Intensified heat transfer in modulated rotating Rayleigh–Bénard convection

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Heat transfer in a Rayleigh–Bénard configuration consisting of a vertical cylinder, which is rotating about its axis, can be intensified considerably when the rotation rate is modulated harmonically in time. Such time-dependent rotation introduces an Euler force into the governing equations which leads to a particular modification of the flow that is shown to support a Nusselt number (Nu) that is considerably higher than in case of constant rotation. We use direct numerical simulation of the incompressible Navier–Stokes equations to perform a comprehensive parameter study of the flow-structuring and associated heat transfer investigating primarily the effect of variations in the frequency with which the rotation rate varies. We consider flow in an upright cylinder of unit aspect ratio which is heated from below and cooled at the top. At sufficiently strong Euler forces the temporal variation of Nu shows a striking dynamics with periods of gradual increase in Nu with more rapid oscillations superimposed, next to rather catastrophic events in which the entire flow-structure that supported high levels of Nu collapses entirely and it returns to a value more similar to that attained at steady rotation. During periods of oscillatory build-up of Nu, high levels of turbulence gradually become more pronounced from the outer cylinder wall inward and a gradually stronger thermal column arises along the centreline of the cylinder. This flow structure can support Nu up to 250% larger than without rotation, a value otherwise achievable only by employing phase transition.

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1. Introduction

Many flows in nature and technology are simultaneously driven by buoyant convection as well as influenced by rotation. Typical examples are found in the large-scale geophysical flows in the atmosphere and the oceans on our Earth. Understanding these is essential for predicting heat and mass transfer, and hence essential for weather and climate predictions. Also in many technological applications buoyancy and rotation are key mechanisms, e.g., in cooling or controlled crystal growth. A simple model that captures the dynamic consequences arising from interactions between these central mechanisms is found in rotating Rayleigh–Bénard convection: a fluid layer enclosed vertically between parallel rotating walls is heated from below and cooled from above (Stevens et al., 2013). In recent years much research was devoted to turbulent convection in Rayleigh–Bénard configurations rotating about the vertical axis. This volume of work deals with experimental, theoretical and simulation studies into effects on the heat transfer at constant rotation rate. Here we extend this study with an hitherto unexplored aspect, i.e., that of effects on the heat transfer due to time-varying rotation rate.

We focus on the question to what extent time-modulated rotation affects the heat transport properties of a cylindrical Rayleigh–Bénard system. A first, unique physical experiment was carried out by Niemela et al. (2010) producing interesting results (Niemela et al., 2010). Heat transport measurements at constant as well as at periodically varied rotation rates were conducted in a wide range of physical conditions, with cryogenic helium gas as working fluid. While no enhancement of heat transport was observed at constant rotation rate, the use of time-periodic variations marked a sharp transition toward much more efficient transport of heat. In these experiments the heat transfer could be measured but no access to associated flow structuring was available. The current numerical study addresses a lower Rayleigh number and adopts water as working fluid. We also establish a significant increase in the heat transport efficiency under time-modulated rotation conditions. At the same time the direct numerical simulations are supported by...
intended to qualitatively complement the experimental work and also shed light on the fluid-mechanical structures that arise during periods of enhanced heat transfer.

The inclusion of a time-dependent rotation rate expressed by the so-called Euler force may qualitatively alter the flow from a condition of a domain filling large-scale circulation (LSC; occuring at low constant rotation rates) or dispersed local thermal plumes (at high constant rotation rates) (Stevens et al., 2013; Kunnen, 2008; Kunnen et al., 2008, 2006, 2010; Stevens et al., 2009), to a more or less segregated situation in which a pronounced thermal column forms along the centreline of the domain and highly sheared structures appear in the boundary layer near the vertical sidewalls. The ability to manipulate these flow structures allows to influence small-scale mixing (Geurts, 2001) and heat transfer characteristics. A dominant Euler force yields very complex flow dynamics in which a long-time build-up of thermal structures arises in an oscillating manner, interspersed by events of very abrupt and considerable collapse with associated strong reduction of the thermal transport efficiency, as quantified by the Nusselt number Nu. This presents an interesting challenge in physical control of such turbulent flow (Kuczaj et al., 2006), aimed at building up high-Nu flow structures by modulated rotation, but avoiding the Nu-collapse.

