Revised budget allocations for fixed-priority-scheduled periodic resources

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Revised budget allocations for fixed-priority-scheduled periodic resources

Martijn M. H. P. van den Heuvel · Pieter J. L. Cuijpers · Johan J. Lukkien · Nathan Fisher

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1 Introduction

Hierarchical scheduling frameworks (HSFs) facilitate a decoupling of development of individual components from their integration on a shared uniprocessor platform. The periodic resource models of Shin and Lee (2008) and Easwaran et al (2007), characterizing periodic resource allocations to components, are complemented with novel methods to abstract timing requirements in the hierarchy of schedulers. HSFs provide temporal isolation between components by allocating a guaranteed resource share, i.e. a budget, to each component.

Dewan and Fisher (2010a) claim a unique, fully polynomial approximation scheme (FPTAS) to calculate a budget for a given task set and resource period. The special case where the approximation parameter \( k = \infty \) should yield an exact budget, which is computed more efficiently than the exhaustive search proposed by Shin and Lee (2008). We show that Dewan and Fisher (2010a) may yield optimistic budgets and we propose a correction for their algorithm.

2 System Model

We use the explicit-deadline periodic resource (EDP) model of Easwaran et al (2007) to specify guaranteed processor allocations to components. The timing interface of a component \( C \) is specified by a triple \( (\Pi, \Theta, \Delta) \), where \( \Pi \in \mathbb{Z}^+ \) denotes its period, \( \Theta \in \mathbb{R}^+ \) its budget and \( \Delta \in \mathbb{R}^+ \) is the relative deadline.
of the EDP resource with $\Theta \leq \Delta \leq \Pi$. The periodic resource model $\Gamma$, proposed by Shin and Lee (2008), is a specialization of the EDP resource $\Omega$ with characteristics $\Gamma(\Pi, \Theta) = \Omega(\Pi, \Theta, \Pi)$.

A component $C$ contains a set $T$ of $n$ sporadic tasks $\tau_1, \ldots, \tau_n$. Each task $\tau_i \in T$ is characterized by a triple $(T_i, E_i, D_i)$, where $T_i \in \mathbb{R}^+$ denotes its minimum inter-arrival time, $E_i \in \mathbb{R}^+$ its worst-case computation time, and $D_i \in \mathbb{R}^+$ its relative deadline, where $0 < E_i \leq D_i \leq T_i$. We assume that tasks are given in priority order, i.e., $\tau_1$ has the highest priority and $\tau_n$ the lowest.

The sufficient schedulability condition presented by Dewan and Fisher (2010a) deems a task set $T$ schedulable on an EDP resource $\Omega$, if

$$\forall 1 \leq i \leq n \exists t \in \tilde{S}_i(k) : \tilde{rbf}(i, t, k) \leq sbf_\Omega(t),$$

where the supply bound function $sbf_\Omega(t)$ computes the minimum processor supply for any interval of length $t$, i.e.

$$sbf_\Omega(t) = \max\left\{0, \left(h_\Omega(t) - 1\right)\Theta, t - \left(h_\Omega(t) + 1\right)\left(\Pi - \Theta\right) + \left(\Pi - \Delta\right)\right\},$$

with $h_\Omega(t) = \left\lceil\frac{t - (\Delta - \Theta)}{\Pi}\right\rceil$ and the cumulative requested processor time by task $\tau_i$ using approximation parameter $k$ is:

$$\tilde{rbf}(i, t, k) \overset{\text{def}}{=} E_i + \sum_{1 \leq j < i} \delta(j, t, k)$$

and the ordered set of testing points is defined as

$$\tilde{S}_i(k) \overset{\text{def}}{=} \{t = b \cdot T_a \mid a = 1, \ldots, i - 1; \ b = 1, \ldots, k; \ t \in (0, D_i]\} \cup \{0, D_i\}.$$

For the special case where $k = \infty$, the schedulability condition in (1) specializes to the exact schedulability condition of Easwaran et al (2007).

**Problem statement:** Given a task set $T$, a period $\Pi$, a deadline $\Delta$ and parameter $k$, we want to determine the minimum budget $\Theta_{\text{min}}$ satisfying Equation (1).

### 3 Revisiting existing budget-allocation algorithms

We present a counter example, considering the optimism in the algorithm by Dewan and Fisher (2010a) for fixed-priority-scheduled components. For EDF-scheduled components, Fisher and Dewan (2009) presented a fundamentally different FPTAS compared to the FPTAS for fixed-priority scheduling reconsidered in this paper.

