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Buckling curves for heavy wide flange QST columns based on statistical evaluation

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A B S T R A C T
This paper proposes the use of existing European buckling curves to check the resistance of heavy wide flange quenched and self-tempered (QST) sections made from high-strength steel, failing by flexural buckling. The buckling curves are evaluated according to the statistical procedure given in Annex D of EN 1990 using finite element analyses. The residual stress model as described in a related paper was used to define the initial stress state of the column in the finite element model. A large database was created containing the ratio between the elastic-plastic buckling resistance obtained from finite element analysis and the buckling resistance obtained from the proposed buckling curve for a wide set of column configurations from which a partial factor γP was deduced. Two different section types were investigated: the stocky HD and more slender HL type, featuring a height-to-width ratio (h/b) of approximately 1.23 and 2.35, respectively. Based on the criterion that the partial (safety) factor γP should not exceed 1.05 it is suggested to check the buckling response of a heavy HD section according to buckling curve “a” or “b” when failing by strong-axis or weak-axis buckling, respectively. HL sections are to be designed according to buckling curve “c” for strong-axis buckling and buckling curve “b” for weak-axis buckling.

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1. Introduction

The advent of quenched and self-tempered (QST) steel sections which combine high strength (i.e. nominal yield stress greater than 430 N/mm²) with good ductility and weldability has led to a broadening of the possibilities in steel construction. This manufacturing method can also be applied to produce heavy wide flange sections, i.e. wide flange sections with flanges thicker than 40 mm.

At the moment, heavy wide flange QST sections are manufactured by ArcelorMittal under the proprietary name of HISTAR (HIgh-STrength ArCelorMittal). Two grades are currently produced: HISTAR 355 and (high-strength) HISTAR 460, which possess a yield stress of 355 N/mm² and 460 N/mm² respectively, not considering reduction of yield stress with increasing material thickness. Heavy wide flange HISTAR 460 sections have already been applied worldwide, with the majority in the United States where the US equivalent of HISTAR 460, Grade 65, is covered by ASTM A913 [1,2].

Besides the high yield stress, HISTAR 460 sections have improved material properties for wide flange sections possessing thick flanges. For HISTAR 460 a smaller reduction in yield stress needs to be incorporated for greater material thicknesses according to ETA-10/0156 [3] when compared to other grades (e.g. S460M and S500M according to EN 10025-4 [4]) as illustrated in Fig. 1.

As such, heavy wide flange HISTAR 460 sections are used to their best advantage when the ultimate limit state is the governing design criterion. This is the case when applied as gravity columns in multistory buildings, beams in short- or medium-span bridges or chord and brace members as part of truss-like structures. In these situations the design is most often controlled by the flexural buckling resistance of the member for which due allowance has to be made according to the relevant design codes.

However, the buckling resistance of these high-strength heavy wide flange sections has not yet been completely facilitated by the European design code EN 1993-1-1 [5]. Table 1 shows the buckling curve classification according to EN 1993-1-1 and the buckling curves are shown in Fig. 2 where the buckling resistance is expressed on the vertical axis as a function of the relative slenderness on the horizontal axis.

HISTAR 460 falls in the category S460 in Table 1. Small and medium-sized HISTAR 460 sections with flange thickness t less than or equal to 40 mm are assigned to buckling curve “a” or “b” depending on the value of the height-to-width ratio h/b. Heavy HISTAR 460 sections which have a flange thickness smaller than or equal to 100 mm can be designed according to buckling curve “a.” Heavy sections possessing a flange thickness in excess of 100 mm and an h/b-ratio smaller than 1.2 are assigned to buckling curve “c.” For heavy HISTAR 460 sections having h/b-ratios greater than 1.2 and flanges thicker than 100 mm no buckling curves are available. However, it seems to be most logical to design these sections also according to buckling curve “c.”

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In order to arrive at buckling curve formulations reflecting the buckling response for heavy HISTAR 460 sections with flange thickness larger than 100 mm and \( h/b \)-ratios greater than 1.2 a combined experimental and numerical study was initiated by ArcelorMittal in Luxemburg and set up and executed by Eindhoven University of Technology in the Netherlands. The experiments consisted of residual stress measurements performed on two different heavy wide flange section types made in steel grade HISTAR 460. A residual stress model was proposed which can be used for heavy wide flange QST sections. The testing procedure and the derivation of this residual stress model are detailed in a related paper [6].

In the present paper, existing ECCS buckling curves are proposed to check the flexural buckling resistance of heavy HISTAR 460 sections. The reliability of the suggested buckling curves is evaluated according to annex D of EN 1990 [7]. The buckling resistance for a wide set of HISTAR 460 columns is evaluated with the finite element method using the residual stress model of Spoorenberg et al. [6] to define the initial stress state of the column and with widely accepted geometric imperfections.

