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Probabilistic model for fatigue crack growth and fracture of welded joints in civil engineering structures

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ABSTRACT

This paper presents a probabilistic assessment model for linear elastic fracture mechanics (LEFM). The model allows the determination of the failure probability of a structure subjected to fatigue loading. The distributions of the random variables for civil engineering structures are provided, and the relative importance of these random variables is determined. An example of a bridge detail is provided in order to show the application of the model. Partial factors are derived for the case of fatigue of welded joints in civil engineering structures. The failure probability appears to be relatively insensitive to the failure criterion (attainment of a through-thickness crack or fracture) when considering the total fatigue life.

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1. Introduction

Since its introduction by Irwin and its application to fatigue by Paris, linear elastic fracture mechanics (LEFM) is a widespread technique used to determine the (residual) life of a structure or component subjected to fluctuating stresses. The method is provided in standards [1,2] and others. In most design procedures, a deterministic approach is applied where the mean value or the mean minus or plus a number of standard deviations is used for the variables such as the crack dimensions and the fatigue crack growth curve. Partial factors are introduced in order to arrive at the required reliability level. These factors are sometimes estimated based on practical experience from the past, or best-guess value are applied. Due to this procedure, the value of the reliability index β in LEFM is unknown in many cases.

Burkedin and Hamour [3,4] have derived partial factors for fracture associated with certain values of β . These partial factors have been incorporated in [1]. However, the standard does not provide partial factors for fatigue. In many other standards for fatigue – notably standards based on S–N curves such as the EN 1993 series [5,6] – the relationship between failure and fracture is not explicitly accounted for, other than providing a minimum Charpy value or a maximum thickness value. This raises the question as to

whether or not the fracture characteristics have a significant influence on the failure probability of a fatigue loaded structure.

Various studies into probabilistic fatigue assessment using LEFM have been conducted. The effects of uncertainties in loading, initial crack width and other factors were assessed in [7]. Probabilistic LEFM was applied to determine the inspection interval of cover plates in bridges in the USA in [8]. A comprehensive overview of more recent probabilistic fatigue assessment models for welded steel structures is provided in [9]. One study, [10], is of special interest, in that it gives a complete overview of the influence of almost all relevant random variables on the failure probability. However, the study is dedicated to stress analysis by measurements for a detail in a specific bridge, so uncertainty factors in models for determining stresses were not considered. Other important differences are the choice of the stress ranges (constant, equivalent range in [10] versus variable amplitude loading in this paper) and the application of statistical methods within the model (used to determine the relative importance of variables in [10] versus used to derive partial factors in this paper).

The study reported in this paper is a follow-up of the earlier probabilistic LEFM studies for fatigue and fatigue followed by fracture. Contrarily to the previous models – the model in [10] as well as all other models described in [9] – it considers the fact that a large stress cycle that can cause fracture, can occur at any moment in time. Thus, the failure probability of the fatigue loaded structure is a combination of the failure probabilities for all time intervals. The LEFM model is based on BS 7910 [1]. For practical

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a	depth of a semi-elliptical crack [mm]
c	half-width at the plate surface of a semi-elliptical crack [mm]
$f(\cdot)$	probability density function [-]
$g(\cdot)$	limit state function [-]
m	Paris Law exponent [-]
p	thickness exponent [-]
t	time [s]
u, v, w	independent standard Gauss variables, representing the physical variables X in a FORM analysis
A	Paris Law parameter (N, mm)
B	plate thickness [mm]
C	model uncertainty [-]
$E[\cdot]$	expectation operator [-]
$F(\cdot)$	cumulative distribution function [-]
K	elastic stress intensity factor [N/mm ^{3/2}]
K_{mat}	material fracture toughness [N/mm ^{3/2}]
K_r	fracture ratio [-]
L	node length [mm]
L_r	plastic collapse ratio [-]
N	number of stress cycles [-]
P	probability [-]
R	failure resistance [-]
T	operating temperature [°C]
T_0	temperature for a median toughness of 3162 N/mm ^{3/2} or 100 MPa m ^{1/2} [°C]
T_k	temperature term that describes the scatter in the Charpy versus fracture toughness correlation [°C]
V	coefficient of variation [-]
W	plate width [mm]
X	random variable vector [-]
Y	stress intensity correction factor [-]
α	FORM sensitivity factor [-]
β	reliability index [-]
γ_{fat}	partial factor for fatigue [-]

μ	mean value
σ	stress, including stress concentrations due to the joint geometry [N/mm ²]
σ_y	yield stress [N/mm ²]
σ_u	ultimate tensile strength [N/mm ²]
σ_r	standard deviation of the stress process [N/mm ²]
λ	Weibull shape parameter [N/mm ²]
ρ	plasticity correction factor or correlation coefficient [-]
ϕ	Cumulative distribution function of the standardised Gauss distribution [-]
Δ	range operator [-]

0	threshold value
a	in depth direction of a semi-elliptical crack
b	bending
c	in width direction of a semi-elliptical crack
d	design value
cr	critical value
f	failure value
$glob$	global stress analysis
k	interval operator or characteristic value
m	membrane
nom	nominal value
o	initial value
p	primary (primary stresses are stresses that could contribute to plastic collapse)
ref	reference value
res	residual
s	secondary (secondary stresses are self-equilibrating stresses necessary to satisfy compatibility)
scf	stress concentration factor
sif	stress intensity factor
tr	transition value
I	tension (mode I)

- The spectrum of highway traffic loads, railway traffic loads, wind loads and wave loads can usually be approximated by a Weibull or closely related distribution – refer to [11–14], respectively. Thus, the fluctuating stress spectrum is approximated as a Weibull distribution.
- The fluctuating stresses are randomly distributed.
- The fluctuating part of the stress is relatively large: its characteristic value is at least of the same magnitude as the permanent stress, e.g. due to the self-weight [15].
- Post welding treatments are not applied.
- Only Charpy-V values are available to assess the material properties for fracture.
- The temperature difference between the minimum operating temperature and the Charpy transition temperature at 27 J is sufficiently large, so that the structure is in the transition range or upper-shelf range.

