatrix} a_2 & 0 & 0 & 0 \\ 0 & a_2 & 0 & 0 \\ a_2-a_1 & 0 & a_1 & 0 \\ 0 & a_2-a_1 & 0 & a_1 \end{bmatrix} \cdot \mathbf{A}. \quad (33)$$

Unlike eq. (32), eq. (33) contains cross terms. In the case of $\alpha_1 = \alpha_2$, new synchronous waves (A_3, A_4) completely correlated with the cyclotron waves (A_1, A_2) are generated. The amplitudes of the cyclotron waves are reduced by $a_2 (I_1/I_0)^{\frac{1}{2}}$ and those of the synchronous waves by $a_1 (I_1/I_0)^{\frac{1}{2}}$. For each individual case the factors a_1 and a_2 have to be calculated.

A simple case is presented by a homogeneous laminar-flow beam and a sufficiently small aperture; in this case α_1 is zero, since the resulting transverse displacement is zero. (Since we are dealing with time dependent quantities (sec. 2.1) only, a possible constant displacement is of no interest.) If the beam possesses no internal rotations, then obviously $a_2 = 1$ (since all the electrons have the same transverse velocity $\dot{\mathbf{r}}$).

In a laminar Brillouin-focussed beam the electrons have an internal rotation, with radian frequency $\omega_c/2$ around the axis. If such a beam is passed through a narrow aperture, then $a_2 \neq 1$ (since at one side more electrons, having a high velocity, are captured). Probably in practice these considerations do not apply, as actual beams are turbulent. Due to the irregular motion of the electrons the probability of capture does not depend on the presence of a signal, and therefore eq. (32) applies instead of the more general eq. (33). As we shall see, the measurements presented in sec. 7.1 provide experimental support for this point of view.

4.2 Scattering

To calculate the effects due to scattering we use the following model. Some electrons are selected at random from the beam. These electrons remain in the beam but their momentum is changed randomly. Then the beam can be thought of as made up of two partial beams, as in fig. 1c. One of the beams having a current I_3 contains only the unscattered electrons. The other partial beam carrying a current I_4 is made up of scattered electrons only. Let $\mathbf{X}_3'$ describe the part of the centre-of-mass motion of the partial beam caused by the waves $\mathbf{B}$ on the beam before the scattering occurs. Since neither the displacement nor the velocity of the unscattered electrons is altered the matrix $\mathbf{X}_3'$ is equal to $\mathbf{X}$, the original coördinate matrix according to eqs (30) and (31). The corresponding coördinate matrix $\mathbf{X}_4'$ of the scattered partial beam is zero, since the electrons are not only separated from the beam but in addition are scattered. This beam carries only noise. A description of these noise waves $\mathbf{B}'''$ as well as of the noise waves $\mathbf{B}''$ of the unscattered beam will be deferred to sec. 5.2.

Having thus obtained the coördinate matrices of the partial beams we must find the corresponding quantities for the whole system, i.e. for the new beam of current I_1 . The wave picture is only relevant in so far as it provides adequate description of both signal propagation along the beam and interaction with external fields. Consequently a general solution of this problem can only be given if the mean axial velocities of the two partial beams are equal and the spread in velocity is only small, or if the velocities of the partial beams are greatly different. The first case may occur if there is elastic reflection against a wall, the second one applies in case of emission of slow secondary electrons or if there is an electron-generating mechanism for instance generation by ions. The axial velocity enters into the considerations because many types of interactions require a longitudinal synchronisation condition to be satisfied.

Let us first calculate the resulting waves of a beam consisting of n partial beams having the same velocity. The corresponding discs of the partial beams can be regarded as n particles having respectively a mass m_m and displacement r_m . If

$$M = \sum_1^n m_m$$

then the centre of mass $\mathbf{r}$ of the resulting beam is given by:

$$M\mathbf{r} = \sum_1^n m_m \mathbf{r}_m.$$

The mass of the corresponding discs is proportional to the d.c. current I_m in the partial beam. Hence

$$I\mathbf{r} = \sum_1^n I_m \mathbf{r}_m, \quad (34)$$

with

$$I = \sum_1^n I_m.$$

After differentiation we find:

$$I\dot{\mathbf{r}} = \sum_1^n I_m \dot{\mathbf{r}}_m. \quad (35)$$

Combining eq. (34) and (35)

$$I\mathbf{X} = \sum_1^n I_m \mathbf{X}_m. \quad (36)$$

In our case of two partial beams with currents I_3, I_4 , and coördinate matrices $\mathbf{X}'_3 = \mathbf{X}$ and $\mathbf{X}'_4 = 0$ we obtain by the use of eq. (36) the resulting matrix $\mathbf{X}'$:

$$\mathbf{X}' = \frac{I_3}{I_1} \mathbf{X}'_3 = \frac{I_3}{I_1} \mathbf{X}, \quad (37)$$

and together with eq. (9) C' , the signal wave matrix of the complete system, is given by

$$C' = \frac{I_3}{I_1} \mathbf{B}. \quad (38)$$

This expression is quite similar to eq. (32), with the exception of the factor I_3/I_1 instead of $(I_1/I_0)^{\frac{1}{2}}$. Hence for the same ratio of beam currents, scattering has a more profound effect. If the scattered beam has a greatly different velocity, so that it is wholly unsynchronised and does not have to be considered any further, then the same reasoning that applies in case of interception can be used. According to sec. 4.1 we can then write

$$C' = \sqrt{I_3/I_1} \cdot \mathbf{B}. \quad (39)$$

4.2.1 A scattered beam in a Cuccia-coupler

In intermediate cases when the velocity of the scattered partial beam is so different from the unscattered beam that there is neither full synchronisation nor that there is complete lack of synchronisation then the interaction process has to be taken into account. Let us do so for the case of a Cuccia-coupler. A Cuccia-coupler¹⁹⁾ consists of a pair of deflection plates placed along the beam over a length b . The length b is short compared to the free-space wavelength of the signal, but several times longer than the cyclotron wavelength $\lambda_c = 2\pi u_0/\omega_c$. A signal voltage on the deflection plates causes a transverse electric field across the beam. If end effects are neglected this field can be considered to be homogeneous. The propagation constant of the field is zero, since over the length that the plates extend in the axial direction, the field can be represented by $E = E_0 \exp [j(\omega t - \phi)]$, neither ϕ nor E_0 depending on z . A given mode of the beam only interacts strongly with external fields if the propagation constant of the waves equals that of the field; as can be deduced, for instance, from eqs (5), (6), (7) or (8). Hence a field as set up by a Cuccia-coupler interacts with the fast wave, only if $\omega \approx \omega_c$ for only then is the phase constant $\beta_1 = (\omega/u_0) (1 - \omega_c/\omega)$ of the wave nearly zero.

As can be calculated²⁰⁾²⁴⁾ the electronic impedance of the beam between the deflection plates can be represented by

$$Z_e \approx \frac{8 d^2 V_0}{b^2 I_0} \left(1 + j \frac{\omega - \omega_c}{3 u_0} b \right), \quad (40)$$

where d is the distance between the plates, b the length of the plates, V_0 the kinetic beam voltage ($e V_0 = \frac{1}{2} m u_0^2$); u_0 the axial electron velocity, and I_0 the beam current. The signal power transfer can be calculated by means of an equivalent circuit. Let us consider the following system (fig. 2). An external signal generator E_u having an internal impedance Z_u is connected in parallel

Fig. 2. Equivalent circuit of a loss free Cuccia-coupler loaded by two partial beams having electronic impedances of respectively Z_1 and Z_2 .

to two impedances Z_1 and Z_2 , which represent active elements. The corresponding voltage sources are denoted by E_1 and E_2 and are independent of E_u . The available power of the external generator is $P_u = E_u E_u^* / 4 \operatorname{Re} Z_u$ and those of the generators with voltage sources E_1 and E_2 are $P_1 = E_1 E_1^* / 4 \operatorname{Re} Z_1$ and $P_2 = E_2 E_2^* / 4 \operatorname{Re} Z_2$. If we use,

$$\alpha = \frac{4 \operatorname{Re} Z_u \cdot \operatorname{Re} Z_p}{|Z_p + Z_u|^2}, \quad (41)$$

$$\zeta = \frac{|Z_p|^2 \operatorname{Re} Z_1}{|Z_1|^2 \operatorname{Re} Z_p}, \quad (42)$$

$$Z_p = \frac{Z_1 Z_2}{Z_1 + Z_2}, \quad (43)$$

we find after a simple but rather cumbersome calculation that the powers P_1' and P_2' delivered by the external generator E_u to the impedances Z_1 and Z_2 are given by,

$$P_1' = \alpha \zeta P_u, \quad (44)$$

$$P_2' = \alpha (1 - \zeta) P_u. \quad (45)$$

The power P_u' delivered by the generators E_1 and E_2 to the external impedance equals

$$P_u' = \alpha \zeta P_1 + \alpha (1 - \zeta) P_2. \quad (46)$$

In the case of the two partial beams we are considering here, Z_1 represents the electronic loading impedance of the unscattered partial beam and Z_2 that of the scattered one. The impedances Z_1 and Z_2 are given by eq. (40), provided that we replace V_0, I_0, u_0 by respectively V_3, I_3, u_3 and V_4, I_4, u_4 ; V_3, V_4, u_3 and u_4 are the respective kinetic beam voltages and velocities. According to the definition of the power flow of a wave being the power required to set up the wave, the available power of the equivalent generator equals the power flow, the latter being the maximum power that can be extracted. Hence we must define the

voltage sources so that $B_1' B_1'^* + B_1'' B_1''^* = E_1 E_1^* / 4 \operatorname{Re} Z_1 = P_1$ and $B_1''' B_1'''^* = E_2 E_2^* / 4 \operatorname{Re} Z_2$, $B_1' B_1'^* + B_1'' B_1''^*$ and $B_1''' B_1'''^*$ being the fast wave power flow of the unscattered and the scattered partial beam respectively.

Considering only the signal on the unscattered beam, i.e. $P_1 = B_1' B_1'^*$, and using eq. (39), we can write

$$P_u = \alpha \zeta B_1' B_1'^* = \alpha \zeta I_3 / I_1 B_1 B_1^*. \quad (47)$$

If we choose $Z_u = Z_p^*$ to provide power matching, then $\alpha = 1$. Let us suppose that ω very nearly equals ω_c , then ζ reduces to $I_3 V_4 / (I_4 V_3 + I_3 V_4)$, as follows from eq. (42), in which the expressions for Z_1 and Z_2 given by eq. (40) are used. In the limiting case of equal partial beam velocities, i.e. $V_3 = V_4$ eq. (47) agrees with eq. (38), as it should; then

$$P_u = \left(\frac{I_3}{I_1} \right)^2 B_1 B_1^*.$$

It is important to note that the expressions (44), (45) and (46) can also be used to describe the behaviour of a coupler network having small losses. In that case we represent the beam loading impedance by $Z_1 = Z_e$ and Z_2 comprises the coupling network impedance as seen from the beam and Z_u the internal impedance of the external source transformed by the network. Transformations like this are described extensively in the literature.

In the case of a lossy coupling network one is often interested in the amount of noise power transferred from the ohmic losses of the network, given by the available (noise-) power P_2 , to the beam, i.e. to Z_1 . A similar calculation as is done in order to derive eqs. (44) and (45) yields for this power P_{21}

$$P_{21} = \alpha \beta \zeta (1 - \zeta) P_2, \quad (48)$$

$$\beta = \frac{|Z_u|^2 \cdot \operatorname{Re} Z_p}{|Z_p|^2 \cdot \operatorname{Re} Z_u}.$$

If the system consisting of coupler and beam is matched to the signal generator, i.e. $Z_u = Z_p^*$, then we have $\alpha = \beta = 1$.

For the sake of completeness it should perhaps be mentioned that the power delivered to Z_2 by the beam is given by

$$P_{12} = \alpha \beta \zeta (1 - \zeta) P_1.$$

4.3 Multivelocity effects

The loss of signal power and the generation of noise by a piece of drifting beam having spread in axial velocity has been described by Lea Wilson ⁷⁾ and in more detail by Gordon ⁸⁾, while measurements have been presented by the author ²⁵⁾. Lea Wilson and Gordon arrived at the result by considering one wave at a time. A description using the technique of the preceding sections

might provide more information, since it considers all the waves simultaneously. Therefore, and since we need the result in chapter 7 dealing with measurements a short description is appropriate to present here.