1.1. Earlier approaches for derivation of buckling curves

From the earliest experiments on pin-ended columns failing by flexural buckling it was observed that the slenderness (ratio between length and radius of gyration) of the member has profound influence on the buckling response. This led to the development of the buckling curve concept, relating the load a column can withstand before instability occurs to its non-dimensional or relative slenderness (slenderness normalized against the steel properties). Important references include Refs. [8] and [9]. The studies underlying the buckling curve concept were often based on a two-fold approach: to obtain the elastic-plastic buckling resistance through full-scale column testing and to conduct theoretical (and later numerical) analyses to replicate and supplement the experimental results. The theoretical analyses were expanded to include a wide set of columns not part of the experimental plan from which design rules (buckling curves) were proposed. The accuracy of the buckling curve was often evaluated through comparison with characteristic values from full-scale tests performed, where “good agreement” between the buckling curve and test justified the selected curve.

1.2. Statistical evaluation of resistance models

The earlier approaches to arrive at buckling curve formulations have become obsolete as with the appearance of EN 1990 Annex D [7] “Design assisted by testing” it is now possible to make a statistical evaluation for new design rules and existing ones and to quantify the variability of salient parameters. In brief, the EN 1990 states that the design resistance \( R_d \) may be obtained directly from the quotient of the characteristic \( R_k \) strength and the partial factor \( \gamma_M \):

\[
R_d = \frac{R_k}{\gamma_M}
\]

Table 1

<table>
<thead>
<tr>
<th>Cross section</th>
<th>Limits</th>
<th>Buckling about axis</th>
<th>Buckling curve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rolled I-sections</td>
<td>( h/b &gt; 1.2 )</td>
<td>( y-y )</td>
<td>S460</td>
</tr>
<tr>
<td></td>
<td>( t_f \leq 40 \text{ mm} )</td>
<td>( b )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( 40 \text{ mm} &lt; t_f \leq 100 \text{ mm} )</td>
<td>( a )</td>
<td>S235</td>
</tr>
<tr>
<td></td>
<td>( h/b \leq 1.2 )</td>
<td>( y-y )</td>
<td>S420</td>
</tr>
<tr>
<td></td>
<td>( t_f \leq 100 \text{ mm} )</td>
<td>( b )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( t_f &gt; 100 \text{ mm} )</td>
<td>( c )</td>
<td>S275</td>
</tr>
</tbody>
</table>

Fig. 1. Decrease of yield stress of HISTAR 460, S460 and S500 with increasing material thickness.

Fig. 2. Buckling curves from Eurocode 3.
where \( \gamma_M \) can be subdivided as follows:

\[
\gamma_M = \gamma_{Rd} \times \gamma_m
\]  

(2)

where:

\( \gamma_m \) is the partial factor for a material property, also accounting for model uncertainties and dimensional variations according to EN 1990 [7] or general partial factor according to EN 1993-1-1 [5];

\( \gamma_{Rd} \) is the partial factor associated with the uncertainty of the resistance model;

\( \gamma_m \) is the partial factor for a material property.

A distinction for the general partial factor \( \gamma_M \) is made depending on the failure mode of the member under investigation. In the present study, columns are investigated for which loss of stability is the governing failure mode. Therefore the general partial factor is – in line with EN 1993-1-1 – denoted by \( \gamma_{M1} \) throughout this paper.

The general partial factor serves as a reduction for the section capacity: high \( \gamma_{M1} \)–values impose a larger reduction on the buckling capacity compared to lower \( \gamma_{M1} \) values.

One of the earliest studies concerning the statistical evaluation of resistance models was carried out by Sedlacek et al. [10] at RWTH Aachen, Germany. Although the investigation was performed prior to the final appearance of EN 1990, it adopted the same methodology. The study was aimed at finding new imperfection factors for the resistance model of Eurocode 3 (EN 1993–1–1) to check the lateral-torsional buckling resistance of rolled and welded beams. The reliability of the old resistance model, originally from the DIN, in addition to the new resistance model, was reevaluated. The statistical evaluation was based on 144 lateral-torsional buckling tests.

A probabilistic assessment of the existing design rules to check the lateral-torsional buckling resistance of beams was performed by the University of Coimbra, Portugal for wide flange beams. The partial factor associated with the uncertainty of the resistance model \( \gamma_{Rd} \) was computed for different load cases and section types using the three different design models for lateral-torsional buckling available in EN 1993–1–1, Rebelo et al. [11].

The evaluation of the partial factors was based on the solution results from finite element analyses conducted on a wide set of beam configurations.

In the accompanying paper by Simoes da Silva et al. [12] the partial factor for the material properties \( \gamma_m \) was determined based on tensile tests conducted on sections made from different steel grades. Using Eq. (2), the factors from Refs. [11] and [12] were used to arrive at \( \gamma_{M1} \)–values for different load cases and steel grades.