2. Deterministic model in BS 7910

The stress intensity factor K for a certain applied stress σ is given as

$$K = \sigma Y \sqrt{\pi a} = (\sigma_m Y_m + \sigma_b Y_b) \sqrt{\pi a} \quad (1)$$

For each principal direction of crack growth, a Paris type expression may be formulated

$$\begin{aligned} \frac{da}{dN} &= A(\Delta K_a)^m \quad \text{for } \Delta K_a > \Delta K_0 \\ \frac{dc}{dN} &= A(\Delta K_c)^m \quad \text{for } \Delta K_c > \Delta K_0 \end{aligned} \quad (2)$$

Fig. 1. A semi-elliptical crack in a steel plate at/near the weld toe.

where the first expression relates to point A and growth in the depth direction. The second expression relates to point C and growth in the width direction (Fig. 1). The stress intensity factor ranges ΔK_a and ΔK_c are given by

$$\begin{aligned}\Delta K_a &= (\Delta \sigma_m Y_{ma} + \Delta \sigma_b Y_{ba}) \sqrt{\pi a} \\ \Delta K_c &= (\Delta \sigma_m Y_{mc} + \Delta \sigma_b Y_{bc}) \sqrt{\pi a}\end{aligned}\quad (3)$$

where $\Delta \sigma$ is the applied stress range, and Y_a and Y_c are stress intensity correction factors for points A and C, respectively. The bi-linear version of the fatigue crack growth model by Paris and Erdogan [17] is considered (Fig. 2).

$$\frac{da}{dN} = \begin{cases} 0 & \text{for } \Delta K \leq \Delta K_0 \\ A_1 (\Delta K)^{m_1} & \text{for } \Delta K_0 < \Delta K \leq \Delta K_{tr} \\ A_2 (\Delta K)^{m_2} & \text{for } \Delta K > \Delta K_{tr} \end{cases}\quad (4)$$

A pair of coupled differential equations is obtained by substitution of Eq. (3) in Eq. (2). With the exception of the material parameters (A, m) and the applied stress range $\Delta \sigma$, all other terms are a function of the crack size (a, c). The equations are evaluated through an incremental numerical procedure, resulting in the relationships between the number of cycles, N , and the crack dimensions, a and c . Starting values are the initial defect dimensions, a_0 and c_0 , and the number of cycles up to crack initiation, N_0 . Since N_0 is usually small for welded connections, its value is set to 0 in this paper.

Fig. 2. Schematic representation of typical crack growth rates with linear and bi-linear approximations.

2.2. Fracture model

In some cases – notably pipelines – failure may be associated with the attainment of a through-thickness crack. In other cases, it is associated with the integrity of the cross-section. In the latter cases, failure occurs when a load cycle exceeds the load bearing capacity of the remaining ligament. This remaining load bearing capacity depends on the actual crack size and the interaction between plastic collapse and fracture. The maximum crack depth that can be sustained by the component is called the critical crack depth, a_{cr} , and it depends on the material's yield stress and fracture toughness, K_{mat} . Several alternatives for the failure assessment curve are provided in [1]. The equations representing the failure assessment line of the often applied method 2A in [1] can be formulated as:

$$K_{r,f} = \begin{cases} 0 & \text{for } L_r > L_{r,max} \\ (1 - 0.14L_r^2)(0.3 + 0.7 \exp(-0.65L_r^6)) & \text{for } L_r \leq L_{r,max} \end{cases}\quad (5)$$

An alternative for Eq. (5) based on the results of large-scale tests is provided by [18]:

$$K_{r,f} = \sqrt{R^2 - L_r^2}\quad (6)$$

Failure is expected when a load cycle causes $K_r > K_{r,f}$. Fig. 3 presents both failure assessment diagrams together with the results of large-scale tests.

Variable R is a normalised resistance parameter, usually taken as 1.0 in deterministic design calculations. Fig. 3 shows that 1.0 is a safe value in most cases, and the mean value is 1.7. Quantities K_r , L_r and $L_{r,max}$ are defined as follows:

$$K_r = \frac{K_I}{K_{mat}} + \rho \quad \text{and} \quad L_r = \frac{\sigma_{ref}}{\sigma_y} \quad \text{and} \quad L_{r,max} = \frac{\sigma_y + \sigma_u}{2\sigma_y}\quad (7)$$

where K_{mat} is the material toughness measured by stress intensity factor (Eq. (8)); K_I is the stress intensity factor: $K_I = (Y_p \sigma_p + Y_s \sigma_s) \sqrt{\pi a}$; ρ is the plasticity correction factor (refer to [1]); and σ_{ref} is the reference stress, which is a function of the primary stress σ_p (refer to [1]).

