Active damping of electric drive train oscillations during ABS-braking

Citation for published version (APA):

Document status and date:
Published: 01/01/2014

Document Version:
Publisher’s PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:
• A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher’s website.
• The final author version and the galley proof are versions of the publication after peer review.
• The final published version features the final layout of the paper including the volume, issue and page numbers.

Link to publication

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

• Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
• You may not further distribute the material or use it for any profit-making activity or commercial gain
• You may freely distribute the URL identifying the publication in the public portal.

If the publication is distributed under the terms of Article 25fa of the Dutch Copyright Act, indicated by the “Taverne” license above, please follow below link for the End User Agreement:
www.tue.nl/taverne

Take down policy
If you believe that this document breaches copyright please contact us at:
openaccess@tue.nl
providing details and we will investigate your claim.
Active damping of electric drive train oscillations during ABS-braking

G Koudijs (0792365)
DC 2014.059

Traineeship report

Coach: M.Sc. J. Dünberger (Daimler AG)
Supervisor: Prof dr. H. Nijmeijer (TU/e)

Daimler AG
Department RD/FFD
71059 Sindelfingen (Germany)

Eindhoven University of Technology
Department of Mechanical Engineering
Dynamics and Control Group

Oscillations are a common problem in electric drivetrains. Compared to a conventional internal combustion engine drivetrain, electric drivetrains carry less damping. The drivetrain eigenfrequencies can be excited by several inputs. Since experience tells that the biggest problems occur during ABS braking, this report focuses on ABS initiated oscillations. Another phenomena making electric drive trains more vulnerable for oscillations, is that during braking no clutch can be applied to reduce the inertia. The increased inertia causes bad operation of the ABS-controller, which is designed for a low inertia drive train. Hence, the braking distance increases and high shaft torques occur. To investigate the problem, the drivetrain is modelled in MATLAB Simulink. It is a central drive configuration: one electric motor drives two wheels via a differential. The model includes shaft flexibilities, damping and some backlash. The drivetrain model is added to a full vehicle model provided by Daimler AG. Since the full vehicle model is a complex, non-linear model, pre-analyses have been done with a simpler linear model, which represents the full vehicle model accurately. The system behaviour is highly dependent on the tire damping, which is mainly influenced by the vehicle speed and tire slip. Two main eigenfrequencies can be distinguished in the side shaft moment response. The most dominant is the eigenfrequency of the electric motor inertia to the rim in the slip dependent range of 8-20Hz. The other is the rotational eigenfrequency of the tire in the 30-60Hz range. While braking this eigenfrequency is less dominant, hence damping of the eigenfrequency of the electric motor inertia is the main objective. In comparison with a single wheel drive configuration, the central drive configuration includes a differential. The differential averages the accelerations left and right, reducing the acceleration of the electric motor inertia and hence shaft moments. However, still high shaft torques can occur. For active oscillation control, no fundamental difference exists between the single wheel drive and the central drive configuration. Only the average of left and right wheel speed has to be taken instead of one wheel speed. Several controllers to actively control drivetrain oscillations have been investigated. A controller on rotational velocities is used since it is the simplest way of controlling. Torque observer controllers need more detailed knowledge of the system parameters. A control method only using the motor speed will per definition increase the peak step response and is therefore not feasible. Hence the $\Delta \omega$-controller is selected, which controls the motor moment such that the drivetrain inertia follows the wheel speed. Time delays have a significant influence on the controller performance. Delay on the motor speed signal reduces mainly the maximum achievable damping. Delay on the wheel speed signal increases the peak step response. For bigger time delays, the controller gain has to be selected smaller to achieve maximum damping. But this will increase the peak step response. Filtering with a forward low-pass filter adds significant delay compared to the maximum allowable time delay for controller performance. Alternative low delay filtering methods could be part of future research. The motor speed signal and wheel speed signal should be filtered with the same filters. Different filtering methods will reduce damping and increase the peak step response. Validation is done by full vehicle simulations. Under all simulated environmental circumstances the controller is effective. The controller effectiveness is reduced for increasing time delays.
Contents

Notation

1 Introduction
   1.1 Background .......................................................... 3
   1.2 Problem definition .................................................. 3
   1.3 Goal and research questions ........................................ 3
   1.4 Method and contents ............................................... 4

2 Modelling
   2.1 Drivetrain ............................................................. 5
   2.2 Tire and vehicle ...................................................... 7
   2.3 Time delay ............................................................ 9
   2.4 Signal filtering ....................................................... 9
   2.5 Summary .............................................................. 10

3 Analysis
   3.1 Problem illustration ................................................ 11
   3.2 Linearisation ........................................................ 11
   3.3 System analysis ...................................................... 13
   3.4 Influence of the differential ...................................... 15
   3.5 Summary .............................................................. 16

4 Control of the linear system
   4.1 State of the art ....................................................... 18
   4.2 Proportional controller on motor speed .......................... 19
   4.3 Proportional controller on speed difference ..................... 19
   4.4 Proportional controller on speed difference with motor moment feedback ........................................ 20
   4.5 PD controller on speed difference ................................ 21
Notation

Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Units</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_{N_{l/r}} )</td>
<td>[N]</td>
<td>variable</td>
<td>Tire normal force</td>
</tr>
<tr>
<td>( F_{x_{l/r}} )</td>
<td>[N]</td>
<td>variable</td>
<td>Tire contact patch force</td>
</tr>
<tr>
<td>( HP(s) )</td>
<td>[-]</td>
<td>Transfer</td>
<td>High-pass filter function</td>
</tr>
<tr>
<td>( J_{EM} )</td>
<td>[kgm^2]</td>
<td>0.075</td>
<td>Inertia of electric motor</td>
</tr>
<tr>
<td>( J_{cp} )</td>
<td>[kgm^2]</td>
<td>0.75</td>
<td>Inertia of the ring (contact patch)</td>
</tr>
<tr>
<td>( J_{rim} )</td>
<td>[kgm^2]</td>
<td>1.0</td>
<td>Inertia of the rim</td>
</tr>
<tr>
<td>( K_m )</td>
<td>[Nms/rad]</td>
<td>variable</td>
<td>Controller feedback gain motor speed</td>
</tr>
<tr>
<td>( K_1 )</td>
<td>[Nms/rad]</td>
<td>variable</td>
<td>Controller feedback gain ( \Delta \omega )</td>
</tr>
<tr>
<td>( K_2 )</td>
<td>[-]</td>
<td>variable</td>
<td>Controller feedback gain motor moment</td>
</tr>
<tr>
<td>( LP(s) )</td>
<td>[-]</td>
<td>Transfer</td>
<td>Low-pass filter function</td>
</tr>
<tr>
<td>( M_{EM} )</td>
<td>[Nm]</td>
<td>variable</td>
<td>Electric motor output moment</td>
</tr>
<tr>
<td>( M_{EM,ECU} )</td>
<td>[Nm]</td>
<td>variable</td>
<td>Desired electric motor moment from ECU</td>
</tr>
<tr>
<td>( M_{EM,des} )</td>
<td>[Nm]</td>
<td>variable</td>
<td>Desired electric motor moment (with power electronics delay)</td>
</tr>
<tr>
<td>( M_{brake} )</td>
<td>[Nm]</td>
<td>variable</td>
<td>Left/right braking input moment</td>
</tr>
<tr>
<td>( M_{ss} )</td>
<td>[Nm]</td>
<td>variable</td>
<td>Moment in the input (central) shaft of the differential</td>
</tr>
<tr>
<td>( M_{ss_{l/r}} )</td>
<td>[Nm]</td>
<td>variable</td>
<td>Moment in the left/right side shaft</td>
</tr>
<tr>
<td>( M_{sw_{l/r}} )</td>
<td>[Nm]</td>
<td>variable</td>
<td>Moment in the left/right tire side wall</td>
</tr>
<tr>
<td>( M_{tire} )</td>
<td>[Nm]</td>
<td>variable</td>
<td>Left/right tire input moment</td>
</tr>
<tr>
<td>( c_{cs} )</td>
<td>[N/m/rad]</td>
<td>1.2e4</td>
<td>Central shaft stiffness</td>
</tr>
<tr>
<td>( c_{ss} )</td>
<td>[N/m/rad]</td>
<td>7.6e3</td>
<td>Side shaft stiffness</td>
</tr>
<tr>
<td>( c_{sw} )</td>
<td>[N/m/rad]</td>
<td>3.9e4</td>
<td>Tire side wall stiffness</td>
</tr>
<tr>
<td>( d_{ss} )</td>
<td>[Nms/rad]</td>
<td>20</td>
<td>Side shaft damping</td>
</tr>
<tr>
<td>( d_{sw} )</td>
<td>[Nms/rad]</td>
<td>10</td>
<td>Tire side wall damping</td>
</tr>
<tr>
<td>( i_{diff} )</td>
<td>[-]</td>
<td>3.2</td>
<td>Gear ratio of the differential</td>
</tr>
<tr>
<td>( i_{tr} )</td>
<td>[-]</td>
<td>2</td>
<td>Gear ratio of the transmission</td>
</tr>
<tr>
<td>( m_{veh} )</td>
<td>[-]</td>
<td>2300</td>
<td>Vehicle mass</td>
</tr>
<tr>
<td>( r_{dyn} )</td>
<td>[-]</td>
<td>0.32</td>
<td>Dynamic tire radius</td>
</tr>
<tr>
<td>( v_{veh} )</td>
<td>[m/s]</td>
<td>variable</td>
<td>Vehicle speed</td>
</tr>
<tr>
<td>( \beta )</td>
<td>[-]</td>
<td>( 1/\sqrt{2} )</td>
<td>Damping of high- and low-pass filters</td>
</tr>
<tr>
<td>( \delta_t )</td>
<td>[Nms/rad]</td>
<td>variable</td>
<td>Tire contact patch damping</td>
</tr>
<tr>
<td>( \Delta \omega )</td>
<td>[rad]</td>
<td>variable</td>
<td>Difference between wheel speed and motor speed on motor level</td>
</tr>
<tr>
<td>( \theta_{EM} )</td>
<td>[rad]</td>
<td>variable</td>
<td>Rotational displacement of electric motor inertia</td>
</tr>
</tbody>
</table>
θ_{EM, meas} \quad [rad] \quad \text{variable} \quad \text{Measured rotational displacement of electric motor inertia (without delay)}