1.3. Present study

Due to their sheer size and high yield stress, the buckling load of heavy wide flange QST sections is far greater than the maximum capacity of most testing machines, which makes it practically impossible to base the buckling curve derivation on experiments. Therefore, finite element analyses form the basis of the present study. The methodology adopted by Rebelo et al. [11] and in line with Annex D of EN 1990 will be used to check the reliability of proposed buckling curves for heavy wide flange HISTAR 460 sections.

A large database is created containing the ratio between the elastic–plastic buckling resistance obtained from non-linear finite element analyses and buckling resistances obtained from the suggested buckling curve for a wide set of column configurations. The residual stress model for heavy HISTAR 460 sections as proposed in a related paper [6] is used to define the initial stress state of the column in the finite element model. It is mentioned that the numerically obtained elastic–plastic buckling resistance serves as a replacement of the ultimate resistance found in a column buckling test. The ratio between both buckling resistances for a specific set of columns is used to compute the partial factor \( \gamma_{Rd} \) associated with the uncertainty of the resistance model.

The paper is confined to the derivation of \( \gamma_{Rd} \), which together with \( a \), for the time being, conservative assumption for \( \gamma_m \) being \( \gamma_m = 1.0 \), can be used to compute the general partial factor \( \gamma_M \) according to Eq. (2). This general partial factor can have a lower value in the nearby future pending the availability of a lower \( \gamma_{M1} \)–value representing the variability in mechanical properties of HISTAR 460 steels. The general partial factor \( \gamma_{M1} \) together with the proposed buckling curve can be used to check HISTAR 460 columns failing by flexural buckling.

The study is limited to heavy wide flange sections manufactured by the quenched and self-tempered method which possess a flange thickness (\( t_f \)) greater than 100 mm and for which the height-to-width ratio (\( h/b \)) is greater than 1.2. The selected sections for the present study are listed in Table 2. In the present study a distinction is made between HD and HL sections, having a \( h/b \)-value of approximately 1.23 and 2.35, respectively.

2. Buckling curve formulation

The theoretical resistance defining the maximum compressive force a column can withstand before failing in a flexural buckling mode is determined using EN 1993–1–1 [5]. The buckling resistance for a column can be verified as follows:

\[
\frac{N_{Ed}}{N_{b,Rd}} \leq 1.0
\]  

(3)

where:

\( N_{Ed} \) is the design value of the compression force;

\( N_{b,Rd} \) is the design buckling resistance.

The design buckling resistance is given by:

\[
N_{b,Rd} = \frac{\gamma A f_y}{\gamma_{M1}}
\]  

(4)

where \( A \) is the cross-sectional area, \( f_y \) is the yield stress, \( \gamma_{M1} \) is the general partial factor for instability and \( \chi \) is the buckling reduction factor. This check is only valid for sections belonging to cross-sectional class 1, 2 or

<table>
<thead>
<tr>
<th>Section name (European)</th>
<th>Section name (American – Imperial)</th>
<th>Weight per m [kg]</th>
<th>( h [\text{mm}] )</th>
<th>( b [\text{mm}] )</th>
<th>( t_m [\text{mm}] )</th>
<th>( t_c [\text{mm}] )</th>
<th>( h/b [-] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>HD 400 × 900</td>
<td>W14 × 16 × 605</td>
<td>900</td>
<td>531</td>
<td>442</td>
<td>65.9</td>
<td>106</td>
<td>1.20</td>
</tr>
<tr>
<td>HD 400 × 990</td>
<td>W14 × 16 × 665</td>
<td>990</td>
<td>550</td>
<td>448</td>
<td>71.9</td>
<td>115</td>
<td>1.23</td>
</tr>
<tr>
<td>HD 400 × 1086</td>
<td>W14 × 16 × 730</td>
<td>1086</td>
<td>569</td>
<td>454</td>
<td>78</td>
<td>125</td>
<td>1.25</td>
</tr>
<tr>
<td>HD 400 × 1202</td>
<td>W14 × 16 × 808</td>
<td>1202</td>
<td>580</td>
<td>471</td>
<td>95</td>
<td>130</td>
<td>1.23</td>
</tr>
<tr>
<td>HD 400 × 1299</td>
<td>W14 × 16 × 873</td>
<td>1299</td>
<td>600</td>
<td>476</td>
<td>100</td>
<td>140</td>
<td>1.26</td>
</tr>
<tr>
<td>HL 920 × 1194</td>
<td>W36 × 16.5 × 802</td>
<td>1194</td>
<td>1081</td>
<td>457</td>
<td>100</td>
<td>140</td>
<td>2.37</td>
</tr>
<tr>
<td>HL 920 × 1269</td>
<td>W36 × 16.5 × 853</td>
<td>1269</td>
<td>1093</td>
<td>461</td>
<td>109</td>
<td>115.1</td>
<td>2.37</td>
</tr>
<tr>
<td>HL 920 × 1377</td>
<td>W36 × 16.5 × 925</td>
<td>1377</td>
<td>1093</td>
<td>473</td>
<td>76.7</td>
<td>115.1</td>
<td>2.37</td>
</tr>
</tbody>
</table>
3. The product of the cross-sectional area and the yield stress is known as the
squash load of the cross section or $N_{pl}$. Using Eqs. (3) and (4), the verification of the buckling resistance can be rewritten as follows:

$$\frac{N_{pl}}{N_{cr}/\lambda} \leq 1.0.$$  \hspace{2cm} (5)

