Epistemologies, beliefs and conceptions of mathematics teaching and learning: the theory, and what is manifested in mathematics teacher's practices in England, France and Germany

Citation for published version (APA):

Document status and date:
Published: 01/01/1999

Document Version:
Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

• A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
• The final author version and the galley proof are versions of the publication after peer review.
• The final published version features the final layout of the paper including the volume, issue and page numbers.

Link to publication

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

• Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
• You may not further distribute the material or use it for any profit-making activity or commercial gain.
• You may freely distribute the URL identifying the publication in the public portal.

If the publication is distributed under the terms of Article 25fa of the Dutch Copyright Act, indicated by the “Taverne” license above, please follow below link for the End User Agreement:
www.tue.nl/taverne

Take down policy
If you believe that this document breaches copyright please contact us at:
openaccess@tue.nl
providing details and we will investigate your claim.

Download date: 23. Jul. 2019
Abstract
This paper firstly explores the issues raised in the literature concerning epistemologies, beliefs and conceptions of mathematics and its teaching and learning. Secondly, it analyses the ways in which mathematics teachers’ classroom practices in England, France and Germany reflect teachers’ beliefs and conception of mathematics and its teaching and learning. Drawing on a recent study of mathematics teachers’ work in England, France and Germany, the findings suggest that teachers’ beliefs and conceptions are manifested in their practices and can be traced back to philosophical traditions of the three countries, to epistemological and educational trends of mathematics and mathematics education, and to personal constructions. It is suggested that teachers’ pedagogical styles are a personal response to a set of assumptions about the subject and its teaching and learning, to a set of educational and philosophical traditions, and to a set of institutional and societal constraints. Thus, it is argued that teachers’ pedagogies need to be analysed and understood in terms of a larger cultural context and in relation to teachers’ conceptions and beliefs, and that a lack of such understanding is likely to inhibit the process of change at all levels of the system.

Introduction
One’s conceptions of what mathematics is affects one’s conception of how it should be presented. One’s manner of presenting it is an indication of what one believes to be most essential in it. … The issue, then, is not, What is the best way to teach? But, What is mathematics really about? (Hersh 1986, p.13)

This quote indicates that what teachers might consider to be desirable ways of teaching and learning of mathematics rests, to a large extent, on their epistemologies, beliefs and conceptions of mathematics. Thom (1973) noted that ‘all pedagogy, even scarcely coherent, rests on a philosophy of mathematics’ (p.204). This philosophy that every teacher constructs for him/herself is likely to be influenced by the epistemologies of mathematics and mathematics education, and by each one’s beliefs and conceptions of mathematics and its teaching and learning.

The literature on teaching and learning has given attention to the conditional or situational factors that shape or colour teachers’ (and pupils’) educational experience in mathematics classrooms (for example, Cole 1990). Within any country and educational community, these factors appear in many forms (for example, physical resources), and they are recognised to be influential. Embedded in the context are the values, beliefs and traditions of a particular education system which may be manifested...
Yet, many of the conditions that exert influence on human thought and practice within classrooms are neither visible nor readily identifiable. Rather, these forces are the unseen, sometimes ‘unperceived’, and often unvoiced principles, philosophies and beliefs that unwittingly penetrate the educational enterprise. For example, Lortie (1975) asserts that teachers’ pedagogical practice, in particular in the early stages of their professional lives, is to a large extent influenced by their own schooling years and during thousands of hours of an ‘apprenticeship of observation’ (Lortie 1975). It is likely that each country gives its teachers and students a different ‘apprenticeship of observation’, which is underpinned by the educational trends and traditions of that particular country. Thus, there exists a complex relationship of forces with many sources of influence at work. One of the quiet but powerful frameworks is the epistemological beliefs and conceptions that teachers (and students) hold. Indeed, the community of educational researchers is becoming increasingly aware of the potential impact that teachers’ beliefs about mathematical knowledge and education have on their classroom practice (Ernest 1988), how they approach the subject they are teaching (Anders and Evans 1994), and interact with their students (Lampert 1990).

In order to understand the complexities of the issue, but at the same time not to lose the rich details of the research, we have chosen to look at the ways in which mathematics teachers’ beliefs and conceptions are manifested in educational practices in England, France and Germany. In the first part of the paper epistemologies, beliefs and conceptions of mathematics and its teaching and learning are discussed as the literature present them. This includes the distinctions and links between epistemologies, beliefs and conceptions, and the main ideas reviewed from the literature. In the second part the empirical data and results from the study of mathematics teachers’ work (Pepin 1997) are explored and discussed.

**Epistemologies, teachers’ beliefs and conceptions**

Epistemology is generally concerned with ‘the theory of knowledge’, especially the critical study of its validity, methods and scope (Hanks et al. 1986; Sierpinska and Lerman 1996). Because of the close connection that exists between beliefs and knowledge, distinctions between them have been difficult to identify and fuzzy (Scheffler 1965). Because it had been noted that teachers frequently treat their beliefs as knowledge, this led researchers who investigated teachers’ knowledge also to consider teachers’ beliefs (Grossman et al., 1989). Indeed, some educationists have argued that it is not useful for educational researchers to search for distinctions between knowledge and belief, but rather to search for whether and how, if at all, teachers’ beliefs (or what they may take to be knowledge) affect their practices (Thompson 1992). Mathematics educators are generally interested in ‘explaining the processes of growth of mathematical knowledge’ and ‘in observing and explaining the processes of mathematical discovery in the making, both in mathematicians and in students’ (Sierpinska and Lerman 1996). Ultimately, as practitioners, they are interested in researching ways of provoking such processes in teaching.

Nevertheless, for the argument in this paper it is important to briefly refer to the distinctions, if there are, between beliefs and knowledge (as knowledge are the basis for epistemologies). A common stance among philosophers is that disputability is associated with beliefs; truth or certainty is associated with knowledge (Scheffler 1965).

Thompson (1992) asserts that:

---

In adopted curricula, educational practices, in systemic features such as pupil organisation, in expectations of students, parents, colleague teachers and administrators, for example.
from a traditional epistemological perspective, a characteristic of knowledge is general agreement about procedures for evaluating and judging its validity; knowledge must meet criteria involving canons of evidence. Beliefs, on the other hand, are often held or justified for reasons that do not meet those criteria, and, thus, are characterised by a lack of agreement over how they are to be evaluated or judged. (p.130)

Nespor (1987) argues that:

Belief systems often include affective feelings and evaluations, vivid memories of personal experiences, and assumptions about the existence of entities and alternative worlds, all of which are simply not open to outside evaluation or critical examination in the same sense that the components of knowledge systems are. (p.321)

However, over time, ‘old theories’ are often replaced by ‘new ones’. Indeed, within the philosophy of science it is commonly accepted that what is referred to as ‘factual’ knowledge is dependent upon current theories (Lakatos 1976; Kuhn 1962). Thus, what may have been regarded as knowledge at one time, may be judged as belief at a another time. Or, once-held beliefs may, in time, be accepted as knowledge in the light of supporting evidence and theories. Thus, there is a temporal quality of theories as canons of evidence (Sierpinska and Lerman 1996). Furthermore, in education there are co-existing and alternative theories that explain the processes in teaching and learning. This may help to explain the difficulty of distinguishing between teachers’ knowledge and beliefs.