The organisation of this paper is as follows. We first present the governing equations and discuss the numerical method in Section 2. Subsequently, in Section 3, the effect of time-modulated rotation is shown in terms of the changes in the turbulent flow structures that arise. The consequences for the transport of heat are discussed afterwards in Section 4 and the paper is completed with concluding remarks in Section 5.

2. Governing equations and numerical method

The main aspects governing flow in a rotating Rayleigh–Bénard cylinder are (i) the effect of buoyancy associated with differences in the mass density arising from differences in the temperature and (ii) the rotation itself, which will be expressed in terms of the commonly included Coriolis force, which expresses the value of the rotation rate, and the less familiar Euler force, which arises due to temporal variations in the rotation rate. In this section we outline the governing equations and describe the mathematical model, including the domain and boundary conditions. Subsequently, we pay attention to the numerical method that was adopted and motivate the spatial resolution that was used for the direct numerical simulation.

We consider cylindrical domains filled with water, of height $H$ and diameter $D = H$, i.e., the aspect ratio $I = D/H = 1$. The domain is allowed to rotate about the vertical ($z$) axis with rotation rate $\Omega(t) = \Omega(t)\hat{z}$ where $\hat{z}$ denotes the unit vector in the $z$ direction. We consider the flow in a co-rotating coordinate frame $\{\hat{x}, \hat{y}, \hat{z}\}$ rotating with $\Omega(t)$ relative to the inertial frame $\{X, Y, Z\}$. The relation between the velocity in the inertial frame of reference (subscript ‘I’) and that in the rotating frame (subscript ‘R’) can be expressed as

$$\left(\frac{d}{dt}\right)_I = \left(\frac{d}{dt}\right)_R + (\Omega(t) \times \mathbf{r}) \quad (1)$$

where the position vector is expressed as $\mathbf{r} = r_x\hat{x} + r_y\hat{y} + r_z\hat{z}$. For the acceleration one may derive

$$\left(\frac{d^2\mathbf{r}}{dt^2}\right)_I = \left(\frac{d^2\mathbf{r}}{dt^2}\right)_R + 2\Omega(t) \times \left(\frac{d\mathbf{r}}{dt}\right)_R + \underbrace{\Omega(t) \times \Omega(t) \times \mathbf{r}}_{\text{centrifugal}} + \underbrace{\frac{d\Omega(t)}{dt} \times \mathbf{r}}_{\text{Euler}} \quad (2)$$

in which we explicitly distinguished the Coriolis, centrifugal and Euler forces respectively. In incompressible flow it is common to absorb the centrifugal force into the pressure term. For rotation about the $z$ axis the Euler force can be shown to yield a contribution to the circumferential momentum transport equation only, while for turbulent 3D flow the Coriolis force is making itself felt more in all coordinate directions.

In the Boussinesq approximation the governing equations that describe incompressible, buoyant flow in a co-rotating frame of reference can be written as $\nabla \cdot \mathbf{u} = 0$ for the conservation of mass, and

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + g\alpha \nabla T + \nu \nabla^2 \mathbf{u} - 2\Omega(t)\hat{z} \times \mathbf{u} - \mathbf{r} \times \frac{d\Omega(t)}{dt} \hat{\theta} \quad (3)$$

Moreover, the transport of heat is characterised by an advection–diffusion equation, given by

$$\partial_t T + (\mathbf{u} \cdot \nabla) T = \kappa \nabla^2 T \quad (4)$$

In this formulation, we absorbed the centrifugal contribution into $\nabla p$ where $p$ denotes the total dynamic pressure. Moreover, $\mathbf{u}$ denotes the velocity field, $g$ the gravitational acceleration, $\alpha$ the thermal expansion coefficient, $T$ the temperature, $\nu$ the kinematic viscosity and $\kappa$ the thermal diffusivity.