θ_{cs} \quad [rad] \quad \text{variable} \quad \text{Rotational displacement of input (central) shaft of the differential}

θ_{cp_{l/r}} \quad [rad] \quad \text{variable} \quad \text{Rotational displacement tire contact patch}

dθ_{cs} \quad [rad] \quad \text{variable} \quad \text{Rotational twist of central shaft}

θ_{rim_{l/r}} \quad [rad] \quad \text{variable} \quad \text{Rotational displacement of the left/right rim}

θ_{rim_{l/r, meas}} \quad [rad] \quad \text{variable} \quad \text{Measured rotational displacement of the rim (without delay)}

θ_{ssl_{l/r}} \quad [rad] \quad \text{variable} \quad \text{Rotational displacement of the left/right side shaft}

θ_{rim_{l/r}} \quad [rad] \quad \text{variable} \quad \text{Rotational displacement of left/right rim inertia}

λ \quad [-] \quad \text{variable} \quad \text{Tire slip}

μ \quad [-] \quad \text{variable} \quad \text{Tire-road friction coefficient}

τ_{ECU} \quad [s] \quad \text{variable} \quad \text{Delay between ECU and electric motor}

τ_{EM} \quad [s] \quad 0.012 \quad \text{Electric motor time constant}

τ_{mot} \quad [s] \quad \text{variable} \quad \text{Delay on motor speed signal}

τ_{rim} \quad [s] \quad \text{variable} \quad \text{Delay on rim speed signal}

ω_{EM} \quad [rad/s] \quad \text{variable} \quad \text{Rotational speed electric motor inertia (˙θ_{EM})}

τ_{d} \quad [Nms^2/rad] \quad \text{variable} \quad \text{Time constant PD-controller}

ω_{HP} \quad [rad/s] \quad \text{variable} \quad \text{Cross-over frequency high-pass filter}

ω_{LP} \quad [rad/s] \quad \text{variable} \quad \text{Cut-off frequency low-pass filter}

### Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABS</td>
<td>Anti-lock Braking System</td>
</tr>
<tr>
<td>ECU</td>
<td>Electronic Control Unit</td>
</tr>
<tr>
<td>ICE</td>
<td>Internal combustion Engine</td>
</tr>
<tr>
<td>PSD</td>
<td>Power Spectral Density</td>
</tr>
<tr>
<td>RMS</td>
<td>Root Mean Square</td>
</tr>
<tr>
<td>P-controller</td>
<td>Proportional controller</td>
</tr>
<tr>
<td>PD-controller</td>
<td>Proportional controller with derivative action</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

1.1 Background

Daimler AG is a German automotive corporation. Part of the corporation is the most valuable premium automotive brand, Mercedes Benz. The project took place in the braking systems team of the Daimler RD/FFD department. The RD/FFD department does vehicle dynamics research and development in the early stages of the design process.

1.2 Problem definition

To reduce emissions, electric and hybrid vehicles are increasingly important. This technology gives rise to new challenges. Electric drivetrains are more sensible for drivetrain oscillations, since they are relatively weakly damped compared to a conventional ICE drivetrain. ICE drivetrains need more damping to damp the highly pulsating torque output [1]. These drivetrains have inherently more system damping, since it includes more friction elements such as the pistons, crankshaft, camshafts, etc. Moreover, these drivetrains are equipped with a clutch or torque converter. Since electric drivetrains deliver torque from standstill a clutch is unnecessary, resulting in a higher inertia during braking and hence higher shaft torques. A torque converter does not influence the inertia, but adds damping to the system.

1.3 Goal and research questions

A typical value for the first eigenfrequency of the drivetrain inertia is 10Hz [1], which is mainly excited during ABS-braking. These oscillations lead to uncomfortable vibrations in the vehicle, high mechanical stresses due to the torque oscillations and an increased braking distance. The goal is to develop a robust solution to damp the electric drivetrain oscillations. The system parameters are arbitrary chosen and do not represent a specific vehicle, however the same methodology could be applied for any electric vehicle. The investigated drivetrain configuration has one electric motor driving the front wheels via a differential. For a rear wheel driven vehicle the methodology is exactly the same. The associated research question is: how to actively damp drivetrain oscillations by controlling the electric motor torque, in an vehicle with one electric motor driving two wheels via a differential? This main question can be split up into the following sub questions:

1. What are the consequences of the drivetrain oscillations during ABS-braking?
2. Can the system be linearised to do linear analysis on the system?
3. How does the passive system behave under varying circumstances?
4. What is the influence of the differential?
5. How to actively damp the oscillations?
6. What is the influence of time delays on the system?
7. How does the developed control algorithm behave in the full vehicle model?

1.4 Method and contents

First step in the process is to develop a drivetrain model in Matlab Simulink (Chapter 2). Since this model includes the non-linear phenomena backlash and tires, the system needs to be linearised in order to be able apply linear tools (Chapter 3). Some system analysis are done and the central drive and single wheel drive configurations are compared in Chapter 3. There is more literature available for a single wheel drive configuration, and therefore it is useful to know how these configurations compare. With the acquired knowledge, different control strategies are compared whereof one is selected (Chapter 4). With the selected controller, the influence of filtering and time delays is investigated (Chapter 5). And finally, all the analysis applied to the linear system are validated with non-linear full vehicle model simulation, which includes an ABS-controller (Chapter 6).

The timeline of the research after the model was developed is depicted in Fig 1.1. The labels on the left side correspond to the level of abstraction, where the full vehicle model has the lowest level, and the root locus plots the highest level of abstraction.

![Fig 1.1: Method analysis](image-url)
Chapter 2

Modelling

In this chapter the modelling of the system is discussed. In Fig 2.1 is the drivetrain the part within the red dotted lines. This drivetrain model is added to the full vehicle model provided by Daimler AG in the Matlab Simulink environment. Because the full vehicle model is a multi-variable, non-linear model, this part has been replaced with a simpler, linear model representing the wheels and tires. The developed model is based on the model used by Rosenberger [9]. Since Rosenberger investigates a single wheel drivetrain configuration, the model is extended with a differential and a second wheel. The bearing dynamics are neglected. Components that have been added are backlash, signal delay and filtering. The state space notation of the full linear system presented in this chapter can be found in the Appendix A.

2.1 Drivetrain

Linear mechanical system

This section concentrates on modelling the part within in red dotted line of Fig 2.1. For convenience the rim inertias are included as well, resulting in a three inertia system. Hence three equations of motion can be derived. First one for the motor inertia:
\[ \dot{\theta}_{EM} = \frac{1}{J_{EM}} (M_{EM} - M_{cs}) \]  (2.1)

Where \( M_{EM} \) is the input torque from the electric motor, \( M_{cs} \) is the moment in the central shaft, which is connected to the motor via a gearbox with gear ratio \( i_{tr} \). The second and third equation of motion are for the left and right wheel:

\[ \dot{\theta}_{rim_{l/r}} = \frac{1}{J_{rim}} (M_{ss_{l/r}} - M_{brake_{l/r}} - M_{l/r}) \]  (2.2)

\( M_{ss} \) is the moment in the side shaft. The tire moment \( M_{tire} \) and braking moment \( M_{brake} \) are inputs. The central shaft is modelled as a rotational spring with backlash neglected for the moment. Hence, the moment in the central shaft can be described by:

\[ M_{cs} = c_{cs} (\frac{\theta_{EM}}{i_{tr}} - \theta_{cs}) \]  (2.3)

The rotation \( \theta_{cs} \) is the rotation of the input shaft of the differential. In the virtual central shaft stiffness \( c_{cs} \), three stiffness's are included. The stiffness of the gears in the gearbox, the stiffness of the input gear of the differential and the stiffness of the central shaft itself.

The side shafts are modeled as a spring \( c_{ss} \) and a damper \( d_{ss} \) (\( c_{sw} \) and \( d_{sw} \) are tire stiffness's which will be discussed further on). The spring represents the stiffness of the side shaft itself and the stiffness of the output gears of the differential. The damper constant represents the damping present in the system.

\[ M_{ss_{l/r}} = c_{ss} (\theta_{ss_{l/r}} - \dot{\theta}_{rim_{l/r}}) + d_{ss} (\dot{\theta}_{ss_{l/r}} - \ddot{\theta}_{rim_{l/r}}) \]  (2.4)

The central shaft is connected to the side shaft by an open differential. For such a differential three relations can be derived:

1. Kinematic relation:

\[ \dot{\theta}_{cs} = \frac{i_{diff} (\dot{\theta}_{ss_{l}} + \dot{\theta}_{ss_{r}})}{2} \]  (2.5)

2. Moment equilibrium about the input cone wheel:

\[ M_{cs} = \frac{M_{ss_{l}} + M_{ss_{r}}}{i_{diff}} \]  (2.6)

3. Moment equilibrium about the differential gear cone wheel:

\[ M_{ss_{l}} = M_{ss_{r}} \]  (2.7)

Goal is to replace the moments \( M_{cs} \) and \( M_{ss} \) in the equations of motion \( (2.1) \) and \( (2.2) \) by a function that depends only on state variables. To do this, the rotation of the central shaft \( \theta_{cs} \) is defined as extra state variable. The array of state variables is then \( [\theta_{EM}, \dot{\theta}_{EM}, \theta_{cs}, \dot{\theta}_{ss_{l}}, \dot{\theta}_{ss_{r}}, \theta_{ss_{r}}, \theta_{ss_{r}}, \theta_{ss_{r}}]^T \). With \( \theta_{cs} \) as state variable, \( (2.3) \) is described by state variables. Hence \( (2.1) \) can be described by only state variables. Because \( (2.7) \) and \( (2.6) \) can be written as

\[ M_{ss} = \frac{M_{cs} * i_{diff}}{2} \]  (2.8)

