Passenger oriented disruption management in railways: by adapting timetables and rolling stock schedules

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Passenger Oriented Railway Disruption Management
By Adapting Timetables and Rolling Stock Schedules

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Abstract In passenger railway operations, unforeseen events require railway operators to adjust their timetable and their resource schedules. The passengers will also adapt their routes to their destinations. When determining the new timetable and rolling stock schedule, the railway operator has to take passenger behavior into account. The capacity of trains for which the operator expects more demand than on a regular day should increase. Furthermore, at locations with additional demand, the frequencies of trains serving that station could be increased.

This paper describes a real-time disruption management approach which integrates the rescheduling of the rolling stock and the timetable by taking the changed passenger demand into account. The timetable decisions are limited to additional stops of trains at stations at which they normally would not call. Several variants of the approach are suggested, with the difference in how to determine which additional stops should be executed.

Real-time rescheduling requires fast solutions. Therefore a heuristic approach is used. We demonstrate the performance of the several variants of our algorithm on realistic instances of Netherlands Railways, the major railway operator in the Netherlands.

Keywords Railways · Disruption Management · Rolling Stock · Timetable

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1 Introduction

In passenger railway operations, unforeseen events (such as infrastructure malfunctions, accidents or rolling stock breakdowns) can make parts of the railway infrastructure temporarily unavailable. Then it is not possible to operate the timetable, rolling stock schedule and crew schedule as planned. Within minutes, or even better, seconds, a new timetable and new resource (rolling stock and crew) schedules must be available. In Cacchiani et al. [2] an overview is given of recovering models and algorithms to solve these rescheduling steps. In this overview it becomes clear that, although the schedules are interdependent, most research focuses on rescheduling one of the schedules at a time. By the complexity of the rescheduling problems, there is not enough time to solve the integrated problem. In this paper we partly integrate the rescheduling of the rolling stock plan and the timetable. Our particular focus lies on passenger service, and we take passenger behavior explicitly into account.

We propose a fast passenger oriented rolling stock rescheduling approach which allows to slightly adapt the timetable. Current literature on integrated rescheduling of the timetable and the rolling stock schedule is scarce. Adenso-Díaz et al. [1] and Cadarso et al. [3] did research on integrated rescheduling of the timetable and rolling stock on cases of the Spanish railway operator RENFE. Like the main focus of our paper, Cadarso et al. [3] take the dynamics of the passenger behavior into account. However, the fundamentals of the approach in the current paper are from Kroon et al. [8]. In Kroon et al. [8] the focus is on improving passenger service by considering passenger behavior while rescheduling the rolling stock. Kroon et al. [8] use an iterative procedure for rescheduling the rolling stock and evaluating the resulting passenger behavior which is inspired by the iterative framework of Dumas and Soumis [5]. Changing the timetable can also improve the passenger service. Therefore we extend the approach of Kroon et al. [8] by allowing the timetable to be slightly adapted as well.

It is important to focus on the passenger service since a disruption does not only affect the timetable and the resource schedules, but also the passengers. However, for railway operators without a seat reservation system it is difficult to reschedule the passengers. The passengers will make their new travel plan by themselves. If they had planned to take a train which is canceled due to the disruption, they will decide not to travel or to reroute themselves. Rerouting of passengers means that they take other trains to their destination than originally planned. This does not necessarily require the passengers to take a detour: They can also take a later train on the same line.

By the changed passenger flows, the disruption causes changes in the demand for seats. Therefore, a rolling stock rescheduling approach to handle a disruption must take the modified passenger flows into account dynamically, and not the passenger flows of a regular day. For example, since some passengers will take a detour, additional capacity on the detour routes is necessary. One way to handle this is to increase the capacity of the trains on this route.
Another option is to increase the capacity by inserting more train services or by letting trains make additional stops.

The consequences of the timetable adaptations may not turn out to be advantageous for all passengers. For example, an additional stop of a train will delay the train with a few minutes. As a consequence, the original passengers of the train will get an additional delay in favor of reducing the delay of the passengers at the station at which an additional stop is made. The small delay of the train can even lead to a large delay for the passengers if they miss their transfer at a later station. The railway operator has to make trade-offs between the different consequences for the passengers.

In this research we limit the timetable decisions to adapting the stopping patterns. Other timetable decisions to influence the passenger flows, for example by inserting additional trains, are left out since an additional train requires the railway operator also to adapt the crew schedules. By adapting the stopping patterns only, the crew schedule remains feasible.

Delay management, which consist of deciding on whether or not trains have to wait for delayed connecting trains, is another problem in which slight timetable adaptations influence the passenger flows. Delay management is a hard problem on its own and thereby not considered in our approach. We refer the interested reader to Schachtebeck and Schöbel [10], Kanai et al. [7] and Dollevoet et al. [4] for recent work on delay management approaches.

The framework of our rolling stock rescheduling approach is discussed in Section 2. In Section 3 we show a relaxation of the model discussed in Section 2. Instead of solving the relaxation, we make use of an iterative procedure of which in Section 4 up to Section 7 the components are explained. Results of different variants of our approach, based on a scenario in the Netherlands are discussed in Section 8, and Section 9 concludes this paper.

2 Rolling stock and timetable rescheduling with dynamic passenger flows

The performance of the disruption management process investigated in this paper arises from the interaction of 3 factors: (i) the timetable, (ii) the rolling stock schedule (seat capacity), and (iii) the passenger behavior.

