Influence of turbulence on the drop growth in warm clouds, Part I: comparison of numerically and experimentally determined collision kernels

Published in:
Meteorologische Zeitschrift

DOI:
10.1127/0941-2948/2014/0566

Published: 01/01/2014

Document Version
Publisher’s PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:
• A submitted manuscript is the author's version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
• The final author version and the galley proof are versions of the publication after peer review.
• The final published version features the final layout of the paper including the volume, issue and page numbers.

Link to publication

Citation for published version (APA):

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

• Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
• You may not further distribute the material or use it for any profit-making activity or commercial gain
• You may freely distribute the URL identifying the publication in the public portal

Take down policy
If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Download date: 12. Sep. 2017
Influence of turbulence on the drop growth in warm clouds, Part I: comparison of numerically and experimentally determined collision kernels

CHRISTOPH SIEWERT\textsuperscript{1,}\textsuperscript{a}, RÓBERT BORDÁS\textsuperscript{2}, ULRIKE WACKER\textsuperscript{3}, KLAUS D. BEHENG\textsuperscript{4}, RUDIE P.J. KUNNEN\textsuperscript{1,}\textsuperscript{a}, MATTHIAS MEINKE\textsuperscript{1}, WOLFGANG SCHRÖDER\textsuperscript{1} and DOMINIQUE THÉVENIN\textsuperscript{2}

1Institute of Aerodynamics, RWTH Aachen University, Germany
2Institute of Fluid Dynamics & Thermodynamics, University of Magdeburg “Otto von Guericke”, Germany
3Alfred-Wegener-Institut Helmholtz-Zentrum für Polar- und Meeresforschung, Bremerhaven, Germany
4Institute for Meteorology and Climate Research, Karlsruhe Institute of Technology, Germany
\textsuperscript{a}Current address: Fluid Dynamics Laboratory, Eindhoven University of Technology, The Netherlands

(Manuscript received December 16, 2013; in revised form June 5, 2014; accepted June 6, 2014)

Abstract

This study deals with the comparison of numerically and experimentally determined collision kernels of water drops in air turbulence. The numerical and experimental setups are matched as closely as possible. However, due to the individual numerical and experimental restrictions, it could not be avoided that the turbulent kinetic energy dissipation rate of the measurement and the simulations differ. Direct numerical simulations (DNS) are performed resulting in a very large database concerning geometric collision kernels with 1470 individual entries. Based on this database a fit function for the turbulent enhancement of the collision kernel is developed. In the experiments, the collision rates of large drops (radius > 7.5µm) are measured. These collision rates are compared with the developed fit, evaluated at the measurement conditions. Since the total collision rates match well for all occurring dissipation rates the distribution information of the fit could be used to enhance the statistical reliability and for the first time an experimental collision kernel could be constructed. In addition to the collision rates, the drop size distributions at three consecutive streamwise positions are measured. The drop size distributions contain mainly small drops (radius < 7.5µm). The measured evolution of the drop size distribution is confronted with model calculations based on the newly derived fit of the collision kernel. It turns out that the observed fast evolution of the drop size distribution can only be modeled if the collision kernel for small drops is drastically increased. A physical argument for this amplification is missing since for such small drops, neither DNSs nor experiments have been performed. For large drops, for which a good agreement of the collision rates was found in the DNS and the experiment, the time for the evolution of the spectrum in the wind tunnel is too short to draw any conclusion. Hence, the long-time evolution of the drop size distribution is presented in a submitted paper by Riechelmann et al.

Keywords: drop collisions, turbulence, collision kernel fit, direct numerical simulation, wind tunnel experiment, drop size spectrum evolution

1 Introduction

A long-lasting challenge in cloud physics is the explanation of the fast growth of drops with radii between 15 and 40 µm (GRABOWSKI and WANG, 2013). One step to answer this question is to understand the impact of cloud turbulence on the cloud microphysics (SHAW, 2003; DEVENISH et al., 2012). This study focuses on the growth of cloud drops by coagulation in a turbulent environment which is an important microphysical process (PRUPPACHER and KLETT, 1997). The collision frequencies of the drops are typically described using so-called collision kernels \( K(\text{BEHENG}, 2010) \) which are effective collection volumes per time.

The increase of computational power in the last decades has enabled the numerical investigation of collision kernels of small and heavy drops under the influence of gravity and turbulence, see GRABOWSKI and WANG (2013) for a recent review. Due to the size of the drops, the radius is less than 100µm, their motion is governed by the smallest scales of the turbulent flow (WANG et al., 2000). Hence, a direct numerical simulation (DNS) of the flow field is required. A relatively large number of studies (e.g. FRANKLIN et al., 2007; AYALA et al., 2008a; WOITIEZ et al., 2009; KUNNEN et al., 2013) have been conducted with the aim of obtaining collision kernels using DNS. However, a direct validation by experimentally determined collisions kernels is still missing. The validation is particularly needed since the aforementioned studies typically neglect the influence of the drops on the flow, leading to so-called geometric collision kernels \( \Gamma \). One notable exception is the
method of Wang et al. (2008), in which the local hydrodynamic interaction between approaching drops is modeled with an approximate method such that the collision kernel \( K \) can be determined. With this method they found that the hydrodynamic interaction can support or prevent the collision of two drops and quantified these effects in the collision efficiency \( E \). In this study, which shows a comparison of experimentally and numerically determined collision kernels, it will be distinguished between the geometric collision kernel \( \Gamma \) and the collision kernel \( K = E \Theta \).

From an experimental point of view, it is already challenging to detect the small water drops in the air stream, i.e., to measure the continuous drop size spectrum accurately. Concerning in-cloud measurements the aircraft speed is typically too high to measure the small scale dynamics with the necessary resolution. One notable exception is the helicopter based measurement platform Actos (Siebert et al., 2006). Nevertheless, the detection of a collision of two of these small drops is even more complex. A major problem is the rarity of these collisions making the measurement time for accurate statistics unacceptably long. In principle, this constraint could be relaxed by increasing the drop number densities in a laboratory experiment. However, the number density or volume loading in clouds is typically low, to such an extent that the influence of drops on the flow field can be neglected. If the volume loading would be increased considerably, the drops would noticeably alter the flow field due to momentum exchange. Hence, there exist only laboratory studies measuring the properties of the dispersed phase like drop size spectra, spatial distributions, and particle trajectories (e.g. Bateson et al., 2012; Saw et al., 2012; Bewley et al., 2013). Vohl et al. (1999) measured the growth of a single drop of 50 \( \mu \)m radius collecting droplets under turbulent conditions in a wind tunnel experiment. So far only Bordás et al. (2011, 2013) have measured the collision rates for an ensemble of drops directly. They overcame the problem of the rarity of collision events by increasing the dissipation rate \( \varepsilon \) of the turbulent kinetic energy \( k \). The consequence is an enhancement of the drop collision frequencies, and the total number of detected collisions is increased.

