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Citation for published version (APA):

Document status and date:
Published: 01/01/2013

Publisher Version:
Publisher’s PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:
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Download date: 20. Sep. 2020
Heat flux scaling in turbulent Rayleigh-Bénard convection with an imposed longitudinal wind

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We present a numerical study of Rayleigh-Bénard convection disturbed by a longitudinal wind. Our results show that under the action of the wind, the vertical heat flux through the cell initially decreases, due to the mechanism of plumes-sweeping, and then increases again when turbulent forced convection dominates over the buoyancy. As a result, the Nusselt number is a non-monotonic function of the shear Reynolds number. We provide a simple model that captures with good accuracy all the dynamical regimes observed. We expect that our findings can lead the way to a more fundamental understanding of the of the complex interplay between mean-wind and plumes ejection in the Rayleigh-Bénard phenomenology.

Thermal convection plays an important role in many geophysical, environmental and industrial flows, such as in the Earth’s mantle, in the atmosphere, in the oceans, to name but a few relevant examples. In particular, the idealized case of Rayleigh-Bénard (RB) convection occurring in a layer of fluid confined between two differentially heated parallel plates under a constant gravitational field has been extensively studied [1–4]. However, in several real-life situations, the picture can be much more complex with horizontal winds perturbing natural convection. In the atmosphere, for instance, this competition plays a crucial role in the formation of thermoconvective storms [5]. On the other side buoyancy effects can be relevant in a number of industrial processes based on forced convection, such as coiled heat exchangers [6]. Similarly, a combination of forced and natural convection is present in indoor ventilation applications [7, 8].

According to the standard picture at the basis of the existing models for the scaling laws of the heat flux [10–12], the RB system is characterized by the multi-scale coupling between large-scale circulation (mean wind) and detaching thermal structures from the boundary layers at the walls (plumes). Besides the above mentioned motivations, applying a mean wind to a natural convection setup can shed light on the effect of bulk flow on the boundary layer dynamics, thus helping to better understand one of the most intriguing feature of RB convection. In this Letter we report on a numerical study of RB convection with an imposed constant horizontal pressure gradient, orthogonal to gravity, that induces the wind (the so-called Poiseuille-Rayleigh-Bénard (PRB) flow setup [13, 14]). We show that the heat transfer from the walls can be dominated by either the buoyancy or by the “forced” convection and that the interplay of the two mechanisms gives rise to a non-trivial dependence of the Nusselt number, Nu, on the parameter space that is spanned by the Rayleigh, Ra, and shear Reynolds numbers, Reτ (quantifying, respectively, the intensity of

FIG. 1. (top panel) Snapshot of the temperature field for the pure RB case (Reτ = 0) at Ra = 1.3 × 10^7. (bottom) Snapshot of the temperature field for the PRB case (Reτ = 92) at Ra = 1.3 × 10^7 and Reτ = 92. Notice that, unlike figure ??, where a buoyant plume detaching from the bottom boundary layer can easily enter the bulk up to the top plate while, here plumes are considerably distorted in the direction of the wind.
buoyancy and of the pressure gradient relatively to viscous forces).

Our main result consists in the observation that, taken a standard RB system as reference, $Nu$ initially decreases and then, when the dynamics is completely dominated by the forced convection regime, it increases again with $Re_T$. A phenomenological explanation for this behaviour is provided together with discussions on the possible implications for the modelling of the $Nu$ vs $Ra$ relation in pure natural convection setup.