Another point to make is about beliefs and belief systems. The notion of belief systems is a metaphor for examining and describing how an individual’s beliefs are organised (Green 1971). As such, they can be conceived of as a cognitive structure, and as dynamic in nature, thus restructuring as individuals change and evaluate their beliefs against their experiences. Green (1971) identified three dimensions of belief systems in the way in which they are related to one another. These dimensions are related to, firstly, the notion that beliefs are not held in total independence of all other beliefs; secondly, to the degree of conviction with which beliefs are held; and, thirdly, to the notion that beliefs are held in clusters (in Thompson 1992).

In addition, there is the notion of conceptions. This is seen here as a more general mental structure, encompassing beliefs, meanings, concepts, for example (Thompson 1992). Thus, though the distinction between conceptions and beliefs might not be distinguishably important, it will be more ‘natural’ at times to refer to teachers’ conceptions of mathematics (as a discipline) than to speak about their beliefs about mathematics.

**Philosophical traditions**

Whilst acknowledging the influence of epistemologies, beliefs and conceptions of mathematics and its teaching and learning, there are other powerful influences that underpin, arguably, teachers’ work. These influences stem from the country’s philosophical school knowledge traditions. They permeate and underlie the individual national systems and influence, to a greater or lesser extent, teachers’ thinking and decision-making and thus their pedagogies (principles and practices) in English, French and German classrooms (Pepin 1997). In this section the underpinning educational traditions of England, France and Germany (McLean 1990) are briefly explained.

The main underpinning philosophy of the English education system is humanism, with its associated principles of individualism and morality, amongst others. English education is said to be child-centred and individualistic, and the interaction between teacher and pupil is greatly emphasised. With respect to morality, there was (and is) the belief that education (originally only for the elite)
should develop qualities such as fairness and integrity, and teachers have traditionally had a pastoral as well as an academic function. The teacher has traditionally been responsible not only for the academic but also for the moral development of the child. Thus, individualism and the moral purpose of education are two of the traditional signposts for the philosophical underpinning of the English education system. One of the claims about humanism is that it is anti-rational and that England has in the past given ‘little weight in education to rational, methodical and systematic knowledge objectives’ (Holmes and McLean 1989). This can be understood in the light of the philosophy of humanism which assumes that to acquire knowledge is not a logical, sequential and standardised process, as rationalists would claim, but that learning is regarded as ‘intuitive’. The acquisition of knowledge was the outcome of the interaction between the inherent qualities of the learner and different materials appropriate to the student’s development. Therefore, the content of education should be selected in the light of individual differences.

There are two features in the philosophy of French education which help in understanding the system and the practices of those who work within it. Firstly, France is seen as one of the heartlands of encyclopaedism, with its main principles of rationality and universality, and the associated principle of égalité, transforming society in the interests of the majority of its members. The principle of rationality encourages the teaching of subjects which are perceived to encourage the development of rational faculties (for example, mathematics). The principle of universality means that students study broadly the same curriculum (at broadly the same time). The associated egalitarian views aspire to remove social inequalities through education and promote equal opportunities for all pupils. Secondly, the principle of laïcité traditionally leaves the social and moral education for the home environment, whereas intellectual and academic work is expected to be placed in school. Thus traditionally, teachers have been responsible for the academic development of the child, the parents and the church for their moral development. However, this has been changing in the sense that changes in the social role of families have transferred a socialising function to schools.

Germany espouses mainly humanistic views, based on Humboldt’s ideal of humanism, combined with naturalistic tendencies. Humboldt’s concept of Bildung searches for ‘rational understanding’ of the order of the natural world. It incorporates encyclopaedic rationalism as well as humanist moralism, and basically promotes the unity of academic knowledge and moral education. Therefore, teachers have traditionally held the two functions, that of academic specialist and, possibly to a lesser extent, that of moral educator. However, the humanist rationale was never allowed to avoid the importance of the study of mathematics and science subjects. The naturalistic view, in the German sense, combines the child-centred approaches with the work-orientated. The ‘wholeness’ of education emphasised the belief that educative experiences are not necessarily intellectual. In Germany there is the cultural view that every occupation has dignity and that work of every occupation should be carried out with maximum commitment and thoroughness.

**Epistemologies of mathematics education**

Whilst it is recognised that epistemologies of mathematics had an important influence on epistemologies of mathematics education, in this paper they are left largely untouched. However, because of their influence on mathematics education, there are some important works that have to be mentioned. For example, in France works of Brunschwig (1912) and Poincaré (1908) were important influences for the works of Bachelard (1938), Piaget (1972) and Dieudonné (1992). Dieudonné, one of the founders of the Bourbaki group, viewed mathematics as a unified whole, in which the meaning and significance of every part is a function of the role it plays in this whole. These ideas found their way into mathematics education at large in the sixties in the ‘Modern maths’ reforms (see Moon 1986).
The works of Wittgenstein (1974) and Lakatos (1976) also influenced mathematics education, perhaps unintentionally, in the sense that heuristics were, so they claim, the essence of mathematics, not the outcomes. Previously, mathematics was identified as a particular body of knowledge, a subset of which is deemed appropriate for school students and a somewhat larger subset for those who may go into higher education. The move to heuristics, which regards the doing of mathematics as the pivotal characteristic of the subject (rather than its content) encouraged problem solving and investigational work as a major focus of school mathematics since the 1970s. This manifested itself by the growth of problem solving and investigational activities in schools by teachers in such groups as the Association of Teachers of Mathematics (ATM) in the UK. As an approach to the teaching of mathematics it was established by academics such as Mason et al. (1984), and as a view of mathematical knowledge by writers such as Lerman (1986), or Ernest (1991), for example.

Turning to epistemologies of mathematics education, there is a basic difference in the viewpoint (compared to the epistemology of mathematics), because mathematics education deals not only with the possible worlds of mathematics itself (as subject matter) but also with the actual minds of students and teachers, which are embedded in a socially complex world of the nation’s education system and the educational institution. Whilst the theories of mathematical knowledge belong to an established science, mathematics education was in need of a generic epistemology and theory of its field of scientific enquiry. These needs are reflected in the interpretations that mathematics educators and researchers have been making of Piaget’s constructivist epistemology, and other epistemological views. In the following sections we shall review some of those interpretations.

There are basically four directions that help us to understand the field: psychometrics; constructivism; socio-cultural views; and interactionist views. A fifth field could be the French didactique.