The buckling reduction factor can be computed according to:

$$\chi = \frac{1}{\psi + \sqrt{\psi^2 - \lambda^2}} \hspace{0.5cm} \text{but} \hspace{0.5cm} \lambda \leq 1.0$$  \hspace{2cm} (6)

where:

$$\psi = 0.5 \left( 1 + \alpha (\bar{X} - 0.2) + \bar{X}^2 \right).$$  \hspace{2cm} (7)

The relative slenderness $\bar{X}$ can be determined as follows:

$$\bar{X} = \sqrt{\frac{N_{pl}}{N_{cr}}}$$  \hspace{2cm} (8)

where $N_{cr}$ is the elastic critical force of the column. The imperfection factor $\alpha$ attains one of the values as listed in Table 3, depending on the cross section, steel grade and buckling case (weak-axis or strong-axis buckling) under consideration.

A graphical representation of the buckling curves is shown in Fig. 2. Based on the selected buckling curve and corresponding imperfection factor a theoretical resistance can be computed for a heavy HISTAR 460 section if the relative slenderness is known. This value will be compared to the elastic–plastic buckling resistance obtained from nonlinear finite element analysis (Section 3).

3. Finite element model

3.1. Elements

The geometrical and material non-linear analyses on the columns containing imperfections (GMNIA) were performed in the ANSYS v.11.0 implicit environment. The columns were modeled with beam elements. The 3D three node finite strain element (BEAM189) was selected for the analyses as it can describe plasticity, large deformations and large strains. A user-defined cross section was modeled based on nominal dimensions, see Table 2. The cross section is subdivided into different cells to capture growth of plastic zones across the cross section. Each cell contains four integration points where the stresses are evaluated (Fig. 4a). Two integration point locations in longitudinal direction of each element describe progressive yielding along the length of the column. A total of 20 elements along the length of the member was considered sufficient. Earlier research studies on column buckling have shown that this element type is able to replicate experimental elastic–plastic buckling tests with good accuracy thereby taking into account the effects of residual stresses [13,14].

3.2. Boundary conditions

All selected column configurations for the present investigations were simply supported. The column was pin-supported and torsionally restrained at the bottom. The same boundary condition was applied at the top with the exception that vertical translation was permitted. For the evaluation of strong-axis buckling, the column was restrained against weak-axis deflections by translational supports along the length (Fig. 3).

3.3. Residual stresses

An individual residual stress value was set for each integration point in the cross section based on the residual stress model from [6]. The stress value specified for each integration point is assigned to the tributary area belonging to that integration point, rendering a step-wise initial stress pattern over the cross section (Fig. 4b). The residual stresses are constant across the flange thickness and web thickness. After inserting the residual stresses into the element, a first solution step was issued to verify internal equilibrium of the residual stress model. Insignificant differences were observed between the residual stress model and the stresses after solving, indicating correct implementation of the residual stress model (Fig. 4c).

3.4. Material model

A bilinear material model was applied to describe the material’s response to loading (Fig. 5). A fixed yield stress value of 450 N/mm$^2$ was used to define the onset of yielding. This value is based on the steel properties, thereby taking into account a reduction in yield stress due to the thickness of the flanges (see also Fig. 1). The nominal yield stress of the material excluding thickness reduction is 460 N/mm$^2$. A more generally accepted value for the Young’s modulus of 200 000 N/mm$^2$ was adopted to define the elastic stage of the material. No strain hardening effects were included.

3.5. Geometric imperfections

The shape of the geometric imperfection was based on the buckling mode belonging to the lowest eigenvalue from a linear buckling analysis. This resulted in a sinusoidal bow imperfection. The amplitude defining the maximum deviation from the ideal geometry was $L/1000$, where $L$ is the height of the column.

3.6. Solution

All elastic–plastic buckling GMNIA are load-controlled. A force with specified magnitude was applied at the top of the column. The Arc-Length method was selected to solve the non-linear equilibrium iterations. The Arc-Length method was selected in preference to the
conventional Newton–Raphson method as the former is able to describe the decreasing load-deflection curve beyond the maximum resistance whereas the latter will abort the solution when the maximum resistance has been reached. The load was divided into four load steps which in turn were further divided into substeps or load increments. For each load-increment a number of equilibrium iterations was performed to arrive at a converged solution. The solution was considered solved when the out-of-balance load vector is smaller than 0.05% of the load increment. Typical load-deflection curves as obtained from the finite element analyses are shown in Fig. 6a. The ultimate strength or flexural buckling resistance of the column \( N_{\text{ult,FEM}} \) was identified as the maximum load on the load-deflection curve. The elastic buckling load \( N_{\text{cr,FEM}} \) is obtained from a linear buckling analysis (LBA) using the Block–Lanczos extraction method of eigenvalues.

