Second-order iterative learning control for scaled setpoints

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Abstract—Iterative learning control (ILC) is a control technique for systems subject to repetitive setpoints or disturbances. However, in many applications, the setpoint is not strictly repetitive, and the learning process should start all over from the beginning if the setpoint changes. In this brief, point-to-point movements with different magnitudes will be considered, which are constructed by scaling a nominal setpoint. Second-order ILC with an adaptive low-pass filter in the trial domain is used to accurately track these scale varying setpoints under the influence of disturbances that are either repetitive or experience the same scaling as the setpoint. Experiments have been carried out to validate the proposed method.

Index Terms—Iterative learning control (ILC), motion control.

I. INTRODUCTION

In many manufacturing processes, production steps are carried out on repetitive structures. Examples of repetitive structures are given in Fig. 1. In many of these production steps, the tool is to be aligned with respect to a feature, perform its task, and move toward the next feature. Most conventional control approaches use a feedback controller for plant stabilization and disturbance rejection in combination with a feedforward controller according to a predefined structure (e.g., mass, damping, and Coulomb friction) to increase the performance. However, there are limitations using this approach. For example, the closed-loop bandwidth can be limited by system dynamics, whereas the fixed-structure feedforward may not be able to capture the disturbances [1], [2]. For tracking a predefined setpoint over and over again, a control technique called iterative learning control (ILC) [1] can be applied. The ILC reduces the tracking error along a trajectory that is traced repeatedly by the iterative refinement of a feedforward signal. Good surveys of recent ILC research can be found in [3]–[6].

One constraint within ILC is that the setpoint needs to be repetitive every trial. In practice, however, the distance between consecutive features may vary (e.g., due to temperature changes). Hence, the setpoint to be tracked is not strictly repetitive but varies due to these variations. Applying ILC for varying setpoints is one of the challenges in current ILC research. Several methods have been developed to use the knowledge from previous ILC trials to construct feedforward signals for new, different setpoints.

In [7] and [8], time-frequency adaptive ILC is proposed, where different setpoints are generated using a constant velocity phase and linear time-invariant. A schematic representation of the plant that is assumed to be discrete and linear time invariant. The authors are with the Department of Mechanical Engineering, Control Systems Technology Group, Eindhoven University of Technology, Eindhoven 5600 MB, The Netherlands (e-mail: j.j.t.h.best@gmail.com; lancheng.liu@gmail.com; m.j.g.v.d.molengraft@tue.nl; m.steinbuch@tue.nl).

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Fig. 1. Repetitive structures: OLED display and diodes on a wafer.

are learned from which a reference trajectory can be constructed. Direct learning control (DLC) [11] and recursive DLC [12] are developed to generate the control signal for a new setpoint using pre-stored setpoints and control signals. The methods presented in [9]–[12] need the converged feedforward signals learned from specific setpoints to construct the new control input for a different setpoint. In this brief, we will present a second-order ILC (SOLIC) algorithm in which scale varying setpoints are applied during the learning process and for which the tracking error will be reduced iteratively.

High-order ILC was studied in [13]–[20]. It is shown that high-order ILC is useful to increase the convergence speed [13], [16], [18], reject disturbances that satisfy an a priori relation from one trial to the next [19] and has robustness in the presence of external disturbances [18]. In this brief, SOLIC will be used to iteratively identify different classes of disturbances, which are used for updating the feedforward signal.

This brief focuses on accurate tracking of setpoints, which are constructed by scaling a nominal setpoint. ILC is used to update the feedforward signal while during iterations scale varying setpoints are applied. We analyze the convergence of the tracking error for situations with and without disturbances. It is assumed that these disturbances are repetitive every trial and/or experience the same scaling as the setpoint. A SOILC strategy will be used to identify these two types of disturbances and compensate for them during iterations. The contributions of this brief are: 1) the design of a SOILC strategy to accurately track scale varying setpoints in which during the learning process these scale varying setpoints are applied and 2) will be implemented on an industrial setup to show the effectiveness.

The rest of this brief is organized as follows. In Section II, the standard ILC method is extended by implementing scaling only, leading to normalized ILC (NILC). SOILC will be derived in Section IV. In Section V, results of experiments will be given, where the different methods will be compared. Section VI will present the conclusions.