Equation \( (2.2) \) can be written only as a function of state variables. An expression for \( \dot{\theta}_{cs} \) can be derived by inserting \( (2.3) \) and \( (2.4) \) into \( (2.6) \)

\[ c_{cs} (\frac{\theta_{EM}}{i_{tr}} - \theta_{cs}) = c_{ss} (\theta_{ss_{l}} + \theta_{ss_{r}} - \dot{\theta}_{rim_{l}} - \dot{\theta}_{rim_{r}}) + d_{ss} (\dot{\theta}_{ss_{l}} + \dot{\theta}_{ss_{r}} - \ddot{\theta}_{rim_{l}} - \ddot{\theta}_{rim_{r}}) \]  (2.9)

By using \( (2.5) \) the following expression can be derived:

\[ c_{cs} (\frac{\theta_{EM}}{i_{tr}} - \theta_{cs}) = c_{ss} (\frac{2 * \theta_{cs}}{i_{diff}} - \theta_{rim_{l}} - \theta_{rim_{r}}) + d_{ss} (\frac{2 * \dot{\theta}_{cs}}{i_{diff}} - \ddot{\theta}_{rim_{l}} - \ddot{\theta}_{rim_{r}}). \]  (2.10)

By some rewriting, \( \dot{\theta}_{cs} \) can be expressed explicitly:

\[ \dot{\theta}_{cs} = \frac{c_{cs} * i_{diff}^2}{2 i_{tr} * d_{ss}} \theta_{EM} + \frac{c_{ss} * i_{diff}^2}{2 * d_{ss}} (\theta_{rim_{l}} + \theta_{rim_{r}}) + \frac{i_{diff} (\dot{\theta}_{rim_{l}} + \dot{\theta}_{rim_{r}})}{2} - \frac{(c_{cs} * i_{diff}^2)}{2 d_{ss}} + \frac{c_{ss} * d_{ss}}{d_{ss}} \theta_{cs} \]  (2.11)
Backlash

Backlash is a non-linear phenomenon. Within the backlash or while going through it, thus for small angular displacements, it is not possible to transmit torque. The backlash is modelled between the center spring and the differential (see Fig 2.1), representing the free play in the transmission gears and the input gear of the differential. Between the side shafts there is assumed to be no backlash because these gears are pre-tensioned. The model presented by Nordin, Galin and Guman \[6\] is considered. But since all shaft damping is added to the side shafts and not to the central shaft, backlash can be modelled by a simple dead-zone model. Hence, the central shaft moment Equation (2.3) will have different states:

\[
\begin{align*}
\text{Compute difference between motor shaft position and input shaft of differential; } & \\
\theta_{cs} = \theta_{EM} - \theta_{cs}; & \\
\text{Left contact; } & \\
\text{if } d\theta_{cs} > \frac{\text{backlash}}{2} & \text{then} \\
M_{cs} = c_{cs} \left( \frac{\theta_{EM}}{i_{tr}} - \theta_{cs} - \frac{\text{backlash}}{2} \right); & \\
\text{end} \\
\text{Right contact; } & \\
\text{if } d\theta_{cs} > \frac{\text{backlash}}{2} & \text{then} \\
M_{cs} = c_{cs} \left( \frac{\theta_{EM}}{i_{tr}} - \theta_{cs} + \frac{\text{backlash}}{2} \right); & \\
\text{end} \\
\text{No contact; } & \\
\text{if } |d\theta_{cs}| < \frac{\text{backlash}}{2} & \text{then} \\
M_{cs} = 0; & \\
\text{end}
\end{align*}
\]

When doing linear analysis, backlash is neglected since this is a non-linear element. Therefore backlash is not included in the state space notations in the appendices. The validity to use linear analysis will be discussed in the next chapters.

Electric motor

If a certain moment is requested from an electric motor, it is not able to deliver that instantaneously. Instead it needs some time to build that moment. This is modelled as a first order differential equation with time constant \(\tau_{EM}\).

\[
\dot{M}_{EM} = \tau_{EM} * M_{EM,des} - \tau_{EM} * M_{EM}
\]  
(2.12)

2.2 Tire and vehicle

The system discussed in the previous section represents the drivetrain with the wheels lifted from the ground. In this section the tire input force in (2.2) will be modelled. Because the drivetrain model only represents the rotational motion, only the rotational dynamics are considered.

As in Pacejka \[7\], the tire and rim are split in two parts. The inner inertia, called the rim represents the rim and the part of the tire side wall. The outer part called ring can be seen as the contact patch mass.
The rim and ring are connected by a torsional spring \((c_{sw})\) and damper \((d_{sw})\) representing the tire side wall dynamics. The moment in the tire side wall can be described by:

\[
M_{sw/l/r} = c_{sw}(\dot{\theta}_{rim/l/r} - \dot{\theta}_{cp/l/r}) + d_{sw}(\ddot{\theta}_{rim/l/r} - \dot{\theta}_{cp/l/r}).
\] (2.13)

The side wall moment \((M_{sw})\) is the tire moment \((M_{tire})\) in (2.2). For the ring the following differential equation can determined:

\[
\ddot{\theta}_{cp/l/r} = \frac{1}{J_{cp}} (M_{sw/l/r} - F_{x/l/r} \ast r_{dyn}).
\] (2.14)

Where \(F_{x/l/r}\) is the force in the contact patch and \(r_{dyn}\) the dynamic tire radius. The contact patch force is determined using the tire slip \((\lambda)\), the road friction coefficient \((\mu)\) and the normal force \((F_N)\).

\[
F_x = tireforce(\mu, F_N, \lambda)
\] (2.15)

An example how the tireforce function behaves is shown in Fig 2.3. Slip \(\lambda\) can be written as a function of the contact patch speed and vehicle speed \(v_{veh}\):

\[
\lambda = \frac{\dot{\theta}_{cp} \ast r_{dyn} - v_{veh}}{|v_{veh}|}
\] (2.16)

As mentioned before, this vehicle model is used for linear analysis only. Since the tire force is a non-linear function, it is linearised around two operation points. First the free rolling tire, and second the tire under ABS-braking. The ABS-control tries to keep the tire slip just within the zone with a positive gradient in order to keep the system stable. This operation point is assumed to be at 6% slip. Fig 2.3 shows both linearisations.

\[
F_x = c_{\lambda} \ast \lambda
\] (2.17)

Where \(c_{\lambda}\) is the gradient of the local linearised slip stiffness shown in Fig 2.3. By defining a new variable \(\delta_{t}\) as

\[
\delta_{t} = \frac{c_{\lambda}}{|v_{veh}|}
\] (2.18)

Equation (2.17) can be rewritten as:

\[
F_x = \delta_{t} \ast (\dot{\theta}_{cp/l/r} \ast r_{dyn} - v_{veh})
\] (2.19)

This means the tire force behaves in a certain operation point like a linear damper between the road and the contact patch with damper constant \(\delta_{t}\).
The vehicle is modelled as a mass that can only move in longitudinal direction. Because the research goal is the investigate drivetrain oscillations during ABS-braking in the range of 5-20Hz, the low frequent vehicle yaw is neglected in this model. Moreover, rolling resistance and air drag are neglected. Hence, the vehicle can be represented by the following simple differential equation:

$$\dot{v}_{veh} = \frac{1}{m_{veh}}(F_x + F_{x_r})$$  \hspace{1cm} (2.20)

The normal force in the tire force function ((2.15)), is dependent on the vehicle deceleration and vehicle geometry, but assumed to be constant in the linear analysis.

### 2.3 Time delay

The speed sensors have a certain communication delay. For the motor speed, the signal communication delay $\tau_{mod}$ is in the range of a few milliseconds. For the wheel speed this can be significantly more. Time delays will be further discussed in Chapter 5. Signal communication delays are for analysis included in the state space model:

$$\dot{\theta}_{EM,meas} = \dot{\theta}_{EM} e^{-\tau_{mod} s}$$  \hspace{1cm} (2.21)

$$\dot{\theta}_{rim,l/r,meas} = \dot{\theta}_{rim,l/r} e^{-\tau_{rim} s}$$  \hspace{1cm} (2.22)

The ECU has a certain computation delay. This delay is modelled on the output of the ECU and is assumed to be 1ms.

$$M_{EM,des} = M_{EM,ECU} e^{-\tau_{ECU} s}$$  \hspace{1cm} (2.23)
2.4 Signal filtering

Because signals in the real world carry noise, filtering is necessary to eliminate these disturbances. Fig 2.4 shows a comparison between a first order filter, a second order filter with damping set to 1, and a second order filter with damping set to $1/\sqrt{2}$. The second order low-pass filter with damping set to $1/\sqrt{2}$ has the best trade-off between phase lag and amplitude behaviour. When using filtering in the next chapters, a second order low-pass filter with damping set to $1/\sqrt{2}$ is used. Such a filter can be described by:

$$\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \omega_{LP}^2 u - \omega_{LP}^2 x_1 - \beta \omega_{LP} x_2 \\
y &= x_1
\end{align*}$$

(2.24)

This describes the relation between the unfiltered input signal $u$ and the output signal $y$. Variable $\beta$ is the filter damping and $\omega_{LP}$ the cut-off frequency.

![Fig 2.4: Filter comparison](image)

2.5 Summary

The drivetrain is modelled as a combination of linear springs, dampers and inertias. The only non-linear element in modelled in the drivetrain is backlash. The drivetrain model is added to the full vehicle model of Daimler AG for full vehicle simulations. Since this is a sophisticated, non-linear and multi-variable model, a simple linear representation of the full vehicle model is developed. This linear model is used for the pre-analysis. In the linear model, the tire contact patch damper stiffness is linearised around a tire operating point. This operating point is dependent on tire slip and vehicle speed. If signals need to be filtered, a second order low-pass filter with damping set to $1/\sqrt{2}$ is used, since it has the best amplitude-phase behaviour.
Analysis

In this chapter, first the problem will be illustrated with a full vehicle simulation of an ABS-braking event. After that the full vehicle model and the linear model are compared, to analyse the validity to do linear system analysis. The system behaviour is discussed with step responses, bode plots and the pole map respectively. Finally the influence of the differential is discussed, to compare the single wheel drive and the central drive configuration.