In the absence of crack tip opening displacement (CTOD) or J -integral tests, the material toughness, K_{mat} , may be estimated from Charpy V tests (units N, mm and °C):

$$K_{mat} = 632 + \left[348 + 2435 \cdot \exp\left(\frac{T - T_0 - T_k}{52}\right) \right] \left(\frac{25}{B}\right)^{1/4} \left[\ln\left(\frac{1}{1-P}\right)\right]^{1/4}\quad (8)$$

where T is the operating temperature, T_0 is the temperature for a median toughness of 3162 N/mm^{3/2}, or 100 MPa m^{1/2}, (which is taken equal to $T_{27J} - 18$ °C, where T_{27J} is the Charpy transition temper-

Fig. 3. K_r – L_r interaction curves according to [1] (Eq. (5)) and [18] (Eq. (6)), with test failure data on mild steel large scale specimens from [18].

ature at 27 J), T_k is the temperature term that describes the scatter in the Charpy versus fracture toughness correlation (which is 25 °C for a standard deviation of 15 °C and 90% confidence level) and P is the probability of K_{mat} being less than estimated (BS 7910 recommends the use of $P = 0.05$ in the absence of experiments that prove otherwise).

3. Probabilistic LEFM model

3.1. Fatigue crack growth

The stress ranges are randomly distributed. Assuming that for $\Delta\sigma$, a sufficiently mixing (ergodic) process, we may use the expectation of the stress range to find the expectation of da/dN

$$E\left(\frac{da}{dN}\right) = A_1 E[\Delta\sigma^{m_1}]_{\Delta\sigma_{th}}^{\Delta\sigma_{tr}} \cdot \left(\left[\frac{B_{nom}}{B}\right]^p C_{load} C_{glob} C_{scf} C_{sif} Y_a \sqrt{\pi a}\right)^{m_1} \dots + A_2 E[\Delta\sigma^{m_2}]_{\Delta\sigma_{tr}}^{\infty} \cdot \left(\left[\frac{B_{nom}}{B}\right]^p C_{load} C_{glob} C_{scf} C_{sif} Y_a \sqrt{\pi a}\right)^{m_2} \quad (9)$$

and

$$E[\Delta\sigma^m]_{s_1}^{s_2} = \int_{s_1}^{s_2} s^m f_{\Delta\sigma}(s) ds \quad (10)$$

For the crack width evaluation (direction c), a similar equation as Eq. (9) is applied. The factors C represent model uncertainties for the fluctuating load model and for various stress calculations (loading, stress analysis of the entire structure, e.g. by the FE model, stress concentration factor and stress intensity factor), and $f_{\Delta\sigma}(s)$ is the probability density function of the stress ranges, $\Delta\sigma$. The parameter B_{nom} is the plate thickness applied in the calculation of the stress – usually taken as the average plate thickness. Thickness exponent p addresses the dependency of the stress on the plate thickness. Its value depends on the type of loading. For bending stress, p is equal to 2. For membrane stress, p ranges between 0 and 1 – depending on the geometry – and is often close to 1. As an example, p is equal to 1 in the case of a single plate in tension, and p is approximately 0.8 in the case of a flange of an I section loaded in bending. Other overall dimensions such as the plate width are considered deterministic because the variation in these other overall dimensions is usually small compared to the variation in plate thickness.

Eq. (10) can be solved numerically using small stress range steps, s . Alternatively, the expectation of the m th moment of the

stress distribution, Eq. (10), can be derived directly depending on the stress spectrum. Three options are detailed here:

1. If the stress process is Gaussian and has a narrow banded spectrum, it can be shown that the peaks of the stress process – and hence the stress ranges – follow a Rayleigh distribution. For this case, the m th moment of the stress range density function can be obtained as

$$E[\Delta\sigma^m] = (2\sigma_r\sqrt{2})^m \Gamma\left(\frac{m}{2} + 1\right) \quad (11)$$

where σ_r is the standard deviation of the Gaussian stress process, and $\Gamma(\cdot)$ is the Gamma function. For a crack growth curve consisting of two slopes Eq. (11) is replaced by:

$$E[\Delta\sigma^m]_{\sigma_{th}}^{\sigma_{tr}} = (2\sigma_r\sqrt{2})^{m_1} \Gamma\left(\frac{m_1}{2} + 1; \left(\frac{\sigma_{tr}}{2\sigma_r\sqrt{2}}\right)^2\right) \\ E[\Delta\sigma^m]_{\sigma_{tr}}^{\infty} = (2\sigma_r\sqrt{2})^{m_2} \Gamma_0\left(\frac{m_2}{2} + 1; \left(\frac{\sigma_{tr}}{2\sigma_r\sqrt{2}}\right)^2\right) \quad (12)$$

where $\Gamma(a; b)$ denotes the incomplete Gamma function and $\Gamma_0(a; b) = \Gamma(a) - \Gamma(a; b)$ the complementary Gamma function.

