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Incorporating space–time constraints and activity-travel time profiles in a multi-state supernetwork approach to individual activity-travel scheduling

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Abstract
Activity-travel scheduling is at the core of many activity-based models that predict short-term effects of travel information systems and travel demand management. Multi-state supernetworks have been advanced to represent in an integral fashion the multi-dimensional nature of activity-travel scheduling processes. To date, however, the treatment of time in the supernetworks has been rather limited. This paper attempts to (i) dramatically improve the temporal dimension in multi-state supernetworks by embedding space–time constraints into location selection models, not only operating between consecutive pairs of locations, but also at the overall schedule at large, and (ii) systematically incorporate time in the disutility profiles of activity participation and parking. These two improvements make the multi-state supernetworks fully time-dependent, allowing modeling choice of mode, route, parking and activity locations in a unified and time-dependent manner and more accurately capturing interdependences of the activity-travel trip chaining. To account for this generalized representation, refined behavioral assumptions and dominance relationships are proposed based on an earlier proposed bicriteria label-correcting algorithm to find the optimal activity-travel pattern. Examples are shown to demonstrate the feasibility of this new approach and its potential applicability to large scale agent-based simulation systems.

1. Introduction

The introduction of activity-based modeling has implied a gradual shift from single trips, via tours, to comprehensive daily activity-travel patterns as the focus of attention. In part, this shift reflects the empirical trend of an increasingly larger share of multi-purpose, multi-stop trips. In addition, the emphasis on activity-travel patterns is the logical consequence of the increased need and necessity to model temporal and spatial interdependencies in the way people organize their daily activities in time and space. Therefore, activity-travel scheduling, which aims to predict which activities are conducted when, where, for how long, with whom, and the transport mode involved, has become a core component of many activity-based models (Arentze and Timmermans, 2004a; Pinjari and Bhat, 2011).

Generally, the scheduling of a given activity program consists of two steps: identifying feasible activity-travel opportunities and finding the (sub-)optimal activity-travel patterns. The identification of feasible opportunities is commonly based on the concept of space–time prism (Hägerstrand, 1970). Scheduling approaches in the literature differ in their mechanisms of deriving feasible or optimum activity-travel patterns. Utility-maximizing models seek the schedule that provides the
highest utility. Examples include Recker (1995), Fuji et al. (1998), Bowman and Ben-Akiva (2001) and Yagi and Mohammadian (2010). In contrast, computational process models assume that individuals pursue satisfactory schedules by building or improving a schedule. Examples include ALBATROSS (Arentze and Timmermans, 2004a) and to some extent TASHA (Miller and Roorda, 2003) and ADAPTS (Auld and Mohammadian, 2009).

To the best of our knowledge, however, most existing approaches fall short of fully representing activity-travel patterns and tend not to consider all options. For example, multi-modal trip chaining between private vehicles and public transport (PT) is often neglected. Likewise, parking is typically omitted so that the impacts of parking policies and park-and-ride (P + R) services on travel behavior cannot be captured (e.g. Gan and Recker, 2008; Horni et al., 2009). Meanwhile, a hierarchical structure downgraded from activity patterns to trips or a sequential structure is often adopted to evaluate choice alternatives; only several global optimization models offer exceptions such as the works of Recker (1995) with mixed integral programming and Jonsson (2008) with dynamic programming, which are still restrictive in terms of the choice dimensions covered. Due to these simplifications, space–time constraints and time dependency in activity-travel patterns are loosely coupled with other choices such as route, mode and parking.

Thus, we argue that substantial improvement to activity-travel scheduling models can be made by fully representing all choice options and capturing the interdependencies in activity-travel trip chaining. As shown (Arentze and Timmermans, 2004b; Liao et al., 2010, 2011), the multi-state supernetwork representation has this potential. However, currently developed multi-state supernetwork models do not systematically accommodate space–time constraints and time-dependency. As pointed out by Pinjari and Bhat (2011), however, the appropriate treatment of the time dimension is probably the most important prerequisite to accurately forecast activity-travel patterns as the temporal aspects are closely interconnected.

Therefore, this paper aims to (i) dramatically improve the representation of the temporal dimension in multi-state supernetworks by embedding space–time constraints into location selection models; and (ii) systematically incorporate time in the disutility profiles of activity participation and parking. This paper will focus on daily activity-travel scheduling at an individual level, although multi-state supernetworks can also be used for travel and joint activity participation (Liao et al., 2013). As a result of this fundamental elaboration, the multi-supernetwork model can more accurately predict highly detailed activity-travel patterns with multi-modal and multi-activity trip chaining. Moreover, to account for the generalized representation, refined behavioral assumptions and dominance relationships are proposed in an earlier proposed bi-criteria label-correcting algorithm to find the optimal activity-travel pattern. Analyses and formal proofs of the scheduling algorithm are also provided.

To that end, the remainder of this paper is organized as follows. In the next section, an overview of the previous multi-state supernetwork model development is presented and the model limitations of them are also discussed. Next, we will discuss the improvements of the supernetwork model. Then, some scheduling examples are presented to illustrate the potential of the approach. We will complete the paper with conclusions and an expose of planned future work.

2. Multi-state supernetwork

2.1. Previous model development

Network-based approaches have a long history in addressing transportation problems. An abstract multiclass-user traffic network was first demonstrated by Dafermos (1972) through the expansion of a road network. The importance of such abstract networks was accentuated by Sheffi and Daganzo (1978) for modeling mode and route choice in a so-called hypernetwork, which was re-termed as supernetwork (Sheffi, 1985). The supernetwork was constructed by adding transfer links at locations in both sub-networks, i.e. car road network and transit network, where an individual can switch between transport modes. A path through this supernetwork expresses the choices of mode and routes. Similar network extensions have been developed for modeling multi-modal trip chaining by Nguyen and Pallottino (1989), Carlier et al. (2003), Lozano and Storchi (2002). The concept of supernetwork began to have a wider interest due to the efforts of Nagurney’s group (2002, 2003, 2005). At a trip-based level, Nagurney and Dong (2002) also introduced transaction links to model one-activity implementation.

From the perspective of activity-based modeling, a fundamental breakthrough was suggested by Arentze and Timmermans (2004b), who suggested the multi-state supernetwork representation for modeling comprehensive activity-travel behavior. A multi-state supernetwork for an activity program is built by interconnecting an integrated land-use multi-modal transport network across every possible combination of activity and vehicle state. In the supernetwork, nodes represent real locations in space. Links are defined in terms of three categories:

- **Travel links**: connecting different nodes of the same activity state, representing the movement of the individual; the modes can be walking, bike, car, or any PT modes.
- **Transition links**: connecting the same nodes of the same activity states but different vehicle states (i.e., parking/picking-up a private vehicle or boarding/alighting PT).
- **Transaction links**: connecting the same nodes of different activity states, representing the implementation of activities.
**Liao et al.** (2010) improved the multi-state supernetwork representation by allowing considerable reduction in network size, without the expense of representation power. The integrated network is split into a set of PVNs (private vehicle networks) and a PTN (PT network). Thus, in the supernetwork, travel links are inside PVNs and PTNs; boarding/alighting PT links are inside PTNs only; parking/picking-up and transaction links are used to interconnect PVNs and PTNs, and PTNs and PTNs respectively. More specifically, inside a PVN, there are only parking locations (including home location) and for each pair of locations there is a PVN connection, which involves only one mode. Inside a PTN, there are parking (if any) and activity locations connected by PTN connections, which include walking, waiting, boarding/alighting, and in-vehicle. The link costs on each component are defined specifically since the time spent on them is perceived differently.

Fig. 1 is an example of a multi-state supernetwork representation for an individual’s activity program, including one fixed activity (at location \(A_1\)) and one flexible activity (at location \(A_2\)), suppose only one alternative location for the sake of simplicity, and two private vehicles (car and bike). \(P_1\&P_2\) and \(P_3\&P_4\) are parking locations for car and bike respectively. Each of them in the first row denotes the specific private vehicle parked at that location. \(P_0\) and \(P_5\) denote car and bike in use respectively. \(s_1s_2\) represents the activity states for \(A_1\&A_2\) (0-unconducted and 1-conducted). Let \(H\) and \(H'\) denote home at the start and end of the activity states respectively; the path denoted by the bold links indicates an activity-travel pattern that the individual leaves home by car to conduct the fixed activity at \(A_1\) with parking at \(P_2\), then returns home and switches to bike to conduct the flexible activity at \(A_2\) with parking at \(P_4\), and finally returns home (undirected links are bi-directed). It can be proved that any path from \(H\) to \(H'\) denotes a possible activity-travel pattern.