The time-modulation of the rotation rate is taken in this paper as

$$\Omega(t) = \Omega_0 + \Delta \Omega \sin (\omega t) \quad (5)$$

where we denote the mean rotation rate by $\Omega_0$, the depth of modulation by $\Delta \Omega$ and the frequency of modulation by $\omega$. This harmonic variation of the rotation rate is a first choice of perturbation protocol. More general acceleration procedures can be selected in order to introduce specific modulation patterns and indirectly achieve some control over the transport properties in the flow. The formulation as given in (5) can equivalently be expressed in terms of three ‘Rossby numbers’. In fact, multiplying (5) by $2H/U$ we find

$$\frac{1}{R_{00}}(t) = \frac{1}{R_{00}} + \frac{1}{R_0} \sin \left(\frac{t}{2R_{00}}\right) \quad (6)$$

where

$$R_{00} = \frac{U}{2H\Omega_0}; \quad R_0 = \frac{U}{2H\Delta \Omega}; \quad R_{00} - \frac{U}{2H\Omega_0} \quad (7)$$

These parameters can be varied independently, giving detailed control over the precise flow regime that dominates the turbulent transport. In this paper the primary interest lies with the dynamics induced by the Euler force and hence we will consider variation of $R_{00}$, keeping the ‘mean’ $R_0$ and ‘modulation depth’ $R_{00}$ fixed. The specific choice for $R_{00}$ and $R_0$ will be motivated momentarily.

It is convenient to present the final computational model in non-dimensional form. We use the convective velocity scale $U = \sqrt{g\alpha \Delta T H}$, the temperature scale $\Delta T$, which is the temperature difference between the bottom and the top wall, and the length scale $H$. The following dimensionless groups can be used to characterise the flow: $Ra = (g\alpha \Delta T H)/(\nu/k)$ the Rayleigh number, $\sigma = V/k$ the Prandtl number, and $R_{00}, R_0, R_{00}$. These dimensionless numbers control the flow. The final dimensionless form of the governing equations can be expressed as:

$$\nabla \cdot \mathbf{u} = 0$$

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + T\hat{z} + \sqrt{\frac{\sigma}{R_{00}}} \nabla^2 \mathbf{u} - \underbrace{\frac{1}{R_{00}} \sin \left(\frac{t}{2R_{00}}\right)}_{\text{constant}} T\hat{z} \times \mathbf{u} - \frac{1}{4R_0 R_{00}} \cos \left(\frac{\pi}{2R_{00}}\right) \mathbf{r} \hat{\theta} \quad (8)$$
These equations are defined in a cylindrical domain of equal height and diameter $H$. The boundary conditions are taken as no-slip $u = 0$ at all the walls of the domain and, $T = 1$ at the bottom plate, $T = 0$ at the top plate and adiabatic conditions $\partial T / \partial z = 0$ at the sidewall.

The dynamics of turbulent flow in a rotating Rayleigh–Bénard cylinder under time-modulated conditions is investigated using direct numerical simulations (DNS). These are based on an extension of the method by Verzicco and Orlandi (1996) and Oresta et al. (2007) to also include the Euler force. We consider as working fluid pure water with Prandtl number $Pr$ and investigate turbulent flow at $Ra = 10^5$, identical to the value used in the study of Kunnen et al. (2008, 2010), in order to facilitate comparison with the constant rotation case.

The rate at which heat can be transferred in a Rayleigh–Bénard setting has been subject of intensive research. A main feature of the method by Verzicco and Orlandi (1996) and Oresta et al. (2007) to also include the Euler force. We consider as working fluid pure water with Prandtl number $Pr$ and investigate turbulent flow at $Ra = 10^5$, identical to the value used in the study of Kunnen et al. (2008, 2010), in order to facilitate comparison with the constant rotation case.

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3. Qualitative flow-structure changes under modulated rotation conditions

We first present illustrations of the flow at constant rotation rate, providing a point of reference for the discussion of alterations in the appearance of the flow due to time-dependent rotation rates. Simulations at various constant rotation rates show that the organisation of the flow into coherent structures is strongly dependent on rotation (Kunnen et al., 2008, 2010). For low rotation rates the domain-filling large-scale circulation (LSC) is the dominant feature. At larger rotation rates an irregular, rapidly changing array of vertically oriented vortices is found. This is illustrated in Fig. 1. The turbulence intensity is reduced by strong rotation, compared to the non-rotating case, and the vertical inhomogeneity increases, reflecting the consequences of the thermal wind balance (Kunnen et al., 2008, 2010). In fact, the slender vortical structures at
$R_o = 0.1$ with a torsional structure are clear illustrations of this (for a further discussion we refer to Kunnen et al. (2008, 2010)).