2. If the stress has a narrow banded spectrum and the peaks follow a Weibull distribution

$$F_{\Delta\sigma}(s) = 1 - \exp\left(-\left(\frac{s}{\sigma_r}\right)^\lambda\right) \quad (13)$$

the m th moment of the stress range density function is equal to

$$E[\Delta\sigma^m] = \sigma_r^m \Gamma\left(\frac{m}{\lambda} + 1\right) \quad (14)$$

and for two slopes

$$E[\Delta\sigma^m]_{\sigma_{th}}^{\sigma_{tr}} = \sigma_r^{m_1} \Gamma\left(\frac{m_1}{\lambda} + 1; \left(\frac{\sigma_{tr}}{\sigma_r}\right)^\lambda\right) \\ E[\Delta\sigma^m]_{\sigma_{tr}}^{\infty} = \sigma_r^{m_2} \Gamma_0\left(\frac{m_2}{\lambda} + 1; \left(\frac{\sigma_{tr}}{\sigma_r}\right)^\lambda\right) \quad (15)$$

Note that a Weibull distribution with $\lambda = 2$ is equal to the Rayleigh distribution.

3. If the stress spectrum is broad-banded, the stress cycles cannot be easily distinguished. In this case, a convention is required for defining stress cycles. There are two approaches available for the modelling of stress cycles of a broad-banded stress process:
- Cycle counting methods. The time history of the stress process can be summarised using rain flow counting [19]. In the case of a crack growth curve with one slope, the stress expectations are equal to:

$$E[\Delta\sigma^m] = \sum_i \left(\frac{N_i}{\sum_i N_i} (\Delta\sigma_i^m) \right) \quad (16)$$

where N_i is the number of cycles corresponding to rain flow stress range $\Delta\sigma_i$ and N_i is the total number of cycles. For a two-slope crack growth curve, Eq. (10) may be used.

- Empirically derived semi-analytical probability distributions for stress ranges of a wide banded stress process such as the 5-parameter mixed Weibull distribution by Zhao and Baker [20].

3.2. Limit state function

Two limit state functions are detailed here:

- g_1 : Failure is associated with the attainment of a certain crack depth, e.g. a through-thickness crack.
- g_2 : Failure is associated with fracture.

The limit state function for failure associated with a critical crack depth a_{cr} can be formulated as

$$g_1(X, t) = a_{cr} - a(t) \quad (17)$$

or, alternatively,

$$g_1(X, t) = N_{cr} - N(t) \quad (18)$$

where N is the number of cycles at time t , corresponding with a crack depth a . N_{cr} is the number of cycles up to the attainment of the critical crack depth a_{cr} . For calculation purposes, the crack depth increase may be divided into, e.g. 1000 intervals k : $\Delta a_k = (a_{cr} - a_0)/1000$. For every crack depth interval Δa_k , the number of cycles, $\Delta N_{cr,k}$, and the crack width interval, Δc_k , are determined. Summation of $\Delta N_{cr,k}$ gives N_{cr} . The failure probability is denoted as

$$P_f = P[g_1(X, t) \leq 0] \quad (19)$$

and it can be calculated directly using a Monte Carlo analysis, a First Order Reliability Method (FORM) or other suitable techniques.

In case failure is associated with fracture, the limit state function may be formulated as:

$$g_2(X, t) = R^2 - K_r^2 - L_r^2 \quad (20)$$

Or, alternatively,

$$g_2(X, t) = N_f - N(t) \quad (21)$$

where N_f is the number of cycles at fracture, i.e. when $R^2 = K_r^2 + L_r^2$. Parameter K_r depends on K_I and K_{mat} (Eq. (7)). The value of K_I is determined in each interval k

$$K_{I,k} = \sigma_{pk} \cdot \left[\frac{B_{nom}}{B} \right]^p C_{load} C_{glob} C_{scf} C_{sif} Y_p \sqrt{\pi a} + \sigma_s \cdot C_{glob}^* C_{sif} Y_s \sqrt{\pi a} \quad (22)$$

where C_{glob}^* is equal to either 1.0 or C_{glob} , depending on the derivation of the secondary stress. Due to the fact that the stress peaks are independent in time, the distribution function of the maximum primary stress level in interval k – called σ_{pk} – is equal to:

$$F_{\sigma_{pk}}(s) = [F_{\sigma_p}(s)]^{\Delta N_{fk}} \quad (23)$$

where ΔN_{fk} is the number of cycles calculated for interval k , and $F_{\sigma_p}(s)$ is the cumulative distribution function of the primary stress, i.e. the sum of the permanent and the fluctuating stress.

A three-parameter Weibull distribution is proposed to describe the fracture toughness K_{mat} by [3,10] (units N, mm, °C)

$$F_{K_{mat}}(x) = 1 - \exp \left[- \left(\frac{x - 632}{K_m^* - 632} \right)^\lambda \right] \quad (24)$$

$$K_m^* = 632 + \left[348 + 2435 \cdot \exp \left(\frac{T - T_0 - T_k}{52} \right) \right] \left(\frac{25}{B} \right)^{1/4} \quad (25)$$

where λ is the shape parameter, taken as 4 on the basis of experiments by Wallin [21].

The failure probability associated with limit state function g_2 can be approximated by one of the two following procedures:

- A lower bound of the failure probability is obtained by assuming that the maximum primary stress level, σ_{pk} , remains unaltered during crack growth. The evaluation is similar to that of limit state function g_1 .
- An upper bound of the failure probability is obtained by: (a) Considering n time intervals for the total number of cycles $N(t)$. The number of cycles considered for crack growth in each time interval is $i \cdot N(t)/n$ with $i = 1 \dots n$. (b) Determine the crack dimensions at the end of each time interval for all $i \cdot N(t)/n$ cycles and the failure probability for the final $1/n \cdot N(t)$ cycles of that time interval. (c) Determine the failure probability of the entire system using the method of Hohenbichler [22], in which all variables are fully correlated between the fractions except for σ_p , which is fully uncorrelated (refer to the Appendix A).