**Counter example:** Consider a fixed-priority-scheduled component $C_1$ with a period $\Pi_1 = \Delta_1 = 67$ and with two tasks $\tau_1 = (169, 1.5, 169)$ and $\tau_2 = (177, 34, 177)$. For this example, Dewan and Fisher (2010a) yield an optimistic budget of 18.5 time units. The required budget for component $C_1$ is,
Revised budget allocations for fixed-priority-scheduled periodic resources

However, 20.33 time units. Task $\tau_2$ therefore violates (1), because $\forall t \in \tilde{S}_2(\infty) : \tilde{r}_b(2, t, \infty) > s_b(167, 18.5, 67)(t)$.

Since the algorithm by Dewan and Fisher (2010a) fails the exact schedulability test (for $k = \infty$) in (1), their budget allocations are optimistic. Fortunately, the source of optimism by Dewan and Fisher (2010a) can be found in their proofs (see the internal report by Dewan and Fisher (2010b)).

Lemma 12 of Dewan and Fisher (2010b) presents a budget candidate, $\Theta_{\min t}$, for each consecutive pair of values $t_a$ and $t_{a+1}$ in the testing set $\tilde{S}_i(k)$ of task $\tau_i$. The definition of $\Theta_{\min t}$ is only valid for a specific domain of $l_1$ and $l_2$ values, i.e. $[1, \lceil l_2 \rceil - 1], [\lceil l_2 \rceil, \lceil l_1 \rceil], \text{ and } [\lceil l_1 \rceil + 1, \infty]$, and the values of $l_1$ and $l_2$ each reconstruct an $h(\alpha, t)$. Lemma 11 and the corollaries 1, 2 and 3 subsequently capture these three regions. Corollary 4 only calculates $\Theta_{\min t}$ at the boundaries of the regions defined by $l_1$ and $l_2$. However, Dewan and Fisher (2010b) have forgotten the remaining case in Corollary 4, i.e. $\lceil l_2 \rceil > \lceil l_1 \rceil$. This also leaves Lemma 12 incomplete.

Reconsidering the example: task $\tau_2$ has a testing set of $\{0, 169, 177\}$ according to (4). This task requires two iterations: one considering interval $[0, 169]$ and one considering interval $[169, 177]$. For interval $[169, 177]$, the algorithm picks the smallest candidate from the following values: $\{20.33; 37; 37; 18.5\}$ which results in 18.5 time units. This value is finally promoted as the optimal budget for component $C_1$. Since in this iteration $\lceil l_2 \rceil > \lceil l_1 \rceil$, the last two budget candidates in the budget-candidate set correspond to an undefined interval. Both values should therefore be discarded, so that 20.33 is returned which coincides with the optimal solution found by an exhaustive search.

4 A revised FPTAS for budget allocations

Algorithm 1 presents a revised FPTAS to calculate budgets for fixed-priority-scheduled tasks on an EDP resource. The final proof of correctness of this algorithm depends on the correctness of $\Theta_{\min t}$ as defined by Dewan and Fisher (2010a) in Lemma 4 and Lemma 5. By including the missing case in the corollaries and lemmas and straightforwardly extending the proofs, we obtain an if-statement at the lines 11-17. As a result, $\Theta_{\lceil l_1 \rceil}$ and $\Theta_{\lceil l_2 \rceil}$ are only conditionally computed. Algorithm 1 (with $k = \infty$) can be used to obtain an exact budget for a given task set and resource period.
Algorithm 1 FPMinimumBudget($T$, $\Pi$, $\Delta$, $k$)

1: $\Theta_{\min} \leftarrow \Pi \cdot \sum_{T_i \in T} \frac{E_i}{T_i}$
2: for all $\tau_i \in T$ do
3: \[ \Theta_{\min} \leftarrow \infty \]
4: for all $t_a, t_{a+1} \in \widehat{S}_i(k)$ do
5: $D_{t_a} \leftarrow \hat{rbf}(i, t_a, k) + \sum_{j < i \land t_a \mod T_a = 0} E_j$
6: $\Theta_{\min} \leftarrow \Theta_{\min} + \hat{rbf}(i, t_{a+1}, k)$
7: \[ l_1 = \frac{(t_a + 1 - \Delta) + \sqrt{(t_a + 1 - \Delta)^2 + 4t_a D_{t_{a+1}}}}{2t_a} \]
8: \[ l_2 = \frac{(t_a - \Delta) + \sqrt{(t_a - \Delta)^2 + 4t_a D_{t_a}}}{2t_a} \]
9: $\Theta_{[l_1+1]} \leftarrow \frac{D_{t_a + l_1 \cdot t_{a+1} + (l_1+1)(t_{a+1}) - t_a}}{t_{a+1}}$
10: $\Theta_{[l_2-1]} \leftarrow \frac{D_{t_a + l_2 \cdot t_{a+1} + (l_2)(t_{a+1}) - t_a}}{t_{a+1}}$
11: if $l_2 \leq l_1$ then
12: $\Theta_{[l_1]} \leftarrow \frac{D_{t_a + l_1 \cdot t_{a+1} + (l_1)(t_{a+1}) - t_a}}{t_{a+1}}$
13: $\Theta_{[l_2]} \leftarrow \frac{D_{t_a + l_2 \cdot t_{a+1} + (l_2)(t_{a+1}) - t_a}}{t_{a+1}}$
14: else
15: $\Theta_{[l_1]}, \Theta_{[l_2]} \leftarrow \infty$
16: end if
17: $\Theta_{\min} \leftarrow \min\{\Theta_{[l_1]}, \Theta_{[l_2]}, \Theta_{[l_1]}, \Theta_{[l_2]}\}$
18: $\Theta_{\min} \leftarrow \min(\Theta_{\min}, \Theta_{\min})$
19: $\Theta_{\min} \leftarrow \max(\Theta_{\min}, \Theta_{\min})$
20: end for
21: return $\Theta_{\min}$
22: end for
23: References