The effect of rotation on the temperature distribution in the domain is equally striking. In Fig. 2 we show snapshots of isosurfaces of the temperature. While the average heat transfer characteristics were found to be nearly identical in terms of the Nusselt number (Kunnen et al., 2010), the structure of the temperature field which underpins this efficiency of heat transfer is very different. In fact, the vortical plumes that are seen in the velocity are also the dominant feature in the temperature field. In contrast, the large-scale circulation does not display a strong influence in terms of flow structuring - the isosurfaces of high and low temperatures are well localised near the upper and lower walls respectively.

The effect of time-modulation of the rotation rate depends strongly on the modulation frequency $R_o$. Taking 'fast' ($R_o = 0.1$) and 'slow' ($R_o = 5$) constant rotation as points of reference, we may traverse from 'slow' to 'fast' rather slowly ($R_o > 1$) or rather quickly ($R_o \ll 1$). This induces quantitative differences in the dominant structures of the flow. In Fig. 3 an overview of temperature and vertical velocity fields is shown characterising the qualitative effects due to changes in $R_o$. The modulation frequency is seen to have only a modest structural effect at $R_o = 0.5$ as can be inferred from Fig. 3(a) and (b). The flow is mainly in a state reminiscent of the LSC state at constant rotation, although an occasional vortical plume is seen in the temperature field and the LSC appears to be somewhat more concentrated near

![Fig. 3. Snapshots of isosurfaces of the temperature (a, c, e) and vertical velocity (b, d, f) displaying the qualitative effect of modulation of the rotation rate. Results are obtained at a mean Rossby number $R_o = 2.45$ and a modulation depth Rossby number $R_o = 2.55$. The Rossby number of the modulation frequency was varied: $R_o = 0.5$ (a, b), $R_o = 0.2$ (c, d) and $R_o = 0.1$ (e, f). Labeling of the contours is as in Figs. 1 and 2.]

![Fig. 4. Time-dependent Nusselt number obtained for time-modulated rotation rates with $R_o = 2.45, R_o = 2.55$ and varying modulation frequencies $R_o$: 0.1 (asterisks), 0.2 (squares), 0.5 (diamonds), 1 (circles). Also included is the constant rotation case at $R_o = 0.1$ (thin solid line). In (a) we show a detailed view for $t \leq 20$ while (b) displays the long-time behaviour, showing for $R_o \leq 0.2$ clear build-up of Nu to rather high values, followed by abrupt collapse.]

the centreline of the cylinder. Increasing the modulation frequency is seen to yield a strong thermal structure centred around the axis of the cylinder, while clear sheared turbulence structures near the vertical sidewalls appear. These shear structures are somewhat aligned horizontally, an effect stimulated by the Euler force which was shown to induce additional flux contributions to the azimuthal momentum transfer only. The ‘middle section’ of the flow domain is occupied by a ‘thermal column’ if \( Ro_{\infty} \) is sufficiently low, in which an almost direct contact between hot and cold fluid arises, thereby creating a basis for an intensified heat transfer to which we return in the next Section. The width of the thermal column is comparable to the domain size. Due to this qualitative change in the flow also effective transport properties such as mixing will be affected. The heat transfer will be quantified in the next Section.

4. Dynamics of heat transfer

We first present the results for the Nusselt number and discuss the peculiar combination of slow and fast dynamics that is observed in case the Euler force is rather strong, i.e., at low values of \( Ro_{\infty} \). Next to a fast modulation on top of a slow, gradually increasing Nusselt number, the simulations display a rather catastrophic collapse of \( Nu \) at certain, quite regularly spaced intervals. We investigate one such collapse of the heat transfer efficiency and show the corresponding qualitative switching that occurs in the flow structures.

