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Particle focusing by AC electroosmosis with additional axial flow

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Abstract ACEO-induced vortices have been used to manipulate fluids and particles for chemical and biochemical analysis in microscale systems. In such applications, a pressure-driven background flow is often applied to deliver the samples or particles into the target region. In the present study, the influence of an axial background flow on ACEO-induced vortices is experimentally investigated. An array of symmetric electrodes is used, perpendicular to the axial flow. Velocity fields of vortical flow are measured using astigmatism μ -PTV. The strength of vortex is quantified as a function of the axial background velocity. Flow measurements with 2- μ m tracer particles reveal that the background flow compresses the ACEO-induced vortices and decreases the vortex size. This vortex region, separated from the background flow, can be seen as an “isolated area” in the microchannel. Next, 5- μ m particle dynamics in such flow with isolated vortical area is investigated. Experimental observations expose that 5- μ m particles do not follow the vortical flow and are focused toward the upper side of the channel. The velocity components of the 5- μ m particles are found to be different with the local flow visualized by the 2- μ m particles. This velocity difference of 5- μ m particles is probably due to the dielectrophoretic force acting on the particles. Furthermore, such particle focusing appears to be strongly related to the background velocity.

Keywords Vortex · 3D velocity measurement · AC electroosmosis · Microflows

1 Introduction

Lab-on-chip devices are widely used in biotechnology and chemical analysis in the last two decades (Stone et al. 2004). The manipulation of components in a sample is an important part of any chemical and biochemical analysis in lab-on-chip devices. Electro-hydrodynamic (EHD) forcing, as a promising technique for the actuation and manipulation of micro-flows, is an interesting option for particle manipulation. EHD forcing, instead of using a pressure gradient or moving parts, drives the flow by way of electrokinetic mechanisms (Laser and Santiago 2004). AC electroosmosis (ACEO), as particular EHD forcing method, generates a fluid motion via a slip velocity above the electrode surface in the presence of an AC electric field (Ramos et al. 1998). The ACEO flow has been applied in various applications of microfluidic systems including micro-pumping (Ajdari 2000), micromixing (Huang et al. 2007) and manipulation of polarizable microparticles (Wu et al. 2007; Felten et al. 2008; Hilber et al. 2009). Combined with other forcings, such as a pressure-driven flow, an ACEO flow generates multi-functional micro-flows (Speetjens et al. 2011). To further improve our understanding of an ACEO-induced flow in such multi-functional micro-flows, in-depth experimental investigations on the 3D velocity field of an ACEO flow in complicated devices are of great importance.

In this study, the flow dynamics of ACEO flow combined with an additional axial flow is experimentally analyzed in a microchannel. In Sect. 2, the experimental setup is described. An array of symmetric electrodes is employed to generate an ACEO flow, and in the channel, an additional axial background flow is enforced by a syringe pump. The velocity field above the electrodes is visualized by 2- μ m tracer particles and is measured using an astigmatism microparticle tracking velocimetry (astigmatism μ -PTV)

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(Liu et al. 2014a; Cierpka et al. 2010, 2011; Cierpka and Kähler 2012). The vorticity is analyzed as function of the axial background velocity and the results are compared to a numerical model developed before (Liu et al. 2014b). In Sect. 3, 5- μm particles are introduced in the observed vortical structure. The particle dynamics is analyzed, and the potential forces acting on the particles are discussed. Finally, in Sect. 4, conclusions are given.

2 ACEO vortex dynamics

In microfluidic systems, a pressure-driven flow is often applied to deliver the sample or particles into the target region. Such background flow causes the ACEO-induced flow to be more complex. In this section, such flow dynamics is experimentally investigated.

2.1 Experimental setup

The micro-device, as shown in Fig. 1, consists of a straight channel. On the bottom of the channel, symmetric electrode pairs are deposited, perpendicular to the axial direction of the channel. The width of each electrode is 56 μm , and the gap between the electrodes is 14 μm . The height and width of the whole channel are 48 μm and 1.0 mm, respectively. The length of the channel is about 23 mm. A pressure-driven background flow is employed, directing from the left side of the channel to the right side.

A potassium hydroxide (KOH) solution was used as the electrolyte, with a concentration of 0.1 mM (Sigma-Aldrich

Co., USA). Such aqueous potassium hydroxide is widely employed as buffer in biological systems. Fluorescent polymer microparticles with a diameter of $d_p = 2 \mu\text{m}$ and a density of 1.05 g/cm^3 (Fluoro-Max, Duke Scientific Corp., Canada) were employed as tracer particles to measure the fluid velocity. The fluorescent microparticle solution in stock was diluted in the KOH solution with a low concentration of about 0.01 % (w/w). The conductivity of the solution after adding the fluorescent microparticle solution was measured to be about 1.5 mS/m (Scientific Instruments IQ170). An AC signal for the electrode array was created by a function generator (Sefram4422, Sefram, the Netherlands), and the amplitude and frequency of the applied voltage were measured using a digital oscilloscope (TDS210, Tektronix, USA). A syringe pump (PHD2000 Syringe Pump, Harvard Apparatus, USA) was used to set up a axial flow and adapted to flow rates of 5–120 $\mu\text{L/h}$. This corresponds to mean background velocities in the microchannel above electrodes of 29 $\mu\text{m/s}$ till 694 $\mu\text{m/s}$. The device was mounted on a chip holder and was connected to a glass syringe (Gastight 1710TLL, Hamilton Co., USA) with an inner diameter of 1.46 mm by using a silicon tube with an inner diameter of 0.79 mm and an outer diameter of 3.99 mm (Masterflex L/S, the Netherlands).