3.1 Problem illustration

As mentioned in the introduction, are the drivetrain dynamics during braking of a conventional ICE drivetrain highly influenced by the clutch or torque converter. Since electric motors deliver maximum torque from standstill, are they normally not equipped with such a device.

In manual drivetrains, normally the clutch is applied by the driver during braking. This reduces the drivetrain inertia, reducing the shaft moments. Automatic transmissions are shifted to neutral, reducing the inertia as well. In electric drivetrains is the inertia during braking is not reduced. This causes high shaft torques and bad operation of the ABS-controller, which is designed to operate with a low inertia drivetrain.

Fig 3.1 shows the full vehicle simulation results, with high drivetrain inertia and with a low inertia. With the full inertia, the ABS-controller is not able to achieve a constant wheel speed deceleration, resulting that the braking distance is increased from 40.2 meters to 41.7 meters, a difference of 1.5 meters. The side shaft moment reaches values up to 1000Nm during this braking event. From this comparison can be concluded that if it is possible to control the electric motor such that is compensates the drivetrain inertia, the problem illustrated can be solved.

3.2 Linearisation

Step response

Fig 3.2 shows the step response comparison between the non-linear simulation model and linear model. The delay at the start of the motor response in the simulation model is due to backlash, which causes more overshoot. The frequency and damping are approximately the same, but the amplitude of the simulation model is higher since it has more overshoot. The initial response in the brake steps are similar. In the
second part of the response the moment oscillates around 0Nm. This means it is running through the backlash, which causes the second part of the brake step response to be different.

Fig 3.2: Step response simulation model and linear model

Frequency response

The frequency response comparison is depicted in Fig 3.3. With the backlash neglected in the simulation model, both models have a very similar amplitude and phase behaviour when choosing the tire operation point correctly.

Based on the frequency responses can be concluded that the simulation model and linear model are very similar when choosing the tire operating point correctly. The step responses show an influence of backlash on the peak step response. This means the linear model can be used for analysis, but the presence of the non-linear backlash phenomenon should always be taken into consideration.
3.3 System analysis

Step response

The step responses are generated with the non-linear simulation model including backlash. Fig 3.4 shows the system step response on several motor and braking moment step heights. The step is activated after one second. The delay before the moment starts to rise is caused by the backlash in the model. The zero moment zones in the brake step response are due to backlash as well. Overshoot does not exactly linearly increase with the input moments. Due to backlash, it is for low amplitude step inputs the overshoot percentage higher.
Frequency response

Figure 3.5 shows the side shaft moment response for different speeds and slip operating points. For the free rolling tire, the slip stiffness is linearised around 0% slip and for ABS-braking around 6% slip. As discussed in Section 2.2, both speed and tire slip influence the tire damping dramatically. The tire damping is crucial for the dynamic drivetrain behaviour, since it determines if the energy is put into the tires or into the drivetrain.

The top left frequency response shows the side shaft moment response to small motor moment variations with a free rolling tire. It shows a peak at 10Hz, which is the rotational eigenfrequency of the motor inertia. The peak is higher for lower speeds. The lower left graph shows the side shaft moment response to small motor moment variations during ABS-braking. Effectively, the tire damping is decreased. By that, the amplitude is reduced and the eigenfrequency increased. For high speeds this peak is higher, which is inverse to the free rolling tire.

The top right Bode amplitude plot shows the side shaft moment response to symmetric brake inputs (on the left and right the same braking moment is applied). It shows two peaks. The first peak at 10Hz, which is the eigenfrequency of the motor inertia. A second peak shows up at 34Hz. This is the eigenfrequency of the wheel, which corresponds to an in-phase rotational eigenfrequency of the tire of about 30Hz mentioned by Pacek [2]. Under ABS-braking both eigenfrequencies are increased. Moreover, the motor eigenfrequency amplitude is increased and the amplitude of the wheel oscillation decreased. The decreased tire damping means less braking energy is put into the tire and more into the drivetrain. This causes the drivetrain inertia to be more dominant in the side shaft moment response.

The latter bode plot represents the conditions where the problem occurs: drivetrain oscillations initiated by ABS-braking. Clearly the most dominant oscillation frequency is 13Hz, that of the electric motor inertia resonance. This is the main resonance that should be damped.

Fig 3.5: Side shaft moment frequency response
Pole map

The pole plot of the linear system linearised for ABS-braking at 100km/h is shown in Fig 3.6. The left pole on the real axis represents the electric motor first order differential equation (2.12). The two poles on the real axis between -30 and -10 are the tire slip poles. The most relevant poles for this project are the poles of the motor inertia resonance, which should be damped. The wheel is represented by two pole sets: one of the two wheels in phase resonance and one of the two wheels in anti-phase resonance. Since (2.5) holds, this means that the anti-resonance of the wheels does not influence the shaft moments.

3.4 Influence of the differential

Mathematical analysis

Since there is more literature available for systems with one electric motor driving one wheel, it is useful to know the behaviour difference between a single wheel drive configuration and a central drive configuration. Because the only difference between them is the existence of a differential, this section will discuss the differential influence. To analyse the differential influence, the naming convention shown in Fig 3.7 will be used. First it is important to note that if the electric motor delivers no input moment, the drivetrain inertia $J$ is only causing the moments in the shaft, as can be seen from (3.1)

$$M_c = J \cdot \dot{\omega}_c$$  \hspace{1cm} (3.1)
Without differential, the central shaft acceleration $\dot{\omega}_c$ would be directly related to the side shaft acceleration. But since it concerns a differential configuration, the differential kinematic relation holds:

$$\dot{\omega}_c = \frac{\dot{\omega}_l + \dot{\omega}_r}{2} \quad (3.2)$$

This means that if one side is accelerated and the other decelerated with the same value, the motor on the central shaft will not be influenced at all. If left side is accelerated and right side hold at a constant speed, the motor is accelerated with only half of the left side acceleration. Only if both sides are accelerated with the same amount, the motor will be accelerated as much as in the case of a single wheel drive configuration. In all other cases the differential acts as a filter, averaging the side shaft accelerations to the central shaft acceleration.

Someone might suggest that although the central shaft moments are zero with an asymmetric acceleration, there might be a moment in the side shafts. But by the moment equilibrium about the central cone wheel the following two equations hold:

$$M_c = M_l + M_r \quad (3.3)$$

$$M_l = M_r \quad (3.4)$$

This means that if the moment in the central shaft is zero, the moment in the side shafts must be zero.

**Frequency response**

For the frequency response comparison, symmetric braking moments left and right are used. If only one side would be braked, the moment amplitude response would be exactly half, as discussed in the previous section.

Since left and right the same braking moments are applied, the averaging function of the differential is eliminated. Fig 3.8 shows that the frequency response of both configurations is very similar in that case. This means that the only fundamental difference between both systems is the averaging function of the differential. The stiffness’s, damping and inertia values can vary for both configurations, but are also vehicle dependent.

### 3.5 Summary

There are two main differences between an electric drivetrain and an ICE drivetrain. First the drivetrain inertia in an ICE drivetrain can be decoupled during braking and second is by definition more damping present in an ICE drivetrain. Simulations show that the drivetrain inertia results in high shaft torques and an increased braking distance.

The simulation model can be represented by a linear model. Most important difference is the backlash. The influence of neglecting backlash in linear analysis should always be taken into consideration.

The shaft moment response consists out of two eigenfrequencies. One of the electric motor inertia to the rim and one of the wheels to the ring. Under ABS-braking the eigenfrequency of the electric motor inertia is causing the highest shaft moment. Damping of these oscillations should be the main controller target.
The difference between a single wheel driven and central driven vehicle is mainly the differential. The only fundamental difference between the two configurations is, that the differential averages the accelerations from the left and right to the central shaft, to which the electric motor is attached. Since this inertia causes the moment in the shafts, this means the differential works as a filter for asymmetric inputs left and right. Hence the central driven vehicle is less sensible for drivetrain oscillations, but they still can occur. Because the systems are, besides that similar, single drive control strategies can be used. The average wheel speed left and right has to be taken in stead of only one wheel speed.

Fig 3.8: Frequency response comparison central drive and single wheel drive
Chapter 4

Control of the linear system

4.1 State of the art

Several methods are used in literature to solve the electric drivetrain oscillation problem. Amann, Böcker and Prenner [1] try to eliminate oscillations due to road disturbances by estimating the shaft moment using Kalman filters. The controller target is to bring the shaft moments to zero by controlling the electric motor moment. The controller is able to operate with and without use of the wheel speed sensors. The drivetrain consists of an electric motor driving the wheels via a differential with backlash.

Bottiglione, Somiotti and Leo [2] try to eliminate drivetrain oscillations initiated by the traction control. Again Kalman filters are used to estimate the torque that should be delivered by the electric motor. The control strategy is applied to a single wheel drive vehicle, without taking into account backlash in the system.

Drivetrain oscillations are, although less dominant, present in ICE drivetrains as well. Lohner [3] controls the clutch of an ICE drivetrain such that it dampens oscillations.

Menne [4] and Rosenberger [9] use speed feedback controllers to damp electric drivetrain oscillations in a single wheel driven vehicle without backlash. Rosenberger concentrates on ABS-braking and suggests to feedback the motor moment with a proportional constant to improve damping.

The controllers can be divided into two groups. One group has as control target to bring the side shaft moment to zero (torque observer), and an other group using the speed as controller objective. The torque observer needs proper knowledge of system parameters like backlash, which is difficult. Moreover, to let this controller work properly under ABS braking, the braking moment should be given as an input. Otherwise the controller is only using the speeds or accelerations, which has no real advantage above a controller on the speeds. But the braking moments are not accurately known, since they are strongly dependent on wear and operating temperature. Therefore there is not a significant advantage to use a torque observer. Therefore the simplest way to control the side shaft moments is selected in this report, using the speed as the controller objective.

Using the speed as controller objective, the following control strategies are investigated in this chapter:

- Proportional controller on motor speed ($\omega_{EM}$);
- Proportional controller on speed difference ($\Delta \omega$);
• Proportional controller on speed difference \((\Delta \omega)\) with motor moment feedback;
• PD controller on speed difference \((\Delta \omega)\).