II. STANDARD ILC AND NILC

In this section, we will briefly discuss the ILC working principle [21], [22]. To explain ILC, consider the block scheme given in Fig. 2 with a controller K and a plant G, both assumed to be discrete and linear time invariant. A schematic representation of the plant that
is considered in this brief is given in Fig. 3. For now, we take the gain $T_k = 1$. The time shift operator in Fig. 2 is denoted by $z$, i.e., $z^{-1}x(t) = x(t-1)$, where $t$ represents the sample number. The trial shift operator is denoted by $w$, i.e., $w^{-1}x_k(t) = x_{k-1}(t)$, where $k$ represents the trial number [19], [20]. The repetitive setpoint is given by $r$, whereas the output at trial $k$ is denoted by $y_k$. During trial $k$, the feedforward signal $f_k$ is applied and the error $e_k$ is measured. Offline, the error signal is filtered with the learning filter $L$ and added to the feedforward signal $f_k$. This filter is chosen as an approximation of the inverse of the process sensitivity $S_p$ and can be designed, for instance, using the zero-phase error tracking controller algorithm [23]. Next, the robustness filter $Q$ is applied, which results in the feedforward $f_{k+1}$ that is applied in the next trial $k+1$. The offline updating is mathematically written as

$$f_{k+1} = Q(f_k + Le_k).$$  

(1)

The tracking error $e$ in trial $k+1$ can be written as

$$e_{k+1} = Sr - S_p f_{k+1}$$  

(2)

where $S = 1/(1 + GK)$ is the sensitivity and $S_p = G/(1 + GK)$ is the process sensitivity. Substitution of the update law (1) into (2) leads to

$$e_{k+1} = Sr - QS_p(f_k + Le_k).$$  

(3)

Similar to (2), we use the fact that $S_p f_k = Sr - e_k$ and substitute this into (3) such that the error at trial $k+1$ becomes a function of the error in the previous trial $k$

$$e_{k+1} = Q(1 - LS_p)e_k + (1 - Q)Sr.$$  

(4)

The above system is called an linear iterative system for which convergence (under the assumption of infinite trajectory length) is obtained when [20]

$$\|Q(e^{j\omega})(1 - L(e^{j\omega})S_p(e^{j\omega}))\|_\infty < 1 \quad \forall \omega \in [-\pi, \pi]$$  

(5)

is satisfied.

As opposed to the repetitive setpoint considered in standard ILC, point-to-point setpoints with varying travel distances will be considered here since small variations are present in the distance between successive features. An example of a nominal setpoint is given in gray in Fig. 4. In general, two ways of setpoint generation for different magnitudes are: 1) include a constant velocity part [7] (dashed line in Fig. 4) and 2) scale the acceleration profile [11], [12] (black line in Fig. 4). In this brief, we handle setpoint variation using the second type and scale a nominal setpoint $r$ by a gain $T_k$, which results in a setpoint $r_k$ that is used in the trial $k$, i.e., $r_k = T_k r$. The value of $T_k$ is assumed to be bounded by $T_k \in [\underline{T}, \overline{T}]$, where $\underline{T}, \overline{T} \in \mathbb{R}^+$ are related to the pitch variation present in the repetitive structure. The scaling factor, $T_k$, can be determined a priori. The center of the camera is initially located above the center of a feature, whereas the next neighboring target feature is already in the field of view. From this, the distance between the features can be determined and $T_k$ can be determined. We scale the setpoint such that the time to reach each target is the same. Moreover, the switching times for the acceleration in this case remain the same, such that we can exploit the use of scaling.

As standard ILC can only cope with a repetitive setpoint, the error will not converge if during iterations these scale varying setpoints $r_k$ are applied. To handle scale varying setpoints, we proceed as follows. Standard ILC can be extended by incorporating the gain $T_k$ before and after the ILC update block as depicted in Fig. 2, which will be referred to as NILC in the remainder of this brief. The learning update uses the normalized error $e_k^* = T_k^{-1}e_k$ to construct a normalized feedforward signal $f_k^{*}$ as shown in the dashed area in Fig. 2, whereas the applied feedforward signal is given by $f_k = T_k f_k^{*}$. The same analysis done in (1)-(5) can be carried out to prove that the error is convergent. However, in case disturbances are present in the system, NILC is likely to fail due to the fact that disturbances will not scale in general.