Let us consider a section of drifting beam comprised between the transverse planes $z = a$ and $z = b$ (fig. 3). The amplitudes of the waves on the beam at

Fig. 3. The signal waves D at $z = a$ are attenuated, due to spread in axial electron velocity at $z = b$; they find expression in the waves D' . Simultaneously the noise waves D'' are generated.

plane a are given by D , hence by the use of eq. (10) the coördinate matrix X_a at $z = a$ can be found. We can divide the beam into n partial beams. The m th partial beam has a current I_m and a nearly uniform axial velocity u_m . In analogy with eqs (28) and (29), the coördinate matrix X_{ma} of the m th partial beam at $z = a$ can be written as

$$X_{ma} = X_a + X_{ma \text{ in}}. \quad (49)$$

The coördinate matrix $X_{ma \text{ in}}$ describes the motion of the centre of mass of the partial beam, relative to the common centre of mass. Hence in analogy with eq. (36)

$$\sum_1^n \frac{I_m}{I_0} X_{ma} = X_a + \sum_1^n \frac{I_m}{I_0} X_{ma \text{ in}} = X_a. \quad (50)$$

Having obtained the coördinate matrix X_a we must find the corresponding matrix X_b in the plane $z = b$. Suppose the wave amplitudes on the m th beam are given by D_m , then the amplitude of the waves D_m' in the plane $z = b$ are given by

$$D_m' = S_m D_m. \quad (51)$$

As follows from the solutions of eqs (5), (6), (7) and (8) the wave propagation matrix S_m is given by

$$S_m = \text{diag} (S_{im}), S_{im} = \exp [-j\beta_{im}L],$$

L is the distance from a to b and β_{im} is the phase constant of the i th mode on the m th partial beam having an axial velocity u_m . Using eq. (9) and $K = K_m$ we rewrite eq. (51)

$$\mathbf{X}_{mb} = \mathbf{U}_m \mathbf{X}_{ma}, \quad (52)$$

with

$$\mathbf{U}_m = \mathbf{N}^{-1} \mathbf{S}_m \mathbf{N}. \quad (53)$$

$\mathbf{U}_m$ describes the transformation of the coördinates along the m th beam. From eqs (52) and (49) it follows that

$$\mathbf{X}_{mb} = \mathbf{U}_m \mathbf{X}_a + \mathbf{U}_m \mathbf{X}_{ma \text{ in}}. \quad (54)$$

Analogously to eq. (34) and in combination with eq. (54) the common centre-of-mass coördinate matrix $\mathbf{X}_b$ at $z = b$ is given by:

$$\mathbf{X}_b = \sum_1^n \frac{I_m}{I_0} \mathbf{U}_m \mathbf{X}_a + \sum_1^n \frac{I_m}{I_0} \mathbf{U}_m \mathbf{X}_{ma \text{ in}}. \quad (55)$$

With the aid of eqs (8) and (53) we obtain

$$\mathbf{D}' + \mathbf{D}'' = \mathbf{S} \mathbf{D} + jKN \sum_1^n \frac{I_m}{I_0} \mathbf{U}_m \mathbf{X}_{ma \text{ in}}, \quad (56)$$

with

$$\mathbf{S} = \sum_1^n \frac{I_m}{I_0} \mathbf{S}_m. \quad (57)$$

The first term of the right-hand side of eq. (56) corresponding to $\mathbf{D}'$ describes the attenuation of the input waves $\mathbf{D}$, the second term the noise introduced by the velocity spread. The second term we will examine more closely in sec. 5.3.

To derive an expression for $\mathbf{S}$ we assume that the number of electrons in the m th partial beam is given by $N_m = N_0 \cdot f(u)du$ where $f(u)$ denotes the normalised axial velocity distribution of the electrons, i.e. $\int_0^\infty f(u)du = 1$. The current z carried by the m th beam is given by $I_m = Ne u_m$; the current of the complete beam is $I_0 = N_0 e \bar{u}$, $\bar{u} = \int_0^\infty u f(u)du$. The integration has to be carried out over all the velocities except the negative ones, as these belong to electrons that do not reach plane $z = b$ (if we exclude reflections). In combination with eq. (57) can we write

$$\mathbf{S} = \sum_1^n \frac{I_m}{I_0} \mathbf{S}_m = \frac{\int_0^\infty \mathbf{S}_u f(u)du}{\int_0^\infty u f(u)du}. \quad (58)$$

Since the propagation constant $\mathbf{S}_u$ of a partial beam having a velocity in the interval u and $u + du$ is a complex matrix, $\mathbf{S}$ will also be a complex one. The power loss of a signal is given by:

$$\mathbf{D}' \mathbf{D}'^\dagger \mathbf{p} = \mathbf{S} \mathbf{D} \mathbf{D}^\dagger \mathbf{S}^\dagger. \quad (59)$$

The diagonal terms of eq. (59) constitute equations describing the loss of signal power of the corresponding wave. They agree with those of Gordon ⁸⁾, however, eq. (59) is slightly more general since it contains also off-diagonal terms describing loss of correlation between the waves. By a slightly different method Gordon has worked out expressions for $S_i S_i^*$ for a number of cases, for instance spread due to potential depression by space charge, due to a lens, and due to thermal emission. For the latter case we find after slight rearrangement of his result

$$S_i S_i^* \approx \exp - \left[\frac{k T_{||}}{m \bar{u}^2} \beta_i L \right]^2. \quad (60)$$

Here $T_{||}$ is the longitudinal electron temperature characterizing the spread of the longitudinal velocity, k is Boltzmann's constant, and L is the length of the drifting beam. The off-diagonal terms of eq. (59) must be calculated by means of eq. (58). However, since they do not appear in our measurements we will refrain from doing this. Perhaps it should be remarked here that the calculations of section 4.3 do not take the synchronisation conditions of interactions into account. It has been tacitly assumed that the spread in axial velocity is small enough to provide complete interaction. Therefore, in case of a spread that is large in comparison with the mean velocity, care should be exercised: possibly the results of section 4.3 do not apply because equation (55) is then not appropriate. The problem to be solved is then quite similar to that encountered in sec. 4.2 and 4.2.1 dealing with scattering, in the case that the scattered beam has a different velocity.

5. The generation of noise

Having thus in the previous sections derived the expressions for the attenuation of the original waves we shall here derive the expressions for the noise waves generated. Before the interception or scattering occurs, the internal motion of the electrons is balanced, that is to say their individual motions are correlated in such a way that there does not result any motion of the beam's centre-of-mass. This balance is upset to some extent during the process involved, as we have seen from the second terms of the right-hand side of eqs (30), (31) and (55).

5.1 Noise generated by interception

In section 4.1 we have calculated the waves A' of the transmitted beam which are due to the waves A on the incident beam. Here we will calculate the power flow of the waves A'' that are likewise associated with the interception.

Let us subdivide the beam of current I_0 into a large number of beamlets n , the m th beamlet carries a current I_m . The elements of the matrix X_m of a particular disc in the m th beamlet are the coördinates of the centre-of-mass of this

disc with respect to the centre-of-mass of the corresponding disc in the complete beam. According to this definition of the internal coördinates, eq. (36) reads:

$$0 = \sum_1^n \frac{I_m}{I_0} \mathbf{X}_m,$$

or

$$\sum_1^n \frac{I_m^2}{I_0^2} \mathbf{X}_m \mathbf{X}_m^\dagger + \sum_{\substack{1 \ 1 \\ m \neq h}}^n \frac{I_m I_h}{I_0^2} \mathbf{X}_m \mathbf{X}_h^\dagger = 0. \quad (61)$$

The first term of eq. (61) corresponds to the r.m.s. fluctuation of the centre of mass when there is no correlation between the motions of the beamlets. We can calculate this term by using the technique described in sec. 3.1. Let this result in a matrix $\mathbf{XX}^\dagger$. The average contribution of each beamlet to $\mathbf{XX}^\dagger$ can be found from

$$\mathbf{XX}^\dagger = \sum_1^n \frac{I_m^2}{I_0^2} \mathbf{X}_m \mathbf{X}_m^\dagger = n \overline{\left(\frac{I_m^2}{I_0^2} \mathbf{X}_m \mathbf{X}_m^\dagger \right)}. \quad (62)$$

The bar denotes the average over all the beamlets. Upon combination of eqs (61) and (62) and after averaging we find

$$\mathbf{XX}^\dagger = - \sum_{\substack{1 \ 1 \\ m \neq h}}^n \frac{I_m I_h}{I_0^2} \mathbf{X}_m \mathbf{X}_h^\dagger = -(n^2 - n) \overline{\left(\frac{I_m I_h}{I_0^2} \mathbf{X}_m \mathbf{X}_h^\dagger \right)}. \quad (63)$$

According to eq. (36) the coördinate matrix of the transmitted beam can be written as

$$\mathbf{X}'' \mathbf{X}''^\dagger = \sum_1^{n'} \frac{I_m^2}{I_0^2} \mathbf{X}_m \mathbf{X}_m^\dagger + \sum_{\substack{1 \ 1 \\ m \neq h}}^{n'} \frac{I_m I_h}{I_0^2} \mathbf{X}_m \mathbf{X}_h^\dagger,$$

n' being the number of transmitted beamlets having a total beam current of I_1 ; I_m and I_h are not changed by the interception. Averaging yields

$$\mathbf{X}'' \mathbf{X}''^\dagger = n' \overline{\left(\frac{I_m^2}{I_1^2} \mathbf{X}_m \mathbf{X}_m^\dagger \right)} - (n'^2 - n') \overline{\left(\frac{I_m I_h}{I_1} \mathbf{X}_m \mathbf{X}_h^\dagger \right)}. \quad (64)$$

Provided that interception occurs at random, we can say that the averages in the right-hand side of eq. (64) are equal to those of eqs (62) and (63). The requirement of random interception is probably met in practice, since practical beams are turbulent and all electrons of the beam have equal probability of being captured. Substitution of eq. (62) and (63) yields

$$\mathbf{X}'' \mathbf{X}''^\dagger = \frac{I_0^2}{I_1^2} \frac{n'}{n} \left(1 - \frac{n' - 1}{n - 1} \right) \mathbf{XX}^\dagger.$$

Remembering that $\mathbf{XX}^\dagger$ refers to internal coördinates of the incident beam and

since $n \gg 1$, $n' \gg 1$, we find with the aid of eqs (9) and (23)

$$\mathbf{A}'' \mathbf{A}''^\dagger \mathbf{p} = \frac{I_2}{I_0} \frac{\omega}{\omega_c} k T_{\text{in}} \mathbf{Q}_1 df, \quad (65)$$

$\mathbf{A}'' \mathbf{A}''^\dagger \mathbf{p}$ represents the noise power flow on the transmitted beam, which carries a current I_1 . The matrix $\mathbf{Q}_1$ has the same form as $\mathbf{Q}$ of eq. (23), except for δ_1^2 which takes the place of δ^2 ; δ_1^2 refers to internal quantities. Hence

$$\delta_1^2 = \omega_c^2 \frac{\overline{r_{j \text{ in}}^2}^\tau}{\dot{r}_{j \text{ in}}^2}^\tau. \quad (66)$$

The internal temperature T_{in} is defined, in analogy to eq. (27), by

$$\overline{\dot{r}_{j \text{ in}}^2}^\tau = \frac{2 k T_{\text{in}}}{m}. \quad (67)$$

Eq. (65) expresses the noise power flow due to the internal motion on the transmitted beam, it contains the unknown T_{in} and δ_1^2 that are contained in the matrix $\mathbf{Q}_1$ according to eq. (23a). The internal temperature and also δ_1^2 can be found if $\overline{r_{j \text{ in}}^2}$ and $\dot{r}_{j \text{ in}}^2$ are known. We shall calculate these quantities in sec. 5.1.1 and 5.1.2.

5.1.1 The mean square displacement

Let us first calculate $\overline{r_{j \text{ in}}^2}$ the mean square displacement of the centre of mass. If we assume that the space-charge smoothing factor of the longitudinal shot noise Γ^2 remains constant during the process, then $\overline{r_{j \text{ in}}^2}$ obeys the same formulae (24), (25) and (26) as $\overline{r_j^2}$ does. Hence (eq. (65) and (66)) we obtain in case of a uniform density beam

$$\delta^2 = \omega_c^2 \frac{m \Gamma^2 D^2}{16 k T_{\text{in}}}. \quad (68)$$

For a beam with a parabolical density distribution and for a hollow beam similar expressions can be derived. However, unless $\Gamma^2 = 1$, we have neglected an important source of noise since the occurrence of a partition current causes a deterioration of the space-charge smoothing. Therefore δ^2 is not equal to δ_1^2 , the quantity in which we are interested. According to the familiar formula of partition noise¹³⁾ the increase of shot current is given by

$$\Delta i^2 = 2 e I_1 (\Gamma_1^2 - \Gamma^2) df, \quad (69)$$

and

$$\Gamma_1^2 = 1 - (1 - \Gamma^2) \frac{I_1}{I_0}. \quad (70)$$

This increase in shot noise causes an extra fluctuation of the displacement of

the centre of mass of the transmitted beam. In accordance with eqs (23) and (65) this corresponds to an extra synchronous wave power flow of

$$A_{3,4}A_{3,4}^* = \mp \frac{\omega}{\omega_c} k T_{in} \delta_{ex}^2 df. \quad (71)$$

The negative sign refers to A_3 , the positive sign to A_4 . Using eqs (24) and (72) we find for a homogeneous beam

$$\delta_{ex}^2 = \omega_c^2 \frac{m (\Gamma_1^2 - \Gamma^2) D_1}{16 k T_{in}}, \quad (72)$$

and similar expressions, with the aid of eqs (25), (26) and (70) for a parabolic distribution and a hollow beam. The diameter D_1 for the transmitted beam takes the place of D , the diameter of the original beam.

To calculate δ_1^2 of the matrix $\mathbf{Q}_1$ of eq. (65) we must combine δ^2 of eq. (68) and δ_{ex}^2 . We observe that eq. (65) contains a factor I_2/I_0 , whereas (71) does not; hence the overall δ_1^2 of a homogeneous beam will be given by

$$\delta_1^2 = \delta^2 + \frac{I_0}{I_2} \delta_{ex}^2 = \omega_c^2 \frac{m \xi_1^2 D_1}{16 k T_{in}} \quad (73)$$

and

$$\xi_1^2 = \frac{D^2}{D_1^2} \Gamma^2 + \frac{I_0}{I_2} (\Gamma_1^2 - \Gamma^2). \quad (74)$$

To obtain some insight into the relative importance of δ_1^2 let us insert some numerical values: $D = 0.4$ mm, $D_1 = 0.3$ mm, $\omega_c = 2\pi 550.10^6$ rad/sec, $T_{in} = 3000$ °K, $k = 1.3 \cdot 10^{-23}$ sec/°K, $m = 9.1 \cdot 10^{-31}$ kg. In the cases of $\Gamma^2 = 1$ and $\Gamma^2 = 0$ we obtain respectively $\delta_1^2 = 2.8$ and $\delta_1^2 = 1.6$. Obviously δ_1^2 is not small enough to be neglected against unity, and in the case of strong space charge smoothing the partition current is rather important. Expressions similar to eq. (73) can be derived in the case of a parabolical distribution and a hollow beam if use is made of the appropriate expressions for δ^2 and δ_{ex}^2 . Having obtained δ_1^2 we must now find the internal temperature.