The variable \(\Delta \omega\) is the difference between the electric motor speed \(\omega_{EM}\) and the wheel speeds \(\omega_{riml/r} \).

\[
\Delta \omega = \omega_{EM} - \omega_{rim} \tag{4.1}
\]

Since this report deals with one electric motor driving two wheels via a differential, the average of the left and right wheel speed has to be taken (Section 3.4).

\[
\bar{\omega}_{rim} = \frac{\omega_{riml} + \omega_{rimr}}{2} \times i_{tr} \times i_{diff} \tag{4.2}
\]

Where \(i_{tr}\) and \(i_{diff}\) are the gear ratios of the transmission and differential respectively (See Fig 2.1).

### 4.2 Proportional controller on motor speed

The first investigated possibility is only using the electric motor speed to damp the oscillations. This has a cost advantage, since no changes have to be made to the hardware. Physically this corresponds to putting a damper on the electric motor speed. The mathematical representation of this method is:

\[
M_{EM,ctrl} = -K_m \times \omega_{EM} \tag{4.3}
\]

Where \(K_m\) is the controller gain (damping constant). This strategy would mean that the electric motor is delivering an opposing moment proportional to the motor speed. At a constant speed, a constant opposing moment would be applied, which is a constant loss. To eliminate this constant moment a high-pass filter can be added so that the controller only reacts to speed changes:

\[
M_{EM,ctrl} = -K_m \times \omega_{EM} \times HP(s) \tag{4.4}
\]

Another possibility would be to use a slow wheel speed signal. But this report focuses on oscillations during braking, which means the vehicle speed and the wheel speeds are dynamically changing.

The root locus plot for this method is shown in Fig 4.1. The diagonal lines are damping lines. The poles of the passive system are depicted in black. The coloured lines show how the pole locations change for increasing controller gains. The poles of the motor oscillation are clearly more damped for an increasing gain. The motor poles become complex due to the high-pass filter. The step response for this control strategy is shown in Fig 4.2. Clearly the peak forces increase. Since this controller is always delivering a moment opposing the speed changes of the electric motor, increasing peak forces is inherent to this controller. Because increasing peak forces can cause damage to the mechanical components, this controller is not a feasible solution.

### 4.3 Proportional controller on speed difference

Another strategy is controlling the motor speed to follow the wheel speed. This physically corresponds to putting a damper between the wheel and the electric motor, or mathematically:

\[
M_{EM,ctrl} = -K_1 \times \Delta \omega \tag{4.5}
\]

Fig 4.3 shows the root locus plot for this controller. Left in case of an ideal electric motor where the moment is put instantaneously on the motor inertia, and on the right in case of a non-ideal electric motor, where the moment is build via a first order differential equation (2.12). For the ideal electric motor, complete damping of the motor oscillation is possible. But as mentioned by Rosenberger \[9\], is the maximum possible damping with in the case of a non-ideal electric motor limited.
Fig 4.1: Root locus of a proportional controller on $\omega_{EM}$

Fig 4.2: Step response of a proportional controller on $\omega_{EM}$

### 4.4 Proportional controller on speed difference with motor moment feedback

Rosenberger [9] proves that the damping can be improved by adding motor moment feedback. The motor moment is not measured, but can be accurately estimated. Fig 4.4 shows the schematic overview of this control strategy. The output motor moment can be written as:

$$M_{EM,ctrl} = -K_1 \Delta \omega - K_2 G_{EM} * M_{EM,ctrl} \quad (4.6)$$

Since there is motor moment feedback, this equation gives the desired motor moment from the controller ($M_{EM,ctrl}$) implicitly. This can be rewritten into an explicit expression:

$$M_{EM,ctrl} = \frac{-K_1}{1 + K_2 G_{EM}} \Delta \omega \quad (4.7)$$
Where the transfer function of the electric motor is in accordance with (2.12).

\[ G_{EM} = \frac{1}{\tau_{EM} * s + 1} \] (4.8)

Putting this into (4.7) results in the following expression that can be rewritten.

\[ M_{EM,ctrl} = \frac{-K_1 * (\tau_{EM} * s + 1)}{\tau_{EM} * s + 1 + K_2} * \Delta\omega = \frac{-K_1}{\frac{K_2 + 1}{\tau_{EM} * s + 1}} * \Delta\omega \] (4.9)

From this equation can be concluded that this controller places a zero at the motor pole. Another pole is added, whose location can be tuned by the motor feedback controller value \( K_2 \). This pole moves to the left with increasing \( K_2 (\tau_{EM}/(K_2 + 1) \) becomes smaller). Since the goal in this project is to keep the controller as simple as possible, it is decided to investigate this control strategy by only putting a zero at the motor pole. This corresponds to an PD-controller and will be discussed in the next section. The advantage of this is, that only one control value should be tuned instead of two.

### 4.5 PD controller on speed difference

As depicted in Fig. 4.3 the motor time constant limits the maximum achievable damping. A PD-controller which puts a zero at the motor pole compensates for the motor time constant. The mathematical
description of this control strategy is:

\[ M_{EM,ctrl} = K_1(\tau_d * s + 1) * \Delta \omega \]  

(4.10)

When \( \tau_d \) is set to \( \tau_{em} \), the system is equivalent to the case of the ideal electric motor in Section 5.3, which means 100% damping is possible. The root locus plot of the P-controller and PD-controller are shown in Fig 4.5. The root locus plot in Fig 4.5 represents the system at 100km/h and 6% slip. Both speed and slip influence the same parameter \( \delta_t \), as discussed in Section 2.2. The root locus plot for varying \( \delta_t \) can be found in Appendix [B]. From the plots in the appendix it can be concluded that the controller in all cases improves stability of the system (poles move to the left). The stability of the tire to ring oscillation should be controlled by the ABS-controller, as long as these are not negatively influenced by the PD-controller.

Someone might suggest to decrease or increase \( \tau_d \) for better performance. However, from Fig 4.6 can be concluded that this might not be always the case, because the amplitude of the 10-20Hz oscillation is increasing in both cases. Therefore the PD-controller discussed in this report will use \( \tau_d = \tau_{EM} \)

### 4.6 Controller selection

A controller only using the motor speed is not feasible since it increases peak forces. Therefore a \( \Delta \omega \) controller is needed. \( \Delta \omega \) is the difference between the wheel speed and motor speed. Is tries to control the motor speed in such a way that it follows the wheel speeds. In the perfect case, the moments in the shaft will be completely eliminated. The torque observer controller has only limited value without feed-forward of the braking moment, which is not accurately known.

For an ideal electric motor (instant torque), a proportional controller on \( \Delta \omega \) is enough to achieve 100% damping. However for a non-ideal electric motor the maximum achievable damping is limited. Adding motor moment feedback can improve the possible damping, but it is less effective compared to an ideal PD-controller, since motor feedback adds a new pole. Therefore the PD-controller is selected.
4.7 Influence of backlash on selected controller

All the analysis in this chapter were performed on the linear system. But as concluded in the previous chapter, backlash should always be considered since it has an influence on the system. The selected control strategy is set up to control the motor speed in such a way that it follows the wheel speed. Braking means an disturbance on the wheel side. Backlash increases the latency between the wheel speed change and the motor speed change. This means backlash gives more time to control the speed difference to zero, without generating any shaft moment. Therefore in this case backlash has some positive influence on the controller performance, as is validated with the simulations shown in Fig 4.7. In the simulations, a brake step is put on the system. The peak step response is in all cases with the gain $\neq 0$ lower in the system with backlash.

Fig 4.7: Influence of backlash on the brake step response for different controller gains
Chapter 5

Time delay and filtering

In most available literature, ideal signals (no filtering) and negligible time delays are assumed. But the communication of the signals will take time and the signals will carry noise making filtering necessary. The influence of time delay and filtering will be investigated in this chapter. The PD-controller discussed in the previous chapter is applied. When mentioning damping, the damping of the motor to rim poles are meant. When discussing the peak step response, the peak in the unit brake step response is meant, as shown in Fig 4.7.

5.1 Time delay

Composition of time delay

The time delay consists of several components.

- Sensing delay
- Communication delay
- Filtering delay
- ECU computation delay
- Power electronics delay

To understand the sensing delay, first it is important to know how the rotational speed generally is measured. A certain amount of pulses is generated during a rotation. By measuring the time between those pulses, the speed is calculated. That means that the speed signal per definition has a delay of half the time between the pulses. The sample time increases with decreasing vehicle speed. Because the motor is rotating at a higher frequency due to the transmission ratios, the delay on the motor speed signal is assumed to be negligible. The wheel speed signal generates 96 pulses per rotation. For instance, at 10 km/h the time between the pulses would be approximately 7ms, meaning a sensing delay of 3.5ms.

The communication delay is the time, that the signal needs to be sent from the sensor to the ECU. Since the motor speed sensor is directly coupled to the ECU, this delay will be minor. But for the wheel speed signal this delay can be significant, dependent on the vehicle architecture. Filtering adds delay to the signals, as will be discussed later in this chapter. Since there is noise on the signals, filtering is necessary. For a forward low-pass filter with a cut-off frequency of 50Hz, this delay is about 4-5ms. The computation
delay is the time between the signal arrives at the ECU, and that the calculated control effort leaves the ECU. This delay is assumed to be equal the the task time of the ECU, which is set to 1ms. The power electronics delay is the time from the ECU until the moment starts to build on the electric motor inertia. Since power electronics are very fast, this delay is assumed to be negligible.

An example of the composition of the time delays is shown in Fig 5.1. The values are dependent on vehicle speed and the vehicle architecture. They should be properly defined in the design process of a new vehicle. Important is that the sum of all these delays is the total time delay which will be discussed in this chapter.

