Existing models of knowledge in teaching: developing an understanding of the Anglo/American, the French and the German scene.

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Abstract
This paper explores existing models of knowledge in teaching in three different environments: the Anglo/American; the French; and the German scene. Particular attention is given to models of mathematics teaching. In each scene it is attempted to develop an understanding of the different aspects of the models, to comprehend their underlying concepts, how they are constructed, and in some cases how they developed historically. By doing this, similarities and differences could be identified. It is concluded, firstly, that what is common to all models in all countries considered is that knowledge in teaching is not seen as static, but as a process of development and change and that experience in the classroom contributes to its growth and change. Secondly, there are differences in terms of origins of concepts, their background in terms of educational traditions. Thus, it is argued that models of knowledge in teaching have to be understood in terms of the countries’ educational and cultural traditions in which they developed. This might usefully guide researchers to the development of a common understanding of what is generally called ‘the science of teaching’ or ‘didactics’, as well as to the identification of research and development projects in the areas of teachers’ knowledge. In order to make the task manageable distinctive models were chosen for the Anglo/American scene, in particular that of Shulman in terms of teachers’ knowledge in general, and that of Ernest in terms of mathematics teaching. For the French scene particular concepts were selected that originate in French research on didactics and mathematics didactics, and these were linked with corresponding didactical theories. For the German scene the research drew almost exclusively on literature that presented and compared the German concept of Didaktik with the Anglo/American concept of research into teaching and curriculum. Therefore, the reader can imagine a grid with three lines (Anglo/American; French and German) and two rows (general didactics; mathematics didactics) which need to be filled with theories and concepts. The main emphasis of the paper is on the Anglo/American aspect, simply because in England the main literature is available in English and it is on the Anglo/American representation of teacher knowledge.

Introduction
In recent years questions relating to teacher knowledge have received an increasing amount of attention by researchers. Researchers have investigated the professional knowledge of teachers from different angles. It is accepted that what teachers know is one of the most important influences on what happens in classrooms and, in turn, on what students learn. Recent studies (for example, Pepin 1997) point out that changes in teaching practices, for example, can only be successful if the context (cultural and structural environment) in which teachers are working is taken into account.
The conceptual tools that teachers possess in order to deal with their work situation depend to a large extent on the cultural (and structural) traditions of the educational environment in which they are working. However, there is no consensus on what teachers need to know in order to ensure that student learning is taking place.

Whilst there does not exist a commonly agreed upon bounded knowledge base for teaching in secondary schools, the body of knowledge from which teachers, teacher educators and curriculum innovators can draw is substantial and growing. With this increasing research the understanding of what teaching consists of and what it requires has been broadened. Teaching is now seen as an ambiguous and complex task requiring the time and willingness to reflect upon and revise actions and decisions, requiring action and decision-making. Teacher judgements therefore are likely to be rooted in a deep understanding of teaching and learning, of the learners and of the subject matter, amongst other factors, and how these components interact in the teaching-learning process in the classroom.

Although it is widely accepted that teachers’ knowledge is an integrated system with each part difficult to isolate, most research has not studied teacher knowledge as integrated. Concerning the interconnected nature of different areas of knowledge, Barnes (1989) asserts the following:

_Talking about teaching is analogous to describing a tapestry that has many threads of different colours woven into complicated textures and patterns. One can remove individual threads and examine them separately, but one cannot appreciate the complexity of the tapestry without seeing how the threads are interwoven to create the whole cloth. As each domain of knowledge is discussed, therefore, one or more areas will also be mentioned._ (p.13)

This reflects the view that areas of knowledge interact and presumes that any attempt to disentangle different areas of knowledge is likely to be both arbitrary and artificial.

As understanding of teaching and learning have grown, views of teaching and learning, of learners, and of subject matter have changed. Views of learning have changed from that of the ‘passive’ to that of the ‘active’ learner. This involves the aim that learners construct meaningful understandings as a result of engagement in activities within an instructional situation that takes into account the context, the learner’s prior knowledge and his/her perceptions of goals for learning. In line with these concepts of learning, views of teaching have changed too, in the sense that researchers have recognised the interactive nature of teacher knowledge. This involves the recognition that teaching is not a linear process of transmitting knowledge from the teacher (or textbooks) to students. The work that dominated educational research on teaching at the beginning of the 19th century to the middle of this century was based on answering questions such as _how best to teach_ and _what methods are best_ in relation to different topics and subjects. In the third quarter of this century research on teaching changed to what teachers and their pupils observably did in classrooms (for example, Brophy and Good 1986). Since the 1970s emphasis has shifted from what teachers (and pupils) do to enquiries about teachers’ thinking, about the knowledge they hold, how this is organised, and how different knowledge sources inform teaching (for example, Shulman 1987). Underpinning this view is the idea that teachers construct their own conceptions of the topics they teach and that they generate their own representations for their students to understand.

