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Equivalent Single-layer Power Cable Sheath for Transient Modeling of Double-layer Sheaths

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Abstract — In this paper a model for cables with a double earth screen layer consisting of unequal conducting materials is developed. The earth screen is modeled by a single-layer with properties such that the transient behavior of the actual cable is reproduced. The first approach aims to equate the impedances and admittances of both models by varying parameters of the single-layer sheath model to optimize the match with the double-layer cable sheath. In the second approach the differences between the ABCD matrices of both models are minimized. By analyzing the ABCD matrices a method is developed to obtain a close match for transient behavior between single- and double-layer models. Analysis of the accuracy of the results shows a good match for the investigated cable.

Index Terms — Power system simulation, Power system transients, PSCAD

INTRODUCTION

In the West of the Netherlands a new transmission grid is planned where part of the system consists of 380 kV cables. These cables have a sheath of a copper layer enclosed by a lead layer. The PSCAD/EMTDC software used to simulate the power system employs a cable model which consists of alternative layers of conducting and insulating material, meaning it only allows a single conducting layer in the sheath. Methods have been developed to describe a core of stranded wires using a solid conductor and to describe a semiconductor layer by adjusting insulating layer parameters by [1] and [2], but no method was previously available to describe the sheath consisting of a lead and a copper layer using a single layer. This paper proposes a method to describe a double-layer sheath, where the resistivity of both layers are unequal, using a single layer sheath. The concerned frequency range is from DC to 500 kHz. By modeling the double-layer sheath using a single conducting layer the considered cable can be simulated using software which is based on four-layer models of power cables, for instance PSCAD/EMTDC.

Section II describes the models with single-layer sheath and double-layer sheath. In Section III an attempt is made to match the impedances of both models by adjusting the single layer sheath model parameters. Section IV gives a method to minimize differences between the ABCD matrix descriptions of both cable sheath models by adjusting single layer sheath model parameters. Section V shows the accuracy of that method, by assessing the similarity of the transient response of the single layer sheath model with parameter values as determined in Section IV and the double layer sheath model. Section VI summarizes the conclusions.

CABLE MODELS

A. Single-Layer Sheath

The cable model with single layer sheath and its parameters (resistivity ρ, permeability μ, permittivity ε, and radius r) are shown in Fig. 1. The double layer sheath model shown in Fig. 2 is discussed later.

\[ \frac{d}{dx} \begin{bmatrix} U_c \\ U_s \\ Z_{cc} \\ Z_{sc} \end{bmatrix} = \begin{bmatrix} Z_{cc} & Z_{cs} \\ Z_{sc} & Z_{ss} \end{bmatrix} \begin{bmatrix} I_c \\ I_s \end{bmatrix} \]

(1)

where \( U_c = U_c(x) \) is the voltage at position \( x \) of the core, \( U_s \) the voltage of the sheath, \( I_c \) the current through the core and
The current through the sheath. The mutual and self-impedances of the core and the sheath are given by

\[ Z_{cc} = z_i + z_s + z_f + z_c + z_f - 2z_s \]
\[ Z_{sc} = Z_{fs} = z_s + z_f + z_f - z_i \]
\[ Z_{ss} = z_s + z_f + z_f. \]  

The impedances \( z_i \) through \( z_f \) are given in [3]; the ground impedance \( z_f \) can be found in [4]. The admittance matrix is given by

\[ Y = \begin{bmatrix} Y_{cc} & Y_{sc} \\ Y_{sc} & Y_{ss} \end{bmatrix} = j\omega \begin{bmatrix} 1/p_c & -1/p_c \\ -1/p_c & 1/p_c + 1/p_s \end{bmatrix} \]

where

\[ p_c = \ln(r_c / r_s) / (2\pi r_c), \quad p_s = \ln(r_s / r_s) / (2\pi r_s). \]  

B. Double-Layer Sheath

The cable model with double-layer sheath is shown in Fig. 2. Only the impedances \( z_3, z_4 \) and \( z_5 \) in (2) of the cable model with single layer sheath depend on the material of this sheath. Therefore, only the equations for the impedances related to the sheath have a different form for the double-layer sheath model compared to the single-layer sheath model. The admittance equations (4) only depend on the radii and insulation layer permittivity and thus they are similar for both models. The values of the parameters are given in Appendix A.

The internal impedance of the inside of the sheath \( z_{3d}, z_{4d} \), mutual impedance between inside and outside of the sheath \( z_{4d}, z_{5d} \), and internal impedance of the outside of the sheath \( z_{5d} \) are given in [5]. The subscript ‘d’ denotes a parameter referring to the cable model with double-layer sheath.