In order to quantify the consequences of time-modulated rotation on the efficiency with which heat can be transferred from the hot bottom plate to the colder top plate, we concentrate on the time-dependent \( Nu \) can be obtained is

\[
Nu(t) = \langle \partial_x T \rangle
\]

where \( \langle \cdot \rangle \) denotes averaging over the top and/or bottom wall. At constant rotation rate the dependence of the Nusselt number on the Rossby number was investigated in Kunnen et al. (2010). It was shown that a maximum arises at \( Ro_0 \approx 2.5 \) as which \( Nu(Ro_0)/Nu(\infty) \approx 1.15 \), comparing the Nusselt number in a rotating system to the Nusselt number without rotation, \( Nu(\infty) \). Moreover, if the rotation rate is increased corresponding to \( Ro_0 < 2.5 \) a monotonously decreasing \( Nu \) is found which is approximately equal to \( Nu(\infty) \) as \( Ro_0 \approx 0.1 \). The long-time averaged value of \( Nu \) in the current system was found to be around 72.

In Fig. 4 we show the evolution of the Nusselt number, starting from a well-developed turbulent state obtained for constant rotation rate at \( Ro = 2.45 \). To create a point of reference, the evolution of \( Nu \) at constant rotation rate \( Ro = 0.1 \) is also included as thin solid line. For the particular initial condition that was selected, we observe a strong transient at \( Ro = 0.1 \) after which the system recovers during a transient state of about 20 dimensionless time units. A similarly long transient period was also observed at other \( Ro \) in case of constant rotation rate.

The effect of modulated rotation is already clearly expressed during the first transient stages. For rather low modulation frequencies, e.g., \( Ro_{\infty} = 1 \) we observe a smoothly oscillating dependence of \( Nu \) on time with a long-time averaged value that differs only little from the constant rotation rate situation as can be inferred from comparison with the constant rotation case at \( Ro = 0.1 \) after the transient stage. We observe, however, that an increase in the modulation frequency, i.e., a decrease in \( Ro_{\infty} \), induces qualitative changes. We observe stronger oscillations in \( Nu \) at \( Ro_{\infty} = 0.5 \) without affecting the long-time averaged value much, but at \( Ro_{\infty} = 0.2 \) and even 0.1 we notice the occurrence of rather fast oscillations with significant amplitude on top of a more gradual development of \( Nu \). This is illustrated clearly in Fig. 4(a) when zooming in on the initial transient. Moreover, we notice from this Figure that also the value of \( Nu \) increases considerably above the value at constant rotation rate. Turning attention to the dynamics of \( Nu \) over longer periods we observe this behaviour in its proper time frame. This expresses one more phenomenon, i.e., that of a very rapid collapse of \( Nu \) during very short bursts, with an averaged behaviour somewhat reminiscent of a saw-tooth curve. Such collapse does not yet express itself clearly as \( Ro_{\infty} = 0.5 \) but for \( Ro_{\infty} = 0.1 \) clearly distinguished events of collapse arise. The time between such two collapse events is seen to increase with decreasing \( Ro_{\infty} \) while the magnitude of the collapse increases with increasing modulation frequency. If the modulation frequency is increased sufficiently, e.g., \( Ro_{\infty} < 0.2 \), we notice that

\[
\text{Fig. 5. Illustration of Nusselt collapse events occurring for a time-modulated rotation rate at } Ro_{\infty} = 0.2. \text{ In (a) the time-history of } Nu \text{ is shown and in (b) we zoom in on a particular example of rapid } Nu \text{ collapse.}
\]
the time-averaged Nu is considerably increased and that the
dynamics of Nu is qualitatively altered. In fact, in case the Euler
forces dominate the dynamics Nu oscillates rapidly during slow
build-up phases, reaches high values, i.e., induces an effective flow
structure for thermal transport and collapses vigorously to initiate
the next build-up phase. As may be observed in Fig. 4(b), in case
$Ro_m = 0.1$ peak values as high as 180–190 can be achieved. This
is up to 250% higher than the time-averaged value attained for
the non-rotating case, found to be 72 in Kunnen et al. (2008) at
the same $Ra = 10^6$ and $\sigma = 6.4$. Considering a rough estimate for
the long-time averaged Nu, obtained from approximately averag-
ing over several periods of rather slow oscillatory growth of Nu fol-
lowed by rapid collapse of Nu, one finds values around 140–150 at
$Ro_m = 0.1$. This represents an increase by about 200% compared to
the long-time average in the non-rotating case.