3.7. Plotting results in buckling curve

For each column configuration for which the ultimate resistance is evaluated through non-linear finite element analyses, the reduction factor is obtained by normalizing the ultimate load against the squash load of the cross section \( N_{\text{pl,FEM}} \).

\[
\chi_{\text{FEM}} = \frac{N_{\text{ult,FEM}}}{N_{\text{pl,FEM}}} \quad (9)
\]

where the squash load of the cross section is computed according to:

\[
N_{\text{pl,FEM}} = A f_y \quad (10)
\]

where \( A \) is the cross-sectional area of the element and \( f_y \) is the nominal yield stress.

This value is labeled as the “experimental” resistance for comparison with the theoretical resistance (Section 4). The relative slenderness of the column can be computed by taking the square root of the ratio between the squash load of the cross section and the elastic buckling load evaluated from a linear buckling analysis:

\[
\lambda_{\text{FEM}} = \sqrt{\frac{N_{\text{pl,FEM}}}{N_{\text{cr,FEM}}}} \quad (11)
\]

Note that Eq. (11) is similar to Eq. (8) but the squash load is now based on that of the FEM model and the elastic buckling load is calculated with a LBA.

In Fig. 6b the ultimate loads from Fig. 6a are plotted in the buckling curve diagram using the Eqs.(10)-(11) in addition to buckling curve “a.” Plotting the ultimate load for a specific group of columns in the buckling curve diagram in addition to a buckling curve allows a first estimate to be made as to whether that specific buckling curve is on the conservative or unconservative side.

4. Statistical evaluation and suggested buckling curves

4.1. Partial factor evaluation procedure

The partial factor evaluation procedure follows Annex D of EN 1990 and is applied here in a similar way as in Ref. [11]. For any heavy QST column \( i \) a comparison can be made between its experimental resistance \( r_{\text{e,i}} \) and its theoretical resistance \( r_{\text{t,i}} \):

\[
R_i = \frac{r_{\text{e,i}}}{r_{\text{t,i}}} \quad (12)
\]

In the present study the experimental resistance refers to \( \chi_{\text{FEM}} \) from Eq. (9) for a heavy HISTAR 460 section failing by flexural buckling as obtained from non-linear finite element analysis, so \( r_{\text{e}} = \chi_{\text{FEM}} \). The theoretical resistance refers to the buckling reduction factor \( \chi \) according to the buckling curve formulation from EN 1993-1-1 (Eq. (6)), so \( r_{\text{t}} = \chi \).
It is noted that in order to arrive at a theoretical resistance a selection for a buckling curve (imperfection factor from Table 3) must already be made. A value of \( R_t \) smaller than 1.0 or larger than 1.0 reflects an unconservative or a conservative theoretical resistance model, respectively. For any group of column configurations belonging to a set with sample size \( n \), the mean value correction factor \( R_m \) and corresponding variance can be determined:

\[
R_m = \frac{1}{n} \sum_{i=1}^{n} R_i \quad \sigma^2 = \frac{1}{n-1} \sum_{i=1}^{n} (R_i - R_m)^2.
\]  

When plotting the experimental resistance on the y-axis and corresponding theoretical resistance on the x-axis for all column configurations belonging to subset \( n \), the points will be distributed around the so-called estimator line: \( r_e = R_m \times r_t \).

For each column configuration belonging to subset \( n \) an error term \( \delta_i \) is introduced:

\[
\delta_i = \frac{r_t \times r_i}{r_e}.
\]

A logarithmic transformation is performed:

\[
\Delta_i = \ln(\delta_i).
\]

For the logarithmic error terms belonging to sample size \( n \), the mean value and corresponding variance are determined as follows:

\[
\bar{\Delta} = \frac{1}{n} \sum_{i=1}^{n} \Delta_i \quad \sigma^2 = \frac{1}{n-1} \sum_{i=1}^{n} (\Delta_i - \bar{\Delta})^2.
\]

The variance can be used to compute the coefficient of variation as follows:

\[
V_b = \sqrt{\exp(\sigma^2/\bar{\Delta}) - 1}.
\]

When using a subset with a sample size \( n > 100 \) the partial factor associated with the uncertainty of the resistance model can be determined as follows:

\[
\gamma_{kd} = \frac{1}{R_m \exp\left(-k_{d,e} Q - 0.5Q^2\right)} \geq 1.0
\]

for which:

\[
Q = \sqrt{\ln(V_b^2 + 1)}
\]

and where \( k_{d,e} \) is the characteristic fractile factor: \( 0.8 \times 3.8 = 3.04 \).