In this study, the collision kernels obtained from experiment and from DNS are juxtaposed. Both setups are designed to match as closely as possible. However, it could not be avoided that the dissipation rates differ significantly. As described above, in the experiment the dissipation rate had to be increased to lower the necessary measurement time. However, currently established DNSs are restricted to small dissipation rates since the necessary resolution and as such the computational effort scales with \( \varepsilon \) (Pope, 2000). Therefore, the numerical data are fitted and extrapolated to the dissipation rates of the experiment and then the data are compared. Firstly, for a direct comparison the total collision rates of the measurements and the fit are contrasted. Since the experimental and numerical total collision rates agree well, a representative collision kernel for the experimental conditions including the drop-size dependence is reconstructed based on the relative distribution of the numerical kernel. Secondly, in an indirect comparison the similarity is tested between the measured evolution of the drop size spectrum and the modeled evolution based on the newly derived collision kernel fit. The main idea and the procedure employed in this paper is similar to the study of Riemer et al. (2007), in which the coagulation growth of a single drop measured by Vohl et al. (1999) is compared with a model calculation based on the kernel fit of Zhou et al. (2001). Unlike the latter study, we consider the evolution of the entire drop size spectrum. Additionally, the newly developed kernel fits are physically more realistic since effects of gravity are included.

The paper is organized as follows. First, the experimental and numerical setup is briefly introduced. Then, the experimental and numerical methods to obtain the collision kernels are described. Thereafter, the numerical results are cast in a fit function by means of a multiplicative term – incorporating all turbulence effects – that is multiplied with the conventional gravitational collision kernel. Next, the fit function is compared to the direct measurements of the collision kernel. Finally, the evolution of a drop ensemble due to coagulation of colliding drops is modeled using the newly derived collision kernel fit for various turbulence scenarios to compare the simulated and experimentally observed changes in the drop size distributions.

2 Methods

2.1 Setup

A sketch of the experimental and numerical setup is shown in Fig. 1. It consists of a rectangular box with the length \( L = 0.8 \) m in the streamwise direction \( x \) and the height \( H = 0.3 \) \( L \) in both cross-stream directions \( y \) and \( z \). The fluid is air with a density \( \rho_f = 1.18 \) kg/m\(^3\) and a viscosity \( \nu = 1.5 \times 10^{-5} \) m\(^2\)/s. The water drops have
a density $\rho_p = 998 \text{kg/m}^3$. The turbulent fluctuations $\mathbf{v'}$ and the drops are created at the inflow. Due to a constant mean velocity $\mathbf{V} = (V, 0, 0)$ the turbulence and drops are transported from left to right. Since there is no mean shear and no perturbations besides at the inflow the turbulence is isotropic and is decaying in the streamwise direction due to viscous damping. For isotropic turbulence it can be shown (Pope, 2000) that the turbulent kinetic energy $k$ decays as

$$k(t) = k_0 \left(\frac{t}{t_0}\right)^{-1.3}$$ \tag{2.1}

where $t_0$ is an arbitrary reference time, and $k_0$ is the value of $k$ at that time. Due to the constant mean velocity $\mathbf{V}$ and the low turbulence intensity ($\mathbf{v'} \ll \mathbf{V}$) this temporal dependence can be replaced by a spatial dependence. In Fig. 2 the measured (experiment M4 Bordás et al., 2011, 2013) and the computed decay of the $k$ are compared with the theoretical scaling of isotropic turbulence (eq. 2.1) in a double logarithmic plot. It is evident that the experimental and numerical data match well. In the measurement section with the length $L_s = 0.4 \text{m}$ located in the rear part of the domain the collision statistics are gathered. Note that streamwise decaying turbulence is present. Hence, all results might not solely depend on the local properties but also on the flow along the previous drop trajectory (Kunnen et al., 2013).

### 2.2 Numerical simulation

#### 2.2.1 Air flow

Due to the small volume loading the influence of the drops on the flow can be neglected (Hagemeier et al., 2011). Hence, the flow field can be computed independent of the suspended drops. The procedure is the same as in Kunnen et al. (2013), Siewert et al. (2014a), and Siewert et al. (2014b). The turbulent fluctuations $\mathbf{v'}$ at the inflow are generated with the method of Batten et al. (2004) and added to the constant mean flow velocity $\mathbf{V}$. The Reynolds number $Re = VL/v$ is 80000. The turbulent flow is practically unbounded since the cross-stream dimension $H$ is at least 25 times larger than the integral length scale. The Navier-Stokes equations are solved by a Finite-Volume solver on a Cartesian grid (Hartmann et al., 2008). The domain is discretized using about 53 million cells such that the cell length $\Delta x$ is of the same size as the smallest scales of turbulence, i.e. the Kolmogorov length $L_k$ ($\Delta x / L_k = 1.65 - 1.0$). Thus a direct numerical simulation is performed (Kunnen et al., 2013).

#### 2.2.2 Drop phase

Drops are constantly released at the inflow. Due to their small radius ($r = 5, 10, \ldots, 100 \mu m < L_k$) and their high density the particles can be treated as point-particles and their equation of motion (Maxey and Riley, 1983) can be simplified to

$$\frac{d\mathbf{v}_d}{dt} = \frac{f_{\text{corr}}}{\tau_d}[\mathbf{v}_f(\mathbf{x}_d) + \mathbf{v}_t - \mathbf{v}_d], \tag{2.2}$$

where $\mathbf{v}_d$ is the instantaneous drop velocity, $f_{\text{corr}}$ a non-linear correction to the classical Stokes drag, $\tau_d$ the drop response time, $\mathbf{v}_f$ the fluid velocity at the drop position $\mathbf{x}_d$, and $\mathbf{v}_t$ the drop terminal velocity. The response time increases with drop size according to

$$\tau_d = \frac{2\pi \rho_d}{9 \nu \rho f} \tag{2.3}$$

The terminal fall velocity $\mathbf{v}_t$ is usually measured in a quiescent fluid. Since the particle Reynolds number $Re_p = 2r|\mathbf{v}_f - \mathbf{v}_d|/v$ attains values from 0.002 to 10 the terminal velocity is significantly smaller than the Stokes solution for drops $r > 30 \mu m$. Hence, it is corrected by the drag correction factor $f_{\text{corr}} = 1 + 0.15 Re_p^{0.687}$ (Clift, 1978) to

$$\mathbf{v}_t = \frac{\tau_d \mathbf{g}}{f_{\text{corr}}} \tag{2.4}$$

where $\mathbf{g}$ denotes gravity.