This upper bound approximation approaches the actual failure probability if many time intervals are considered.

The two approximations are visualised in Fig. 4. The black curve represents the stress level at the limit state. This value is decreasing due to the increasing crack growth. The grey line indicates the maximum primary stress level σ_{pk} .

3.3. Distribution functions of the random variables

The distribution functions for the random variables in Table 1 are taken from literature. Expert opinion values are used when the distributions are not available in literature. The table also provides characteristic values X_k that can be used in deterministic calculations.

4. Example of a cover plate on a bridge girder

4.1. Description of the geometry and load model

The example described here is a transverse weld in a cover plate detail, located in a main girder of a three span bridge as indicated in Fig. 5. The crack is growing from the weld toe into the girder flange. The part of the stress intensity correction factor, Y , considering the effect of the weld is taken as the 2-dimensional approximation by Maddox and Andrews, incorporated in [27,1].

The fluctuating part of the global stress σ is modelled as a stationary zero mean Gaussian process with a standard deviation σ_r (Fig. 6). The peak values $\hat{\sigma}$ are assumed to follow a Rayleigh distribution with a mean value $\mu_r = \sigma_r \Gamma(1 + 1/2) = \sigma_r \sqrt{\pi/2} = 1.25\sigma_r$.

$$f_{\hat{\sigma}}(x) = \frac{x}{\sigma_r^2} \exp \left(- \frac{x^2}{2\sigma_r^2} \right) \quad (26)$$

Fig. 4. Visualisation of the lower bound and the upper bound approximations for g_2 in case of n equally divided fractions of $N(t)$ (a) Lower bound for P_f , (b) Upper bound for P_f .

Table 1

Distribution functions for random variables (units: mm, N, °C).

Variable		Distribution	μ	V	Ref.	X_k
A_1 (air) ^a	Paris Law parameter of stage I	Lognormal	4.80×10^{-18}	1.70	[1]	$\mu(1+2V)$
A_2 (air) ^a	Paris Law parameter of stage II	Lognormal	5.86×10^{-13}	0.60	[1]	$\mu(1+2V)$
m_1 (air)	Slope value of stage I	Deterministic	5.10	–	[1]	μ
m_2 (air)	Slope value of stage II	Deterministic	2.88	–	[1]	μ
ΔK_0 (air)	Threshold value for ΔK	Lognormal	140	0.40	[23]	$\mu(1-2V)$
A_1 (marine) ^a	Paris Law parameter of stage I	Lognormal	5.37×10^{-14}	1.10	[1]	$\mu(1+2V)$
A_2 (marine) ^a	Paris Law parameter of stage II	Lognormal	5.67×10^{-7}	0.16	[1]	$\mu(1+2V)$
m_1 (marine)	Slope value of stage I	Deterministic	3.42	–	[1]	μ
m_2 (marine)	Slope value of stage II	Deterministic	1.11	–	[1]	μ
ΔK_0 (marine)	Threshold value for ΔK	Deterministic	0.0	–	[1]	μ
a_o	Initial crack depth ^d	Lognormal	0.15	0.66	[24,25]	μ
a_o/c_o	Initial aspect ratio ^d	Lognormal	0.62	0.40	[24,25]	μ
B	Plate thickness	Uniform	$\mu(1 \pm 0.05)$	–	[26]	μ
σ_{res}	Residual stresses ^c	Lognormal	300	0.20	EO ^f	μ
σ_y	Yield stress ^c	Lognormal	381	0.07	[26]	355
R	Resistance fracture toughness	Lognormal	1.7	0.18	[18]	1
K_{mat}	Fracture toughness	Weibull	Eqs. (24) + (25)	–	[21]	Eq. (8) with $P = 0.05$
T_k	Scatter in the Charpy versus fracture toughness correlation	Normal	0	$\sigma = 15$	[21]	25
C_{load}	MU fluctuating load model ^b	Lognormal	1.0	– ^e	EO	μ
C_{glob}	MU global stress model	Lognormal	1.0	0.10	EO	μ
C_{scf}	MU stress concentration	Lognormal	1.0	0.20	EO	μ
C_{sif}	MU stress intensity factor (hand calculation or 2D-FEM)	Lognormal	1.0	0.20	EO	μ
C_{sif}	MU stress intensity factor (3D-FEM)	Lognormal	1.0	0.07	EO	μ
T	Operating temperature	Normal	– ^e	– ^e	–	$\mu(1-2V)$
$\hat{\sigma}$	Maximum value of the fluctuating stress	Weibull	– ^e	– ^e	–	$\mu(1+2V)$

^a Stress ratio $\sigma_{min}/\sigma_{max} \geq 0.5$.

^b MU = Model Uncertainty.

^c Values are provided for S355.

^d Values are for an as-welded structure.

^e Case-dependent value.

^f EO = Expert Opinion.

The stress ranges $\Delta\sigma$ (double amplitude) follow a Rayleigh distribution with a mean of $2.5\sigma_r$. The probability density function may be written as

$$f_{\Delta\sigma}(x) = \frac{x}{4\sigma_r^2} \exp\left(-\frac{x^2}{8\sigma_r^2}\right) \quad (27)$$

The values for the case-specific parameters considered in this example are listed in Table 2.