5.1.2 The mean square velocity

In a beam having transverse thermal velocities only, the internal temperature T_{in} is equal to the transverse electron temperature $T_\perp$ according to eqs (27) and (67). The temperature $T_\perp$ is related to the cathode temperature T_c as follows. The imperfections at the cathode surface cause the transverse electron temperature at the cathode plane to be somewhat higher than T_c , say a value $T_c'{}^{17}$. On the basis of the Liouville theorem²⁶) it can be deduced that at any other cross-section of the beam $T_\perp$ can be given by

$$T_{\perp} = \frac{D_1^2}{D_0^2} T_c'. \quad (75)$$

The quantity D_0 denotes the diameter of the cathode area emitting the electrons which arrive in the cross-section of diameter D_1 under consideration. It has been tacitly assumed that no other electrons, for instance secondary ones are present in the beam. As has been shown by Wessel-Berg and Blötekjaer ²⁷), T_{in} is only equal to $T_{\perp}$ if there are no non-thermal rotations present. Non-thermal rotations can be caused by heterogeneous fields, for instance, on account of lenses or of expansion of the beam by its own space charge in the axial homogeneous magnetic field. It seems impossible to present a general expression for T_{in} in case of a practical and turbulent beam. However, we can perhaps find an approach by considering a laminar Brillouin beam ²⁸). Also we shall try to obtain an expression for the maximum temperature to be expected in case of a turbulent beam.

In case of a laminar flow, and if thermal velocities are not taken into account the transverse velocity of an electron is given by $\dot{r}_{j\ in} = (\omega_c/2) r_{j\ in}$ ²⁸). Let us assume that thermal velocities are present, and that the following assumptions hold. The velocity distribution of the electrons is a normal distribution having a spread given by $T_{\perp}$ and an average velocity of $\bar{\dot{r}}_{j\ in} = \sigma \omega_c/2$. Here σ denotes the distance of the electron to the disc centre-of-mass. Hence the probability distribution of the number of electrons with a velocity in lying between v and $v + dv$ can be written

$$n(v) dv = \frac{1}{2\pi\sigma^2} \exp - \left[\frac{(v - \omega_c \rho/2)^2}{2\sigma^2} \right] \cdot dv, \quad (76)$$

with

$$\sigma^2 = \frac{2kT_{\perp}}{m}. \quad (77)$$

The number of electrons comprised within the annulet between ρ and $\rho + d\rho$ is given by

$$dn(\rho) = \frac{8\rho d\rho}{D^2} N, \quad (78)$$

where N is the total number of electrons in the disc. If we assume that the velocity and the displacement of the electrons is uncorrelated, the mean square velocity $\dot{r}_{j\ in}^2$ can be found by

$$\begin{aligned} \dot{r}_{j\ in}^2 &= \frac{8}{D^2} \frac{1}{2\pi\sigma^2} \int_0^{D/2} \rho d\rho \int_{-\infty}^{+\infty} v^2 \exp - \left[\frac{(v - \omega_c \rho/2)^2}{2\sigma^2} \right] \cdot dv = \\ &= \frac{2k}{m} \left(T_{\perp} + \frac{m\omega_c^2 D^2}{64} \right). \end{aligned} \quad (79)$$

The integral can be found with the aid of an integral table ²⁹⁾. From eq. (79) and eq. (67) we obtain

$$T_{\text{in}} = T_1 + \frac{m\omega_c^2 D^2}{64 k}. \quad (80)$$

From this result we see that the internal temperature consists of a thermal and a non-thermal part. The non-thermal part agrees with the result of Wessel-Berg and Blötekjaer ²⁷⁾ obtained for a zero-temperature laminar-flow Brillouin beam. Putting numbers into (77) we find for a beam of $D = 0.4$ mm, $\omega_c = 2\pi f_c$ and $f_c = 550$ Mc/s for the non-thermal part $T_{\text{in}} - T_1 = 1970$ °K, hence the contribution of rotations proves to be considerable. As has been remarked before, practical beams are turbulent and laminar Brillouin flow is never attained. We shall now derive an expression for the maximum temperature to be expected in the case of a beam possessing uniform density. It has been shown both theoretically ²⁶⁾³⁰⁾ and by experiment ³¹⁾³²⁾³³⁾ that the internal motion of a beam having a uniform density is composed of two rotations, with radian frequencies ω_+ and ω_- .

$$\omega_{+,-} = \frac{\omega_c}{2} \pm \frac{\omega_c}{2} \sqrt{1 - \frac{2\omega_p^2}{\omega_c^2}}, \quad (81)$$

$$\omega_p = \sqrt{\frac{\eta I_0}{\varepsilon_0 u_0 A}}.$$

Here ε_0 is the dielectric constant of vacuum, I_0 the d.c. beam current, u_0 the axial electron velocity, and A the beam's cross-section area.

The internal displacement vector of an electron can be written as $\mathbf{r}_j = \mathbf{r}_+ \exp[j\omega_+ t] + \mathbf{r}_- \exp[j\omega_- t]$; $\mathbf{r}_+$ and $\mathbf{r}_-$ are complex vectors independent of t . To calculate the internal temperature we must know the distributions of $\mathbf{r}_+$ and $\mathbf{r}_-$. In general these distributions are not given. However, we can ascertain for ourselves that quite high temperatures are possible by considering the limiting case, $\omega_p \approx 0$, i.e. no space-charge in the beam. Then $\omega_+ \approx \omega_c$ and $\omega_- \approx 0$, which implies that the electron rotates with the frequency ω_c around $\mathbf{r}_-$, i.e. around some point which may be off the beam axis. The displacement is then given by $\mathbf{r}_j \approx \mathbf{r}_- + \mathbf{r}_+ \exp[j\omega_c t]$. The maximum velocity of the electrons comprised within the annulet between ρ_m and $\rho_m + d\rho_m$ is $[(\rho_m + R)/2] \omega_c$ since $(\rho_m + R/2)$ is the maximum value of r_+ (otherwise the electron would not always remain in the beam). If we assume that all the electrons have their corresponding maximum velocity we obtain, analogously to the derivation of eq. (80) for the internal temperature of a homogeneous beam

$$T_{\text{in}} = T_1 + \frac{17 m \omega_c^2 D^2}{96 k}. \quad (82)$$

The non-thermal contribution to T_{in} is about 11 times as large as the corresponding one of eq. (80). Although in practice not all electrons will have maximum transverse velocity, we can conclude that quite high temperatures can be encountered in turbulent beams due to the non-thermal rotations.

5.1.3 The noise waves due to interception

To summarize the results of the preceding sections, the noise-power flow of the transmitted beam is given by eq. (65)

$$\mathbf{A}''\mathbf{A}''^\dagger \mathbf{p} = \frac{\omega}{\omega_c} \frac{I_2}{I_1} kT_{in} \mathbf{Q}_1 df \quad (65)$$

T_{in} is equal to T_1 in case of a beam having transverse thermal velocities only; otherwise T_{in} consists of a thermal and a non-thermal part, as shown by eqs (80) and (82). The latter expression denotes the maximum value to be expected.

$\mathbf{Q}_1$ is given by eq. (23), and δ_1^2 by eq. (73).

5.2 Noise due to scattering

In sec. 4.2 we have derived expressions relating the waves $\mathbf{C}'$ on the beam and the original waves $\mathbf{B}$, fig. 1c. In this section we will consider the noise waves $\mathbf{C}''$ generated by the scattering. The waves $\mathbf{C}''$ are composed of a contribution by the noise waves $\mathbf{B}''$ of the unscattered partial beam and by a contribution of the noise waves $\mathbf{B}'''$ of the scattered partial beam. First of all we shall consider the waves $\mathbf{B}''$ and $\mathbf{B}'''$, then we shall calculate the resulting waves $\mathbf{C}''$.

5.2.1 The waves $\mathbf{B}''$

The mechanism which generates the waves $\mathbf{B}''$ is inherently the same as that of interception. The electrons are simply taken from the main beam and $\mathbf{B}''$ represents the noise waves due to this effect. However, the electrons remaining in the beam after having been scattered give rise to the noise waves $\mathbf{B}'''$. Hence, according to eq. (65) we obtain

$$\mathbf{B}''\mathbf{B}''^\dagger \mathbf{p} = \frac{I_4}{I_1} \frac{\omega}{\omega_c} kT_{in} \mathbf{Q}_3 df. \quad (83)$$

The matrix $\mathbf{Q}_3$ equals $\mathbf{Q}$ of eq. (23), only δ_3^2 takes the place of δ^2 . We can obtain an expression for δ_3^2 by appropriate substitutions in eq. (75) of δ_1^2 of the interception. If we substitute ξ_3 , D_1 , D_3 , I_1 , I_1 , I_4 and I_3 instead of ξ_1 , D , D_1 , I , I_0 , I_2 and I_1 respectively, eq. (73) yields

$$\delta_3^2 = \frac{m\xi_3^2 D_3^2}{\omega_c 16 kT_{in}}, \quad (84)$$

$$\xi_3^2 = \frac{D_1^2}{D_3^2} \Gamma^2 + \frac{I_1}{I_4} (\Gamma_3^2 - \Gamma_1^2). \quad (85)$$

Here D_3 represents the diameter of the unscattered partial beam, eq. (84) applies in the case of a homogeneous beam. The space-charge smoothing factor Γ_3^2 of the unscattered beam is given by eq. (70)

$$\Gamma_3^2 = 1 - (1 - \Gamma_1^2) \frac{I_3}{I_1}. \quad (86)$$

5.2.2 The waves $\mathbf{B}'''$

The scattered electrons constitute the scattered partial beam and their motion is fully uncorrelated, since they are selected at random from the main beam and scattered in addition. Hence the noise waves can be calculated on the basis of the central limit theorem (sec. 3.1). The distributions of $\mathbf{r}_j'''$ and $\dot{\mathbf{r}}_j'''$ of the scattered electrons may be greatly different from that of the original beam. Therefore a general answer cannot be given, and each scattering process involved should be considered anew. Let us first consider a simple case: suppose scattering occurs by elastic reflection of electrons from a wall parallel to the beam axis. Since the reflections are elastic the transverse energy of the electrons is not modified, so that $\overline{\dot{\mathbf{r}}_j'''^2}$, and hence T_{in} , is the same as the corresponding quantity of the original beam. Suppose that the diameter of the scattered beam is D_4 and the electrons are uniformly distributed over the cross-section. In this case we obtain with the aid of eq. (65)

$$\mathbf{B}'''\mathbf{B}'''\dagger \mathbf{p} = \frac{\omega}{\omega_c} k T_{in} \mathbf{Q}_4 df, \quad (87)$$

with $\mathbf{Q}_4 = \mathbf{Q}$ provided we substitute δ_4^2 instead of δ^2 . Using eq. (68) we find

$$\delta_4^2 = \omega_c^2 \frac{m \Gamma_4^2 D_4^2}{16 k T_{in}}, \quad (88)$$

and Γ_4^2 , the space-charge smoothing factor, according to eq. (70) is given by

$$\Gamma_4^2 = 1 - (1 - \Gamma_1^2) \frac{I_4}{I_1}. \quad (89)$$

The assumption of T_{in} being equal to that of the original beam is probably not correct in most cases. For instance, if some electrons are reflected at a wall and if there is an equilibrium between the space-charge in the beam and the forces exerted by the external magnetic field, then expansion of the scattered beam will occur. The expansion does not only find expression in an increased diameter D_4 , but additional rotations acquired may increase T_{in} considerably over that of the original beam. This fact is clearly illustrated by eq. (83), which contains

both the beam diameter and the square of the rotation frequency. Expansion of the unscattered partial beam, however, has no influence on the result; its centre-of-mass motion is unaffected. If the interception considered is being followed by another one or if the interception extends over a distance that is large compared to the ripple wavelength ²⁶⁾ of the beam then of course the increase in T_{in} must find expression in the generated noise.

5.2.3 Combination of the waves

The problems associated with the calculation of the waves on the complete beam have been discussed in secs 4.2 and 4.2.1. Therefore we can be brief here. In case that both partial beams have approximately equal axial velocity eq. (36) applies. Rearranging eq. (36) and using eq. (9) we obtain for the noise power flow of the complete beam

$$C''C''^\dagger p = \frac{I_3}{I_1} B''B''^\dagger p + \frac{I_4}{I_1} B'''B'''^\dagger p. \quad (90)$$

Let the internal temperature of the unscattered beam be given by $T_{in\ 3}$ and that of the scattered beam by $T_{in\ 4}$. Then upon substitution of eq. (83) and (87) in eq. (90) we find

$$C''C''^\dagger p = \frac{\omega}{\omega_c} \left(\frac{I_3 I_4}{I_1^2} Q_3 T_{in\ 3} + \frac{I_4}{I_1} Q_4 T_4 \right) k df. \quad (91)$$

If the scattered beam has an axial velocity so much different from that of the unscattered beam that it completely falls out of synchronism, eq. (83) applies directly, and hence

$$C''C''^\dagger p = \frac{\omega}{\omega_c} \frac{I_4}{I_1} Q_3 T_{in\ 3} k df. \quad (92)$$

In an intermediate case, a general expression cannot be given. For the special case of a fast wave and a Cuccia-coupler we find with the aid of eq. (46)

$$C_1''C_1''^* = a \zeta \frac{\omega}{\omega_c} k T_{in\ 3} df + a (1 - \zeta) \frac{\omega}{\omega_c} k T_{in\ 4} df. \quad (93)$$

5.3 Noise due to multivelocitv effects

The noise waves D'' due to spread in axial velocity can be calculated by examination of the second term of the right hand side of eq. (56) (due to the different phase shifts the sum of the internal coördinates $X_{mx\ in}$ is not equal to zero after the electrons have been allowed to drift over some length). Rather than doing this, we shall try to arrive at the result by simple reasoning.