III. EXISTENCE OF DISTURBANCES

The assumptions behind NILC are that the closed-loop system is LTI and that the error scales with the same gain as the setpoint, which in the presence of disturbances does not hold [24]. In this brief, three types of disturbances will be considered.
In this section, first the principle of SOILC will be explained. Then, it will be analyzed under which conditions the proposed SOILC approach is convergent and what the influence of sensor noise and applying similar setpoints is. Finally, improvements will be presented by adding an adaptive low-pass filter in the trial domain for SOILC.
A. Principle of SOILC

Consider the control scheme in Fig. 8 at this moment without sensor noise \( n_k = 0 \), where the goal is to design a feedforward signal \( f_{k+1} \) in such a way that the error \( e_{k+1} \) in trial \( k+1 \) is zero. Assume we have two trials \( k-1 \) and \( k \), for which the errors of these two trials can be written as

\[
e_{k-1} = \dot{T}_{k-1}g + h - S_p f_{k-1}
\]

\[
e_k = \dot{T}_k g + h - S_p f_k.
\]

After these two trials, the repetitive terms \( g \) and \( h \) can be estimated similar to (8)

\[
\dot{g} = \frac{e_k - e_{k-1} + S_p(f_{k-1} - f_k)}{T_{k-1} - T_k}
\]

\[
\dot{h} = \frac{T_k e_k - T_{k-1} e_{k-1} + S_p(T_{k-1} f_k - T_k f_{k-1})}{T_{k-1} - T_k}.
\]

Assume the feedforward signal for trial \( k+1 \) is \( f_{k+1} \), then the error for trial \( k+1 \) can be estimated as

\[
e_{k+1} = \dot{T}_{k+1}g + \dot{h} - S_p f_{k+1}
\]

\[=(1-\alpha)e_{k-1} + a e_k + S_p((1-\alpha)f_{k-1} + a f_k) - S_p f_{k+1}\]

with \( \alpha \) defined as

\[
\alpha = \frac{T_{k-1} - T_{k+1}}{T_{k-1} - T_k}, \quad T_{k-1} \neq T_k.
\]

Since we want to design a feedforward signal \( f_{k+1} \) in such a way that \( e_{k+1} = 0 \), from (14) we solve \( f_{k+1} \)

\[
f_{k+1} = (1-\alpha)f_{k-1} + a f_k + \frac{1}{S_p}((1-\alpha)e_{k-1} + a e_k).
\]

As in standard ILC, the inverse of the process sensitivity \( S_p \) is approximated by \( L \) and a robustness filter \( Q \) can be added (Section II), leading to the second-order update law

\[
f_{k+1} = Q((1-\alpha)f_{k-1} + a f_k + L((1-\alpha)e_{k-1} + a e_k)).
\]

Update law (17) can be used for the situation, where types 1 and 2 disturbances are present in the system. However, at this point, three questions remain to be answered. First, is the new linear iterative system convergent? Second, what happens in case \( T_k = T_{k-1} \)? Third, how does SOILC deal with type 3 disturbances?

B. Analysis of SOILC

In this section, the SOILC approach is analyzed with respect to the three previous mentioned questions.

1) Convergence: From (10) and (11), we obtain

\[
S_p f_{k-1} = \dot{T}_{k-1}g + h - e_{k-1}
\]

\[
S_p f_k = \dot{T}_k g + h - e_k.
\]

In (14), substitute \( f_{k+1} \) by (17) and together with (18) and (19), we obtain

\[
e_{k+1} = Q(1 - LS_p)(a e_k + (1 - \alpha)e_{k-1}) + (1 - Q)T_{k+1}g + h.
\]

To analyze the convergence of error, the system is constructed as a linear iterative system. From (20), it can be seen that \( e_{k+1} \) is related to \( e_k \) and \( e_{k-1} \). We define \( \Sigma_k = (e_k e_{k-1})^T \) and \( \mu_k = T_{k+1}g + h \) such that

\[\Sigma_{k+1} = A \Sigma_k + B \mu_k\]

with

\[A = \begin{pmatrix} a_1 & a_2 \\ 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 - Q \\ 0 \end{pmatrix}\]

\[(21)\]

where \( a_1 = Q(1 - LS_p)a \) and \( a_2 = Q(1 - LS_p)(1 - a) \). Convergence of this linear iterative system is assessed in the frequency domain using the work of [20]. Transforming the linear iterative system to the frequency domain leads to

\[\bar{\Sigma}_{k+1}(\omega) = A(\omega)\bar{\Sigma}_k(\omega) + B(\omega)\bar{\mu}_k(\omega)\]

\[(23)\]

where the signals \( \bar{\Sigma}, \bar{\mu} \) and \( \bar{\mu} \) are transformed to the frequency domain using

\[X(\omega) = \sum_{l=0}^{\infty} x(l)e^{-j\omega l}\]

\[(24)\]