The equations of motion for the fluid velocity, $u$, and temperature, $T$, are:

$$\partial_t u + u \cdot \nabla u = -\frac{1}{\rho} \nabla P + \nu \nabla^2 u + \alpha g T + f \quad (1)$$

$$\partial_t T + u \cdot \nabla T = \kappa \nabla^2 T, \quad (2)$$

in addition to the incompressibility condition, $\nabla \cdot u$. The properties of the fluid are $\rho$ the (assumed constant) fluid density, $\nu$ the kinematic viscosity, $\alpha$ the thermal expansion coefficient, and $\kappa$ the thermal diffusivity. $P$ is the pressure field, $\mathbf{g} = g\hat{\mathbf{z}}$ the gravity and $f$ a forcing term of the form $f = (F/\rho)\hat{\mathbf{z}} \equiv \vec{F} \hat{\mathbf{z}}$ ($\hat{\mathbf{z}}$ is the direction parallel to the walls, or stream-wise direction). Equations (1) and (2) are evolved using a 3d thermal lattice Boltzmann algorithm [15][16] with two probability densities (for density/momentum and for temperature, respectively). As mentioned in the introduction, to characterize the dynamics we need two parameters: the Rayleigh number, $Ra$, quantifying the strength of buoyancy (with respect to viscous forces),

$$Ra = \frac{\alpha g \Delta H^3}{\nu \kappa},$$

(where $\Delta = T_{\text{hot}} - T_{\text{cold}}$ is the temperature drop across the cell and $H$ the cell height), and the shear Reynolds number $Re_T$,

$$Re_T = \frac{H}{2\nu} \sqrt{\frac{\dot{F} H}{2}}.$$

We performed several runs (in a computational box of size $256 \times 128 \times 128$, uniform grid; see figure for snapshots of the temperature field in the simulation cell), exploring the two dimensional parameter space $(Ra, Re_T)$, within the ranges $Ra \in [0; 1.3 \times 10^6]$ and $Re_T \in [0; 205]$; the Prandtl number $Pr = \nu / \kappa$ is kept fixed and equal to one.

A typical key question in RB studies is how the dimensionless heat flux through the cell, $Nu$, varies as a function of the Rayleigh number:

$$Nu(z) = \frac{\overline{w_z T}(z) - \kappa \partial_z \overline{T}(z)}{\kappa \Delta} = \text{const} \equiv Nu \quad (3)$$

with $Ra$; here and hereafter the overline indicates a spatial (over planes $z = \text{const}$) and temporal (over the statistically stationary state) average. The second and third equalities (which state that $Nu$ is constant with $z$) follow from taking the average of equation (2). In our setup in addition to buoyancy there is the longitudinal pressure gradient which affects the heat flux. We therefore focus on the dependence of $Nu$ on the two-dimensional parameter space $(Ra, Re_T)$; in figure we plot $Nu$ as a function of $Re_T$ for various fixed $Ra$. We find that, for moderate/high $Ra$, the effect of the lateral wind is to quench the buoyancy driven convection, and thus $Nu$ decreases with $Re_T$. For very low $Ra$, below the critical Rayleigh number $Ra_c$, the dynamics of the flow is instead completely dominated by the forced convection and thus $Nu$ increases with the $Re_T$. In figure we show the $Ra = 0$ case. Correspondingly, at increasing $Re_T$ the mean temperature profiles (see figure) show a bending in the bulk and a decrease of the gradient in the boundary layer. Our interpretation of these observations is that, for small $Re_T$/high $Ra$ (i.e. in the natural convection dominated regime), the wind acts essentially sweeping away thermal plumes (which are mixed and lose their coherence closer

\[ insert figure \]
to the walls) and hence the heat flux is depleted. Increasing \( Re_\tau \) more and more we eventually reach a state where buoyancy becomes irrelevant. Here, \( Nu \) starts to increase again by resuspension of temperature puffs in the bulk due to bursts from the wall emerging because of the turbulent channel flow. To give an indication of the validity of such a conjecture we have measured the following quantity:

\[
\phi_i(z) \equiv \frac{\langle \partial_z u_i \rangle^2}{\left( \frac{\partial_z u_i}{} \right)^2},
\]