Studies of interactive teaching (for example, Clark and Peterson 1986), examinations of knowledge used by teachers in ‘routine’ situations (for example, Shavelson and Stern 1981), and studies of how experts and novices use knowledge differently (for example, Leinhardt 1989) support the perspective that teacher reasoning depends on the richness of their knowledge about their pupils,
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They also suggest that experts organise their knowledge in meaningful clusters and that they connect these in a network of coherent relations. This organisational knowledge is believed to guide teachers’ intentions, beliefs and practices (Shavelson and Stern 1981).

But not everyone agrees with this ‘information-processing’ model of thinking (Barnes 1989). For example, Halkes and Olson (1984, in Barnes 1989) assert that teacher thinking is too subtle and complex to be reduced to models based on this theory. In their view the information-processing theory (information from many sources organised by individuals into cognitive structures) is useful for explaining teacher thinking, but the ways the clusters interact is likely to be as important. Toulman (1972) proposes the metaphor of ‘intellectual ecology’. He contends that, as in an ecosystem, patterns are constantly changing as the system evolves; in the same way in the classroom multiple ‘interwoven structures’ (rather than single static structures) operate to inform teacher judgement. Therefore, according to him, knowledge is organised into multiple interacting clusters of related concepts, and these change according to the teaching situation.

The emerging view of teaching as thought-in-action has repercussions for teacher education in the sense of how to transform knowledge into professional education. Typically teacher educators have provided student teachers with components of knowledge from all the domains of knowledge that can inform teaching. But deciding which knowledge and experience to include raises the question of which concepts of teaching and learning and of the nature of the subject itself underpin the selection of content.

In terms of conceptions of teaching and teaching expertise, Kennedy (1987 in Barnes 1989) outlines four ways that different professions have defined the expertise crucial to professional performance, because she believes that conceptions of the fundamental nature of the work influence formulations of the knowledge (and thus the expertise) that is expected of the expert in that profession. Firstly, she suggests ‘technical skill’ as an expertise, but argues that programmes based on skill training have been largely abandoned because training in performance skills is no longer seen as an adequate preparation for actual practice. Secondly, she draws the comparison with medical schools when outlining the ‘application of theory to practice’. Kennedy (1987) and also Schön (1987 in Barnes 1989) argue that knowing a principle is insufficient preparation for teaching, because student teachers must firstly recognise a classroom event as one where the principle applies. Therefore the ability to identify events and distinguish among cases is the difficulty in this model. Thirdly, Kennedy refers to ‘critical analysis’, as in law schools where students learn how to analyse and critique cases that have already been tried. Whilst this model appears to be successful in developing students’ analytical skill expertise, it does not teach them to improve the situation (once they have analysed it). Fourthly, as an alternative to these concepts of professional expertise, Kennedy (1987, in Barnes 1989) suggests that teaching can be viewed as ‘deliberate action’. She argues that

the expertise needed by professionals is the capacity to make sense of a situation by comparing it to situations that one has previously encountered. Based on knowledge gained from reflection on and critical analysis of previous experiences, professionals can postulate alternative interpretations of a situation and ways to respond. They can then reflect and revise their initial understandings. (p.19)

Therefore, and contrary to the ‘epistemology of technical rationality’ in education (Smyth 1987 in Barnes 1989), theory is not separated from practice. Student teachers are encouraged to reflect upon and reformulate assumptions about the nature, purposes and conditions of their work as teachers. Teaching expertise is thus the capacity for principled thought about the details of the situation in the
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classroom for example, that leads to action. It also includes the capacity and willingness to reflect on one’s decisions and actions in the light of their potential impact on learning (Barnes 1989). The goal is to help teachers to develop for themselves meaningful frameworks for teaching. Featherstone (1987) mentions that ‘knowledge does not mean anything until it is remade in the present’.

The Anglo/American scene

It is widely accepted that what teachers know is one of the most important influences on what is happening in the classroom in terms of teaching and learning. However, there is not one body of knowledge that has been identified as necessary to ensure that learning is taking place. Indeed, many components of teachers’ professional knowledge have been identified.

This section will provide a review of influential perspectives in recent research that have dealt with the questions of teacher knowledge. In order to make the task manageable, first a general overview is given, then examples of well-conceived studies that address what are regarded as important research themes are identified, and these are presented together with a brief discussion of the models.

The research to be undertaken in this project falls generally into the category of research into teachers’ thinking, and especially teachers’ thinking whilst teaching. In this field a wide variety of theoretical perspectives has been adopted in recent years, and a brief outline of other work will explain the context in which this study is undertaken.

As mentioned earlier, it was not until the 1970s that researchers started systematically to study teachers’ thinking. The work of Clark and Peterson (1986) was influential in the sense that it highlighted that what teachers do depends a great deal on what they think. Research into teachers’ thinking is therefore necessary to develop an understanding of teaching. Calderhead (1981) also takes a psychological approach, but rejects models that construe teachers’ decision-making as an information-processing activity.

