Vibration control in coupled building-like structures

Janssen, S.E.M.; Nijmeijer, H.

Published: 01/01/2014

Document Version
Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

• A submitted manuscript is the author's version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
• The final author version and the galley proof are versions of the publication after peer review.
• The final published version features the final layout of the paper including the volume, issue and page numbers.

Link to publication

Citation for published version (APA):

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

• Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
• You may not further distribute the material or use it for any profit-making activity or commercial gain
• You may freely distribute the URL identifying the publication in the public portal?

Take down policy
If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Download date: 15. Dec. 2018
Vibration control in coupled building-like structures

S.E.M. Janssen

DCT 2014.051

Traineeship report

Coach(es): prof.dr. G. Silva
Supervisor: prof.dr H. Nijmeijer

Technische Universiteit Eindhoven
Department Mechanical Engineering
Dynamics and Control Group

Eindhoven, December, 2014
Abstract

The project is about attenuating vibrations in two building-like structures under ground floor disturbances. The structures are models of a three story and a five story building. Only two sensors can be used in controlling the buildings and for evaluation of the results. The outputs of the system are the displacements of the top floors. The control effort should be kept as small as possible by adding passive control between the connected floors.

The building-like structures are analyzed and a Simulink file is created for simulations. Passive and active control options are analyzed with the model and show stable and satisfactory results.

An experimental setup is created and the model assumptions are verified. The model parameters are adjusted after the experiments. The model fits the data sufficiently close.
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>D  Simulation with the same stiffness at different floors</td>
<td>53</td>
</tr>
<tr>
<td>E  Simulation with different stiffness at the same floor</td>
<td>54</td>
</tr>
<tr>
<td>F  Multiple Positive Position Feedback with eight absorber frequencies</td>
<td>55</td>
</tr>
<tr>
<td>G  Online active controlling</td>
<td></td>
</tr>
<tr>
<td>G.1 MRAC theory</td>
<td>59</td>
</tr>
<tr>
<td>G.2 MRAC theory in online model verification</td>
<td>60</td>
</tr>
<tr>
<td>G.3 MRAC theory in active control</td>
<td>61</td>
</tr>
<tr>
<td>G.4 Conclusion</td>
<td>61</td>
</tr>
</tbody>
</table>
1 Introduction

For many years buildings and civil structures in modern cities suffer from vibrations caused by road traffic, irregularities in railways, gas and oil drilling, wind and earthquakes. In Mexico City especially earthquakes form a serious issue.

Vibration absorbers are used to suppress or at least attenuate vibrations in mechanical structures. There are three basic methodologies described as passive, semi-active and active control. In passive control a system with constant stiffness and damping is used to control vibrations in a narrow frequency band. Semi-active vibration control uses adaptive stiffness or damping characteristics. In active vibration control degrees of freedom can be added, feedforward and feedback controllers can be used and a better performance can be achieved at the cost of energy usage. These three methodologies are used to control vibrations in single buildings in general by adding an extra mass at the top floor.

This report considers the problem of controlling two buildings; a three story and a five story building. This is done by controlling the stiffness and damper characteristics of a coupling at the optimal floor; first, second or third floor. No extra mass is added.

The buildings are modeled as an eight degree of freedom system with concentrated masses at the floors. The masses are assumed to only move in one direction. The floors of each building are connected by four metal strips that can be modeled as a spring and a damper. In the experiments the base of the building-like structures is excited with an electromagnetic shaker. An acceleration sensor is added at the top floor of each building to provide input for the feedback control and as a measure for the output of the system.

The main objective is to attenuate the frequency response of the system focusing on the frequency band that is excited by earlier mentioned causes of vibrations in buildings and civil structures.

1.1 Description of the building-like structure

The coupled building-like structure consists of eight masses. The connection in the individual buildings are denoted by equivalent stiffness and equivalent linear viscous damping as shown in Figure 1.1. Both buildings exhibit the same noise at ground level denoted by $z$. The figure also shows the notation that is used in the entire report. $m_{12}$ denotes for example the second floor of the three story building, the first building.

![Figure 1.1: Three different control configuration possibilities](image)

Figure 1.1: Three different control configuration possibilities
The used material is aluminum with a density of \(2.6989 \cdot 10^3\) kg/m\(^3\) and a Young's Modulus \((E)\) of 68 GPa. The links are assumed to be massless and to have sizes of 150 mm x 12.7 mm x 1.3 mm. The floors have sizes 100 mm x 150 mm x 6.3 mm. Their masses are simply determined by multiplying the volumes with the density.

\[
k = \frac{12EI}{l^3} \tag{1.1}
\]

\[
l = \frac{wd^3}{12} \tag{1.2}
\]

\[
k_e = k_1 + k_2 + k_3 + k_4 \tag{1.3}
\]

The stiffness of a single beam is calculated with (1.1) and (1.2) and springs in parallel can be added as in (1.3). In these equations \(l\) is the second moment of inertia, \(l\) is the length of the strip between two floors, \(w\) is the width of the strip and \(d\) is the depth. \(k_e\) is the equivalent stiffness between two floors. This results in an equivalent mass of 0.257 kg and an equivalent stiffness of 4.1 kN/m.

The base of the structure 'Ground' is affected by lateral motion as indicated with \(z\) in Figure 1.1. This motion represents the moving ground under a building and is represented with an electromechanical shaker in the experiments.

1.2 Plan of approach

With the discussed assumptions and the determined values a theoretical analysis is done that shows the resonance frequencies and modes. Then a Simulink model is discussed and Fast Fourier Transforms (FFT) of the accelerations of all masses are shown. This is done for the uncoupled system and the three coupled system configurations and passive control is discussed. The equivalent damping is not calculated and is added to the model after experiments. Then three different active control options are discussed.

Then the model is realised and several measurements are done. The data is compared to the results of the analytical calculations and the outcomes of the simulations with Simulink.

At last conclusions are drawn and recommendations for the rest of the project at CINVESTAV are done.

1.3 Organisation of the report

The report is organized as follows; in Chapter 2 the buildings are modelled and the resonance frequencies and modes are analytically determined. In Chapter 3 the buildings are simulated with an Simulink model and passive coupling in analyzed. In Chapter 4 the same simulation model is used to analyze different active and semi-active controllers. In Chapter 5 validation experiments are discussed. Conclusions are recommendations are placed in Chapter 6.
2 Modelling and theoretical results

The discussed model is transferred to a system of matrices using the equations of motion. The main objective is to determine the resonance frequencies and modal shapes for comparison to later discussed simulations and measurements. First this is done for the uncoupled system and then for the coupled system. Since no coupling parameters are yet determined, the coupling stiffness is assumed to be equal to the stiffness between the floors of the buildings.

\[ M \ddot{x}(t) + C \dot{x}(t) + Kx(t) = (c_{11}x_1 + c_{21}x_2)\ddot{z}(t) + (k_{11}e_1 + k_{21}e_2)z(t) \]  \hspace{1cm} (2.1)

The equations of motion for the system without control and under ground floor acceleration \( \ddot{z} \) are given by (2.1). In Appendix A the equations of motion are visible for the separate masses. These are used to determine the matrices. The used masses are 0.257 kg en the stiffnesses are 4.1 kN/m as determined in Chapter 1.

2.1 Uncoupled buildings

In the uncoupled situation \( x = [x_{11} \ x_{12} \ x_{13}]^T \) is the generalized vector with displacements with respect to the nonactuated situation for the three story building. For the five story building \( x = [x_{21} \ x_{22} \ x_{23} \ x_{24} \ x_{25}]^T \). \( M \) is the mass matrix, \( C \) is the damping matrix and \( K \) is the stiffness matrix.

\[ M_3 = \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{12} & 0 \\ 0 & 0 & m_{13} \end{bmatrix}, C_3 = \begin{bmatrix} c_{11} + c_{12} & -c_{12} & 0 \\ -c_{12} & c_{12} + c_{13} & -c_{13} \\ 0 & -c_{13} & c_{13} \end{bmatrix}, \]
\[ K_3 = \begin{bmatrix} k_{11} + k_{12} & -k_{12} & 0 \\ -k_{12} & k_{12} + k_{13} & -k_{13} \\ 0 & -k_{13} & k_{13} \end{bmatrix}, e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, e_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \]  \hspace{1cm} (2.2)

\[ M_5 = \begin{bmatrix} m_{21} & 0 & 0 & 0 & 0 \\ 0 & m_{22} & 0 & 0 & 0 \\ 0 & 0 & m_{23} & 0 & 0 \\ 0 & 0 & 0 & m_{24} & 0 \\ 0 & 0 & 0 & 0 & m_{25} \end{bmatrix}, C_5 = \begin{bmatrix} c_{21} + c_{22} & -c_{22} & 0 & 0 & 0 \\ -c_{22} & c_{22} + c_{23} & -c_{23} & 0 & 0 \\ 0 & -c_{23} & c_{23} + c_{24} & -c_{24} & 0 \\ 0 & 0 & -c_{24} & c_{24} + c_{25} & -c_{25} \\ 0 & 0 & 0 & -c_{25} & c_{25} \end{bmatrix}, \]
\[ K_5 = \begin{bmatrix} k_{21} + k_{22} & -k_{22} & 0 & 0 & 0 \\ -k_{22} & k_{22} + k_{23} & -k_{23} & 0 & 0 \\ 0 & -k_{23} & k_{23} + k_{24} & -k_{24} & 0 \\ 0 & 0 & -k_{24} & k_{24} + k_{25} & -k_{25} \\ 0 & 0 & 0 & -k_{25} & k_{25} \end{bmatrix}, e_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, e_2 = \begin{bmatrix} 1 \end{bmatrix} \]  \hspace{1cm} (2.3)

The corresponding matrices for the three story building are in (2.2) and for the five story building in (2.3). Note that all mass, damping and stiffness matrices are symmetric and positive definite for all positive values of mass, stiffness and damping.