We consider disruptions where passenger behavior has a large impact on the performance of the railway system if the timetable and rolling stock schedule are not changed. Examples are disruptions where certain tracks are blocked for a number of hours. Passengers react to these disruptions by finding alternative routes to their destinations. However, the capacity on these alternative routes can be limited, resulting in overcrowded trains and thereby longer dwell times and delays.

Two ways to handle the increased demand on the alternative routes are to enlarge the capacity of the trains and to adapt the timetable. Adapting the capacity of the trains alone is not always enough. For example, it can be impossible to increase the capacity of a train by lack of time and/or reserve
rolling stock or due to limited platform lengths. Therefore, timetable adap-
tations such as adding extra train services, rerouting trains or adding extra
stops for trains are worthwhile as well.

By adapting the timetable, the railway operator can influence the passenger
flows by providing new alternative travel routes and by influencing the demand
for certain trains. For example, a train can make an additional stop at a station
to give passengers at that station an additional, earlier, travel option to their
destination and to decrease the demand for the next train calling at that
station and travelling in the same direction.

In this research we limit the timetable adaptations to adaptations of the
stopping patterns of trains. A stopping pattern of a train indicates the stations
where the train makes a stop. A stopping pattern can contain, next to the
scheduled stops, also new stops at stations where the train did not have a
scheduled call.

To make a new rolling stock schedule and timetable for the remainder of
the day, we assume that the complete characteristics of the disruption are
revealed at the moment the disruption starts. For example, at that time, the
exact duration of the disruption is known.

Then, a general framework for rescheduling the rolling stock and timetable
by considering the passenger behavior based on the model of Kroon et al. [8],
with the difference that now also decision variables for the timetable decisions
are included, can be stated as follows:

\[
\begin{align*}
\min & \ c(x) + d(y) + e(z) \\
\text{subject to} & \ z \in Z \\
x & \in X_z \\
y & = f(x, z) \in Y
\end{align*}
\] (1)

(2)

(3)

(4)

Here Z is the set of all possible timetables given the disruption, X_z is the set of
all possible rolling stock schedules matching with timetable z, and Y is the set
of feasible passenger flows. The function f(x, z) returns the emerging passenger
flow for a given timetable z \in Z and rolling stock schedule x \in X_z. Note that
the chosen timetable z and rolling stock schedule x uniquely determine the
passenger flow y by the function f. This means that the only real decision
variables are the rolling stock schedule x and the timetable z.

The objective function consists of three terms. The function c(x) gives the
system related costs of a rolling stock schedule, which can also be seen as the
rolling stock rescheduling costs. The function d(y) gives the service related
costs of a passenger flow. The function e(z) that gives the system related costs
of a timetable, so the timetable rescheduling costs. The highest priority is
given to assigning at least one rolling stock unit to each train, to prevent the
train to be cancelled by lack of rolling stock. Such cancellations will not only
have a large negative influence on the passenger flows, but also make the crew
schedule infeasible.
2.1 Iterative Procedure

The optimization model (1)-(4) is very difficult to solve directly, mainly due to the complex structure of the objective function $f$. We are not aware of any algorithmic framework that would be able to handle realistic instances of (1)-(4). Therefore we propose an extension to the iterative heuristic of Dumas and Soumis [5] and Kroon et al. [8]; the approach is sketched in Figure 1.

Fig. 1 Iterative procedure for solving the rolling stock rescheduling problem with dynamic passenger flows.

In each iteration, we evaluate the passenger flows by using a simulation algorithm. The simulation is based on the previous iteration’s timetable and rolling stock schedule. Here the rolling stock schedule is only needed because it determines the capacities of the trains. We use the simulation model introduced by Kroon et al. [8]. The details of this simulation model are summarized in Section 4. Note that the first iteration uses the initially modified timetable, and assumes that each train has the same capacity as in the original schedule.

The passenger simulation pinpoints the trains with insufficient capacities. The rolling stock rescheduling model computes a new schedule based on these findings, balancing it with other criteria, such as operational costs. For details we refer to Section 5.

After another round of passenger simulation, we evaluate which adaptations of the timetable could potentially improve the service quality. Each individual adaptation is a minor change, such as requiring a train to make an extra stop. Therefore we can assume that the just computed rolling stock schedule remains feasible. We describe several variants for finding the most promising timetable adaptation in Section 6. Having decided on the timetable, the next iteration will start by launching a passenger simulation.

Our method differs from the framework of Kroon et al. [8] by adding the timetable adaptation step to the loop. Since the passenger flows can be heavily...
impacted both by a new rolling stock schedule and by an adapted timetable, we carry out passenger simulations after each of them.

The iterative approach is purely heuristic; it does not necessarily converge, and has no optimality guarantee. Motivated by the limited time in real-life applications, we terminate our algorithm after a certain number of iterations, and we report the best solution found. In addition, we compute lower bounds, described in Section 7, in order to be able to judge the quality of the solution.

3 Operator control

The rescheduling process may result in a better outcome if the operator can directly influence the passengers’ behavior by appropriately assigning them to the train services (rather than letting the passengers choose their routes). We call this situation operator control. In this section we describe an optimization model for operator control which is a relaxation of the model (1)-(4). We are not going to use this model in our computational tests, since it is a computationally large model and we do not want to assume operator control. However, we still want to present this relaxation of the model to give an idea about its complexity, and thereby justifying our use of an iterative procedure.