**Psychometrics**
Historically, before the 1960s almost all educational research was within the discipline of psychology. In mathematics education and within this psychometric paradigm, pupils were considered to possess differing amounts of a number of traits (different ‘abilities’) which in turn allowed pupils’ intelligence to be measured by testing (Spearman 1972). A radical change took place between 1950 and 1970 when Piaget’s work was translated into English. The effect was particularly strong in mathematics education, because some of his works focused on logical and mathematical thinking (Piaget 1952). This shifted the focus from psychometrics to developmental cognitive psychology (although strictly speaking, and Piaget himself admitted to it, his work was in the area of generic epistemology (of mathematics) rather than educational psychology or mathematics education). In Britain there were a number of profound changes within teacher education, and subsequent curriculum changes (for example, Nuffield Mathematics Project), which led to changes in how mathematics was presented first in primary schools, and later in secondary schools.

**Constructivism**
From the constructivist point of view, there are no direct connections between teaching and learning, since the teacher’s knowledge cannot be conveyed to the students, the teacher’s mind is inaccessible to the students and vice versa. This supports the notion that pupils actively construct their own learning through assimilation and accommodation of cognitive structures, a process which is influenced by the experiences of the pupil, but is dependent upon whether the existing nature of structures is such as to allow the concepts to be acquired. Within constructivism some now nearly independent strands have developed: social constructivism; and radical constructivism.

Social constructivists argue for a process of enculturation, separate from and in addition to the child’s constructions. Cobb (1989) claims that children’s mathematical constructions are ‘profundely
influenced’ by social and cultural conditions. Bauersfeld (1995) suggests that ‘the core part of school mathematics enculturation comes into effect on the meta-level and is ‘learned’ indirectly’. Vygotsky also focused on the role of language in learning, thus introducing the discipline of linguistics and mathematical communication into mathematics education research (see for example, Pimm 1987, Durkin and Shire 1991). Vygotsky’s work is further discussed under ‘socio-cultural views’ (see below).

For radical constructivists, the first principle is that the teacher recognises that s/he is not teaching students about mathematics, s/he is ‘teaching them how to develop their cognition’ (Confrey 1990, p.110), and that s/he is ‘a learner in the activity of teaching’ (Steffe and D’Ambrosio 1995, p.146). Thus, and as von Glasersfeld put it, teaching is ‘a task of inferring models of the students’ conceptual constructs and then generating hypotheses as to how the students could be given the opportunity to modify their structures so that they lead to mathematical actions that might be considered compatible with the instructor’s expectations and goals’ (von Glasersfeld 1990, p.34). At the level of groups of students, Steffe and D’Ambrosio (1995) describe constructivist teaching as interacting with students in a learning space whose design is based, at least in part, on a working knowledge of students’ mathematics. This learning space consists of three elements: the posing of situations; the encouragement of reflection; and interactive mathematical communication.

Socio-cultural views

This label is given to theories which espouse the view that the individual is situated within cultures and social situations such that it makes no sense to speak of the individual or of knowledge unless seen through context or activity. Knowledge is cultural knowledge taken as socially produced, bound up with social values and socially regulated. Referring back to the epistemologies of mathematics, it is only relatively recently, and following Lakatos (1976), that mathematics has been accepted, not as a universal body of knowledge independent of local cultures, but as itself a social construction (see also a comprehensive review of differing ideologies in the philosophy of mathematics by Ernest 1991).

There has been growing interest in and focus on the social context of the mathematics classroom (for example, Bishop 1988; Keitel 1989; Lerman 1994). What is of current interest is a move away from the identification of social factors as the realm of the affective to a concern with the part that the social and cultural environment plays as a whole in the development of the child. In terms of knowledge this moves away from ‘knowledge a priori’, and also away from ‘knowledge as it is individually constructed’ to ‘knowledge as socially constructed and justified’ (Sierpinska and Lerman 1996). Lave (1988) developed a notion of knowledge-in-action in contrast to a cognitive perspective, and located mathematics in various contexts in which people act (everyday and workplace situations), but she did not engage in any depth with pedagogical issues. Vygotsky, on the other hand, was centrally concerned with learning (and teaching). Being influenced by Marxist theories, he regarded consciousness (and thus the individuals who compose it) as a product of time and space, and in particular of one’s cultural situation. Vygotsky’s concern was with the nature of consciousness and its development. For him, communication drove consciousness, and the process of learning was integral to communication. The psychology of the individual, consciousness, is formed through the mediation of tools, which are in themselves expressions of the socio-historical-cultural situation. This brings subject and object together, and new knowledge and knowledge structures lead to a shift of the ‘world’.

Vygotsky (1978) identified the ‘zone of proximal development’ which is the difference between what a child can do on her/his own and what s/he can do with the aid of a more experienced peer/adult/teacher. The child is assisted through a process which lies in the student’s ‘zone of proximal
development’ until the ‘scaffolding’ can be removed and the child can act alone. This is a fundamental shift in the sense that all learning is viewed as taking place with others. The theory that ‘learning leads development’ stands in direct contrast to the writings of Piaget for whom development, in the form of the child’s stages of development, led learning. Whilst the Piagetian model is based on the ‘lone learner’, the learners construct their own understanding, social constructivism claims that the role of the teacher (or parent/peer) is crucial in ‘scaffolding’ the learning.

Another fundamental feature of Vygotsky’s theories was the process of internalisation. ‘The process of internalisation is not the transferral of an external to an … internal plane of consciousness, it is the process in which this plane is formed’ (Leont’ev 1981, p.57). Thus, there is unification of teaching and learning.

**Interactionist views**

In this paradigm interactions are not regarded as mere auxiliary and helpful factors of development, but interactions and development are seen as inseparable. The focus of study is not the individual but interactions between individuals within a culture (Bruner 1985). Language (and ‘languaging’) becomes very important, which is seen as the ‘active moulder of experience’ and not a ‘passive mirror of reality’ (Bauersfeld 1995). Wittgenstein is often quoted saying that ‘speaking of language is … a form of life’ (cited in Bauersfeld 1995).

For an interactionist mathematics educator, learning is not just an endeavour of the individual mind trying to adapt to an environment, nor can it be reduced to a process of enculturation into a pre-established culture. In the mathematics classroom, the individual construction of meanings takes place in interaction with the culture of the classroom while at the same time it contributes to the constitution of this culture (Cobb and Bauersfeld 1995, p.9). This property is called ‘reflexivity’ which is quite central to interactionist approaches.

In this approach meanings are elaborated through negotiations whereby the group comes to agree on certain conventions in the interpretation of signs, situations, and behaviours. Through this interaction, the individual contributions may add up to something nobody in particular has thought about and anticipated (Voigt 1995). An important issue is that people learn indirectly, through participating in a culture and its discursive practices. For example, pupils learn what counts as mathematical thinking by observing what is addressed and what kind of solutions are distinguished by the teacher and other students as ‘simple’ or ‘non-acceptable’. 

The view on language is different from the previously discussed paradigms. For Piaget, language is an expression of thought, for Vygotsky a medium of cultural transmission. Interactionism ceases to see language as a separate object (tool) that can be used for one purpose or another, but they regard language as creating a reality, ‘languaging’ (Bauersfeld 1995). Relating to this, mathematics is seen a special type of discourse, where discourse is interpreted as ‘language-in-action’, a ‘vehicle for doing things with and to others’ (Bruner 1985). Thus, mathematics becomes a way of seeing the world and thinking about it.