4.2. Fatigue life and fracture calculation

Fig. 7 presents the crack growth and the failure assessment diagram determined with a deterministic approach. Two cases are considered: one with mean values for all random variables and one with a ‘design set’ of random variables, defined according to the 4th and 7th column of Table 1, respectively. The thick curves in the left-hand graph are valid for failure associated with fracture, and the dashed curves are valid for $a_{cr} = B$. The figure shows that

the number of cycles to failure, N_{cr} , in case of fracture is almost equal to N_{cr} of a through-thickness crack in the case of the average set of variables. For this case, the difference in N_{cr} between the two failure criteria is less than 2%. Thus, N_{cr} appears to be insensitive to the failure criterion (either a through-thickness crack or fracture) in the case of the average set of variables. On the contrary, the figure shows that the failure criterion is important in the case of the ‘design set’ of variables.

The probabilistic model is programmed in Fortran, using FORM to determine the reliability index β (Ref. Appendix A), where

$$P_f = \Phi(-\beta) \quad (28)$$

Five cases are considered: the critical crack depth assumed to be: (1) equal to B ; (2) equal to $0.5 B$; (3) equal to $0.2 B$; (4) associated with fracture applying the lower bound solution (lb); (5) associated with fracture applying the upper bound solution (ub). The value of the crack depth at fracture in the 4th case appears to be approximately $a_f \approx 0.6 B$. In all cases, the Paris law parameters A

Fig. 5. Geometry considered in the example.

Fig. 6. Fatigue load model.

Table 2

Values for case-specific parameters considered in the example (units: mm, N, °C).

Parameter		Value
B	Plate thickness	40
W	Plate width	800
L	Node length	2000
p	Thickness exponent	0.80
μ_r	Mean value of the Gaussian stress process	50
σ_r	Standard deviation of Gaussian stress process	40
σ_{perm}	Permanent stress	100
	Steel grade	S355
T_{27J}	Temperature corresponding to a CVN of 27 J	–50
μ_T	Mean value of the operating temperature (Gauss distribution)	10
σ_T	Standard deviation of the operating temperature (Gauss distribution)	8
V_{load}	Coefficient of variation of the uncertainty with respect to the load model	0.10

Fig. 7. Result of the deterministic calculation (a) Crack growth, (b) Failure assessment diagram.

and model uncertainties C are considered to be fully correlated in depth and in width direction. Fig. 8 presents the resulting value of β as a function of the number of cycles.

The figure indicates that the lower bound and the upper bound solutions of the fracture criterion give the same failure probability. A larger parametric study with other values for T_{27J} , T , σ_r and σ_{perm}

has been carried out. In all cases, the difference in β between the lower and upper bound solutions was negligible. Hence, it is sufficient to only consider the lower bound solution – which is considerably easier to program than the upper bound solution.

Further, the figure shows that the failure criterion has almost no effect on β . This is attributed to the progressive crack growth

Fig. 8. β as a function of N for various failure criteria.

(Fig. 7) where the number of cycles between $a = 0.2 B$ and $a = B$ is only 20 % of the total number of cycles between $a = a_0$ and $a = 0.2 B$. Based on the parametric study with other values for T_{27J} , T , $\Delta\sigma$ and σ_{perm} , the relative insensitivity to the failure criterion appears to be typical for the entire fatigue life of a welded joint, provided that the critical crack depth at fracture is larger than approximately $0.2B$. This is generally the case if the following conditions are satisfied:

- The fluctuating stress is relatively large (we considered cases with $\sigma_r \geq 10 \text{ N/mm}^2$).
- The permanent stress is not larger than half the yield stress, in agreement with the findings in [15].
- The temperature difference between the minimum operating temperature T_{min} and the Charpy transition temperature at 27 J is sufficiently large, so that the structure is in the transition range or upper-shelf range. The required temperature difference is: $(T_d - T_{27J}) \geq 40^\circ\text{C}$ for $40 \text{ mm} < B \leq 100 \text{ mm}$ and $(T_d - T_{27J}) \geq 20^\circ\text{C}$ for $B \leq 40 \text{ mm}$.

These conditions are satisfied for most civil engineering structures. For these conditions, and for these conditions only, a reasonable estimate of the failure probability is obtained by considering the criterion $a_{cr} = B$. If these conditions are not met, the fracture criterion needs to be considered in determining the failure probability.

The FORM sensitivity factors α resulting from the analysis indicate the relative importance of the random variables at the design point (Ref. Appendix A). The crack growth parameter A and the model uncertainties C appear to be the most important variables. The sensitivity factors for A , C_{load} , C_{glob} , C_{scf} and C_{sif} are $\alpha = 0.5$, 0.3 , 0.3 , 0.5 and 0.5 , respectively.

The relationship between β and N in Fig. 8 can be compared with the number of cycles to failure according to the deterministic sets of random variables (vertical lines in Fig. 8) in order to determine the failure probability for these deterministic sets. As an example, the deterministic analyses with the average variables gives a failure probability of $P_f = 0.38$. The reason that the average set does not result in $P_f = 0.5$ is due to the fact that not all random variables have a Gauss distribution.