We split all timetable services into trips $t \in T$ representing a movement of a train between two consecutive planned stops. The main decision for the rolling stock schedule is to assign compositions to trips, where a composition consists of one or more combined train units. Let $G_t$ be the set of all compositions $g$ which can be assigned to trip $t$, and the capacity of composition $g$ is denoted by $Cap_g$. Binary variables $x_{t,g}$ indicate whether composition $g$ is used ($x_{t,g} = 1$) for trip $t$ or not ($x_{t,g} = 0$).

For the timetable decisions every trip $t \in T$ has a set $J_t$ of possible stopping patterns for calls at the intermediate stations. Here a stopping pattern indicates a sequence of intermediate stations at which the train makes an additional stop. Binary variables $z_j$ indicate whether stopping pattern $j$ is used ($z_j = 1$) or not ($z_j = 0$).

A passenger $p \in P$ should take a path from its origin to its destination within his/her proposed deadline, where a path itself is a sequence of rides on trains between two stations. Let $K^p$ be the set of all paths that passenger $p \in P$ could take and let $K^p_t \subset K^p$ be all paths which passenger $p$ could take with (part of) trip $t$ in it. Note that the paths in $K^p$ and $K^p_t$ can be based on every possible stopping pattern. Let $J_k$ be the set of all stopping patterns $j$ matching with path $k$. The binary variable $y_{kp}^j$ is 1 if passenger $p$ picks path $k$ and 0 otherwise. The parameter $d_{kp}^j$ indicates the associated cost (delay) of passenger $p$ taking path $k$. 


Then, in case of operator control, the model of (1)-(4) can be relaxed by:

\[
\min c(x) + \sum_{p \in \mathcal{P}} \sum_{k \in K_p} y_{pk}^p d_{pk} + e(z) \tag{5}
\]

s.t.

\[
x \in \bar{X}_z \tag{6}
\]

\[
\sum_{j \in J_t} z_j = 1 \quad \forall t \in T \tag{7}
\]

\[
\sum_{k \in K_p} y_{pk}^p = 1 \quad \forall p \in \mathcal{P} \tag{8}
\]

\[
y_{pk}^p - z_j \leq 0 \quad \forall p \in \mathcal{P}, \forall k \in K_p \text{ and } \forall j \in \tilde{J}_k \tag{9}
\]

\[
\sum_{p \in \mathcal{P}} \sum_{k \in K_p} y_{pk}^p \leq \sum_{g \in G} x_{t,g} \text{Cap}_g \quad \forall t \in T \tag{10}
\]

\[
y_{pk}^p \in \{0, 1\} \quad \forall p \in \mathcal{P} \text{ and } \forall k \in K_p \tag{11}
\]

\[
z_j \in \{0, 1\} \quad \forall t \in T \text{ and } \forall j \in \tilde{J}_t \tag{12}
\]

The objective function (5) is to minimize the total costs of the rolling stock rescheduling, the passenger flows (sum of delays) and the timetable rescheduling. Constraints (6) compactly summarize the constraints on the underlying rolling stock rescheduling problem. These rolling stock decisions are influenced by the chosen timetable \(z\) since there are some minimum process times required in the rolling stock schedule. So, if some trips take longer than planned certain processes can become infeasible. Constraints (7) determine that for every trip exactly one stopping pattern must be selected. Every passenger must pick exactly one path, which is modeled by Constraints (8). Constraints (9) ensure that only matching paths and stopping patterns can be chosen. The chosen paths by the passengers should also match with the available capacity in the trains which is modeled by Constraints (10).

Even this relaxation of the model of (1)-(4) is a complex model to solve in a real-time environment by the interdependence between the rolling stock and the timetable via the passenger flows. Therefore, we will solve the model in (1)-(4) by an iterative procedure as discussed in Section 2.1.

4 Passenger flow simulation

The iterative procedure starts with a simulation of the passenger flows, and each time the timetable or rolling stock schedule is updated a new simulation of the passenger flow is necessary.

To keep the simulation tractable, all passengers with the same characteristics (origin, destination and arrival time at the origin) are aggregated into passenger groups.

To simulate the passenger flows we use the simulation algorithm as described in Kroon et al. [8]. It is important to mention that this is a deterministic simulation algorithm to calculate the emerging passenger flows. This
means that, given a timetable and rolling stock schedule, there is a uniquely defined resulting passenger flow. Here we shortly summarize the assumptions of the model as described in Kroon et al. [8]. For more details we refer to that paper. We emphasize that the approach is modular, which allows us to replace the simulation model by any other simulation model to model the passenger behavior.

4.1 Assumptions

For the simulation of passenger behavior, Kroon et al. [8] make assumptions on three fundamental issues: (i) What information is available to the passengers? (ii) Which traveling strategy do passengers apply to the available information? and (iii) How do passengers interact?

Information available for the passengers

It is assumed that passengers always know the most recent timetable. This means that if the timetable is updated due to a disruption, they know which trains are canceled, which trains make additional stops, and which trains are delayed. Passengers do not know the future timetables, so they cannot anticipate on cancellations, delays and additional stops before the disruption occurs. Furthermore, they do not know anything about whether or not they fit in the trains they would like to take.