To further reduce the required size of supernetwork representations, **Liao et al.** (2011) proposed a heuristic approach for constructing personalized PVNs and PTN, based on the notion that only a small set of locations are of interest to individuals. The approach involves activity and parking location choice models. PVN and PTN connections are generated and extracted by a route choice model. Hence, a multi-state supernetwork is decomposed into a concatenation of selected locations and connections distributed at different activity-vehicle states. Every link can be defined in a state-dependent and personalized way as follows:

\[
\text{dis}U_{i,m,s} = \beta_{i,m} \times X_{i,m,s} + \epsilon_{i,m,s}
\]  

(1)

where \(\text{dis}U_{i,m,s}\) denotes the disutility on link \(l\) for individual \(i\) in activity state \(s\) with transport mode \(m\), \(X_{i,m,s}\) denotes a vector of attributes, \(\beta_{i,m}\) is a weight vector, and \(\epsilon_{i,m,s}\) is an error term. A key parameter is the number of selected activity locations, on which the scale of the supernetwork is contingent. The larger this parameter the more likely the optimal locations are covered by the choice model. Sensitivity analysis showed that the optimal locations could be selected out by setting low values of this parameter.

Since Eq. (1) is not able to capture travel dynamics and time constraints, **Liao** (2011) incorporated time dependent components into multi-state supernetworks. Given a constructed personalized multi-state supernetwork, car travel speed profiles, PT timetables, linear parking cost profiles and time windows at activity locations are well-embedded. After adding these elements, the supernetwork topology (e.g. Fig. 1) remains the same, whereas most of the link costs are defined time-dependently. In particular, PVN connections look up a time-dependent road network, while PTN connections look up time-expanded PT network (Pyrga et al., 2008). A side product is that the network structure may fail the FIFO (first-in-first-out) property (Dean, 2004). Therefore, a bicriteria label-correcting scheduling algorithm was developed based on certain behavioral assumptions.

2.2. **Model limitations**

Through this sequence of improvements, different aspects of the multi-state supernetwork approach for modeling multimodal, multi-activity travel have been developed in a stage-wise manner. Yet, they cannot be considered as full-fledged complete activity-travel scheduling models because solutions do not necessary satisfy space–time constraints, both at the level of consecutive episodes and the level of higher order destination pairs, and timing profiles/preferences underlying
activity-travel choices were not fully addressed. In particular, two key components concerning the multi-state supernetwork representation are still missing.

First, space–time constraints are not embedded in the process of selecting relevant locations in previous models. A number, $N_d$ (a parameter), of alternative locations was selected for each flexible activity with the least disutilities in terms of the associated travel disutility and attractiveness of the locations. Globally optimal flexible activity locations could be found in the selected subsets by setting a small $N_d$. However, this is not a very rigorous approach. In addition, when considering space–time constraints in the scheduling process but not in the location selection process, $N_d$ needs to be relatively large when the globally optimal flexible activity locations are included in the selected subsets. It is because distant alternative locations with higher attractiveness have the tendency to be selected in the subsets; whereas, they tend to violate the space–time constraints. As $N_d$ gets larger, the number of parking locations (vehicle states) increases accordingly, and consequently the scale of the supernetwork increases considerably, which may lead to unacceptable computation times. A similar logic applies to the ensuing parking location selection process. Thus, space–time constraints should be incorporated into the location selection process to remove infeasible and inferior locations and unnecessary travel connections in the multi-state supernetwork representation.

Second, duration/search-time and disutility profiles of activity participation and parking respectively were not taken into account in the previous multi-state supernetwork models. However, time dependency of duration/search-time and disutility is a common phenomenon of activity participation and parking. Without taking their time-dependency into account, the model tends to output inaccurate predictions in the temporal dimension and even wrong predictions in activity patterns and locations. Beside travel speed profiles, profiles of activity participation and parking should also be incorporated into the multi-state supernetwork representation.

The incorporation of these two components in the multi-state supernetworks represents a significant improvement in the sense that it will make the representation for activity-travel behavior fundamentally more accurately. Although these mechanisms have been included in other types of activity-based models such as AMOS (Kitamura et al., 1996), GISICAS (Kwan, 1997), ALBATROSS (Arentze and Timmermans, 2004a), SimTRAVEL (Pendyala et al., 2010), and SimAGENT (Bhat et al., 2012), they have been treated in a simplified, limited manner, even in the most recent publications (e.g. Farber et al., 2013). Most models construct the activity-travel pattern sequentially or adapt it iteratively without integrating route and mode choice. In the construction procedure, when one activity episode is added, space–time constraints are checked only for the adjacent episodes through the concept of space–time prism, which, however, cannot guarantee the constraints are satisfied across the full activity-travel pattern. In the adaptation procedure, which is not common to start with, modifications keep occurring on the base activity-travel pattern until some conditions are satisfied; during the process, space–time constraints can be checked for the full activity-travel schedules. However, without explicitly integrating routing choice potentially involving multi-modal trip chaining, the validity is still limited. In the temporal dimension, the time-dependency of travel, activity participation and parking is only loosely coupled.

### 3. Extensions of the supernetwork model

The model improvements are discussed in this section to address the above model limitations. As shown in Fig. 2, the multi-state supernetwork approach for activity-travel scheduling consists of three main steps with inputs of scenario of land use and transportation system at LHS and individual choice heuristics and preferences at RHS. In the following part of this section, we will firstly discuss how space–time constraints can be embedded into location choice models (Step 1). In addition to travel speed profiles, time-dependent profiles of activity participation and parking will also be integrated in the supernetwork representation to improve the space–time resolution (Step 2). Subsequently, the property and performance of the activity-travel scheduling algorithm will be discussed (Step 3). These improvements are meant to better capture the space–time constraints, and more accurately represent and predict activity-travel patterns and behavior.

We define an activity program – AP as follows:

1. There is at least one out-of-home activity and at most three departing home modes: walking, bike, and car.
(2) At a time, an individual leaves home with at most one private vehicle (bike or car) to conduct at least one activity out-of-home.
(3) The individual can take PT after parking a private vehicle if any, and must return home with all private vehicles at home and all the activities conducted in the end.
(4) Each activity is associated with the attribute indicating whether it is fixed that must be conducted at a fixed location or flexible that can be conducted at one of multiple locations.
(5) Each activity is also associated with an ideal minimum duration which is derived when the individual conducts the activity at an ideal location. The real duration at a specific location should be no less the ideal minimum duration. And each activity location is associated with a time window constraint.
(6) Systemic sequential relationships among activities are only determined by space–time constraints, but personal sequential relationships are assigned by the individuals.

This definition extends the previous one (Liao et al., 2010) by allowing multiple home-based tours during the day and tending the sequencing with space–time constraints. Let $i$ be the individual concerned and $x$ denote an activity in the AP. Suppose $c(x)$ is the activity location for $x$ if $x$ is fixed, $c(x)$ the $j$th ($j \geq 1$) alternative location if $x$ is flexible, $D(x)$ the ideal minimum duration, $[u_L(x), u_H(x)]$ the time window that $i$ can stay out-of-home, and $[u_L(y), u_H(y)]$ the largest time window range with $u_L(x) = \min(u_H, \min(u_L, \max(v_c(x))))$ and $u_R(x) = \min(u_L, \max(v_c(x)))$.

Moreover, two types of time windows are identified. T1: $i$ must arrive at the activity locations no later the opening time; T2: $i$ can arrive after the opening time, but has to finish the activity before the closing time. Assume that if $i$ has to wait, $i$ suffers linear disutility in terms of the waiting time, and that if $i$ cannot finish the activity before the closing time, $i$ suffers infinite disutility.

3.1. Selection of activity and parking locations

Selecting relevant activity and parking locations is essential for the construction of a personalized multi-state supernetwork. To implement an AP, $i$ would reflect on where to engage in the activities, how and when to get there, and where to park the private vehicles (if any). These elements are interwoven and have an impact on each other. If using the original location sets without any selection, there is possibly a combinatorial explosion on the supernetwork scale and the scheduling problem becomes intractable. On the other extreme, if randomly selecting a few locations, the desired activity-travel pattern may not be achievable. Hence, designing an approach with fine balance between scale and precision is important. In Liao et al. (2011), locations are selected in terms of the trade-off between estimated travel disutility and attractiveness of the activity locations, in which no space–time constraint is considered. The following part discusses the refinement that space–time constraints are combined with individual choice heuristics and preferences.

3.1.1. Selection of activity locations

The first step is to determine the sequencing among the activities to reduce the solution space. The final sequencing is the union of personal and systemic ones. It is trivial to determine the personal one, which is the input from $i$ in the form of whether $i$ prefers to conduct one activity before another. The systemic one is determined by space–time constraints. All activities must be conducted after departing home (at $H$) at the first time and before the final home-returning (at $H'$). If there is only one activity in AP, it is trivial to do so. Otherwise, for any two activities $\alpha$ and $\gamma$, the sequential relationship before ($\rightarrow$) or after ($\leftarrow$) can be checked by the time window constraints: if $u_L(\alpha) \leq u_L(\gamma)$ or $u_R(\alpha) - D(\alpha) \leq u_L(\gamma) + D(\gamma)$, $\alpha$ is before $\gamma$ and vice versa. For instance, consider the case of two activities in an AP, escorting a child to school with duration 2 min and time window [8:30 am, 9:30 am], and working at the office with duration 8 h and time window [8:00 am, 8:00 pm]. Then, the first activity should be before the second.