Moreover, the process of construction of knowledge is based on interpretations that have their source not in the individual alone but in his/her interpretation with others within a culture. Constructivism is the point of view of the individual as s/he makes sense of the world, interactionism is the point of view of an observer of the social life, and looks at people sharing meaning and at the functioning of language as it creates meanings. For Bauersfeld, and according to interactionism, meanings are generated neither by the individual minds nor are they attributed to some historically founded ‘collective mind’ of a society, but they are continually constituted in interactions whose
patterned character accounts for the relative stability of cultures. Bauersfeld (1995), in rehabilitating some of the ‘old-fashioned’ values in education, has stressed the role of the quality of the culture in which one lives for personal upbringing. He reminds researchers that imitative learning ‘is the most common form of learning in a culture’ (p.283). Thus, the role of the teacher becomes paramount in the educational process.

French ‘Didactique’

Since the mid-seventies mathematics educators have devoted much time in their work on the epistemology of mathematics education and on the nature of that knowledge involved in mathematics education. French research on ‘didactique des mathématiques’, the issue of preparing mathematics for students, can be broadly divided into two not independent but nonetheless distinct theoretical fields: the field of ‘didactical transposition’ developed by Chevellard (1991, 1992); and the theory of ‘didactical situations’ initiated by Brousseau (1986).

The theory of didactical transposition concentrates on the analysis of those processes that are based on reference knowledge, in particular the processes involved when transposing ‘scholarly knowledge’ (savoir savant) to that of ‘taught knowledge’ (savoir enseigné). It is assumed that there exists some identifiable knowledge called ‘savoir savant mathématique’, against which the mathematics taught in schools could be judged or ‘legitimised’. Another assumption of didactic transposition is that what is taught will ultimately be learnt by students, and that there is some expert knowledge. These notions are foreign from a constructivist point of view, in the sense that there is no knowledge existing outside individuals’ minds, and thus no distinction between expert and novice knowledge. There has been much criticism of the vagueness of the notion of ‘savoir savant’ (Freudenthal 1986), which created a response to it. It is argued that society recognises the existence of a group of professionals who produce knowledge which is considered as ‘knowledgeable’ (savant). More recently Chevellard (1991) looked at relations between the social practice of research in mathematics and social practice of institutionalised teaching and learning of mathematics at school. He subsequently extended his theory and assumed that all knowledge is knowledge of an institution.

Brousseau’s (1986) theory, the theory of didactical situations, is situated at a more local level. It aims to model teaching situations so that they can be developed and managed in a controlled way. At the basis of this theory is the assumption that ‘knowledge exists and makes sense for the cognising subjects only because it represents an optimal solution in a system of constraints’ (p.368). According to Artigue (1994) it is based on a constructivist approach and operates on the principle that knowledge is constructed through adaptation to an environment that appears problematic to the student. Von Glasersfeld (1995) writes:

*From the constructivist perspective, as Piaget stressed, knowing is an adaptive activity. This means that one should think of knowledge as a kind of compendium of concepts and actions that one has found to be successful, given the purposes one has in mind.* (p.7)

Brousseau’s theory aims to become a theory for the control of teaching situations in their relationship with the production of mathematical knowledge. The didactic systems are therefore made up of three mutually interacting components: the teacher, the student, the knowledge. The aim is to develop the conceptual and methodological means to control the interacting phenomena and their relation to the construction and functioning of mathematical knowledge in students.

The basic assumption of Brousseau’s theory of situations is that knowledge constructed or used in a situation is defined by the constraints of this situation, and that, therefore, by creating certain artificial constraints the teacher is able to provoke students to construct a certain type of knowledge.
Teachers’ beliefs and conceptions

In response to the question ‘What is mathematics?’ Hersh (1986) offers the following answer:

Mathematics deals with ideas. Not pencil marks or chalk marks, not physical triangles or physical sets, but ideas (which may be represented or suggested by physical objects). What are the main properties of mathematical activity or mathematical knowledge, as known to all of us from daily experience? (1) Mathematical objects are invented or created by humans. (2) They are created, not arbitrarily, but arise from activity with already existing mathematical objects, and from the needs of science and daily life. (3) Once created, mathematical objects have properties which are well-determined, which we may have great difficulty discovering, but which are possessed independently of our knowledge of them. (pp.22–23)

Hersh here adopts the idea of the practising mathematician and, in line with other philosophers such as Lakatos (1986), challenges the basic assumption that mathematical knowledge is a priori and infallible. An assumption underlying Hersh’s view of mathematics is that knowing mathematics is making mathematics, its creative activities and processes. This view of mathematics is reflected in documents such as The Cockcroft Report (Committee of Inquiry into the Teaching of Mathematics in Schools, 1983) in England, for example. The conception of mathematics teaching that is reflected in this document is one in which students engage in purposeful activities that grow out of a problem situation, requiring reasoning and creative thinking, gathering and applying information, discovering, inventing, and communicating ideas, and testing those ideas through critical reflection and argumentation. This view of mathematics teaching is in sharp contrast to alternative views in which the mastery of concepts and procedures is the ultimate goal of instruction, although it does not deny the value of concepts and procedures in the mathematics curriculum. The National Council of Teachers of Mathematics (NCTM, 1989) writes that ‘instruction should persistently emphasise “doing” rather than “knowing that”’ (p.7).

The nature of teachers’ beliefs about the mathematics and its teaching and learning, as well as the influence of those beliefs on teachers’ classroom practices, are relatively new areas of study. A number of studies in mathematics education (for example, Lerman 1983; Thompson, 1984) have suggested that teachers’ beliefs about mathematics and its teaching and learning significantly influence the ‘modelling’ of teachers’ characteristic pedagogies. Ernest (1988) noted that among the key elements that influence teachers’ practices, three are most influential: (1) teachers’ system of beliefs concerning mathematics and its teaching and learning; (2) the social context of the teaching situation (constraints, opportunities, etc.); (3) teachers’ level of reflection (p.1). He contends that the research literature on mathematics teachers’ beliefs indicate that teachers’ approaches to mathematics teaching depend basically on their systems of beliefs (in particular on their conceptions of the nature of mathematics) and on their mental models of teaching and learning mathematics.

Thompson (1992) views a teacher’s conception of the nature of mathematics as ‘that teacher’s conscious or subconscious beliefs, concepts, meanings, rules, mental images, and preferences concerning the discipline of mathematics’ (p.132), which constitute the rudiments of a philosophy of mathematics. Ernest (1988) distinguished three conceptions of mathematics:

First of all, there is a dynamic, problem-driven view of mathematics as a continually expanding field of human creation and invention, in which patterns are generated and then distilled into knowledge. Thus, mathematics is a process of enquiry and coming to know, adding to the sum of knowledge. Mathematics is not a finished product, for its results remain open to revision (the problem-solving view).
Secondly, there is the view of mathematics as a static but unified body of knowledge, a crystalline
realm of interconnecting structures and truths, bound together by filaments of logic and meaning. Thus, mathematics is monolith, a static immutable product. Mathematics is discovered, not created
(the Platonist view).