Let us assume that at the cross-section a the transverse electron motion is random. The power flow of the noise waves D_a at a that are associated with the centre-of-mass motions of the discs is given (eq. (23)) by

$$\mathbf{D}_a \mathbf{D}_a^\dagger \mathbf{p} = \frac{\omega}{\omega_c} k T_{\text{in}} \mathbf{Q} df, \quad (94)$$

in which $\mathbf{Q}$ and T_{in} are given by eqs (24) and (27). If we neglect electron-electron interaction in a section of drifting beam, no correlation of the electron motion will built up. Therefore at the cross-section b at some axial distance from a the power flow $\mathbf{D}_b$ of the noise waves must equal that at a , which is expressed by eq. (94). However, from eq. (59) it follows that the waves $\mathbf{D}_a$ at a are attenuated by $\mathbf{S}$ after having travelled from a to b . Consequently we can write

$$\mathbf{D}_b \mathbf{D}_b^\dagger \mathbf{p} = \mathbf{S} \mathbf{D}_a \mathbf{D}_a^\dagger \mathbf{S}^\dagger \mathbf{p} + \mathbf{D}'' \mathbf{D}''^\dagger \mathbf{p},$$

and hence the noise power flow of the waves $\mathbf{D}''$ is given by:

$$\mathbf{D}'' \mathbf{D}''^\dagger \mathbf{p} = \frac{\omega}{\omega_c} k T_{\text{in}} (\mathbf{Q} - \mathbf{S} \mathbf{Q} \mathbf{S}^\dagger) df. \quad (95)$$

The diagonal terms agree with the expressions derived by Gordon ⁸⁾, except for a minor difference in the case of synchronous waves. In the latter case we must substitute for the synchronous wave noise temperature of Gordon, that of Wessel-Berg and Blötekjaer ¹¹⁾ in order to obtain complete agreement.

The internal temperature T_{in} and δ^2 in $\mathbf{Q}$ are independent of z since the beam does not expand and we neglect ripple. T_{in} should be determined in accordance with the considerations of sec. 5.1.2 and δ^2 in $\mathbf{Q}$ with those of 5.1.1 and 5.1.2. It is important to note that δ^2 does not contain a partition noise term as the velocity of a given electron is constant and therefore it will always remain in the same partial beam.

6. Experimental verification with a fast wave

In this chapter we shall discuss the experiments performed with fast waves in order to verify the theory presented in the preceding chapters 4 and 5. The measurements were performed with the aid of electron beam tubes incorporating Cuccia-couplers ¹⁹⁾. A discussion of experiments involving synchronous waves will be presented in chapter 7.

6.1. Fast wave tube

An exploded view of a tube used for interception measurements with fast waves is shown in fig. 4. This tube is a modified experimental low-noise quadrupole amplifier ³⁾. Between the input coupler and the quadrupole section a movable vane having a number of holes punched into it, and a second signal coupler are inserted. The vane is suspended by means of a pivot joint and is weighted at the other end. This enables us, by rotating the tube around an axis parallel to the beam, to adjust the position of the vane relative to the beam, the

Fig. 4. Exploded view of a tube used for interception measurements. The moveable vane can be made to penetrate into the beam by rotation of the tube parallel to the beam. The inset shows the connections of the quadrupole.

beam axis being horizontal. Any desired amount of interception can be secured by permitting the edge of a particular hole to penetrate more or less deeply into the beam. (Zero interception, of course, can only be obtained if the diameter of the hole is larger than the beam diameter). To minimize reflection of electrons at the vane its surface is covered with a spongy layer of chromium. (In one tube (109) as discussed in sec. 6.2.2 other materials have been tried.)

The insertion of the second coupler enables us to take off or to bring on a signal immediately after interception. The use of this coupler will be described in sec. 6.2.1 in greater detail.

Amplification in this tube can be accomplished by a quadrupolar electric field within the quadrupole section set up by an external pump generator. This pump generator has a frequency $\omega_g = 2\omega_c$ and supplies the energy needed for the amplification. During passage of the beam through this field the fast wave signal (and noise) of frequency ω is amplified, and simultaneously an idler of frequency $\omega_i = 2\omega_c - \omega$ is generated. As a consequence of the fact that a "signal" of frequency ω_i will give rise to an "idler" of frequency ω , noise on the beam of frequency ω_i will also contribute to the noise output at frequency ω . If the amplification is large enough, the amplitudes of signal and idler are nearly equal³⁾. For instance, in the case of a power gain of 10 db they differ only by about 10 percent. To understand the main operation of the tube let us consider fig. 5. For the moment we assume that the vane is placed in such a position that no interception occurs. The second coupler is detuned to such an extent that energy exchange with the signal on the beam is prevented.

The beam as it emerges from the gun carries a fast noise wave $A_1(\omega)$ of frequency ω and also $A_1(\omega_i)$ of frequency ω_i . (The other noise waves in the

Fig. 5. Block diagram of the operation of the tube (fig. 4). Coupling only takes place with the fast waves $A_1(\omega)$ and $A_1(\omega_i)$ of respectively frequency and $\omega_i = 2\omega_c - \omega$.

modes A_2, A_3 and A_4 are of no interest in this tube, since neither the couplers nor the amplifying section provide first-order coupling to these modes.) Unless the input coupler is not properly loaded, nearly all the fast wave noise power of frequencies ω and ω_i is extracted by the input coupler. Measurements show that less than 0.1 percent (attenuation more than 30 db) of the fast noise wave power will survive. Simultaneously the input coupler excites a fast signal wave and also a new set of fast noise waves on the beam.

The latter noise waves are caused by the noise sources contained within the signal generator and by the thermal noise of the coupler system losses. The coupler system comprises a matching network to provide power matching of the Cuccia-coupler and the signal generator. The network consists of an adjustable balun ³⁴⁾ connected to the Cuccia-coupler by a short length of 300 Ω twin line. The balun matches the coupler to a 50 Ω coaxial cable providing the connection with the signal generator. After amplification in the quadrupole section the signal and the noise are taken from the beam by the output coupler. The output coupler system is identical to the input coupler system.

If now the vane is allowed to penetrate more or less deeply into the beam then the picture of the tube operation does not change substantially; only the waves on the beam emerging from the input coupler are attenuated and new noise waves are generated, as has been described in the previous sections 4 and 5.

For completeness' sake it should be mentioned that the principal dimensions of the tube of fig. 4 are as follows: Each Cuccia-coupler has an axial length of 11 mm, a plate width of 3 mm and a spacing of 0.8 mm, the d.c. coupler voltage with respect to the cathode is 6 V. The axial length of the quadrupole is 11 mm, the cross-section of the quadrupole is rectangular, the spacing of the opposite plates is 2 mm and the d.c. voltage about 5.3 V with respect to the cathode. A scale drawing of the electron gun is shown in fig. 6 ³³⁾. The first anode is made of iron and shields the cathode magnetically. The temperature T_c of the L-cathode employed is $T_c = 1420$ °K. Depending upon the gun voltages the beam diameter is 0.4 to 0.5 mm. Typical beam current and gun voltages are:

Fig. 6. A scale drawing of the electrode arrangement of the gun employed in the tubes.

$I_0 = 29 \mu\text{A}$ (fig. 8), $V_{a1} = 6 \text{ V}$, $V_{a2} = 130 \text{ V}$, $V_{a3} = 40 \text{ V}$, $V_{a4} = 0 \text{ V}$, $V_{a5} = 10 \text{ V}$ and $V_{a6} = 6 \text{ V}$.

6.2 Measurement of the attenuation

The attenuation due to interception can be found from the measurement of the insertion gain of the tube as a function of the interception ratio I_1/I_0 . The current I_0 represents the incident and I_1 the transmitted current, as in sec. 4.1, fig. 1. Here we shall define insertion gain to be the ratio of available signal output power to available signal input power. The insertion gain can be measured by standard methods³⁵⁾ with a signal generator, a sensitive receiver and a calibrated attenuator.

During these measurements we switch off the pump generator in order to avoid difficulties caused by beam defocussing effects³²⁾³³⁾ and the occurrence of the idler signal. For each value of the interception ratio both the input and output coupler have been matched carefully to provide maximum signal transmission. The second coupler is still detuned. Since ω is approximately equal to ω_c , loss of signal power due to a spread in the axial electron velocities is negligible (eqs (59) and (60); $S_1 S_1^* = 1$), so that the overall insertion gain of the tube, complete with its input and output coupler matching networks, can be written as

$$g = g_{c1} \cdot g_{c3} \cdot g_v, \quad (96)$$

g_{c1} , g_{c3} and g_v represent the insertion gains of respectively the input system, the output system and the intercepting vane. The quantities g_{c1} , g_{c3} and g_v are smaller than one since they actually constitute a loss.

We are particularly interested in g_v , since g_v equals the factor to which the fast wave power is reduced on account of interception. From our measurements we obtain g as a function of I_1/I_0 . To determine g_v we must also know g_{c1} and g_{c3} . The determination of g_{c1} and g_{c3} will be discussed first of all.

6.2.1 Determination of g_{c3}

The insertion gain of a coupler depends upon the beam current passing through the coupler. A decrease of current corresponds to a decrease of insertion gain because the electronic beam impedance (eq. 40) increases and hence network losses have a more profound influence. This can be deduced from eqs. 42 and 46 thereby indentifying ζ with g_{c1} or g_{c3} respectively, α is always unity since the couplers are kept in matched condition. Let us examine g_{c3} in the first place. We observe that the current I_1 which flows through both the input coupler and the output coupler, undergoes drastic changes as a consequence of the interception. Therefore g_{c3} will vary also.

In order to determine the variation of g_{c3} a measurement of the insertion gain $g_{c2} \cdot g_{c3}$ was performed with the second coupler now used as the input of the tube shown in fig. 4. The insertion gain of the second coupler is denoted by g_{c2} . The beam current I_1 flows through both the second and the output coupler. Therefore, and since the couplers and their matching networks are made identical, $g_{c2} = g_{c3}$ holds to a good degree of approximation, so that $g_{c3} = \sqrt{g_{c2}g_{c3}}$. Results are shown in fig. 7. We observe that only in the case of large interception a large departure from the zero interception value does occur.

As a check, the measurement of g_{c3} is also performed in a different way. The insertion gain of the tube is measured again but now with the first instead of the second coupler as the input coupler. The second coupler is detuned. The vane is placed in such a position that no interception occurs and hence the tube insertion loss, eq. 95, is given by $g_{c1} \cdot g_{c3}$. Now the beam current is reduced from the value I_0 to a value I_1 by lowering the cathode temperature.

The insertion loss $g_{c1}g_{c3}$ is measured as a function of I_1/I_0 . Since input and output coupler are made equal we can say that $g_{c3} = g_{c1}/g_{c3}$. Results are also plotted in fig. 7. We see that there is good agreement between the values of g_{c3} determined by quite different methods.

6.2.2 Determination of g_{c1}

Let us now consider the behaviour of g_{c1} . Since the current I_0 flowing through the input coupler is nearly constant, we might expect g_{c1} to be also nearly constant during the measurements. This g_{c1} equals the zero interception value of g_{c3} since both the input and output coupler are identical. We obtain $g_{c1} = 0.84$ as follows from fig. 7 at $I_1/I_0 = 1$. To establish the fact that g_{c1} is nearly constant, we shall consider the mechanism causing the variations of g_{c1} and estimate the errors associated with these variations.

Fig. 7. The overall insertion gain g and output coupler gain g_{c3} as a function of I_1/I_0 .

Notwithstanding the fact that the vane has been covered with a material of poor reflecting capacity, such as spongy chromium, some backward reflection of electrons inevitably occurs. In a two-fold way this affects a reduction of g_{c1} . In the first place the signal energy to be supplied to the beam by the coupler fields is divided over the incident beam I_0 and the reflected beam I_r travelling towards the gun. In the second place the electrons reflected back into the gun might upset the gun adjustment, resulting in a decrease of I_0 and thus causing an additional decrease of g_{c1} .

First we shall determine the loss of signal on account of reflection. A Cuccia-coupler is insensitive to the direction of flow of the beam. This property enables us to identify the incident and the reflected beams with respectively the unscattered and the scattered partial beams and to use the corresponding formulae presented in section 4.2.1. From eq. (44) it follows that ζ equals the factor by which g_{c1} is reduced on account of reflection since in eq. (44) ζ is equal to the ratio of signal power P_1 on the incident beam and the available signal power P_u ($\alpha = 1$ as the coupler is matched).

The electronic impedances Z_1 and Z_2 of, respectively, the incident and the reflected beam follow from eq. (40). To calculate Z_2 ; I_0 is substituted by I_r . The absolute values of the axial velocities of the beams can be assumed to be

equal, as owing to their low impact energy (about 4 eV) the electrons will be mainly elastically reflected. Having thus obtained Z_1 and Z_2 we find, using eq. (42)

$$\zeta = \frac{I_0}{I_0 + I_r} \quad (96)$$

Provided the reflection coefficient ε is small we can say $I_r \approx \varepsilon I_2$ where I_2 is the intercepted current. To insert some numerical values in eq. (97), let us assume that full interception takes place with a vane of uncontaminated silver. Then ³⁹⁾ $\varepsilon = 0.1$, $I_2 = 0.9 I_0$ and $\zeta \approx 0.9$ which means that ten percent of the input power is lost by the reflection. In practice this reduction will be much smaller since in many cases ε is smaller and full interception does not occur.

We must now consider the second source of reduction of g_{c1} i.e. the reduction of I_0 due to deterioration of the gun adjustment by reflected electrons. Since input and output couplers are identical in construction we can, in order to determine the reduction, use fig. 7 directly, provided we identify the reduced beam current with I_1 and read g_{c1} instead of g_{c3} . For example, suppose that I_0 is reduced by 20%, corresponding to $I_1/I_0 = 0.8$, then (fig. 7) we see that g_{c1} will be reduced by only 2 percent. In the actual case, with the vane covered with spongy chromium, I_0 determined from the sum of $I_1 + I_2$ is constant within 8 percent, as may be seen from fig. 8 (tube 88), so that g_{c1} can be considered virtually constant during the measurements.