So, finally Eq. (18) gives the partial factor belonging to a suggested buckling curve based on a set of column configurations.

4.2. Partial factor for suggested buckling curves

Non-linear finite element analyses were carried out for eight different heavy wide flange cross sections, see Table 2. For each cross section the weak-axis and strong-axis buckling response was evaluated. The relative slenderness of the investigated columns was in the range between 0.31 and 3.3.

Plotting the finite element results in a buckling curve diagram permits a first judgment on the suitability of the buckling curve to represent the column strength for heavy HISTAR 460 sections. In case the chosen buckling curve is positioned below the finite element results, it will provide conservative column strength values. The buckling curve can be regarded as unconservative when the finite element data is below the buckling curve. Fig. 7 (left) shows the finite elements results for a HL 400 × 1202 section in HISTAR 460 buckling about its weak axis in a buckling curve diagram in addition to buckling curve “a.”

Similar trends are found when plotting the theoretical column strength against its numerical counterpart such as shown in Fig. 7 (right). In case the buckling curve produces column strengths similar to the finite element results, the data is positioned on the line \( r_e = r_t \). Data distributed above the line \( r_e = r_t \) indicates that the buckling curve provides conservative values for the column strength. Unconservative columns strengths are found when the data points are below \( r_e = r_t \). When the buckling curve formulation represents column strengths different from those obtained with finite element analyses the data points will be distributed around the line \( r_e = R_m \times r_t \), where \( R_m \) is mean value correction factor according to Eq. (13) and Refs. [7] and [15].
This line will give a better description of the correlation between the theoretical and numerical values in comparison to \( r_e = r_t \).

The partial factor associated with the uncertainty of the resistance model is evaluated for each buckling curve. The corresponding \( \gamma_{Rd} \)-values for each section type and buckling axis are presented in Table 4. For unfavorable buckling curves the \( \gamma_{Rd} \)-value is lower in comparison to more favorable buckling curves for a majority of the investigated cases. Hence, relating the elastic–plastic buckling response of a heavy HISTAR 460 section to a more favorable buckling curve is at the expense of a higher partial factor \( \gamma_{Rd} \).

The most favorable buckling curve selected for heavy HISTAR 460 sections failing by flexural buckling is based on the criterion \( \gamma_{Rd} \geq 1.05 \), as denoted with an asterisk in Table 4. Values of \( \gamma_{Rd} \) smaller than 1.05 can be fairly rounded off to 1.00 and values greater than 1.05 cannot be accepted.

The differences between sections belonging to the same type (HD or HL) and buckling around the same axis (weak or strong) are relatively small, indicating that section geometry for the same section type has little influence on the partial factor.

In general the partial factors for an identical buckling curve are greater for the weak-axis buckling case than those for the strong-axis buckling case. This reflects the more detrimental influence of residual stresses for columns failing by weak-axis buckling. Assuming that a \( \gamma_{Rd} \)-value is smaller than 1.05 allows \( \gamma_{Rd} = 1.05 \) to be used, HD and HL sections failing by weak-axis buckling should be assigned to curve “b.” Curve “a” is assigned to HD sections failing by strong axis buckling. HL sections buckling about the strong axis should be checked by buckling curve “a.” The results are summarized in Table 5.

5. Discussion

5.1. Partial factor evaluation

The previous statistical analyses rely on the assumption that the data points can be described by a uni-modal function (or single probability density function). In Ref. [11] the suitability of describing the data points with a uni-modal function was checked by displaying the data points against the theoretical quantiles belonging to a normal distribution, evaluated using the mean and variance from the full set of data points. Although good agreement between both was observed it was decided to use data points belonging to the “lowest part” of the distribution i.e.

---

**Table 4**

Partial factor.