Around 45 million drops are advanced by a Lagrangian solver simultaneously with the flow. Thereby the fluid velocity $\mathbf{v}_f$ is interpolated by a third order least square interpolation at the particle position $\mathbf{x}_d$ (see eq. 2.2). Since the influence of the drops on the flow is
neglected, i.e., the drops do not hydrodynamically interact, a ghost particle approach is used in which the drops can pass through each other. However, collisions are detected by the proactive method (Sundaram and Collins, 1996).

The number (per volume) of drops with radii within an interval $\Delta r_i$ around $r_i$ is denoted by $n_i$. The collision rate $\dot{N}_{ij}$, that is the number (per volume and per time) of collisions of drops with radii from the intervals $\Delta r_i$, $\Delta r_j$, is detected. Then, the geometric collision kernel $\Gamma_{ij}$ can be calculated by the known number densities $n_i$ and $n_j$

$$\Gamma_{ij}(r_i, r_j) = \frac{\dot{N}_{ij}}{n_i n_j},$$

(2.5)

Since the turbulent flow is spatially decaying the drops are advected through regions with different turbulence intensities. Hence, the geometric collision kernel (eq. 2.5) depends not only on the radii of the involved drops, but also on the dissipation rate $\varepsilon$. To provide a qualitative estimate of the relative importance of fluid and particle inertia, turbulence, and gravity on the particle motion, the particle Stokes number $\tau_s/\tau_K$ attains values from 0.004 to 5, the non-dimensional terminal velocity $v_t/v_K$ ranges from 0.1 to 50, and the squared Galileo number $(\rho_d/\rho_s - 1)8g r^3/v^2$ has values from 0.037 to 294. Thus the drop dynamics vary with the drop size from fluid tracer-like behavior to inertia and gravity dominated behavior within the size range investigated here. Statistics are gathered in 6 regions characterized by mean dissipation rates $\varepsilon = 250, 200, \ldots 50, 30 \text{cm}^2/\text{s}^3$ and a Taylor-scale based Reynolds number $R_\lambda = 20$. The Reynolds number is relative small due to the streamwise confinement of the turbulent length scale in the streamwise decaying turbulent flow field. Hence, the lack of scale separation might be an issue. However, Kunnen et al. (2013) showed that with the new streamwise decaying setup the collision statistics of the typical periodic setup can be reproduced. Thus, the new setup is a valuable alternative. The collision statistics database of Kunnen et al. (2013) is extended and now forms the largest database so far for turbulent geometric collision kernels, containing 1470 independent entries, see sec. 3.1. In Fig. 3 a) exemplary results of $\Gamma_{ij}$ ($\varepsilon = 100 \text{cm}^2/\text{s}^3$) over $r_i$ are given for $r_j = 30 \mu m$ and 50 $\mu m$. The results of Ayala et al. (2008a) are plotted for comparison. The error bars indicate the standard deviation of the kernel as a measure for the sampling uncertainties (Ayala et al., 2008a). It can be seen that the standard deviation of the kernel is at least two orders of magnitude lower than the obtained kernel, i.e., the sampling errors are negligible. However, it has to be kept in mind that in contrast to the equations of the flow phase the drop equations are simplified based on several assumption. These simplifications introduce model errors which cannot be quantified.

2.3 Results from the M4 experiment

2.3.1 Air flow

The turbulent two-phase flow for the experimental measurements is produced in the wind tunnel of the University of Magdeburg, “Otto von Guericke”, using the configuration M4 described in Bordás et al. (2011) and in Bordás et al. (2013). The turbulence is produced by a cylindrical bluff body (radius 10 mm) and a passive turbulence grid (grid spacing 24 mm and rod radius 2.5 mm) located 150 mm upstream of the measurement section. Additionally, the drop injection (pipe with radius 9 mm inserted from above 630 mm upstream of the measurement section) also adds turbulent fluctuations. To reduce the influence of these disturbances...
on the results only the lower half of the measurement section is considered, but still the turbulent flow field is only approximately isotropic. The Reynolds number \( \text{Re} = \frac{VL_0}{v} \) is 80000. For the current study, particle-image velocimetry (PIV) data in the measurement plane \( y = 0 \) are used to derive the mean turbulent kinetic energy \( k \) and its mean dissipation rate \( \varepsilon \). Additional flow characterizations are reported in Bordás et al. (2011), where the energy dissipation rate is estimated by means of PIV measurements and is presented as an average value for the whole domain, and in Bordás et al. (2013), where the specific values are shown for the \( x = 0 \) positions derived from Laser-Doppler velocimetry measurements. It is clear that the smallest turbulent scales are not resolved by PIV measurements. However, a correction procedure for the measured data based on the Smagorinsky large-eddy simulation model suggested by Tanaka and Eaton (2007) is applied. Even after this correction, we expect that the measured dissipation rates underestimate the real values; yet they are still at least one order of magnitude larger than those in the present numerical simulations. At the positions of the drop collision detection the dissipation rate ranges from 17250 to 71100 cm\(^2\)/s, the resulting Kolmogorov length scale \( L_k = 220 \ldots 150 \mu\text{m} \), the integral length scale \( 0.3 \ldots 3 \text{ cm} \), and the Taylor-scale based Reynolds number \( R_{\lambda} = 30 \ldots 140 \).

2.3.2 Drop phase

Water drops are constantly released at the inflow using air-assisted atomizers (Lechler, type 154.104.16.14 and 166.208.16.12). The dispersed phase is measured by means of phase-Doppler anemometry, delivering drop velocity and diameter as a function of time with a typical temporal resolution of several kHz. Knowing the drop arrival time and the size of the detection volume, which depends on the drop size and the drop trajectory as described by Roisman and Tropea (2001), the drop number density can be derived. Since the measurement volume is typically smaller than 1 mm\(^3\), the measurement data can be considered as local information in the flow.

Fig. 4 shows the observed drop size distributions \( f_r(r) \) with drop radius \( r \) as internal coordinate and \( n_i = f_i(r_i)\Delta r_i \) being the number of drops (per volume) in the radius interval \( \Delta r_i \) around \( r_i \). The three curves give the size distributions at the inlet, at the beginning of the measurement region and at its end (outlet). Using the constant mean velocity \( V \), the three observations are interpreted as traveling times of \( t = 0 \text{s}, 0.15 \text{s}, \) and \( 0.3 \text{s} \), respectively. Most of the drops have radii below 7.5 \( \mu\text{m} \), and no drops are present with \( r \geq 50 \mu\text{m} \). The number of small drops is nearly halved during the flow; any change for the large drops is within the plotting accuracy. Size distributions from previous experiments have already been presented in Bordás et al. (2013).