Either $\leftarrow$ or $\rightarrow$ is transferable, e.g. if $\alpha \rightarrow \gamma$ and $\gamma \rightarrow \theta$ ($\theta$ is another activity), then $\alpha \rightarrow \theta$ and vice versa. However, if the sequence cannot be determined, the relationship is either before or after, e.g. if $\alpha \leftarrow \gamma$ and $\alpha \rightarrow \theta$, then $\alpha \rightarrow \gamma$ or $\gamma \rightarrow \theta$. When a personal sequencing is in conflict with the systemic one, the former must obey the latter. For instance, if $\alpha \rightarrow \gamma$ due to the individual's preference and $\alpha \rightarrow \gamma$ in terms of space–time constraint, $\alpha$ should be after $\gamma$. Following these logics, the sequencing can be determined among all the activities.

The next step is to locate activities. As defined, departing $H$ and returning $H'$ are regarded as extra fixed activities in an AP. It is trivial to locate the fixed activities. Locating the flexible ones needs to take into account the fixed locations. For two fixed activities $\alpha$ and $\gamma$, we define a direct sequential pair, $\alpha \rightarrow \gamma$, as no other fixed activity $\theta$ exists satisfying and only satisfying $\alpha \rightarrow \theta \rightarrow \gamma$ or $\alpha \rightarrow \theta \rightarrow \gamma$. Otherwise, $\alpha$ and $\gamma$ cannot form a direct sequential pair. Then, any flexible activity must fall between at least a direct sequential pair.

If only one flexible activity $\delta$ is between and only between a direct sequential pair $\alpha \leftrightarrow \gamma$, one of these sequencing options, $\alpha \rightarrow \delta \rightarrow \gamma$ or $\alpha \rightarrow \delta \rightarrow \gamma$ or both, is feasible. For the sake of simplicity, we consider the case of $\alpha \rightarrow \delta \rightarrow \gamma$. Any feasible location $c(\delta)$ for $\delta$ should meet the time window constraints as follows:

$$u_L(\delta) + D(\delta) + u_H(\delta) + \sum_{D(\delta) \in A_P} (D(\delta) + T_{hi}) \leq v_c(\delta) - D(\delta) - T_{hi}$$

(2)
\[ u_{ij}(\delta) + D_i(\delta) + T_{i\delta} + tt_{c_j(\delta)\alpha_{ij}} + \sum_{\delta \in AP} (D(\delta') + \tilde{w}) \leq v_{i\alpha(\gamma)} - D_i(\gamma) - T_{i\gamma} \]

where \( tt_{c_j(\delta)\alpha_{ij}} \) and \( tt_{c_j(\delta)\alpha_{ij}} \) denote the travel time from \( c(\alpha) \) to \( c(\delta) \) and from \( c(\delta) \) to \( c(\gamma) \) respectively, and \( T_i(= \{ \alpha, \delta, \gamma \}) \) is the threshold of extra time that \( i \) reserves for an activity to cope with uncertainty in travel and activity participation. \( \delta (\delta \neq \delta) \) denotes another flexible activity falls between and only between \( \alpha \leftrightarrow \gamma \). \( T_{i\alpha} \), to \( \forall \alpha \), can be used as a parameter to subtly adjust the opportunity zone. The larger \( T_{i\alpha} \) is, it is more likely that the selected locations in a later stage are feasible for the whole activity program, but it is less likely that the optimal locations are covered by the selected sets. The upper bound for \( T_{i\alpha} \) can be set when the lowest velocity and the highest activity duration are perceived by \( i \), while the lower bound is obtained with the setting from the other way round. Eqs. (2) and (3) represent the lower bound space–time constraints that \( \delta \) is derived by the following:

\[ \text{Feasible locations for flexible activity.} \]

\[ \text{Fig. 3. Feasible locations for flexible activity.} \]
optimal locations can be selected with a small value of $N^d$. Like $T_{ua}$, $N^d$ can also be used to adjust the action space. The larger $N^d$ the more likely the optimal locations are covered, which is computationally more time-consuming instead.

3.1.2. Selection of parking locations

After locating all the activities, parking locations are selected in terms of the available private vehicles. Individuals use private vehicles to access activity locations directly, or park them at transport hubs (TH) (e.g., train stations for bike and car parking) or P + R facilities (P + Rs) to switch to PT for avoiding long distance riding or congestion and difficulty of parking in city centers. These three types of locations are potential options for parking. In this paper, we combine the heuristic rules in Liao et al. (2012) with space–time constraints to select potential parking locations.

For a private vehicle $p$, $p = c$ (car) or $b$ (bike), two distance circles with centers at home are set for $i$, acceptance distance $E_{ip}^a$ and limit distance $E_{ip}^b$, satisfying $E_{ip}^a < E_{ip}^b$ and $E_{ip}^b = +\infty$. The rules are: (1) if $p$ will not drive over a distance of $E_{ip}^a$ away from home but may drive over $E_{ip}^b$, $i$ must park $p$ inside $E_{ip}^b$, and if it lies between $E_{ip}^a$ and $E_{ip}^b$, $i$ may park $p$ inside circle $E_{ip}^b$; otherwise, $i$ will drive directly to the activity location. Fig. 4 is an example, in which TH/1 is potential for bike parking, and TH/1, TH/2, P + R/1, P + R/2 and A are potential for car parking.

The number of potential parking locations may still be large so that a parking location choice model is necessary. The above rules already filter out some activity locations as potential parking locations. Thus, the parking location choice model is tailored for THs and P + Rs. Assume $P_k$ is a feasible location of such. Without loss of generality, for a private vehicle $p$, $P_k$ should not only be in accordance with the heuristic rules, but also satisfy the time window constraints formulated as follows:

$$u_i + t_{ip} + tt_{P_k, c_j(x)} \leq v_{c_j(x)} - D_i(x) - T_{t_a} \tag{6}$$

where $c_j(x)$ denotes an activity location for $x$ covered by $P_k$. The coverage is defined as:

1. if $P_k$ is a TH inside the circle $E_{ip}^a$, it covers the activity locations outside $E_{ip}^a$;
2. if $P_k$ is a TH inside the circle $E_{ip}^b$, it covers the activity locations outside $E_{ip}^b$;
3. if $P_k$ is a P + R facility, it covers the activity locations inside its hosted city center.

From H to $P_k$, the involved mode is $p$. From $P_k$ to $c_j(x)$, we assume the used mode is $m_i$ if $P_k$ is a TH, and $m_s$ if $P_k$ is a P + R facility. Thus, $tt_{ip}^p$ and $tt_{P_k, c_j(x)}$ can be estimated in the same way as explained in Section 3.1.1. The parking location choice model is specified as follows:

$$disU_{ip} = disU_{ip}^p + disU_{ip}^{T_a} + disU_{im_s}^{T_b} \tag{7}$$

where $disU_{ip}^p$, choosing disutility of $i$ for parking $p$ at $P_k$; $disU_{ip}^{T_a}$, disutility of parking $p$ at $P_k$ with duration $\sum_i^{T_a} (x$ is covered by $P_k)$; $disU_{im_s}^{T_b}$, disutility of travel from H to $P_k$ with mode $p$; $disU_{im_s}^{T_b}$, average travel disutility from $P_k$ to different $c_j(x)$ covered by $P_k$ with mode $m$ ($m = (m_s, m_i)$).

With Eq. (7), we select a small number $N^p$ of potential parking locations with the lowest level of choosing disutility from feasible THs and P + Rs.

Following the above procedures of selecting activity and parking locations, a heavily reduced multi-state supernetwork (step 2 in Fig. 2) can be constructed to a given AP as described in Section 2. Note that the purpose of the selection of locations is to rule out the most irrelevant locations rather than directly pick out the optimal locations, which is, nevertheless, done in Step 3 of Fig. 2.

In the full scale representation (e.g. Fig. 1), when a private vehicle is parked at a location, the individual can conduct multiple activities successively without switching parking locations. As a TH or P + R facility has its own coverage, when $p$ is parked at such a location, it is logical to restrict that $i$ can only conduct those activities at the locations covered by this parking location. For example, when the car is parked at a P + R, it is not allowed to conduct activities at locations not in the hosted city center. In addition, when $p$ is parked at an activity location of a flexible activity, it is not allowed to conduct this activity at other alternative locations. These two reasonable restrictions result in reductions in the number of possible activity-vehicle states, and the scale of the supernetwork is consequently reduced.