<table>
<thead>
<tr>
<th>Heavy section</th>
<th>Buckling axis</th>
<th>Sample size</th>
<th>( a_0 )</th>
<th>( a )</th>
<th>( b )</th>
<th>( c )</th>
<th>( d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>HD 400 × 900</td>
<td>Weak-axis</td>
<td>104</td>
<td>1.161</td>
<td>1.054</td>
<td>0.994(^a)</td>
<td>0.978</td>
<td>0.947</td>
</tr>
<tr>
<td></td>
<td>Strong-axis</td>
<td>119</td>
<td>1.015(^a)</td>
<td>1.000</td>
<td>0.998</td>
<td>0.984</td>
<td>0.950</td>
</tr>
<tr>
<td>HD 400 × 990</td>
<td>Weak-axis</td>
<td>104</td>
<td>1.160</td>
<td>1.054</td>
<td>1.003(^a)</td>
<td>0.991</td>
<td>0.968</td>
</tr>
<tr>
<td></td>
<td>Strong-axis</td>
<td>111</td>
<td>1.016(^a)</td>
<td>1.010</td>
<td>1.017</td>
<td>1.015</td>
<td>0.995</td>
</tr>
<tr>
<td>HD 400 × 1086</td>
<td>Weak-axis</td>
<td>109</td>
<td>1.159</td>
<td>1.053</td>
<td>1.000(^a)</td>
<td>0.989</td>
<td>0.964</td>
</tr>
<tr>
<td></td>
<td>Strong-axis</td>
<td>109</td>
<td>1.015(^a)</td>
<td>1.012</td>
<td>1.022</td>
<td>1.021</td>
<td>1.004</td>
</tr>
<tr>
<td>HD 400 × 1202</td>
<td>Weak-axis</td>
<td>110</td>
<td>1.160</td>
<td>1.053</td>
<td>0.997(^a)</td>
<td>0.983</td>
<td>0.955</td>
</tr>
<tr>
<td></td>
<td>Strong-axis</td>
<td>134</td>
<td>1.016(^a)</td>
<td>1.009</td>
<td>1.017</td>
<td>1.013</td>
<td>0.992</td>
</tr>
<tr>
<td>HD 400 × 1299</td>
<td>Weak-axis</td>
<td>110</td>
<td>1.160</td>
<td>1.053</td>
<td>0.999(^a)</td>
<td>0.985</td>
<td>0.957</td>
</tr>
<tr>
<td></td>
<td>Strong-axis</td>
<td>137</td>
<td>1.017(^a)</td>
<td>1.009</td>
<td>1.011</td>
<td>1.003</td>
<td>0.978</td>
</tr>
<tr>
<td>HL 920 × 1377</td>
<td>Weak-axis</td>
<td>100</td>
<td>1.275</td>
<td>1.146</td>
<td>1.017(^a)</td>
<td>0.982</td>
<td>0.953</td>
</tr>
<tr>
<td></td>
<td>Strong-axis</td>
<td>166</td>
<td>1.053</td>
<td>0.992(^a)</td>
<td>0.983</td>
<td>0.964</td>
<td>0.922</td>
</tr>
<tr>
<td>HL 920 × 1194</td>
<td>Weak-axis</td>
<td>101</td>
<td>1.298</td>
<td>1.167</td>
<td>1.031(^a)</td>
<td>0.974</td>
<td>0.933</td>
</tr>
<tr>
<td></td>
<td>Strong-axis</td>
<td>106</td>
<td>1.073</td>
<td>0.991(^a)</td>
<td>0.967</td>
<td>0.940</td>
<td>0.887</td>
</tr>
<tr>
<td>HL 920 × 1269</td>
<td>Weak-axis</td>
<td>103</td>
<td>1.247</td>
<td>1.158</td>
<td>1.030(^a)</td>
<td>0.985</td>
<td>0.953</td>
</tr>
<tr>
<td></td>
<td>Strong-axis</td>
<td>101</td>
<td>1.056</td>
<td>0.993(^a)</td>
<td>0.985</td>
<td>0.969</td>
<td>0.931</td>
</tr>
</tbody>
</table>
values below $R_{\text{m}} - \sigma_R$ for evaluation of the partial factor associated with the uncertainty of the resistance model. For this data group, labeled as $n_{\text{lin}}$, a mean ($R_m$) and standard deviations ($\sigma_R$ and $\sigma_{\Delta}$) were computed which were subsequently inserted into Eqs. (17)-(19) to arrive at a $\gamma_{Rd}$-value. The sample size of $n_{\text{lin}}$ was set at a minimum value of 20. A similar approach was used in Ref. [16].

For the present study it is investigated whether there exists a substantial difference between $\gamma_{Rd}$-values computed using the full set of data points and $\gamma_{Rd}$-values based on a smaller subset or lower tail in accordance with [11].

In Fig. 8 the data points are plotted on the vertical axis and their theoretical quantiles for a normal distribution on the horizontal axis for HD
400 × 1202 and HL 920 × 1377 sections failing by weak-axis and strong-axis buckling. The theoretical values are based on the proposed buckling curve as listed in Table 5. The corresponding partial factors as deduced from the full set and lower tail are presented in Table 6. Based on the small difference between $\gamma_{Rd}$-values from the full data set and the lower tail it can be concluded that the former set can be used to evaluate the partial factors associated with the uncertainty of the resistance model.

### 5.2. Statistical characteristics

From the selected buckling curves as presented in the previous section, histograms are constructed for the ratio between the finite element solution results and the theoretical flexural buckling resistance for the full data set to gain insight into its statistical characteristics. The corresponding histograms are given in Fig. 9 for all HD and all HL sections in addition to the corresponding mean value $\gamma_m$ and standard deviation $\sigma_t$ for a normal distribution.