Partial factors are introduced which allow the designer to assess the structure with the deterministic model while meeting the required reliability index. The partial factors are seen as reflecting both the level of uncertainty in a calculation as well as the level of consequences of failure. The procedure followed in this article allows for calibrating partial factors. The partial factor for fatigue γ_{fat} is derived with which Eq. (3) should be multiplied:

$$\begin{aligned} \Delta K_a &= \gamma_{fat} \cdot (\Delta\sigma_m Y_{ma} + \Delta\sigma_b Y_{ba}) \sqrt{\pi a} \\ \Delta K_c &= \gamma_{fat} \cdot (\Delta\sigma_m Y_{mc} + \Delta\sigma_b Y_{bc}) \sqrt{\pi a} \end{aligned} \quad (29)$$

Note that separate partial factors are usually defined for the load and for the resistance side. For simplicity, only one overall partial factor γ_{fat} is used here which covers scatter in the load and in the resistance. The values for the partial factor in Eq. (29) are determined with the deterministic model using the 'design set' of variables and applying the failure criterion $a_{cr} = B$. The number of load cycles to failure for the case $\gamma_{fat} = 1.0$ according to this deterministic model is $N_{cr} = 1.4 \times 10^5$ (Fig. 7). Fig. 8 indicates that the corresponding reliability index is equal to $\beta = 1.1$. A deterministic calculation can be made with a certain value for γ_{fat} , resulting in a certain value for N_{cr} . Subsequently, the reliability index corresponding with this value can be subtracted from Fig. 8. This allows for establishing the partial factor for a certain required reliability index. As an example, analyses with the model for the geometry and loading parameters in this chapter result in $\gamma_{fat} = 1.2, 2.0, 2.6$ and 3.1 for $\beta = 1.5, 3.1, 3.8$ and 4.3 , respectively.

5. Partial factors for fatigue LEFM

Failure probabilities and partial factors for fatigue of welded details are derived with the procedure described in Section 4.2. The applied failure criterion is the attainment of a through-thickness crack $a_{cr} = B$. The values for γ_{fat} should be applied according to Eq. (29). The values for γ_{fat} are only valid for an initial weld defect in an as-welded structure – with distributions for the defect size according to Table 1 – and design values for all variables. In total, 48 combinations of geometries and load spectra are considered according to Table 3 and Fig. 9. The combinations considered are expected to cover most welded details in civil engineering structures.

For structures in reliability class 2, the European standard EN 1990 recommends a target value of β for the entire fatigue life ranging from 1.5 to 3.8, depending on the degree of inspectability, reparability and damage tolerance.

The number of cycles to failure varies considerably between the considered cases. However, the variation in partial factors for all combinations of geometry and load spectrum appears to be small: V is between 0.01 and 0.04. This means that the partial factor is

Table 3

Geometric variables considered for deriving partial factors for fatigue LEFM.

Variable	Values
B	10 mm, 25 mm, 40 mm
W	800 mm
L	25 mm, 40 mm, 60 mm, 2000 mm
θ	45°

Fig. 9. Probability density functions of load spectra considered for deriving partial factors for fatigue LEFM.

Table 4

Partial factors γ_{far} for fatigue LEFM for as-welded structures (distributions according to Table 1, deterministic values according to Table 3) as a function of the reliability index for the entire fatigue life.

V_{Cloud}	V_{Cglob}	V_{Cscf}	β P_f	1.5	3.1	3.8	4.3
				7×10^{-2}	10^{-3}	7×10^{-5}	10^{-5}
0.0	0.1	0.2	0.2	1.1	2.0	2.6	3.1
0.0	0.1	0.2	0.07	1.0	1.7	2.1	2.5
0.0	0.1	0.1	0.07	1.0	1.5	1.8	2.0
0.1	0.1	0.2	0.2	1.1	2.1	2.7	3.3
0.1	0.1	0.2	0.07	1.1	1.8	2.3	2.7
0.1	0.1	0.1	0.07	1.0	1.5	1.9	2.2
0.2	0.1	0.2	0.2	1.2	2.3	3.1	3.9
0.2	0.1	0.2	0.07	1.2	2.1	2.7	3.2
0.2	0.1	0.1	0.07	1.1	1.9	2.3	2.7

relatively independent of the geometry and load spectrum. Since the considered geometries and load spectra differ significantly, it is expected that the derived partial factors are generally applicable. The resulting partial factors as a function of β are provided in Table 4, for various sets of coefficients of variations for the model uncertainties. These factors serve as an example: In the authors' view, the main contribution of this paper is the probabilistic model that allows for deriving partial factors. The model can also be used to determine separate partial factors for the load and resistance side, to determine the inspection interval, and to determine the partial factors in case of the residual fatigue life of structures after inspection. In the latter case, the average value and coefficient of variation of the initial crack dimensions a_o and c_o should be based on the probability of detection curve of the inspection technique. Note that the failure criterion – either $a_{cr} = B$ or fracture – may influence the residual life in that case and should thus be incorporated into the model as shown in Sections 3 and 4.

6. Conclusions

This paper presents a probabilistic linear elastic fracture mechanics model to be used for fatigue loaded civil engineering structures. The application of the model on a bridge detail is presented, and partial factors are derived. The following conclusions result from the work:

1. Model uncertainty factors appear to be important in the assessment of the failure probability. Thus, it is important to account for model uncertainties. In addition, the design life can be increased if it can be demonstrated that the model uncertainties are smaller than assumed in this paper.
2. A lower bound and an upper bound solution are provided for the failure probability in case of fatigue crack growth followed by fracture. These solutions give almost equal failure probabilities for the distribution functions of variables considered in this paper. Hence, it is sufficient to only consider the lower bound solution – which is considerably easier to program.
3. The failure probability is relatively insensitive to the failure criterion ($a_{cr} = B$ or a_{cr} is associated with fracture) for the entire fatigue life of most civil engineering structures. This justifies the fact that the fracture criterion is not explicitly accounted for in standards based on $S-N$ curves.
4. For civil engineering structures, a partial factor ≥ 1.0 needs to be applied in order to meet the required reliability for fatigue of a civil engineering structure for all values of the reliability index β recommended in EN 1990.
5. Partial factors for fatigue LEFM are derived for various values of β . These partial factors appear to be insensitive to the detail geometry and the load spectrum.