![Fig. 4. Example of potential parking locations.](https://example.com/fig4.png)
3.2. Profiles of activity participation and parking

As aforementioned, travel speed profiles of private vehicles and PT timetables have been integrated in the multi-state supernetwork. However, the duration and disutility of activity participation and search time of parking are still assumed fixed at different time of the day, which is not true in reality. The purpose of the remainder of this subsection is to incorporate time dependent profiles of activity participation and parking.

3.2.1. Activity participation profiles

To keep consistency, this paper adopts the concept of disutility for activity participation, even if conducting an activity as a rule produces utility. Disutility for conducting an activity at a location refers to a loss of utility compared to a hypothetical ideal location scoring the highest utility with the perceived ideal minimum duration. At a real location, the duration and disutility of an activity should be measured based on a total bundle of the attributes. Some of the attributes are stable for a long period (e.g. quality and price level), but some of them are short-term time-dependent (e.g. crowd-ness at different time of the day) (Lam and Yin, 2001). In total, the duration and disutility of activity participation should also depend on the time-of-the-day in the context of daily activity-travel scheduling.

As each activity $a$ is associated with $D_i(a)$, there is an increment in the duration at a real activity location. One part is from the static factors and the other is due to time-dependency. We can make the safe assumption that the duration for conducting an activity is dependent on the static attributes of the locations and the start time. It means that the duration is fixed given a start time. Likewise, we assume this rule also applies to the disutility at a location. Hence, profiles of duration and disutility of activity participation can be drawn in terms of the attributes of a location and the ideal minimum duration. With fixed and time-dependent components, coupled with personalized and state-dependent information, the duration for conducting activity $a$ at $cJ(a)$ is formulated as:

$$durr_{icJ(a)}(t) = D_i(a) + t_{iscJ(a)}^s + t_{iscJ(a)}^d(t)$$

where $durr_{icJ(a)}(t)$, duration of $i$ conducting $a$ at $cJ(a)$ at activity state $s$ at arrival time $t$; $D_i(a)$, ideal minimum duration of $a$; $t_{iscJ(a)}^s$, extra duration caused by the static attributes of $cJ(a)$; $t_{iscJ(a)}^d(t)$, extra duration caused by time-dependency.

Similarly, Eq. (1) for conducting an activity should be extended as:

$$disU_{icJ(a)}(t) = b_{CA} × X_{icJ(a)}^{CA} + e_{icJ(a)}^{CA} + disU^d_{icJ(a)}(t)$$

where $disU_{icJ(a)}(t)$, disutility of $i$ conducting $a$ at $cJ(a)$ at activity state $s$ at arrival time $t$; $b_{CA} × X_{icJ(a)}^{CA}$, disutility of fixed component at $cJ(a)$, which include price level, quality, level and duration of $D_i(a) + t_{iscJ(a)}^s$; $e_{icJ(a)}^{CA}$, error term of $i$ conducting $a$ at $cJ(a)$; $disU^d_{icJ(a)}(t)$, extra disutility caused by $t_{iscJ(a)}^d(t)$.

Fig. 5a shows an example of the composition of the activity duration. Fig. 5b shows an example profile of $disU^d_{icJ(a)}(t)$, which is not necessarily in the same shape of $t_{iscJ(a)}^d(t)$. Taking grocery shopping for example, longer duration and higher disutility are generally engendered in the peak time than non-peak time. Differently, a work activity often has a fixed duration.
but the extra disutility is quite contingent on the start time. Other activities such as having lunch or dinner are also inclined to be time-dependent due to physiological needs.

The disutility of conducting an activity in Eq. (4) is different from the one in Eq. (9) because the former is only a rough estimation for the selection process while the latter is more accurate for the scheduling process. Profiles of $D_i(x) + t_{beg,i}(x)$ are considered as the of free-flow durations at $c_i(x)$ in terms of $D_i(x)$, and $t_{beg,i}(c_i(x), t)$ can be estimated in terms of the occupancy profile and capacity of $c_i(x)$, which is similar to the form of BPR (Bureau of Public Roads) function (Lam et al., 2006). Profile of $dist^d_{bic,j}(t)$ and personalized parameters, i.e. $\beta^d_{i,j}$, can be investigated and estimated with revealed and stated data. (These estimations are beyond the scope of this paper.)

Just as the travel time profiles of private vehicles, the duration profiles of activity locations theoretically satisfy time-FIFO property ("non-overtaking condition"). This property states that an individual arriving earlier at an activity location should finish conducting the activity no later than arriving at a later time. Formally, in the concise form, let $(t_n, d_n) \in \mathbb{N}, t_n > 0, d_n > 0), durr_i(t_n) and dist_i(t_n)$ denote the arriving label (time, disutility), duration and associated disutility respectively at the start of a transaction link at whatever activity states. After the activity participation, at the other end of the transaction link, the label $(t_n', d_{n}')$ is updated as $(t_n + durr_i(t_n), d_n + dist_i(t_n))$. If there are two possible arriving labels at the entry node of a transaction link that render the individual to implement activity, $(t_1, d_1)$ and $(t_2, d_2)$, with the condition of $t_1 \leq t_2$ denoted as C1, the time-FIFO property is formulated as:

$$ t_1 + durr_i(t_1) \leq t_2 + durr_i(t_2) \quad (10) $$

With Eqs. (8) and (10), we can find that the extra activity duration profiles should meet:

$$ t_{beg,i}(t) \geq -1, \quad \forall i, \quad \forall s, \quad \forall x \text{ and } \forall c_j(x) \quad (11) $$

Meanwhile, in reality, if the arrival time is within the time window, the individual would hate to wait until a later time to conduct the activity. Then, we can obtain:

$$ disU_i(t_1) \leq \beta_{iw} \times (t_2 - t_1) + disU_i(t_2) \quad (12) $$

where $\beta_{iw}$ is the disutility parameter on waiting time. If only $t_1$ or both $t_1$ and $t_2$ are before the opening time, the individual has to wait until the opening time. In either case, Eq. (12) still holds. According to Eqs. (9) and (12), we can arrive the following:

$$ dist_{beg,i}(t_1) - dist_{beg,i}(t_2) \leq \beta_{iw} \times (t_2 - t_1), \quad \text{to } \forall i, \quad \forall s, \quad \forall x \text{ and } \forall c_j(x) \quad (13) $$

With Eq. (13), we can find that the extra activity disutility profiles should meet:

$$ dist^d_{beg,i}(t) \geq -\beta_{iw}, \quad \text{to } \forall i, \quad \forall s, \quad \forall x \text{ and } \forall c_j(x) \quad (14) $$

Furthermore, if C1 and the condition of $d_1 \leq d_2 - \beta_{iw} \times (t_2 - t_1)$ denoted as C2 are both satisfied, with Eqs. (10) and (12), we can obtain:

$$ t_1' \leq t_2 \quad \text{and} \quad d_1' \leq d_2' \quad (15) $$

$$ (d_1' = d_1 + dist_i(t_1) \leq d_2 - \beta_{iw} \times (t_2 - t_1) + dist_i(t_1) \leq d_2 + dist_i(t_2) = d_2') \quad \text{Eq. (15) is in line with the new dominance relationship defined in Liao (2011)} \text{ that } (t_1, d_1) \text{ dominates } (t_2, d_2) \text{ if C1 and C2 hold. This dominance relationship manifests that behaviorally individuals do not wait until a time with extra higher disutility to start the next episode of activity participation.}$$

### 3.2.2. Parking profiles

As parking space in urban areas is becoming a scarce resource, there is a need to model parking choice in activity-travel scheduling systems, which is often missing in the literature. Multi-state supernetworks can model parking choice in a unified fashion as other choices. Unlike the disutility of activity participation that is only assigned to transaction links, the disutility of parking private vehicles is related to three components, i.e. parking, picking-up and parking duration. Fig. 6, an extract from Fig. 1, depicts a chain of parking process, in which $P_1$ and $P_2$ denote two parking locations, $R_1 & R_2$ and $R_2 & R_3$ represent examples of alternative routes going through PTNs and transaction links, and then parking duration equals to the time spent on these routes.

In most travel behavior studies, parking duration is not considered, and the time and disutility of parking and picking-up stage are treated as fixed according to Eq. (1). In Liao et al. (2012), parking duration has been modeled by incorporating linear parking fare profiles in the form of $y_p = a_p + b_p \times t$, in which $y_p$ and $t$ denotes monetary cost and duration at parking location $P_i$ respectively. Constant $a_p$ is treated in parking links and the linear unit $b_p$ in terms of parking duration is uniformly distributed on the routes when the vehicle is parked. While bike parking may be possible anytime, car parking is getting increasingly difficult at some time of the day in urban areas so that the search time in the parking stage is even comparable to the travel time of the trip. In contrast, the same problem seldom occurs in the picking-up stage. Therefore, it is reasonable to incorporate search time profiles of parking and consider fixed time elapse and disutility in the picking-up stages.