Convergence is now obtained if

\[\rho = \max_{\omega \in [0, \pi]} \rho(A(\omega)) < 1\]

\[(25)\]

with \( \rho(A(\omega)) \) denoting the spectral radius of \( A(\omega) \) defined as

\[\rho(A(\omega)) = \max_{i=1,2} |\lambda_i(A(\omega))|\]

\[(26)\]

The eigenvalues of the matrix \( A(\omega) \) are given by

\[\lambda_{1,2}(\omega) = a_1(\omega) \pm \sqrt{a_1(\omega)^2 + 4a_2(\omega)}\]

\[(27)\]

Therefore, convergence of the linear iterative system (21) is guaranteed if the condition

\[\left|\frac{a_1(\omega) \pm \sqrt{a_1(\omega)^2 + 4a_2(\omega)}}{2}\right| < 1 \quad \forall \omega\]

\[(28)\]

is satisfied. Since the phase of \( L(\omega)S_p(\omega) \) is zero for all frequencies and \( Q(\omega) \) is a zero-phase low-pass filter, \( a_1(\omega) \) and \( a_2(\omega) \) are real-valued, \( \forall \omega \). It will be shown that there is a tradeoff between the designed \( Q \) and \( L \) filter and the maximum allowable values of \( a \), which guarantee convergence. In Fig. 9, the gray area indicates the allowable values of \( a \) for different values of \( \rho(Q(\omega)(1 - L(\omega)S_p(\omega))) \). From this figure, we have the following observations:

1) for convergence, \( \rho(Q(\omega)(1 - L(\omega)S_p(\omega))) \leq 1 \) is necessary;
2) if \( \rho(Q(\omega)(1 - L(\omega)S_p(\omega))) = 1 \), then \( 0 \leq a \leq 2 \);
3) however, from the definition of \( a \), it is possible that \( a \) is negative depending on the values of the gains;
4) the smaller the value of \( \rho(Q(\omega)(1 - L(\omega)S_p(\omega))) \), the larger the range of possible values of \( a \).
Note that the presented convergence analysis is very strict, since it is expected that the error in the next trial is always smaller than the current error, irrespective of the applied gain with which the setpoint is scaled. However, if the setpoint in the next trial is obtained by scaling with a larger gain than the current setpoint, the error in the next trial is also expected to be larger than the current error. As a result, it is harder to always obtain a smaller error in the next trial than the error in the current trial.

2) Perfect Pitch: If in the previous two trials, the gains $T_k$ and $T_{k-1}$ are the same, by definition, the value of $\alpha = \pm \infty$. Hence, if $\|Q(e^{i\omega})((1-L(e^{i\omega})S_p(e^{i\omega}))\|_{\infty} > 0$, then convergence can not be guaranteed. This is caused by the fact that we can not estimate $g$ and $h$ after two trials with the same setpoint. In fact, after having designed the learning filter $L$ and the low-pass filter $Q$, the quantity $\|Q(e^{i\omega})((1-L(e^{i\omega})S_p(e^{i\omega}))\|_{\infty}$ can be calculated. Using this result in combination with Fig. 9, the allowable values of $\alpha$ can be determined. Improvements regarding this issue will be discussed Section IV-C.

3) Type 3 Disturbances: If $T_{k-1}$ is close to $T_k$, then type 3 disturbances highly affect the estimations of $g$ and $h$. This is explained as follows. By considering sensor noise $n_k$ in Fig. 8, the errors $e_{k-1}$ and $e_k$ can be written as

$$e_{k-1} = T_{k-1}g + h - S_p f_{k-1} - S_n n_{k-1}$$
$$e_k = T_k g + h - S_p f_k - S_n n_k.$$  (29)

Substitution of (29) and (30) into (12) and (13) leads to

$$\hat{g} = g - S \frac{n_{k-1} - n_k}{T_{k-1} - T_k}, \quad \hat{h} = h - S \frac{T_k n_{k-1} - T_{k-1} n_k}{T_{k-1} - T_k}.$$  (31)

Therefore, if $T_k$ is close to $T_{k-1}$, there will be large estimation errors, since the noise terms are amplified. In this brief, we will deal with this sensor noise by iteratively estimating $g$ and $h$ such that these noise terms will not be amplified. This is done by introducing an adaptive low-pass filter in the trial domain on the estimates of $g$ and $h$.