2.1.1 Resonance frequencies

To determine the resonance frequencies of this multi-degree-of-freedom linear system with symmetric matrices a state space formulation is used. The second order equations of motion are reformulated to a set of first order differential equations. This is done by using a new variable \( y = [x \ \dot{x}]^T \)

\[ Ay(t) + By = r(t) \]  \hspace{1cm} (2.4)

The used matrices to ensure the equivalence are

\[ y(t) = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}, A = \begin{bmatrix} D & M \\ M & 0 \end{bmatrix}, B = \begin{bmatrix} K \\ 0 \end{bmatrix} \]  \hspace{1cm} (2.5)
In control engineering the state-space equations are generally written as
\[ \dot{y}(t) = \hat{A}x(t) + \hat{B}u(t) \] (2.6)

This equation is equivalent to (2.6) if we take
\[ y(t) = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}, \hat{A} = A^{-1}B, \hat{B} = A^{-1} \begin{bmatrix} I \\ 0 \end{bmatrix} \] (2.7)

Which results in the following matrices
\[ \hat{A} = \begin{bmatrix} 0 & -I \\ M^{-1}K & M^{-1}D \end{bmatrix}, \hat{B} = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix} \] (2.8)

The eigenvalues of \( \hat{A} \) are the resonance frequencies times the imaginary unit. If the damping is nonzero these eigenvalues are complex.

The damping is put to zero since this does not influence the resonance frequencies of the system and the damping is unknown. The resulting frequencies are visible in Table 2.1.

<table>
<thead>
<tr>
<th>Resonance frequency [Hz]</th>
<th>Three story</th>
<th>Five story</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.9</td>
<td>5.7</td>
<td></td>
</tr>
<tr>
<td>25.0</td>
<td>16.7</td>
<td></td>
</tr>
<tr>
<td>36.2</td>
<td>26.3</td>
<td></td>
</tr>
<tr>
<td>-</td>
<td>33.8</td>
<td></td>
</tr>
<tr>
<td>-</td>
<td>38.5</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.1: Theoretical resonance frequencies of uncoupled system

2.1.2 Mode shapes

The eigenvectors of \( \hat{A} \) are determined to find the modal shapes. The modes are visible in Figure 2.1, the first mode is corresponding to the lowest eigen frequency et cetera. These modes are also visible during the experiments and show at which floor which mode shows the largest and smallest vibration amplitude.

The highest mode of both buildings is a swinging mode, an inverse pendulum which is easily measurable at the top floors. Some modes show one or several nodes. Around these nodes the resonance may not be measurable. None of the nodes is at the top floor, so it should be possible to measure all resonances at the top floor and therefore determine an active controller that uses only feedback from the top floors.
Figure 2.1: Visualization of the eight different modes of the separate buildings

### 2.2 Coupled buildings

The buildings are coupled as is visible in Figure 1.1. In (2.9) the matrices are shown for the coupled system. Here \( \mathbf{x} = \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{21} & x_{22} & x_{23} & x_{24} & x_{25} \end{bmatrix}^T \) is the generalized vector of displacements with respect to the nonactuate position. Only one of the floors has a passive vibration absorber so if for example \( c_{11} \) and/or \( k_{11} \) is nonzero all other coupling parameters are zero.
\[ M_8 = \begin{bmatrix}
m_{11} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & m_{12} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & m_{13} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & m_{21} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & m_{22} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & m_{23} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & m_{24} \\
0 & 0 & 0 & 0 & 0 & 0 & m_{25}
\end{bmatrix}, \]
\[ C_8 = \begin{bmatrix}
c_{11} + c_{12} + c_{11} & -c_{12} & 0 & -c_{11} & 0 & 0 & 0 & 0 \\
-c_{12} & c_{12} + c_{13} + c_{21} & -c_{13} & 0 & -c_{22} & 0 & 0 & 0 \\
0 & -c_{13} & c_{13} + c_{22} & 0 & 0 & -c_{31} & 0 & 0 \\
-c_{11} & 0 & 0 & c_{21} + c_{22} + c_{11} & -c_{22} & 0 & 0 & 0 \\
0 & 0 & -c_{22} & 0 & -c_{22} + c_{23} + c_{2} & -c_{23} & 0 & 0 \\
0 & 0 & 0 & 0 & -c_{23} & c_{23} + c_{24} + c_{3} & -c_{24} & 0 \\
0 & 0 & 0 & 0 & 0 & -c_{24} & c_{24} + c_{25} & -c_{25} \\
0 & 0 & 0 & 0 & 0 & 0 & -c_{25} & c_{25}
\end{bmatrix}, \]
\[ K_8 = \begin{bmatrix}
k_{11} + k_{12} + k_{11} & -k_{12} & 0 & -k_{11} & 0 & 0 & 0 & 0 \\
-k_{12} & k_{12} + k_{13} + k_{21} & -k_{13} & 0 & -k_{22} & 0 & 0 & 0 \\
0 & -k_{13} & k_{13} + k_{22} & 0 & 0 & -k_{31} & 0 & 0 \\
-k_{11} & 0 & 0 & k_{21} + k_{22} + k_{11} & -k_{22} & 0 & 0 & 0 \\
0 & -k_{22} & 0 & -k_{22} + k_{23} + k_{2} & -k_{23} & 0 & 0 & 0 \\
0 & 0 & -k_{23} & 0 & -k_{23} + k_{24} + k_{3} & -k_{24} & 0 & 0 \\
0 & 0 & 0 & 0 & -k_{24} & k_{24} + k_{25} & -k_{25} & 0 \\
0 & 0 & 0 & 0 & 0 & -k_{25} & k_{25} & 
\end{bmatrix}, \]
\[ e_1 = \begin{bmatrix}1 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \end{bmatrix}, \quad e_2 = \begin{bmatrix}0 \\
1 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \end{bmatrix}. \]
2.2.1 Resonance frequencies

The analytical resonance frequencies are determined in the same way as is done for the uncoupled variant. They are visible in Table 2.2. These frequencies are depending on the coupling stiffness. The coupling stiffness is equal to the stiffness between the floors.

<table>
<thead>
<tr>
<th>Coupling first floor</th>
<th>Coupling second floor</th>
<th>Coupling third floor</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.0</td>
<td>6.4</td>
<td>6.6</td>
</tr>
<tr>
<td>10.0</td>
<td>12.0</td>
<td>14.2</td>
</tr>
<tr>
<td>17.8</td>
<td>20.1</td>
<td>18.2</td>
</tr>
<tr>
<td>25.7</td>
<td>25.7</td>
<td>25.7</td>
</tr>
<tr>
<td>29.6</td>
<td>26.7</td>
<td>30.4</td>
</tr>
<tr>
<td>34.8</td>
<td>34.3</td>
<td>34.0</td>
</tr>
<tr>
<td>38.0</td>
<td>37.6</td>
<td>36.7</td>
</tr>
<tr>
<td>42.5</td>
<td>43.6</td>
<td>42.4</td>
</tr>
</tbody>
</table>

Table 2.2: Theoretical resonance frequencies of passively coupled system

2.2.2 Mode shapes

The eigenmodes or resonance modes are also determined analytically for the coupled situation. The results are visible in figure 2.2. In many modes the amplitude of the displacement of the two top floors of the five story building is higher than the other amplitudes. The two top floors are uncoupled and swing like an inverted pendulum.

Figure 2.2: Visualization of the eight different modes with passive coupling at the third floor
3 Simulation and passive control

The objective of the simulations is to determine the coupling effects of passive and later active control. Before that the model is validated by comparison with the analytical results.

In this chapter first the model itself is discussed. Then simulations are done without coupling and compared to the analytical results. More important are the simulations with coupling since here also the possibilities of passive coupling are shown.

3.1 Model description

The Simulink model consists of nine subsystems, representing the eight floors and the ground. To determine the accelerations and positions of the floors, all forces on the floor are needed. Therefore the velocity and position of the surrounding floors are needed. In Appendix B an overview of the system and one of the subsystems are shown and explained.

The input of the system is a chirp signal, another option to determine all the resonance frequencies is to use a white noise. The chirp signal is preferred since the noise level in the results is significantly lower.

The model is used for both the coupled and the noncoupled situation. In the corresponding mfile the coupling parameters are simply put to zero to create the noncoupled situation. The values for masses and stiffnesses used in the simulations are the ones analytically determined; 0.257 kg and 4.1 kN/m respectively. Values for damping are not analytically determined. The damping in the simulations is tuned to improve the visibility of the graphs. All damping parameters in a single simulation are equal and the value is mentioned below the corresponding graph.

3.2 Noncoupled simulation

First the noncoupled situation is simulated. The acceleration output of all the floors is converted to the frequency domain with a FFT. The results are visible in Figures 3.1 and 3.2.

There should be three resonance peaks visible in Figure 3.1 due to the three masses; these are visible at all floors although sometimes the peaks are low. The visibility is mainly affected by the height of the first peak, which influences the scale of the graphs. At the second floor the second mode has a low amplitude. This coincides with the analytically computed mode shapes. The simulated resonance frequencies are $\omega_1 = 8.9$ Hz, $\omega_2 = 25.1$ Hz and $\omega_3 = 36.5$ Hz.