MATCHING IMPEDANCES

To match the impedances of the cable model with single-layer sheath to the cable model with double-layer sheath, the most direct way is to set the parameters shown in Fig. 1 as follows:

- \( \rho_s \) and \( \mu_s \) are parameters whose values can be optimized for a best match.
- All other parameter values of the single-layer sheath model are taken equal to those of the corresponding parameters of the double-layer sheath model.

With these assumptions all corresponding impedances and admittances of the two models are equal, except \( z_3 \neq z_{3d}, z_4 \neq z_{4d} \), and \( z_5 \neq z_{5d} \). If there are values of \( \rho_s \) and \( \mu_s \) for which \( z_3 = z_{3d}, z_4 = z_{4d} \) and \( z_5 = z_{5d} \), they would give an exact match between the two models.

In case of a perfect match, both cable models have equal behavior for any frequency. In particular, a DC current through the sheath should give an equal voltage over the sheath for both models. So in case of a perfect match the DC resistances \( R_{DC} \) of both sheaths are equal. As a first approach, the equivalent resistivity of the single-layer sheath \( \rho_s = \rho_{eq} \) is calculated such that the DC resistances of both models are equal. The double-layer sheath can be seen as two parallel resistances, which gives

\[ R_{sc} = \frac{\rho_s l}{A_s + A_l} = \frac{(\rho_s l / A_s)(\rho_l l / A_l)}{\frac{A_s l}{A_s} + \frac{A_l l}{A_l}} \]

where \( l \) is the length of the cable, and

\[ A_1 = \pi(r_s^2 - r_a^2), \quad A_2 = \pi(r_s^2 - r_f^2). \]

The condition for equivalent resistivity becomes:

\[ \rho_{eq} = \frac{\rho_s\rho_{eq}(A_1 + A_2)}{\rho_s A_1 + \rho_{eq} A_2}. \]

Although the equivalent resistivity gives an equivalent model at DC, its performance is not guaranteed at higher frequencies (e.g. due to skin effect in the conductors). Therefore, a modified resistivity may result in a better match of the behavior over the entire frequency range. As a measure for accuracy, the Mean Error (ME) is calculated as

\[ \text{ME} = \left| z_{3d} - z_3 \right| + \left| z_{4d} - z_4 \right| + \left| z_{5d} - z_5 \right| \]

This parameter measures the average distance between the double layer and equivalent complex impedances \( z_3 - z_5 \) with respect to \( z_{3d} - z_{5d} \). The ME is zero when there is a perfect match between the cable models. The minimum of \( \text{ME} \) found by varying \( \rho_s \) is about \( 2.46 \times 10^{-4} \) (\( \Omega \cdot \text{m} \)), which occurs at \( \rho_s = \rho_{eq} \). This is shown in Fig. 3.

![Figure 3. Mean error ME as function of $\rho_s/\rho_{eq}$](image-url)

Minimizing the ME by varying \( \rho_s \), \( \mu_s \), and \( r_3 \) all at once does not give a significantly lower minimum.

Errors in the impedance values propagate to errors in transient currents and voltages in the cable connection.
Moreover, \( ME \) does not approach zero for any choice of parameters for the single-layer sheath. Therefore it is uncertain whether the parameter values for which the \( ME \) is minimal also minimize errors in transient simulations, where all other parameters of (1)–(4) also participate. For this reason the ABCD matrix, which has a direct relationship to transient simulations, is investigated in the following section.

**MATCHING ABCD MATRIX**

**C. Description of the ABCD Matrix**

Combining the matrix equations involving impedance and admittance gives:

\[
\frac{d}{dx} \begin{bmatrix} U \\ I \end{bmatrix} = \begin{bmatrix} O & -Z \\ -Y & O \end{bmatrix} \begin{bmatrix} U \\ I \end{bmatrix}. \tag{9}
\]

The relationship between currents and voltages on a point \( x = q \) on a cable to the voltages and currents on \( x = p \) can be expressed using the ABCD matrix:

\[
\begin{bmatrix} U_p \\ I_p \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} U_q \\ I_q \end{bmatrix}. \tag{10}
\]

where

\[
\begin{bmatrix} A & B \\ C & D \end{bmatrix} = (T e^{-\omega T^{-1}})^{-1}.
\]

Here, \( T \) is the matrix consisting of the eigenvectors of (9) as columns, and \( \Lambda \) is a square matrix with the corresponding eigenvalues along its main diagonal.

In the system under investigation the sheath of the cable is grounded at point \( p \) with impedance \( Z_{pS} \) and at point \( q \) with impedance \( Z_{qS} \) and the cable length is \( D \), as shown in Fig. 4.