Morine-Dershimer provides an overview of recent research on teachers’ thinking. She outlines ‘four of the most influential ... alternative interpretations of what it might mean to think like a teacher’ and labels them as ‘thinking through schemata’, ‘reflecting in/on practice’, ‘formulating pedagogical content knowledge’, and ‘perceiving practical arguments’ (Morine-Dershimer 1990, in Brown and McIntyre 1993).

As an example of Morine-Dershimer’s ‘thinking through schemata’ category, Brown and McIntyre (1993) present the model of Gaea Leinhardt who explored, in great detail and very systematically, the nature and structure of teachers’ knowledge in the area of mathematics teaching. The model is explained in more detail in the section on ‘further models’.

Each of the other three approaches to teachers’ thinking identified by Morine-Dershimer is based to a large extent on the work and writing of Schön (1983 and 1987), but it has been most influential in relation to ‘reflecting in/on practice’, according to Brown and McIntyre (1993). Schön argued that thinking and expertise is based to a large extent on experienced-based knowledge and on thinking about what is applicable in particular situations. However, he did not base his research on classroom teaching and it is therefore not yet empirically established how helpful his ‘reflection-in-action’ is in relation to classroom teaching. His work has been important in relation to teaching in the sense that it has highlighted the importance of professional practice and shown that professional practice of high intellectual quality does not depend on articulated theory or on analytical thinking.
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Morine-Dershimer’s third category refers to the work of Shulman and associates. His model of teachers’ knowledge is explained in detail below. However, Shulman does not claim that he attempts to describe teachers’ thinking, but that he attempts to highlight important aspects of teaching. Brown and McIntyre (1993) assert that the notion of ‘pedagogical content knowledge’ (as well as the ‘reflection in/on action’ notion) suggests a way of interpreting teaching, possibly even certain approaches to teaching (such as transmission style). McEwan and Bull (1991) argue on the same grounds that there is not a distinction between subject knowledge and pedagogic content knowledge, that ‘all knowledge is, in varying ways, pedagogic’.

The fourth of Morine-Dershimer’s categories (‘perceiving practical arguments’) refers to her own work based on an approach proposed by Gary Fenstermacher (1986 in Brown and McIntyre 1993). She quotes Fenstermacher when reasoning that ‘using research to help teachers change their minds about the ends and means of instruction was a more defensible application of research than merely training them to imitate the effective behaviour patterns identified by that research’ (pp.13 and 14 Morine-Dershimer 1990, in Brown and McIntyre 1993).

Because of the emphasis of many researchers on exploring teachers’ knowledge in terms of its components, the approach for presenting individual models has been to examine first research that has dealt with individual components (Shulman model, Ernest model), and then proceed to models of integrated knowledge (for example, the model of Brown and McIntyre).

The Shulman model

Shulman (1986b) asserts that ‘where the teacher cognition programme has clearly fallen short is in the elucidation of teachers’ cognitive understanding of the subject matter content and the relationships between such understanding and the instruction teachers provide for students’. He terms the role of subject matter knowledge in teaching the ‘missing paradigm’ in research on teacher cognition, and argues that knowledge of teaching will not advance until this lack is addressed. His interest is mainly in the realm of teachers’ subject matter knowledge and the role it plays in teaching, whilst acknowledging that teachers need to possess a ‘specialised understanding of the subject matter, one that permits them to foster understanding in most of their students’ (Wilson, Shulman and Richert 1987). Shulman (1986b) proposes a framework for analysing teachers’ knowledge that distinguishes between different categories of knowledge, and he mainly distinguishes between three kinds of knowledge: subject matter knowledge; pedagogical knowledge and curricular knowledge. In later publications (Wilson, Shulman and Richert 1987; Shulman 1987) he offers other kinds of knowledge:

knowledge of subject matter;
pedagogical content knowledge;
knowledge of other content;
knowledge of the curriculum;
knowledge of learners and their characteristics;
knowledge of educational aims (purposes and values and their philosophical and historical backgrounds);
knowledge of educational context (character of school communities and cultures); and
general pedagogical knowledge (broad principles and strategies of classroom management and organisation).

Some of these components overlap with what Shulman had proposed earlier.

Shulman (1986a) defines ‘subject matter knowledge’ as ‘the amount and organisation of the knowledge per se in the mind of the teacher’ (p.9). The important part of Shulman’s work is the
acknowledgement of ‘pedagogical content knowledge’ which helps to fill the gap of the ‘missing paradigm’. He describes ‘pedagogical content knowledge’ as that knowledge ‘which goes beyond knowledge of subject matter per se to the dimension of subject matter knowledge for teaching’ and asserts:

Within the category of pedagogical content knowledge I include, for the most regularly taught topics in one’s subject area, the most useful forms of representation of ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject that make it comprehensible to others. Since there are no single most powerful forms of representation, the teacher must have at hand a veritable armamentarium of alternative forms of representation, some of which derive from research whereas others originate in the wisdom of practice. Pedagogical content knowledge also includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons. If these preconceptions are misconceptions, which they so often are, teachers need knowledge of the strategies most likely to be fruitful in reorganising the understanding of learners, because those learners are unlikely to appear before them as blank slates. (Shulman 1986a, p.9)