The velocity field was measured using astigmatism microparticle tracking velocimetry (astigmatism $\mu\text{-PTV}$). The basic principle of astigmatism $\mu\text{-PTV}$ is that due to the anamorphic effect by inserting a cylindrical lens, particle images are deformed into ellipses. This ellipticity is directly related to the particle position normal to the focal plane (Liu et al. 2014a; Cierpka et al. 2010, 2011; Cierpka and Kähler 2012). A fluorescence microscope with a 20 $\times$ Zeiss objective lens (numerical aperture of 0.4 and focal length of 7.9 mm) was used. To illuminate the fluorescent tracer particles, a pulsed monochromatic laser beam with a wavelength of 532 nm was produced by a Nd:YAG laser (ICE450, Quantel, USA). A CCD camera (12-bit SensiCam qe, PCO, Germany) was used with a resolution of $1,376 \times 1,040 \text{ pixel}^2$. In front of the camera sensor, a cylindrical lens with a focal length of 150 mm (LJ1629RM-A, Thorlabs, USA) was used. A digital delay generator (DG535, Stanford Research Systems, USA) controlled the timing of the laser and camera simultaneously. The double-exposure setting of the camera was used with the data acquisition of consecutive frames with two alternating time delays (a long time delay and a short time delay). Based on earlier experiments (Liu et al. 2014a), the long time delay has the limitation of measurable range of velocity, and thus, the velocity data measured in the short time delay were used in the present study. The short time delay was set to be $\Delta t = 0.02$ or 0.03 s , depending on the measured velocity. The non-slip boundary condition on the side walls locally introduces a dependence of the flow on the y -coordinate, resulting in an essentially 3D velocity field: $u = u(x, y, z)$.

Fig. 1 Schematic diagram of the micro-device (a) and the *top-view* of channel geometry (b). An array of symmetric electrode pairs is added to the *bottom* of the micro-channel. Each electrode is 56 μm in width, and the gap between the electrodes is 14 μm in width. The height and width of the whole channel are 48 μm and 1.0 mm, respectively. The direction of the axial background flow is indicated

However, sufficiently far away from these side walls, this effect diminishes and the flow becomes 2D: $u = u(x, z)$ (Liu et al. 2014b). Performing flow measurements in the center of the channel avoids wall effects and thus yields the quasi-2D bulk velocity.

2.2 Velocity measurements

Figure 2 depicts the 3D velocity vectors of 2- μm tracer particles at a voltage of 4 V_{pp} and a frequency of 600 Hz for an additional background flow with a flow rate of 20 $\mu\text{L/h}$ (resulting in an average background velocity $U_{axi} = 116 \mu\text{m/s}$). Three components of the particle velocity (u_x, u_y and u_z) are represented in colors. The x -component of the velocity u_x varies from -400 to $400 \mu\text{m/s}$, and the maximum of $|u_x|$ is close to the electrode edges (see Fig. 2a). When particles approach the electrode surfaces, the magnitude of $|u_z|$ increases significantly (see Fig. 2c).

Fig. 2 3D Particle velocity vectors at applied voltage of 4 V_{pp} , frequency of 600 Hz and average background velocity $U_{axi} = 116 \mu\text{m/s}$, where arrow colors indicate the magnitude of the different velocity components: **a** u_x , **b** u_y and **c** u_z . Yellow areas indicate the position of the electrodes (color figure online)

Compared with $|u_x|$ and $|u_z|$, which vary in a wide range above the electrodes, $|u_y|$ of most tracer particles remains small everywhere, varying in the range from -30 to $30 \mu\text{m/s}$, as shown in Fig. 2b. This indicates that the tracer particles can be considered to be in a quasi-two-dimensional (quasi-2D) flow, similar to the observations in Liu et al. (2014a, b). Therefore, the raw measured 3D particle velocity vectors are projected in the (x, z) plane. The quasi-2D nature of the flow field advances a detailed picture of the flow field in arbitrary y -planes.

Figure 3 shows the projected particle vectors in the (x, z) plane. According to the magnitude of u_x and u_z , the periodic structure of the local flow is clearly seen, which is consistent with the spatial period of the electrode pattern. The velocity distribution in each spatial period is in the same range; the vortex size and shape above each electrode are similar. As the flow is close to the Stokes limit (the Reynolds number $Re = \frac{u_m D}{\nu} \sim 10^{-3}$, where D is the electrode width, u_m the characteristic velocity ($\sim 100 \mu\text{m/s}$) and ν the fluid kinematic viscosity), this spatial periodicity in the flow is attributed to the periodic boundary conditions and geometry.

Since the flow field above the electrodes is periodic along the x -axis, the velocity field can be studied in only one period, which corresponds to a single electrode and half a gap at both edges. Spatial periodicity admits transformation of the original (selectively low density) data into

Fig. 3 Side view of the particle velocity vectors at applied voltage of 4 V_{pp} , frequency of 600 Hz and average background velocity of $U_{axi} = 116 \mu\text{m/s}$. The black solid lines indicate the electrode positions

Fig. 4 Quasi-2D particle velocity vectors for three average axial background velocities and two applied voltages of 4 and 6 V_{pp} at frequency of 600 Hz. The colors indicate the velocity magnitude in the x -direction, u_x . For clarity of the figures, the color bars are chosen differently for the 4 V_{pp} and the 6 V_{pp} cases. The black solid line indicates the electrode position (color figure online)

a high-density data set above one electrode. Hence, the projected velocity vectors from all electrodes in Fig. 3 are overlaid and combined into a single data set that describes the flow field on an individual electrode.

Figure 4 shows the projected velocity vectors $u_{x,z} = (u_x, u_z)$ in the (x, z) plane at frequency of 600 Hz and voltages of 4 and 6 V_{pp} for different averaged axial background velocities ($U_{axi} = 58, 116$ and $174 \mu\text{m/s}$). Two counter-rotating vortices are clearly seen over one electrode, which are asymmetric. Compared with the case without the background flow in which symmetric counter-rotated vortices were observed (Liu et al. 2014a), the axial flow leads to symmetry breaking of the vortices: The vortex at the left side of electrode appears to be “opened up” by the axial flow, and one at the right side is enveloped. An obvious boundary can be seen between the axial

background flow and the vortex region, for example, at 4 V_{pp} and $U_{axi} = 58 \mu\text{m/s}$.