![Figure 4. Cable segment used for calculating reduced ABCD matrix](image)

The voltages and currents in case of two conducting layers are

\[
\begin{bmatrix} U_{\rho S} \\ U_{\rho S} \\ I_{\rho S} \\ I_{\rho S} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & B_{11} & B_{12} \\ A_{21} & A_{22} & B_{21} & B_{22} \\ C_{11} & C_{12} & D_{11} & D_{12} \\ C_{21} & C_{22} & D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} U_{\rho S} \\ U_{\rho S} \\ I_{\rho S} \\ I_{\rho S} \end{bmatrix}. \tag{11}
\]

In the configuration shown in Fig. 4 we also have

\[
U_{\rho S} = -Z_{\rho S} I_{\rho S}, \quad U_{\rho S} = Z_{\rho S} I_{\rho S}. \tag{12}
\]

Applying these relations the ABCD matrix equation (11) can be written as the reduced ABCD\(_{noS}\) matrix equation (13), where only the core currents and voltages are calculated; the influences of the sheath are incorporated in the elements with subscript noS. The reduced matrix equation is

\[
\begin{bmatrix} U_{\rho S} \\ I_{\rho S} \end{bmatrix} = \begin{bmatrix} A_{noS} & B_{noS} \\ C_{noS} & D_{noS} \end{bmatrix} \begin{bmatrix} U_{\rho S} \\ I_{\rho S} \end{bmatrix}. \tag{13}
\]

**D. Matching of Models**

To minimize the errors in transient simulations, we minimize the differences between the ABCD\(_{noS}\) matrix of the single-layer sheath model and the ABCD\(_{dSnoS}\) matrix of the double-layer sheath model. For matching the ABCD\(_{noS}\) matrices the error for each element \( E_X \) (\( X \) refers to either the \( A, B, C \) or \( D \) matrix component) is given by

\[
E_X = |X_{dSnoS} - X_{noS}|. \tag{14}
\]

The following parameter values are chosen for analyzing both matrices: \( D=1000 \) m, \( Z_{S5} \) and \( Z_{p5} = 1 \) mΩ.

First, the errors for the situation \( \rho_S = \rho_{S1} \), and \( \rho_S = \rho_{eq} \) are analyzed. Results for the error in \( A_{noS} \) are shown in Fig. 5. From this figure we conclude that the equivalent resistivity gives the smallest error for frequencies below about 4 kHz, while \( \rho_S = \rho_{S1} \) gives the smallest error for frequencies above about 4 kHz.

Based on the previous conclusions an ‘equivalent radius’ method is proposed to get a low error over the entire frequency range. The concept of the equivalent radius method is to take \( \rho_S \) equal to that of the best conducting layer of the double layer sheath to achieve low errors at high frequencies, and then calculate \( r_3 = r_{eq} \) such that the DC resistivity of the single layer sheath is equal to that of the double layer sheath to achieve low errors at low frequencies. This leads to the following parameter values:

- \( \rho_S = \rho_{S1} \)
- \( r_2 = r_{d} \)
- \( r_3 = r_{eq} \)
- All other parameters of the single-layer sheath model are taken equal to those of the double layer sheath model.

To obtain \( r_{eq} \), first the area of the single layer sheath \( A_{eq} \) for which the DC resistance has the desired value is calculated:

\[
A_{eq} = A_1 + \frac{\rho_{S1}}{\rho_{d}} A_1. \tag{15}
\]
From the area of the single layer

\[ A_{eq} = \pi r_{eq}^2 - \pi r_{sd}^2 \]  

(16)

the desired radius \( r_{3eq} \) is obtained:

\[ r_{3eq} = \frac{\rho_{S1}}{\rho_{S2}} \left( \frac{r_{sd}^2 - r_s^2}{r_s^2} ight) + r_s. \]  

(17)

Fig. 5 shows error \( E_A \) when \( r_3 = r_{3eq} \) and \( \rho_S = \rho_{S1} \). This figure shows that the equivalent radius method gives a lower \( E_A \) compared to the other two configurations up to 75 kHz. Above 75 kHz, using the equivalent radius method, a marginally higher \( E_A \) is obtained than if \( \rho_S = \rho_{S1} \) is taken with \( r_3 = r_{sd} \). Appendix B shows that similar conclusions can be drawn for the other elements of the ABCD matrix.