In Figure 3.2 there should be five resonance frequencies visible. It is possible to distinguish them in the acceleration of $m_{21}$; the first floor of the second (five story) building. This is partly caused by the low amplitude of the first mode that is again influencing the scale. The simulated resonance frequencies are $\omega_1 = 5.7$ Hz, $\omega_2 = 16.7$ Hz, $\omega_3 = 26.3$ Hz, $\omega_4 = 34.0$ Hz and $\omega_5 = 38.9$ Hz.
Figure 3.1: FFT of accelerations of all masses of the noncoupled three story building

Figure 3.2: FFT of accelerations of all masses of the noncoupled five story building
In Table 3.1 the resonance frequencies from the uncoupled analytical calculations and the simulations are put together. Since the resonance frequencies found in simulation with the Simulink model are very similar to the frequencies found analytically it is concluded that the Simulink model of the noncoupled buildings is correct. The difference can be caused by the limited measurement time or the sample frequency, it is also possible that very small errors that Matlab creates in every step eventually lead to small differences.

<table>
<thead>
<tr>
<th>Resonance frequency [Hz]</th>
<th>Three story</th>
<th>Five story</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Theoretical</td>
<td>Simulation</td>
</tr>
<tr>
<td>8.9</td>
<td>8.9</td>
<td>5.7</td>
</tr>
<tr>
<td>25.0</td>
<td>25.1</td>
<td>16.7</td>
</tr>
<tr>
<td>36.2</td>
<td>36.5</td>
<td>26.3</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>33.8</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>38.5</td>
</tr>
</tbody>
</table>

Table 3.1: Theoretical and simulated resonance frequencies of uncoupled system

The found resonance frequencies itself are not reliable since the masses and stiffnesses have to comply with measurements.

### 3.3 Coupled simulation

The same Simulink file is used but now the spring and damper characteristics between one of the floors is nonzero.

First simulations are done with the same coupling parameters as in the analytical calculations. The results are compared to validate the coupling in the Simulink model. Then vibration attenuation results with coupling at different floors are discussed and also with different coupling parameters. At the end it is concluded if the model is correct and what the possibilities of passive control are.

#### 3.3.1 Comparison with analytical results

The same stiffness as between the floors of 4.1 kN/m is used for the coupling stiffness. The values for the masses are unchanged and the damping is again tuned to improve the visibility of the graphs per simulation. Graphs of the acceleration response in frequency domain are visible in Appendix C and show the accelerations at all eight floors for the three different coupling configurations; coupling at the three different floors as is visible in Figure 1.1.

For all three configurations the eight resonance frequencies are determined and compared to the ones found in the theoretical analysis. The frequencies can be found in Table 3.2. The differences between the theoretical frequencies and the frequencies from the Simulink simulations are small. At higher frequencies the differences increase. The measurement time and sample frequency influence the accuracy of the simulation.
Table 3.2: Theoretical and simulated resonance frequencies of passively coupled system

<table>
<thead>
<tr>
<th>Coupling first floor</th>
<th>Coupling second floor</th>
<th>Coupling third floor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theoretical</td>
<td>Simulation</td>
<td>Theoretical</td>
</tr>
<tr>
<td>6.0</td>
<td>6.0</td>
<td>6.4</td>
</tr>
<tr>
<td>10.0</td>
<td>10.0</td>
<td>12.0</td>
</tr>
<tr>
<td>17.8</td>
<td>17.8</td>
<td>20.1</td>
</tr>
<tr>
<td>25.7</td>
<td>25.8</td>
<td>25.7</td>
</tr>
<tr>
<td>29.6</td>
<td>29.9</td>
<td>26.7</td>
</tr>
<tr>
<td>34.8</td>
<td>35.0</td>
<td>34.3</td>
</tr>
<tr>
<td>38.0</td>
<td>38.3</td>
<td>37.6</td>
</tr>
<tr>
<td>42.5</td>
<td>42.9</td>
<td>43.6</td>
</tr>
</tbody>
</table>

The differences between the resonances found in theory and in simulations are small enough not to reject the earlier conclusion that the model is correct.

3.3.2 Coupling at different floors

Eventually a choice has to be made to couple at a certain floor. Their are two effects distinguishable: resonance frequency shift and amplitude change at the resonance frequencies.

The frequency change is visible in Table 3.2. The first two resonance peaks increase in frequency if the spring is put at a higher floor. For the other modes it is not possible to distinguish such a clear trend. Keep in mind that the frequencies are depending on the coupling stiffness chosen. Therefore it is possible that also this trend changes.

The change in amplitude for different configurations can not be determined from the graphs in Appendix C since a different damping is used for the different configurations. As expected a higher damping leads to overall lower amplitudes. A graph has been created with the acceleration results for the three different coupling configurations with the same damping in one plot. This graph with accelerations for all masses is visible in Appendix E, a plot with the accelerations of the top floors is visible in Figure 3.3. The top floors are the outputs of the system and will eventually be the only ones with acceleration sensors. Also the top floors show the highest resonance amplitudes and are therefore more important to attenuate. In these plots the stiffness is equal to the stiffness between the floors and the damping is put to 2.1 Ns/m.

The biggest effect is visible in the first three modes. In both buildings the frequency of the first mode only changes little and the resonance frequency of the second and third mode changes up to 4.2 Hz.

The amplitude change in the first two modes of the five story building is small, in the three story building this is not the case. The most interesting is that the amplitude of the first mode increases with coupling at a higher floor and the amplitude of the second mode decreases with the coupling at a higher floor. Only when coupling is placed at the first floor, the amplitude of the second mode is higher than the amplitude of the first mode. Note that in general the amplitude in the five story building is higher than in the smaller building.

There is also an effect visible at the third mode. If the coupling is placed at the third floor, this mode is hardly visible. This coincides with expectations from the mode shapes in Figure 2.2 where the top floors show little displacement. The choice of coupling causes a clear frequency shift, the amplitude is depending on the building.
The frequency shift is in this case not very useful since the first mode does not change much and it also does not separate two modes at the same frequency. Coupling at the third floor causes the most positive effect at the second and third mode. The first mode should then be attenuated with well chosen damping and stiffness and probably (semi-) active control.

3.3.3 Coupling stiffness

To determine the effects of the coupling stiffness, the change in amplitude and shift in frequency are discussed of a building coupled at the third floor. The results for three different stiffnesses are visible in Figure 3.4. In this figure the damping in the three cases is the same and is equal to 2.45 Ns/m.

The first and second mode of both buildings have a smaller amplitude for the lowest stiffness. So a large stiffness seems to increase the vibrations in the buildings instead of attenuating or decreasing them. Another interpretation is that a higher stiffness has to be accompanied by a higher damping. This means that the stiffness can be chosen and the damping follows from this choice.

Also for both first modes of both buildings the frequency of the resonance increases with the stiffness. This is of course not depending on the damping. The frequency shift in the second mode is much larger than the shift in the first mode. This is similar to the effects of the change of the coupling floor.

---

Figure 3.3: FFT of accelerations of top floors with coupling at different floors

The first mode does not change much and it also does not separate two modes at the same frequency. Coupling at the third floor causes the most positive effect at the second and third mode. The first mode should then be attenuated with well chosen damping and stiffness and probably (semi-) active control.

3.3.3 Coupling stiffness

To determine the effects of the coupling stiffness, the change in amplitude and shift in frequency are discussed of a building coupled at the third floor. The results for three different stiffnesses are visible in Figure 3.4. In this figure the damping in the three cases is the same and is equal to 2.45 Ns/m.

The first and second mode of both buildings have a smaller amplitude for the lowest stiffness. So a large stiffness seems to increase the vibrations in the buildings instead of attenuating or decreasing them. Another interpretation is that a higher stiffness has to be accompanied by a higher damping. This means that the stiffness can be chosen and the damping follows from this choice.

Also for both first modes of both buildings the frequency of the resonance increases with the stiffness. This is of course not depending on the damping. The frequency shift in the second mode is much larger than the shift in the first mode. This is similar to the effects of the change of the coupling floor.
Different stiffness in coupling at third floor

It seems that in general the spring influences the vibrations negatively. It should be kept in mind that a coupling is needed to attenuate vibrations in both buildings at the same time. In this configuration, where the coupling is placed at the third floor, the stiffness can only be used to suppress the third mode.

**3.3.4 Conclusion**

Different simulations are done. First the resonance frequencies are compared to the theoretically determined frequencies and no significant differences are found. Then different couplings are compared to find the optimal coupling parameters and the possibilities of passive control.

It is concluded that the addition of a spring at any of the floors is not a solution that will attenuate or decrease the effects of the disturbance at the ground. A spring and a damper are more likely to be used successfully next to active or semi-active coupling to decrease the needed controller input.
4 Active control

To attenuate vibrations in a wide frequency range active control is necessary. Traditionally a mass is added that is used as a vibration absorber. In this case no masses are added and vibrations in both buildings have to be attenuated at the same time, which complicates the control significantly.

Three different active controllers are discussed here. The first uses a connection in one of the floors and a control force at this floor. The latter two use a controller that consist of one or several virtual vibration absorbers to attenuate one or several frequencies.

Two cases can be distinguished; the collocated and the non-collocated case. In the collocated case the accelerometers are placed at the same floor as the controller. In the non-collocated case the accelerometers are placed at both top floors and the controllers are placed at a different floor. This increases the complexity of the controller but decreases the swinging of the top two floors of the five story building-like structure.

At the end of the chapter a online adaptive controller is suggested but not finished and tested.

4.1 Stiff coupling with a control force

A simple way to control the coupled buildings with only one input is by connecting two floors and perform a controlling force at this floor. The connection should have a very high stiffness so that the system becomes a seven degree of freedom system. The three possible situations are sketched in Figure 4.1.