Increasing the background velocity at a fixed applied voltage enhances the symmetry breaking, as shown in Fig. 4. The left vortex is further broken down and integrated into the background flow. By contrast, the right vortex is confined more closely with the increase of the axial flow. Additionally, this symmetry breaking of the vortices is found to be reduced by increasing the applied voltage, see Fig. 4. This is due to the increase of the vortex strength with the voltage (Liu et al. 2014b).

2.3 Circulation variation as function of axial flow rate

By using the Gaussian averaging algorithm Liu et al. (2014a), the velocity vectors are interpolated on a Cartesian

grid with equidistant spacing $\Delta x = \Delta z = 1 \mu\text{m}$. The velocity $\bar{u}_{x,z}(x_i, z_i)$ at each point (x_i, z_i) of the Cartesian grid is calculated by using the Gaussian averaging algorithm. $\bar{u}_{x,z}$ is calculated using the Gaussian average:

$$\bar{u}_{x,z}(x_i, z_i) = \sum_{j=1}^N w'_{ij} u_{x,z}(x_j, z_j), \quad (1)$$

where w'_{ij} is the normalized weight of the neighboring particles and N is the number of the particles in the range with a diameter of $2\sigma_d$ (σ_d the standard deviation of the weight in the entire data (Liu et al. 2014a)). The weight w_{ij} of the neighboring particle is determined by the relative distance from the neighboring particle j to the particle i :

$$w'_{ij} = \frac{w_{ij}}{\sum_{j=1}^N w_{ij}}, \quad \text{with} \quad (2)$$

$$w_{ij} = \exp\left(-\left(\frac{(x_i - x_j)^2}{2\sigma_d^2} + \frac{(z_i - z_j)^2}{2\sigma_d^2}\right)\right)$$

where x_j and z_j are the coordinates of neighboring particle j . Based on the previous study using the similar setup, σ_d is set to be $0.5 \mu\text{m}$. And if $N < 2$, the data point at (x_i, z_i) is considered to be empty, indicating that there is no particle present at this position. The spurious vectors are removed by using a global outlier detection algorithm based on the global standard deviation σ ,

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^N e_i^2, \quad (3)$$

where $e_i^2 = u'(x_i, z_i) \cdot u'(x_i, z_i)$ and $u'(x_i, z_i) = u_{x,z}(x_i, z_i) - \bar{u}_{x,z}(x_i, z_i)$. When $e_i > 2\sigma$, the vectors are removed from the data set (Liu et al. 2014a).

Figure 5 shows the interpolated velocity field $\bar{u}_{x,z}(x, z) = (\bar{u}_x, \bar{u}_z)$ in the (x, z) plane. Global velocity maxima are seen at the electrode edges, where the electric field strength is the highest. The velocity field exposes a vortical structure of the flow above one electrode. It is interesting to note that there is a white area between the vortical and unidirectional flows, see Fig. 5. This is attributed to the streamlines surrounding the vortex, which acts as a transport barrier to particle migration and effectively divides the flow into two disconnected flow domains (background flow and vortex). As a result, once tracer particles are entangled in the vortical region, they cannot escape into the adjacent axial flow. A distinction of the particle movement between the vortical and unidirectional flows can be seen by the white area in Fig. 5. With the increase of the vortex strength by increasing the voltage, such entangling effect increases, which is indicated by the increase of white area.

Figure 6 shows the streamlines of the flow field at $4 V_{pp}$ and 600 Hz for U_{axi} of 58 and $116 \mu\text{m/s}$, respectively. The

calculation of these streamlines was performed by using a second-order Runge–Kutta integration of the velocity field. The streamlines are shown to be concentric wave-like, indicating that the axial flow envelops the vortex. The coexistence of the main flow and vortex flow, shown by the closed streamlines, indicates a creation of a coherent structure. With increasing vortex strength, the streamlines become more confined.

As the diameter of tracer particles is $2 \mu\text{m}$, the fluid velocity measured at $z \approx 1 \mu\text{m}$ can be incorrectly measured due to the interaction between the particles and wall, for example, the particles could rebound at the bottom wall due to large $\bar{u}_z$ or roll over the wall surface due to the large variation of $\bar{u}_x$ in the z -direction. Besides, the sticking particles at the wall may locally influence the electric and flow fields. As a result, the following analysis will focus on the velocity field for $z \geq 3 \mu\text{m}$.

Using the interpolated velocity field $\bar{u}_{x,z}(x, z)$ ($z \geq 3 \mu\text{m}$), the spanwise component of the mean vorticity is calculated, $\omega_y = \partial u_x / \partial z - \partial u_z / \partial x$. Due to the large fluctuation of the measured velocity, the interpolated velocity field in Fig. 4 is further smoothed by the method following (Liu et al. 2014a). Figure 7 depicts the spanwise vorticity at voltage of $4 V_{pp}$ and frequency of 600 Hz for different average background velocities U_{axi} of 58 and $116 \mu\text{m/s}$. Two strong vortical regions are indicated close to the electrode edges. This is in accordance with the measured velocity field in Fig. 4, i.e., the particles roll close to the electrode edges. The circulation, i.e., strength, of the mean vortex is given via $\Gamma = \int_A \omega_y dA$, where the area A is determined via eigenvalues of the strain-rate tensor (λ_2 -method) (Jeong and Hussain 1995; Vollmers 2001). The boundary of the vortex ($\lambda_2 = -10$) reveals that the vorticity distribution is asymmetric due to the induced flow. The circulation for the vortices is calculated for different axial background velocities. In Fig. 7, the circulation for the left vortex is $\Gamma_1 = 1,899 \mu\text{m}^2/\text{s}$ and the one for the right vortex is $\Gamma_2 = -2,828 \mu\text{m}^2/\text{s}$ for $U_{axi} = 58 \mu\text{m/s}$ and $\Gamma_1 = 1,328 \mu\text{m}^2/\text{s}$ and $\Gamma_2 = -4,168 \mu\text{m}^2/\text{s}$ for $U_{axi} = 116 \mu\text{m/s}$, respectively.