![Fig 5.1: Composition of time delays](image)

**Symmetric time delay**

In this section equal (symmetric) time delay on the motor speed and wheel speed signal is assumed. The damping and peak step response as function of the controller gain for different delays are depicted in Fig 5.2. For minimal delay (1ms), it is possible to achieve 100% damping and even increase the gain further creating an over-damped system, without the system becoming unstable. But with increasing delay, the maximum achievable damping decreases. Moreover, the gain at which maximum damping is achieved becomes smaller. With 6ms delay the maximum achievable damping is 53% at a gain of 5. With 6ms time delay the maximum achievable damping is only 24% at a gain of 2.5. The peak step response is only limited influenced by the time delays, but reduced by increasing the controller gain. As mentioned before, maximum damping is achieved at smaller gains for bigger time delay. Hence, the peak step response is indirectly influenced by increasing the time delay.

**Asymmetric time delay**

There might be different delays on the motor speed and wheel speed signal. Since the motor speed sensor is directly coupled to the ECU, the delay on the wheel speed is assumed to be equal or higher compared to the motor speed signal. Fig 5.3 shows the damping and peak step response as function of the controller gain for varying delays on the wheel speed signal. Time delay on the wheel speed signal reduces damping and reduction of the peak step response. In case of 1ms delay on motor speed and 11ms on wheel speed, the peak step response will not decrease any more for increasing gains. For bigger time delay the peak step response even increases.

**Handling asymmetric delays**

A phenomena not discussed so far, is that if the wheel speed has more delay compared to the motor speed during constant deceleration, the wheel speed is measured too high. This means a constant offset in $\Delta \omega$. 

<table>
<thead>
<tr>
<th>Delay on motor speed</th>
<th>Delay on wheel speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensing delay (10km/h)</td>
<td>Filtering delay (50Hz low-pass filter)</td>
</tr>
<tr>
<td>Task rate ECU</td>
<td>Communication delay</td>
</tr>
</tbody>
</table>
which the controller tries to compensate by increasing the motor speed delivering positive (driving) moment. This is an unwanted phenomena during braking, which is solved by using a high-pass filter (HP(s)) on $\Delta\omega$. This changes the controller equation \[4.10\] to:

$$M_{EM,ctrl} = K \ast (1 + \tau_d \ast s) \ast HP(s) \ast \Delta\omega$$ \[5.1\]

Where the high-pass filter is described by:

$$HP(s) = \frac{\omega_{HP}^2 \ast s^2}{\omega_{HP}^2 \ast s^2 + \beta \ast \omega_{HP} \ast s + 1}$$ \[5.2\]

Damping $\beta$ is selected $1/\sqrt{2}$ for best amplitude and phase behaviour. The cross-over frequency $\omega_{HP}$ is put at 1Hz, low enough that it does not influence the controller performance for controlling $\Delta\omega$ in the relevant range.
Combined time delay

The results for maximum achievable damping and the peak step response are shown in Fig 5.4. The gains are selected to achieve the best possible damping. The color range for damping is limited from 20% to 70%. For motor delays more than 6ms, good damping is unachievable. The maximum delay on wheel speed is mainly limited by the peak step response. Since it should at least not increase, the delay on the wheel speed should not exceed 11ms. These values are for the system and vehicle parameters used in this report. They can vary with changing system parameters.

![Fig 5.4: Damping and peak step response reduction for varying delays](image)

5.2 Low-pass filtering

Cut-off frequency

Filtering of signals is needed in practice to filter noise from the measured signals. A low-pass filter is the most used filtering method in practice. There are other filtering methods which are not discussed in this section.

As discussed in Section 2.4, a second order low-pass filter is used with a damping constant of $1/\sqrt{2}$ for the best phase and amplitude properties. Fig 5.5 shows the amplitude, phase and time delays for this filter with different cut-off frequencies. The most important range is from 10 to 20Hz, since the eigenfrequency
of the motor inertia has a value in that range. The delay for the 25Hz filter in this range is 9-10ms. As discussed in the previous section, it is difficult to achieve reasonable improvement with that much delay. Moreover, the amplitude is significantly reduced in the 10-20Hz range. For the 50Hz and 100Hz low-pass filter the amplitude is hardly influenced. The time delay for the 50Hz filter in the 10-20Hz range is 4.5ms and for the 100Hz filter 2.2ms.

Fig 5.5: Amplitude, phase and time delay behaviour for second order low-pass filter with different cut-off frequencies

Symmetric filtering

The damping and peak step response as function of the controller gain for different filters is depicted in Fig 5.6. It shows the big influence of filtering on the achievable damping. As for the symmetric time delay in the previous section, symmetric filtering has limited influence on the peak step response.

Fig 5.6: Damping and peak step response reduction for different symmetric filters
Asymmetric filtering

Since generally the motor speed signal has a higher resolution, someone might think about filtering the motor speed with a higher cut-off frequency compared to the wheel speed. But from Fig 5.7 can be concluded that asymmetric filtering has a bad influence on damping and the peak step response. Therefore, the same filtering method should be used for both signals.

![Damping and peak step response reduction for asymmetric filters](image)

**5.3 Conclusions**

Time delay has a significant influence on the maximum achievable damping. With increasing delays, the maximum damping is reduced and achieved at lower controller gains. Since the peak step response is increased for smaller gains, more time delay will indirectly increase the peak step response. Vehicle architecture determines that the wheel is is probable to have more time delay compared to the motor speed signal. This delay will reduce damping as well, although not as significant if the motor speed would be delayed with the same amount. However, asymmetric delay has a negative influence on the peak step response.

Both signals should be filtered with the same filter. Using different filters will increase the peak step response and reduce damping. Filtering with a second order low-pass filter is a big part of the total delay that is feasible for the controller. Therefore filtering should be further researched in the future. Filters with minor delay, like the Kalman filter, could be very useful.
Chapter 6

Simulated validation

The results of the previous chapters are validated with full vehicle model simulations. The full vehicle model was provided by Daimler AG in the MATLAB Simulink environment. The part within the red dotted lines in Fig 2.1 is added to this full vehicle model. The model includes an ABS-controller.

The increased braking distance and high shaft torques of the passive system are already shown in Fig 3.1. The controller designed in Chapter 4 will be applied in this chapter. The effectiveness of the controller under varying environment conditions is investigated. In this chapter only braking for high and low friction coefficient is discussed. Additional simulation results for other conditions can be found in Appendix C. The validation of the controller effectiveness in combination with time delays is discussed in the second part of the chapter. With every simulation the passive (standard) system, the system with low inertia and the results of the controlled system will be compared. All simulations are ABS-braking from 100km/h.

6.1 Braking for high $\mu$

The results from a braking event from 100km/h on a friction coefficient $\mu = 1$ are depicted in Fig 6.1. The top left graph shows the wheel speeds during the event. With the controller (green) the wheel is decelerated more fluently compared to the passive system (black), where the wheels are nearly locked after 13 meters. The top right plot shows the vehicle speed as function of the travelled distance. The point where this speed is zero, is the braking distance. In red, the distance for the passive system with low inertia is illustrated. The middle left plot shows the moments in the side shafts, which are reduced significantly with the controller. The middle right plot shows the electric motor output moment which is applied by the controller. The electric motor is modelled as a first order differential equation, without taking additional constraints like maximal deliverable torque into consideration. The bottom plot shows the power spectral density (PSD) of $\Delta \omega$. The controller objective is to minimize $\Delta \omega$. With controller, $\Delta \omega$ is reduced to nearly zero. Hence it can be concluded that the controller operation is good.

Some values are shown at the bottom of Fig 6.1. With controller, the braking distance, RMS value of the side shaft moment and the peak moment are reduced compared to the passive system. The braking distance of the low inertia system and the controlled system are approximately the same. That the braking distance for the controlled system is slightly shorter compared to the low inertia system, has to do with the not adapted interaction between the $\Delta \omega$-controller and the ABS-controller. There can be some slight variations in braking distance, sometimes decreasing the braking distance and sometimes increasing the braking distance. But as for this simulation, these deviations are small.

Fig 6.2 shows the braking moments left and right of the front (driven) axle. Clearly, the braking moment
with controller is much more constant. This results in a better slip control, decreasing the braking distance.

\[
\begin{align*}
\text{Angular velocities} & \quad \text{Vehicle velocity} \\
\omega [\text{rad/s}] & \quad v_x [\text{km/h}] \\
\text{s [m]} & \quad \text{s [m]} \\
0 & \quad 0 \\
20 & \quad 100 \\
40 & \quad 50 \\
60 & \quad 20 \\
80 & \quad 10 \\
100 & \quad 0 \\
\end{align*}
\]

\[
\begin{align*}
\text{Side shaft moment} & \quad \text{Motor output moment} \\
M_{ss} [\text{Nm}] & \quad M_{EM} [\text{Nm}] \\
\text{s [m]} & \quad \text{s [m]} \\
0 & \quad 0 \\
20 & \quad 500 \\
40 & \quad 0 \\
60 & \quad -500 \\
80 & \quad 0 \\
100 & \quad -1000 \\
\end{align*}
\]

\[
\begin{align*}
\text{PSD } \Delta \omega & \\
|Y| & \quad f [\text{Hz}] \\
5 & \quad 5 \\
10 & \quad 10 \\
15 & \quad 15 \\
20 & \quad 20 \\
25 & \quad 25 \\
30 & \quad 30 \\
35 & \quad 35 \\
40 & \quad 40 \\
45 & \quad 45 \\
50 & \quad 50 \\
\end{align*}
\]

<table>
<thead>
<tr>
<th>Braking dist. [m]</th>
<th>(M_{ss,RMS} [\text{Nm}])</th>
<th>(M_{ss,peak} [\text{Nm}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard</td>
<td>41.7</td>
<td>312</td>
</tr>
<tr>
<td>Low inertia</td>
<td>40.2</td>
<td>-</td>
</tr>
<tr>
<td>Controlled</td>
<td>40.0</td>
<td>68</td>
</tr>
</tbody>
</table>

![Fig 6.1: Straight braking on \(\mu=1.0\)](image)

### 6.2 Braking for low \(\mu\)

The results for braking on a road with a low friction coefficient are shown in [Fig 6.3](image). Since the achievable accelerations are obviously smaller on low-\(\mu\), the side shaft moments due to the inertia are lower. Hence the braking distance is less influenced by the inertia at low-\(\mu\) braking. But although the amplitude of side shaft moment is rather low, there is clearly an oscillation which might result in noise or uncomfortable vehicle vibrations. The system free play has an negative influence in this region, since the system in constantly running through the backlash. With the controller these oscillations and the braking distance are reduced. And since the amplitude of the PSD of \(\Delta \omega\) is significantly reduced, it can be concluded that the controller works for low-\(\mu\) ABS-braking.
Fig 6.2: Braking moments during straight braking on $\mu=1.0$