Included in this category would be the specific knowledge of how the subject might be interpreted for learners, and the knowledge of how pupils think within specific (mathematical) domains (work within the cognitive science tradition readily fits into this category). Curricular knowledge is the knowledge of instructional materials available for teaching various topics. In essence Shulman proposes that the important components of teachers’ knowledge are what they know about the subject, about how learners think about the subject, and about instructional materials developed to teach the subject. It is increasingly recognised that pedagogical content knowledge forms the essential bridge between academic subject matter knowledge and the teaching of the subject matter (Ernest 1989). For it is that knowledge which determines how the subject matter is represented to students in their learning experiences, and it includes the practical skills for transforming subject matter for teaching and the pedagogical knowledge and skills for teaching it. Interestingly, McEwan and Bull (1991) assert that all content knowledge, whether held by scholars or teachers, has a pedagogical dimension, and that therefore Shulman’s distinction between content knowledge and pedagogic content knowledge cannot be supported.

Shulman’s classification of teachers’ knowledge has been proven to be very stimulating for research into teachers’ cognitions. For example, Bromme (1994) who worked in Germany on mathematics teaching and learning took up Shulman’s suggestions, but extended them by two further concepts: the ‘philosophy of content knowledge’; and by distinguishing between the knowledge of the academic discipline and that of the subject in school. Within ‘school knowledge’ he includes goals about school and concepts of general education into the meanings of the subject-specific concepts. Banks and Leach (1996) also propose that school knowledge ‘relates to the way subject knowledge is specific to schools’. By the ‘philosophy of school mathematics’ Bromme refers to ‘the epistemological foundations of mathematics and mathematics learning and about the relationship between mathematics and other fields of human life’, in other words teachers’ perceptions on the nature of mathematics and its teaching. I show in my study (Pepin 1997) that, although teachers are not explicit about their perceptions on the nature of mathematics, teachers’ beliefs on its nature are manifested in their classroom practices and that they are different in England, France and Germany.
The Ernest model

Ernest (1989) explores teachers’ knowledge in mathematics teaching, and his model includes teachers’ attitudes and beliefs. What is notable in Ernest’s model is the importance ascribed to teachers’ beliefs, in particular those concerning the nature of the particular subject (mathematics), and concerning the process of teaching and learning of the subject. He tries to develop a more fundamental understanding of how mathematics teachers’ knowledge, beliefs and attitudes provide a basis for classroom teaching approaches. He specifies the essential knowledge, beliefs and attitudes of mathematics teachers (and the ways in which these affect the teaching of mathematics) as follows:

Knowledge
- of mathematics;
- of other subject matter;
- of teaching mathematics (mathematics pedagogy, mathematics curriculum);
- of classroom organisation and management of mathematics teaching;
- of the context of teaching mathematics (school context, students taught);
- of education (educational psychology, general education, mathematics education);

Beliefs
- conception of the nature of mathematics
- models of teaching and learning mathematics;
- principles of education;

Attitudes
- attitude to mathematics;
- attitude to teaching mathematics.

Although this model is more detailed than that of Wilson, Shulman and Richert (1987), it shares many of its components with Shulman’s model. The comparison provides some measure of support for Ernest’s model, since there is evidently a great deal of overlap. Ernest’s pedagogical knowledge refers to Shulman’s knowledge ‘which a teacher uses to transform and represent knowledge of mathematics for teaching’ or pedagogical content knowledge (Wilson, Shulman and Richert 1987). A key difference between Shulman’s and Ernest’s models is the apparent neglect of attitudes and beliefs in Shulman’s model. However, it seems that beliefs about subject matter are to some extent incorporated into Shulman’s ‘knowledge of subject matter’.

However, there are other models in which knowledge has been studied as an integrated phenomenon. There is a research tradition in education that studies teachers as experts. This approach analyses the connections between the professional knowledge and professional activity of good performers within a certain field of activity.