Figure 8a shows the variation of Γ_1 and Γ_2 as function of the mean additional axial velocity U_{axi} at 600 Hz and voltages of 4 and $6 V_{pp}$. With the increase of the background velocity from 58 to $174 \mu\text{m/s}$, $|\Gamma_1|$ decreases from $1,899$ to $1,203 \mu\text{m}^2/\text{s}$ while $|\Gamma_2|$ increases from $2,828$ to $5,415 \mu\text{m}^2/\text{s}$ at $4 V_{pp}$. It suggests that the background flow decreases the strength of the left vortex, while enhancing the strength of the right vortex. The same tendency of $|\Gamma_1|$ and $|\Gamma_2|$ as function the background velocity is found for $6 V_{pp}$. When the voltage increases from 4 to $6 V_{pp}$ at 600 Hz , the circulation increases approximately by a factor of 2.5 and 4.5 for Γ_1 and Γ_2 , respectively. Figure 8b shows the variation of $|\Gamma_2/\Gamma_1|$ as function of the background velocity. It

Fig. 5 Interpolated velocity fields for three average background velocities and two applied voltages of 4 and 6 V_{pp} at frequency of 600 Hz. The colors indicate the interpolated velocity magnitude in the x -direction, $\bar{u}_x$. Black solid line indicates the electrode position (color figure online)

Fig. 6 Streamlines at voltage of 4 V_{pp} and frequency of 600 Hz for different average background velocities U_{axi} : **a** 58 $\mu\text{m/s}$ and **b** 116 $\mu\text{m/s}$. The black solid line indicates the electrode position

Fig. 7 Vorticity and circulation at 4 V_{pp} and 600 Hz for different average background velocities U_{axi}: **a** 58 μm/s and **b** 116 μm/s, where pink solid lines indicate the boundary of the ACEO vortex, λ₂ = −10. The black solid line indicates the electrode position (color figure online)

Fig. 8 **a** Circulation of the vortices above one electrode as function of the average background velocity at applied voltages of 4 and 6 V_{pp} and frequency of 600 Hz. **b** Circulation ratio of the two vortices $|\Gamma_2|/|\Gamma_1|$ as function of the average background velocity

reveals that the difference between Γ_1 and Γ_2 increases with the increase of the background velocity.

2.4 Comparison to numerical models

It is interesting to see to what extent the numerical model described in our previous study for a situation without axial flow (Liu et al. 2014b) is able to describe the ACEO flow combined with the axial flow. In Liu et al. (2014b), a correction factor Λ was introduced on the numerical velocity in order to quantitatively match the experimental results, based on the deviation of the velocity profile at $z = 3 \mu\text{m}$ above the electrode surface. A value $\Lambda = 0.35$ was obtained for the numerical results for KCl solutions, yielding the closest match between numerical and experimental results. In the present study, the same correction factor of $\Lambda = 0.35$ is applied with respect to the applied voltage of 4 V_{pp} for KOH solutions. The results reveal that the corrected numerical slip velocity at $z = 3 \mu\text{m}$ is close to the experimental one for average background velocities of $U_{\text{axi}} = 58$ and $116 \mu\text{m}/\text{s}$, see Fig. 9. This suggests that the correction factor for ACEO can also be used for a complicated flow and a different electrolyte.

Compared with the experimental observations in Fig. 5, the corrected predictions precisely describe the measured velocity fields as shown in Fig. 10, including the

size and position of the vortices. Furthermore, the spanwise component of the vorticity is calculated and shown in Fig. 11. The predicted vortex area A and circulation are consistent with the experiment: For $U_{\text{axi}} = 58 \mu\text{m}/\text{s}$, the circulations of two vortices are $\Gamma_1 = 2,188 \mu\text{m}^2/\text{s}$ and $\Gamma_2 = -4,228 \mu\text{m}^2/\text{s}$, which are close to the experimental ones ($\Gamma_1 = 1,899 \mu\text{m}^2/\text{s}$ and $\Gamma_2 = -2,828 \mu\text{m}^2/\text{s}$), and for $U_{\text{axi}} = 116 \mu\text{m}/\text{s}$, the predicted circulations are $\Gamma_1 = 1,457 \mu\text{m}^2/\text{s}$ and $\Gamma_2 = -5,017 \mu\text{m}^2/\text{s}$, close to the experimental ones ($\Gamma_1 = 1,328 \mu\text{m}^2/\text{s}$ and $\Gamma_2 = -4,168 \mu\text{m}^2/\text{s}$).