<table>
<thead>
<tr>
<th></th>
<th>Braking dist. [m]</th>
<th>$M_{ss,RMS}$ [Nm]</th>
<th>$M_{ss,peak}$ [Nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard</td>
<td>137.2</td>
<td>97</td>
<td>911</td>
</tr>
<tr>
<td>Low inertia</td>
<td>134.0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Controlled</td>
<td>134.8</td>
<td>37</td>
<td>502</td>
</tr>
</tbody>
</table>

Fig 6.3: Straight braking on $\mu = 0.3$
6.3 Signal time delay

Symmetric delay

Fig 6.4 show the cases that the motor speed signal and wheel speed signal are delayed with 6ms. The simulation results for 3ms and 9ms delay on both signals can be found in Appendix C. The controller gain is adapted to the varying delays. As discussed in Chapter 5, the controller gain should be reduced to achieve maximum damping with increasing delays. Moreover, the achievable damping and peak step reduction are decreased with increasing delays. At 6ms delay, a low amplitude oscillation starts to occur. This low amplitude oscillation might generate noise or uncomfortable vibrations in the vehicle. The braking distance and peak moments are still reduced compared to the passive system without any control.

Asymmetric delay

As discussed in Chapter 5, time delay on the wheel speed signals has a negative influence on damping and the peak step response. Fig 6.5 shows the case when the motor speed signal is delayed with only 1ms and the wheel speed with 11ms. The simulation for 6ms delay on wheel speed can be found in Appendix C. Especially for the 11ms case, the first peak in the side shaft moment is not reduced, but increased. Moreover the achieved damping is decreased.
Due to the high-pass filter, there is no constant moment offset as a result of the time difference between the motor and wheel speed signal, as discussed in Section 5.1.

Fig 6.5: Braking with 1ms delay on motor speed and 11ms on wheel speeds

Combined delay

Fig 6.6 shows the braking distances, peak moments and RMS-moments in the side shaft for varying delays with controller. The braking distance is only for time delays at the borders of the plot significant increased. In all other cases helps the controller to stop the vehicle faster. From the peak moment color plot can be concluded that the delay on the wheel speeds should not exceed 11ms, in order to keep the peak moments low. Exactly the same conclusion was drawn in Chapter 5. The RMS values in the side shaft show significant improvement if the time delays do not become too large.

When comparing these plots with the damping plot in Fig 5.4 can be concluded that damping is the biggest constraint for the controller. Although the RMS-moment, peak moment and braking distance are not really influenced, there can be low amplitude vibrations which can be highly uncomfortable. Hence can be concluded that if sufficient damping and peak step response reduction in the linear model are achieved, the braking distance and side shaft moments will significantly improve in the full vehicle simulation.
Fig 6.6: Braking distance, peak- and RMS moments in the side shaft for varying time delays

6.4 Summary

The worst case simulated environmental condition is braking for high friction coefficients, since there the accelerations are the highest. Because the electric motor inertia is causing the moments in the shaft, high accelerations result in high shaft moments. These high moments can cause uncomfortable oscillations, damage the mechanical system and increase the breaking distance. Under all simulated environmental conditions the controller is effective.

Time delays have a negative influence on the controller performance. Although the braking distance is not really influenced, relatively low amplitude oscillations in the shaft moment occur. This can lead to uncomfortable vibrations in the vehicle. These oscillations are due to the limited damping that can be achieved in the system with signal time delay. With limited time delay, the controller improves braking distance and shaft moments under all simulated environmental conditions.
Conclusions and outlook

7.1 Conclusions

Oscillations are a common problem in electric drivetrains, because they are equipped with less damping compared to conventional ICE drivetrains. This research concentrates on oscillations during ABS-braking. Another phenomena improving the performance of the ICE drivetrain, is that during braking the clutch can be applied. This reduces the inertia of the drivetrain coupled to the wheels. The increased inertia of the electric drivetrain results in bad operation of the ABS-controller. Bad operation of the ABS-controller causes high shaft moments and an increased braking distance. The goal is to develop a robust solution to damp electric drivetrain oscillations.

To investigate the problem, the drivetrain is modelled in MATLAB Simulink and added to a full vehicle model provided by Daimler AG. For the given purpose the non-linear full vehicle model can accurately be represented by a linear model in specific operating conditions. The drivetrain is a central drive configuration: one electric motor drives two wheels via a differential. The system behaviour is highly dependent on the tire contact patch dampings, which is mainly influenced by the vehicle speed and tire slip. Two main eigenfrequencies can be distinguished in the side shaft moment response. The most dominant is the eigenfrequency of the electric motor inertia to the rim in the 8-20Hz range. The other one is the rotational eigenfrequency of the tire in the 30-60Hz range. This eigenfrequency is during ABS braking less dominant, hence damping of the eigenfrequency of the electric motor inertia is the main objective.

A single wheel drive configuration with one electric motor driving one wheel has no differential. The differential averages the accelerations left and right, reducing the acceleration of the electric motor inertia and hence shaft moments. However, still high shaft torques can occur. For controlling these oscillations no fundamental difference exists between both configurations. Only the average of left and right wheel speed has to be taken instead the single wheel speeds.

Several controllers are investigated. A controller on rotational velocities is used since it is the simplest way of controlling. Torque observer controllers need more detailed knowledge of the system parameters. A control method only using the motor speed will per definition increase the peak step response, and is therefore not feasible. Hence the $\Delta\omega$-controller is selected, which uses the difference between the wheel speed and the motor speed as controller objective. By adding a PD-controller with a zero at the electric motor pole, the time constant of the electric motor can be completely compensated.

Time delays have a significant influence on the controller performance. Time delay consists out of a sensing delay determined by the sensor architecture, a communication delay dependent on vehicle network
architecture, a filtering delay dependent on filtering method, and a computation delay determined by the task time of the ECU. Delay on the motor speed reduces mainly the maximum achievable damping, while delay on the wheel speed signal increases the peak step response. For bigger time delays, the controller gain has to be selected smaller to achieve maximum damping. This will increase the peak step response as well. Using different filtering methods for both signals has a negative influence on damping and peak step response. A higher filtering frequency will add less time delay to the system, but the filtering frequency should be selected low enough to filter noise.

Validation is done by full vehicle simulations. Under all simulated environmental circumstances the controller is effective. The controller effectiveness is reduced significantly for increasing time delays.

### 7.2 Outlook

Since time delays limit the controller performance dramatically, reducing time delays should be the main objective for further research. Because filtering delay is a significant part of the total delay, signal filtering should be investigated further. An option could be to use a Kalman filter, which has only minor delay. Moreover the communication delays should be kept to a minimum.

Since in this project no specific vehicle is determined, the constraints of the electric motor are not taken into account. In practice the motor moments and gradients are limited. The influence of these on the controller performance should be further investigated.

If in the future a new car should be designed, the process in Fig 7.1 should be followed. First some vehicle parameters should be estimated, from where time constraints can be determined. After the hardware is designed, the exact system parameters can be determined. With these parameters the controller can be properly tuned using the linear model. This controller should then be validated first doing a full vehicle simulation, followed by a test track validation.

![Fig 7.1: Design process](image)
Bibliography


Appendix A

State space model

Because the model presented in Chapter 3 includes time delays, a state space model as shown in Fig A.1 is created. The model consists out of the following equations.

\[ \dot{x} = Ax + B_1u + B_2w; \]
\[ y = C_1x + D_{11}u + D_{12}w; \]
\[ z = C_2x + D_{21}u + D_{22}w; \]
\[ w = z(t - \tau); \]

(A.1)

Output matrix \( y \) determines the outputs of the system. Since the outputs are dependent on the user desires, the matrices determining the output \( y \) (matrices \( C_1, D_{11} \) and \( D_{12} \)) will not be further discussed. All matrix entries of the other entries are zero if not mentioned.
\[ u = \begin{bmatrix} M_{EM,\text{extern}} \\ M_{brake} \\ M_{brake,\text{c}} \end{bmatrix} \]  \hspace{1cm} \begin{align*} \text{(A.2)} \end{align*} \hspace{1cm} \begin{bmatrix} w \\ \omega_{EM,\text{meas}} \\ \omega_{rim,\text{meas}} \end{bmatrix} = \begin{bmatrix} M_{EM,\text{des}} \\ \omega_{EM,\text{meas}} \\ \omega_{rim,\text{meas}} \end{bmatrix} \]  \hspace{1cm} \begin{align*} \text{(A.3)} \end{align*} \hspace{1cm} x = \begin{bmatrix} \theta_{EM} \\ \omega_{EM} \\ \theta_{rim} \\ \omega_{rim} \\ \theta_{cs} \\ \theta_{cp} \\ \omega_{cp} \\ \theta_{cp} \\ \omega_{cp} \\ v_{eh} \end{bmatrix} \]  \hspace{1cm} \begin{align*} \text{(A.4)} \end{align*}

State matrix

\[
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

Electric motor inertia

\[ a_{2,1} = -\frac{1}{J_{EM}} \frac{c_{cs}}{i_r} \]

\[ a_{2,7} = \frac{1}{J_{EM}} \frac{c_{cs}}{i_tr} \]

\[ a_{2,13} = \frac{1}{J_{EM}} \]

Left rim inertia

\[ a_{4,1} = \frac{1}{J_{rim}} \frac{c_{sw} \cdot i_{diff}}{2 \cdot i_{tr}} \]

\[ a_{4,3} = -\frac{1}{J_{rim}} \frac{c_{sw}}{i_{tr}} \]

\[ a_{4,4} = \frac{1}{J_{rim}} \frac{d_{sw}}{i_{tr}} \]

\[ a_{4,7} = -\frac{1}{J_{rim}} \frac{c_{cs} \cdot i_{diff}}{2} \]
\[ a_{4,8} = \frac{1}{J_{rim}} * c_{sw} \]
\[ a_{4,9} = \frac{1}{J_{rim}} * d_{sw} \]