Figure 4.1: Three different control configuration possibilities with connected floors

Here only the first situation is discussed further since it assumed that this leads the simplest controller and the best performance.

4.1.1 Collocated

In the collocated case the accelerometer should be placed at the same floor as the controller. Since the controller is placed at the two connected floors only one accelerometer is needed. The equations of motion are transferred to a transfer function. The used controller consists of

1. Integrator
2. Notch at 6.7 Hz
3. Notch at 15.4 Hz
4. Leadlag filter at bandwidth
5. Gain

The two notches are placed at the first two resonance frequencies to suppress the vibrations with highest amplitudes. The integrator and the leadlag filter are used to increase the phase at the bandwidth for stability and robustness. The gain is used to increase the bandwidth of the system. This results in a bandwidth of 2.3 Hz and the Bode and Nyquist plots of the open loop are visible in Figures 4.2 and 4.3.

Figure 4.2: Bode diagram of the open loop from input the force to the velocities at the third floor
The bandwidth indicates the highest frequency the system is able to follow. Here the system of buildings should not move at all, only the noise added through the ground is to be controlled. The noise is allowed to consist of frequencies above 2.3 Hz since vibrations at these frequencies are suppressed at the third floor. These figures do not show that the vibrations at all other floors of the buildings are also damped.

Another issue is the generation of the control force. It can not be generated with an extra mass and should not create counter forces on other floors. Therefore this control option is not realistic.

4.1.2 Non-collocated

The corresponding matrices for the set of second and first order differential equations are determined. Then two transfer functions are derived from the input at, in this case, the connected third floor to the output of the system; the displacement at the top floors of both buildings.

To determine a stable controller Bode diagrams and Nyquist diagrams of the open loop are used. The same reasoning is used as for the collocated case to create the controller. Therefore it consists of the same parts. The controller consists of

1. Integrator
2. Notch at 6.7 Hz
3. Notch at 15.4 Hz
4. Leadlag filter at bandwidth
5. Gain

The resulting Bode diagrams and Nyquist plots are visible in Figures 4.4 and 4.5. The bandwidth is 2.4 Hz.
Figure 4.4: Bode diagrams of the open loop from the input force to the velocities at the top floors

Figure 4.5: Nyquist plots of the open loop from the input force to the velocities at the top floors

The controller can be optimized further which is not done since it is too time consuming considering the fact the tuning should be redone when the correct model parameters are used. Also the infinite stiff coupling is not realistic and probably not the most energy efficient since no passive damping is used. In this case the earlier discussed problem how the control force is generated is also present.
4.2 Positive Position Feedback

A Positive Position Feedback (PPF) control scheme is applied to compensate the system response under ground motion. This is done next to passive control consisting of a spring with a stiffness of 4.1 kN. This control scheme adds an additional virtual degree-of-freedom to the original mechanical system, considered as a virtual passive absorber or as a second order filter.

The positive position terminology comes from the fact that the position coordinate of the primary system is positively fed to the filter, and the position (displacement) coordinate of the compensator (secondary) system is positively fed back to the primary system. In most cases the secondary system is positively fed back to the filter, and the position (displacement) coordinate of the compensator system, considered as a virtual passive absorber or as a second order filter.

\[ M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = f(z, z) + B_f u(t) \]

\[ \ddot{\eta}(t) + 2\zeta_f \omega_f \dot{\eta}(t) + \omega_f^2 \eta(t) = g\omega_f^2 B_f^T \dot{x}(t) \]

\[ u(t) = g\omega_f^2 \eta(t) \]  

(4.1)

\[ B_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \]

(4.2)

\[
\begin{bmatrix}
M & 0 & 0 \\
C & 2\zeta_f \omega_f & 0 \\
0 & 0 & \omega_f^2
\end{bmatrix}
\begin{bmatrix}
\dot{\chi} \\
\dot{\eta} \\
\eta
\end{bmatrix}
= 
\begin{bmatrix}
K & -B_f g\omega_f^2 \\
-g\omega_f^2 B_f^T & \omega_f^2
\end{bmatrix}
\begin{bmatrix}
\dot{y} \\
\eta
\end{bmatrix}
= [0] \quad \text{or} \quad \begin{bmatrix}
p_1 \\
p_2
\end{bmatrix} = (K - g^2 \omega_f^2 B_f B_f^T) \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}
\]

(4.3)

In (4.1) the equations of motion for PPF are shown. \( f(z, z) \) represents the influence of the ground born vibrations and is described in Chapter 2. The input force \( u(t) \) is put at one of the floors of both buildings. In (4.2) the different input matrices are shown. \( B_i \) represents a coupling at floor \( i \). In this case the mechanical structure has only two available accelerations sensors at the top floors, \( x_{13} \) and \( x_{25} \), and one control force acting positively on the floors that are connected of both buildings. The resulting closed loop structure is visible in (4.1) and in compact matrix form in (4.3).

The virtual passive absorber \( \eta \) has relative damping \( \zeta_f \) and absorber natural frequency \( \omega_f \). It is coupled to the primary system by \( g\omega_f^2 B_f^T y(t) \). The PPF control force is described by \( u(t) \) where \( g \) is the control gain.

Since \( M \) is symmetric and positive definite the overall mass matrix in (4.3) is also symmetric and positive definite. The proportional damping matrix \( C \) is symmetric and positive definite and the overall damping matrix has similar properties. To guarantee closed loop stability the overall stiffness matrix should also be positive definite and symmetric. This restricts the choice of \( g \) and \( \omega_f \).

\[
p^T K p = \begin{bmatrix} p_1^T \\ p_2^T \end{bmatrix} \begin{bmatrix} K & -B_f g\omega_f^2 \\
-g\omega_f^2 B_f^T & \omega_f^2
\end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}
= p_1^T (K - g^2 \omega_f^2 B_f B_f^T) p_1 + \omega_f^2 (g B_f^T p_1 - p_2)^T (g B_f^T p_1 - p_2)
\]

(4.4)

The second term in (4.4) is always nonnegative, so for closed-loop asymptotic stability \( g \) and \( \omega_f \) must be chosen such that \( (K - g^2 \omega_f^2 B_f B_f^T) \) is positive definite.
Here $\omega_f$ is chosen after reviewing Figures C.5 and C.6. The most common resonance frequency in the output of both coupled buildings is used; $\omega_f = 6.6$ Hz.

To determine the values for the control gain that result in an asymptotically stable system the eigenvalues of $\left(K - g^2 \omega_f^2 B_f B_f^T \right)$ are plotted as a function of $g$ in Figure 4.6. Only values between -0.6 and 0.6 result in a stable system.

![Figure 4.6: Eigenvalues of $(K - g^2 \omega_f^2 B_f B_f^T)$ as function of the control gain](image)

The control gain is chosen to be 0.4. To determine the optimal value for the damping characteristic $\zeta$, Figure 4.7 and 4.8 are made. If the damping in the controller is high, the effect at the resonance peak is less. If the damping is too low new problems arise before and after the peak as is visible in Figure 4.7 and 4.8. The chosen damping is 0.1.
Figure 4.7: Accelerations of the three story building with different damper values

Figure 4.8: Accelerations of the five story building with different damper values
The resulting controller is stable and the attenuation results for this single resonance are good. But the other seven peaks are still present. It is preferred to at least attenuate the first two resonances and if possible all resonances.

### 4.3 Multiple Positive Position Feedback

This control scheme based on multiple virtual passive absorbers, Multiple Positive Position Feedback (MPPF), is an extension of PPF. More absorber natural frequencies can be chosen to attenuate more peaks at the same time.

\[
\begin{align*}
M\ddot{x}(t) + C\dot{x}(t) + Kx(t) &= -f(\dot{z}, z) + BU(t) \\
\dot{N}(t) + 2Z\Omega N(t) + \Omega^2 N(t) &= G\Omega^2 B^TN(t) \\
u(t) &= G\Omega^2 N(t)
\end{align*}
\]

The control scheme is visible in (4.5). The B matrices have not changed with respect to the ones of PPF in (4.2). The compact form of the closed-loop system is expressed in (4.6).

Again closed loop stability can be guaranteed if the overall stiffness matrix is positive definite and symmetric. This restricts the choice of \(G\) and \(\Omega\).

\[
p^T \hat{K} p = \begin{bmatrix} p_1^T & p_2^T \end{bmatrix} \begin{bmatrix} K & -BG\Omega^2 \\ -G\Omega^2 B^T & \Omega^2 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = p_1^T (K - BG^2\Omega^2 B^T) p_1 + \Omega^2 (GB^T p_1 - p_2)^T (GB^T p_1 - p_2)
\]

The second term in (4.7) is again always nonnegative, so for closed-loop asymptotic stability \(G\) and \(\Omega\) must be chosen such that \((K - G^2\Omega^2 BB^T)\) is positive definite.

Again values for the natural absorber frequencies in \(\Omega\) are chosen after reviewing Figures C.5 and C.6. The most common resonance frequencies in the output of both coupled buildings are used; \(\omega_1 = 6.6\) Hz, \(\omega_2 = 14.2\) Hz and \(\omega_3 = 18.2\) Hz.