C. Improving SOILC

In this section, we will improve the principle of SOILC with respect to: 1) sensor noise and 2) for cases in which $T_k = T_{k-1}$. By introducing an adaptive low-pass filter in the trial domain, SOILC is first made less sensitive to sensor noise. Incorporating sensor noise $n_k$ in the previous analysis leads to the update law

$$f_{k+1} = Q((1-\alpha)f_k + \alpha f_k + L((1-\alpha)e_{k-1} + \alpha e_k + S((1-\alpha)n_{k-1} - S_n n_k)).$$  (32)

If the previous two gains, $T_k$ and $T_{k-1}$ are close to each other, the absolute value of $\alpha$ can be large. Hence, the sensor noise is amplified by a large gain and becomes part of the next feedforward signal $f_{k+1}$, which is not desired and may cause a large error.

In the trial domain, the random type 3 disturbances, such as sensor noise, are changing from trial to trial, whereas the repetitive type 2 disturbances remain the same. Therefore, the sensor noise can be seen as a high-frequency signal in the trial domain, whereas the repetitive disturbances can be seen as a low-frequency signal in the trial domain [26]. This implies that we can use a low-pass filter in the trial domain to reject the sensor noise. We use SOILC with an adaptive low-pass filter in the trial domain to smooth out the sensor noise. In this way, the estimations of $g$ and $h$ are obtained iteratively and filtered and are then used in the generation of the new feedforward signal $f_{k+1}$. Define the terms $g$ and $h$ as the true values and $\hat{g}_k$ and $\hat{h}_k$ to represent the corresponding estimations after trial $k$

$$\hat{g}_k = \frac{e_{k-1} - e_k + S_p f_{k-1} - f_k}{T_{k-1} - T_k}.$$  (33)
$$\hat{h}_k = \frac{T_{k-1}e_{k-1} - T_k e_k - T_{k-1} f_{k-1} + S_p f_{k-1} - f_k}{T_{k-1} - T_k}.$$  (34)

The first-order adaptive low-pass filters in the trial domain are chosen as

$$\hat{g}_{k+1} = (1 - \gamma_k) \hat{g}_k + \gamma_k \tilde{g}_k$$  (35)
$$\hat{h}_{k+1} = (1 - \gamma_k) \hat{h}_k + \gamma_k \tilde{h}_k.$$  (36)

where $\tilde{g}_k$ and $\tilde{h}_k$ are the low-pass filtered outputs of the estimations, which are going to be used in the update law. The value of $\gamma_k$ can be tuned to give a weighting on how much the current estimates of $g$ and $h$ are used for the construction of the new feedforward signal. For stability of these low-pass filters, it is required that $0 \leq \gamma_k \leq 1$. To prevent sensor noise amplification when $T_k \approx T_{k-1}$, we choose $\gamma_k$ as $\gamma_k = \beta |T_{k-1} - T_k|$, where now the scalar $\beta$ should satisfy $0 \leq \beta \leq 1/\|T - T_k\|$. In such a way, we can cancel out the denominator $(T_{k-1} - T_k)$ in (31), so only $\beta S(n_{k-1} - n_k)$ and $\beta S(T_k n_{k-1} - T_{k-1} n_k)$ are considered in the estimations of $g$ and $h$. A tradeoff is present in this case between convergence speed and sensitivity to random disturbances, as is also discussed in [27]. A larger value of $\beta$ results in faster convergence but results in a system that is more sensitive to noise and vice versa.

In case $T_{k-1} = T_k$, the value of $\gamma_k$ becomes zero. As a result, the estimates $\hat{g}_{k+1}$ and $\hat{h}_{k+1}$ are not updated and equal the previous ones $\hat{g}_k$ and $\hat{h}_k$ such that no learning is performed. For the cases where $T_{k-1} \neq T_k$ and learning is to be performed in trial $k+1$ with $T_{k+1} = T_k$, two options are considered.

