Revised budget allocations for fixed-priority-scheduled periodic resources

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ISSN 0926-4515

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Computer Science Reports 12-03
Eindhoven, February 2012
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Received: date / Accepted: February 21, 2012

1 Introduction

Hierarchical scheduling frameworks (HSFs) facilitate a decoupling of development of individual components from their integration on a shared uniprocessor platform. The periodic resource models of Shin and Lee (2008) and Easwaran et al (2007), characterizing periodic resource allocations to components, are complemented with novel methods to abstract timing requirements in the hierarchy of schedulers. HSFs provide temporal isolation between components by allocating a guaranteed resource share, i.e. a budget, to each component.

Dewan and Fisher (2010a) claim a unique, fully polynomial approximation scheme (FPTAS) to calculate a budget for a given task set and resource period. The special case where the approximation parameter $k = \infty$ should yield an exact budget, which is computed more efficiently than the exhaustive search proposed by Shin and Lee (2008). We show that Dewan and Fisher (2010a) may yield optimistic budgets and we propose a correction for their algorithm.

2 System Model

We use the explicit-deadline periodic resource (EDP) model of Easwaran et al (2007) to specify guaranteed processor allocations to components. The timing interface of a component $C$ is specified by a triple $(\Pi, \Theta, \Delta)$, where $\Pi \in \mathbb{Z}^+$ denotes its period, $\Theta \in \mathbb{R}^+$ its budget and $\Delta \in \mathbb{R}^+$ is the relative deadline.
of the EDP resource with $\Theta \leq \Delta \leq \Pi$. The periodic resource model $\Gamma$, proposed by Shin and Lee (2008), is a specialization of the EDP resource $\Omega$ with characteristics $\Gamma(\Pi, \Theta) = \Omega(\Pi, \Theta, \Pi)$.

A component $C$ contains a set $T$ of $n$ sporadic tasks $\tau_1, \ldots, \tau_n$. Each task $\tau_i \in T$ is characterized by a triple $(T_i, E_i, D_i)$, where $T_i \in \mathbb{R}^+$ denotes its minimum inter-arrival time, $E_i \in \mathbb{R}^+$ its worst-case computation time, and $D_i \in \mathbb{R}^+$ its relative deadline, where $0 < E_i \leq D_i \leq T_i$. We assume that tasks are given in priority order, i.e. $\tau_1$ has the highest priority and $\tau_n$ the lowest.

The sufficient schedulability condition presented by Dewan and Fisher (2010a) deems a task set $T$ schedulable on an EDP resource $\Omega$, if

$$\forall 1 \leq i \leq n \ \exists t \in \hat{S}_i(k) : \hat{r}bf(i, t, k) \leq sbf(\Omega)(t),$$

(1)

where the supply bound function $sbf(\Omega)(t)$ computes the minimum processor supply for any interval of length $t$, i.e.

$sbf(\Omega)(t) = \max \{ 0, \ (h(\Omega, t) - 1)\Theta, \ t - (h(\Omega, t) + 1)(\Pi - \Theta) + (\Pi - \Delta) \}$,

(2)

with $h(\Omega, t) = \left\lceil \frac{t - (\Delta - \Theta)}{\Pi} \right\rceil$ and the cumulative requested processor time by task $\tau_i$ using approximation parameter $k$ is:

$$\hat{r}bf(i, t, k) \overset{\text{def}}{=} E_i + \sum_{1 \leq j < i} \delta(j, t, k)$$

and $\delta(j, t, k) \overset{\text{def}}{=} \begin{cases} \frac{t}{T_j} E_j & \text{if } t \leq (k - 1)T_j \\ E_j + \frac{t}{T_j} E_j & \text{otherwise} \end{cases}$

(3)

and the ordered set of testing points is defined as

$$\hat{S}_i(k) \overset{\text{def}}{=} \{ t = b \cdot T_a \mid a = 1, \ldots, i - 1; \ b = 1, \ldots, k; \ t \in (0, D_i) \} \cup \{0, D_i\}.$$

(4)

For the special case where $k = \infty$, the schedulability condition in (1) specializes to the exact schedulability condition of Easwaran et al (2007).

**Problem statement:** Given a task set $T$, a period $\Pi$, a deadline $\Delta$ and parameter $k$, we want to determine the minimum budget $\Theta_{\text{min}}$ satisfying Equation (1).

### 3 Revisiting existing budget-allocation algorithms

We present a counter example, considering the optimism in the algorithm by Dewan and Fisher (2010a) for fixed-priority-scheduled components. For EDF-scheduled components, Fisher and Dewan (2009) presented a fundamentally different FPTAS compared to the FPTAS for fixed-priority scheduling reconsidered in this paper.

**Counter example:** Consider a fixed-priority-scheduled component $C_1$ with a period $\Pi_1 = \Delta_1 = 67$ and with two tasks $\tau_1 = (169, 1.5, 169)$ and $\tau_2 = (177, 34, 177)$. For this example, Dewan and Fisher (2010a) yield an optimistic budget of 18.5 time units. The required budget for component $C_1$ is,
however, 20.33 time units. Task $\tau_2$ therefore violates (1), because $\forall t \in \hat{S}_2(\infty) : \tilde{r}_{bf}(2, t, \infty) > s_{bf}(67, 18.5, 67)(t)$.