In the present study, also a vortex region separated from the main background flow was found. The formation of such vortical structures in a straight channel was also numerically analyzed in Speetjens et al. (2011). Their numerical investigation showed that in the Stokes limit ($Re = 0$) the flow topology due to ACEO forcing results in the formation of tori in the microchannel. This flow topology can change into chaotic streamlines for sufficiently strong fluid inertia. In the present experiments, such tori were not observed due to two reasons. Firstly, the measurement resolution is not yet sufficient to distinguish individual tori in the continuous family of tori. Secondly, the secondary circulation that causes tracers to describe a torus is extremely slow, requiring excessive measurement times to fully visualize these

Fig. 9 Comparison of corrected numerical ($\Lambda = 0.35$) and experimental velocity profiles above one electrode at voltage of $4 V_{pp}$ and frequency of 600 Hz for different average background velocities U_{axi} : **a** $58 \mu\text{m/s}$ and **b** $116 \mu\text{m/s}$. The black solid line indicates the electrode position

Fig. 10 Predicted velocity field, with a correction factor of $\Lambda = 0.35$, at voltage of $4 V_{pp}$ and frequency of 600 Hz for different average background velocities U_{axi} : **a** $58 \mu\text{m/s}$ and **b** $116 \mu\text{m/s}$. The black solid line indicates the electrode position

Fig. 11 Predicted vorticity and circulation, with a correction factor of $\Lambda = 0.35$, at voltage of $4 V_{pp}$ and frequency of 600 Hz for different average background velocities U_{axi} : **a** $58 \mu\text{m/s}$ and **b** $116 \mu\text{m/s}$. Pink solid lines indicate the boundary of the ACEO vortex, $\lambda_2 = -10$. The black solid line indicates the electrode position (color figure online)

structures. The current experimental setup cannot yet track tracers sufficiently long. However, the numerical study in Speetjens et al. (2011) showed that an axial pressure-driven flow yields a flow topology that is consistent with the experimental observations. Good agreement is found between two key features: symmetry breaking of the vortical structure above each electrode and the formation of a net axial throughflow region on top of the vortices (Fig. 6).

3 Particle focusing

In the previous section, the vortex region induced by ACEO was found to be separated from the main flow. This creates

a coherent tube-like structure. Since the ACEO flow is under laminar flow conditions ($Re \sim 10^{-3}$), such tubes can act as “isolated area” in the flow domain. This causes the targeted particles near the bottom wall to migrate away from the wall or the ones far away from the wall to move toward the bottom. This phenomenon of “isolated area” can be utilized to enhance particle focusing. In this section, the possibility of focusing particles by using the ACEO vortex will be investigated.

3.1 Experimental procedure

Fluorescent polystyrene particles with diameter of $5 \mu\text{m}$ (2.5 % solid, Micro Particles GmbH, Germany) were used.

The density of particles is $\rho_p = 1.05 \text{ g/cm}^3$, which is very close to that of water 1.00 g/cm^3 . The particles were added to the 0.1 mM KOH solution, with a mass concentration of about 0.02% . The conductivity of the solution is measured to be 1.5 mS/m (IQ170, Scientific Instruments, USA). The astigmatism μ -PTV technique was employed to measure the particle displacement.

3.2 5- μm particle velocities

Experimental data show that $|u_y|$ of 5- μm particles is in general less than $30 \mu\text{m/s}$, which is quite small compared with $|u_x|$ and $|u_z|$. Similar velocity component ratios are also observed for the 2- μm tracer particles. This confirms the earlier observation that in the present study ACEO flow in combination with additional background flow can be simplified to a quasi-2D flow. Hence, the measured 3D particle velocity vectors are, as before, projected in the (x, z) plane.

Figure 12 shows the projected velocity vectors of 5- μm particles in the (x, z) plane at $6 V_{pp}$ and 600 Hz for different background velocities ($U_{axi} = 232, 347$ and $464 \mu\text{m/s}$). Since the tracer particles are randomly distributed in the solution, the particle concentration is expected to be periodic before entering the ACEO effect region. According to the previous experiments using 2- μm particles, a concentration of the particles was expected to be periodic outward along the electrodes. However, the result of the 5- μm particles shows that after passing over 2 electrodes in the microchannel, particles move in a moderate trend toward the top of the channel. Specifically, at the initial stage, the particle movement is significant, whereas this movement becomes relative small in the final stage. This tendency of particle focusing is reduced with the increase of the background

velocity (Fig. 12). As a result, particle focusing in the microchannel is achieved in the present experiments.

To quantify 5- μm particle behavior, the projected velocity vectors were interpolated on a Cartesian grid with equidistant spacing $\Delta x = \Delta z = 1 \mu\text{m}$, as described in Sect. 2.2. Based on each data point in the grid, the tracer particles in the range with a diameter of $2\sigma_d = 2 \mu\text{m}$ (the standard deviation of the weight is now set to be $1 \mu\text{m}$ instead of $0.5 \mu\text{m}$ for the smaller-2 μm particles) were selected, and then, the interpolated velocity $\bar{u}_{x,z}$ of 5- μm particles is calculated. Figure 13 shows the interpolated velocity field $\bar{u}_{x,z}$ of 5- μm particles at different background velocity rates.

The minimum observed z -position of the 5- μm particles along the electrodes $z_{min}(x)$ is calculated based on the velocity fields shown in Fig. 13. In the domain $\Omega : [x, z] = [0, 420] \times [3, 45] (\mu\text{m} \times \mu\text{m})$, the lowest data point with relevant velocity information for a fixed x -position is regarded as the minimum position of 5- μm particles. Figure 14 shows $z_{min}(x)$ for different background velocities. A larger minimum z -position of the particles means that the particles are focused toward the upper side of the channel. The particle focusing decreases with the increase of the additional flow. For instance, at the right side of the fifth electrode $x = 387 \mu\text{m}$, $z_{min}(x)$ is about 29 and $24 \mu\text{m}$ for the additional axial velocity of 232 and $464 \mu\text{m/s}$, respectively.

To evaluate the particle focusing as function of background velocity, the average minimum z -position of the particle presence above one electrode is calculated as $\bar{z}_{min} = \frac{1}{L} \int_{L=70 \mu\text{m}} z_{min}(x) dx$. Figure 15a shows the variation of $\bar{z}_{min}$ for different background velocities above each electrode. A large $\bar{z}_{min}$ means a high effect of the particle focusing. Similar as in Fig. 14, the results show that $\bar{z}_{min}$ decreases with the increase of background velocity.