Thirdly, there is the view that mathematics, like a bag of tools, is made up of an accumulation of
facts, rules and skills to be used by the trained artisan skilfully in the pursuance of some external
end. Thus, mathematics is a set of unrelated but utilitarian rules and facts (the instrumentalist
view). (p.10)

Lerman (1983) identified two alternative conceptions of the nature of mathematics, which he named
‘absolutist’ and fallibilist’, and which, according to him, correspond to two competing schools of
thought in the philosophy of mathematics: Euclidean and Quasi-empirical (Lakatos 1978). From
the absolutist perspective, mathematics is based on universal and ‘true’ foundations, and as such is
‘the paradigm of knowledge, certain, absolute, value-free, and abstract’. From the fallibilist
perspective mathematics develops through conjectures, proofs and refutations, and uncertainty is
inherent in the discipline (Lerman 1983). There are obvious parallels between Lerman’s absolutist
and fallibilist views and Ernest’s platonic and problem-solving views.

Skemp (1978) proposed that two conceptions of mathematics account for sharp differences in
classroom practices and emphases: ‘relational mathematics’ and ‘instructional mathematics’. According to Skemp, instrumental knowledge of mathematics is knowledge of a set of ‘fixed’
plans for performing mathematical tasks (step-by-step procedure), whereas relational knowledge
of mathematics is characterised by the possession of conceptual structures that enable the teacher/
pupil to construct several plans for performing a given task.

**Teachers’ conception of mathematics teaching and learning**

Whilst differences in teachers’ conceptions of mathematics appear to be related to differences in
their views about mathematics teaching, teachers’ conceptions of mathematics teaching are also
likely to reflect their views of how students learn mathematics and of students’ mathematical
knowledge (Carpenter et al. 1988). There seems to be a logical, natural connection between teachers’
teaching ‘models’ and their underlying theories of how students learn mathematics. However, the
literature claims that for most teachers the two have not developed into a coherent theory of
instruction. Clark (1988) suggested that conceptions of teaching and learning tend to be eclectic
collections of beliefs and views that appear to be more the result of their years of classroom experience
than any type of formal approach. He says:

Research on teacher thinking has documented the fact that teachers develop and hold implicit
theories about their students…. about the subject matter that they teach … and about their roles
and responsibilities and how they should act … These implicit theories are not neat and complete
reproductions of the educational psychology found in textbooks or lecture notes. Rather, teachers’
implicit theories tend to be eclectic aggregations of cause-effect propositions from many sources,
rules of thumb, generalisations drawn from personal experience, beliefs, values, biases, and
prejudices. (p.6)

In studying the source of teachers’ beliefs about teaching and learning, it has been noted that those
beliefs are, to a large extent, formed during teachers’ schooling years and are shaped by their own
experience as pupils. Teachers have spent thousands of hours in an ‘apprenticeship of observation’
(Lortie, 1975) which is likely to lead to the development of a body of values, commitments,
orientations and practices. The literature suggests that these established values and orientations
persist despite the efforts of training institutions (Lacey 1977; Haggarty 1995).
In terms of models of mathematics teaching, Kuhs and Ball (1986) identified ‘at least four dominant distinctive views of how mathematics should be taught’:

1. **Learner-focused**: mathematics teaching that focuses on the learner’s personal construction of mathematical knowledge;
2. **Content-focused with an emphasis on conceptual understanding**: mathematics teaching that is driven by the content itself but emphasises conceptual understanding;
3. **Content-focused with an emphasis on performance**: mathematics teaching that emphasises student performance and mastery of mathematical rules and procedures; and
4. **Classroom-focused**: mathematics teaching based on knowledge about effective classrooms. (p.2)

The learner-focused view of mathematics teaching is underpinned by a constructivist view of mathematics learning (von Glasersfeld 1987). From this perspective of teaching, the teacher is viewed as facilitator and stimulator of pupil learning. Students are ultimately responsible for judging the appropriateness of their own ideas. The content-focused with emphasis on understanding is the view that follows from the Platonic view (Ernest 1988) of the nature of mathematics. This view of teaching emphasises students’ understanding of the logical relations among various mathematical ideas and the concepts and logic underlying mathematical procedures. In this model the content is organised according to the structure of mathematics, and the content is organised according to a hierarchy of skills and concepts. From this perspective, the role of the teacher is to demonstrate, explain and present the content in an expository style, and that of the pupils to listen, participate and do exercises that have been set by the teacher. Central to the fourth view is the notion that classroom activity must be well-structured and efficiently organised. The assumption here is that students learn best when the lessons are clearly structured and the teacher follows principles of effective instruction.

Ernest (1991) identified five categories of educational ideologies of mathematics education: ‘industrial trainer’; ‘technological pragmatist’; ‘old humanist’; ‘progressive educator’; and ‘public educator’. Briefly, for the ‘industrial trainer’ mathematics is a ‘clear body of knowledge and techniques’. His/her theory of mathematics teaching is authoritative and teaching is seen as ‘passing on a body of knowledge’ (Lawlor 1988, p.9, in Ernest, 1991). According to the ‘technological pragmatist’s’ ideology of mathematics education, knowledge has two parts: pure mathematical skills, procedures and facts; and applications and uses of mathematics. The theory of mathematics learning associated with this theory is comparable to an apprenticeship in the sense that knowledge and skills are acquired through practical experience. The ‘old humanist’ regards mathematics as a ‘pure, hierarchically structured’ body of objective knowledge. The teacher’s role is that of ‘lecturer and explainer’, communicating the structure of mathematics meaningfully. Within the theory of the educational ideology of the ‘progressive mathematics educator’, mathematics is ‘a vehicle for developing the whole child’, where the emphasis is not the curriculum but the child. Ernest asserts that ‘the process of mathematical problem-solving and investigating, such as generalising, conjecturing … figure more prominently than specification of mathematical content. The teaching of the subject consists of encouragement, facilitation, and the arrangement of carefully structured situations for investigation. For the ‘public educator’, school mathematics must reflect mathematics as a social construction, and therefore not be seen as alienated from the student’s world. Mathematical knowledge is expected to provide ‘an understanding of and power over both the abstract structures of knowledge and culture, and the mathematised institutions of social and political reality’. The teaching of mathematics includes a number of components:

1. ‘Genuine discussion, both student-student and student-teacher, since learning is the social construction of meaning;
2. Co-operative groupwork, project work and problem solving, for confidence, engagement and mastery;
3. Autonomous projects, exploration, problem posing and investigative work, for creativity … and engagement through personal relevance;
4. Learner questioning of course content, pedagogy and modes of assessment used, for critical thinking; and
5. Socially relevant materials, projects and topics, including race, gender and mathematics, for social engagement and empowerment.’ (pp.208, 209, Ernest 1991)

**Relationship between beliefs about teaching and instructional practice**

The literature suggests that teachers’ conceptions of teaching and learning mathematics are not related in a simple cause-and-effect way to their instructional practices. The relationship is not a simple one. Yet, an assumption that appears to underlie many investigations is that the relationship is one of linear causality, where first come the beliefs and then follows the practice. The literature suggests that the relationship is more complex, involving a give and take between beliefs and experience and thus is dialectical in nature. (Thompson 1992). There exists a complex relationship with many sources of influences at work. For example, one such source is the social context in which mathematics teaching takes place. Embedded in this context are the values, beliefs and expectations of students, parents, colleague teachers and administrators, perhaps the adopted curriculum, the educational practices of assessment and pupil organisation, and the values and philosophical leanings of the educational system at large.