In order to appreciate the observed collapse of the Nusselt num-
er in the rapidly modulated rotation case, we illustrate the case
$Ro_m = 0.2$ in Fig. 5, displaying an overall recurring collapse of Nu
after an extended period of more gradual oscillatory build-up.
The precise time history of the Nusselt number in Fig. 5(b) shows
a reduction in Nu of about 80 units from a high value around 135
to a low value of approximately 55, in about 7 dimensionless time
units. This is quite fast compared to the general period of build-up
of Nu which may be estimated around 35 time units at the selected
parameters.

The reason for this strong reduction in $Nu$ may qualitatively
understood by turning to the large changes in the flow structure,
j ust prior to a collapse, e.g., at $t = 117$, and just after a collapse,
e.g., at $t = 125$. Fig. 6 illustrates the associated qualitative changes
in the flow structure, concerning temperature and velocity. We
notice by comparing Fig. 6(a) and (b) that the thermal column,
which developed over the past $\approx 35$ time units, almost completely
disintegrates and the associated high value of $Nu$ reduces to values
typical of the non-rotating situation. This event coincides with a
situation of high turbulent activity throughout the domain
(Fig. 6(c)) to one in which again most activity is located in the
shear layers near the side walls only (Fig. 6(d)). Currently, we do
not have a full mechanistic interpretation of the complete dynam-
ics associated with such an event of Nu-collapse but present only

![Illustration of a Nusselt collapse event occurring around $t = 120$ for a time-
modulated rotation rate at $Ro_m = 0.2$. In (a, b) the instantaneous solution for the
temperature is depicted in a side view, just before at $t = 117$ (a) and just after at
$t = 125$ the collapse (b). Similarly, in (c, d) we show top views of contours of the
vertical velocity, corresponding to $t = 117$ and $t = 125$ respectively.](image)

the qualitative features. This striking dynamics requires further study,
aimed at understanding how such dramatic collapse can be
prevented, ultimately leading to sustained, much higher levels
of heat transfer, provided suitable physical flow control can be
established.

5. Concluding remarks

In this paper we presented DNS results of time-modulated
rotating Rayleigh–Bénard convection in a cylindrical domain of
unit aspect ratio. The inclusion of a time-dependent rotation rate
introduces an additional term in the equations which represents
the so-called Euler force. This force acts in the circumferential
direction only and may qualitatively alter the flow from a condi-
tion of a domain filling large-scale circulation (at low constant rotation rates) or dispersed local thermal plumes (at high constant
rotation rates), to a more or less segregated situation in which a
pronounced thermal column forms along the centreline of the
domain and highly sheared structures appear in the boundary
layer near the vertical sidewalls. Situations displaying this thermal
column were found to support a strongly increased Nusselt num-
ber, indicating a striking qualitative similarity with findings from
the experimental work reported in Niemela et al. (2010). The
experimental work was conducted at different flow conditions,
e.g., a higher Rayleigh number and a different Prandtl number,
which are currently not accessible for DNS. Ongoing research into
space–time parallel algorithms is dedicated to increase the range of
Rayleigh numbers with the aim to connect also quantitatively to
the experimental findings.

A dominant Euler force was shown to yield very complex flow
dynamics in which a long-time build-up of thermal structures
arises in an oscillating manner, interspersed by events of very
abrupt and considerable collapse with associated strong reduction
of the thermal transport efficiency, as quantified by the Nusselt
number. This presents an interesting challenge in physical control
of such turbulent flow, aimed at building up high-$Nu$ flow struc-
tures by modulated rotation. This is subject of ongoing research
and new agitation procedures will be developed that prevent too
high levels of turbulent velocity fluctuations near the centreline of the
cylinder.

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