Fig. 12 Projected velocity vectors of 5- μm particles at $6 V_{pp}$ and 600 Hz in the case of different background velocities, $U_{axi} = 232, 348$ and $463 \mu\text{m/s}$, where the *black solid lines* indicate the electrode position

Fig. 13 Interpolated velocity field of 5- μm particles at $6 V_{pp}$ and 600 Hz in the case of different background velocities, $U_{axi} = 232, 348$ and $463 \mu\text{m/s}$, where the *black solid lines* indicate the electrode positions

Fig. 14 Minimum position of 5- μm particle position ($z_{\min}(x)$) as function of x above the electrode array at 6 V_{pp} , 600 Hz and at various average background velocities

Additionally, $\bar{z}_{\min}$ is found to increase with the number of electrodes, as shown in Fig. 15b.

To discuss the 5- μm particle dynamics, the velocity profiles of 5- μm particles are compared to the local flow which is visualized by using 2- μm particles. Figure 16 shows the x -component of interpolated velocities $\bar{u}_x$ for 5- μm particles and local flow at 6 V_{pp} , 600 Hz and a background velocity of 232 $\mu\text{m/s}$. $\bar{u}_x$ is taken at different positions: 0, 14, 28, 42 and 56 μm away from the left edge of the electrodes. It is seen that the $\bar{u}_x$ values of the 5- μm particles are approximately the same as the ones of the local flow. However, a significant deviation of the vertical component $\bar{u}_z$ between the local flow and particles is observed, as shown in Fig. 17. At the left edge of electrodes ($x = 0 \mu\text{m}$), the local flow has a large negative $\bar{u}_z$, which is due to the ACEO-induced vortex. In contrast, for the 5- μm particles, the magnitude of such negative $\bar{u}_z$ appears to be smaller. For example, at $x = 0 \mu\text{m}$ and $z = 15 \mu\text{m}$, $\bar{u}_z$ of the local flow above electrode 3 is $-311.0 \mu\text{m/s}$ while for the 5- μm particles $\bar{u}_z = -63.2 \mu\text{m/s}$. Furthermore, at $x = 14 \mu\text{m}$ above the electrode, the difference of $\bar{u}_z$ between the local flow and particles becomes small compared with the one at

$x = 0 \mu\text{m}$. Additionally, this difference decreases with the increase of z . The same tendency of the velocity profiles ($\bar{u}_x$ and $\bar{u}_z$) of the local flow and 5- μm particles is also found in the cases of a background velocity of 348 and 464 $\mu\text{m/s}$, respectively.

3.3 Analysis of the 5- μm particle dynamics

To explain the behavior of 5- μm particles, an analysis of the forces acting on them is necessary. Several forces have been discussed in the previous study Liu et al. (2014a). In general, 2 μm polystyrene particles are subjected to drag force from the local flow, while dielectrophoretic (DEP) force (except close to the electrode edge where the DEP force has an influence on the particle movement), buoyancy, electrothermal flow and Brownian motion are considered to be negligible. Due to that, we assume that the velocity measured by 2- μm particles represents the velocity of the local flow. However, in case the particle size is increased to 5 μm , the forces that can be neglected for 2- μm particles have to be re-evaluated.

3.3.1 Response time

If a sudden change in fluid velocity or direction occurs, it takes a small amount of time for tracer particles to catch up with the flow. The response time of a spherical particle in a stationary flow is mainly subjected to the particle size. The governing equation on a spherical particle is given as (Werely and Meinhardt 2005):

$$m_p \frac{du_p}{dt} = -3\pi\mu_f d_p (u_p - u_f) \quad (4)$$

where m_p is the particle mass ($m_p = \frac{1}{6}\pi d_p^3 \rho_p$ with ρ_p the particle density), μ_f the viscosity of the fluid, u_p the velocity of the particle and u_f the velocity of the local fluid, and $3\pi\mu_f d_p (u_p - u_f)$ is the Stokes drag force F_{drag} . With an initial condition $u_p(t = 0) = 0$, u_p after the time t is given by

Fig. 15 **a** Variation of the average minimum z -position of the particle presence above one electrode, $\bar{z}_{\min} = \frac{1}{L} \int_{L=70\mu\text{m}} z_{\min}(x) dx$, as function of different background velocities. **b** Variation of $\bar{z}_{\min}$ for each electrode at different background velocities. The position of electrodes is shown in Fig. 12

Fig. 16 The x -component of the interpolated velocity component $\bar{u}_x$ of the local flow (visualized by using $2\text{-}\mu\text{m}$ particles and $5\text{-}\mu\text{m}$ particles at $x = 0, 14, 28, 42$ and $56\text{ }\mu\text{m}$ away from the left edge of the electrode. The applied voltage is 6 V_{pp} , the frequency is 600 Hz , and the axial background velocity is $232\text{ }\mu\text{m/s}$. The black lines indicate the electrodes

$$u_p = u_f \left(1 - \exp\left(-\frac{t}{\tau}\right) \right), \text{ with } \tau = \frac{d_p^2 \rho_p}{18 \mu_f}. \quad (5)$$

For a particle, with $d_p = 5\text{ }\mu\text{m}$ and $\rho_p = 1.05\text{ g/cm}^3$, in a solution with $\mu_f = 10^{-3}\text{ kg/ms}$ and $\rho_f = 1.00\text{ g/cm}^3$, the time constant is $\tau = 1.4\text{ }\mu\text{s}$. This means that after $t = 1.4\text{ }\mu\text{s}$ the particle will have the same velocity as the fluid, and the particle will completely follow the flow. The convective time in our measurement ($\tau = W/u$, with W the electrode width and u the ACEO slip velocity) is in the order of 10 ms , so much larger than the response time.