**Findings**

The research that forms the basis of the empirical work reported in this paper (Pepin 1997) sought to develop an understanding of mathematics teachers’ work at secondary level in three European countries: England, France and Germany. The original question underlying the study was whether it would be possible for mathematics teachers at secondary level in England, France and Germany to work in a country other than their own (Pepin, 1999b). Twelve mathematics teachers, four in each country, were ‘shadowed’ for two weeks each, in order to develop an understanding of their beliefs concerning teaching and learning, and their classroom practices. The work was carried out within the framework of an ethnographic approach, in combination with stimulated recall, in order to explore the context in which teachers were working; and how they conceived of and carried out their tasks in schools. Five theoretical conclusions were generated from the study. Those theories were concerned with commonalities amongst mathematics teachers in the three countries; with the influence of cultural educational traditions on teachers’ pedagogies (Pepin, 1999a); with the influence of varying ranges of teachers’ tasks and responsibilities on their beliefs and practices (Pepin 1998); with terms and conditions under which teachers work with respect to people in the wider community; and with the influence of teachers’ different beliefs about mathematics on their practices.

The latter theoretical conclusion is the focus of this paper. Firstly, findings on teachers’ perceptions of the nature of mathematics are discussed. Secondly, it is argued that teachers’ beliefs and conceptions of mathematics and its teaching and learning are manifested in their practices, and the practices encouraged in textbooks.

**Perceptions of mathematics**

Not all teachers chose to talk about the nature of mathematics in an explicit way. Some explained their views on mathematical reasoning, rigorous proof and mathematical expressions, which in turn gave the researcher indications of their beliefs and conceptions concerning the nature of mathematics.
There were three lines of perception about the nature of mathematics: mathematics as a tool; as ‘training the mind’ with its logic; and as a criterion for selection.

Most teachers who chose to comment on the nature of mathematics explicitly saw it as a tool or utensil (at the 11–16 age level). Some English teachers pointed to the ‘skill side’, mathematics as a tool for other subjects, which was also reiterated by German and French colleagues.

From what teachers mentioned concerning mathematical reasoning, it was clear that they also acknowledged the more transcendent nature of it (‘training of the mind’) and that it had a high priority in their view. German and, more particularly French, teachers felt that logic was the principal element of mathematics, and their classroom practice reflected these beliefs. In England, curiously, there were inconsistencies between what English teachers said and what they did in the classroom. Although English teachers talked about logic and reasoning quite extensively, they rarely practised it in their classrooms. The English teachers all mentioned logic and reasoning, the development of the mind through logical ways of thinking, as part of the nature of mathematics. This was surprising to the researcher, because in none of the lessons observed did the researcher see an emphasis on logical reasoning. Teachers seemed to assume that logical skills would be learnt from activities, such as investigations where reasoning was asked. Furthermore, the researcher speculates that teachers were interested in results (the piece of course work) and not in the process of how pupils discovered, investigated or their reasoning. For example, when commenting on course work and investigations, one teacher explained that pupils did their investigations (investigational tasks for course work) ‘under exam conditions’. In her opinion it was ‘nicely organised’, it was ‘cut and dried’ and ‘finished’ within a reasonable time.

This suggested that there was a inconsistency between some English mathematics teachers’ view of the nature of mathematics and its manifestation in their practices. One possible explanation could be that teachers themselves were educated in mathematics to give reasons for what they did, but in school, with time constraints and a busy working schedule, found it hard to comply with their own expectations. Another explanation would be that for them it was not worth emphasising the logic for most pupils and only appropriate for the most able. However, they did not mention this dichotomy in any way, and it is speculated that English teachers were not critical of this aspect of their work.

Regarding mathematical expression, some English teachers did not view formal mathematical notation or expressions as a means by which to educate their pupils to think in a logical way. They tried to adjust their vocabulary to pupils’ level of understanding. One teacher commented that she tried ‘to work out what language (was) useful’ for pupils’ understanding. In terms of mathematics notation, her colleague believed that ‘the idea of being lazy as a reason for mathematical notation’ was ‘one that the kids (could) generally connect to’ (in the sense of not writing a lot of words, but rather expressing it in a simple way).

One German teacher chose to talk about the nature of mathematics explicitly. He differentiated between the pure side of mathematics (the logic) and the tool side of it (as utensil for other sciences) which was regarded by him as the basis and which the teacher of his school form (Hauptschule) had to deal with most of the time. His colleague working in the Gymnasium emphasised that the subject material should be prepared in such a way that it was ‘orientated’ towards ‘logic’ and she regarded it as ‘correct’ to treat a topic in an ‘abstract’ way. This indicated that German teachers of both school forms considered logic as the principal element of mathematics, and they tried to include it into their practices.

Some French teachers perceived mathematics as a criterion for selection (for further education or jobs, for example), others regarded it as a tool or utensil in science, for example. But they all
emphasised that there was another side to it, with something of a ‘transcendent’ nature, and one teacher summed it up by saying that the logical reasoning in mathematics served as ‘training of the mind’. In France, rigorous proof was part of the curriculum in years 9 and 10 and one teacher commented that the ‘aim’ of rigorous proof was ‘logical thinking’. From the ways teachers conducted their lessons and the type of exercises they provided, the emphasis on justification and proof in the curriculum documents and from the interview with the inspector, the researcher concluded that in France logical reasoning was regarded as the main element of mathematics, in practice as well as in theory.

**Teachers’ beliefs on the nature of mathematics are manifested in their practices and they are different in the three countries**

This section is concerned with teachers’ beliefs and conceptions of mathematics teaching and learning. It is argued that teachers had different views about the nature of mathematics, the aims of teaching mathematics and the ways it could be learnt, which were manifested in their practices. In the first instance, teachers’ conceptions of teaching and learning mathematics are compared with ‘views’ of the literature. In the second instance, a classification was developed concerning the knowledge base of mathematics. The three dimensions that were identified were concerned with conceptual links, with process integration (into teaching) and with completeness of pupils’ mathematical experiences. In the third instance it is claimed that traces of those dimensions and ideologies of mathematics education can be traced in the textbooks used, which in turn helped to develop an understanding of teachers’ practices. Textbooks reinforced those particular positions on those three dimensions. The researcher argues that the ways mathematics was explained and presented in textbooks helped to understand teachers’ practices in the classroom.