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10/02 F.E.J. Kruseman Aretz
<table>
<thead>
<tr>
<th>Date</th>
<th>Title</th>
<th>Authors</th>
</tr>
</thead>
<tbody>
<tr>
<td>10/03</td>
<td>On Rule Formats for Zero and Unit Elements</td>
<td>Luca Aceto, Matteo Cimini, Anna Ingolfsdottir, MohammadReza Mousavi and Michel A. Reniers</td>
</tr>
<tr>
<td>10/04</td>
<td>Towards Model-Based Testing of Electronic Funds Transfer Systems</td>
<td>Hamid Reza Asaadi, Rantin Khosravi, MohammadReza Mousavi, Neda Noroozi</td>
</tr>
<tr>
<td>10/05</td>
<td>Schedulability analysis of synchronization protocols based on overrun without payback for hierarchical scheduling frameworks revisited</td>
<td>Reinder J. Bri, Uğur Keskin, Moris Behnam, Thomas Nohte</td>
</tr>
<tr>
<td>10/06</td>
<td>Locally unique labeling of model elements for state-based model differences</td>
<td>Zvezdan Protić</td>
</tr>
<tr>
<td>10/07</td>
<td>Converting existing analysis to the EDP resource model</td>
<td>C.G.U. Okwudire and R.J. Bri</td>
</tr>
<tr>
<td>10/08</td>
<td>Reconstruction and verification of group membership protocols</td>
<td>Mohammed Atif, Sjoerd Cranen, MohammadReza Mousavi</td>
</tr>
<tr>
<td>10/09</td>
<td>A linear translation from LTL to the first-order modal $\mu$-calculus</td>
<td>Sjoerd Cranen, Jan Friso Groote, Michel Reniers</td>
</tr>
<tr>
<td>10/10</td>
<td>Extending an Open-source Real-time Operating System with Hierarchical Scheduling</td>
<td>Mike Holenderski, Wim Cools, Reinder J. Bri, Johan J. Lukkien</td>
</tr>
<tr>
<td>10/11</td>
<td>1st Doctoral Symposium of the International Conference on Software Language Engineering (SLE)</td>
<td>Eric van Wyk and Steffen Zschaler</td>
</tr>
<tr>
<td>10/12</td>
<td>3rd International Software Language Engineering Conference</td>
<td>Pre-Proceedings</td>
</tr>
<tr>
<td>10/13</td>
<td>Discrimination Aware Decision Tree Learning</td>
<td>Faisal Kamiran, Toon Calders and Mykola Pechenizkiy</td>
</tr>
<tr>
<td>10/14</td>
<td>Specification Guidelines to avoid the State Space Explosion Problem</td>
<td>J.F. Groote, T.W.D.M. Kouters and A.A.H. Osaiweran</td>
</tr>
<tr>
<td>10/15</td>
<td>GEM: a Distributed Goal Evaluation Algorithm for Trust Management</td>
<td>Daniel Trivellato, Nicola Zannone and Sandro Etalle</td>
</tr>
<tr>
<td>10/17</td>
<td>Decompositional Reasoning about the History of Parallel Processes</td>
<td>L. Aceto, A. Birgisson, A. Ingolfsdottir, and M.R. Mousavi</td>
</tr>
<tr>
<td>10/18</td>
<td>Robustness of Behavioral Equivalence on Open Terms</td>
<td>P.D. Mosses, M.R. Mousavi and M.A. Reniers</td>
</tr>
<tr>
<td>10/19</td>
<td>Desynchronisability of (partial) closed loop systems</td>
<td>Harsh Beohar and Pieter Cuijpers</td>
</tr>
<tr>
<td>11/01</td>
<td>Refinement of Synchronizable Places with Multi-workflow Nets - Weak termination preserved!</td>
<td>Kees M. van Hee, Natalia Sidorova and Jan Martijn van der Werf</td>
</tr>
<tr>
<td>11/02</td>
<td>Using a DSL and Fine-grained Model Transformations to Explore the boundaries of Model Verification</td>
<td>M.F. van Amstel, M.G.J. van den Brand and L.J.P. Engelen</td>
</tr>
<tr>
<td>11/03</td>