3.3.2 Centrifugal force

The particles in a rotating flow are subjected to a centrifugal force. The centrifugal force drives the particle away from the center of rotation. The centrifugal force acting on the particle is:

$$F_{cen} = (m_p - m_l) \alpha_e = (m_p - m_l) R \omega^2, \quad (6)$$

where $\alpha = R \omega^2$ is the acceleration of the particle from centrifugal force, with R the distance to the center of the swirl flow and $\omega = u_\theta / R$ the rotation velocity with u_θ the radial angular velocity. Consider only the centrifugal force and

Fig. 17 The z -component of the interpolated velocity component $\bar{u}_z$ of the local flow (visualized by using 2- μm particles) and 5- μm particles at $x = 0, 14, 28, 42$ and $56 \mu\text{m}$ away from the *left* edge of the electrode. The applied voltage is $6 V_{pp}$, the frequency is 600 Hz, and the axial background velocity is $232 \mu\text{m/s}$. The *black lines* indicate the electrodes

the drag force ($F_{drag} = -3\pi\mu_f d_p(u_p - u_f)$) acting on the particles. Assuming that the radial fluid velocity is zero, the magnitude ratio of the radial particle velocity $u_{p,\theta}$ and radial angular velocity is given as

$$\left| \frac{u_{p,\theta}}{u_\theta} \right| = \frac{d_p^2 u_\theta}{18\nu R} \left(\frac{\rho_p}{\rho_l} - 1 \right), \quad (7)$$

Assume u_θ is equal to the mean fluid velocity of the vortex. For a 5- μm particle in the vortex flow with a characteristic length scale of $20 \mu\text{m}$ and mean fluid velocity of about $100 \mu\text{m/s}$, the magnitude ratio of the radial difference ($u_{p,\theta}$)

and angular velocity is less than 10^{-6} . So, the centrifugal force on the particle is negligible.

3.3.3 Lift force

Beside the drag forces from the fluid, particles experience a lift force acting over their surfaces in a flow field with a spatial velocity gradient (Carlo 2009). The lift force is perpendicular to the direction of streamlines and causes the particles to migrate away from the streamlines. The lift forces have been found to be important in many microfluidic systems (Carlo 2009; Park et al. 2009; Xuan et al. 2010).

The lift force acting on the particles in a confined domain is mainly influenced by several parameters: the fluid velocity u_f , particle velocity u_p , channel dimension aspect, particle diameter d_p and velocity gradient perpendicular to the streamlines ∇u_f . Based on Saffman's theory, the magnitude of the lift (or Saffman lift) force is given by Saffman (1965):

$$F_{\text{lift}} = 1.615 \rho_f \nu_f^{1/2} d_p^2 (u_p - u_f) \sqrt{|\nabla u_f|} \text{sign}(\nabla u_f) \quad (8)$$

where $\nu_f = \mu_f / \rho_f$ is the fluid kinematic viscosity. For a sphere, the ratio of the Saffman lift force to the viscous drag force can be given as

$$\left| \frac{F_{\text{lift}}}{F_{\text{drag}}} \right| = 0.171 \frac{d_p}{\nu_f^{1/2}} \sqrt{|\nabla u_f|}. \quad (9)$$

In a vortical flow with a characteristic size of $20 \mu\text{m}$ and mean fluid velocity of about $500 \mu\text{m/s}$, the Saffman lift force on a $5 \mu\text{m}$ diameter particle is typically about 1 % of the drag force. As a result, the lift force can also be neglected.

3.3.4 Dielectrophoretic force

In a non-uniform electric field, the particle due to its polarization experiences a dielectrophoretic force (DEP) (Green and Morgan 1999). The magnitude of the DEP force is determined by the spatial variation of the electric field, which reaches a maximum at the electric field maxima. The dielectrophoretic force is given by Castellanos et al. (2003)

$$F_{\text{DEP}} = \frac{1}{4} \pi d_p^3 \epsilon \text{Re}(\chi_{\text{CM}}) \nabla |E_{\text{RMS}}|^2, \quad (10)$$

where $\nabla |E_{\text{RMS}}|^2$ is the gradient of the square of the RMS electric field and $\text{Re}(\chi_{\text{CM}})$ the real part of the complex Clausius–Mossotti (CM) factor. The complex CM factor is $\chi_{\text{CM}} = (\tilde{\epsilon}_p - \tilde{\epsilon}_m) / (\tilde{\epsilon}_p + 2\tilde{\epsilon}_m)$, where $\tilde{\epsilon}$ is a complex permittivity given by $\tilde{\epsilon} = \epsilon - i(\sigma\omega)$ with $i = \sqrt{-1}$, and the subscripts p and m refer to the particle and suspending medium, respectively (Green and Morgan 1999). The CM factor exhibits a frequency dependence and varies between +1 and −0.5. Dependent on the sign of $\text{Re}(\chi_{\text{CM}})$, the dielectrophoretic force is directed toward the region of high gradient of the electric field (positive DEP) or away from (negative DEP) (Zhang et al. 2009). Considering the polystyrene particle properties are $\sigma_p = 10 \text{ mS/m}$ and $\epsilon_p = 2.55\epsilon_0$ (ϵ_0 the absolute permittivity of vacuum), and the suspending medium has a permittivity of $\epsilon_f = 78\epsilon_0$ and conductivity of $\sigma_f = 1.5 \text{ mS/m}$, $\text{Re}(\chi_{\text{CM}})$ is expected to have a value of 0.65 at 600 Hz (Green and Morgan 1999; Zhang et al. 2009).

Fig. 18 Electric field lines between the symmetric electrodes in semicircular system

Assuming that the movement of particles is only due to DEP forces, the particle velocity is given by

$$u_{\text{DEP}} = \frac{d_p^2 \epsilon \text{Re}(\chi_{\text{CM}}) \nabla |E_{\text{RMS}}|^2}{12\mu}, \quad (11)$$

with μ the dynamic viscosity of the bulk solution.