where \( \delta u_i \equiv u_i(x + \ell, y, z; t) - u_i(x, y, z; t) \). The observable \( \phi_i(z) \) is the ratio of a generalized second order transverse over longitudinal structure function and, as such, it serves as a sort of scale-dependent anisotropy indicator: a large value of \( \phi_i(z) \) means a coherent motion in the wall-normal direction. In figure 4, we plot \( \phi_i(z) \) on a large scale (\( \ell \approx H \)) and on a scale of the order of the thermal boundary layer thickness (\( \ell = \lambda_0 \)), which gives an estimate of a characteristic size of plumes, for natural convection (\( Ra = 6.5 \times 10^6, Re_\tau = 0 \)) and for a case with the wind (\( Ra = 6.5 \times 10^6, Re_\tau = 205 \)). For the pure RB case \( \phi_{\ell=H}(z) \) grows to large values in the bulk, due to the thermal wind, while \( \phi_{\ell=\lambda_0}(z) \) goes to the isotropic value \( \phi \approx 2 \) in the bulk and it is larger than \( \phi_{\ell=H}(z) \) close to the wall, pointing out the presence of detaching plumes. The same quantity \( \phi_{\ell=\lambda_0}(z) \) in the wall-proximal region is significantly smaller for \( Re_\tau = 205 \), indicating the depletion of plumes ejection.

With this picture in mind we are now going to build a model to recover the numerical findings. Our argument goes as follows. As shown in figure 3 under the action of the lateral wind the temperature profile ceases to be flat in the bulk. This permits us to write a first order closure for the turbulent heat flux of the kind:

\[
\bar{u}_z T = -\kappa_T \partial_z T,
\]

where \( \kappa_T \) is a turbulent diffusivity. When writing [5], where \( \kappa_T \) is constant with \( z \), we are implicitly restricting ourselves to the bulk region (where the mean temperature gradient is basically constant); we are allowed to do that by [3], i.e. the constancy of the heat flux through planes parallel to the walls. The Nusselt number will assume the form:

\[
Nu \sim \left( 1 + \frac{\kappa_T}{\kappa} \right) \left| \frac{\partial_z T}{\Delta/ H} \right|.
\]

We consider that two types of structures contribute to turbulent diffusion, namely buoyant plumes (\( \kappa_T^{(P)} \)) and bursts (\( \kappa_T^{(B)} \), triggered by the turbulent channel flow), so that we may write

\[
\kappa_T = \kappa_T^{(P)} + \kappa_T^{(B)}.
\]

As previously discussed we attribute the heat flux reduction to the sweeping of plumes by the wind; we model this saying that the plume loses its coherence (or else, it releases its heat content) after travelling a distance from the wall of the order of the kinetic boundary layer thickness (\( \lambda_u \)), i.e. we suggest that we can adopt a Prandtl mixing length (\( \ell_m \)) theory kind of approach, using

\[
\ell_m \sim \lambda_u;
\]