The shadowgraph imaging technique is employed to detect drop collisions in the flow. Collision events are identified based on a geometrical evaluation of the drop projections for drops with radii larger than 7.5 \( \mu\text{m} \) (Bordás et al., 2013). All collision events are assumed to result in coalescence of the involved drops. It should be noted here that this method is a local but statistical method (10 Hz recording rate), resulting in a local drop collision probability instead of a temporal information.

From the estimated drop number density per unit time and from the collision probability, the collision rate can be derived. Unfortunately, not enough collision events were registered to provide directly a statistically reliable distribution of the collisions into size classes of the participating drops. Instead the total collision rate \( \dot{N}_{\text{tot}} \) is given, i.e., the sum of all collisions per unit volume and unit time \( \dot{N}_{ij} \)

\[
\dot{N}_{\text{tot}} = \sum_i \sum_{j \leq i} \dot{N}_{ij}. \tag{2.6}
\]

3 Experimental-numerical comparison

3.1 Curve fit of the numerical results

DNS simulations provide only collision rates for discrete input parameters, i.e., discrete drop radii. However, in clouds as well as in the wind tunnel continuous drop spectra are present (cf. Fig. 4). Hence, the DNS data are not directly applicable. Therefore, a curve fit to the discrete DNS results is developed in the following.

For drops settling due to gravity in a quiescent atmosphere the gravitational collision kernel \( K_{\text{grav}} \) reads

\[
K_{\text{grav}}(r_i, r_j) = E_{\text{grav}}(r_i, r_j) \Gamma_{\text{grav}}(r_i, r_j) = E_{\text{grav}}(r_i, r_j) \pi (r_i + r_j)^2 |v_i - v_j|.
\tag{3.1}
\]

Here the subscript \( \text{grav} \) indicates settling in a quiescent atmosphere in contrast to a turbulent atmosphere \( \text{turb} \). The quantity \( \Gamma_{\text{grav}} \) denotes the gravitational geometric collision kernel, \( E_{\text{grav}} \) is the collision efficiency comprising all influences due to the effects of the flow fields surrounding the approaching drops in an otherwise quiescent environment, and \( v_i \) is the terminal fall velocity of the drop. Note that when analyzed in more detail \( E_{\text{grav}} \) should be formally multiplied by another term, the coalescence efficiency describing the fact that the two colliding drops are not unified after collision. However, in this study it is assumed that the outcome of a collision of two drops is a single larger drop, hence the coalescence efficiency equals one. Pinsky et al. (2001) found that the collision kernel depends significantly on the altitude since the drop terminal velocities \( v_i \) depend on the air density and viscosity.

Wang et al. (2005) suggested to express the turbulent enhancement of the collision frequencies as a factor \( \eta_g \) relative to the geometric gravitational kernel

\[
\Gamma_{\text{turb}} = \eta_g \Gamma_{\text{grav}}. \tag{3.2}
\]
The usage of $\eta_g$ has the advantage that differences in the atmospheric conditions, e.g., in the air density and viscosity, cancel since both $\Gamma_{\text{grav}}$ and $\Gamma_{\text{turb}}$ are affected. This can be supported by analyzing different DNS calculation results. Ayala et al. (2008a) used linear Stokes drag in the main part of their study. In the present study, however, a non-linear correction to Stokes drag is used, which significantly lowers the terminal velocities of droplets with $r > 30 \mu m$ (see eq. 2.4). Additionally, the atmospheric conditions are slightly different leading to approximately 4 percent different terminal velocities, also for small drops. Hence, the absolute values for the collision kernels differ greatly, especially for large drops (Fig. 3 a). However, if the collision kernel is normalized by the corresponding gravitational kernel, these differences vanish (Fig. 3 b).

To keep the fit of the turbulent geometric collision kernel $\Gamma_{\text{turb}}$ independent of the actual atmospheric conditions that it is applied to, it is developed relative to the geometric gravitational collision kernel $\Gamma_{\text{grav}}$. The values of the simulated collision kernel can be cast into a symmetric matrix with the different radius classes as row and column indices. The resulting fit is visualized in such a matrix form, see Fig. 6 b). From this perspective the DNS simulation results are 7 matrices, one for each dissipation rate $\varepsilon$ including the gravitational case with $\varepsilon = 0$, each containing 210 independent entries of the 20 considered radii. The turbulent kernel in Fig. 5 a) spans over several orders of magnitude, i.e., it is a multi-scale problem within the considered drop size range as indicated in sec. 2.2.2. In contrast, the turbulent enhancement $\eta_g$ is on the order of one, see Fig. 5 b) where all secondary diagonals ($R - r = \text{const}$, with $R = \max(r_i,r_j)$, $r = \min(r_i,r_j)$) of the $\varepsilon = 100 \text{cm}^2/s^3$ matrix normalized by the gravitational $\varepsilon = 0 \text{cm}^2/s^3$ matrix are plotted. This is an additional advantage of fitting $\eta_g$ instead of directly fitting the kernel. From the $\eta_g$ plot it is assumed that the turbulent enhancement is normally distributed for each secondary diagonal. Hence, the mean value $\mu$, the standard deviation $\sigma$, and the amplification factor $\alpha$ of these normal distributions are fitted depending on the radius $r$ [\mu m], the secondary diagonal $R - r$ [\mu m], and the dissipation rate $\varepsilon$ [\text{cm}^2/s^3].

$$\eta_g(r, \alpha(R-r, \varepsilon), \mu(R-r, \varepsilon), \sigma(R-r, \varepsilon)) = 1 + \frac{\alpha}{(2\pi\sigma^2)^{0.5}} \exp \left( -\frac{(r-\mu)^2}{2\sigma^2} \right)$$  \hspace{1cm} (3.3)

$$\alpha(R-r, \varepsilon) = 16.88 \exp(-0.184(R-r)) \varepsilon^{0.2852}$$ \hspace{1cm} (3.4)

$$\mu(R-r, \varepsilon) = (-0.0052(R-r)^2 + 0.145(R-r) + 3.5) \frac{3.8 \varepsilon + 1915}{\varepsilon + 85}$$ \hspace{1cm} (3.5)

$$\sigma(R-r, \varepsilon) = \frac{(R-r) + 155}{(R-r) + 25} \frac{\varepsilon + 1300}{\varepsilon + 166}$$ \hspace{1cm} (3.6)

This fit approach with 13 adjustable parameters leads to continuous curves through all 1330 available off-diagonal data points with a maximum deviation of 10 percent. It is ensured that the case of the gravitational kernel ($\varepsilon = 0$) is exactly recovered.