Right rim inertia

\[ a_{6,1} = \frac{1}{J_{rim}} * \frac{c_{cs} * i_{diff}}{2 * i_{tr}} \]
\[ a_{6,5} = -\frac{1}{J_{rim}} * c_{sw} \]
\[ a_{6,6} = -\frac{1}{J_{rim}} * d_{sw} \]
\[ a_{6,7} = -\frac{1}{J_{rim}} \]
\[ a_{6,10} = \frac{1}{J_{rim}} * c_{sw} \]
\[ a_{6,11} = \frac{1}{J_{rim}} * d_{sw} \]

Central shaft rotational velocity

\[ a_{7,1} = \frac{c_{cs} * i_{diff}^2}{2 * d_{ss} * i_{tr}} \]
\[ a_{7,3} = \frac{c_{cs} * i_{diff}}{2 * d_{ss} * i_{tr}} \]
\[ a_{7,4} = \frac{i_{diff}}{2} \]
\[ a_{7,5} = \frac{c_{cs} * i_{diff}}{2 * d_{ss} * i_{tr}} \]
\[ a_{7,6} = \frac{i_{diff}}{2} \]
\[ a_{7,7} = \frac{c_{cs} * i_{diff}^2}{2 * d_{ss}} - \frac{c_{cs}}{d_{ss}} \]

Left side wall inertia

\[ a_{9,3} = \frac{1}{J_{cp}} * c_{sw} \]
\[ a_{9,4} = \frac{1}{J_{cp}} * d_{sw} \]
\[ a_{9,8} = -\frac{1}{J_{cp}} * c_{sw} \]
\[ a_{9,9} = -\frac{1}{J_{cp}} * (d_{sw} + \frac{\delta_{slip} * r_{dyn}^2}{v_{veh}}) \]
\[ a_{9,12} = \frac{\delta_{slip} * r_{dyn}}{v_{veh}} \]

Right side wall inertia

\[ a_{11,5} = \frac{1}{J_{cp}} * c_{sw} \]
\[ a_{11,6} = \frac{1}{J_{cp}} * d_{sw} \]
\[ a_{11,10} = -\frac{1}{J_{cp}} * c_{sw} \]
\[ a_{11,11} = -\frac{1}{J_{cp}} * (d_{sw} + \frac{\delta_{slip} * r_{dyn}^2}{v_{veh}}) \]
\[ a_{11,12} = \frac{\delta_{slip} * r_{dyn}}{v_{veh}} \]
Vehicle mass
\[
a_{12.9} = \frac{1}{m_{veh}} \delta_{slip} \tau_{dyn} \frac{v_{veh}}{v_{veh}}
\]
\[
a_{12.11} = \frac{1}{m_{veh}} \delta_{slip} \tau_{dyn} \frac{v_{veh}}{v_{veh}}
\]
\[
a_{12.12} = -\frac{2}{m_{veh}} \delta_{slip} \frac{v_{veh}}{v_{veh}}
\]

Electric motor PT1
\[
a_{13.13} = -\frac{1}{\tau_{EM}}
\]

Motor speed low-pass filter
\[
a_{15.14} = -\omega_{LP,EM}^2
\]
\[
a_{15.15} = -2 \beta \omega_{LP,EM}
\]

Left wheel speed low-pass filter
\[
a_{17.16} = -\omega_{LP,rim}^2
\]
\[
a_{17.17} = -2 \beta \omega_{LP,rim}
\]

Right wheel speed low-pass filter
\[
a_{19.18} = -\omega_{LP,rim}^2
\]
\[
a_{19.19} = -2 \beta \omega_{LP,rim}
\]

Controller
\[
a_{21.14} = -K
\]
\[
a_{21.15} = K \tau_d
\]
\[
a_{21.16} = K \frac{i_{diff} \cdot i_{tr}}{2}
\]
\[
a_{21.17} = K \tau_d \frac{i_{diff} \cdot i_{tr}}{2}
\]
\[
a_{21.18} = K \frac{i_{diff} \cdot i_{tr}}{2}
\]
\[
a_{21.19} = K \tau_d \frac{i_{diff} \cdot i_{tr}}{2}
\]
\[
a_{21.20} = -\omega_{HP}^2
\]
\[
a_{21.21} = -2 \beta \omega_{HP}
\]

Input to state matrix

\[
Brake input
b_{14.2} = -\frac{1}{J_{rim}}
\]
\[
b_{16.2} = -\frac{1}{J_{rim}}
\]

Delay to state matrix

\[
Motor moment input
b_{21.1} = \frac{1}{\tau_{EM}}
\]
\[
Filter inputs (measured signals with delay)
b_{15.2} = \omega_{LP,EM}^2
\]
\[ b_{2,7,2} = \omega_{LP,EM}^2 \]
\[ b_{2,19,2} = \omega_{LP,EM}^2 \]

State to delay matrix

**Controller input**
\[ c_{2,14} = -K \]
\[ c_{2,15} = K \tau_d \]
\[ c_{2,16} = K \frac{i_{diff} \tau_{tr}}{2} \]
\[ c_{2,17} = K \frac{i_{diff} \tau_{tr}}{2} \]
\[ c_{2,18} = \frac{i_{diff} \tau_{tr}}{2} \]
\[ c_{2,19} = \frac{i_{diff} \tau_{tr}}{2} \]
\[ c_{2,20} = -\omega_{HP}^2 \]
\[ c_{2,21} = -2 \beta \omega_{HP} \]

**Measured signals**
\[ c_{2,2} = 1 \]
\[ c_{2,3,4} = 1 \]
\[ c_{2,4,6} = 1 \]

Input to delay matrix

**Activation delay on extern desired motor moment**
\[ d_{2,1,1} = 1 \]
Appendix B

Controller robustness

In this appendix the robustness of the controller with varying slip stiffnesses of the tire is investigated. Therefore the tire damping constant $\delta_t$ is varied. The derivation of $\delta_t$ can be found in Section 2.2:

$$F_x = \delta_t \ast (\dot{\theta}_{cp} \ast r_{dyn} - v_{veh})$$  \hspace{1cm} (B.1)

$\delta_t = -400$
\[ \delta_t = -50 \]

\[ \delta_t = 0 \]
\( \delta_t = 50 \)

\[\begin{align*}
\text{Pole-Zero Map} & \quad \delta_t = 50 \\
\delta_t = 400 \\
\text{Imaginary Axis} & \quad \text{Real Axis}
\end{align*}\]
\[ \delta_t = 3200 \]

\[ \delta_t = 1 \times 10^9 \]
Appendix
C

Additional validation simulations

Environmental conditions

Braking for $\mu$-step

Braking on $\mu$-step means a sudden change in friction coefficient during the braking event. The friction coefficient is changed after approximately 20 meters. A negative step (from 1.0 to 0.3) and a positive step (from 0.5 to 1.0) are simulated. The results are shown in Fig C.1 and Fig C.2 respectively.

For the passive system, the wheels lock as soon as the negative step in friction coefficient is applied. For the controlled system the wheel speed is reduced as well, but the ABS-controller adapts faster to the new friction coefficient, so that the wheels do not completely lock. The controlled side shaft response is characterized by a peak as soon as the friction coefficient changes. This peak is similar to the peak step response discussed in the previous chapter. The braking distance and $\Delta \omega$ are reduced significantly compared to the passive system.

For the positive friction coefficient step values of 0.5 and 1.0 are selected. For bigger steps the ABS-controller used in the simulation has difficulties to adapt to the changed friction coefficient. With the controller the braking distance and the shaft torques are reduced significantly. Remarkable high electric motor moments are needed to control $\Delta \omega$. Here the electric motor constraints can be exceeded depending on the applied motor.
Fig C.1: Straight braking on $\mu = 1.0\text{-}0.3$
Fig C.2: Straight braking on $\mu = 0.5-1.0$
Braking for $\mu$-split

The $\mu$-split braking event is shown in Fig C.3. On the one side a friction coefficient of 1.0 and on the other side of 0.5 is applied. The moments in the side shafts are lower than in case of a high friction coefficient, but higher than in case of a low friction coefficient. This is due to the filtering function of the differential, discussed in Section 3.4, and the reduced braking moments to keep the vehicle straight. The influence of the inertia on the braking distance is small. But the controller reduces the side shaft moment oscillations significantly.

![Graphs showing angular velocities, vehicle velocity, side shaft moment, motor output moment, and PSD $\Delta\omega$.]

<table>
<thead>
<tr>
<th></th>
<th>Braking dist. [m]</th>
<th>$M_{ss, RMS}$ [Nm]</th>
<th>$M_{ss, peak}$ [Nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard</td>
<td>65.7</td>
<td>170</td>
<td>523</td>
</tr>
<tr>
<td>Low inertia</td>
<td>65.5</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Controlled</td>
<td>65.8</td>
<td>50</td>
<td>310</td>
</tr>
</tbody>
</table>

Fig C.3: Straight braking on $\mu$-split (1.0-0.5)
Corner braking

The main function of the differential is compensating for different rotational speeds left and right. Fig C.4 shows that during cornering the controller is effective. As concluded in Section 3.4 has the differential no fundamental influence on the performance of the controller.

Fig C.4: Corner braking on $\mu=1.0$

<table>
<thead>
<tr>
<th></th>
<th>Braking dist. [m]</th>
<th>$M_{ss,RMS}$ [Nm]</th>
<th>$M_{ss,peak}$ [Nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard</td>
<td>42.6</td>
<td>298</td>
<td>1030</td>
</tr>
<tr>
<td>Low inertia</td>
<td>41.0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Controlled</td>
<td>40.9</td>
<td>73</td>
<td>330</td>
</tr>
</tbody>
</table>
Controlling with time delay

Symmetric delay

Fig C.5: Braking with 3ms delay on motor and wheel speeds
Fig C.6: Braking with 9ms delay on motor and wheel speeds
Fig C.7: Braking with 1ms delay on motor speed and 6ms wheel speeds.