The model of Brown and McIntyre

Recent studies have sought to understand teachers on their own terms and in their own language by attempting to elicit the often implicit and only partially articulated elements of teacher knowledge that guide teachers’ actions in specific situations. In this work on teachers’ implicit theories researchers try to get teachers to talk about their work, combined with extended observations of teachers’ work. The researchers attempt, on the basis of these verbal and observational data, to draw inferences about the content, the organisation and the development of teachers’ craft knowledge.
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For example, Brown and McIntyre (1993) developed a model based on the assumption that ‘over a period of time experienced teachers have acquired substantial practical knowledge about teaching, largely through their classroom experience rather than their formal training’ (p.12). Underlying this approach is the notion that teaching is a craft (rather than a science-based technology) and that experienced teachers have ‘craft knowledge’ which is accessible to others. In terms of teachers’ professional development this means that teachers can share their diverse and successful approaches, a ‘building on strength model’ (Brown and McIntyre 1993). The purpose is to explore ‘that part of (teachers’) professional knowledge which teachers acquire primarily through their practical experience in the classroom ... which guides their day-to-day actions in classrooms, which is for the most part not articulated in words and which is brought to bear spontaneously, routinely and sometimes unconsciously on their teaching’ (Brown and McIntyre 1993, p.17). They refer to this aspect of teachers’ knowledge as ‘teachers’ professional craft knowledge’. This knowledge is studied by developing an understanding of how teachers themselves make sense of the knowledge and the thought that they use in their everyday classroom practice. Brown and McIntyre (1993) found out that teachers commonly judged their teaching in terms of ‘the achievement or maintenance of states of pupil activity which they took to be normally desirable for particular phases and types of lessons (Normal Desirable States-NDS)’ and each teacher had their own NDSs for their lessons and for phases of their lessons.

Further models

Another interesting model in the area of cognitive models of the skills of teaching has been the work of Leinhardt and her colleagues. Their goal has been to describe in depth the mental structures of skilled teachers. The underpinning belief here is that teaching is ‘a complex cognitive skill amenable to analysis in a manner similar to other skills described by cognitive psychology’ (Leinhardt and Greeno 1986, p.75, in Grouws 1992). After hypothesising a model of mental structures, they examined and contrasted the teaching behaviour of expert and novice teachers to see whether their behaviour fits their model. According to Leinhardt and her colleagues, the skill of teaching is to a large extent determined by two inter-related systems of knowledge: subject matter (content knowledge) and lesson structure (practical knowledge). The methodology involved observing teachers and then preparing detailed semantic nets of the mathematics presented (Leinhardt and Smith 1985 in Grouws 1992). According to Leinhardt and her colleagues teacher knowledge is made up of the skills and abilities needed to run classrooms well and to adequately interpret and explain certain procedural mathematical ideas in order that the students are able to acquire the mathematical skills that the teacher feels are important. Knowledge of content was important in Leinhardt’s work and one of the major results was that expert teachers’ knowledge appeared to be organised into a hierarchical structure, they used richer systems of representations and they tended to present more detailed conceptual and procedural knowledge.

Elbaz (1983) has provided another kind of model, that of knowledge as practical and personal. Her ideas stress the growing dynamic nature of teachers’ knowledge. She suggested that it is necessary to recognise that knowledge is always changing and never static.

The work of Elbaz relates to a further kind of model, that of regarding teachers’ knowledge as situated. In this model it is asserted that in-school knowledge is acquired by working alone, whereas out-of-school knowledge is acquired by working in a social situation to construct one’s own learning. It is assumed that all knowledge is situated and that it is to a large extent a result of the activity undertaken, the context and the culture in which it is developed (Brown et al. 1989 in Grouws 1992). The concept of situated knowledge suggests that school-knowledge is different from real-world knowledge. Furthermore, it is suggested (Fennema and Franke 1992 in Grouws 1992) that the components of teachers’ knowledge appear to be situated in the narrow environment of the
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Comparing all these models of teachers’ knowledge, one consistency across all conceptions is the view that knowledge is never static but continually changing and developing. Teachers’ knowledge grows through interaction with the subject and with the students in the classroom, but also through professional experiences, amongst other components. Once this can be accepted, the challenge is to develop an understanding of this process and to discover what experiences contribute to the growth and changes.

The French scene

In France a distinction is made between la pédagogie (educational theory or pedagogy) and la didactique of a certain subject (for example, the didactics of mathematics is regarded as the science of teaching and learning mathematics). In simple terms, pedagogy is more general than didactics. The term pedagogy is generally used in terms of education and, according to Houssaye (1994, p.13), includes general educational theory, such as socio-psychological theory, for example. Didactics is, according to Henry (1990), in general terms ‘the study of phenomena of the teaching and learning in one discipline’ which specifically includes not only the teaching but also the learning of the subject. Thus, the term pedagogy is used in a much wider sense and not specialised on one subject, whereas didactics refers more precisely to the teaching and learning of a specific subject. In France the discipline of didactics of mathematics is a recognised discipline in education and is always associated with the teaching of the subject. Robert (1988) explains:

Une des ambitions de la didactique, a contrario, est d’essayer de préciser le plus scientifiquement possible les véritables marges de manoeuvre de tout enseignant de mathématiques dans sa classe, en analysant le fonctionnement de l’ensemble du système et de chaque composante, puis de développer et d’étudier certains choix, jugés optimaux dans la gestion globale et locale de la classe [One of the ambitions of didactics is to try to specify in the most scientific way the real possibilities for manoeuvre for all mathematics teaching in the class by analysing the functioning of the totality of the system and its components, and then to develop and study certain choices which are regarded to be optimal in the sense of general and individualised classroom management] (p.2).