**Philosophies of mathematics and mathematics education underpin teachers’ practices**

According to Ernest’s conceptions of mathematics (1988) there are three philosophies, and in another of Ernest’s work (1991) five ideologies of mathematics education are suggested, which he proposes as tentative categories for groups of teachers working in the British context. However, he asserts that the ideologies need not be bounded by nationality. It was found in the comparative research (Pepin 1997) that all French, German and English teachers studied, consciously or subconsciously, ascribed to one or several of three of Ernest’s theories: the ‘technological pragmatist’; the ‘old humanist’; and the ‘progressive educator’.

However, although all teachers appeared to subscribe to one or several of the three categories, there were different weightings and emphases in the different national systems. In England the emphasis in the classroom was on the utilitarian and pragmatic side (‘technological pragmatist’ view) combined with the individualistic and child-centred view. However, teacher educators in England espoused the ‘progressive educator’ or ‘public educator’ (Ernest 1991) philosophy, and there were traces of what would be described as ‘humanist’ traditions in what teachers said, but neither tradition was recognisable in English teachers’ classroom practices. In France teachers traditionally regarded mathematics teaching as important for ‘training the mind’ (‘old humanist’) and for work preparation (‘technological pragmatist’), whereas more recently theories of mathematics teaching were encouraged where personal exploration was to be facilitated (‘progressive educator’). Therefore, French teachers showed a mixture of three philosophies. In Germany it depended on the school type which ideology was adopted. Whereas in the Gymnasium the ‘classic’ view of mathematics as a body of structured knowledge prevailed (‘old humanist’), the Hauptschule adopted a more pragmatic view where ‘useful knowledge’ was to be transmitted (‘technological pragmatist’ view).

Ernest’s attempts to classify philosophies looked helpful, and it was interesting to note where teachers appeared to be on this classification. However, although Ernest’s classification was useful to a certain extent, it was not entirely appropriate in order to develop an understanding of the twelve
teachers in the three countries. There were additional dimensions and it was therefore decided to develop a new way of looking at the issue of teachers’ beliefs and conceptions in relation to their practices and in terms of the rich data that were available.

**Three ‘original’ dimensions that underpin teachers’ practices**

The first classification that was developed was concerned with the coherence of the mathematics taught, which was concerned with conceptual links, the ‘inter-connectedness’ of concepts, and with ideas of a body of mathematical knowledge or a set of beliefs about the coherence of mathematical concepts. There seemed to have been a range of interpretations, from the emphasis of the conceptual link between the (mathematical) knowledge base to no emphasis of the conceptual link, and teachers could be put somewhere within that range. For example, in France teachers expected from themselves (and were expected by the inspector) to have a certain distance from the content they were teaching, in order to be able to see its links to other areas of mathematics and subsequently be able to identify effective ways of teaching the content.

Secondly, a process dimension about teachers teaching mathematics was identified, in which it was either neglected (as in Germany) or was seen as integral to the learning of the mathematics (as in France). The whole idea about logical thinking was generally also part of that dimension. For example, in France teachers emphasised the process element by preparing cognitive activities for pupils. The idea of ‘letting pupils discover’ was linked to the teaching of the content, and therefore combining process and content. In England investigations appeared to be done separately, as a separate issue which seemed to be almost like another area of content.

Thirdly, there was the dimension concerning the coherence of pupils’ mathematical experiences. For example, in Germany and in France pupils were expected to reach certain levels at the end of every school year, otherwise they had to repeat the year. On the other hand, in England pupils reached levels of the National Curriculum and some progressed further than others within the same year. This led to a particularity which was not evident in France and Germany, in the sense that English pupils could leave school after year 11 whichever level they had reached.

Other examples for the three dimensions were given by French teachers. The reasoning and training-of-the-mind aspect of mathematics was repeatedly emphasised by French teachers (and the inspector) and the researcher could see this conviction in practice in the classroom. Pupils had to reason (sometimes with rigorous proof) their results and they were given cognitive activities (problem-solving) to discover notions of mathematics for themselves. The emphasis was on the process and not the result. French teachers were genuinely concerned about the essence of the lesson and how to teach it best, what would enhance pupil understanding, and that all pupils were able and entitled to learn the whole of mathematics (taught at that age level). French teachers focused on developing mathematical thinking. They tried to pose thought-provoking problems and expected students to struggle with them. They drew together ideas from the class and the whole class discussed solutions. Teachers tried to forge links between ideas, skills and ‘cognitive activities’ (small investigations) on the one hand, and concepts on the other. Therefore, it is firstly argued that French teachers’ perception of different facets of mathematics (inter-connectedness of concepts, process-orientated, entitlement) resulted in a picture of mathematics as a whole. It is secondly argued that teachers’ perceptions of how mathematics was structured, its unifying concepts and methods, in other words its ‘wholeness’, influenced their teaching in such a way that various cognitive approaches were used to provide a learning-enriched environment. Thirdly, by expecting the whole class to move forward together, French teachers’ practice reflected egalitarian views, and their emphasis on mathematical reasoning reflected the cultural tradition of rationality, one of the encyclopaedic principles (embodied in the notion of formation d’esprit – training of the mind).
In Germany, the view of mathematics which teachers revealed was relatively formal and included logic and proof (‘old humanist’ view). It included a view of the teacher’s role as that of the explainer who taught the structure of mathematics through an ‘exciting’ delivery and by adapting the structured textbook approach meaningfully. German teachers’ views of mathematics also included teachers’ aspiration to treat each topic in relative depth (notion of quality) which in turn meant that they spent considerable time on each topic. This view of mathematics went hand in hand with their view of teaching, in the sense that they used traditional (in Germany) front-teaching approaches (Frontalunterricht) combined with the conversational interactive style. The conversational style allowed them to discuss topics in relative depth and to monitor pupils’ misunderstandings by involving the whole class in discussions. In terms of the three dimensions, the dimension of conceptual links was reported by teachers in discussions. Although it was not made explicit in their classroom practices, through spending extended time on each topic (i.e. less fragmentation) there seemed to be conceptual links. The process-orientation was not detectable. Teachers tried to transmit their knowledge to pupils as effectively as possible. The third dimension, at which point pupils were allowed ‘to jump off’, depended on the school forms. In general pupils were expected to be taught and to learn the curriculum of the particular year and of the particular school type.