the latter relation should be interpreted as a scaling (or proportionality) relation rather than an order of magnitude. The characteristic velocity of a rising plume reaching a height \( \sim \ell_m \) can be estimated as \( u \sim \sqrt{\alpha g \Delta T_m} \).
hence the contribution to the turbulent diffusion will be
\[ \kappa_T^{(p)} = \sqrt{\alpha g \Delta \lambda_u^{3/2}} \tag{9} \]
and assuming a laminar boundary layer of Blasius type of thickness \( \lambda_u \sim \frac{H}{\Re} \tag{10} \)
we get
\[ \kappa_T^{(p)} \sim \sqrt{\alpha g \Delta \frac{H^{3/2}}{\Re^{3/2}}} \tag{11} \]
For a turbulent burst one can also assume that \( \ell_m \sim \lambda_u \), but the expression for the characteristic advecting velocity requires some more care. In a pure forced convection setup (or in our case when the wind is dominant) there is no buoyancy, so we cannot use the expression of the free-fall velocity; instead convection is driven by turbulent fluctuations from the wall. Invoking again the mixing length theory for a first order closure for the velocity we can write \( u_z \sim \ell_m \partial_x \bar{U}_x \), whence
\[ \kappa_T^{(b)} \sim \ell_m^2 \partial_x \bar{U}_x \sim \lambda_u^2 \partial_x \bar{U}_x \tag{12} \]
estimating the shear as \( \partial_x U_x \sim U_c/\lambda_u \) (\( U_c \) being the centreline velocity) we get
\[ \kappa_T^{(b)} \sim \lambda_u U_c \tag{13} \]
In the limited range of \( \Re \) that we span it is reasonable to assume that the friction coefficient goes as \( C_f \sim \Re^{-2} \), hence that \( U_c \) scale as \( U_c \sim (\nu/H) Re_f^2 \tag{19} \). Inserting this scaling law together with the relation \( \Re = \frac{\Re \lambda^2}{\nu} \) inside the expression for \( \kappa_T^{(b)} \) we obtain
\[ \kappa_T^{(b)} \sim \nu \cdot \Re \tag{14} \]
Putting the expressions \( \Re = \frac{\Re \lambda^2}{\nu} \) and \( \Re = \frac{\Re \lambda^2}{\nu} \) inside equation \( \Re \) we end up with
\[ Nu - 1 \sim \left( A_1 \frac{\sqrt{\alpha g \Delta H^{3/2}}}{\kappa Re_f^{3/2}} + A_2 \frac{\nu Re_f}{\kappa} \right) \frac{|\partial_x T|}{(\Delta/H)} \tag{15} \]
which can be recast, introducing the dimensionless numbers \( Ra \) and \( Pr \), into the following form:
\[ Nu - 1 \sim \left( A_1 \frac{Ra^{1/2} Pr^{1/2}}{Re_f^{3/2}} + A_2 Pr Re_f \right) \frac{|\partial_x T|}{(\Delta/H)} \tag{16} \]
where \( A_1 \) and \( A_2 \) are two free parameters of the model. Some comments on equation \( \Re \) are in order. Firstly, it reproduces the non-monotonic dependence of the heat flux, \( Nu \), on the applied wind, \( \Re \), and it turns out to be in fair agreement with the numerical data (see figure \( \Re \)). Secondly, it provides an argument for the scaling \( Nu \sim \Re \) for the case of pure forced convection (\( Ra = 0 \), see figure \( \Re \)). It is interesting to note that, for very small \( Re \), our model would give a scaling \( Nu \sim Ra^{1/2} \), i.e. what expected for Kraichnan’s ultimate regime of convection \( \Re \).

The phenomenology behind it suggests that the lower heat flux observed in the standard RB convection (with respect to \( Ra^{1/2} \)) may be seen as the result of a negative feedback of the shear, due to the large scale circulation, on the plumes detaching from the boundary layer. Indeed, if we imagine the Nusselt number to follow a Kraichnan scaling on an effective Rayleigh \( Ra_{eff} \), renormalized by turbulent viscosity and thermal diffusivity (behaving as \( \Re \)), that is
\[ Ra_{eff} = \frac{Ra}{(\nu/T)(\kappa_T/\kappa)} \tag{17} \]
we end up with the following relation
\[ Nu \sim Ra_{eff}^{1/2} \equiv Ra^{1/2}/Re \cdot \Re \tag{18} \]
If we now insert into \( \Re \) the ultimate regime scaling for Reynolds \( Re \sim Ra^{1/4} \), we obtain
\[ Nu \sim Ra^{1/4} \tag{19} \]
a well known scaling, predicted theoretically and found in a vast number of experiments (see \( \Re \) and references therein). Let us, finally, remark that equation \( \Re \) should not be expected to be valid for \( Re \rightarrow 0 \), since in this case the mean temperature gradient is zero and a closure like \( \Re \) does not apply \( \Re \). In particular we detect a region where the sweeping mechanism is not yet effective and \( Nu \) decreases slowly with \( Re \); we denote the shear Reynolds number at which the crossover between such region and the \( Nu \sim Re_{\epsilon}^{-3/2} \) regime takes place as \( Re_{\epsilon} \). We argue that such crossover can be determined under the condition that the characteristic velocity of a rising plume, \( U_{\epsilon} = \sqrt{g \Delta H} \), be of the same order of the centreline velocity of the Poiseuille flow, \( U_{\epsilon} = \sqrt{F H^2/\nu} \). Equating these two latter relations we have
\[ \sqrt{\alpha g \Delta H} \sim \sqrt{F H^2/\nu} \tag{16} \]
which gives, in dimensionless form and introducing the crossover Reynolds,
\[ Re_{\epsilon} \sim Ra^{1/4} \tag{19} \]
This results is compared with the numerical data in the inset of figure \( \Re \). For \( Re \geq Re_{\epsilon}^{*} \) the longitudinal flow is still laminar (notice that the Nusselt number for \( Ra = 0 \) remains equal to one) and represents just a small disturbance to the buoyant circulation. The initial fall-off of \( Nu vs Re \) can be captured by looking at the conservation equation for the total energy, which can be derived from \( \Re \) and \( \Re \) to be \( \Re \)
\[ \varepsilon = (Nu - 1) Ra + 8Re_{\epsilon}^2 (u_z), \tag{20} \]
where $\langle \cdots \rangle$ denotes an average over the entire volume, $\varepsilon = \langle (\partial_i u_j)^2 \rangle$ is the kinetic energy dissipation rate and we set $Pr = 1$. It is clear that $\langle u_2 \rangle \sim U^{(P)} \sim Re_\tau^2$ ($\langle U^{(RB)} \rangle \sim 0$). Since $U^{(RB)} \gg U^{(P)}$ the longitudinal wind perturbs the RB dynamics only slightly so that $\varepsilon \approx \varepsilon^{(RB)}$. From (20) we therefore derive

