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Citation for published version (APA):

Silitonga, S., Maljaars, J., Soetens, F., & Snijder, H. H. (2014). Numerical simulation of fatigue crack growth rate and crack retardation due to an overload using a cohesive zone model. In G. Clark, & C. H. Wang (Eds.), *11th International Fatigue Congress, 2-7 March 2014*, (pp. 777-783). (Advanced Materials Research; Vol. 891-892). Trans Tech Publications. <https://doi.org/10.4028/www.scientific.net/AMR.891-892.777>

DOI:

[10.4028/www.scientific.net/AMR.891-892.777](https://doi.org/10.4028/www.scientific.net/AMR.891-892.777)

Document status and date:

Published: 01/01/2014

Document Version:

Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
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Numerical simulation of fatigue crack growth rate and crack retardation due to an overload using a cohesive zone model

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Keywords: fatigue crack growth rate, damage mechanics, fatigue crack propagation, cohesive zone model, crack retardation.

Abstract. In this work, a numerical method is pursued based on a cohesive zone model (CZM). The method is aimed at simulating fatigue crack growth as well as crack growth retardation due to an overload. In this cohesive zone model, the degradation of the material strength is represented by a variation of the cohesive traction with respect to separation of the cohesive surfaces.

Simulation of crack propagation under cyclic loads is implemented by introducing a damage mechanism into the cohesive zone. Crack propagation is represented in the process zone (cohesive zone in front of crack-tip) by deterioration of the cohesive strength due to damage development in the cohesive element. Damage accumulation during loading is based on the displacements in the cohesive zone.

A finite element model of a compact tension (CT) specimen subjected to a constant amplitude loading with an overload is developed. The cohesive elements are placed in front of the crack-tip along a pre-defined crack path. The simulation is performed in the finite element code Abaqus. The cohesive elements behavior is described using the user element subroutine UEL. The new damage evolution function used in this work provides a good agreement between simulation results and experimental data.

Introduction

Prediction of the crack propagation period is crucial in the assessment of existing structures. Such knowledge is required so that appropriate repair or replacement of the structural components can be timely conducted before any failure occurs. The crack propagation in ductile metals mainly occurs through the mechanism of plastic blunting (accompanied by crystallographic slip) and sharpening of the crack-tip that leads to the formation of striations.

A very important engineering method used today to predict fatigue crack growth is based on the work of Paris [1]. The method is developed in the framework of linear elastic fracture mechanics (LEFM) where the far-field stresses are relatively small so that plasticity is limited to a small region near the crack tip. Under this condition, the majority of the material remains in the elastic range. However, fatigue crack propagation in metals is directly related to crack tip plasticity where irreversible plastic flow induces damage accumulation at the crack tip in each cycle. A number of modifications on the Paris law have been proposed over the years to also account for large-scale yielding [2], crack retardation [3] and plasticity-induced crack closure [4]. Despite the extensive use of these models, the essential physical background of fatigue crack growth is not completely described by the theory.

A cohesive zone model is an alternative approach to simulate fatigue crack propagation by means of a finite element simulation. The cohesive zone model (CZM), firstly introduced by Dugdale [5] and Barenblatt [6], regards fracture as a gradual phenomenon in which separation takes place between two adjacent virtual surfaces across an extended crack tip (cohesive zone) and is resisted by the presence of cohesive forces. In this paper, a damage-based cohesive zone model is used to model fatigue

crack growth with a damage mechanism based on energy dissipation. Damage accumulation due to loading deteriorates the cohesive element stiffness. The crack extension is defined by fully diminished cohesive strength.

This paper is organized in the following manner. *Cohesive zone for fatigue crack growth* section focuses on the formulation of the proposed cohesive zone model. A brief overview of the basic mechanics of the model is presented. The damage evolution function used in this work is also given in this section. The finite element simulation results of the model are given in *Results* section. The final section consists of the remarks on the model simulation and conclusion of the work.

Cohesive Zone for Fatigue Crack Growth

Cohesive Zone Approach. A cohesive zone is placed in front of the physical crack tip (Fig.1a) at a predefined crack path. The cohesive traction is related to the separation of cohesive surfaces by a cohesive law which determines the constitutive behavior of the cohesive zone model. Upon the application of external loads to a cracked body (Fig.1b) shows Traction-Separation Laws (TSL) in normal direction), the cohesive surfaces separate gradually leading to an increase in traction (T_n) until a maximum value ($\sigma_{max,0}$) is reached (cohesive strength). The traction decreases to approximately zero as the separation (Δ_n) reaches a critical value (δ_{sep}). The area under the TSL is known as the cohesive energy.