In England, the emphasis was on the skill side of mathematics and results. The notion of ‘training-the-mind’ for logic reasoning was missing, except for the high ability sets and some investigative tasks. This, combined with teachers’ determination to keep pupils busy and entertained, led to an impoverished mathematical diet for some low achieving children. Although teachers talked about logic and proof as their aims, their teaching generally did not include these aspects. They were concerned with covering the content of the curriculum. English teachers spent relatively little time explaining concepts to the entire class, and they introduced and explained a concept or skill to pupils, gave examples on the board, and then expected pupils to practise on their own while they attended to individual pupils. Situations where pupils discovered multiple solutions or investigated new solutions that required reasoning were rare and usually reserved for ‘investigation’ lessons. Therefore, firstly it can be claimed that notions of justification and proof (to be taught according to Attainment Target 1, level 7 and 8) were only taught to the high achieving children in their respective sets, and with the lower achieving set children being deprived of this experience. Secondly, investigations and content were taught at different times, resulting in the separation of process and content. Thirdly, process-orientated teaching through investigations and problem-solving became an activity in its own right, where teachers tried to teach pupils to ‘behave mathematically’ rather than ask them to think about the structures their patterns might illuminate. Although the texts (National Curriculum non-statutory guidelines, or the Cockcroft Report, for example) emphasised the connection of process and content, most English teachers did not practise according to the texts. They regarded it ‘difficult’ to teach ‘investigatively’. It appeared that either lessons focused on process elements (i.e. ‘doing’ investigations for course work) or on content elements (as in teaching AT2, AT3 and AT4 of the National Curriculum), but with no links between the two. In addition, English children were presented with different topics at close intervals. Notions, such as percentages, for example, were taught for a relatively short period of time, and expected to be revisited at a later stage. It was assumed that, if pupils revisited a topic often enough, they would finally understand. This also led to relatively erratic jumps from one topic to another and the notion of the ‘spiral curriculum’ supported this idea.

In terms of inter-connectedness of concepts, this gave the impression of a fragmented view of mathematics (which the teachers surely did not have) and there was a coherence missing. Concerning the completeness of experiences, it was accepted that some pupils (because of their low achievement) would have access to only part of the curriculum. Pupils were generally grouped in achievement sets and lower achievement set pupils worked through the National Curriculum at a slower pace.
than the higher achievement sets – all strategies that derive from an individualistic approach (as part of the humanistic philosophy). This, in turn, meant that each pupil’s mathematical experience was determined by the level they reached in the National Curriculum, whether or not this led to a coherence of mathematical experience or not.

**Mathematics textbooks reinforce particular positions on those three dimensions**

In all three countries textbooks were mediated, to a greater or lesser extent, by teachers. It is argued here that the ways mathematics was explained and presented in textbooks helped to understand teachers’ practices and that they were in line with the practices that were observed in the classroom. It was not clear the extent to which textbooks could either challenge or modify existing practices in a country, but certainly in England, where most teachers followed textbooks and reported little time for preparation, one could speculate that their potential was not fully exploited by curriculum developers.

In Germany (especially in the *Gymnasium*) there was one textbook for algebra and one for geometry, each covering two years. Interestingly, algebra and geometry were taught quite separately. This indicated that there was hardly any inter-connectedness between the two domains. For every chapter in the books there was a short introduction to the notion for each topic, followed by routine exercises leading to relatively demanding problems. Topics were explained in depth (although with relatively short and formal explanations) and there was relatively little ‘revisiting’ of topics. Textbooks emphasised the mathematical content in its structured and pure form, with a hierarchical structure connecting the topics. This reinforced teachers’ views that knowledge was to be taught in a structured way. The mathematics was given to teachers and they knew that they were to convey the content to pupils. The textbooks neither suggested nor encouraged particular teaching approaches and therefore did not attempt to provide pedagogic stimulus and guidance to teachers.

In France, textbooks were chosen by teachers and schools. Those books integrated specific (cognitive) activities in order to encourage teachers to teach in the ways encouraged by the inspectors. The books provided teachers with support for the preparation of their lessons, in terms of introductory activities as well as in the selection of appropriate exercises. Teachers in France were encouraged by their inspectors to prepare their lessons carefully (to step back and think about the ‘best’ way of teaching a topic), and approved textbooks suggested how to introduce topics with cognitive activities.

In England, as many as six textbooks covered the content for two years, and topics were often revisited from one year to the next. Textbooks were usually presented with brief explanations, cartoons and pictures in the introduction followed by exercises. In theory, teachers were expected to follow the departments’ schemes of work, and these comprised a list of topics to be taught to the year group and set, with reference to chapters in various textbooks. In practice, teachers ‘ran through’ those suggested topics which reinforced the notion that each part of the mathematics programme was separate, unless the departmental schemes of work provided for the inter-connectedness of topics. Activities were not integrated in the sense that teachers taught a chapter and then did an investigation which might or might not have a connection with the chapter that had been taught. Topics were revisited in the textbooks, in line with the notion of the spiral curriculum where it is assumed that children gain a deeper understanding of a topic if they were introduced to the notions on several occasions. It was difficult to find a textbook in England which promoted the kind of cognitive activities that might help teachers to teach their lessons ‘investigatively’ (investigations are given at the end of chapters, as side-aspects of the main content teaching).
Conclusions
The findings of the research demonstrate that teachers’ classroom practices in the three countries reflected their beliefs and conception of mathematics and its teaching and learning. Teachers’ beliefs and conceptions could be traced back to philosophical traditions of the three countries, and to epistemological and educational trends of mathematics and mathematics education. Looking at the literature there is a powerful argument that systems are the main determinant for different pedagogies practised in different countries. In this paper it is argued that there are subtle and ‘non-visible’ forces at work in classrooms of our schools. They are the often unvoiced principles, philosophies and beliefs that penetrate the educational setting. It is suggested that teachers’ pedagogical styles are a personal response to a set of institutional and societal constraints (e.g. curricular organisation), to a set of educational and philosophical traditions, and a set of assumptions about the subject and its teaching and learning. Thus, it is argued that teachers’ pedagogies need to be analysed and understood in terms of a larger cultural context and in relation to teachers’ conceptions and beliefs, and that a lack of such understanding is likely to inhibit the process of change at all levels of the system.

References
Glasgow: Collins.
Chicago: The University of Chicago Press.
Skills, and Dispositions. East Lansing: Michigan State University, Center of Teacher Education.
University Press.
LAMPERT, M. (1990) When the problem is not the question and the solution is not the answer: mathematical knowing 
University Press.
LERMAN, S. (1986) Alternative Views of the Nature of Mathematics and their Possible Influence of the Teaching of 
Education, University of London.
the BERA '98 Conference in Belfast, Northern Ireland, August 1998.
Sage Publications.
Educational Researcher, forthcoming issue.
Sage Publications.
Educational Researcher, forthcoming issue.
SCHEFFLER, I. (1965) Conditions of Knowledge: An Introduction to Epistemology and Education. Glenview: Scott, 
Foresman and Company.
BISHOP, K. CLEMENTS, C. KEITEL, J. KILPATRICK and C. LABORDE (eds.) International Handbook of 


Centre for Research and Development in Teacher Education
School of Education
The Open University
Walton Hall
Milton Keynes, MK7 6AA, UK
Tel.: ++ 44 (0) 1908 652438
Fax: ++ 44 (0) 1908 652218
E-mail: B.E.Pepin@open.ac.uk