$$Nu \approx Nu_0(Ra) - (A_3/Ra)Re_\tau^3,$$  \hfill (21)

where $Nu_0$ is the Nusselt number for $Re_\tau = 0$, i.e. pure RB, and $A_3$ is an order one constant. Equation (21) is plotted in figure 2 for three different $Ra$ showing good agreement with the numerics up to the expected crossover shear Reynolds $Re_\tau^*$. We have performed direct numerical simulation of Rayleigh-Bénard convection with an imposed longitudinal pressure gradient inducing a mean wind. We found that the Nusselt number has a non-monotonic dependence on the shear Reynolds number based on the applied pressure drop: to an initial decrease (justifiable in terms of a mechanism of sweeping of plumes by the longitudinal wind) an increase follows, when the dynamics is dominated by the turbulent “forced convection” regime. Based on these empirical concepts, we provided a correlation which proved able to recover the numerical findings with reasonable accuracy. The observations and the modelling give a hint that, in standard RB convection, the shear due to the large scale circulation may act back onto the boundary layer against the ejection of plumes to the bulk (thus being a possible mechanism for the depletion of heat transfer respect to the ultimate state of turbulent convection). Our work is a first attempt to look directly at the effect of disturbing in a controlled manner the dynamics of the boundary layer in such a way to give an insight of its role in natural convection. A possible follow-up of the present study is to use a perturbation other than a simple Poiseuille flow.

Acknowledgements. We thank R. Benzi, P. Roche, R.P.J. Kunnen, F. Zonta and P. Ripesi for useful discussions and L. Bouhali for careful reading of the manuscript. AS and AG acknowledge financial support from the Icelandic Research Fund. AS acknowledges FT and the Department of Mathematics and Computer Science of the Eindhoven University of Technology for the hospitality.

[17] We approximate that the plume still feels an acceleration proportional to the temperature difference $\Delta$, i.e. that the bending of the thermal short-cut is small.
[19] More precisely our simulations suggest something closer to $U_\tau \sim Re_\tau^{1.9}$.
[20] The scaling is $Re \sim Ra^{1/2}$ and then, since $Re_\tau \sim Ra^{1/2}$, we get $Re_\tau \sim Ra^{1/4}$.
[21] In principle there would be an extra dependence of $Nu$ on $Re_\tau$ in equation (16) stemming from $\partial T / (\Delta / H) \sim f(Re_\tau)$, which, however, turns out to be a subdominant correction, only becoming relevant for very low $Re_\tau$.
[22] We assume that the energy dissipation rate is not affected by the longitudinal wind, at least in its boundary layer contributions which are dominant in this regime (our numerical simulations confirm this picture).