Didactics (and pedagogy) are often presented in the form of a triangular model (see Henry 1990, or Houssaye 1994). Although this model is likely to be limited, it nevertheless establishes more clearly the objectives of the study of didactics.

The triangle has as its vertices the teacher, the pupil and the knowledge. These terms are of course generic in the sense that, for example, the term ‘teacher’ carries with it all the components of the educational system that assigns this role to him/her. ‘Knowledge’ means all available knowledge in mathematics, for example, including the school mathematics communicated through the curriculum, but also knowledge of its transformation for the purpose of teaching. (See also Chapter 8, p.110)

The three axes of the triangle which link, for example, the teacher to the knowledge on the one hand and to the pupil on the other hand (and also the pupil to the knowledge), are represented by processes and conceptions which are now explained.

Teaching is represented by the link between the teacher and the knowledge. Teachers with their education and professional experiences take with them into the classroom a number of perceptions about their work, about the subject, about the teaching of the subject and about the pupils. These, in turn, influence the teaching of the subject. This axis also includes the epistemology of teachers.
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concerning their teaching. For example, in the 1960s, mathematics teachers in France were influenced by the Bourbaki movement and the ‘New Maths’ (see Moon 1986).

The relationship in terms of teaching between teacher and pupil is represented by the axis which links pupil and teacher. For example, studies on traditional teaching (in the sense of traditional transmission style) and, opposed to it, a more active approach explain some of the possible relationships.

The link between the pupil and the knowledge is represented by the process of learning. The cognitive psychologists have contributed a lot to the understanding of this link. For example, Jean Piaget asserts in his Theory of Equilibrium that a child periodically reorganises his/her knowledge by means of overcoming situations where s/he was confronted with problems. He therefore puts the action as the determining factor for learning. For the mathematics educators, the action consists of problem solving and this is regarded as the essence for the learning of mathematics. But in this link between the pupil and the knowledge, one also has to understand the role of barriers to learning which manifest themselves in mistakes. Mistakes are important in the sense that they produce a break and in turn allow pupils to review their knowledge or procedures. Moreover, the teacher’s attitude towards mistakes reveals his/her perceptions of the learning of mathematics.

Research into general didactics, and the didactics of mathematics in particular, has developed certain concepts around which all present work evolves. In the French research on the didactics of mathematics, there are two distinct, but interrelated, theoretical fields: the theory of didactical transposition, based on the work of Chevallard since the 1980s (Chevallard 1991); and the theory of didactical situations, developed by Brousseau (1986) since the beginning of the 1970s and developed by subsequent researchers since. The didactics of mathematics is seen here as the study of the issues of the preparation of mathematics for students.

The two theoretical approaches concern fundamental but different levels of didactical analysis. Artigue (1994) asserts:

... the theory of didactical transposition concentrates on the analysis of these processes that are based on reference knowledge, particularly on the reference knowledge produced by the legitimizing mathematical institution (scholarly knowledge), that lead to objects of teaching (knowledge to be taught) that are found in the daily life of the class (taught knowledge). It tries to go beyond particular studies and highlight certain laws and regularities in these complex transposition processes. ... To a certain extent, the theory of didactical situations is situated at a more local level. It aims to model teaching situations so that they can be developed and managed in a controlled way. (p.28)

However, both theories emphasise the need to view the study of didactics by acknowledging the systemic features. Therefore, to prepare mathematics for students is perceived as a didactical task that requires a more global systemic analysis (Artigue 1994).

The concept of the transposition didactique (Chevallard 1991) is explained as the process from the savoir savant (scientific knowledge – knowledge which is accessible through books and magazines and generally accepted as knowledge by the research community) to the savoir enseigné (taught knowledge – knowledge proposed to pupils in the form of textbooks, for example, or that adopted in class in order to foster pupil learning). This process of transforming subject knowledge into taught knowledge is, of course, the very process that Shulman calls for when referring to pedagogical content knowledge. Therefore, by adopting the didactical transposition approach, one acknowledges the institutions at the source of knowledge.
On the other hand, the approach via the theory of didactical situations concentrates on a narrower system: the didactical system built around the teacher and his/her students, and situated within the society in which the teaching system is located. This theory is based on a constructivist approach, in the sense that knowledge is constructed through adaptation to a ‘problematic’ environment (‘disequilibrium’ stage). It aims to become a theory for the control of teaching situations in their relationship with the production of mathematical knowledge. The didactical situation is, therefore, made up of mainly three components: the teacher, the pupil and the knowledge. The aim is to develop the conceptual and methodological means to control the interacting phenomena and their relation to the construction and functioning of mathematical knowledge in the student (Artigue 1994).

Two of the main concepts in this field are the contrat didactique and the contrat disciplinaire. The concept of the contrat didactique (initiated by G. Brousseau in the early 1980s) concerns the relationship between teacher and pupils. He specifies:

... nous appelons contrat didactique l’ensemble des comportements (spécifiques) du maître qui sont attendus de l’élève et l’ensemble des comportements de l’élève qui sont attendus du maître [... We call didactical contract all the (specific) behaviour of the teacher which is expected of him/her by the pupil, and all the behaviour of the pupil which is expected of him/her by the teacher] (in Houssaye 1994, p.41).