However, fitting $\eta_g$ instead of the kernel also has the disadvantage that the case of monodisperse collisions has to be treated differently. For equal-sized drops ($R-r=0$) the kernel is zero in the gravitational case.
and rj first 5 secondary diagonals (nel conditions (eq. 3.8) ensures a smooth, monotonic transition. The chosen value \( r + 5 \) is solely determined by the spacing between two radius classes in the DNS calculation. It should only alter the parameters in \( \chi \), not the kernel itself. The fit for the monodisperse case is less accurate than the bidisperse case but also the underlying DNS data are less certain. On the one hand, the statistical accuracy of the collision kernel \( \Gamma \) scales with the number of collisions that can be averaged. Since monodisperse collisions are less frequent by orders of magnitude, the statistical uncertainties are much greater, see Fig. 3 a). On the other hand, there is no consensus in the literature about the physics of monodisperse clustering under the combined influence of gravity and turbulence. Although the results of Ayala et al. (2008a) and this study match nearly perfectly for the bidisperse case, the monodisperse clustering scales differently, see the discussions in Kunnen et al. (2013) and Siewert et al. (2014a). Hence, the accuracy of the fit is considered acceptable also for the monodisperse case. Note that at \( \varepsilon = 0 \), i.e., no turbulence (= reference case later on), it yields \( \Gamma_{\text{turb}} \equiv \Gamma_{\text{grav}} \) as desired.

It must be pointed out that the fit itself consists of purely empirical curve fitting. Only the underlying DNS data are based on a physical description of the turbulent collision problem. However, due to the large number of available data points (1470) and their smoothness it seems reasonable that the fit can also be applied outside of the parameter space for which it has been developed. Nevertheless, the reader must keep in mind that an extrapolation of the dissipation rate \( \varepsilon \) over several orders of magnitude is carried out in order to compare the DNS data to the experimental wind tunnel data. In Fig. 6 b) the turbulent enhancement factor \( \eta_{\varepsilon} \) (eq. 3.3) is shown for DNS conditions (\( \varepsilon \leq 250 \text{cm}^2/\text{s}^3 \)) and for wind tunnel conditions (\( \varepsilon \sim 5000 \text{cm}^2/\text{s}^3 \)). At DNS conditions the turbulent enhancement of the collision kernel is only moderate and restricted to nearly equal sized particles. In contrast, at wind tunnel conditions the enhancement is restricted to collisions of small drops, since the mean value of the distributions has moved to smaller radii (eq. 3.5), the peak value is increased (eq. 3.4), and the standard deviation is smaller (eq. 3.6). In Fig. 6 a) these differences are amplified by choosing logarithmic radii axes.

For real cloud environments such an extrapolation of the dissipation rate \( \varepsilon \) is typically not needed given the lower values of \( \varepsilon \) encountered (Siewert et al., 2006).
However, the Reynolds number $R_\lambda$ is much larger at cloud environment than in the DNS and the experiment. There is no consensus in the literature how the Reynolds number affects the collision kernel. On the one hand Shaw (2003) noted that the turbulent intermittency increases the Lagrangian acceleration, on the other hand Ayala et al. (2008a) concluded from their DNS simulations that $\varepsilon$ is of primary importance and the dependence on $R_\lambda$ is only secondary. However, in the theoretical model of Ayala et al. (2008b) the collision kernel differs greatly when it is applied to DNS conditions ($R_\lambda \sim 70$) or to cloud conditions ($R_\lambda \sim 20000$). Recent results of the same group do not show a strong influence of $R_\lambda$ up to 500 (see Rosa et al., 2013, Fig. 15). However, Falkovich et al. (2002) proposed the activation of a different physical collision process at high Reynolds numbers. Its existence was recently confirmed by experiments (Bewley et al., 2013). Grabowski and Wang (2013) argued that the fraction in space occupied by highly intermittent regions has an insignificant effect on the collision kernel as confirmed by the saturation of the currently available DNS and kinematic simulation. In conclusion, the Reynolds number dependency of the collision kernel remains an open question. Since no DNS data at different Reynolds numbers is available, there is no Reynolds number dependence of the current fit. Still the turbulence enhancement by this fit should be considered as a lower bound for the collision probabilities in a cloud environment with $R_\lambda$ up to three orders of magnitude larger.

### 3.2 Comparison of the collision kernels

In this section the numerically and experimentally determined collision kernels will be compared. For distinction, we will mark values coming from the fit function by the subscript $Fit$ and values coming from the experiment by the subscript $Exp$. In the last section the kernel fit was developed to extrapolate the numerical kernel to the experimental conditions. As explained before the measurements provide drop number densities $n_{i,Exp}$ and the total collision rates $N_{tot,Exp}$. However, a disproportionately high measurement effort would be needed for a statistically reliable distribution of the collisions into size classes of the participating drops, i.e., into size-dependent collision rates $N_{ij,Exp}$.

For a possible comparison either the total collision rates $N_{tot}$ (eq. 2.6) can be juxtaposed, which was the main topic of Bordás et al. (2013), or an experimental collision kernel $K_{ij,Exp}$ can be reconstructed by the adoption of its drop size dependence from the numerical simulations. The present paper considers both approaches.

The first comparison is done for the measured total collision rate $N_{tot}$ as a function of the dissipation rate $\varepsilon$ as presented in Fig. 7. It can be seen that $N_{tot,Exp}$ is more or less constant for all dissipation rates. However, at different measurement positions not only the dissipation rate is different but also the drop number densities. Hence, although it is expected that in principle higher dissipation rates result in higher collision frequencies, this cannot be observed here due to the differences of the drop number densities. The measured collision rates are confronted with those derived by the kernel fit based on DNS. The total number of collisions is calculated from eqs. 2.5 and 2.6 for the measured conditions as

$$N_{tot,Fit} = \sum_{j=1} \sum_{i=1} \Gamma_{ij,Fit} E_{ij,turb} n_{i,Exp} n_{j,Exp}$$

(3.9)

with the collision kernel fit $\Gamma_{ij,Fit}$ as a function of the dissipation rate and the turbulent collision efficiency $E_{ij,turb}$ according to Wang et al. (2008). In the same way, the number of measured collisions is calculated from

$$N_{tot,Exp} = \sum_{j=1} \sum_{i=1} n_{i,Exp} n_{j,Exp}$$

(3.10)

In conclusion, the Reynolds number dependency of the collision kernel as confirmed by the saturation of the currently available DNS and kinematic simulation. In conclusion, the Reynolds number dependency of the collision kernel remains an open question.