Similar to the profiles of travel time and activity participation, the search time also theoretically satisfies the time-FIFO property that the one arriving at a parking location earlier should not find the parking place later than arriving later. In the
meantime, after arriving at a parking location, the individual would have to wait until another time to execute parking. These two features about time-dependency are exactly the same as those of activity participation. Let $\Delta st_{i,pk}^s(t)$ and $disU_{i,pk}^s(t)$ denote the search time and search disutility for individual $i$ with private vehicle $p$ at parking location $P_k$ at arrival time $t$ respectively. $\Delta st_{i,pk}^s(t)$ can also be estimated by free-flow searching time, occupancy profiles and capacity of $P_k$. Thus, Eqs. (11) and (14) also apply to the profiles of parking to any private vehicle and parking location. Also, we can derive the following:

$$\Delta st_{i,pk}^s(t) \geq -1 \text{ and } disU_{i,pk}^s(t) \geq -\beta_{iw} \forall i, \forall s, \forall p \text{ and } \forall P_k$$

(16)

With the constant component of parking cost, the disutility $disU_{i,pk}^s(t)$ of parking link (from PVN to PTN) at arrival time $t$ is updated as:

$$disU_{i,pk}^s(t) = \beta_e \times \alpha_t + \beta_i^p \times X_{ispP}^k + \epsilon_{ispP}^k + disU_{i,pk}^s(t)$$

(17)

where $\beta_e$ is the parameter on monetary cost of $i$, $\beta_i^p \times X_{ispP}^k + \epsilon_{ispP}^k$ is the static component of parking $p$ at $P_k$.

Let $(t_1, d_1)$ and $(t_2, d_2)$ denote two arrival labels at a parking location in a PVN, and $(t_1', d_1')$ and $(t_2', d_2')$ two labels at the same parking location in a PTN after parking. If $C1$ and $C2$ are satisfied to $(t_1, d_1)$ and $(t_2, d_2)$, respectively, $\Delta st_{i,pk}^s(t)$ and $disU_{i,pk}^s(t)$ hold. Similarly, it applies to picking-up links as fixed time and disutility are assumed on them. It shows that the profiles of parking can also be incorporated in a unified way as activity participation.

3.3. Optimization algorithm

Based on the above treatments, more accurate multi-state supernetworks can be constructed. Any path from $H$ to $H'$ (e.g. Fig. 1) still represents an activity-travel pattern since the topology remains the same. However, such a path can be infeasible when it fails to satisfy a time window constraint, resulting in infinite disutility. In the location selection process, $T_w$ for and $N^p$ can be adjusted to ensure that the selected locations are feasible to implement the AP. In the following part, we discuss the optimization algorithm to a multi-state supernetwork, where at least one feasible activity-travel pattern exists.

The optimization algorithm involves finding a path through the supernetwork, which differs with the objectives and the properties of the network. If the objective is to minimize the total time in a time-FIFO network, the label setting procedures (e.g. Dijkstra, 1959) can find the optimal pattern. If it is to minimize disutility in a disutility-non-FIFO network, it needs to extend the network in space–time (Dean, 2004) or adopt label correcting procedure (Powell and Chen, 1998; Skriver and Andersen, 2000). In this context, a least disutility path from $H$ to $H'$ ought to be sought under time window constraints and profiles of parking and activity. Moreover, the disutilities and compositions of PVN and PTN connections are defined on-the-fly, looking up in road network and time-expanded PT network respectively. The disutility-FIFO property is strongly broken in the multi-state supernetwork.

Rather than further extending the supernetwork, the algorithm in this paper adopts the bicriteria label correcting routine in Liao (2011) but bases on two different refined behavioral assumptions. The first assumption (A1) is that $i$ always seeks the fastest connections in PVNs and PTNs. The second (A2) is that for two labels at a node, i.e. $(t_1, d_1)$ and $(t_2, d_2)$, $i$ will not consider $(t_2, d_2)$ if $(t_1, d_1)$ dominate $(t_2, d_2)$, i.e. $(t_1, d_1) \succeq (t_2, d_2)$. Traditionally, if $t_1 \leq t_2$ and $d_1 \leq d_2$, then $(t_1, d_1) \succeq (t_2, d_2)$. This condition is relaxed in (Liao, 2011) that only if $C1$ and $C2$ both hold, then $(t_1, d_1) \succeq (t_2, d_2)$. As discussed in subsection 3.2, if $C1$ and $C2$ hold for two arrival labels at the entry node of a transaction, parking or picking-up link, Eq. (15) holds after the traverse. It means that departing with a dominated label to execute one episode of them will not benefit to save time or decrease disutility. However, this feature may be invalid for PVN and PTN connections under A1. Although the fastest connections are the same as the least disutility ones in many cases, for a PVN or PTN connection, a dominated label cannot save time but may lessen disutility after the traverse because there are multiple factors in Eq. (1) except time components. Therefore, C2 should be further relaxed in A2 to allow more labels in the non-dominated sets for PVN and PTN connection queries.

For a PVN connection $PV_c$, a good relaxation margin at RHS of $C2$ is the disutility range between the best and the worst case of travel with the involved private vehicle. Then, for PVN connections, $C2$ is replaced by $C3$ as:

$$d_1 \leq d_2 - \beta_{iw} \times (t_2 - t_1) - d_{ispP}^k$$

(18)
where $d_{ipPVc}^b (d_{ipPVc}^b \geq 0)$ is the disutility interval with private vehicle $p$ on $PV_c$, and $d_{ipPTc}^b = 0$ when $t_1 = t_2$. With $C1$ and $C3$, Eq. (15) still holds. This twist meets the dominance condition with any parameter settings on travel component. The larger $d_{ipPTc}^b$, the deeper the solution space is exploited than $C2$. Similarly, for a PTN connection $PT_c$, a good relaxation margin at RHS of $C2$ is the disutility range between the best and the worst case of travel by PT. Then, for PTN connections, $C2$ is replaced by $C4$ as:

$$d_1 \leq d_2 - \beta_{iw} \times (t_2 - t_1) - d_{ipPTc}^b$$

where $d_{ipPTc}^b (d_{ipPTc}^b \geq 0)$ is the disutility interval with PT on $PT_c$, and $d_{ipPTc}^b = 0$ when $t_1 = t_2$. Other properties apply in the same as a PVN connection.

The multi-state supernetworks are highly sparse networks by considering a PTN or PVN connection as a special “link”. This is especially true after the reduction mentioned in the end of Section 3.1.2. Thus, label correcting procedure is selected for finding the optimal activity-travel pattern since it as a rule performs better than label setting procedure for sparse networks and those optimal paths potentially involving many links. However, for PTN and PVN connection queries, label setting procedure is applied since the optimal routes potentially involves few links and there are mature speeding-up techniques in the literature.

Based on the new dominance relationship, each node of the supernetwork preserves a non-dominated set of labels. Let $n$ be a node in the multi-state supernetwork, $B_n$ the non-dominated set of labels, $b_n(t, d)$ a member of $B_n$. Note that the condition for dominance relationship is different at an entry point of PVN (Eq. (18)) or PTN (Eq. (19)) connection from other nodes. The algorithm proceeds with each non-dominated label at an entry node to sequentially traverse a link and correct the non-dominated set of the exit node. This process terminates if no new node that can be corrected.

To allow the choice of departure time, a limited non-dominated label set $B_{it}$ is generated at $H$ in the beginning, and the non-dominated label sets at other nodes are initialized empty, which may change during the execution. $n$ is re-considered for scanning whenever $B_{it}$ is changed. The algorithm stops when no node is in the list for scanning. After the algorithm ends, the optimal label can be found in $B_{it}$ in terms of the objective, thereafter, the optimal path or activity-travel pattern can be backtracked. In this study, we choose the label with $\min(d_{it} + \beta_{iw} \times (t_{it} - \min(t|t \in B_{it})))$ of $B_{it}$ as the optimal label at $H$.