All possible combinations of control gains resulting in a stable system are determined. The chosen control gains are; \(g_1 = 0.4\), \(g_2 = 0.15\) and \(g_3 = 0.1\). The chosen damping factors are determined from Figures 4.9 and 4.10. The results are \(\zeta_1 = 0.9\), \(\zeta_2 = 0.3\) and \(\zeta_3 = 0.3\).
Figure 4.9: Accelerations of the top floor of the three story building with different damper values zoomed at the resonance frequencies
Figure 4.10: Accelerations of the top floor of the five story building with different damper values zoomed at the resonance frequencies

At this moment three of the resonance peaks are damped by the virtual absorber. In total there are eight resonance peaks since there are eight degrees of freedom. The same procedure is repeated for all resonance frequencies as visible in Tabel 4.1. The optimal dampings are determined from Figures F.1 till F.3.

<table>
<thead>
<tr>
<th>Resonance</th>
<th>Gain</th>
<th>Damping</th>
</tr>
</thead>
<tbody>
<tr>
<td>first</td>
<td>0.15</td>
<td>0.7</td>
</tr>
<tr>
<td>second</td>
<td>0.05</td>
<td>0.4</td>
</tr>
<tr>
<td>third</td>
<td>0.05</td>
<td>0.2</td>
</tr>
<tr>
<td>fourth</td>
<td>0.05</td>
<td>0.4</td>
</tr>
<tr>
<td>fifth</td>
<td>0.05</td>
<td>0.2</td>
</tr>
<tr>
<td>sixth</td>
<td>0.05</td>
<td>0.4</td>
</tr>
<tr>
<td>seventh</td>
<td>0.05</td>
<td>0.4</td>
</tr>
<tr>
<td>eighth</td>
<td>0.05</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Table 4.1: Stable gains and optimal damping values for all eight resonance frequencies

For PPF and MPPF the same problem arises as for the controller applied to the stiff connected floors. How can the controller output, the force at the third floor, be generated without the addition of an extra mass. Here there are still a possibilities. An option is to add two controllable springs; one under positive and one under negative prestress.
4.4 Model Reference Adaptive Control

MRAC is a form of controlling where the controller is created and updated online. In this way it is not a problem if the stiffness and damping of the buildings change a little over time. The first steps in the controller design are done and explained in Appendix G.

4.5 Conclusion

Different controllers are tested on the analytically determined transfer functions. The controller that uses a stiff connection between the buildings and a single control force, creates a stable system. This option is not energy efficient due to the lack of passive control.

The addition of a virtual absorber results in nice attenuation of the resonances and a stable system. Also there is still a possibility for passive control which improves the efficiency.

It is not easy to attenuate vibrations in two building-like structures at the same time without the extra mass that is traditionally used. The generation of the control force at the third floor is not easy. Controllable springs and dampers can be used to generate the PPF or MMPF controller output possibly.

Adaptive control might be necessary to control buildings over an extensive period. The controller design should be finished and tested.
5 Experiments

An experimental setup is created to check the models and test passive control. The setup consists of a shaker, a slider, a ground plate, the two building-like structures and several accelerometers. The setup with the shaker and the five story building is visible in Figure 5.1. The shaker is mounted on a heavy plate that has four legs with anti slip rubber. On this plate a slider mechanism with low resistance placed. On the slide a plate is placed that will be referred to as ‘ground’. This plate is attached to the shaker with a pin and its acceleration is measured. On the ground plate one or two of the buildings can be placed with screws. To check the models an accelerometer is placed at every floor. For the control only the acceleration of the top floors can be used.

The input of the shaker is a sine sweep up to 45 Hz in 120 seconds. The input of the shaker is not measured since the acceleration of the ground plate is used as an input for the system. For the third floor of the three story building, $x_{13}$, a time and frequency result of an experiment are visible in Figure 5.2. At very low frequencies small vibrations are found due to friction in the slider. At higher frequencies some small peaks other then the three main resonance peaks are visible. There may be several reasons for these; internal resonances in the shaker, resonances in other parts of the setup or unaccounted for resonances in the building. This last option can be the beams connecting the floors that are assumed to be massless. Also if the screws are a bit loose the beams can move relatively to the masses.

Figure 5.1: Experimental setup with the shaker and the five story building
The recorded acceleration data is fed to a program written at CINVESTAV that filters it and determines the locations of the peaks. Also the raw data is saved to create graphs as in Figure 5.2 to compare to the simulation data.

The first measurements that are made are to check some assumptions in the model and to visualize the noise. Then the uncoupled situation is measured to determine the characteristic stiffnesses and dampings and compare them to the analytically calculated values. The new values are put in the simulation model and the results are compared to the measurement output. Then a spring is added and the coupled situation is analyzed. This spring is used to validate the coupled simulation model and is chosen for its availability and not its passive damping results.

5.1 Noise measurements

In the simulations the only reason for noise is rounding errors. This is not the case in the experiments. Noise in the measurements has many causes, some are inevitable like the measurement resolution. Some can be reduced with a controller, this is for example the case with the internal vibrations in the shaker. Two possible noise causes are experimentally tested; transversal movement of the buildings and the influence of a second building without coupling. These are assumed to be zero in the simulation and if this is not the case the model needs to be adjusted.

5.1.1 Transversal movement

It is assumed that the buildings only move in one direction. This is a major simplification since now there are only as many resonance frequencies as there are masses. Extra accelerometers are placed in the transversal direction on the top floors of the buildings to check this assumption. $x_{13}$ and $x_{25}$ represent the top floor of the three story and the five story building respectively.
The results of the experiment are visible in Figure 5.3. All data is acquired in a single measurement. Note the scale of the axes. The top left graph shows the transversal acceleration in the three story building. There is one clear peak at 23 Hz. This peak is also visible in the FFT of the lateral acceleration at the same floor. The lateral resonance is likely the cause of the transversal vibrations. At the top right graph the FFT of the transversal acceleration of the top floor of the five story building is visible. There is a clear peak at 8 Hz visible, which does not coincide with a lateral resonance peak. The exact reason of the transversal vibration is unknown, but it is not really relevant due to the low amplitude. The amplitude of the transversal acceleration in the three story building is ten times bigger than the amplitude of the five story building.

![Figure 5.3: Transversal and lateral acceleration response of the two top floors](image)

### 5.1.2 Vibrations through ground plate

A possible reason for noise are the resonances in the other building. The vibrations in the ground plate are not supposed to change due to the resonances. To check this two measurements are compared. Two times the acceleration of the first floor \(x_{11}\) and of the third floor \(x_{13}\) of the three story building are measured, once with the presence of the five story building and once without. The results are visible in Figure 5.4. At first glance the two top graphs seem to fit nicely on each other. The two bottom graphs show the difference between the two measurements. A small difference at the acceleration peaks is expected due to high uncontrolled accelerations. There are no clear peaks in the difference graph at the resonance frequencies of the five story building. Therefore the model is not adjusted and the influence of the other building through the ground is not further discussed.
Figure 5.4: Noise caused by presence of the other building

Figure 5.5: Repeatability of measurements
5.1.3 Repeatability

Every measurement might be a little bit different although nothing is changed in the set-up. Two measurements are performed with the same input and no changes in the setup. The two acceleration responses of the top floor of the three story building are compared in Figure 5.5.

The first thing is that all smaller peaks are present in both measurements. This means that these are not noise but actual resonances in parts of the setup. In this particular measurement the input gain is a bit low, so the three main resonance peaks are not very clear in the top graph. The peaks are more clear in the graph showing the difference. This means that the differences are mainly at the three resonance peaks as expected. The change is in amplitude and not in frequency. It seems that the second resonance peak (about 23 Hz) is not measured in the second experiment. This is probably caused by the low input signal.

The repeatability is not perfect but it should be sufficient for validation of the model and control.

5.2 Peak-picking method

Peak picking is a simple method to determine the characteristic damping of a vibration mode. The mass of the system is estimated by weighting and the stiffness is adjusted by hand to fit the measurements well.

The theory of peak picking is based on the assumption that the mass and stiffness are diagonalizable and the damping is proportional. With this assumption the different modes can be decoupled. The damping is calculated with the frequencies and is independent of the used units. The resonance frequency is determined $(\omega_r)$ and the two frequencies at which the amplitude is $\frac{1}{\sqrt{2}}$ of the amplitude at the resonance frequency $(\omega_1$ and $\omega_2)$. The damping is then determined with the following formula.

$$\zeta_r = \frac{\omega_2 - \omega_1}{2\omega_r}$$ (5.1)

All peaks are visible in all acceleration measurements of all floors and two measurements are used. This means that there are six damping coefficients for each peak in the three story building and ten for each peak in the five story building. The resulting damping coefficients are averaged and also a standard deviation is calculated. This is visible in Table 5.1. It was expected that the damping coefficients of the different floors are similar since the damping is caused by the same four beams. The standard deviations are high and sometimes even exceed the average. The standard deviations for the five story building are somewhat smaller and the damping coefficients of this building are therefore more reliable.

<table>
<thead>
<tr>
<th>Damping coefficient</th>
<th>Standard deviation</th>
<th>Damping coefficient</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0141</td>
<td>0.0173</td>
<td>0.0097</td>
<td>0.0071</td>
</tr>
<tr>
<td>0.0072</td>
<td>0.0064</td>
<td>0.0011</td>
<td>0.0002</td>
</tr>
<tr>
<td>0.0015</td>
<td>0.0016</td>
<td>0.0032</td>
<td>0.0024</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>0.0003</td>
<td>0.0001</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>0.0003</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

Table 5.1: Found damping coefficients with existing program which uses peak-picking method

5.3 Uncoupled system

First the uncoupled system is checked. Here it is easier to see results of small adjustments. Also the coupling itself can cause errors due to the attachment, et cetera.
5.3.1 Matching model parameters to data

To find the stiffness of each resonance frequency the masses are weighted and the peak is matched to the peak found in the data. The found values for damping and stiffness are inserted in the model. The results are visible in Figure 5.6 and 5.7.