#### Figure 6: Colored contour plots using the kernel fit dependent on the participating drop radii and the dissipation rate. The dashed contourlines correspond to the colorbar values. The upper left triangle corresponds to the DNS conditions ($\varepsilon = 250 \text{cm}^2/\text{s}^3$). The lower right triangle corresponds to the wind tunnel conditions ($\varepsilon = 50000 \text{cm}^2/\text{s}^3$) a) geometric collision kernel $\Gamma$ (eq. 3.7) over logarithmic radii classes b) turbulent enhancement factor $\eta_f$ (eq. 3.3) over linear radii classes.
Figure 7: Comparison of the total number of collisions per volume and time $\dot{N}_\text{tot} \quad [1/(\text{cm}^3\text{s})]$ as a function of the dissipation rate $\varepsilon \quad [\text{cm}^2\text{s}^{-3}]$. The measured collision rates from the experiment are shown as circles with error bars. Additionally, the geometric collision kernels of the fit of Ayala et al. (2008b) (triangles) and this study (squares) are calculated and multiplied by the measured drop densities $n_i, \text{Exp}$ and the collision efficiency $E_{ij,\text{turb}}$ of Wang et al. (2008).

Figure 8: Comparison of the experimental (circles with grey shaded error bars) and computational (squares) collision kernels $K_{\text{turb}}$ as a function of the radius $r_i$ in a semi-log plot for $r_j = 30 \mu$m. The experiment and the fit are equal, i.e., $\text{RCR}_{ij,\text{Exp}} = \text{RCR}_{ij,\text{Fit}}$. Therewith we can estimate the distribution of the experimental collision rates

$$\dot{N}_{ij,\text{Exp}} = \dot{N}_{\text{tot,Exp}} \text{RCR}_{ij,\text{Fit}} = \dot{N}_{\text{tot,Exp}} \frac{\dot{N}_{ij,\text{Fit}}}{\dot{N}_{\text{tot,Fit}}}. \quad (3.11)$$

From the collision rates $\dot{N}_{ij,\text{Exp}}$ and the number densities $n_i, \text{Exp}$ the experimental collision kernel $K_{ij,\text{Exp}}$ can be calculated (eq. 2.5). Obviously, the dependence of the experimental kernel on the drop radius is influenced by the applied fit function and the applied radii resolution. The collision kernels $K_{\text{turb}}$, derived from the experimental measurements and the kernel fit, are depicted in Fig. 8 for a selected radius $r_j = 30 \mu$m and the measured dissipation rate $\varepsilon = 42635 \text{cm}^2\text{s}^{-3}$. As known from other studies, in principle the collision kernel grows at increasing drop radius. However, there is a broad local minimum for equal-sized drops since the difference in the gravitational settling velocity vanishes, which is otherwise the dominant contribution to the collision kernel (see eq. 3.1). Within the broad local minimum a thin local maximum occurs due to the clustering effect (Ayala et al., 2008a) and the hydrodynamic interaction (Wang et al., 2008).

### 3.3 Comparison of the drop spectrum evolution

In this section, the evolution of a drop size distribution (DSD) for conditions as observed in experiment M4 is...
In general, it changes due to advection, sedimentation in the field of gravity, nucleation, condensation/evaporation, coagulation, and breakup. According to the experimental setup, we neglect all microphysical processes except coagulation. Then, the prognostic equation for the DSD \( f_m(m, t) \) reads (e.g. Hu and Sristavasta, 1995; e.g. Pruppacher and Klett, 1997)

\[
\frac{\partial f_m(m, t)}{\partial t} = \frac{1}{2} \int_{0}^{m} K(m - m', m') f_m(m', t) f_m(m - m', t) \, dm' - \int_{0}^{\infty} K(m, m') f_m(m', t) f_m(m, t) \, dm', \tag{3.12}
\]

where \( m \) denotes the drop mass and \( f_m(m) \, dm \) is the number (per volume) of drops having drop mass in an interval \( dm \) around \( m \). The term on the left-hand side denotes the rate of change of the DSD. On the right-hand side, the first term denotes the gain of drops by coagulation of two smaller drops having masses \( m' \) and \( m - m' \) such that the resulting drop has mass \( m \), ensuring mass conservation. The second term describes the loss of drops due to collision of a drop of mass \( m \) with any other drop. The quantity \( K(m', m - m') \) represents the collision kernel. Eq. 3.12 is solved numerically for given \( K \) in the framework of a 1D-model (with coordinate height) called RAINSHAFT (cf. Seifert and Beheng (2001), Seifert (2008)).

With regard to the comparison between the simulated evolution of the DSD and the observed DSD, we assume steady state conditions in the experiment and interpret the changes of the measured DSDs in \( x \)-direction as changes with time, \( t = x/V \) with \( V \) being the average advection velocity in \( x \)-direction. This gives a timescale of less than 1 s. Within such a small time span, the observed droplets settle in the gravitation field of the Earth only over a vertical distance of a few centimeters, hence we neglect this transport together with all other processes except coagulation in the simulation. Yet, the effect of the drops’ differential fall velocity is considered in the collision kernel.

We will model the evolution of the drop size distribution from experiment M4, see Fig. 4, assuming a collision efficiency \( E = 1 \), hence \( K = \Gamma \). and using the geometric collision kernel \( \Gamma_{\text{geom}} \) from eq. 3.7.

Figure 9 shows the evolution of the DSD \( f_s(r) \). In the case of spherical drops \( f_s(r) \) and \( f_m(m) \) are uniquely related by the transformation \( f_s(r) \, dr = f_m(m) \, dm \). That is the same number of drops is found in the radius interval \( dr \) around \( r \) as in the mass interval \( dm \) around \( m \) (for spherical drops \( m = m(r) = 4\pi \rho_r r^3/3 \)).

In the RAINSHAFT simulations we initialize the DSD by using the observed DSD (cf. Fig. 4). This has been measured at increments of \( \Delta r = 2 \mu m \) for small drops and increasing \( \Delta r \) for large drops, emphasized by the thick black line in Fig. 9, and using the boundary condition \( f_s(r = 0) = 0 \). The initial DSD for the simulations is found by linear interpolation of the observed DSD on a very fine \( r \)-grid.

All observed DSDs are extrapolated for \( r < 2 \mu m \), see Fig. 4.