Colomb (in Houssaye 1994) describes two models of this didactical contract and asserts that teaching now is situated somewhere within the range between those two models which represent the two extreme ends of the range. In the first model the teacher is the guardian of a body of knowledge, and the pupil is expected to adapt his/her knowledge to the body of knowledge presented to him/her. This implies most of the time that the pupil has to suppress and negate his/her own understanding. In the second model the teacher delegates the whole responsibility in terms of knowledge to the pupil who is expected to construct knowledge from his/her own understanding.

The second concept, that of the contrat disciplinaire, is described in a similar form as the contrat didactique. Colomb (in Houssaye 1994) defines it in the following way:

... nous avons introduit le concept de contrat disciplinaire: ‘Ensemble des comportement du maître qui sont attendus de l’élève et ensemble des comportements de l’élève qui sont attendus du maître dans une discipline au cours d’une année scolaire’. [... the whole behaviour of the teacher expected by the pupil and the whole behaviour of the pupil expected by the teacher, in one subject over the course of a school year] (p.46).

In terms of more concrete concepts of teaching and learning there exists a wide literature in the area of mathematics education in France. According to the literature, at the beginning of the 20th century the traditional concepts of teaching were based on perfect exposition (and transmission), repetition and calling on pupils’ memory rather than understanding. Traditionally teaching has been based on the ‘empty vessel’ (or ‘incremental steps’) views, but the literature now supports and encourages the ‘constructivist’ view. In other words, there has been a shift in the literature towards a constructivist view of learning.

Mante (1989) has produced a critique of the traditional perception and presented a constructivist concept in terms of their limits and espoused views, which represents this shift well. He also proposes the triangular relationship between the knowledge taught, the pupil and the teacher, and asserts that behind every learning situation there are certain perceptions of learning involved. He defines three
perceptions of learning: the perception of the ‘empty vessel’; that of ‘little steps’; and the constructivist view. It would be too lengthy to explain the three concepts in detail, but it might be interesting to connect them to concepts of teaching which are commonly known in England, France and Germany.

At the extreme lies the *cours magistral* (traditional teacher-led, lecture-type lesson) where the teacher transmits the content without concern about what is learnt. The traditional French *cours magistral* is an example of the concept of the ‘empty vessel’.

An example of the concept of ‘little steps’ is the lesson where the teacher gives the pupils a large number of relatively easy examples and exercises, in order for the pupils to understand the essence of the lesson. Some computer-assisted teaching schemes work in this way.

Examples of the constructivist concept are situations where teachers provide pupils with investigative activities. However questions emerge of, for example, initial perceptions of pupils about the subject matter. Mante (1989) suggests that mistakes can help teachers to discover pupils’ initial perceptions. There are also questions over which role the teacher takes in this scenario. These relatively recently emerging concepts of teaching in the constructivist way are perhaps rare, but teachers are encouraged (by inspectors, in-service courses, journals and textbooks) to try them out (see Pepin 1997).

By looking at these theories and concepts of didactics in France one can detect at least two underpinning beliefs: firstly, that for a teacher just knowing the subject matter is not sufficient for teaching the subject; and secondly, that there are underlying common processes in learning in school (common to most pupils) which can be made evident. These two assumptions give rise to the demand for investigating into teaching and learning (a specific subject), and for making this a scientific discipline, *la didactique*.

When comparing France (and the French cultural traditions, such as egalitarian views, for example) with England (and the English cultural traditions, such as individualism, for example) one starts to understand why there have not been developments towards didactics in Britain. Didactics is based on the assumption that there are common processes involved in human learning. If this assumption is negated, because of individualistic views, this explains in part why didactics, or a science of teaching, could not develop in Britain. This was earlier described as the ‘missing paradigm’. Simon (1994) argues that for ‘a combination of social, political and ideological reasons (for example, the British public schools which dominated educational institutions for a long time had no regard for pedagogy in the sense of a systematic and rational approach to teaching a certain subject’) (p.14) pedagogy has ‘never taken root and flourished in Britain’. Based on the recognition that there is a human capacity for learning and that the process of learning among human beings is similar across the human species as a whole, he calls for ‘a renewed understanding both of the power of education to effect human change and especially cognitive development, and of the need for the systematisation and structuring of the child’s experiences in the process of learning’ (pp.16, 17). As a result he accuses the child-centred approaches (for example, propagated by the Plowden Report (1967) in England) of starting from the ‘wrong’ position (individual differences). Simon (1994) explains that in order ‘to develop effective pedagogy means starting from the opposite standpoint, from what children have in common as members of the human species, to establish the general principles of teaching and, in the light of these, to determine what modifications of practice are necessary to meet specific individual needs’ (p.18). He believes that the aim is to develop skills and abilities in all children which in turn involves ‘importing a definite structure’ into the teaching and thus into the learning experiences of pupils. In his view and from the social point of view, the means of promoting human qualities and characteristics cannot be left to individual teachers ‘on the grounds that each individual child is unique so that the development of pedagogy is both impracticable and
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According to him, teachers need ‘assistance in the pursuit of their common objective – the education of a new generation of pupils’ which requires ‘carefully defined goals, structure and adult guidance’ (p.20).