E. Correction Admittance Matrix

When taking \( r_3 = r_{3eq} \neq r_{sd} \), as is the case when using the equivalent radius method, the admittance matrix of the single layer sheath model becomes unequal to the admittance matrix of the double layer model. The potentials \( \rho_S \) and \( \rho_{sd} \) of the single and double layer model, which are used in the admittance matrix, are given by:

\[ p_s = \frac{\ln(r_s / r_{eq})}{(2\pi \varepsilon_2)} \]

\[ p_{sd} = \frac{\ln(r_{sd} / r_s)}{(2\pi \varepsilon_2)}. \]  

(18)

To obtain equal potentials, and thereby admittance matrices, \( \varepsilon_2 \) can be corrected, since no other impedance or admittance depends on this parameter. The corrected permittivity \( \varepsilon_{2c} \) for which \( \rho_S = \rho_{sd} \) is given by

\[ \varepsilon_{2c} = \varepsilon_{pe} \frac{\ln(r_s / r_{eq})}{\ln(r_{sd} / r_s)}. \]  

(19)

Fig. 6 shows \( E_A \) for the equivalent radius method with and without corrected permittivity. Correcting the permittivity does not give significantly lower \( E_A \). The maximum difference between \( E_A \) for corrected and uncorrected \( \varepsilon_2 \) is only 5·10^{-5} %. From this we conclude that it is superfluous to correct the admittance matrix.

F. Correction \( z_6 \)

When taking \( r_3 = r_{3eq} \neq r_{sd} \), also the impedances \( z_6 \) of both models differ. The equations for both models are given by

\[ z_k = \frac{j \omega \mu_2}{2\pi} \ln\left(\frac{r_s}{r_{eq}}\right). \]  

\[ z_{6d} = \frac{j \omega \mu_2}{2\pi} \ln\left(\frac{r_{sd}}{r_s}\right). \]  

(20)

To obtain equal impedances \( z_6, \mu_2 \) can be corrected, since no other impedance or admittance depends on this parameter. The corrected permeability \( \mu_{2c} \) for which \( z_6 = z_{6d} \) is given by

\[ \mu_{2c} = \frac{\ln(r_{sd} / r_s)}{\ln\left(\frac{r_s}{r_{eq}}\right)}. \]  

(21)
Fig. 7 shows the mean error $E_A$ for corrected and uncorrected permeability of the outer insulation layer. This figure shows that for frequencies below about 30 Hz, uncorrected $\mu_2$ gives the lowest $E_A$, while above 30 Hz $\mu_2$ gives slightly lower $E_A$; above 1 kHz this difference is negligible. From this we can conclude that it is best to use the uncorrected value for $\mu_2$.

G. Optimizing Parameters

To confirm that the equivalent area method gives the best approximation of the double layer sheath transient behavior, the minimum difference between the ABCD matrices of both models is determined for a range of values of $r_3$ and $\rho_S$. To determine the difference a Mean Error for the ABCD matrix ($ME_{ABCD}$) is introduced

$$ME_{ABCD} = |A_{d,eq} - A_{aw}| + |B_{d,eq} - B_{aw}| + |C_{d,eq} - C_{aw}| + |D_{d,eq} - D_{aw}|$$

(22)

The value of $ME_{ABCD}$ for varying $r_3$ and $\rho_S$ is shown in Fig. 8.

![Fig. 8. Mean error $ME_{ABCD}$ as function for $r_3$ and $\rho_S$. The minimum value of $ME_{ABCD}$ is indicated by the dot.](image)

This figure shows that a minimum exists for the $ME_{ABCD}$- The parameter values at the minimum $\rho_S = \rho_{S,min}$ and $r_3 = r_{3,min}$ are approximately equal to $\rho_{cu}$ and $r_{3,eq}$:

$$r_{3,min}/r_{3,eq} = 1.002$$

$$\rho_{S,min}/\rho_{S,eq} = 1.011.$$  

(23)

From this observation, and from the fact that the differences between $\rho_{S1}$ and $\rho_{S2}$ and between $r_{3,eq}$ and $r_3$ are much greater, we conclude that it will be challenging to find a method which gives values closer to $r_{3,min}$ and $\rho_{S,min}$ than the equivalent area method. Whether the equivalent radius method is also applicable when the outer layer of the sheath has a lower resistivity than the inner layer has not been investigated.

**TRANSIENT RESPONSE OF EQUIVALENT MODEL**

To test the accuracy of the match between cable model with single layer sheath with equivalent radius and the cable model with double-layer sheath, a switching surge voltage response is calculated. The setup of the surge calculation is acquired by adding a voltage source in Fig. 4 at terminal $p$. Terminal $q$ is left open, so the relationship between $U_p$ and $U_q$ is given by:

$$U_q = A^T U_p.$$  

(24)

This test only checks the $A$ component of the ABCD matrix.