Appendix A. The system analysis by Hohenbichler and Rackwitz [22]

In the First Order Reliability Method (FORM), the limit state function is linearised in the design point. This design point is the point on the curve $g(X) = 0$ with the highest probability density. The design point is determined using a numerical iteration procedure. FORM uses a set of independent standard Gauss variables u . The physical variables X are obtained by transformation

$$X = \mu_X + u\sigma_X \quad (30)$$

$$F_X(x) = \Phi(u) \quad (31)$$

The linearised limit state function can be written as

$$g_L = B + A_1 u_1 + A_2 u_2 + \dots \quad (32)$$

Since $\mu(g_L) = B$ and $\sigma(g_L) = \sqrt{\sum_i A_i^2}$, the reliability index and failure probability can be obtained through

$$\beta = \frac{\mu(g_L)}{\sigma(g_L)} = \frac{B}{\sqrt{\sum_i A_i^2}} \quad (33)$$

$$P(g < 0) \approx P(g_L < 0) = \Phi(-\beta) \quad (34)$$

Dividing g_L by $\sqrt{\sum_i A_i^2}$ gives

$$g_L^* = \beta + \alpha_1 u_1 + \alpha_2 u_2 + \dots \quad (35)$$

where $\alpha_i = \frac{A_i}{\sqrt{\sum_i A_i^2}}$.

The design point values for the random variables are

$$u_{d,i} = -\alpha_i \beta \quad (36)$$

$$X_{d,i} = F^{-1} \Phi(u_{d,i}) = F^{-1} \Phi(-\alpha_i \beta) \quad (37)$$

The method of Hohenbichler is used in this paper to determine the failure probability of a system consisting of two limit state functions. In the case of a series system

$$P(F) = P\{g_a < 0 \cup g_b < 0\} \quad (38)$$

where we assume that g_a and g_b are Gauss distributed. The failure probability is rewritten according to

$$P(F) = P\{g_a < 0\} + P\{g_b < 0\} - P\{g_a < 0\} \cdot P\{g_b < 0 | g_a < 0\} \quad (39)$$

For the derivation below, g_a and g_b should be selected such that $\beta_a > \beta_b$. The problem is reduced to determining the conditional probability $P\{g_b < 0 | g_a < 0\}$. This conditional probability depends on β_a , β_b and correlation coefficient ρ_{ab} .

$$\rho_{ab} = \sum_{i=1}^n \alpha_{ai} \cdot \alpha_{bi} \cdot \rho_{abi} \quad (40)$$

where α_{ai} is α of X_i in g_a ; α_{bi} is α of X_i in g_b and ρ_{abi} is the correlation of variable X_i in element g_a and the same variable in element g_b . Values for β_a , β_b , α_{ai} , α_{bi} follow from the FORM analysis. Limit state functions g_a and g_b are now written as

$$g_a = \beta_a - u \quad (41)$$

$$g_b = \beta_b - v \quad (42)$$

where u and v are standard Gauss distributed. Since β_a and β_b are constant: $\rho(uv) = \rho_{ab}$. Let us replace the dependent Gauss variables u and v by independent Gauss variables u and w . Limit state function g_b can then be written as

$$g_b = \beta_b - \rho_{ab} u - w \sqrt{1 - \rho_{ab}^2} \quad (43)$$

The conditional probability is written as

$$P\{g_b < 0 | g_a < 0\} = P\left\{\beta_b - \rho_{ab}u - w\sqrt{1 - \rho_{ab}^2} < 0 | \beta_a - u < 0\right\} \quad (44)$$

Hohenbichler proposed replacing this conditional probability by the following unconditional probability

$$P\{g_b < 0 | g_a < 0\} = P(g'_b < 0) = P\left\{\beta_b - \rho_{ab}u' - w\sqrt{1 - \rho_{ab}^2} < 0\right\} \quad (45)$$

where u' is the original variable u in which is considered that $u > \beta_a$.

$$u' = \Phi^{-1}\{1 - p \cdot \Phi(u)\} \quad (46)$$

$$p = P\{u > \beta_1\} \quad (47)$$

The limit state function g'_b can now be written as:

$$g'_b = \beta_b - \rho \cdot \Phi^{-1}\{1 - p \cdot \Phi(u)\} - w\sqrt{1 - \rho_{ab}^2} \quad (48)$$

The failure probability $P\{g'_b < 0\}$ can be determined using the standard FORM.

In this paper, we have used the method of Hohenbichler to determine the failure probability in the case of a series system of more than two elements. As an example, in the case of a series system with three elements g_a , g_b and g_c , the following steps are carried out:

- Determine the combined failure probability of the first two elements g_a and g_b using the procedure of Hohenbichler described above.
- Determine the values for α_i and β of the combined limit state function.
- Use the procedure of Hohenbichler to determine the combined failure probability of the two elements (g_a and g_b) and g_c .

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