Strategy of the passengers

Each passenger has a traveling strategy. This strategy decides for the passenger what will be the preferred path to the destination given the most recent timetable. In Kroon et al. [8] all passengers have the same strategy. In our research we also use this single strategy. The used strategy is that passengers want to reach their destination as early as possible. If several paths have the same earliest arrival time, the passengers prefer the path with the least transfers between trains. If we have multiple paths with the same earliest arrival time and the same minimum number of transfers, the passengers will take the path with the earliest departure time. It is worthwhile to mention that in practice there is a more balanced trade off between transfers and travel time. It seems to be highly unrealistic that passengers are willing to transfer 2 times to save 1 minute of travel time. Note that one could easily include other strategies as well.

Each passenger wants to reach his destination before a certain deadline. If a passenger is not able to reach his destination before the deadline, it is assumed that the passenger gives up travelling by train. In this way it is modeled that passengers are not willing to wait endlessly.
Interaction between the passengers

If a train arrives, first passengers who want to leave the train get the option to do so, then the passengers waiting at the platform who want to enter the train compete for the available capacity in the train. It can happen that there is not enough capacity for all passengers. Then it is assumed that the number of passengers from each passenger group who actually board a train is relative to the size of the group. This could lead to a fractional number of passengers but it is assumed that the contribution of fractional flows are neglectable.

It is possible that not all members of a passenger group are able to board the train: Some of them have to stay behind. We say that these passengers are rejected by the train. In case of rejections, the passenger group is split into two: those passenger who were able to board the train and those who were not. The rejected passengers must find a new preferred path from their current location, while the boarded passengers can just follow their previously computed preferred path.

4.2 Evaluating the passenger flow

In this research we evaluate the passenger flows by the delays which passengers face in comparison with their original expected arrival times and by the number of passengers who did not reach their destination within their set deadline. In our experiments we try different ways to penalize delay minutes. In one setting the delay minutes are penalized uniformly and in another setting longer delays are penalized more, since one may argue that longer delays are worse than several small delays. For passengers who are not able to reach their destination within their deadline we penalize passengers leaving the system by the difference between the deadline and the expected arrival time of the intended traveling path. For each passenger, his delay or penalty for not reaching his destination within his deadline is called his inconvenience.

5 Rolling stock rescheduling

The rescheduling of rolling stock follows the procedure of Kroon et al. [8]. In this procedure the rolling stock is rescheduled based on the model described in Nielsen et al. [9] (which is an extension of Fioole et al. [6]). The basic decisions in the model are to assign a rolling stock composition to each trip such that as many of the passengers are accommodated. The difficulty of the rolling stock rescheduling is that a composition consists of multiple combined train units.

During operations the operator can change the compositions by decoupling or coupling units in the front or the back of the train. These operations are called shunting operations. Shunting personnel must be arranged to facilitate these operations. Therefore, changing the shunting operations also includes new tasks for the shunting personnel, which is not preferred.
Since a composition can consist of different types of train units, the order in which they are combined within a composition matters (i.e. one could not decouple a unit in the middle).

As discussed, the main objective of our approach is to prevent cancellations caused by lack of rolling stock. Therefore we once determine how many trains need to be cancelled due to lack of rolling stock. To do this we run the rolling stock rescheduling approach on the initially modified timetable with the single goal to find a feasible rolling stock schedule by minimizing the number of trains without rolling stock. This means that we have only a penalty for trains which do not get rolling stock assigned to them. All other penalties are set equal to 0. The result shows how many trains need to be cancelled inevitably by lack of rolling stock. In the rolling stock rescheduling steps we enforce the number of cancelled trains to be equal to this to ensure that no more trains than necessary are cancelled. Still the rolling stock rescheduling approach has freedom in which trains it does not assign rolling stock to.

For all remaining rolling stock rescheduling steps, the model has two objectives: It consists of a trade of between minimizing the rolling stock rescheduling costs and the inconvenience for the passengers. The rolling stock rescheduling costs are mostly based on how much the rolling stock schedule is changed. For example one does not want to make too many new shunting operations, since these new shunting operations must be communicated (with a certain probability of miscommunication) and it requires personnel to perform them.

The inconvenience for passengers is based on the latest simulation run with the timetable and rolling stock schedule of the last iteration. Penalties are defined for assigning a certain composition to a trip. The penalties are determined by estimating the effect of the train capacities on the total passenger inconvenience measured as discussed in Section 4.2.

Per trip the average inconvenience per passenger who was not able to board the train is computed. To do this, per passenger group the difference in inconvenience between passengers who were not able to board and passengers who were able to board is determined. Then, the weighted (based on group size) average of these differences is considered as the average inconvenience per passenger who is not able to board the train. In the objective function the number of seat shortages is multiplied with this average inconvenience per rejected passenger. For more details we refer to Kroon et al. [8].

Kroon et al. [8] reported that the approach of updating the objective function could lead to cyclical behavior if the feedback from earlier iterations is ignored. We follow the described exponential smoothing procedure in Kroon et al. [8] (which is based on Dumas and Soumis [5]) to take feedback from earlier iterations into account as well. We use the setting which performed best in their case. This setting means that feedback from earlier iterations is weighted for 35 percent.
6 Timetable adaptations

The disruption management process admits timetable decisions in order to better facilitate the passenger flows. In this paper we limit the allowed timetable modifications to adding stops to timetable services.