<td>Reconciling Operational and Epistemic Approaches to the Formal Analysis of Crypto-Based Security Protocols</td>
<td>H.R. Mahrrooghi and M.R. Mousavi</td>
</tr>
<tr>
<td>11/05</td>
<td>Semantics, bisimulation and congruence results for a general stochastic process operator</td>
<td>Jan Friso Groote and Jan Lanik</td>
</tr>
<tr>
<td>11/06</td>
<td>Moore-Smith theory for Uniform Spaces through Asymptotic Equivalence</td>
<td>P.J.L. Cuijpers</td>
</tr>
<tr>
<td>11/07</td>
<td>Transforming SOS Specifications to Linear Processes</td>
<td>F.P.M. Stappers, M.A. Reniers and S. Weber</td>
</tr>
<tr>
<td>11/08</td>
<td>A Component Framework where Port Compatibility Implies Weak Termination</td>
<td>Debiyoti Bera, Kees M. van Hee, Michiel van Oseh and Jan Martijn van der Werf</td>
</tr>
<tr>
<td>11/09</td>
<td>Model, analysis, and improvements for inter-vehicle communication using one-hop periodic broadcasting based on the 802.11p protocol</td>
<td>Tsesesuren Batsuuri, Reinder J. Bri and Johan Lukkien</td>
</tr>
<tr>
<td>Date</td>
<td>Authors</td>
<td>Title</td>
</tr>
<tr>
<td>------</td>
<td>---------</td>
<td>-------</td>
</tr>
<tr>
<td>11/10</td>
<td>Neda Noroozi, Ramtin Khosravi, MohammadReza Mousavi and Tim A.C. Willemse</td>
<td>Synchronizing Asynchronous Conformance Testing</td>
</tr>
<tr>
<td>11/11</td>
<td>Jeroen J.A. Keiren and Michel A. Reniers</td>
<td>Type checking mCRL2</td>
</tr>
<tr>
<td>11/12</td>
<td>Muhammad Atif, MohammadReza Mousavi and Ammar Osaiweran</td>
<td>Formal Verification of Unreliable Failure Detectors in Partially Synchronous Systems</td>
</tr>
<tr>
<td>11/13</td>
<td>J.F. Groote, A.A.H. Osaiweran and J.H. Wesselius</td>
<td>Experience report on developing the Front-end Client unit under the control of formal methods</td>
</tr>
<tr>
<td>11/15</td>
<td>John Businge, Alexander Serebrenik and Mark van den Brand</td>
<td>Eclipse API Usage: The Good and The Bad</td>
</tr>
<tr>
<td>11/17</td>
<td>M.F. van Amstel, A. Serebrenik and M.G.J. van den Brand</td>
<td>Visualizing Traceability in Model Transformation Compositions</td>
</tr>
<tr>
<td>11/18</td>
<td>F.P.M. Stappers, M.A. Reniers, J.F. Groote and S. Weber</td>
<td>Dogfooding the Structural Operational Semantics of mCRL2</td>
</tr>
<tr>
<td>12/01</td>
<td>S. Cranen</td>
<td>Model checking the FlexRay startup phase</td>
</tr>
<tr>
<td>12/02</td>
<td>U. Khadim and P.J.L. Cuijpers</td>
<td>Appendix C / G of the paper: Repairing Time-Determinism in the Process Algebra for Hybrid Systems ACP</td>
</tr>
<tr>
<td>12/03</td>
<td>M.M.H.P. van den Heuvel, P.J.L. Cuijpers, J.J. Lukkien and N.W. Fisher</td>
<td>Revised budget allocations for fixed-priority-scheduled periodic resources</td>
</tr>
<tr>
<td>12/04</td>
<td>Ammar Osaiweran, Tom Fransen, Jan Friso Groote and Bart van Rijnsoever</td>
<td>Experience Report on Designing and Developing Control Components using Formal Methods</td>
</tr>
<tr>
<td>12/05</td>
<td>Sjoerd Cranen, Jeroen J.A. Keiren and Tim A.C. Willemse</td>
<td>A cure for stuttering parity games</td>
</tr>
<tr>
<td>12/06</td>
<td>A.P. van der Meer</td>
<td>CIF MSOS type system</td>
</tr>
</tbody>
</table>