1) In case $T_{k-1} = T_k$, a standard ILC update could be applied for trial $k+1$, which updates the feedforward signal and decreases the next tracking error. The update law is in that case given by $f_{k+1} = Q(f_k + Lc_k)$. A disadvantage, however, is that the estimates $\hat{g}$ and $\hat{h}$ are not updated while standard ILC is applied. Whenever future gain values differ from $T_k$ such that SOILC can be applied again, the error might increase significantly, since old and possibly nonconverged values of $\tilde{g}_{k+1}$ and $\tilde{h}_{k+1}$ are used.

2) In order to keep learning and update $\tilde{g}_{k+1}$ and $\tilde{h}_{k+1}$ while $T_{k+1} = T_k$, we propose the following. Instead of using information of the previous two trials $k$ and $k-1$ to update $\tilde{g}_k$ and $\tilde{h}_k$ in SOILC, we use the information of the previous trial $k$ and trial $k - p$, where $p \geq 1$ is the smallest number for which $T_{k-p} \neq T_k$ to update $\hat{g}$ and $\hat{h}$. Therefore, the update can be written as

$$\tilde{g}_{k+1} = (1 - \gamma_k) \tilde{g}_k + \gamma_k \tilde{g}_k$$  (37)
$$\tilde{h}_{k+1} = (1 - \gamma_k) \tilde{h}_k + \gamma_k \tilde{h}_k.$$  (38)

with now $\gamma_k = |T_{k-p} - T_k|$ and

$$\tilde{g}_k = \tilde{g}_{k+1} - e_k - S_p f_{k+1} - f_k$$  (39)
$$\tilde{h}_k = \tilde{h}_{k+1} - e_k - S_p f_{k+1} - f_k$$  (40)

As an example, consider the gains depicted in Fig. 10. The gains are different for each iteration except for iteration five to eight, where the gains are the same. Hence, for $k = 6, 7$, and $8$ the value $T_{k-1} = T_k = 0$. Therefore for iteration $k = 6, 7$ and $8$, the values of $p$ in (39) and (40) are $2, 3$, and $4$, respectively. For iteration $k = 2, 3, 4, 5, 9, 10$, the value $T_{k-1} - T_k \neq 0$. Therefore, the value of $p$ in
the updated estimations still use a zero-phase low-pass filter $Q$ to filter out the high-frequency components in the measured error, we derive the new update law

$$
\tilde{g}_{k+1} = (1 - \beta |T_{k-1} - T_k|)\tilde{g}_k + \beta \operatorname{sgn}(T_{k-1} - T_k)\times(\tilde{e}_k - e_k + T_{k-1}\tilde{g}_{k-1} + \tilde{h}_{k-1} - T_k\tilde{g}_k - \tilde{h}_k).
$$

Similarly

$$
\tilde{h}_{k+1} = (1 - \beta |T_{k-1} - T_k|)\tilde{h}_k + \beta \operatorname{sgn}(T_{k-1} - T_k)(T_{k-1}e_k - T_k\tilde{e}_k + T_{k-1}\tilde{g}_{k-1} + \tilde{h}_{k-1} - T_k\tilde{g}_k - \tilde{h}_k)).
$$

To filter out the high-frequency components in the measured error, we still use a zero-phase low-pass filter $Q$ as the robustness filter after the updated estimations $\tilde{g}_{k+1}$ and $\tilde{h}_{k+1}$. Therefore, the first-order low-pass filters in the trial domain (35) and (36) now also include the low-pass filtering in frequency domain, i.e., the new filters now are

$$
\begin{align*}
\hat{g}_{k+1} &= Q((1 - \gamma_k)\hat{g}_k + \gamma_k\tilde{g}_k) \\
\hat{h}_{k+1} &= Q((1 - \gamma_k)\hat{h}_k + \gamma_k\tilde{h}_k).
\end{align*}
$$

V. RESULTS

In this section, the performance of standard ILC, NILC, SOILC, and SOILC with an adaptive low-pass filter in the trial domain for scale varying setpoints will be compared. The proposed methods are validated on a xy-wafer stage, where the task is to move from one feature to the next (Fig. 5). The frequency response function (FRF) from the input of the motor to the position output is given in Fig. 11. The plant is modeled by a mass-damper system with delay. The obtained model given by

$$
G = 1 \times 10^{-4} \frac{1 \times 10^{-3} z^{-1} + 1.34z^{-1} + 5.14z^{-1} + 1.23}{z^{-1} + 0.85z^{-1}}
$$