Figure 18 shows the schematic diagram of the electric field above a symmetric electrode pair. Assuming the electric field is simplified to be semicircular, given $E_{\text{RMS}} = V_d \sqrt{2\pi r}$, where $r = \sqrt{x^2 + z^2}$ is the distance to the center of the gap and V_d the potential difference applied in the bulk solution. Therefore, the magnitude of the velocity component u_{DEP} in radial direction with origin in the center of the gap simplifies to

$$|u_{\text{DEP}}| = \frac{d_p^2 \epsilon \text{Re}(\chi_{\text{CM}}) |V_d|^2}{12\mu\pi^2 r^3}. \quad (12)$$

Due to the charging of the electric double layer (EDL), V_d in the bulk is generally much lower than the voltage difference applied between the electrodes. Assuming V_d is half of the voltage difference applied on the electrodes, the magnitude of the particle velocity due to the DEP force is calculated as function of z at the different x -positions for an applied voltage of $6 V_{pp}$ with $\text{Re}(\chi_{\text{CM}}) = 0.65$, and the result is shown in Fig. 19. At the electrode edge ($x = 0 \mu\text{m}$ and $z = 0 \mu\text{m}$), the magnitude of $|u_{\text{DEP}}|$ reaches a maximum. With the increase of z at $x = 0 \mu\text{m}$, $|u_{\text{DEP}}|$ significantly reduces, which is 117.2 and $22.4 \mu\text{m/s}$ at $z = 10$ and $20 \mu\text{m}$, respectively. When the distance from the electrode edge increases, the gradient of the electric field decreases. As a result, $|u_{\text{DEP}}|$ decreases, for example, $|u_{\text{DEP}}|$ at $z = 0 \mu\text{m}$ reduces from about 621.6 to $77.7 \mu\text{m/s}$ when x increases from 0 to $7 \mu\text{m}$. Similar to the case at $x = 0 \mu\text{m}$, $|u_{\text{DEP}}|$ at $x = 7 \mu\text{m}$ decreases with the increase of z , which is 41.9 and $14.7 \mu\text{m/s}$ at $z = 10$ and $20 \mu\text{m}$, respectively. The theoretical velocity magnitude of $5\text{-}\mu\text{m}$ particles due to DEP appears to be close

Fig. 19 Magnitude of particle velocities $|u_{DEP}|$ (2- μm particles in black and 5- μm particles in red) due to the DEP at a distance of $x = 0, 7$ and $14 \mu\text{m}$ away from the electrode edge. In the experiments, only velocities are measured for $z \geq 3 \mu\text{m}$ (color figure online)

to the experimental observation of the velocity difference between the flow field and the 5- μm particles in Figs. 16 and 17. This suggests that the DEP force can be a reason for the 5- μm particle focusing. However, it should be noted that if the DEP force is responsible for the particle movement in the ACEO flow, the particle moving away from the electrode surface suggests that they experience a negative DEP force, which is different with the theoretical prediction with $\text{Re}(\chi_{CM}) = 0.65 > 0$. In experiments with no or with a very low axial velocity, the 5- μm particles move toward the electrode edges following the ACEO flow field. However when they approach the bottom plate, they start to deviate toward to center of the vortices indicating that indeed an extra negative DEP force is acting on them.

In addition, Fig. 19 shows the velocity magnitude of 2- μm particles due to the DEP force, with the same physical parameter as 5- μm particles. As expected, the DEP velocity of 2- μm particles is much smaller compared to that of 5- μm particles. This is due to the decrease of the DEP force as function of d_p^2 .

In the experimental observations, the minimum position of particles over the electrodes is observed to be dependent of the additional axial background flow (see Fig. 14). This can be explained with the acting time of the DEP force on the particle above one electrode. In principle, a large axial velocity means a short detention time above each electrode, leading to the short acting time of the DEP force. As a result, the acceleration displacement of a particle above one electrode is reduced. As shown earlier in Fig. 15, one can considerably increase the particle focusing by increasing the electrode number. However, it should be noted that the increase of $\bar{z}_{\min}$ becomes smaller with the increase of

the electrode number. This is because the magnitude of the DEP force significantly reduces when the particle moves toward the top of the channel. As a result, after a certain number of the electrodes, this particle focusing will not have a significant improvement.

4 Conclusions

In this study, the effect of additional axial flow on the ACEO vortex is experimentally investigated on a symmetric electrode pattern perpendicular to the axial direction of the channel. Flow measurements reveal that the background flow results in breakdown of one vortex and confinement of the other vortex above one electrode (Fig. 4). Such axial-wise symmetry breaking is enhanced by an increase of the background velocity. This leads to the coexistence of a vortical and a unidirectional flow in the micro-channel. The circulation of the two vortices above one electrode has a different tendency as function of the additional axial background velocity: One is reduced while the other is increased due to symmetry breaking. The circulation ratio of two vortices increases with the increase of the axial background flow. This can also be predicted by the nonlinear electrokinetic model (Liu et al. 2014b), with a correction factor $\Lambda = 0.35$.

Experiments show that 5- μm particles are repulsed from the electrode surface and thereby are focused toward the upper side of the channel after passing over the ACEO-induced vortices (Figs. 12, 13, 14). The accumulation of 5- μm particles has been found to be related to the velocity difference between the velocity components of the 5- μm particles and the local flow visualized by the 2- μm particles. First order of magnitude estimates on the forces acting on the particles suggest that this particle accumulation is caused by dielectrophoresis. For large particle sizes, the DEP force becomes significantly important, especially when the particle is close to the electric field maxima. As the acting time of the DEP force over each electrode is inversely proportional to the background flow rate, for a fixed number of electrodes the particle focusing decreases with the increase of the flow rate. Compared with the traditional DEP particle focusing, where the DEP force affects a limited region, the present method uses the ACEO vortex to transport the sample toward the DEP-affected region.

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