The RAINSHAFT simulations are performed for several variants to calculate the collection kernel \( \Gamma \). In the reference case without turbulence (Fig. 9 \( \varepsilon = 0 \)), the collisions are caused only by the difference in the terminal fall velocities of the drops. Since the fall velocities are small for small drops, the evolution of the DSD is very slow. In the second case, \( \varepsilon = 250 \text{cm}^2\text{s}^{-3} \) is assumed, that is the upper limit for which the kernel fit eq. 3.7 is valid. Yet, the evolution is still too slow to match with the experiment M4. A similar result follows for the extrapolation of formula 3.7 to \( \varepsilon = 50000 \text{cm}^2\text{s}^{-3} \), a value of the order of the dissipation rate observed in the experiment M4.

Obviously, the RAINSHAFT simulation of the DSD exhibits an evolution that is much too slow compared to the M4 results when using this \( \Gamma \)-fit (eq. 3.7), despite the apparent agreement of the kernels as derived from experiment and DNS-simulation. Their conditions are now re-inspected. (i) The curve fit (eq. 3.7) has been derived for dissipation rates \( \varepsilon \leq 250 \text{cm}^2\text{s}^{-3} \). The extrapolation up to \( \varepsilon = 50000 \text{cm}^2\text{s}^{-3} \) as in the third case study, is certainly doubtful and has been used simply as a sensitivity study. (ii) During the experiment, no information on the kernel is available for drops of radius \( r < 7.5 \mu m \) (Bordás et al., 2013). Thus the fit function for \( \Gamma \) as derived from DNS cannot be validated for small drops. (iii) The derivation of the fit is performed at radius intervals of 5 \( \mu m \), with \( r = 5 \mu m \) being the smallest drop radius under consideration. This is not a severe problem for comparison of the fitted kernel with data from the experiment. In particular we find good agreement in Fig. 8, when one of the collision partners has a radius of 30 \( \mu m \). For the kernel to be applied to the RAINSHAFT simulations of the DSD, however, no confirmation is available for the extrapolation of the fitted kernel data to smaller radii. To conclude, neither the experiment M4 nor the DNS simulations provide any resilient information on the kernel for the small drops as required for the RAINSHAFT simulations.

By trial and error, we found reasonable agreement between the RAINSHAFT simulation results and experiment M4 when a lower limit of the kernel of \( \Gamma_{\text{min}} = 5 \times 10^{-4} \text{cm}^3/\text{s} \) is prescribed for colliding drops. This value is highlighted by the thick dashed line in Fig. 6 a). The lower limit increases the kernel if the larger participating drop has a radius smaller than roughly 30 \( \mu m \). In the size range with abundant drops, this choice means for drops of e.g., \( r_i = 2 \mu m \) and \( r_j = 10 \mu m \) an increase of \( \Gamma \) by a factor of roughly 100. The respective faster evolution of the DSD is shown in the lower right graphic in Fig. 9.

The question remains which processes may be responsible for the rapid loss of the smallest drops. A first
Figure 9: Evolution of the drop size distribution \( f_r(r,t) \) [cm\(^4\)] vs. the drop radius [µm] for experiment M4 and current simulations. Thick black line: Initial DSD (inlet \( x = 0 \)). Green (red) dashed line: DSD as measured at the center at \( x = 0.4 \) m (outlet at 0.8 m) (cf. Fig. 4), which corresponds to an elapsed time of \( t \approx 0.15 \) s (0.3 s). Other lines: DSD as simulated for \( t = 0 \) s, 0.2 s, 0.4 s, 1.0 s. Upper left: \( \varepsilon = 0 \), upper right: \( \varepsilon = 250 \) cm\(^2\)/s\(^3\), lower left: \( \varepsilon = 50000 \) cm\(^2\)/s\(^3\), lower right: \( \varepsilon = 50000 \) cm\(^2\)/s\(^3\) and lower limit of the kernel \( \Gamma_{\text{min}} = 5 \times 10^{-4} \) cm\(^3\)/s.

Candidate is the condensational growth, which has been disregarded in the RAINSHAFT simulation. The drop number is conserved during this process, but the DSD is shifted and narrowed along the \( r \)-coordinate (ROGERS and YAU, 1989), however, no such indication is seen in the observed DSDs. A second possibility is the neglect of the vertical transport due to sedimentation. Simulations show that owing to the abundance of small drops no effect is seen during 2 s simulation time in most of the vertical region. Only in the uppermost part, the “larger” drops fall out, whereby the depletion of the smaller ones is even reduced. A third candidate is a missing mechanism for drop collisions in the formulation of the kernel, such as Brownian motion which is known to be important for the collisional growth of aerosol particles (HERBERT and BEHENG, 1986). Although this effect will increase the total collision kernel \( K \) for the smallest drops, it is still far too weak to explain the observed rapid evolution. BORDÁS et al. (2012) and SCHMEYER et al. (2014) compare the results for the DSD evolution from a numerical fluid model with those from another experiment with the same wind tunnel as described above. They use a different approach for the coagulation kernel than in this paper, namely the sum of a standard Brownian motion kernel and a standard shear kernel. Each contribution is multiplied by a tuning factor. The authors find reasonable agreement with the measurements of the DSD only if the parameter for the Brownian contribution is multiplied by a factor of \( 1.5 \times 10^{6} \) and \( 10^{5} \), respectively. This confirms our finding, that the neglect of Brownian motion in our simulation is not responsible for the found discrepancy.
A physical argument for the requirement of the strong enhancement of the collection kernel for small drops is missing. Therefore, we will look for a potential explanation arising from the DNS simulations or the experiment. Generally, multi-particle effects for small drops could be more significant in the highly turbulent environment than expected such that the neglect of the hydrodynamical interaction in the DNS is not valid. Maybe an additional unknown physical process is active for small drops in flows at high turbulence intensity. For instance, high shear rates could lead to drop break-up. Also the truncated model equations used in the DNS may become invalid for such high turbulence intensities. However, it remains an open question why this should only affect the smaller drops. In the experiment wall effects may be present. Since the drops are injected opposite to the mean flow direction, they may not have fully adapted to the flow such that the interpretation of the streamwise coordinate as traveling time by use of the constant mean velocity V is not accurate. Additionally, the drop injection could lead to significant temporal variations of the flow properties, while we used average values in this study. Variations in the dissipation rate $\varepsilon$ may alter the turbulent kernel, but in view of the weak dependency of the results in Fig. 9 on $\varepsilon$, this effect is not very likely. Also the DSD in the wind tunnel is subject to fluctuations, see the standard deviation given in Fig. 4, and measurement errors in the size distribution may be caused by a drop-diameter dependence of the Doppler instrument. Therefore, another sensitivity study for the evolution of the DSD was performed by using the observed DSD but increased by twice the standard deviation. This is an increase of about 10% of the abundant small drops, but an increase of more than 100% for the rare large drops. Since eq. 3.12 is a nonlinear function in the DSD, we expect a faster collection mechanism. The result is given in Fig. 10. Indeed, we see this effect, though for $\varepsilon = 250 \text{cm}^2/\text{s}$ it is still too slow. With the prescribed minimum value of the kernel $\Gamma_{\text{min}}$, the collection is likewise sped up, and after 1 s elapsed time, the DSD from the simulation that had started with the increased drop number, falls below the DSD from the reference case. This shows that locally varying drop number concentrations may contribute to the mis-estimation of the observed DSD. Still, this cannot fully explain the discrepancy.