In this paper, adapting the stopping patterns means that trains may call at stations where they normally just pass through. Making an additional stop results in new traveling options for some passengers but also in an increased travel time for others. Therefore it is necessary to make a trade off between the positive and negative effects of the changed stopping pattern. The objective of this research is to minimize the sum of the delays of all passengers. Therefore, we only allow timetable changes that do not increase the total delay of all passengers. We assume that an additional stop will delay all further trips of a train by a fixed number of minutes and that those delays will not influence other train traffic.

The (greedy) procedure to adapt the stopping patterns goes as follows: (i) we have a list of candidate timetable adaptations, (ii) we evaluate for each candidate the consequences, (iii) we apply the timetable adaptation with the most positive consequences.

First of all, this approach requires that in step (i) a list of candidate timetable adaptations is given. The dispatchers can give this as input to the approach. In the extreme case, every timetable service is allowed to make an additional stop at every station it passes.

The effect measured in step (ii) indicates how much the total delay of the passenger will change if only that single timetable adaptation will be applied. Therefore, in step (iii) we limit ourselves to allow only one timetable adaptation per iteration of the solution approach. If no candidate timetable adaptation reduces the total delay of the passengers, no timetable change is made.

For all candidate timetable adaptations, the consequences of applying the adaptation needs to be computed. In Sections 6.1-6.4 we discuss several methods and approximations to compute these consequences. In Section 6.1 we discuss a method to compute the exact effect of the additional stop. The exact effect can be computed since we use a deterministic passenger flow simulation. However, computing the exact effect can be time consuming. Therefore, we also suggest a faster approximation algorithm in Section 6.2. Furthermore, we introduce two heuristics in Sections 6.3 and 6.4 which are more transparent for use in practice.

In the different variants we evaluate the effects of different candidate timetable adaptations. We refer to train $i$ as the train of which the stopping pattern an be adapted within the candidate timetable adaptation. Furthermore, the station at which train $i$ will make the additional stop within the candidate timetable adaptation is called station $b$. 
6.1 Exact effect of an additional stop (EXACT)

To determine the exact consequences of an additional stop, we need to run the simulation algorithm which is discussed in Section 4 twice. First the simulation is performed with the current timetable and then the simulation is performed with the timetable which results from the timetable adaptation. From both simulations we get the total delay minutes of the passengers. The difference between these two total delay minutes shows the consequences of the candidate timetable adaptation. This approach measures the exact effect of the additional stop and is called EXACT.

A variant of EXACT, denoted by EXACT*, will not use the current capacity of train $i$ in the simulation but the capacity of the largest possible composition allowed for train $i$. The difference between this simulation and the simulation of the current timetable will then not measure the exact effect of the extra stop but the potential (which can be larger) effect of the additional stop. In this case it is left to the rolling stock rescheduling phase to check whether it is possible to increase the capacity of train $i$.

6.2 Estimated effect of an additional stop (EST)

In this section we introduce an approach to estimate the effect of an additional stop. In this variant, both the positive and the negative effects of the additional stop are considered. For all passengers their preferred path to their destination is known. If we change the timetable by including an additional stop, some passengers might get another preferred path to their destination. Some passengers arrive earlier at their destination due to an additional stop at their origin or destination. Other passengers might profit from the delay of the train caused by the additional stop, since due to the delay they were able to catch this train which departs earlier than the train they were intending to take. All these passengers originally did not have train $i$ on their preferred path, but in case the additional stop is executed they have train $i$ on their preferred path.

However, since the additional stop takes some time it also causes some delays for passengers who had train $i$ on their preferred path in the current timetable. Due to the delay of train $i$ it can be that their preferred path changes. Also if the preferred path does not change it can still mean that the passengers are delayed if the trip on train $i$ was the final trip in their path.

To estimate the effect of the additional stop we compute for each passenger his preferred path in the current timetable and the preferred path in the timetable in which train $i$ makes an additional stop. The sum of all these differences is our estimate of the consequences of the additional stop. Note that this is an estimate since this method assumes that every passenger can take his preferred path which might not be true by the capacity of the rolling stock.

We make variants of this approach by assuming different durations of the additional stop in the determination of the shortest path. This means that
we can use for the estimation of the effect another duration of the additional stop than the real duration of the additional stop. For example, if we assume a shorter duration of the additional stop, we over-estimate the potential effect of the additional stop. However, maybe this will lead to a good optimization direction. The duration of the extra stop used in the estimation approach will be called the *extra stop penalty*.

6.3 Rule of thumb: Do not pass passengers who did not fit in a previous train (PRACT1)

If passengers did not fit in a train, then they have to wait for the next train in the same direction. Especially for these passengers, since they were already rejected, it is very frustrating if a train in their direction passes them without stopping. A rule in practice could be that it is not allowed to pass a group of passengers who were rejected by a previous train. This is an easy to use rule of thumb. We will evaluate the performance of this rule which is called PRACT1.

To decide whether or not train $i$ needs to make an additional stop we have to consider two other trains. The first considered train $h$ is the last train which arrived at station $b$ before the passing time of train $i$ at station $b$. If there were passengers with destination $b$ rejected to board train $h$ at the departure from the last station $a$ before station $b$, train $i$ will make an additional stop at station $b$ to let these rejected passengers travel from station $a$ to $b$.