In order to see whether some influence of the vane coating could be detected, a tube incorporating a vane carrying a number of small washers covered with different materials was constructed (tube 109). The materials tried were gold, silver, constantan, spongy chromium, and soot. The diameter of each washer is 3 mm, it has a hole of 0.8 mm and a thickness of 0.03 mm. Figure 8 shows I_0 measured with tube 109 as a function of I_1/I_0 .

With the soot coating I_0 appears to be reduced by only 8 percent. Therefore g_{c1} can be considered a constant in this case. With the other materials the beam current is reduced by 12 and 20 percent at respectively $I_1/I_0 = 0.5$ and $I_1/I_0 = 0.1$. A current reduction of 20 percent corresponds to 2 percent reduction of g_{c1} (using fig. 7). Assuming $\varepsilon = 0.1$ the corresponding reductions on account of reflection are respectively 4 and 8 percent. This results in an overall reduction of 6 percent at intermediate interception and at large interception ($I_1/I_0 = 0.1$) of 10 percent.

6.2.3 Determination of g_v

The values of g_v of tube 88 calculated from the measured values of g using the values of g_{c3} of fig. 7 are indicated in fig. 9 thereby assuming g_{c1} constant. The corresponding values of g_v of tube 109 are shown in fig. 10, here also g_{c1} is assumed constant. With exception of soot the values of g_v in fig. 10 are

Fig. 8. The beam current $I_0 = I_1 + I_2$ as a function of I_1/I_0 .

possibly 6 percent too low at intermediate values of interception and 10 percent at large interception. However, application of the correction does not alter the picture presented by fig. 10 appreciably. Therefore and since we do not know ε for the various materials the correction is not applied.

6.3 Interpretation of attenuation measurements

When we consider a fast wave with $\omega \approx \omega_c$, the power-flow in case of interception not accompanied by scattering can, according to eq. (32) and eq. (65), be given by

$$B_1 B_1^* = \frac{I_1}{I_0} A_1 A_1^* + \frac{I_2}{I_1} k T_{in} df. \quad (98)$$

Since the ratio of transmitted power flow $B_1 B_1^*$ to incident power flow $A_1 A_1^*$ equals I_1/I_0 , we have $g_v = I_1/I_0$. Therefore, if there is no scattering, the measured values of g_v should all be on a straight line representing $g_v = I_1/I_0$, since g_v is plotted against I_1/I_0 . An average experimental curve, the broken line in fig. 9, is found lying somewhat below the expected one, the maximum dis-

crepancy being about 20 percent. This indicates that although the reduction of g_v due to interception can be largely explained by interception alone, some scattering must be present. Since the electron impact energy is very low, we can assume that the scattered electrons are mainly elastically reflected. Consequently the relation (38) applies, as in this case the scattered electrons have on the average the same axial velocity as the other electrons in the beam. Combination of eqs (38), (91) and (96) yields for the power-flow

$$C_1 C_1^* = \frac{I_1}{I_0} \left(\frac{I_3}{I_1} \right)^2 A_1 A_1^* + \left[1 - \frac{I_1}{I_0} \left(\frac{I_3}{I_1} \right)^2 \right] k T_{in}' df$$

and

$$T_{in}' = \frac{(I_0 - I_1) I_3^2 T_{in 1} + I_0 I_4 (I_3 T_{in 3} + I_1 T_{in 4})}{I_0 I_1^2 - I_1 I_3^2}. \quad (97)$$

Consequently we have $g_v = I_1/I_0 (I_3/I_1)^2$. Since we know I_1/I_0 we can calculate the amount of scattering given by $(I_3/I_1)^2$. For instance, (fig. 9) at $I_1/I_0 = 0.8$ about 10 percent of I_1 is scattered.

Some support for the assumption that scattering is present is provided by measurements of the beam diameter by orbit resonance ³²⁾³³⁾. These measurements ⁶⁾ indicate that the beam is defocussed by the interception. Also it is well known in electron optics that a gun having a limiting aperture in one

Fig. 9. Insertion gain g_v of the vane as a function of I_1/I_0 . Vane material spongy chromium.

Fig. 10. Insertion gain g_v as a function of I_1/I_0 for different vane coatings.

electrode often produces a flared spot. Inspection of fig. 10 reveals that the values of g_v , measured with a soot covering, follow the curve $g_v = I_1/I_0$ more accurately than the points obtained with a silver covering do. This indicates that in the case of a soot covering less scattering occurs. This is to be expected; silver has a much higher reflecting capacity than soot has, as can also be seen from fig. 8. In the case of soot, I_0 is not reduced so much as in all other cases. Surprisingly (fig. 10), the results obtained with the constantan washer with a hole of 0.8 mm are similar as those obtained with soot, whereas those with the constantan washer having a hole of 1.5 mm are not. Microscopic inspection showed, however, that the edge and the walls of the holes show many scratches and grooves due to machining. This, and possibly contamination of the surface, might explain the different behaviour of the constantan washers. Spongy chromium causes only slightly more scattering than soot as shown in fig. 10, but the backward reflection proves to be more serious as exemplified by fig. 8, tube 109, Possibly the face of the washer directed towards the gun might be contaminated during mounting or processing of the washer. Note that in fig. 8 tube 88 does not show such a strong reduction of I_0 . The measured values of g_v obtained with spongy chromium in tube 109 agree, as they should, with those obtained with the aid of tube 88. According to fig. 10 and in accordance with expectations gold causes a larger scattering than soot does. Except for soot and silver a very

marked deviation does not occur. For instance, at $I_1/I_0 = 0.8$, the values g_v of soot and silver differ by about 15 percent and 30 percent respectively from the values of g_v which would occur if there is more scattering. (Earlier measurements ²⁵) did not reveal these effects).

It might be interesting to repeat the measurements with a tube having very smooth holes in the vane and covered by materials free from contaminations. However, in view of the excessive amount of time and labour required to develop such a tube and the relative smallness of the effect this work has not been repeated. A difficult problem which then arises is to keep the materials free from contaminations during the measurements which take at least a few hours; otherwise a technique as has been outlined by Bruining ³⁹) would probably be very useful here. In this technique, immediately prior to the measurements the metal is evaporated by a heated filament and then precipitated on the target.

6.4 Fast wave noise measurements

In this section a description of the noise measuring technique will be given, and the results will be discussed in section 6.5. The noise due to interception can be determined by measuring either the increase of the noise figure or measurement of the noise output of the tube, both as a function of the interception. The noise figure of the tube can be measured by well known techniques ¹³) using a 10 P noise diode ³⁶)³⁷), a 3 db attenuator and a sensitive receiver. Provided the amplification of the tube under test is sufficiently large, the noise figure equals that of the tube, the noise generated by the subsequent receiver being negligible in this case. As we know, the amplification of the tube can be increased by increasing the pumping field strength in the quadrupole. The upper limit is set by the defocussing of the beam ³²). This limit decreases with a decreasing space-charge density in the beam. In our case a lower value of I_1 corresponds to a lower space-charge density, as some defocussing occurs at the vane (sec. 6.1.3). Consequently unwanted extra interception at lower values of I_1 may result if the gain is high. To overcome this difficulty without being forced to measure the receiver noise separately and apply corrections afterwards, we employed a technique ⁶) similar to that of a Dicke receiver ¹³).

The pump oscillator was amplitude-modulated by a tone of 400 c/s and synchronous detection was used. This enabled us to measure the noise figure of the tube alone even at low values of gain, as only signals which had been amplified were modulated and hence detected. Beam defocussing was prevented altogether by a reduction of the pumping field amplitude to provide only about 4 db of electronic gain, resulting in an overall gain of about 2.5 db. The noise figures measured are shown in figs 11, 12 and 13 as a function of $\Omega = (1 - g_v)/g_v$. To calculate the noise figure we must consider the noise in the input system and also that due to the vane (fig. 14). The input coupler extracts fast-wave noise power from the beam whose equivalent temperature is T_b . The equivalent noise

Fig. 11. Noise figure F as a function of $\Omega = (1 - g_v) g_v^{-1}$. Spongy chromium vane.

temperature is defined as usual in terms of the available noise power P_{av} by the relation $P_{av} = kT_b df$.

Due to imperfect noise extraction a residual noise wave with an equivalent temperature T_r remains on the beam. The input coupler is matched by the network to a 50Ω load of temperature T_c . The thermal noise of this load impresses a fast noise wave of temperature $g_{c1} T_c$ on the beam, where g_{c1} , the coupler loss, is equal to $0.84 \pm 3\%$ (0.75 db).

Upon identifying g_{c1} with ζ of eq. (48) we find that the thermal noise of the coupler losses is responsible for another fast wave having a temperature $g_{c1} \cdot (1 - g_{c1}) \cdot T_1$; T_1 is the temperature of the coupler. The coupler is assumed to be uniform of temperature. In combination with eq. (99) we find that the power flow at the input of the quadrupole is given by

$$C_1 C_1^* = k \{ g_v [T_r + g_{c1} T_0 + g_{c1} (1 - g_{c1}) T_1] + (1 - g_v) T_{in'} \} df. \quad (100)$$

If no noise were generated in the tube and by the coupler losses (i.e. $T_r = T_1 = 0$) the power flow would be given by

$$C_1 C_1^* = k g_v g_{c1} T_0 df. \quad (101)$$

Remembering that $\omega \approx \omega_c \approx \omega_i$ we note that eqs (98) and (99) describe the power flow in both the signal and the idler channels. Consequently, provided the measuring receiver is sensitive at ω only, (and not at ω_i) the double channel noise figure is given by ¹³⁾³⁸⁾

$$F = 1 + \frac{T_r}{g_{c1}T_0} + (1 - g_{c1}) \frac{T_1}{T_0} + \frac{1 - g_v}{g_v g_{c1}} \frac{T_{in}'}{T_0} = F_0 + \frac{\Omega}{g_{c1}} \frac{T_{in}'}{T_0} \quad (102)$$

with

$$\Omega = \frac{1 - g_v}{g_v}.$$

The first term F_0 of the right-hand equation represents the zero interception noise figure, whereas the second describes the increase due to interception. In practice $T_1 \approx T_0 = 290^\circ \text{K}$. From a direct measurement of the output noise equivalent temperature ¹³⁾ of the tube we can also determine T_{in}' . In this case no pump power is applied to the quadrupole and hence possible errors due to defocussing and second order coupling are eliminated. However, the method proves to be more tedious, since we must account, by means of g_{c3} , for the

Fig. 12. Noise figure F as a function of Ω ; here larger values of Ω are also indicated (fig. 11).

$$T_{eq\ 3} = g_{c3} \frac{C_1 C_1^*}{kdf} + (1 - g_{c3}) T_3 . \quad (103)$$

Here $T_{eq\ 3}$ is the equivalent output noise temperature and T_3 is the temperature of the output coupler and associated network. In practice $T_3 \approx T_1 \approx T_0 = 290^\circ\text{K}$. Results are shown in figs 15 and 16.

Fig. 15. Transverse temperature T_{in}' as a function of Ω .

Fig. 16. Transverse temperature T_{in}' as a function of I_1/I_0 for two different gun adjustments.

6.5 Interpretation of the noise measurements

In this section we shall discuss the results obtained with the aid of the technique described in the previous section. Measured noise figures are shown in figs 11, 12 and 13. From eq. (102) it can be concluded that, provided T_{in}' can be considered a constant, F as a function of Ω should follow a straight line intersecting the ordinate at $F = F_0$. According to eq. (97), T_{in}' comprises $T_{in\ 1}$, $T_{in\ 3}$ and $T_{in\ 4}$.

If the temperatures $T_{in\ 3}$ of the unscattered partial beam and $T_{in\ 4}$ of the scattered partial beam are equal to $T_{in\ 1}$, the internal temperature of the incident beam, then $T_{in'} = T_{in\ 1}$, eq. (97). We may expect this to be the case at low values of Ω , i.e. at low interception, since then the beam is not strongly disturbed. In fact, initially the measured values of F follow a straight line, figs 11 and 12. The slope of the line corresponds to $T_{in'} = 2100\ ^\circ\text{K}$ ($\pm 10\%$). With larger values of Ω a departure occurs, indicating an increase of $T_{in'}$. This is to be expected, since at the vane expansion occurs which causes extra rotations and an increase in temperature (cf. sec. 5.2.2) of the scattered beam. Probably another effect may also play a role: The electrons reflected back into the gun by the vane alter the focussing conditions of the gun and thereby increase the non-thermal rotations and hence increase $T_{in'}$, i.e. $T_{in\ 1}$ and $T_{in\ 2}$ as well as $T_{in\ 3}$. Fig. 15 shows the values of $T_{in'}$ obtained from the measured output noise. The average value $T_{in'} = 2280\ ^\circ\text{K}$ agrees reasonably well with $T_{in'}$ found from figs 11 and 12.

According to fig. 13, F as measured with different vane materials initially follows a straight line also. However, the slope of the line and hence $T_{in'}$ is different for different materials. Possibly this is caused by the backward reflected electrons upsetting the gun adjustment resulting in modifications of $T_{in'}$.

The focussing voltages of the electron gun have not been selected to provide a minimum value of $T_{in'}$ but rather to minimize the noise figure at zero interception which is not directly related to $T_{in'}$. Therefore, depending upon the details of the reflection and the gun focussing conditions, $T_{in'}$ can either be raised or lowered.

Comparison of $T_{in'}$ measured with different materials and $T_{in'}$ in the case of soot, having the least reflection, shows that both cases occur. For instance; $T_{in'} = 2140\ ^\circ\text{K}$ in the case of silver, and $T_{in'} = 3600\ ^\circ\text{K}$ in the case of constantan, (1.5 mm hole) are respectively lower and higher than $T_{in'} = 2870\ ^\circ\text{K}$ obtained with a soot covering. With other materials the deviations encountered are much smaller.

The largest amount of scattering takes place with a silver covering (fig. 10). A large scattering in conjunction with an increase of $T_{in\ 4}$ due to beam expansion (sec. 5.2.2) gives rise to larger deviations of the locus of F from a straight line. Fig. 13 shows that although the effect is not very distinct, this indeed happens.