Since the algorithm by Dewan and Fisher (2010a) fails the exact schedulability test (for $k = \infty$) in (1), their budget allocations are optimistic. Fortunately, the source of optimism by Dewan and Fisher (2010a) can be found in their proofs (see the internal report by Dewan and Fisher (2010b)).

Lemma 12 of Dewan and Fisher (2010b) presents a budget candidate, $\Theta_{t_a}^{min}$, for each consecutive pair of values $t_a$ and $t_{a+1}$ in the testing set $\hat{S}_i(k)$ of task $\tau_i$. The definition of $\Theta_{t_a}^{min}$ is only valid for a specific domain of $l_1$ and $l_2$ values, i.e. $[1, [l_2] - 1], [[l_2]_1, [l_1]],$ and $[[l_1]+1, \infty]$, and the values of $l_1$ and $l_2$ each reconstruct an $h_{(t, t)}$. Lemma 11 and the corollaries 1, 2 and 3 subsequently capture these three regions. Corollary 4 only calculates $\Theta_{t_a}^{min}$ at the boundaries of the regions defined by $l_1$ and $l_2$. However, Dewan and Fisher (2010b) have forgotten the remaining case in Corollary 4, i.e. $[l_2] > [l_1]$. This also leaves Lemma 12 incomplete.

Reconsidering the example: task $\tau_2$ has a testing set of $\{0, 169, 177\}$ according to (4). This task requires two iterations: one considering interval $[0, 169]$ and one considering interval $[169, 177]$. For interval $[169, 177]$, the algorithm picks the smallest candidate from the following values: $\{20.33; 37; 37; 18.5\}$ which results in 18.5 time units. This value is finally promoted as the optimal budget for component $C_1$. Since in this iteration $[l_2] > [l_1]$, the last two budget candidates in the budget-candidate set correspond to an undefined interval. Both values should therefore be discarded, so that 20.33 is returned which coincides with the optimal solution found by an exhaustive search.

### 4 A revised FPTAS for budget allocations

Algorithm 1 presents a revised FPTAS to calculate budgets for fixed-priority-scheduled tasks on an EDP resource. The final proof of correctness of this algorithm depends on the correctness of $\Theta_{t_a}^{min}$ as defined by Dewan and Fisher (2010a) in Lemma 4 and Lemma 5. By including the missing case in the corollaries and lemmas and straightforwardly extending the proofs, we obtain an if-statement at the lines 11-17. As a result, $\Theta_{[l_1]}$ and $\Theta_{[l_2]}$ are only conditionally computed. Algorithm 1 (with $k = \infty$) can be used to obtain an exact budget for a given task set and resource period.
Algorithm 1 FPMinimumBudget($T$, $\Pi$, $\Delta$, $k$)

1: $\Theta_{min} \leftarrow \Pi \cdot \sum_{\tau_i \in T} \frac{E_i}{T_i}$
2: for all $\tau_i \in T$ do
3: $\Theta_{min}^{i} \leftarrow \infty$
4: for all $t_a, t_{a+1} \in \tilde{S}_i(k)$ do
5: $D_{t_a} \leftarrow \hat{r}b(i, t_a, k) + \sum_{j < i \land t_a \mod T_j = 0} E_j$
6: $D_{t_{a+1}} \leftarrow \hat{r}b(i, t_{a+1}, k)$
7: $l_1 = \frac{(t_a - \Delta) + \sqrt{(t_a - \Delta)^2 + 4H D_{t_{a+1}}}}{2H}$
8: $l_2 = \frac{(t_{a+1} - \Delta) + \sqrt{(t_{a+1} - \Delta)^2 + 4H D_{t_a}}}{2H}$
9: $\Theta_{\lfloor l_1 \rfloor + 1} \leftarrow D_{t_a} + \frac{l_1}{\Pi \cdot \tau_i + \Delta}$
10: $\Theta_{\lceil l_2 \rceil} \leftarrow D_{t_{a+1}} + \frac{l_2}{\Pi \cdot \tau_i + \Delta}$
11: if $|l_2| \leq |l_1|$ then
12: $\alpha = \sum_{t_{\tau_i} \in T \cdot t_{\tau_i} \geq D_{t_{a+k-1}} - T_i} \frac{E_i}{T_i}$
13: $\Theta_{\lceil l_1 \rceil} \leftarrow \frac{D_{t_a} + \alpha(t_{\lceil l_1 \rceil} + \Delta - t_a)}{t_{\lceil l_1 \rceil} + \alpha}$
14: $\Theta_{\lceil l_2 \rceil} \leftarrow \frac{D_{t_{a+1}} + \alpha(t_{\lceil l_2 \rceil} + \Delta - t_a)}{t_{\lceil l_2 \rceil} + \alpha}$
15: else
16: $\Theta_{\lceil l_1 \rceil}, \Theta_{\lceil l_2 \rceil} \leftarrow \infty$
17: end if
18: $\Theta_{min}^{\lceil t_a \rceil} \leftarrow \min\{\Theta_{\lceil l_1 \rceil}, \Theta_{\lceil l_2 \rceil}, \Theta_{\lceil l_1 \rceil}, \Theta_{\lceil l_2 \rceil}\}$
19: $\Theta_{min}^{i} \leftarrow \min\{\Theta_{min}^{i}, \Theta_{min}^{\lceil t_a \rceil}\}$
20: end for
21: $\Theta_{min} \leftarrow \max(\Theta_{min}, \Theta_{min}^{\lceil t_a \rceil})$
22: end for
23: return $\Theta_{min}$

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