The pseudo-code for the algorithm can be written as follows:

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>input: $(AP, B_{it},$ personalized parameters, scenario setup)</td>
</tr>
<tr>
<td>2.</td>
<td>execute step 1 and step 2 in Fig. 2 to construct a supernetwork $- SNK$</td>
</tr>
<tr>
<td>3.</td>
<td>initialization: scanList ${H}$, $B_n = \emptyset$ for $n \in SNK \setminus {H}$</td>
</tr>
<tr>
<td>4.</td>
<td>while scanList $\neq \emptyset$</td>
</tr>
<tr>
<td>5.</td>
<td>choose first node $n$ from scanList, and scanList $\Leftarrow$ scanList $\setminus {n}$</td>
</tr>
<tr>
<td>6.</td>
<td>for each link $n \rightarrow w (w \in SNK)$</td>
</tr>
<tr>
<td>7.</td>
<td>for each label $b_n(t, d) \in B_n$ that did not traverse $n \rightarrow w$ before</td>
</tr>
<tr>
<td>8.</td>
<td>update $b_n(t, d)$ in terms of link type and arrival time at $n$</td>
</tr>
<tr>
<td>9.</td>
<td>if $t \leq t_{it}$</td>
</tr>
<tr>
<td>10.</td>
<td>merger $B_w$ and $b_n(t, d)$ into a non-dominate set</td>
</tr>
<tr>
<td>11.</td>
<td>end if</td>
</tr>
<tr>
<td>12.</td>
<td>end for</td>
</tr>
<tr>
<td>13.</td>
<td>end for</td>
</tr>
<tr>
<td>14.</td>
<td>if $B_n$ changes and $w \notin$ scanList</td>
</tr>
<tr>
<td>15.</td>
<td>scanList $\Leftarrow$ scanList $\setminus {w}$</td>
</tr>
<tr>
<td>16.</td>
<td>end if</td>
</tr>
<tr>
<td>17.</td>
<td>end while</td>
</tr>
<tr>
<td>18.</td>
<td>output optimal label and backtrack the path</td>
</tr>
</tbody>
</table>

The above activity-travel scheduling algorithm considers location, timing, mode and route choice simultaneously; whereas, in most peer activity-travel scheduling models, pure time-dependent shortest paths are sought on the route choice level after fixing the location sequencing, timing and modes. In this paper, the choice space can be explored more extensively than as done in the sequential fashion. Meanwhile, as there are multiple factors in the link costs, later arrival time at an entry node may overtake in the disutility at the exit node. The new dominance relationship (Eqs. (18) and (19)) allows this overtaking more adequately. In the following part, analyses of the supernetworks and the scheduling algorithm are presented.

**Lemma 1.** With A1, we can derive the proposed supernetworks satisfy time-FIFO.

**Proof.** Let $(t_1, d_1)$ and $(t_2, d_2)$ be two non-dominated labels at an entry node of a link, and after the traverse of this link, the labels are updated as $(t_1', d_1')$ and $(t_2', d_2')$ respectively. In the supernetworks, there are five types of “links”. For transaction, parking and picking-up links, if $t_1 \leq t_2$, then we have $t_1' \leq t_2'$ as discussed in Section 3.2. According to A1, for PVN and PTN “links”, we can also get $t_1' \leq t_2'$ if $t_1 \leq t_2$ since road network and PT time-expanded network are generally time-FIFO. Thus, the proposed supernetworks are time-FIFO. $\square$
While A1 prohibits waiting except being forced to wait for PT and time windows, A2 allows overtaking in terms of disutility among the arrival labels with the relaxed conditions. As the supernetworks are time-FIFO, the label with the fastest arrival time at a node is always in the non-dominated label. With A2, we can also arrive at the following:

**Lemma 2.** The proposed algorithm outputs behaviorally the optimal activity-travel patterns.

**Proof.** Consider any directed path \( p \) from node \( H \) to node \( H' \). Let \( p \) consist of a sequence of nodes \( H = n_1, n_2, n_3, \ldots, n_j = H' \), and \( T_{n_j}(b_{n_j}(t, d)) \) be an incremental vector of time and disutility after the traverse of link \( n_{j-1} \rightarrow n_j \) with label \( b_{n_{j-1}}(t, d) \) at \( n_{j-1} \). Based on the algorithm, we have:

\[
B_{n_j} \geq \text{merge}(b_{n_j}(t, d) + T_{n_j}(b_{n_j}(t, d))), \forall b_{n_j}(t, d) \in B_{n_j}
\]

(20)

\[
B_{n_j} \geq \text{merge}(b_{n_j}(t, d) + T_{n_j}(b_{n_j}(t, d))), \forall b_{n_j}(t, d) \in B_{n_j}
\]

(21)

\[
\ldots
B_{n_j} \geq \text{merge}(b_{n_j}(t, d) + T_{n_j}(b_{n_j}(t, d))), \forall b_{n_j}(t, d) \in B_{n_j}
\]

(22)

For \( \forall b_{n_j}(t, d) \in B_{n_j} \) during the label correcting process, the caused label at a node along \( p \) is either behaviorally dominated or survive in \( B_{n_j} \). In either case, the label at \( H' \) of \( p \) will not dominate any label of \( B_{H'} \). Thus, there is no path causing the label(s) at \( H' \) to dominate any label of \( B_{H'} \). With A2, the proposed algorithm outputs the optimal activity-travel patterns.

A best-case running time can be achieved if every link in the supernetwork is only visited once when the labels updated by the second visit are always dominated. Let \( P \) and \( Q \) denote the number of PTN and PVN connections respectively, \( M \) and \( N \) the number of nodes in PT time-expanded network and road network respectively, and \( V_{\text{SNK}} \) and \( E_{\text{SNK}} \) the number of nodes and links in the supernetwork. Given that for practical daily activity programs, the inequality \( E_{\text{SNK}} \leq \max(M, N) \) holds. In this case, we can obtain the following:

**Lemma 3.** The best-case time complexity of the algorithm is \( O(P \cdot M \cdot \log M + Q \cdot N \cdot \log N) \) with using Fibonacci priority queue for PTN and PVN queries.

**Proof.** As the PT expanded network and road network in general are very sparse graphs (Pyrga et al., 2008; Schultes, 2008), a PTN and PVN connection query require time \( O(M \cdot \log M) \) and \( O(N \cdot \log N) \) respectively with using Fibonacci priority queue. During the process of labeling, links except PTN and PVN connections, like parking, picking-up and transaction are treated in constant steps, i.e. \( O(1) \). Since every link is visited only once in the multi-state supernetwork, the time complexity for the algorithm is \( O(T(E_{\text{SNK}})) \), where \( T(E_{\text{SNK}}) \) represents the time to traverse all the “links” in the supernetwork. \( T(E_{\text{SNK}}) \) equals to \( O(P \cdot M \cdot \log M + Q \cdot N \cdot \log N + E_{\text{SNK}} - P - Q) \), which can be reduced to \( O(P \cdot M \cdot \log M + Q \cdot N \cdot \log N) \) due to \( E_{\text{SNK}} \leq \max(M, N) \).

Given the purpose of daily activity-travel scheduling in this paper, \( t \) is discretized into one minute per unit. With A1 and A2, this algorithm terminates in finite steps since \( t \) is positive integral bounded by \( \theta_H (\theta_H \leq 1440) \) and \( d \) is positive. We can obtain the worst-case running time complexity as follows.

**Lemma 4.** The worst-case time complexity of the algorithm is \( O(T_H \cdot V_{\text{SNK}} \cdot (T(E_{\text{SNK}}) + T_H \cdot E_{\text{SNK}})) \), where \( T_H \) is the number of discretized time steps in range \( (u_H, \theta_H) \).

**Proof.** Because of the dominance relationship, there are no two labels with the same time in any non-dominated label sets. Thus, there are at most \( T_H = \theta_H - u_H \leq 1440 \) in this paper) non-dominated labels at a node. According to Lemma 2, the algorithm finds all the non-dominated label sets of all nodes after at most \( V_{\text{SNK}} - 1 \) passes. A pass is defined as scanning all links in the supernetwork. Thus, the time for labeling correcting procedure is \( O(T_H \cdot V_{\text{SNK}} \cdot T(E_{\text{SNK}})) \), where \( T(E_{\text{SNK}}) \) equals to \( O(P \cdot M \cdot \log M + Q \cdot N \cdot \log N) \) according to Lemma 3. Meanwhile, it takes at most linear time \( O(T_H) \) for each label to merge with the non-dominated set. The total time for merging is \( O(T_H^2 \cdot V_{\text{SNK}} \cdot E_{\text{SNK}}) \). To sum up, the worst-case time complexity is \( O(T_H \cdot V_{\text{SNK}} \cdot (T(E_{\text{SNK}}) + T_H \cdot E_{\text{SNK}})) \).

In reality, the proposed algorithm terminates very fast for daily activity programs even without any speeding-up techniques. Although it is difficult to obtain the average running time complexity for label correcting procedures, they in general run fast in sparse networks, to which the proposed multi-state supernetworks belong.