In the case of spongy chromium (tube 109) $T_{in'} = 2870\ ^\circ\text{K}$, which is rather high in comparison with the results obtained in the case of tube 88 having also a spongy chromium covered vane. However, the gun of tube 109 is not a very good one, its beam current appears to be lower and its noise figure higher ($F_0 = 2$) than the corresponding quantities of tube 88.

Fig. 16 gives $T_{in'}$, as determined from the output noise of tube 123, in which the vane is covered with spongy chromium and has a hole of 1 mm diameter.

Two different focussing conditions of the gun are used. (Tube 88 did not permit this experiment, since a_1 and a_6 have been interconnected.) The lower curve of fig. 16 corresponds to a gun setting with voltages as have been mentioned in section 6.1. The upper curve is obtained with a slightly different set of voltages: $V_{a1} = 6.4$ V, $V_{a2} = 147$ V, $V_{a3} = 44$ V, $V_{a4} = 0$ V, $V_{a5} = 11.2$ V and $V_{a6} = 6$ V. We observe that $T_{in'}$ indeed depends strongly upon the gun focussing. This fact provides some support for the hypothesis that electrons reflected back into the gun modify $T_{in'}$, since they alter the focussing conditions. Also it can be clearly seen from fig. 16 that $T_{in'}$ increases with increasing interception, i.e. with lower values of I_1/I_0 . The temperature $T_{in'} \approx 2100$ °K found from the lower curve of fig. 15 in the region of $0.5 \leq I_1/I_0 \leq 1$ agrees very well with the values of $T_{in'}$ obtained with the aid of tube 88; this is to be expected since both vane and gun adjustment are the same in both cases.

7. Experimental verification with synchronous waves

The previous experiments involved exclusively fast waves, we now shall consider some measurements involving synchronous waves as well. The main design of the tube used is shown in fig. 17. To impress a synchronous wave on the beam a combination of a Cuccia-coupler and a quadrupole system is employed.

Except for the pumping frequency being ω_c instead of $2\omega_c$ the quadrupole is identical to the one used to obtain amplification, sec. 6.2.3. According to theories presented by a number of authors²²⁾²³⁾¹⁸⁾ the waves are not amplified in this case but instead a periodic transformation of a signal into an "idler" wave and vice versa takes place.

Fig. 17. Exploded view of the tube used for the synchronous wave measurements.

The axial dependence of the transformation is sinusoidal and its axial period depends upon the pumping field strength, cyclotron frequency and electron velocity. In our case the signal wave is a fast wave and according to the theory the "idler" is a synchronous wave. The synchronous wave is a positive energy wave if $\omega < \omega_c$ and a negative energy wave if $\omega > \omega_c$. (By chance during these measurements the magnetic field was negative, for a positive field we would have a positive energy wave for $\omega > \omega_c$ and a negative one for $\omega < \omega_c$). Consequently we can study either one of the synchronous waves by a proper choice of the signal frequency. Since we use Cuccia-couplers, $\omega \approx \omega_c$ and therefore $\omega_i = |\omega - \omega_c|$ is a rather low frequency. Hence the propagation constants $\beta_3 = \beta_4 = \omega_i/u_0$ are small, so that attenuation due to multivelocity effects will be negligible. Let us insert numerical values in eq. (60); with $L = 2.10^{-3}$ m being the spacing between the quadrupoles, $T_{II} \approx 2500^\circ \text{K}^{25}$, $u_0 = 11.9.10^5$ m/sec ($V_b \approx 4$ V), $f_c = 550$ Mc/s, $f_i = 20$ Mc/s we obtain $S_3 S_3^* > 0.999$ resulting in a negligible loss.

The main operation can be understood from fig. 18. The input coupler removes

Fig. 18. Block diagram of the operation of the tube shown in fig. 17. Input and output Cuccia-couplers interact only with a fast wave of frequency ω , and not with $\omega_i = |\omega_c - \omega|$ the latter frequency being rather low. The quadrupoles convert the fast wave into a synchronous wave and vice versa.

fast-wave beam noise and impresses a fast wave on the beam. During passage of the first quadrupole the fast wave is converted into a synchronous wave, and synchronous wave noise into fast wave noise. If interception at the vane occurs, both waves are attenuated and new noise in both modes is introduced. By the second quadrupole the synchronous wave is converted into a fast wave, and finally it is taken off by the output coupler. The synchronous wave due to the fast wave at the input of the second quadrupole remains undetected by the output coupler.

The tube is adjusted as follows: First of all the gun and the Cuccia-couplers are adjusted to provide maximum signal transmission; the overall insertion gain is about 0.7 times. No interception at the vane should occur during the tube

adjustment. Then pump power is applied to the first quadrupole and its amplitude is increased until the output signal disappears. Very large attenuations can be obtained in this way. However, beam defocussing and associated errors which are likely to occur in the measurements put an upper limit to the useful attenuation obtainable. Having adjusted the fields in the first quadrupole, we apply a pump signal originating from the same generator to the second quadrupole also and adjust its amplitude and phase to restore the signal transmission.

It is interesting to note that at the same time the second quadrupole restores the signal, it refocusses the beam, so that almost no beam current is lost. This can probably be explained as follows; A quadrupole field is known to provide focussing of the beam in one transverse direction and to defocus it in the other; this causes the beam to assume a twisted ribbon-like shape which is restored to a cylindrical one by the second quadrupole. For instance, to provide an attenuation of 16.5 db by means of the first quadrupole alone the beam current decreases from 26 to 12 μA ; after adjustment of the second quadrupole the signal transmission proved to be only 0.7 db worse and the collector current rose to 24 μA , so that only 2 μA were lost. The adjustment used during the measurements, providing a signal loss of 13 db, results in a beam current loss

Fig. 19. Insertion gain of a positive-energy synchronous wave versus I_1/I_0 , spongy chromium vane, for different hole diameters.

of only $1 \mu\text{A}$. A signal loss of 13 db means that except for about 5% all the fast wave power is converted into synchronous wave power and vice versa. When both quadrupoles were put into operation a reduction of 10% of the overall insertion gain was observed. Results of the measurements of g_v , the insertion gain of the vane in the case of a positive energy synchronous wave, are shown in figs 19 and 20. Negative energy synchronous wave measurements are shown in fig. 21.

Fig. 20. Insertion gain of a positive-energy synchronous wave versus I_1/I_0 , spongy chromium vane.

7.1 Interpretation of the attenuation measurements

First we shall discuss the insertion gain measurements. Eqs (22) and (38) predict the same attenuation for the same amount of interception for every single wave. Comparison of figs (19), (20) and (21) with fig. 9 shows that this is indeed the case for the fast cyclotron and both the synchronous waves within a few percent. Implicitly this provides experimental support for the assumption that the probability of electron capture does not depend upon the signal. This point is stressed even more when we consider in fig. 19 the insertion gain caused by the 0.3 mm hole. Here we see that g_v is not very much lower than the value measured with the larger holes. On the basis of the laminar flow model one

Fig. 21. Insertion gain of a negative-energy synchronous wave versus I_1/I_0 , spongy chromium vane.

would rather expect g_v to be very low, since in this case the hole diameter of 0.3 mm is much smaller than the beam diameter, the latter being 0.4 to 0.5 mm. Probably the fact that actually they are somewhat lower might be explained by an accidental contamination of the 0.3 mm hole. Experimental support for this point of view that contamination is present is provided by measurements with this tube and with the fast cyclotron wave. The fast wave measurements (tube 127) with the 0.3 mm hole are also shown in fig. 19 and we observe that the same deviation occurs. In fig. 9 with tube 88 such an effect is not observed.

7.2 The synchronous wave noise

The relationship between the tube output noise temperature T_{out} measured at the output coupler and the noise generated by the interception can be calculated as follows: Let the beam carry a fast wave of temperature T_b and a synchronous wave of temperature T_s when it enters the first quadrupole. The completeness of the conversions by the quadrupole is given by the insertion gain g_q ; a complete conversion results in $g_q = 1$ and in our case, as 5 percent is not converted, $g_q = 0.95$. Consequently the first quadrupole output consists of a fast wave having $T_b' = g_q T_s + (1 - g_q) T_b$ and of a synchronous wave

of $T_s' = g_q T_b + (1 - g_q) T_s$. According to eqs (32), (38), (65) and (91) the temperatures of the fast and synchronous waves beyond the vane are given respectively by $T_b'' = g_v T_b' + (1 - g_v) T_{in}'$ and $T_s'' = g_v T_s' + (1 - g_v) T_{in}''$, and

$$T_{in}'' = \frac{(I_0 - I_1) I_3^2 (1 + \delta_1^2) T_{in1} + I_0 I_4 [I_3 (1 + \delta_3^2) T_{in3} + I_1 (1 + \delta_4^2) T_{in4}]}{I_0 I_1^2 - I_1 I_3^2}$$

If $\delta_1^2 \approx \delta_2^2 \approx \delta_3^2 \approx \delta^2$, then

$$T_{in}'' = (1 + \delta^2) T_{in}' \quad (104)$$

T_{in}' is given by eq. (99).

The temperature T_b''' of the fast wave which is presented to the output coupler is $T_b''' = g_q T_s'' + (1 - g_q) T_b''$ and the output noise temperature $T_{out} = g_{c3} T_b''' + (1 - g_{c3}) T_0$, where g_{c3} represents the output coupler's insertion gain. Thus the relationship of T_{out} and T_{in}'' has been established, and if we know T_b , T_s , g_q , g_v and g_{c3} , we can calculate T_{in}'' from the measured values of T_{out} . (Note that, since we consider only temperatures as "seen" at ω , the above equations do not contain the ratio ω_i/ω .)

The measurements are done with tube 127 and tube 133. Tube 127 did not allow measurements at $f = 570$ Mc/s owing to parasitic coupling of the quadrupoles and the couplers. Therefore, with tube 127 only measurements on the

Fig. 22. T_{in}'' as measured both with positive and negative energy synchronous waves.

positive energy synchronous wave were done. The temperatures T_b and T_{in}' are measured as described in sec. 6.4, while T_s is determined from the output noise if only a single quadrupole is in operation. In the latter case the synchronous noise wave is converted into a fast wave which appears at the output. Having obtained these temperatures we can find T_{in}'' . Results are indicated in fig. 22.

7.3 Interpretation of the noise measurements

We shall now consider the noise measurements. A very accurate discussion of the values of T_{in}'' (fig. 22) is impossible since we do not know the exact beam diameters and the current density distributions over the cross-sections of the incident, the unscattered and the scattered partial beams. To estimate the value of T_{in}'' to be expected we use the expression $T_{in}'' = (1 + \delta^2) T_{in}'$, as given in eqs (102) and (68). The space-charge suppression factor Γ^2 is about 0.6, as has been measured by the usual method of comparing the beam's shot current with that of a saturated diode¹³). The values of δ^2 and T_{in}'' obtained with $T_{in}' = 2200$ °K (sec. 6.5) and $\Gamma^2 = 0.6$ are shown in table I. Since the current distribution of the beam will be something between a homogeneous and a parabolical distribution both distributions have been considered in the table.

TABLE I

Current distribution	$D \cdot 10^{-3} \text{ m}$	δ^2	$T_{in}'' \text{ °K}$
Homogeneous	0.4	2.88	7200
„	0.5	3.56	10000
Parabolical	0.4	0.91	4200
„	0.5	1.27	5000

Inspection of fig. 22 and comparison with table I shows that, at low interception, T_{in}'' measured with the negative energy wave (tube 133) is indeed of the right order of magnitude. However, T_{in}'' measured with a positive energy wave (tubes 127 and 133, 1 mm hole) is considerably higher. We shall discuss this discrepancy in greater detail in section 7.3.1.

From fig. 22 we see that T_{in}'' first increases with increasing interception and then decreases. This can be explained by consideration of $T_{in}'' = (1 + \delta^2) T_{in}'$ and $\delta^2 = m\omega_c^2 \Gamma^2 D^2 / 16 k T_{in}'$ (eq. 68) and where $T_{in} \approx T_{in}'$. From fig. 16 it follows that T_{in}' initially remains constant and then rises with larger values of interception. Increasing interception (fig. 22) results in an increase of Γ^2 towards 1 due to the partition noise. With small interception D^2 is probably

not very much reduced and therefore the increase of I^2 results in an increase of T_{in}'' . Ultimately, at larger interception the decrease of D^2 prevails over the increase of I^2 and also over the increase of T_{in}' , so that an overall decrease of T_{in}'' results. The effect of a reduction of D is clearly illustrated by the values of T_{in}'' measured with the 0.3 mm hole, which are lower than any T_{in}'' measured with the same interception and larger holes.

7.3.1 Temperature difference of the synchronous waves

We shall now explain why T_{in}'' of the positive energy wave is higher than that of the negative energy wave. Repeated measurements using slightly different sets of gun voltages showed the difference to be consistent and the figures presented the lowest obtainable. Moreover, it has been measured that T_s , being the temperature of the synchronous waves emerging from the gun, also shows this effect. The positive energy wave has $T_s \approx 5300$ °K which is larger than $T_s \approx 3600$ °K for the negative energy wave.

As has been verified by connecting the pumping oscillator directly to the receiver input the difference could not be caused by the noise side-bands of the oscillator. (The receiver bandwidth is 0.1 Mc/s and the separation of the signal frequency and pump frequency either 15 or 20 Mc/s.) Also, noise "spilled" by a space-charge wave or by higher harmonics waves would have been a possibility if there was a sufficiently strong longitudinal field component ²²⁾⁴⁰⁾. Because infinite phase velocity couplers are used and since the beam has been centered carefully such a field is not present.