Fig. 1: (a) Cohesive zone approach to crack tip [7] (b) Traction-Separation law

Cohesive Zone Formulation. Using the principle of virtual work, the mechanical equilibrium considering the effect of the cohesive tractions is written as

$$\int_V \boldsymbol{\sigma} : \delta \boldsymbol{\epsilon} dV - \int_{S_{int}} \mathbf{T}_{CZ} \cdot \delta \Delta dS = \int_{S_{ext}} \mathbf{T}_e \cdot \delta \mathbf{u} dS \quad (1)$$

where V is the specimens volume, S_{int} is the internal/cohesive surface and S_{ext} is the external surface, $\boldsymbol{\sigma}$ is the Cauchy stress tensor, $\boldsymbol{\epsilon}$ is the strain tensor, $\mathbf{u}$ is the displacement vector, $\mathbf{T}_{CZ}$ denotes the cohesive traction vector and $\mathbf{T}_e$ is the external traction vector and Δ is a vector representing the separation displacement across the two adjacent cohesive surfaces. The cohesive tractions consist of normal and tangential components, $\mathbf{T}_{CZ} = T_n \mathbf{n} + T_t \mathbf{t}$. The $\mathbf{n}$ and $\mathbf{t}$ are the unit vectors normal and tangent to the surface, respectively. The separation displacement vector, $\Delta = \Delta_n \mathbf{n} + \Delta_t \mathbf{t}$, is calculated

from the displacements (u^{top} and u^{bot} in Fig.1b) of the opposing cohesive surfaces. The Δ_n and Δ_t are the normal and tangential separation displacements, respectively. The cohesive tractions used in this work are based on Needleman [8] given as follows,

$$\begin{aligned} T_n &= \sigma_{max,0} e \exp\left(-\frac{\Delta_n}{\delta_0}\right) \left\{ \frac{\Delta_n}{\delta_0} \exp\left(-\frac{\Delta_t^2}{\delta_0^2}\right) + (1-q) \frac{\Delta_n}{\delta_0} \left[1 - \exp\left(-\frac{\Delta_t^2}{\delta_0^2}\right)\right] \right\} \\ T_t &= 2\sigma_{max,0} e q \frac{\Delta_t}{\delta_0} \left(1 + \frac{\Delta_n}{\delta_0}\right) \exp\left(-\frac{\Delta_n}{\delta_0}\right) \exp\left(-\frac{\Delta_t^2}{\delta_0^2}\right) \end{aligned} \quad (2)$$

where δ_0 is the amount of material separation required to reach the cohesive strength in normal loading (also called the characteristic length), $e = \exp(1)$, q is the coupling representation between normal and shear tractions i.e. the ratio between the area under the functions of pure tangential and pure normal traction. In this work, the cohesive strengths for normal and tangential directions are equal, which implies $q = 0.429$.

Degradation of the material integrity ahead of the crack tip under cyclic loading is represented in the cohesive zone through deterioration of the cohesive strength. The amount of material degradation can be quantitatively represented by a damage variable ($0 \leq D \leq 1$), where the value of zero represents intact material and a value of one represents complete failure of the cohesive element. A value in between results in a reduced cohesive stiffness.

The cohesive traction behavior depends on the current state of damage as well as the current separations which leads to an irreversible and history dependent traction-separation equation. Using the effective stress concept [9], the damage variable is incorporated into the TSL of Eq. 2 by replacing the initial cohesive strength ($\sigma_{max,0}$) with the current cohesive strength given as [10],

$$\sigma_{max} = \sigma_{max,0}(1 - D) \quad (3)$$

In order to properly describe fatigue crack propagation under cyclic loading, the unloading/reloading paths also need to be considered. It is assumed that the unloading of the normal separation occurs to the origin of the traction-separation function as illustrated in Fig.2. For unloading/reloading path in tangential direction, a residual separation is introduced to take into account the interaction between the crack surfaces [11], though, it is considerably small in mode I problem. The paths in normal and tangential loading direction are given as [11],

$$\begin{aligned} T_n &= T_{n,max} + \left(\frac{T_{n,max}}{\Delta_{n,max}}\right) (\Delta_n - \Delta_{n,max}) \\ T_t &= T_{t,max} + \sqrt{2e} \left(\frac{\sigma_{max}}{\delta_0}\right) (\Delta_t - \Delta_{t,max}) \end{aligned} \quad (4)$$

where $\Delta_{n,max}$ is the maximum value of normal separation before unloading and $T_{n,max}$ is the corresponding normal traction (Eq. 2) with $\sigma_{max,0}$ replaced by σ_{max} . An identical definition is also applied to the tangential direction.