It can be argued that the two components in Sections 3.1 and 3.2 are better treated in the supernetwork context due to the two main features of the multi-state supernetwork approach. First, alternative activity-travel patterns of an activity program are all represented as paths at a high level of detail. Especially, parking, detailed PT connections and multi-modal and multi-activity trip chaining are consistently represented, which are more or less missing in other studies. Second, the choice of location, mode and route are modeled simultaneously rather other in a sequential or hierarchical fashion that puts routing at last. Thus, in the multi-state supernetwork approach, the space–time constraints can be examined along the full activity-travel pattern. The time continuity and dependency of travel, activity participation and parking in the time dimension are closely linked.
4. Illustration

This section applies the improved multi-state supernetwork approach to the activity-travel scheduling problem for an individual. The approach is executed with C++ in Windows environment running at a PC using one core of Intel® CPU Q9400@ 2.67 GHz, 8 G RAM. The study area concerns the Eindhoven-Helmond corridor of the Netherlands (Fig. 7), which is about 15 km long and shares the largest volume of mobility in the Eindhoven region. Suppose that an individual \( i \), living in Helmond, has an activity program on a typical day. Fig. 7 and other related data are described as follows:

1. Two black dots (TH/1 and TH/2) denote PT stations (also THs). In between, there are an intercity and a slow train connection which take 10 and 12 min respectively in every 30 min. There are also two bus connections, which take 44 min in every 20 min. Fare for train and bus are 0.15 €/km and 0.3 €/km respectively. The PT timetable is provided by a PT routing company, 9292OV, for the purpose of scientific research. In the PT time-expanded network, there are 176,163 nodes and 309,979 links.

2. Two gray circles define the border of Eindhoven and Helmond city centers. There is a P + R facilities at the south edge of Eindhoven city center. Inside the circles, the roads are urban roads. Four types of roads are identified, i.e. urban, local, regional, national. Average speeds for bike and walking are assumed as \( (14, 16, 17, 0) \) and \( (5, 6, 0, 0) \) respectively in km/h, and the fuel cost for car is set as \( (0.18, 0.16, 0.12, 0.1) \) in €/km. In the road network, there are 28,734 nodes and 81,360 links. Speed profiles of car are assumed in Fig. 8a, in which there are two even peaks.

3. Boxed H and O denote \( i \)'s home and office respectively. Small dots represent the locations of grocery shopping, which are extracted from employee data in this study area. As for illustration, shopping locations are classified into three types, denoted as \{1, 2, 3\} labeled in brackets with the ID on the left, in terms of time window, quality level and price level.

![Eindhoven–Helmond corridor (scale: 1:100000).](image)

![Profiles of car use.](image)
Table 1
Personalized parameters.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Travel</th>
<th>Transition</th>
<th>Transaction</th>
<th>Cost (€)</th>
<th>Quality (All)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Walk</td>
<td>$\beta^{w}_{in}$</td>
<td>$\beta^{w}_{AT}$</td>
<td>$\beta^{w}_{CT}$</td>
<td>1.25</td>
<td>1.20</td>
</tr>
<tr>
<td>Bike</td>
<td>$\beta^{b}_{in}$</td>
<td>$\beta^{b}_{AT}$</td>
<td>$\beta^{b}_{CT}$</td>
<td>1.15</td>
<td>1.00</td>
</tr>
<tr>
<td>Bus</td>
<td>$\beta^{b}_{in}$</td>
<td>$\beta^{b}_{AT}$</td>
<td>$\beta^{b}_{CT}$</td>
<td>0.75</td>
<td>0.60</td>
</tr>
<tr>
<td>Slow train</td>
<td>$\beta^{s}_{in}$</td>
<td>$\beta^{s}_{AT}$</td>
<td>$\beta^{s}_{CT}$</td>
<td>0.8</td>
<td>0.50</td>
</tr>
<tr>
<td>Fast train</td>
<td>$\beta^{f}_{in}$</td>
<td>$\beta^{f}_{AT}$</td>
<td>$\beta^{f}_{CT}$</td>
<td>0.65</td>
<td>0.30</td>
</tr>
<tr>
<td>Car</td>
<td>$\beta^{c}_{in}$</td>
<td>$\beta^{c}_{AT}$</td>
<td>$\beta^{c}_{CT}$</td>
<td>1.2</td>
<td>1.00</td>
</tr>
<tr>
<td>Board &amp; wait</td>
<td>$\beta^{b}_{in}$</td>
<td>$\beta^{b}_{AT}$</td>
<td>$\beta^{b}_{CT}$</td>
<td>0.0</td>
<td>1.20</td>
</tr>
<tr>
<td>Alight</td>
<td>$\beta^{a}_{in}$</td>
<td>$\beta^{a}_{AT}$</td>
<td>$\beta^{a}_{CT}$</td>
<td>1.20</td>
<td>1.00</td>
</tr>
<tr>
<td>Park</td>
<td>$\beta^{p}_{in}$</td>
<td>$\beta^{p}_{AT}$</td>
<td>$\beta^{p}_{CT}$</td>
<td>1.00</td>
<td>1.20</td>
</tr>
<tr>
<td>Pick</td>
<td>$\beta^{i}_{in}$</td>
<td>$\beta^{i}_{AT}$</td>
<td>$\beta^{i}_{CT}$</td>
<td>1.00</td>
<td>1.20</td>
</tr>
<tr>
<td>Activity (All)</td>
<td>$\beta^{a}_{in}$</td>
<td>$\beta^{a}_{AT}$</td>
<td>$\beta^{a}_{CT}$</td>
<td>13.1</td>
<td>0.83</td>
</tr>
</tbody>
</table>

(4) Parking locations are differentiated by parking facility type and parking cost. Assume that bike parking is always possible and free. For car parking, potential parking locations are activity location, P + Rs and THs. ($\alpha_{p}, \beta_{h}$) is set in unit of ($€/h$) as $(0.8, 0.18)$ for P + Rs and THs. ($\alpha_{p}, \beta_{h}$) in other locations is dependent on the zoning, which is $(1.0, 0.6)$ if within 1 km to the city center points of Eindhoven and Helmond, $(0.0)$ if more than 2 km to the city center points, and otherwise, $(0.5, 0.3)$. The search time profiles are shown in Fig. 8b, which are drawn based on the function and attributes of the parking locations. We assume $disU_{i_{app}}(t) = \beta_{hp} \times \Delta t_{i_{app}}(t)$.

(5) For the sake of simplicity, we assume that activity states do not affect link costs. Personalized parameters are set in Table 1, in which time and monetary cost are the main components for travel, while monetary cost and quality are main components for locations. Other general parameters are set as: $s_{w} = 20 \text{ km/h}$, $s_{f} = 40 \text{ km/h}$, $E_{i} = 5 \text{ km}$, $E_{i} = 10 \text{ km}$, $E_{i} = 30 \text{ km}$, $E_{i} = +\infty$, $T_{ix} = 10 \text{ min}$ to $\forall a$, $d_{i_{app}} = 5$ to $\forall PV_{c}$ and $\forall PT_{c}$ in the corridor. $N^{h}$ is set as 3, which means that the parking location choice model is not needed here.

4.1. Example 1

This example concerns the activity program, which includes: (1) two activities, i.e., working at the office and grocery shopping, with ideal minimum durations of 310 and 10 min respectively; (2) ownership of a car; (3) personal sequencing: working immediately prior to shopping; (4) with $\forall a$, $\forall PT$ and $\forall Y$, $\forall d_{i_{app}} = 5$ to $\forall PV_{c}$ and $\forall PT_{c}$ in the corridor. $N^{h}$ is set in unit of $[7:30 \text{ am } + Y, 0] | Y = 5 \cdot X$, $0 \leq X < 8, X \in \mathbb{Z}$. There are 52 alternative locations in total for shopping (small dots in Fig. 7). If without any reduction in the choice set, the supernetwork scale becomes very large and the scheduling query cannot be answered in an acceptable time. Given the personal sequencing, there is only one direct sequential pair for shopping between working and returning home. After applying space–time constraints (Eqs. (2) and (3)), only eight alternatives are feasible including an ID set of {2,7,11,14,18,23,34,41}. Meanwhile, this approach can model the choice of departure time at home. The number of departure labels does not affect the computational performance of the label correcting algorithm since most of the source labels will soon be dominated in a later stage if $\beta_{hp} > 0$. The non-dominated label set at home, i.e. $B_{ih}$, is initialized as $[(7:30 \text{ am } + Y, 0) | Y = 5 \cdot X$, $0 \leq X < 8, X \in \mathbb{Z}$. Based on the setting above, the activity-travel scheduling algorithm is executed with different values of $N^{h} (1 \leq N^{h} \leq 8)$ for shopping. When $N^{h} = 1$, the non-dominated set at H’ is $[(6:58 \text{ pm }, 745.57), (6:36 \text{ pm }, 732.91)]$. By backtracking, a detailed activity-travel schedule including all the choice facets can be found. The first label is derived when $i$ leaves home by walking at 8:05 am and then taking PT; the latter is derived by departing with car at 8:10 am to TH/1 for parking and then walk to the office. Overall, i choose the second label based on the final objective. The running results of different $N^{h}$ are shown in Table 2. The results show that the optimal location is selected out when $N^{h} = 1$. By backtracking, the shopping location with ID 14(1) close to TH/1 (as the parking location) is selected.