The damping values are already determined with the peak-picking method. These values do not fit the data well. The values are adjusted to show both the model and the data in the same graph. Damping found with the peak-picking method of three story building is increased with a factor 200 and the damping of the five story building is increased with a factor 100. These errors may be caused by the fact that actual top of the peak, so the maximum acceleration, in the data is not easily measured.

Another problem with the damping values is that the match is different for every resonance peak. The first peak is much higher in the model. In the measurement all frequencies are measured the same amount of time. This means that lower frequencies have less oscillations to reach their maximum amplitude. At these frequencies the amplitude will probably increase if the total measurement time is increased.

Another reason for the need to increase the damping is that no filtering is used in the program. The reason for this is that the filtering decreases the height of all peaks. The problem now is that the program takes the first points at its calculated amplitude from the resonance frequency. In Figure 5.3.1 this is visible. So the measurement noise is not averaged which causes the approximation to have narrower peaks than the data.

![Model matched with data](image)

**Figure 5.6**: FFT of accelerations of adjusted model and data of three story building
Figure 5.7: FFT of accelerations of adjusted model and data of five story building

Figure 5.8: Interface of program used to find damping coefficients
At some peaks the model has a higher amplitude at other peaks the data has a higher amplitude as is visible in Figure 5.6 and 5.7. This, together with the relatively big standard deviations and the fact that the values are way too low, results in the conclusion that the found damping coefficients are not reliable.

To determine the stiffness parameters also the resonance frequencies in the data should be checked. The resonances of every peak are visible in every floor. The same measurements are used so there are again six values for every peak in the three story building and ten values for every peak in the five story building. Again a standard deviation is determined. These standard deviations are smaller and the resonance frequencies seem therefore more reliable than the damping coefficients. It is easier to measure the location of the peaks correctly then the amplitude.

<table>
<thead>
<tr>
<th>Three story building</th>
<th>Five story building</th>
</tr>
</thead>
<tbody>
<tr>
<td>resonance frequency [Hz]</td>
<td>standard deviation</td>
</tr>
<tr>
<td>7.7</td>
<td>0.094</td>
</tr>
<tr>
<td>22.9</td>
<td>0.184</td>
</tr>
<tr>
<td>34.9</td>
<td>0.262</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 5.2: Resonance frequencies and their standard deviations determined from the measurements

5.3.2 Comparison to theoretical values

The theoretical stiffnesses and masses were determined by simple calculations. Now these values are also determined from simple measurements and the frequency response data. The results are visible in Table 5.3 and 5.4.

After production the floors are weighted. Their weight is a little bit higher than the theoretical values due the addition of the screws and accelerometers and a small deviation in the size.

The experimental stiffness is more interesting. It was expected that the stiffness between every floor was the same, this is clearly not the case. The reason for this is not further investigated. It might be related to the fact that in the theoretical model the beams are not connected and non-neighbouring floors do not influence each other directly. Since the four beams in reality reach from the ground floor to the top floor this is not the case.

<table>
<thead>
<tr>
<th>Mass [kg]</th>
<th>Stiffness [kN/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theoretical</td>
<td>Experimental</td>
</tr>
<tr>
<td>0.257</td>
<td>0.283</td>
</tr>
<tr>
<td>0.257</td>
<td>0.283</td>
</tr>
<tr>
<td>0.257</td>
<td>0.283</td>
</tr>
</tbody>
</table>

Table 5.3: Difference between theoretical and experimental mass and stiffness of three story building
Table 5.4: Difference between theoretical and experimental mass and stiffness of five story building

<table>
<thead>
<tr>
<th>Mass [kg]</th>
<th>Stiffness [kN/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theoretical</td>
<td>Experimental</td>
</tr>
<tr>
<td>0.257</td>
<td>0.283</td>
</tr>
<tr>
<td>0.257</td>
<td>0.283</td>
</tr>
<tr>
<td>0.257</td>
<td>0.283</td>
</tr>
<tr>
<td>0.257</td>
<td>0.283</td>
</tr>
<tr>
<td>0.257</td>
<td>0.283</td>
</tr>
</tbody>
</table>

The stiffnesses are not the same between the floors but on average it is close to the theoretical values. In Table 5.5 the theoretically calculated resonance frequencies are put together with the ones found in the data. In the second column the resonance frequencies are visible that are found with the model after inserting the experimentally found parameters for mass and stiffness. For the three story building these are very close to the experimental frequencies. For the five story building there are small differences. These differences are caused by unmodelled phenomena like the earlier discussed influence of non-neighbouring floors.

Table 5.5: Difference between theoretical and experimental resonance frequencies with adjusted model parameters from the data

<table>
<thead>
<tr>
<th>Frequency [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Three story building</td>
</tr>
<tr>
<td>Theoretical</td>
</tr>
<tr>
<td>8.9</td>
</tr>
<tr>
<td>25.0</td>
</tr>
<tr>
<td>36.2</td>
</tr>
<tr>
<td>-</td>
</tr>
<tr>
<td>-</td>
</tr>
</tbody>
</table>

5.3.3 Conclusion

At this moment the cause of the deviation in expected values for damping and stiffness is not known. This does not mean that the measurements are not suitable to draw conclusions on different control options. More and other measurements could be made to determine the cause of the differences, but this is not done. The control can be performed on the resonances visible in the data and tuned by hand. The model can be used to generate an effective controller and estimate the raw controller parameters. It is even possible to generate a controller model that adjusts itself after a deviation in the resonance frequencies or measured amplitudes. Such a controller model still lacks the unmodelled phenomena, but may be sufficient to generate the effective controller variables to attenuate the main resonances.

5.4 Coupled system

A spring is added between two floors of the buildings. The measurements are repeated in the same manner as described earlier for the uncoupled situation.

The results of only one spring are discussed here. The stiffness of the spring is determined by measuring the change in length of the spring for six different forces. The spring is assumed to be linear and the stiffness is then determined with a root-least-square method. The stiffness of the spring is 352.3 N/m. Three measurements are done and the results are again compared to the results in the model.
During the measurements resonance modes in the spring are visible. This is kept in mind while discussing the differences between results from the model and the data. Also no damping is assumed between the buildings.

Figure 5.9: FFT accelerations of top floor of three story building for different spring configurations

Figure 5.10: FFT accelerations of top floor of five story building for different spring configurations
In Figure 5.9 and 5.10 the measurements are visible together with the model with the adjusted parameters. The accelerations are measured at the top floors of the buildings. It is expected that due to the connection at every floor the eight resonances of the system are visible. This is not the case.

In Figure 5.9 it is visible that the third resonance frequency is not easily visible in the measurement although it is visible in the Simulink simulation results. When the spring is placed at the top floor this resonance frequency is not visible in both the measurement and the simulation results. Again the amplitudes of the data at frequencies below 15 Hz are lower than the amplitudes of the model. This may again be caused by the short measurement time, or actually the short time a mode is actuated.

In Figure 5.10 the second mode is not visible in the data of all three measurements. The seventh mode is not visible when the spring is connected at the third floor. Besides that, the graphs coincides well and the model seems also accurate for a spring connection.

The deviations in Figure 5.9 are bigger than in Figure 5.10. The resonance frequencies are put in Table 5.6. In the first column the theoretical values are placed. These are still with the theoretical estimated values for mass and stiffness. This is the reason for the small deviation from the other two columns. The coupling stiffness is put to the actual value.

In theory there should be eight resonance frequencies. In the model these should also be visible. This is not the case as is visible in the second column of the table. In the data more peaks are missing. This might be caused by the fact that resonances are close to each other; the fourth and fifth mode are only separated by 1 Hz. The resonances that are found in both the data and the model with adjusted parameters are close; less than 1 Hz difference.

<table>
<thead>
<tr>
<th>Frequency [Hz]</th>
<th>Three story building</th>
<th>Spring at second floor</th>
<th>Spring at third floor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theory</td>
<td>Adjusted model</td>
<td>Data</td>
<td>Theory</td>
</tr>
<tr>
<td>5.8</td>
<td>4.5</td>
<td>4.5</td>
<td>6.0</td>
</tr>
<tr>
<td>9.1</td>
<td>8.0</td>
<td>8.0</td>
<td>9.6</td>
</tr>
<tr>
<td>16.9</td>
<td>15.6</td>
<td>-</td>
<td>17.1</td>
</tr>
<tr>
<td>25.4</td>
<td>22.8</td>
<td>23.3</td>
<td>25.1</td>
</tr>
<tr>
<td>26.6</td>
<td>23.4</td>
<td>-</td>
<td>26.3</td>
</tr>
<tr>
<td>33.9</td>
<td>32.3</td>
<td>31.2</td>
<td>33.9</td>
</tr>
<tr>
<td>36.4</td>
<td>35.1</td>
<td>34.8</td>
<td>36.4</td>
</tr>
<tr>
<td>38.6</td>
<td>-</td>
<td>-</td>
<td>38.7</td>
</tr>
</tbody>
</table>

Table 5.6: Difference between theoretical and experimental resonance frequencies with adjusted model parameters from the data

From the earlier presented test with the simulation model it was expected that the effects of the coupling would only influence the first and the second mode significantly (Figure 3.3). The amplitude of the first mode and the frequency of the second mode should increase with increasing connection floor for the three story building. In the five story building the main effect is the increase in frequency of the second mode.

The frequency shift is indeed visible in the data, from 8.0 to 8.9 Hz in the three story building. In the five story building this mode was not found in the data at all.