However, earlier in this chapter it has been mentioned that the beam is defocussed by the pumping field in the first quadrupole and refocussed by that in the second one. Although the amplitude of the pumping field has been reduced to such an extent that interception on account of defocussing is prevented, probably the electrons still acquire a coherent motion in the first quadrupole. This motion is not yet erased by the turbulency of the beam when the beam enters into the second quadrupole and therefore certainly not at the vane. (Otherwise a refocussing by the second quadrupole-field would have been impossible.) Consequently, with respect to this new motion, the assumption that $\dot{x}y^{*T}$ and similar terms occurring in $\mathbf{Q}$ (eq. 23) and also in $\mathbf{Q}_1$, $\mathbf{Q}_3$ and $\mathbf{Q}_4$, which refer to the internal motion, are zero is incorrect. Moreover the interception cannot be considered to be completely at random as the vane intercepts the outer electrons first of all.

However, with respect to the original motions the assumptions are probably still correct. In section 7.1 it was found that the probability of capture does not depend upon the signal. Actually this does not prove that the interception occurs completely at random, as it only says that the electrons in the proximity of the edge of the vane have almost equal probability of being captured. However, any coherent internal motion, as for instance caused by the lenses of the gun will be

erased by the turbulency of the beam by the time the beam hits the vane. The distance of the vane from the gun is 28 mm whereas the distance from the first quadrupole is only 1 mm, corresponding respectively to 14 and 0.5 cyclotron wavelengths.

Let us consider what happens with the temperature of the waves if $\dot{x}\dot{y}^{*\tau}$ and similar terms of $\mathbf{Q}$ are unequal to zero. By writing out the expression $\mathbf{N} \mathbf{X} \mathbf{X}^\dagger \mathbf{N}^\dagger$ the diagonal terms of $\mathbf{Q}$ divided by $\dot{x}\dot{x}^* + \dot{y}\dot{y}^{*\tau}$ are found to be

$$(q_{11}, q_{22}, q_{33}, q_{44}) =$$

$$(1 + \delta', -1 + \delta', -1 - \delta^2 - \delta' - \delta'' - \delta''', 1 + \delta^2 - \delta' - \delta'' + \delta''').$$

with

$$\varepsilon^2 = \omega_c^2 \frac{\overline{xx^* + yy^{*\tau}}}{\dot{x}\dot{x}^* + \dot{y}\dot{y}^{*\tau}}, \quad \delta' = -j \frac{\overline{\dot{x}\dot{y}^* - \dot{x}^*\dot{y}^\tau}}{\dot{x}\dot{x}^* + \dot{y}\dot{y}^{*\tau}}, \quad (105)$$

$$\delta'' = j \frac{\omega_c (\dot{x}\dot{x}^* + \dot{y}\dot{y}^* - \dot{x}^*\dot{x} - \dot{y}^*\dot{y}) + \omega_c^2 (x^*y - xy^{*\tau})}{\dot{x}\dot{x}^* + \dot{y}\dot{y}^{*\tau}} +$$

$$\delta''' = \frac{\omega_c (\dot{x}\dot{y}^* + \dot{x}^*\dot{y} - \dot{y}\dot{x}^* - \dot{y}^*\dot{x})}{\dot{x}\dot{x}^* + \dot{y}\dot{y}^{*\tau}}.$$

From eq. (105) it follows that if $\delta' + \delta'' \neq 0$ the temperatures of respectively the negative and the positive energy synchronous noise wave are different since then $|q_{33}| \neq |q_{44}|$. Similarly the temperatures of respectively the fast and the slow cyclotron wave are unequal if $\delta' \neq 0$ since then $|q_{11}| \neq |q_{22}|$.

It should be noted that the off-diagonal terms as well as the diagonal terms of $\mathbf{Q}$ do change in case of correlation. Consequently also the correlation of the cyclotron and the synchronous waves is affected. As we do not need the result we shall not discuss the off-diagonal terms here.

Unfortunately we do not know the motion of the electrons of the beam emerging from the quadrupole to allow a calculation of δ' and δ'' directly. However, we can find an approach by considering the following rather special case. Let us assume that the quadrupole field causes a number of electrons to rotate with frequency ω_r around the beam axis. We shall first consider these electrons only. The frequency ω_r depends upon the space-charge conditions of the beam, sec. 5.1.2, eq. (81). This implies (fig. 23) that if we specify x we also specify $\dot{y} = \omega_r x$ and similarly $\dot{x}$ and y are related by $\dot{x} = -\omega_r y$. Since the coördinates x and y are independent all cross-correlation terms except those involving x and $\dot{y}$ or y and $\dot{x}$ are zero. Let us write $\dot{x} = |\dot{x}| \exp[j\phi]$, $\dot{y} = |\dot{y}| \exp[j\phi_2]$, $x = |x| \exp[j\phi_3]$ and $y = |y| \exp[j\phi_4]$. Assuming symmetry and since $\dot{x} = -\omega_r y$ and $\dot{y} = \omega_r x$ we find that $|\dot{x}| = |\dot{y}| = \omega_r |x| = \omega_r |y|$, $\phi_4 = \pm \phi_1 \pm 2m\pi$ and $\phi_3 = \pm \phi_2 \pm (2n + 1)\pi$, n and m are integers. Inserting

Fig. 23. The electrons in the beam rotate with frequency ω_r around the beam axis. The triangles with respectively $\mathbf{r}_{jin}$ and $\hat{\mathbf{r}}_{jin}$ are equiangular. Consequently $\dot{x} = -\omega_r y$ and $\dot{y} = \omega_r x$.

x , y , $\dot{x}$ and $\dot{y}$ into the expressions for δ^2 , δ' , δ'' and δ''' , (eq. (105)), and remembering that ϕ_1 and ϕ_2 are independent since x and y are, we find upon averaging that $\delta^2 = \omega_c^2/\omega_r^2$, $\delta' = 0$, $\delta'' = 0$ and $\delta''' = 2\omega_c/\omega_r$. Because $\delta' = \delta'' = 0$ the internal temperatures of respectively the cyclotron-waves and the synchronous waves remain equal. However, if we do not longer assume x and y independent and write $\phi_4 = \phi_3 + \pi/2$ then the picture alters profoundly. We then obtain: $\delta^2 = \omega_c^2/\omega_r^2$, $\delta' = -1$, $\delta'' = -2\omega_c/\omega_r - \omega_c^2/\omega_r^2$ and $\delta''' = 2\omega_c/\omega_r$. Hence:

$$(q_{11}, q_{22}, q_{33}, q_{44}) = (0, -2, 0, 2 + 2\omega_c^2/\omega_r^2 + 4\omega_c/\omega_r). \quad (106)$$

Obviously now the temperature of the positive energy synchronous wave is higher than that of the negative energy synchronous wave as $|q_{44}| > |q_{33}| = 0$. By considering x and y it can be concluded that the relation $\phi_4 = \phi_3 + \pi/2$ means that the locus of the electrons rotating with ω_r around the center-mass of the disc is a straight line. Hence the (partial) beam assumes the shape of a ribbon, possibly a twisted one, which rotates with ω_r around the beam axis.

We shall now calculate the increase of the complete beam's internal temperature on account of the ribbon beam consisting of n electrons out of N the total number of electrons in every disc of the beam. We observe that the problem is essentially the same as the calculation of the resulting powerflow of a beam consisting of two partial beams; here the electrons of one partial beam show the rotations. The noise powerflow generated by disturbing the beam is proportional to the internal temperature ($\mathbf{Q} \cdot T_{in}$). Consequently, if all the electrons have the same axial velocity we can use eq. (90), provided we identify the diagonal terms of $\mathbf{C}''\mathbf{C}''^\dagger \mathbf{p}$ and $\mathbf{B}''\mathbf{B}''^\dagger \mathbf{p}$ with the corresponding internal temperatures of respectively the complete beam and the partial beam of the rotating

electrons. (We consider only the increase of the internal temperature.) Further $I_3/I_1 = n/N$. Hence we obtain

$$\Delta T_{in} = \frac{n}{N} q_{44} T_{in r}. \quad (107)$$

The temperature $T_{in r}$ is defined in similarity with T_{in} and relates to the ribbon beam consisting of the rotating electrons. In order to derive an expression for $T_{in r}$ of the ribbon beam we must know the density distribution of the electrons. Earlier in this chapter it has been observed that a further increase of the pumping field strength will produce a strong defocussing. This indicates a large electron density at the edges of the ribbon. Keeping this in mind we shall represent the number of electrons encountered in the disc between the radii r and $r + dr$ by $dn = 8 n r dr / D_r^2$ rather than by a constant number. Here D_r denotes the width of the ribbon beam. This distribution yields the same value for $\bar{r}^2$ as has been found in the case of a homogeneous beam having a circular cross-section. Consequently we can find $T_{in r}$ from eq. (80) by substituting $\frac{1}{2}\omega_c$ by ω_r and putting $T_1 = 0$ (we do not consider thermal rotations). We obtain

$$T_{in r} = \frac{m\omega_r^2 D^2}{16 k}. \quad (108)$$

Suppose that the original motion and the formation of the ribbon beam by the pumping are independent. Then the beam's internal temperature is expressed by the sum of the original temperature and the increase given by eq. (108). Let us do so for the synchronous waves and calculate the ratio of the temperatures. We obtain for the ratio of the internal temperature $T_{in'' \text{ pos}}$ of the positive energy waves and that of the negative energy wave $T_{in'' \text{ neg}}$:

$$\frac{T_{in'' \text{ pos}}}{T_{in'' \text{ neg}}} = \frac{T_{in'' \text{ neg}} + \frac{n}{N} q_{44} T_{in r}}{T_{in'' \text{ neg}}} \quad (109)$$

and

$$q_{44} = 2 + 2 \omega_c^2 / \omega_r^2 + 4 \omega_c / \omega_r.$$

If we assume that the interception is not random, and that the outer electrons are intercepted first of all, then the beam diameter will gradually decrease with interception. Consequently, if $D \approx D_r$, we can say that $D_i^2/D^2 = I_1/I_0$ where D_i is the beam diameter after interception. Hence $T_{in r} = (I_1/I_0) T_{in r0}$ where $T_{in r0}$ is the value of $T_{in r}$ at zero interception.

Let us now insert some numerical values in eq. (109). In a beam having a uniform density, two values ω_+ and ω_- (eq. (81)) of ω_r are possible. Using $I_0 = 29 \mu\text{A}$, $D = 0.5 \text{ mm}$, $u_0 = 11.9 \times 10^5 \text{ m/sec}$ and $f_0 = 550 \text{ Mc/s}$ we obtain $f_r = 250 \text{ Mc/s}$, $\omega_+/\omega = 0.87$ and $\omega_-/\omega = 0.13$. In the case of $\omega_r = \omega_+$

we obtain $q_{44} = 9.24$ and $T_{in\ r0} = 9900$ °K. If $\omega_r = \omega_+$ we obtain $q_{44} = 151$ and $T_{in\ r0} = 220$ °K. Upon substitution into eq. (109) we find:

$$\frac{T_{in''\ pos}}{T_{in''\ neg}} \approx \frac{T_{in''\ neg} + c \frac{n}{N} \frac{I_1}{I_0}}{T_{in''\ neg}} \quad (110)$$

with $c = 9.2 \times 10^4$ and $c = 3.3 \times 10^4$ if $\omega_r = \omega_+$ and $\omega_r = \omega_-$ respectively. From fig. (22) with $I_1/I_0 = 0.8$ (1 mm hole) it follows that $T_{in''\ pos}/T_{in''\ neg} = 2.96$. According to eq. (110) this corresponds to $cn/N = 1.08 \times 10^4$ yielding $n/N = 0.12$ and $n/N = 0.33$ if $\omega_r = \omega_+$ and $\omega_r = \omega_-$ respectively. Let us use this particular value of cn/N and calculate the ratio of the temperature for other values of I_1/I_0 . The results are shown in Table II. (We consider only measurements in the range $0.8 \geq I_1/I_0 \geq 0.2$ because these are more accurate than those outside this range).

TABLE II

I_1/I_0	$T_{in''\ neg}$ °K	$T_{in''\ pos}/T_{in''\ neg}$	
		calculated	measured
0.8	4400	2.96	2.96
0.6	6000	2.08	2.14
0.4	7000	1.62	1.55
0.2	6400	1.34	1.40

According to table II the calculated values of $T_{in''\ pos}/T_{in''\ neg}$ agree very well with those determined from the measurements in fig. 22 (1 mm hole).

Therefore we can conclude that the difference of the internal temperatures of the synchronous waves can adequately be explained by the presence of a ribbon shaped partial beam. The physical appearance of the actual beam corresponding to this model will be a beam having a more or less elliptical cross-section with a concentration of electrons near the ends of the major axis.

As further independent data are not available and since the available quadrupole theory only predicts that under space-charge conditions the internal motion is hardly affected, we cannot determine whether the ribbon beam rotates with ω_+ or ω_- . This point could have been settled by a measurement of the difference of the internal temperature of the cyclotron waves. According to eq. (106) this difference is given by $2 I_1 n T_{in\ r} I_0^{-1} N^{-1}$, the slow cyclotron wave having the highest temperature. The maximum temperature difference being 49 °K and 1000 °K in the case of a rotation at respectively the frequency ω_- and ω_+ . Unfortunately, at present no slow cyclotron wave couplers which are sufficiently

free from side-effects, such as coupling to the fast synchronous and to the space-charge waves, are available.

Finally we shall explain qualitatively the difference observed in T_s , the temperature of the synchronous waves emerging from the gun, their ratio being about 1.5. In the gun there is a region in which the beam is strongly decelerated ($V_{a1} = 0V$). In this region the spread in axial velocity is not negligible causing an attenuation of the original synchronous noise waves (sec. 4.3) and a generation of new synchronous noise waves (sec. 5.3). Although the temperatures of the original synchronous waves starting from the cathode are equal, a correlation in the electron motion causes a difference in temperature of the new synchronous wave. Probably this correlation builds up as follows. A minor mechanical misalignment of the gun electrodes or a small non-uniformity of the cathode surface causes the beam to have a non-circular cross-section or to have an off-centered density distribution. (We effectively use a spot of only 0.25 mm of the cathode surface, see fig. 6). Consequently, in similarity with the ribbon beam a number of electrons show a correlation of their x and y displacement. In order to obtain Brillouin focussing the cathode of the gun is shielded by the first anode which is made of iron. Immediately beyond this anode the electrons will perform concentric rotations with the frequency $\frac{1}{2}\omega_c$. The subsequent beam expansion will increase the rotation frequency to a value ω_+ given by eq. (81). In the case of the ribbon beam we have seen that if the rotations occur at the frequency ω_+ the number of electrons having x and y correlation need to be only small. As here the temperature ratio observed is much smaller than that caused by the ribbon beam at $I_1/I_0 = 0.8$, we can be rather sure that the effects mentioned actually are responsible for the temperature difference.