At a longer simulation time, one indeed finds an enhancement in evolution of the DSD due to turbulence even in the case without the prescribed $\Gamma_{\text{min}}$, however, no observations are available. Xue et al. (2008) study the sensitivity of the evolution to various approaches for the turbulent kernel for an idealized case, and they find the expected acceleration. Due to the long integration time of 30 min their results are not comparable to the current findings. However, we present the long-time evolution of cloud drop ensembles in a submitted paper by Riechelmann et al.

4 Conclusion

This study provides a parametrized collision kernel for water drops in a turbulent environment. It is developed as a function of the radii of the colliding drops and the dissipation rate as a measure for turbulence intensity and is based on a very large database coming from new DNS simulations. The setup of those simulations was designed to match closely the corresponding windtunnel experiment, although a difference in the dissipation rate could not be avoided due to the dissimilar capabilities of the experiment and the simulation. Nevertheless, the measured collision rate for drops with radii larger than 7.5 $\mu$m can be compared with the kernel fit. They match well over the whole investigated range of dissipation rates such that for the first time an experimental
collision kernel could be constructed by enhancing the reliability with the kernel fit. Additionally, the evolution of the drop spectrum due to turbulent coagulation is simulated using the new kernel parametrization and compared with the measured spectra. The depletion of small drops with radii smaller than 7.5 µm turns out too slow. The observed changes can only be explained by a drastic increase of the collision kernel. The reason for this remains unclear since neither the experiment nor the DNS considered the collisions of such small drops. For larger drops \((r > 7.5 \mu m)\) the observed number concentration is too low to identify significant changes of those few drops over the short wind tunnel measurement distance.

The impact of turbulence on the evolution of cloud drop ensembles for time scales of natural clouds is presented in the submitted paper by Riechelmann et al.

### Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>Parameter for fit of (\eta_\nu), see eq. (3.3)</td>
</tr>
<tr>
<td>(\chi)</td>
<td>Transition factor of fit in eq. (3.7)</td>
</tr>
<tr>
<td>(\Delta x)</td>
<td>Cell length</td>
</tr>
<tr>
<td>ε</td>
<td>Dissipation rate of turbulent kinetic energy (k)</td>
</tr>
<tr>
<td>(\eta_\varepsilon)</td>
<td>Turbulent enhancement factor of geometric collision kernel ((\Gamma_{\text{turb}} = \eta_\varepsilon \Gamma_{\text{grav}}))</td>
</tr>
<tr>
<td>(\Gamma)</td>
<td>Geometric collision kernel</td>
</tr>
<tr>
<td>(\mu)</td>
<td>Parameter for fit of (\eta_\nu), see eq. (3.3)</td>
</tr>
<tr>
<td>ν</td>
<td>Fluid kinematic viscosity</td>
</tr>
<tr>
<td>(\rho)</td>
<td>Density</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>Parameter for fit of (\eta_\nu), see eq. (3.3)</td>
</tr>
<tr>
<td>(\tau)</td>
<td>Time Scale</td>
</tr>
<tr>
<td>E</td>
<td>Collision efficiency</td>
</tr>
<tr>
<td>(f_m)</td>
<td>Drop size distribution (coordinate (m))</td>
</tr>
<tr>
<td>(f_r)</td>
<td>Drop size distribution (coordinate (r))</td>
</tr>
<tr>
<td>(f_{\text{corr}})</td>
<td>Correction factor for Stokes drag</td>
</tr>
<tr>
<td>(H)</td>
<td>Height</td>
</tr>
<tr>
<td>(K)</td>
<td>Collision kernel ((K = E\Gamma))</td>
</tr>
<tr>
<td>(k)</td>
<td>Turbulent kinetic energy</td>
</tr>
<tr>
<td>(L)</td>
<td>Length</td>
</tr>
<tr>
<td>(m)</td>
<td>Drop mass</td>
</tr>
<tr>
<td>(N)</td>
<td>Collision rate</td>
</tr>
<tr>
<td>(n)</td>
<td>Number of drops per volume</td>
</tr>
<tr>
<td>(r)</td>
<td>Drop radius</td>
</tr>
<tr>
<td>(R_{\text{L}})</td>
<td>Larger drop with radius (r = \max(r_i, r_j))</td>
</tr>
<tr>
<td>(R_{\text{r}})</td>
<td>Smaller drop with radius (r = \min(r_i, r_j))</td>
</tr>
<tr>
<td>(RCR)</td>
<td>Relative collision rate (eq. 3.10)</td>
</tr>
<tr>
<td>(Re)</td>
<td>Reynolds number</td>
</tr>
<tr>
<td>(R_{\text{L}})</td>
<td>Taylor length-scale based Reynolds number</td>
</tr>
<tr>
<td>(t)</td>
<td>Time</td>
</tr>
<tr>
<td>(v)</td>
<td>Velocity vector</td>
</tr>
<tr>
<td>(V, V)</td>
<td>Constant mean flow velocity</td>
</tr>
<tr>
<td>(v')</td>
<td>Turbulent fluctuation velocity</td>
</tr>
<tr>
<td>(v_t)</td>
<td>Terminal settling velocity</td>
</tr>
<tr>
<td>(x, y, z)</td>
<td>Cartesian coordinates</td>
</tr>
</tbody>
</table>

### Acronyms

- DNS: Direct numerical simulation
- DSD: Drop size distribution
- M4: Name of wind tunnel experiment
- PIV: Particle-image velocimetry

### Acknowledgments

The funding of this project in the framework of the priority program SPP1276 METSTROEM by the German Research Foundation (DFG) is gratefully acknowledged. C. Siewert and R.P.J. Kunnen were funded under grant number SCHR 309/39, R. Bordás under TH 881/13, U. Wacker under WA 1344/8, and K.D. Beheng under BE 2081/10. The authors are grateful for the computing resources provided by the High Performance Computing Center Stuttgart (HLRS). We also thank the reviewers for their many helpful comments.

### References