It can be concluded that after attaching the spring to the buildings, the data from the measurements still coincides with the model fairly well. However the effects of the spring are not clearly visible.
5.5 Conclusion

First several measurements are made to check assumptions made in the model. The transversal movement is very little and therefore neglected. The effect of the vibrations of the other building through the ground floor is also negligible. The last measurement checks the repeatability of the measurements; the two sequent measurements show only small differences and mainly at resonances.

A peak-picking method is used to determine the damping form the data. This method is used here at CINVESTAV and an internally created Matlab program is used. Probably due to the measurements these results are not accurate.

Measurements are performed on the uncoupled model and from the data the parameters of the Simulink model are adjusted. The stiffness turned out to be non-equally distributed between the different floors. The found damping values are increased significantly to match the data. After these adjustments the model fits the data well and the assumptions made seem justified.

Ultimately the buildings are connected with a spring and again the results of the model and the data are compared. Due to the lack of accurate measurements at low frequencies the measured effects of the spring are minimal.

The adjusted model seems accurate enough for active controlling. If all parameters of the model are determined nicely from the measurements, they may still change over time. Especially in the real-life situation of the buildings. Therefore it is preferred to have a model that updates itself online as the MRAC controller discussed in Appendix G.
6 Conclusion and recommendations

6.1 Conclusion

The traditional way of controlling a building under vibrations is by adding an extra mass and using it as an vibrations absorber. This uses several floors of a building and is probably not the most energy efficient. Therefore the use of a connection between two buildings for attenuating vibrations is analyzed.

This report uses the example of two different buildings; a three story and a five story building. It is assumed that the floors only move in one direction which results in an eight-degree-of-freedom system. First the resonance frequencies and mode shapes are determined analytically. Then a Simulink model is created and validated with the analytical results with and without coupling.

The Simulink model is used to simulate passive controller options. The choice of the coupling floor and the coupling stiffness influences the resonance frequency and the amplitude of some of the lower modes.

The same model is used for the simulation of active control options. If two floors are connected very stiff and a control force is applied at this floor the resonances can be attenuated well. The energy efficiency is not well since no passive control is used. Positive Position Feedback (PPF) damps only the resonances at one frequency. Multiple Positive Position Feedback (MPPF) can be used to reduce vibrations at all eight resonance peaks. All these control options are based on a control force acting on one or several floors. For PPF and MPPF there are possibilities to generate the control force without the addition of an extra mass.

To check the used assumptions and the simulations of the uncoupled, passively controlled and actively controlled buildings, the setup is manufactured. For the control only the accelerations of the top floors should used, but for the validation experiments accelerometers are placed at all floors. The model is correct.

The model parameters are determined from the data and differ somewhat from the analytically calculated values. The difference in damping is likely caused by the errors in the measurement and the difference in stiffness due to unmodelled phenomena. The Simulink file is updated to the new parameters and the output fits the measurement output well.

6.2 Recommendations

It is recommended to further investigate the cause of the differences in stiffness parameters between the analytical values and the values determined from the data. It was not expected that the stiffness between different floors was different and the modeling of the beams might be incomplete.

The damping between the floors is at this point unknown. The found values had standard deviations that exceed the average and had to be increased with a factor 100 or 200. The correct damping should be determined to optimize the simulations.

A problem that may arise is the change in model parameters over time. Therefore it is recommended that the control is adjusted online. The theory of Model Reference Adaptive Control (MRAC) can be used to determine the model parameters and the instant optimal coupling stiffness and damping. This way of controlling can be used next to passive control to minimize the controller output. Another advantage is the possibility to use the same controller on different but similar buildings.
References


A  Equations of motion

To determine the equations of motions mass balance is used on the simplified system of Figure 1.1. Also the coupling is added in the equations below. For the uncoupled situation it is sufficient to put the couplings stiffnesses and dampings to zero. Then the two systems are decoupled.

In (A.1) the force balance equations are visible and in (A.2) and (A.3) the matrix forms are shown which result in the matrices of (2.2) and (2.3) for the uncoupled situation and (2.9) for the coupled situation.

\[
m_{11}\ddot{x}_{11} = -c_{11}(\dot{x}_{11} - \dot{z}) - c_{12}(\dot{x}_{11} - \dot{x}_{12}) - c_{13}(\dot{x}_{11} - \dot{x}_{13}) - k_{11}(x_{11} - z) - k_{12}(x_{11} - x_{12}) - k_{13}(x_{11} - x_{13})
\]

\[
m_{12}\ddot{x}_{12} = -c_{12}(\dot{x}_{12} - \dot{x}_{11}) - c_{13}(\dot{x}_{12} - \dot{x}_{13}) - c_{21}(\dot{x}_{12} - \dot{x}_{21}) - k_{12}(x_{12} - x_{11}) - k_{13}(x_{12} - x_{13}) - k_{22}(x_{12} - x_{22})
\]

\[
m_{13}\ddot{x}_{13} = -c_{13}(\dot{x}_{13} - \dot{x}_{12}) - c_{23}(\dot{x}_{13} - \dot{x}_{23}) - k_{13}(x_{13} - x_{12}) - k_{23}(x_{13} - x_{23})
\]

\[
m_{21}\ddot{x}_{21} = -c_{21}(\dot{x}_{21} - \dot{z}) - c_{22}(\dot{x}_{21} - \dot{x}_{22}) - c_{23}(\dot{x}_{21} - \dot{x}_{23}) - k_{21}(x_{21} - z) - k_{22}(x_{21} - x_{22}) - k_{23}(x_{21} - x_{23}) - k_{11}(x_{11} - x_{21})
\]

\[
m_{22}\ddot{x}_{22} = -c_{22}(\dot{x}_{22} - \dot{x}_{21}) - c_{23}(\dot{x}_{22} - \dot{x}_{23}) - c_{24}(\dot{x}_{22} - \dot{x}_{24}) - k_{22}(x_{22} - x_{21}) - k_{23}(x_{22} - x_{23}) - k_{24}(x_{22} - x_{24})
\]

\[
m_{24}\ddot{x}_{24} = -c_{24}(\dot{x}_{24} - \dot{x}_{23}) - c_{25}(\dot{x}_{24} - \dot{x}_{25}) - k_{24}(x_{24} - x_{23}) - k_{25}(x_{24} - x_{25})
\]

\[
m_{25}\ddot{x}_{25} = -c_{25}(\dot{x}_{25} - \dot{x}_{24}) - k_{25}(x_{25} - x_{24})
\]
\[
\begin{bmatrix}
 m_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & m_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & m_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & m_{21} & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & m_{22} & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & m_{23} & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & m_{24} & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{25} & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{25} \\
\end{bmatrix}
\begin{bmatrix}
x_{11} \\
x_{12} \\
x_{13} \\
x_{21} \\
x_{22} \\
x_{23} \\
x_{24} \\
x_{25} \\
\end{bmatrix}
= \begin{bmatrix}
x_{11} \dot{z} + k_{11} \dot{z} \\
x_{12} \dot{z} \\
x_{13} \dot{z} \\
x_{21} \dot{z} + k_{21} \dot{z} \\
x_{22} \dot{z} \\
x_{23} \dot{z} \\
x_{24} \dot{z} \\
x_{25} \dot{z} \\
\end{bmatrix}
\]  
(A.2)

\[
\begin{bmatrix}
c_{11} + c_{12} + c_{13} & -c_{12} & 0 & -c_{13} & 0 & 0 & 0 & 0 & 0 \\
-c_{12} & c_{12} + c_{13} + c_{2} & -c_{13} & 0 & -c_{2} & 0 & 0 & 0 & 0 \\
0 & -c_{13} & c_{13} + c_{3} & 0 & 0 & -c_{3} & 0 & 0 & 0 \\
-c_{13} & 0 & 0 & c_{21} + c_{22} + c_{1} & -c_{22} & 0 & 0 & 0 & 0 \\
0 & -c_{2} & 0 & -c_{22} & c_{22} + c_{23} + c_{2} & -c_{23} & 0 & 0 & 0 \\
0 & 0 & -c_{3} & 0 & -c_{23} & c_{23} + c_{24} + c_{3} & -c_{24} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -c_{24} & c_{24} + c_{25} & -c_{25} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -c_{25} & c_{25} & 0 \\
\end{bmatrix}
\begin{bmatrix}
x_{11} \\
x_{12} \\
x_{13} \\
x_{21} \\
x_{22} \\
x_{23} \\
x_{24} \\
x_{25} \\
\end{bmatrix}
(A.3)

\[
M_{k} \dot{x} = \begin{bmatrix}
c_{11} \dot{z} + k_{11} \dot{z} \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix} - C_{k} \dot{x} - K_{k} \dot{x}
\]
B  Simulink implementation of equations of motion

The basic scheme of the Simulink model is shown in Figure B.1. In Figure B.2 a substructure is visible. This is the substructure of the second floor of the five story building; so one of the blocks with the most possible inputs.

![Figure B.1: Overview of the Simulink simulation model](image)

The basic idea is to sum all the forces from all springs and dampers and calculate the acceleration; one of the results of the simulation. This is integrated two times to find the position; needed as result of the simulation and to calculate the spring forces. The velocity is only used in the simulation to calculate the damper forces.

The lower two ‘lanes’ of the model in Figure B.2 can only be nonzero if the coupling is placed at the second floor.
Figure B.2: Simulink model of the second floor of the five story building
C Simulations with coupling at different floors

Simulations of the uncoupled and coupled situation are compared for the three different couplings configurations. This results in six graphs, one of the three story building and one of the five story building for each configuration. In Figures C.1 and C.2 coupling in the first floor of both buildings is visible, in Figures C.3 and C.4 the second floor is coupled and in C.5 and C.6 coupling is placed at the third floor.