For HD sections failing by strong-axis buckling the standard deviation and mean are lower when compared to the same sections for weak-axis buckling. Although the dispersion is greater for the weak-axis buckling case for HD sections, all values are on the safe side (i.e. $t_e/\tau_t > 1.0$). The proposed buckling curves for HL sections yield a greater dispersion and lower mean for the weak-axis buckling case. As for HL sections failing by strong-axis buckling no values for $r_e/\tau_t < 1.0$ are found.

### 5.3. General partial factor

The presented analyses were limited to the computation of $\gamma_{Rd}$-values. No definite buckling curve can be suggested as no expression for $\gamma_m$, the partial factor for the material properties, is yet available which is necessary to arrive at the general partial factor $\gamma_{M1}$ (Eq. (2)). The $\gamma_{M1}$-value can be obtained from statistical analyses conducted on a database containing the yield stress for a wide set of coupon tests on HISTAR 460 sections.

As the yield stress of a single coupon can never be lower than the nominal value for HISTAR 460, as this would lead to the member being rejected, it can be reasonably assumed that a $\gamma_{M1}$-value of 1.0 is a conservative value to account for the variability of the material properties. As soon as a database becomes available containing the yield stress from a wide set of coupon tests a more accurate value of $\gamma_{M1}$ can be obtained.

Assuming that $\gamma_{M1} = 1.0$ is the target value for the general partial factor, then the criterion for choosing a buckling curve is that $\gamma_{M1} < 1.05$. Together with the conservative assumption of $\gamma_m = 1.0$, the buckling curves as presented in Table 5 are proposed for the design of heavy wide flange HISTAR 460 sections for which flexural buckling is the governing design criterion.

### 5.4. Application to other steel grades

The residual stress model from Ref. [6] as used in the present analyses is representative for any heavy wide flange section having similar cross-sectional dimensions and made with the quenched and self-tempered process. As such, the residual stress model can be used to define the initial stress state in heavy sections made from grade S460 and S500 as these are manufactured with identical methods by ArcelorMittal as HISTAR 460 steel. Grade S500 also has the same nominal yield stress after reduction to account for material thickness effects.
(i.e. 450 N/mm², see Fig. 1) and therefore the presented γrd-values are equally applicable for heavy wide flange sections made from this grade.

For heavy wide flange sections made from grade S460 a lower yield stress (i.e. 385 N/mm²) must be taken into account as a greater reduction in material properties is present for large thicknesses when compared to HISTAR 460. Provided the manufacturing process is the same for both steels, the residual stresses will have similar magnitudes in absolute terms, which implies that the normalized residual stress (i.e. residual stress normalized against nominal yield stress) is greater for S460 than for HISTAR 460. As the flexural buckling resistance is strongly related to normalized residual stress values at critical locations (i.e. flange tips) it can be expected that the current buckling curves and accompanying γrd-values for HISTAR 460 are too optimistic for heavy wide flange S460 sections.

The aforementioned extension of research results to other grades only pertains to the derivation of the partial factors associated with the uncertainty of the resistance model γm. Each grade will have its own unique characteristics in terms of material variability, and will therefore attain different γm-values, which will affect the general partial factor γM1.

6. Conclusions

In this paper buckling curves are proposed to check the flexural buckling resistance of heavy wide flange quenched and self-tempered (QST) columns. These sections, currently manufactured under the proprietary name HISTAR (HIgh-STrength ARcelorMittal) by ArcelorMittal, which have a flange thicker than 100 mm and a height-to-width (h/b) ratio greater than 1.2 are not covered by Eurocode 3 (EN 1993–1–1). A database was created containing the elastic-plastic buckling resistance for a wide set of heavy HISTAR 460 columns (both the stocky HD type and slender HL type, having a nominal yield stress of 450 N/mm²) failing by weak-axis and strong-axis buckling. The buckling resistance was evaluated using non-linear finite element analyses where an earlier proposed residual stress model as described in a related paper was used to define the initial stress state.

The numerical buckling loads were compared against theoretical values, where the latter corresponds to the buckling resistances for a selected buckling curve according to EN 1993–1–1. Based on the ratio between both values, a partial factor γrd associated with the uncertainty of the resistance model was evaluated according to Annex D of EN 1990 for each of the five buckling curves. Aiming at a target value γrd = 1.0, meaning that the resulting γrd-values should not exceed 1.05, it was found that the elastic–plastic buckling resistance of HD sections failing by weak-axis buckling is best represented by buckling curve “b.” HD sections failing by strong-axis buckling were assigned to buckling curve “a.” HL sections should be designed according to buckling curve “a” or “b” when failing by strong-axis buckling or weak-axis buckling, respectively.

Pending the availability of a database containing the yield stress of a wide set of coupon tests from heavy wide flange HISTAR 460 sections a conservative value of 1.0 for γm is suggested to render the suggested buckling curves applicable for buckling checks. In that case the computed partial factor γrd equals the general partial factor γM1.

References