8. Conclusions

We can conclude that the theory predicts correctly the attenuation on account of interception and scattering of the fast cyclotron and both synchronous waves. The noise generated in the fast cyclotron and negative energy synchronous mode is satisfactorily accounted for by the theory. The noise generated in the positive energy synchronous mode was found to be much larger than expected by the simple theory. The discrepancy could be adequately accounted for by the presence of a correlation of the displacement and also of the displacement and the velocity of the internal electron motion.

Literature

- 1) R. Adler, Proc. Inst. Radio Engrs **46**, 1300-1301, 1958.
- 2) R. Adler, G. Hrbek, G. Wade, Proc. Inst. Radio Engrs **46**, 1756-1757, 1958.
- 3) R. Adler, G. Hrbek, G. Wade, Proc. Inst. Radio Engrs **47**, 1713-1723, 1959.
- 4) E. I. Gordon, Bell Syst. T. J. **39**, 1603-1616, 1960.
- 5) A. E. Siegman, J. appl. Phys. **31**, 17-26, 1960.
- 6) P. A. H. Hart, J. appl. Phys. **33**, 2401-2408, 1962.
- 7) C. P. Lea Wilson, Proc. Inst. Radio Engrs **48**, 255-256, 1960.

- ⁸⁾ E. I. Gordon, Proc. Inst. Radio Engrs **49**, 1208, 1961.
- ⁹⁾ R. Kompfner, Nachr. techn. Fachber. **22**, 403-405, 1960.
- ¹⁰⁾ H. A. Haus, F. N. H. Robinson, Proc. Inst. Radio Engrs **43**, 981-991, 1955.
- ¹¹⁾ T. Wessel-Berg, K. Bløtekjaer, Agard low-noise conf. Oslo 1961.
- ¹²⁾ T. Wessel-Berg, K. Bløtekjaer, Trans. Inst. Radio Engrs **ED9**, 388-399, 1962.
- ¹³⁾ A. van der Ziel, "Noise" Prentice Hall Inc., New Jersey 1954.
- ¹⁴⁾ G. Wade, K. Amo, D. A. Watkins, J. appl. Phys. **25**, 2514-2520, 1954.
- ¹⁵⁾ A. M. Mood, Introduction to the Theory of Statistics, McGraw Hill Inc. 1950.
- ¹⁶⁾ K. Bløtekjaer, J. appl. Phys. **33**, 2409-2414, 1960.
- ¹⁷⁾ G. W. Preston, J. appl. Phys. **27**, 627-630, 1956.
- ¹⁸⁾ R. W. Gould, C. C. Johnson, J. appl. Phys. **32**, 248-258, 1961.
- ¹⁹⁾ C. L. Cuccia, RCA Rev. **10**, 270-303, 1949.
- ²⁰⁾ L. P. Smith, C. I. Suhlmann, Proc. Inst. Radio Engrs **35**, 644-657, 1947.
- ²¹⁾ P. A. H. Hart, C. Weber, Nachr. techn. Fachb. **22**, 358-361, 1960.
- ²²⁾ E. I. Gordon, A. Ashkin, J. appl. Phys. **32**, 1137-1144, 1961.
- ²³⁾ K. Bløtekjaer, T. Wessel-Berg, Nachr. techn. Fachber. **22**, 372-382, 1960.
- ²⁴⁾ T. J. Bridges, C. P. Lea Wilson, J. C. Vokes, Nachr. techn. Fachb. **22**, 352-355, 1960.
- ²⁵⁾ P. A. H. Hart, Paper read at the IVth Int. Congress on microwave tubes, Scheveningen, Netherlands, Sept. 1962.
- ²⁶⁾ G. Hermann, J. appl. Phys. **29**, 127-133, 1957.
- ²⁷⁾ T. Wessel-Berg and K. Bløtekjaer, Paper read at the IVth Int. Congress on microwave tubes, Scheveningen, Netherlands, Sept. 1962.
- ²⁸⁾ A. H. W. Beck, Space charge waves, Pergamon Press 1958.
- ²⁹⁾ W. Gröbner, N. Hofreiter, Integraltafel II, Springer Verlag 1961.
- ³⁰⁾ K. J. Harker, J. appl. Phys. **28**, 645-650, 1957.
- ³¹⁾ A. Ashkin, J. appl. Phys. **28**, 564-569, 1957.
- ³²⁾ E. Adler, A. Ashkin, E. I. Gordon, J. appl. Phys. **32**, 672-675, 1961.
- ³³⁾ P. A. H. Hart, C. Weber, Philips Res. Repts **16**, 376-388, 1961.
- ³⁴⁾ H. Meinke, F. W. Gundlach, Taschenbuch der Hochfrequenztechnik, Springer Verlag 1962.
- ³⁵⁾ E. L. Ginzton, "Microwave Measurements", McGraw-Hill Inc., New York 1957.
- ³⁶⁾ H. Groendijk, Philips tech. Rev. **20**, 108-110, 1958/1959.
- ³⁷⁾ P. A. H. Hart, Philips tech. Rev. **23**, 293-309, 1961/1962.
- ³⁸⁾ J. T. Jager, B. J. Robinson, Proc. Inst. Radio Engrs **49**, 1603-1616, 1960.
- ³⁹⁾ H. Bruining, Physica **10**, 913-917, 1938.
- ⁴⁰⁾ J. W. Klüver, J. appl. Phys. **329**, 1111-1114, 1961.

Samenvatting

Op een door een axiaal en homogeen magneetveld gefocusseerde electronenbundel kunnen zich vier transversale onafhankelijke en ongedempte golven voortplanten. De bundeldiameter is klein gedacht ten opzichte van de inhomogeniteit van de erop werkende velden. Het onderwerp van dit proefschrift is na te gaan wat er met deze golven gebeurt, en hoeveel ruis er opgewekt wordt als de bundel gedeeltelijk onderschept of verstrooid wordt. Zijdelings is er ook onderzocht wat er gebeurt als de bundel een spreiding in axiale electronen snelheid heeft.

De niet oneindig dunne bundel wordt gedacht als te zijn opgebouwd uit een groot aantal dunne en niet met elkaar gekoppelde transversale schijfjes. Het verband tussen de amplitudes van de golven en de transversale verplaatsings- en snelheidscoördinaten van het zwaartepunt van de schijfjes ten opzichte van de rusttoestand, blijkt eenvoudig in een matrix vergelijking uit te drukken te zijn. Behalve dat de electronen in een schijfje hun bijdrage aan de zwaartepuntsbeweging van dat schijfje leveren kunnen ze ten opzichte van dit zwaartepunt nog bewegingen uitvoeren, de interne beweging genaamd. Zwaartepunts en interne beweging zijn slechts onafhankelijk wanneer dezelfde electronen deel blijven uitmaken van hetzelfde schijfje en wanneer er tevens geen inhomogene krachten op het schijfje worden uitgeoefend.

In de hoofdstukken 2 en 3 worden de bovengenoemde begrippen besproken en voor zover nodig uitgebreid en in een geschikte wiskundige vorm gebracht.

Treedt er onderschepping op, dan wordt het aantal electronen in de schijfjes verminderd, bij verstrooiing wordt de plaats en de snelheid van sommige electronen op een willekeurige manier veranderd, en door axiale snelheidsspreiding zullen sommige electronen uittreden en door anderen vervangen worden. Tijdens deze processen zijn de zwaartepuntsbeweging en de interne beweging niet meer onafhankelijk. De beweging van het nieuwe zwaartepunt kan men gesplitst denken in een component afkomstig van de oorspronkelijke beweging en een component veroorzaakt door de interne beweging.

In hoofdstuk 4 wordt de verzwakking van de golven tengevolge van deze processen onderzocht. Het blijkt daarbij belangrijk te zijn dat de kans dat een electron aan het proces deelneemt niet van de amplitude of faze van de golven afhangt. Zelfs gedurende het proces treedt dan geen koppeling van de golven op, en de component van de zwaartepuntsbeweging als gevolg van de oorspronkelijke beweging is onafhankelijk van de component voortkomende uit de interne beweging. De laatste component stelt dan de bij deze processen opgewekte ruis voor die onderzocht is in hoofdstuk 5. Tussen de aldus ontstane ruisgolven blijkt correlatie te bestaan. Bij het afleiden van de ruisformules is aangenomen dat de kans voor alle electronen om aan het proces deel te nemen ongeveer gelijk is, en dat er geen correlatie bestaat tussen de transversale snelheid en de

plaats van de electronen ten opzichte van het zwaartepunt. Omdat in de praktijk de bundels turbulent zijn is deze aanname redelijk.

De meetresultaten verkregen met behulp van de zogenaamde snelle cyclotron golf blijken goed in overeenstemming met de theorie te zijn. De experimenten zijn gedaan met een electronenbuis waarin de koppeling met uitsluitend de snelle cyclotrongolf tot stand wordt gebracht met deflectieplaatkoppelingen. Tussen de ingangs- en de uitgangskoppeling bevindt zich een draaibaar vaantje dat meer of minder ver in de bundel gebracht kan worden en aldus een regelbare onderschepping veroorzaakt. Om de nauwkeurigheid van de metingen te kunnen verhogen is er een kwadрупool-versterkersysteem tussen het vaantje en de uitgangskoppeling aangebracht. Hoewel het vaantje met een reflectiearm materiaal is bedekt is gebleken dat er altijd wat verstrooiing gelijktijdig met de onderschepping plaats vindt. Beide effecten moesten dus gelijktijdig worden onderzocht.

De metingen besproken in hoofdstuk 7 hebben betrekking op de beide synchrone golven. Eerst wordt het signaal met een deflectieplaatkoppeling als een snelle cyclotrongolf op de bundel gebracht en dan met een kwadрупool systeem omgezet in een synchrone golf. Een tweede kwadрупool systeem zet de synchrone golf later weer om in een snelle cyclotrongolf die met een deflectieplaatkoppeling uitgekoppeld kan worden. Het draaibare vaantje dat de onderschepping veroorzaakt zit tussen de beide kwadрупolen. Door de signaalfrequentie hoger of lager dan de frequentie van het kwadрупoolveld te kiezen kan men aldus met de ene of met de andere synchrone golf koppelen. De resultaten zijn in overeenstemming met de eenvoudige theorie behalve, dat de ruis van de synchrone golf met de positieve energie stroom veel hoger is dan die van de golf met negatieve energiestroom. Dit is te wijten aan concentrische rotaties van electronen om de as en aan een inhomogene stroomdichtheid. Ten aanzien van deze rotaties kan men de onderschepping niet willekeurig noemen en ook zijn de plaats en snelheid van de electronen gecorreleerd. Brengt men deze effecten in rekening, dan is de overstemming met de experimentele resultaten zeer goed.

STELLINGEN

STELLINGEN

I

De gewoonlijk voor thermische stralingsdetectoren aangegeven onderste grens voor de nog te meten stralingsenergiestroom is noch een praktische, noch een principiële onderste grens.

(J. M. W. Milatz en H. A. van der Velden, *Physica* 10, 368-380. 1943)

II

Wanneer men het warmtecontact van een thermische stralingsdetector verkleint dan neemt zijn tijdconstante toe. Meestal wordt de detector dan onbruikbaar traag. De bruikbaarheid voor metingen van met tamelijk hoge frequentie onderbroken straling hoeft hier echter niet onder te lijden.

III

De nog meetbare stralingsenergiestroom van een thermische detector kan worden verkleind door het selectief maken van de detector voor die straling. Het is echter niet zinvol dergelijke berekeningen uit te voeren zonder tevens het conductie- en eventueel convectie-warmtegeleidingsvermogen in de berekening te betrekken. Dit met het oog op de tijdconstante van het systeem en op de ruis gepaard gaande met de omzetting van de temperatuursverhoging van de detector in een waarneembare grootheid.

(E. H. Putley, *Infrared Physics* 4, 1-8, 1964)

IV

Als uitgangspunt voor studies betreffende verhitting van een plasma is het plasma gevormd in een elektronenbundel-plasma ontlading even interessant als dat gevormd door de „C Stellarator”. Het is bovendien veel goedkoper.

V

Afgezien van de vraag of zonwering binnen of buiten het raam moet hangen, is het nuttig de binnenzijde van goed reflecterend materiaal zoals aluminium te maken, en de buitenzijde liefst wit te schilderen.

(A. S. Bishop, *Physics Today*, March, 19-26, 1964)

VI

Het is wel mogelijk om door bundelexpansie afkoeling van de snelle cyclotron-golf te krijgen mits de bundel voldoende laminair is.

(T. Wessel-Berg en K. Blötckjaer „Microwaves” voordracht IVe Int. congres over microgolfbuizen in Scheveningen Sept. 1962)

VII

Voor het ontruisen van parametrische versterkers met een electronenbundel kan met voordeel gebruik gemaakt worden van een halfgeleider diode.

VIII

Let men op de afwezigheid van neiging tot spontane blokkering van schijfremmen, dan is het argument dat schijfremmen op de voorwielen behoren onjuist en geheel te zien als reclameargument.

IX

Het zou het aantal ernstige verkeersongelukken kunnen verminderen als bij de rijproef ook geëist werd dat de kandidaat, rijdend met tamelijk hogesnelheid, de wagen in zo kort mogelijke tijd tot stilstand kan brengen, liefst met enig begrip over de te verwachten instabiliteiten.