In these simulations a coupling stiffness of 4.1 kN is used. This is equal to the stiffness between two neighbouring floors. Damping is used to increase the visibility of the graphs, but is kept constant for all simulations. The damping of course does not change the resonance frequencies.

It should be noted that the resonance frequencies have shifted and that resonances of the neighbouring building are visible in the FFT of the other. This means that a total of eight peaks should be visible, these are not easy to distinguish.

FFT acceleration response

![Graphs showing FFT acceleration response for uncoupled and coupled situations](image)

Figure C.1: FFT of accelerations of all masses of first coupled three story building
Figure C.2: FFT of accelerations of all masses of on first floor coupled five story building

Coupling at the first floor does not effect the FFT of the five story building much and the effect on the three story building is mainly at the first resonance peak.
Figure C.3: FFT of accelerations of all masses of on second floor coupled three story building
The effect of coupling at the second floor seems to be positive for the three story building. Especially the peak of the lowest resonance frequency of the original building has decreased in amplitude. In the five story building the coupling does not have a positive effect for all floors.
Figure C.5: FFT of accelerations of all masses of on third floor coupled three story building
Coupling at the third floor seems to have positive effect for the three story building-like structure; peaks are lower than without coupling. Also for the five story building coupling seems positive but results are less convincing.

These results are depending on the values for stiffness and damping and these coupling characteristics can be tuned for the three situations separately. This is not done since the model parameters are not verified at this moment and tuning is depending on these parameters. The goal was to compare the results of coupling at different floors with the same coupling parameters.
D  Simulation with the same stiffness at different floors

The used coupling stiffness is the same as between the floors; 4.1 kN. The damping is put to 2.1 Ns/m to decrease the noise in the measurements.

![Acceleration responses for different modes](image)

Figure D.1: FFT of accelerations of all masses with coupling at different floors

The main effects are on the lowest three modes. The first mode changes the most in amplitude, the second mode shows the largest frequency shift. The effects on the third mode are not easily visible on this scale but show both a small frequency shift and a small change in amplitude.
E Simulation with different stiffness at the same floor

The coupling is placed at the third floor and the damping is put to 2.1 Ns/m to decrease the noise in the measurements.

Due to the high amplitudes of the first mode of the higher floors of the five story building, the visibility of the last three graphs is low. The main effects are on the lowest three modes. The first mode changes the most in amplitude, the second mode shows the largest frequency shift.

Figure E.1: FFT of accelerations of all masses with coupling at the third floor with different stiffnesses
Multiple Positive Position Feedback with eight absorber frequencies

To determine the optimal damping parameters of the Multiple Positive Position Feedback (MPPF) for eight resonance frequencies plots are made for three different dampings. In Figure F.1 and F.2 the overall acceleration responses are visible for each floor. In Figure F.3 and F.4 the accelerations responses of the top floors are visible, but now zoomed at the different resonance frequencies. The first two figures are used for the general response that shows the importance of the difference resonances. The last two are used to actually choose the damping parameters. The chosen parameters are in Table 4.1.

FFT acceleration response for different controller dampings

Figure F.1: Accelerations of the three story building for passive coupled and active coupled system with different damper values
Figure F.2: Accelerations of the five story building for passive coupled and active coupled system with different damper values
Figure F.3: Accelerations of the top floor of the three story building for passive coupled and active coupled system with different damper values zoomed at the resonance frequencies
Figure F.4: Accelerations of the top floor of the five story building for passive coupled and active coupled system with different damper values zoomed at the resonance frequencies
**G Online active controlling**

Since it is possible that model parameters change over time and it is preferable that the controller works on several models and even in reality, online adaption is preferred. A correct model is needed, but the parameters can be guessed. The theory of a MRAC (Model Reference Adaptive Controller) can even be used to determine the optimal coupling stiffness and damping, so to determine the needed active controller input.

In a normal situation MRAC is used to create a controller almost equal to the inverse plant to be able to follow a reference trajectory created by a similar plant. This is not all applicable in the current situation. Only the part where the inverse plant in the controller is adapted to fit the real plant as good as possible.

In this chapter the theory of MRAC is explained briefly. In the situation of the control of the buildings it can be used in two ways. The first is only the verification of the model. In the second application it generates the parameters for the model of the buildings that would ensure a stable control and vibration attenuation. This model is then partly put to reality by adjusting the coupling parameters. Due to time restrictions the main part of this chapter is only in theory.

**G.1 MRAC theory**

The MRAC scheme is visible in Figure G.1. The use of the reference model is to create a plausible reference trajectory. Therefore this model is similar to the actual plant but does not need to have the same model parameters. The controller consists for the main part of the inverse plant. This plant is determined online on the base of an initial estimation of the plant. If this estimation is not too far off and the system is asymptotically stable the estimated plant in the controller will converge to the real plant.

![Figure G.1: Normal MRAC control scheme](image)

\[
\begin{align*}
u &= \hat{m}\ddot{q}_m + \hat{b}\dot{q} + \hat{k}q - \hat{m}(b_r\dot{e}_q + k_r e_q) \\
&= \hat{\theta}^T v \\
e_q &= q - q_m \\
\dot{\hat{\theta}} &= -\Gamma^{-1} v e_q \\
v &= \begin{bmatrix} \hat{q}_m - b_r\dot{e}_q - k_r e_q & \dot{q} + q \end{bmatrix} \\
\dot{e}_q + b_r\dot{e}_q + k_r e_q &= 0 \\
\hat{\theta} &= -\Gamma^{-1} v e_q
\end{align*}
\]

(G.1) (G.2) (G.3)
MRAC uses the control law in (G.1), in this formula $b$ and $k_r$ are variables of the freely chosen error model that is visible in (G.2). In this case $m_r$ is chosen to be one and therefore not visible in the equations. In $v$ there is a difference between $q$ and $q_m$; here $q_m$ is from the reference model and $q$ is from the plant. $\hat{\theta}$ and $v$ are, in the simplest case of one mass, the size 1 by 3. In the case of this eight mass system both matrices are 8 by 3. The adaption law is visible in (G.3). It consists of the parameter $\Gamma$, this is the weighting factor.

### G.2 MRAC theory in online model verification

In this case the theory is not exactly the same. The scheme visible in Figure G.2 is used in stead of the scheme in Figure G.1. The controller does not provide an input for the plant in this case. The only input for the plant is the ground disturbance. Also the reference trajectory is absent. In this case the estimated, theoretical model is supposed to follow the real model by adjusting its parameters.

![Figure G.2: Verification scheme based on MRAC](image)

The same update law from (G.3) is used but the parameters are a little bit different. Now $q_m$ is from the real plant and $q$ is from the estimated model. At start the weighting factor is chosen to be the identity matrix, this can be tuned later to increase the speed or resolution of the update process.

For the actual use and the simulation of the verification method a Simulink file is created. The simulation variant consists of two models of the system. One should be replaced with the measurement inputs in the actual experiments (the real plant) and one representing the model that will be used in the control (the estimated model).

The verification of the Simulink file consists of three steps: 1. perfect initial guess, 2. perfect initial guess with noise, 3. close initial guess with noise. In the first step the errors between the real plant and the estimated model should be zero at all times. This is true except for small computational errors around the resonance peaks which result in small deviations in the model parameters $\hat{\theta}$. This problem is solved with low pass filtering of the error used as the input of the adaptation law in (G.3).

In the second step extra noise is added to the simulation. This represents the measurement errors and the numerical differentiation errors caused by the fact that only the acceleration is measured and also the velocity and position are needed in the adaptation law. The effect of this noise in $q_m$ is decreased by the low-pass filtering of the errors but is still present in the results. These problems are solved by creating a minimum error before the adaption law should react to the errors. This decreases the resolution of the resulting model parameters.

In the last step the adaptation should actually converge to the real parameters of the model. For this to happen the initial guesses of all parameters should not be too far off. Since the theoretical model is present and the masses and stiffnesses are not expected to differ more than 50 percent from the calculated values, this is not a problem. Unfortunately due to time restrictions this last step is not performed with succes yet. It is left for future work if MRAC is preferred in the control of the buildings.
G.3 MRAC theory in active control

It would be even better if not only the model is updated online but also the control. In this way the buildings can change over time without the need of performing the tuning of the control again.

The optimal spring and damper follow from the estimated plant determined by the MRAC theory. The coupling should be able to be adjusted fast enough for active control at the time of an earthquake or vibrations due to any other cause. The control scheme is visible in Figure G.3, the $c$ and the $k$ represent the coupling damping and stiffness relatively.

![Figure G.3: Active control scheme based on MRAC](image)

Measurements have to show what range of coupling values are needed for the control. Unadjustable passive control can be applied next to the adjustable connection to reduce the control energy.

Although this control option is not tested in simulation or with actual measurements, is this the most realistic option to actually control two real live buildings for a significant amount of time. Also the force generation is not an issue.

The main issue is that the parameters will only converge to the correct values if vibrations are present. It is not possible to excite real-life buildings for this cause. So experiments with a model should result in parameters for the estimated models close enough to the reality to ensure stability at the first real-life test.

G.4 Conclusion

Online active controlling is likely to be necessary due to the change of the buildings over time. The fact that the same controller can be used on several similar buildings without the need to perform experiments on all of them is also positive. Passive control can be used next to the active control to reduce the controller input.

The MRAC theory can be used to update the model only or to generate the active controller output. Both options are discussed briefly without simulations or experiments due to time restrictions.

More experimenting should be done but at this time this way of actively determining the coupling parameters seems a good option as well as the earlier discussed PPF and MPPF.