Damage Mechanism. In this work, a modified damage formulation based on Roe and Siegmund [10] is used. The damage evolution function is related to the displacement and the dissipated cohesive energy in the cohesive zone. The function is written as,

$$\dot{D} = \gamma \frac{|\dot{\bar{\Delta}}|}{\delta_{sep}} \left(\frac{\Gamma_i}{\Gamma_0}\right) H(\bar{\Delta} - \kappa \bar{\Delta}_{max}) \quad (5)$$

where γ and κ are material parameters. The resultant $\bar{\Delta}$ and its increment are expressed as,

$$\bar{\Delta} = \sqrt{\Delta_n^2 + \Delta_t^2}, \quad \dot{\bar{\Delta}} = \bar{\Delta}_i - \bar{\Delta}_{i-1} \quad (6)$$

where $\bar{\Delta}_i$ and $\bar{\Delta}_{i-1}$ are the $\bar{\Delta}$ at current and previous increment, respectively. The parameter $\bar{\Delta}_{max}$ is the maximum resultant separation at the material crack tip. The dissipated cohesive energy (Γ_i) is described in Fig.3 . In this work, due to considerably small dissipated cohesive energy in tangential direction for mode I problem, only normal dissipated cohesive energy is considered. The total available cohesive energy (Γ_0) is equal to the area under the TSL curve in normal direction.

Fig. 2: Schematic representation of the unloading/reloading paths in normal direction during cyclic loading

Fig. 3: Definition of dissipated cohesive energy in each cycle during loading

Results

Specimen and loading. The finite element implementation of the model described above is conducted on a standard Compact Tension (CT) specimen made of aluminum alloy AL7075-T651. The details on its dimensions, mechanical properties and geometry are given in Table 1, Table 2 and Fig.4, respectively. In this work, the applied maximum force is equal to $P_{max} = 3$ kN with force ratio ($R = P_{min}/P_{max}$) equal to 0.1.

Table 1: Specimen dimensions

H	W	B	r	a
[mm]	[mm]	[mm]	[mm]	[mm]
60.96	50.8	5	6.35	13.25

Table 2: Specimen mechanical properties

E	ν	σ_y	K_c
[MPa]	[-]	[MPa]	[MPa $\sqrt{m}$]
71200	0.33	420.3	45.7

Fig. 4: CT specimens details

Cohesive and damage parameters. The cohesive energy is equal to the fracture energy of the material ($\Gamma_0 = G_c = K_c^2/E$). The cohesive strength ($\sigma_{max,0}$) is set to be equal to $E/100$. The characteristic length is given as, $\delta_0 = \Gamma_0 / (\sigma_{max,0}e)$. The maximum separation, δ_{sep} , is approximately equal to $8\delta_0$.

The values of γ and κ are, at this moment, chosen such that the model simulation of crack growth rate fits the experimental results. The complete set of the cohesive parameters is given in Table 3.

Table 3: Cohesive law parameters

$\sigma_{max,0}$ [MPa]	δ_0 [μm]	δ_{sep} [μm]	γ [-]	κ [-]
712	14.2	113.6	11	0.75

Finite element simulation. The CT specimen is modeled in Abaqus with plane stress elements CPS8, which are elements with 8-node and 3x3 integration points. The constitutive relationship attributed to these elements is a standard elastic-plastic stress-strain relationship. The active plastic zone and the plastic wake shown in Fig.1a are described in the model through the continuum elements which surround the cohesive elements.

In front of the initial crack tip, 150 cohesive elements are placed, each having a length of 0.05 mm. The cohesive zone model described in the second section has been implemented through the user element subroutine UEL. Simulations are conducted to produce a crack extension of 7.25 mm. Due to geometrical symmetry of the CT, only half of the geometry is simulated.

The results of the simulation are compared to the crack growth rate data (da/dN) obtained from [12] in Figure 7. The crack growth behavior due to overload is also shown in Fig.6. The results of the model simulation are in a good agreement with experimental results. Note that the rate of the initial growth is tuned using parameters γ and κ , but the crack growth rate development as the crack progresses, and the overload effect, are a direct result of the model without additional tuning. The crack retardation in this model is caused by a reduced separation in cohesive zone due to residual compression stress produced in the continuum elements around the crack tip after the overload. The reduced separation leads to a smaller damage increment (Eq. 5) which in turn results in a larger number of cycles to reach the critical damage ($D = 1$). The reduced separation after the overload, at an integration point in a cohesive element in front of the material crack tip, is shown in Fig. 5.

Fig. 5: Reduced separation in a cohesive element after a 100% overload

Conclusions

A damage-based cohesive zone is used to describe crack extension due to cyclic loading by decreasing of the cohesive strength with the number of cycles. The dissipative mechanisms through the deterioration of the cohesive strength during unloading-reloading can be used to phenomenologically represent the accumulated damage that leads to complete decohesion and crack extension in materials. The model introduces a damage mechanism as a function of the separation resultant and the dissipated cohesive energy in the cohesive zone.

The developed theoretical cohesive zone model is used to simulate fatigue crack growth of a CT specimen using Abaqus through the user element subroutine UEL. The model is used to simulate crack retardation due to an overload.

Fig. 6: Crack propagation behavior with 100%OL

Fig. 7: Crack growth rate behavior with 100%OL

Acknowledgments

This research was carried out under project number MC1.1.09323 in the framework of the Research Program of the Materials innovation institute M2i (www.m2i.nl)

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