The voltage at terminal $q$ can be obtained by first transforming the input surge to the frequency domain, then calculate the phasor quantities of the output voltage using (24) and finally transferring the phasor quantities back to the time domain. The resulting voltage $U_{q,f}$ is shown in Fig. 9.

![Fig. 9. Surge voltage calculated using ABCD matrices of various configurations; inset zooms in on the 26th oscillation](image)

This figure clearly shows that the cable model with single layer sheath using parameter values given by the equivalent radius method (continuous grey line; $\rho_S = \rho_{cu}, r_3 = r_{3,eq}$) has the best match of transient behavior to the cable model with double layer sheath (dotted line). The inset shows still no observable deviation after 25 oscillations.

**CONCLUSIONS**

A method has been found to describe a cable model with double-layer sheath using a cable model with single-layer sheath which has almost equal transient behavior. Minimizing differences in impedances $z_3$, $z_4$, and $z_5$ of both models does not minimize differences in transient behavior. To obtain most accurate results, differences in the ABCD matrices of both models should be minimized. In this case, differences can be minimized by giving the single-layer sheath the resistivity of the better conductor of the double-layer sheath, and the radius of the single-layer sheath should be changed such that the single-layer sheath and double-layer sheath have equal DC resistance.

The accuracy of the proposed method is confirmed by performing a switching surge and comparing results between the original double layer sheath model and various single layer sheath approximations. In this comparison the single
layer sheath cable with parameters following from the proposed method clearly outperforms other options.

REFERENCES


APPENDIX A– PARAMETER VALUES OF THE DOUBLE LAYER SHEATH MODEL

<table>
<thead>
<tr>
<th>Par.</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_{l1} )</td>
<td>Radius of core</td>
<td>0.0306 m</td>
</tr>
<tr>
<td>( r_{2d} )</td>
<td>Outer radius of inner insulation layer</td>
<td>0.0631 m</td>
</tr>
<tr>
<td>( r_{3d} )</td>
<td>Outer radius of outer sheath layer</td>
<td>0.0655 m</td>
</tr>
<tr>
<td>( r_{4d} )</td>
<td>Outer radius of outer insulation layer</td>
<td>0.0715 m</td>
</tr>
<tr>
<td>( r_{5d} )</td>
<td>Outer radius of inner sheath layer</td>
<td>0.0635 m</td>
</tr>
<tr>
<td>( \rho_{c1} )</td>
<td>Resistivity of core</td>
<td>( 1.98 \times 10^8 ) ( \Omega ) m</td>
</tr>
<tr>
<td>( \rho_{31} )</td>
<td>Resistivity of inner sheath layer</td>
<td>( 1.68 \times 10^7 ) ( \Omega ) m</td>
</tr>
<tr>
<td>( \rho_{51} )</td>
<td>Resistivity of outer sheath layer</td>
<td>( 2.20 \times 10^7 ) ( \Omega ) m</td>
</tr>
<tr>
<td>( \rho_{e1} )</td>
<td>Resistivity of earth</td>
<td>100 ( \Omega ) m</td>
</tr>
<tr>
<td>( \varepsilon_{1d} )</td>
<td>Permittivity of inner insulation layer</td>
<td>( 2.51 \times 10^{11} ) ( \text{F/m} )</td>
</tr>
<tr>
<td>( \varepsilon_{2d} )</td>
<td>Permittivity of outer insulation layer</td>
<td>( 1.99 \times 10^{11} ) ( \text{F/m} )</td>
</tr>
</tbody>
</table>

APPENDIX B – ERRORS FOR VARIOUS SINGLE LAYER SHEAT PARAMETERS FOR OTHER ELEMENTS OF THE ABCD MATRIX

![Figure 10](image_url) Error in \( B \) as function of frequency with \( r_{l1} = r_{2d} \) and \( \rho_{e1} = \rho_{31} \) or \( \rho_{e1} = \rho_{eq} \) and for \( r_{l1} = r_{3d} \) and \( \rho_{51} = \rho_{31} \).

![Figure 11](image_url) Error in \( C \) as function of frequency with \( r_{l1} = r_{2d} \) and \( \rho_{e1} = \rho_{31} \) or \( \rho_{e1} = \rho_{eq} \) and for \( r_{l1} = r_{3d} \) and \( \rho_{51} = \rho_{31} \).

![Figure 12](image_url) Error in \( D \) as function of frequency with \( r_{l1} = r_{2d} \) and \( \rho_{e1} = \rho_{31} \) or \( \rho_{e1} = \rho_{eq} \) and for \( r_{l1} = r_{3d} \) and \( \rho_{51} = \rho_{31} \).