The second considered train $j$ (which is in most cases the same as train $h$), is the last train which departed from station $b$ in the same direction as train $i$ before train $i$ passes station $b$. If there were passengers rejected to board train $j$ at station $b$, train $i$ will make an additional stop at station $b$ to let these rejected passengers in.

Since we allow one timetable adaptation per iteration we have to make a comparison on how effective the additional stop will be. Therefore, we sum up the advantages for all passengers who were rejected to board train $h$ in station $a$ or train $j$ in station $b$. For the passengers rejected to board train $h$ in station $a$ the advantage is measured by the difference between the arrival time of train $i$ at station $b$ and the arrival time of the first train from $a$ to $b$ after train $i$. For the passengers rejected to board train $j$ in station $b$ the advantage is estimated by the arrival time of train $i$ at the first station after station $b$ where both trains $h$ and $i$ call and the arrival time at the same station of the first train departing after train $i$ from station $b$.

In this measurement we assume that all passengers are able to board train $i$, so that train $i$ is assumed to have infinite capacity. We do not use the actual capacity since in the rolling stock rescheduling phase the capacity of train $i$ could be increased.
6.4 Rule of thumb: including the negative effects \textit{PRACT2}

The approach \textit{PRACT1} based on a rule of thumb only considers the positive effects of an additional stop. This results in a situation that even if only one passenger may profit from the additional stop, the stop will be executed. In another rule of thumb, \textit{PRACT2}, the delay for passengers traveling by train $i$ caused by the additional stop is included. In this approach, the advantages for passengers are measured in the same way as approach \textit{PRACT1}, and the inconvenience per passenger who travels by train $i$ at the moment train $i$ makes an additional stop at station $b$ will be equal to a fixed parameter. This parameter is equal to the duration of the extra stop plus an eventual penalty. Note that for practitioners this rule requires more knowledge. In \textit{PRACT1} the dispatchers only need monitor whether there are trains where some passengers did not fit in the train. In \textit{PRACT2} the dispatchers also need to know how many passengers did not fit in the train, and how many passengers are in the next train passing station $b$.

7 Lower bound

The proposed approach does not guarantee to converge to an optimal solution. To consider the quality of our solutions, we check the gap between a lower bound and the value of our solution. Depending on the nature of the disruption, the lower bound on the rolling stock rescheduling costs will not differ that much from 0, but the lower bound on the passenger delays can be quite interesting. In this section we come up with a lower bound which takes the positive effects of an additional stop into account.

This lower bound can be reached by assuming infinite capacity on all trains, together with assuming that all extra stops are executed and that an extra stop does not cause any arrival delay.

To be more precise, in this lower bound all extra stops are executed and the departure times at stations after the additional stop are delayed by the time an extra stop will take, and all arrival times are kept the same. This way we ensure that no passenger faces an arrival delay caused by the additional stop and that passengers who may profit from a delayed train caused by an additional stop still have the opportunity to enter the train. Then a simulation run with infinite capacity on the trains and with the timetable as described above gives a lower bound on the total passenger delay.

It can happen that passengers have an advantage by a delayed train since they can pick a train earlier than their planned train. This must be considered in the lower bound. Therefore we cannot just add the extra stops and leave all departure and arrival times the same.

By delaying the departure times we have a lower bound which is valid for both cases, with and without the extra stop. No one gets an arrival delay, and some passengers arrive earlier since they have an extra travel opportunity by the extra stop.
This lower bound represents the delay of the passengers which the operator cannot prevent by increasing the rolling stock capacities or adapting the timetable. This delay is caused by the train services that are inevitably cancelled due to the unavailability of infrastructure caused by the disruption.

8 Computational Results

We tested the proposed approach on instances based on cases of Netherlands Railways (NS) which is the major railway operator in the Netherlands. In these instances, a disruption, due to some blocked switches, caused that fewer trains than normally can be operated on certain tracks.

8.1 Detailed case description

The instances take the busiest part the Dutch railway network into account which is represented in Figure 2. The original passenger flows are constructed conform a regular weekday of Netherlands Railways, which resulted in 15064 passenger groups with a total of about 450,000 passengers.

In almost each of the trajectories we have four Intercity trains per hour in each direction. An Intercity train is a train which only calls at the larger stations. All intercity trains in this network are considered, furthermore the regional trains, that stop at every station, between The Hague (Gv) and Utrecht (Ut) are also considered.

For the rolling stock rescheduling, four types of rolling stock are available; two types for regional trains and two types for intercity trains. The regional train types can be coupled together, which leads to 5 possible compositions, and the Intercity train types can also be coupled together in 10 different compositions.

In Figure 2 the dotted line represents the disrupted area. On those tracks on a normal day each hour 4 Intercity trains and 4 regional trains run in each direction. We constructed two instances with a disruption in the rush hours between 7:00 A.M. and 10:00 A.M. In the first instance (ZTM1), 2 regional trains per hour per direction are canceled. In the second instance (ZTM2) also 2 Intercity trains per hour per direction are canceled. This means that in instance ZTM1 in each direction 6 trains per hour still run between Gouda (Gd) and The Hague (Gv), and only 4 trains per hour in each direction in instance ZTM2.