![Fig. 9. Profiles of 10-min shopping.](image-url)
In this example, with Eqs. (2) and (3), only those shopping locations with longer opening time and within the narrow ellipse drawn based on office and home can be candidates in the location selection process. If without Eqs. (2) and (3), infeasible locations will compete for the candidacy, which possibly leads to wrong prediction of location choice and hinders the algorithm to converge. For example, location with ID 51(3) is also close to TH/1; however, it is not in the candidate list with Eqs. (2) and (3). In addition, profiles of activity and parking also make a difference. Location with ID 14(1) is selected also because its extra duration is shorter; and the individual parks the car at TH/because of the higher parking cost and higher searching time in the city center.

4.2. Example 2

Based on Example 1, we add one more private vehicle, i.e. bike. Since adding one private vehicle does not change the possible direct sequential pairs, the selected activity locations for a specific $NA^d$ are the same as in example 1. Unlike car, there is a limited riding distance for bike. The potential parking locations for bike are THs and alternative shopping locations within the distance of $ELib$ away from home. In this case, TH/2 and shopping locations inside Helmond are potential for bike parking.

We run the scheduling algorithm at different $NA^d$ again. When $NA^d = 1$, the non-dominated set at $H^0$ is $\{(6:36 \text{ pm}, 732.91)\}$. The same shopping location as Example 1 is selected. The optimal label is the first one, which implies that $i$ leaves home by bike to TH/2 and after parking takes PT to the office. The second label is obtained in the same way as in Example 1. For comparison, the results of different values of $NA^d$ are shown in Table 3. The optimal shopping location is selected out when $NA^d = 1$. By backtracking, it is found that $i$ would leave home at 8:10 am with bike and conduct shopping after working at location with ID 14(1); and then walk to TH/1 to take PT to TH/2, pick-up the bike and finally returns home.

This example shows that the multi-state supernetwork approach can still systematically assess the choices of departure time, route, mode and parking location after the incorporation of time-dependent components.

4.3. Example 3

Based on Example 2, this example relaxes the personal sequencing relationship between working and shopping. Thus, there is another two direct sequential pair for shopping, i.e. departing home and working, and departing home and returning home. The second pair implies that a second tour would occur. After applying Eqs. (2) and (3), there are 19 and 13 feasible alternative locations for these two direct sequential pairs respectively. The activity location selection procedure, i.e. Eqs. (4)
Comparison with different value of \( N_d \) of example 3.

<table>
<thead>
<tr>
<th>( N_d )</th>
<th>Supernetwork scale</th>
<th>Queries</th>
<th>Optimal label at ( H^t )</th>
<th>Running time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nodes</td>
<td>Links</td>
<td>PVN</td>
<td>PTN</td>
<td>All</td>
</tr>
<tr>
<td>1</td>
<td>132</td>
<td>222</td>
<td>94</td>
<td>58</td>
</tr>
<tr>
<td>2</td>
<td>204</td>
<td>277</td>
<td>123</td>
<td>70</td>
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<td>4</td>
<td>396</td>
<td>401</td>
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<tr>
<td>5</td>
<td>518</td>
<td>474</td>
<td>217</td>
<td>124</td>
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<tr>
<td>8</td>
<td>972</td>
<td>673</td>
<td>339</td>
<td>160</td>
</tr>
<tr>
<td>12</td>
<td>1804</td>
<td>977</td>
<td>523</td>
<td>220</td>
</tr>
<tr>
<td>16</td>
<td>2596</td>
<td>1222</td>
<td>682</td>
<td>262</td>
</tr>
<tr>
<td>19</td>
<td>3204</td>
<td>1398</td>
<td>798</td>
<td>292</td>
</tr>
</tbody>
</table>

Each row with the bold values denotes the optimal activity-travel pattern is found.

and (5), should be applied to all the valid locations. When \( N_d = 1 \), the selected activity location is the same as in example 1 and 2. The non-dominated label set consists of \{\((6:39\ pm,\ 697.86)\), \((6:06\ pm,\ 742.42)\), \((6:09\ pm,\ 705.53)\)\}. The first label is obtained in the same way as in Example 2. The second label is derived when \( i \) departs with car, parks it at TH/1 and then does shopping at location 14(1) before walking to the office, while the third is derided when departing with bike and parking it at TH/2. Although \( i \) does not prefer to shopping in the morning as shown in Fig. 9b, shopping in the morning can result in earlier home-returning in the end. Apparently, \((6:09\ pm,\ 705.53)\) is the best label, for which \( i \) needs to depart home at 7:45 am and conduct shopping before walking to the office. However, if \( i \) does not mind too much arriving home later, which means \( p_{d0}^i \) is set a very lower value, \( i \) will do shopping after working.

For comparison, the results of different values of \( N_d \) are shown in Table 4. When \( N_d > 1 \), the optimal label at \( H^t \) is no longer improved. Thus, when \( N_d = 1 \), the optimal shopping location is selected.

The above three examples demonstrate that multi-state supernetwork approach to activity-travel scheduling is still feasible after incorporating the finer treatments of time dimension mentioned in Section 3. The optimal locations for flexible activities can be picked out by setting low values of \( N_d \). This argument holds especially when the timeslots are tight between different direct sequential pairs even if with more activities in the AP.

According to MON at year of 2007 and 2008 (Dutch national travel diary), around 45% of the individuals have no more than 1 out-of-home activities and around 73% no more than two. In Example 3, there are two activities working and shopping in the activity program, which is quite typical daily activity program. If with more activities involved, the time budget for flexible activities is getting less; as a result, there are fewer alternatives for flexible activities. The algorithm can also terminate fast. Meanwhile, there is little difference on the query time per PTN or PVN connection between a small corridor like in the illustration and a large area, for instance, the whole country of The Netherlands. Thus, the scale of the examples is reasonably set.

With those finer time components, this approach can output more accurate activity-travel patterns with higher level of choice dimensions in a reasonable time. As shown, the number of queries is far more than the number of links in the supernetwork. It is because the label correcting procedure allows overtaking with C1, C3 and C4. The number of queries also increases with the increment of \( p_{d0}^i \), \( d_{ip}^{PV} \), and \( d_{ip}^{PT} \). The running time is mainly spent on PVN and PTN queries. It means that the response time to activity-travel scheduling can be heavily decreased by applying advanced speeding-up techniques, such as SHARC (Bauer et al., 2011) and highway hierarchies (Schultes, 2008), with which the speeding up factors can be up to 1000 times for PVNs and 100 times for PTNs. In example 3, if \( N_d = 5 \), the supernetwork includes unnecessary locations and connections, and the total computation time is 4.17 s with the raw algorithm. As an activity program with two activities is typical, the average computation time can be estimated as 4.17 s if setting \( N_d = 5 \) for a general activity program. If the average speeding-up ratio is achieved as 100 after adopting the techniques, the running time per individual can be reduced to 0.04 s, which stays in the same magnitude order as peer activity-travel scheduling models. Thus, this approach is not only of implication to the next generation of activity navigation system, but also of potential usage for large-scale accessibility analysis and dynamic activity-travel simulation system.

5. Conclusions and future work

Multi-state supernetwork model is a systemic approach that integrates networks of multi-modal transport and locations of facilities/services. Personal preferences from the demand side and network dynamics from the supply side can be represented in the supernetworks. This paper has incorporated (1) space–time constraints into the selection models for personalized networks, and (2) time/duration and disutility profiles of activity participation and the search phrase of parking into the representation and optimization process. Trade-off at a higher space–time resolution along the multi-modal and multi-activity chains can be modeled. A twist in the label correcting algorithm is proposed to guarantee behaviorally optimal solutions. Formal proofs are also provided. Examples demonstrated the feasibility and powerfulness of the improved multi-state supernetwork approach. This paper develops the previous supernetwork models fundamentally from the static context to
the time-dependent context. In conclusion, this paper represents the integral supernetwork model for activity-travel scheduling.

Nevertheless, several issues are worth considering. First, individuals' preferences and choice heuristics should be accurately estimated as a variety of personalized parameters. The pattern of generalized link cost is applied in the supernetwork to weigh between different choice facets and different factors. The precision of the predicted activity-travel patterns is contingent on accurate parameter settings. Second, advanced speedup-up techniques need incorporated into the routing of PVN and PTN connections so that any an activity-travel scheduling query can be answered in a real-time fashion. Third, new activity-travel patterns and behavior are emerging from observations with the widespread use of ICT and social network media. Thus, the integration of other choice dimensions such as ICT use and joint activity-travel in the multi-state supernetwork is also of great interest. These issues will be addressed in future research.

Acknowledgement

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References