is also shown in Fig. 11 by its FRF and shows a good match until $\sim 60$ Hz. As a consequence, a mismatch between the measured process sensitivity and its model is expected after 60 Hz. Therefore, the $Q$ filter gets a cutoff frequency of 50 Hz. A controller $K$ is tuned which consists of a lead filter with a zero at 6 Hz and a pole at 100 Hz and a second-order low-pass filter with a cutoff frequency of 250 Hz and a damping of 0.6. Finally, a notch is added at 80 Hz. The discrete controller is given by

$$
K = 1 \times 10^4 \frac{3.3z^{-5} - 2.3z^{-4} - 7.8z^{-3} + 11z^{-2} - 3.5z^{-1} - 0.82}{z^{-5} + 2.4z^{-4} + 2.4z^{-3} + 1.2z^{-2} + 0.38z^{-1} + 0.070}.
$$

The number of iterations that will be performed is 30. For the sake of comparison, the arbitrary gains are chosen the same for the four methods. The bounds are given by $T = 0.75$ and $T = 1.25$. Fig. 12 shows the applied gains. The proposed methods are applied on the xy-wafer stage with the value of $\beta$ chosen as 1 in this case, such that $0 \leq \beta \leq 1/T = 2$ is satisfied. The maximum errors for each iteration are given in Fig. 13. Since standard ILC does not incorporate the scaling of the setpoint, it is expected that the final error oscillates depending on the applied gains, which also shows in Fig. 13. NILC does incorporate the scaling of the setpoint, however, it is based on the absence of type 2 disturbances. Dry friction is one of the major disturbances present in the experimental setup. It can be seen that if two successive gains are quite different the error of NILC increases, which is caused by the inappropriate scaling of the error. To a priori, determine the performance for standard ILC versus NILC for the case given in Fig. 8 consider the standard ILC case where: 1) a fixed disturbance $d$ is present and 2) a setpoint $r$ is applied with (for now) a fixed gain, i.e., $T_1$. The error of each iteration is given by $e_k = T_1Sr - S_Pd - S_Pf_k$. After convergence...
depends on the plant.

In both cases, non-zero errors appear. Which one is the least directly

\[ \alpha \]

compare the performance between standard ILC and NILC.

If the frequency content of

\[ G \]

are obtained using SOILC with an adaptive low-pass filter in the trial

domain. After 11 iterations, the error is converged to maximum errors

are zero as well. As opposed to standard ILC and NILC, the SOILC

solution in case

\[ T_k \]

would be applied to the setpoint then for standard ILC together

with the converged feedforward signal

\[ f_{\infty} \]

what is captured within the signal

\[ d \]

should be known. The proposed solution in case

\[ T_{k-1} = T_k \]

in SOILC with adaptive low-pass filtering in the trial domain is investigated next. During the iterations, the applied gains in this case are the same as in Fig. 12, except that the gains of iterations 11 through 20 are kept the same in this case and equal to the gain of iteration 11. Furthermore, the value of \( \beta \) is taken as 0.5, such that learning is slower. If \( \beta = 1 \), we saw in the previous results that the error and therefore also the estimates \( \hat{g} \) and \( \hat{h} \) already converged within 11 iterations. The effect that we want to visualize here is that learning is still present even when the gain values of two successive iterations are the same. The results are given in Fig. 14. The gray hatched area indicates that the gains are the same for these iterations. A first observation is that the error converges slower. This was expected due to the lower value of \( \beta \). Second, the error converges even when the gains of iterations 11 through 20 are the same. Furthermore, after iteration 20, when the gains deviate again, learning is still present as can be seen by the further reduction of the error. This result shows that the proposed solution in the case where two successive gains are the same is effective.

VI. CONCLUSION

Three methods, NILC, SOILC, and SOILC with an adaptive low-pass filter in the trial domain have been investigated to handle scale varying setpoints in ILC. Experiments are carried out to validate these methods. NILC achieves a good performance when there is no disturbance at all. SOILC is sensitive to nonrepetitive noise when the previous applied setpoints are almost the same. SOILC with an adaptive low-pass filter in the trial domain can handle the situation when both repetitive disturbances and nonrepetitive noise exist and achieves a good performance. After convergence, the error is reduced to \(< 5 \mu m\). The investigated methods consider disturbances that experience the same scaling as the setpoint, and trial-independent repetitive disturbances. Another class of disturbances are position-dependent disturbances, e.g., cogging. This kind of disturbances cannot be handled in this brief, since scaling cannot be applied. This will, therefore, be subject for future research.

REFERENCES