8.2 Parameter settings

The objective function consists of system related costs for the timetable adaptation and the rolling stock rescheduling, and costs for the passenger delays. For the timetable adaptations we do not consider any penalties other than that we assume that an additional stop will delay a train by 3 minutes.
Table 1  Rolling stock rescheduling costs

<table>
<thead>
<tr>
<th>Type of costs</th>
<th>value</th>
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</thead>
<tbody>
<tr>
<td>New shunting operation</td>
<td>500</td>
</tr>
<tr>
<td>Changed shunting operation</td>
<td>500</td>
</tr>
<tr>
<td>Canceled shunting operation</td>
<td>100</td>
</tr>
<tr>
<td>Off balances at the end of the day, per unit</td>
<td>200</td>
</tr>
<tr>
<td>Seat shortage per seat per kilometer</td>
<td>0.1</td>
</tr>
<tr>
<td>Carriage Kilometers</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

The rolling stock rescheduling costs are given in Table 1. Most important is that the rolling stock schedule should not change too much from the original plan, since changed plans require communication between the dispatchers and the personnel, and a failure in this communication is easily made. Therefore we introduce costs for having other shunting operations than planned. Changing the shunting operations also includes new tasks for the shunting personnel, which is not preferred. We consider the carriage kilometers as least important.

The passenger service costs consist of the passenger delay minutes as discussed in Section 4.2, where the penalties for passengers who left the system because of not reaching their end station within their deadline are also measured in delay minutes.

The approach will make at maximum 15 iterations.

To solve the composition model of the rolling stock rescheduling we used CPLEX 12.5. The test instances are run on a laptop with a Intel(R) Core(TM) i7-3517U 1.9/2.4 Ghz and 4.0 GB RAM.
8.3 Results

This section provides the results of the two test instances. For the timetable rescheduling part we had different approaches to decide which Intercity trains should make an additional stop. We compare the effect of the different approaches on the final solution. We also compare our approach (which includes the option to adapt the timetable) with the method of Kroon et al. [8] (which does not have an option to adapt the timetable) referred to as \((NO\_STOP)\). In the approach \((EST)\) we estimated in the timetable rescheduling step the effect of an additional stop. Within this estimation we discussed that we could assume different lengths of the additional stops. This assumed length of the additional stop is also called the extra stop penalty. In our experiments we used 0, 1, 2 and 3 minutes for the extra stop penalty. Note that an extra stop penalty of 0 minutes means that it is assumed that nobody faces negative effects of the additional stop. Furthermore, note that the realized timetable adaptation always includes a 3 minute delay caused by the additional stop.

In Table 2 we provide the best result found in the iterative procedure for each of the variants of the approach. Note that the iterative procedure does not necessarily converge to an optimal solution and thus the best solution can be found at any iteration. Therefore, we included the number of the iteration where the best solution was found. For each approach the lower bound as discussed in Section 7 is given. Furthermore, the table contains the value of the objective function which consist of the sum of the rolling stock rescheduling costs (by considering the parameters in Table 1) and the passenger inconvenience (measured in passenger delay minutes as discussed in 4.2). We also
report how many extra stops are included in the timetable of the best result. The computation time is measured in seconds and reports the computation time over all 15 iterations, and not just the computation time up to the moment the best solution is found. The latter would not be fair, since beforehand it is not known at which iteration the best solution will be found.

Performance

In the approach NO_STOP based on Kroon et al. [8], timetable adaptations were not allowed. Our results show that allowing the stopping patterns to be adapted can reduce the passenger delays dramatically by about 25 to 35 percent.

From Table 2 we can deduce that the approach EXACT led in both cases to the lowest passenger delay minutes and the lowest rolling stock rescheduling costs. EXACT*, the variant of the approach EXACT, reaches the same solutions, but it takes longer to get there. The estimation approach works well as long as we overestimate the positive effects of the additional stop by having a lower extra stop penalty (0 or 1 min) than the realized delay (3 min).

The performance of the approach EST 0min is surprising. It underestimates the negative effects and overestimates the positive effects of the additional stop but it is still able to reach solutions which do not differ much from the solutions reached by the approach EXACT. In deciding on which train should make an additional stop, the approach EST 0min assumes that an additional stop does not cause any delay and thereby no one faces negative effects of the additional stop. In every iteration an additional stop is introduced (by assuming that every additional stop has at least some positive effect).

On the other hand, the bad performance of the approach EST 3min is also surprising. Especially since in this approach the duration of the additional stop in the estimation approach matches the realized duration of an additional stop. However, this approach finds it never worthwhile to make an additional stop. Since this approach does not consider the capacities of the trains, it does not take rejected passengers into account. The EST approaches, thereby underestimate the positive effect the additional stop could have for rejected passengers. It seems that in EST 0min and EST 1min this underestimation is balanced by the overestimation of the other positive effects, but in the EST 3min approach the underestimation is not corrected by another overestimation.

The rules of thumb approaches PRACT1 and PRACT2 are outperformed by our exact approach EXACT and by our estimation approaches EST 0min and EST 1min. This shows that our more complex approaches are able to come to better solutions.

Iterative behavior

In Figures 3 - 8 we show for six of the variants how the solution of the case ZTM1 changes over the iterations. The black dot indicates the first solution
Fig. 3 EXACT case ZTM1

Fig. 4 EST 0min case ZTM1

Fig. 5 EST 1min case ZTM1

Fig. 6 EST 2min case ZTM1

Fig. 7 EST 3min case ZTM1

Fig. 8 PRACT1 case ZTM1
and by arrows we indicate how the solution evolves. On the horizontal axis we have the rolling stock rescheduling costs and on the vertical axis we have the passenger delays. With the rolling stock rescheduling both the rolling stock rescheduling costs and the passenger delays can change. However a timetable adaptation only influences the passenger delays, and therefore, a vertical drop or increase in the figure can most of the time be associated with a timetable adaptation.

The \textit{EXACT} approach has a quite clear converging path to its best solution by decreasing passenger delays and rolling stock rescheduling costs. The solution of the approaches \textit{EST 0min} and \textit{EST 1min} first goes to solutions with low passenger delays and low rolling stock rescheduling costs, but from a certain moment, the passenger delays are increasing again. The approaches \textit{EST 2min} and \textit{PRACT1} converge like the \textit{EXACT} approach to their best solution, but especially the solution of \textit{PRACT1} does not come close to the solution of \textit{EXACT}. The approach \textit{EST 3min} has in every iteration the same solution.

In Figures 9 and 10 the iterative behavior of our best performing variants (\textit{EXACT} and \textit{EST 0min}) on case ZTM2 are given. All variants did not show converging behavior for this case. The figures demonstrate that the \textit{EXACT} approach explores a smaller region of solutions. The approach \textit{EST 0min} first goes to solutions with large passenger delays, then gets to solutions with low passenger delays and in the end it goes again into the direction of solutions with high passenger delays.

Computation time

Our approach added a module which adapts the timetable to the iterative procedure of the approach \textit{NO\_STOP} of Kroon et al. [8]. This means that we assume that by the additional computations our approach cannot be faster than the \textit{NO\_STOP} approach.
### Table 3 Results with passenger flow costs times 10

<table>
<thead>
<tr>
<th>Instance</th>
<th>Solution Method</th>
<th>Lower bound</th>
<th>Objective</th>
<th>Passenger delay rescheduling costs</th>
<th>Rolling stock rescheduling costs</th>
<th>Extra stops</th>
<th>Iteration of best solution</th>
<th>Computation time (sec)</th>
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</thead>
<tbody>
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<td>ZTM1</td>
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<td>558793</td>
<td>55874</td>
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<td>7</td>
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</table>

If we use the EXACT approach, an instance is solved in about double the time of the NO_STOP approach. The other variants of the approach solve the instances faster (within 6 minutes).

The first rolling stock rescheduling step, to determine the number of trains without rolling stock, is carried out in about 80 seconds. Then, next rolling stock rescheduling steps take 4 to 5 seconds per iteration. The timetable rescheduling phase takes 8 to 15 seconds per iteration within the EXACT approach, since multiple simulations must be carried out. The computation time of the timetable rescheduling phase drops to 1 to 6 seconds per iteration for the estimation approaches EST and to less than 1 second for the approaches PRACT1 and PRACT2.

### 8.4 Additional tests

To see what happens with the solutions if we put more weight on the passenger delays, we run all approaches also with an objective in which the costs of the passenger flow is multiplied by 10. The results are presented in Table 3. The results and the performance are almost similar to the results in Table 2. In one third of the cases the rolling stock rescheduling costs are slightly higher to reach lower passenger delays.

In a third test we experiment on how the approaches behave if we give additional penalties to longer delays. In these tests we penalize delays between 15 and 30 minutes with an additional 5 minutes delay and delays longer than 30 minutes with an additional 10 minutes delay. Again one can see from the
Table 4 Results with 5 minutes additional penalty for delays larger than 15 minutes and 10 minutes additional penalty for delays larger than 30 minutes

<table>
<thead>
<tr>
<th>Instance</th>
<th>Method</th>
<th>Lower bound</th>
<th>Objective</th>
<th>Passenger delay minutes</th>
<th>Rolling stock rescheduling costs</th>
<th>Extra stops</th>
<th>Iteration of best solution</th>
<th>Computation time (sec)</th>
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results in Table 4 that these additional penalties do not influence the solutions. In more than half of the cases, the best solution found is the same as in the situation without these additional penalties (as presented in Table 2). For the other cases the differences were not large.

These two additional tests show that our approach is not sensitive to changes in the evaluation of the passenger inconvenience.

9 Conclusions and further research

In this paper we proposed a disruption management approach which integrates the rescheduling of rolling stock and the adaptation of stopping patterns with the aim of improving passenger service.

Computational tests are performed on realistic large scale instances of the Dutch railway network. The two tested instances show that allowing the timetable to be adapted can reduce the total delay of passengers by more than 20 percent without increasing the rolling stock rescheduling costs. We suggested several variants of the approach, with the difference lying in the way of how the timetable changes are evaluated. These variants lead to different results and different computation times, but the results per variant are not quite sensitive to the exact cost parameter settings.

Our solution approach does not necessarily converge to an optimal solution. The lower bounds indicate that the gap between the solution and the lower bound is decreased by allowing stopping pattern adaptations. However the gap
is still significant, which is probably caused by the weak lower bound. This is a topic for future research.

In future research we will incorporate other timetable decisions as well, for example reroutings of trains. Furthermore we want to proceed with integrating delay management decisions into the model, which will be quite challenging since the delay management approach is already a difficult problem to solve on its own.
References