Therefore, according to Simon’s theory, didactics are necessary on the basis of egalitarian assumptions and for the purpose of educating the greatest number in the ‘best possible’ way. What is interesting in this sense is that recently in France where the education system has been centralised and didactics has been well developed, the tendency is to ‘go individual’ (in the sense of attending to the needs of the individual child and to see the class as being constituted of 30 individuals). In England, where individualism in education has been well-rooted over the decades and no didactics has been developed, the recent tendency is to ‘go common’ (in the sense that whole-class teaching is advocated, therefore to see the class as a whole).

Another point to be made here is that by looking at the models of teaching and learning in England and France, it seems that the Anglo/American research has been more empirically based than the French. The theoretical conclusions drawn from the Anglo/American research appeared to have emerged straight out of the empirical data. This belief in empiricism, research and theoretical conclusions on the ‘here and now’ (together with the belief in individualism) did not appear to allow researchers in England to develop a construct such as that of didactics. In French didactical research it seems as if there has been another layer of abstraction, in order to organise the thinking (for example, constructs such as transposition didactique), although didactical constructs are informed by empirical research. This point is taken up later when comparing German didactics with American research in teaching.

Furthermore, another issue concerns the distinction between didactics and pedagogy, and their links. As didactics does not appear to exist in England, one cannot look for clues in the ‘pure’ British or American literature. The French literature, however, has given a starting point (that pedagogy is wider cut than didactics). It asserts that didactics mainly concentrates on the teaching and learning process of a particular subject in class, in the sense that it does not encompass pedagogical problems such as discipline problems in class, or general motivation. Tochon and Munby (1993) give an interesting explanation in this respect: they define didactics as ‘the organisation of subject-matter knowledge through time as a preactive or postactive anticipation (before or after the classroom interaction synchrony), whereas pedagogy stems from the interactive management of time’ (p. 206). Leinhardt and Greeno (1986) called didactics and pedagogy the teacher’s ‘double agenda’. In this sense didactics deals with ‘content processing which implies planning a sequential time, while pedagogy is concerned with students’ relationships to knowledge and behavioural actualisation of teaching within real time’ (Tochon and Munby 1993). In other words, didactics is concerned with the preparation and teaching of a specific subject (even topic) which also implies considerations on the learning process (see Piagetian ideas in France, for example), whereas pedagogy is concerned about the ‘here and now’ of teaching in order to accommodate the varying context of teaching itself.

The German Didaktik

There have been several authors (for example, Kansanen 1995a, Hopmann and Riquarts 1995, Hamilton and Gudmundsdottir 1994) who have written on issues concerning the dialogue between the German Didaktik on the one hand and the American research on teaching and the curriculum traditions on the other hand. In this section the different views are explored in connection with what are regarded as significant curriculum-didactics debate, and how these compare with the particular traditions.
As in French didactics literature, in Germany a core concept in the development of didactics has been the \textit{Didactic triangle} with its three components of the content, the learner and the teacher, in order to structure the field of didactic research and theory. This shows that the common strand is the belief in an integrative approach, in the sense of including the subject to be taught, the student and the teacher into the research on teaching and learning.

According to Kansanen (1995a) there are several theoretical models which developed over the centuries and which can be regarded as the main traditions of the German \textit{Didaktik}. At the beginning of the 17th century there was the ‘founder’ model of Ratke and Comenius, which regarded didactics as the art or study of teaching (\textit{Lehrkunst}). During the following centuries the influence of Herbart altered the position of didactics, in the sense that he turned didactics into a discipline of its own (instruction under the conditions of schooling as distinct from general educational theory, such as education in the family). Whereas Herbart himself argued for an active interplay between the three components (teacher, student and content), Herbart’s followers changed his analytical tools into formal stages of instruction. Every hour and minute of the lesson was supposed to follow the same pattern (an example is the Prussian school routine). At the beginning of the 20th century ‘reform pedagogy’ influenced the new modes of thinking. The aim was an ‘animated, multifaceted education’ (Hopmann and Riquarts 1995) with child-centred activities. Kerschensteiner, Gaudig and Petersen were its main representatives. On the theoretical side, a theory of education and teaching emerged, \textit{geisteswissenschaftliche Didaktik}, which built on the philosophy and pedagogy of Dilthey. Its main representatives were Nohl, Weniger and Klafki. This new didactics did not find many followers in non-German speaking countries and therefore became an almost entirely German concept. In the early 1960s the Berlin School (Heimann, Schulz, Otto) proclaimed an empirical-analytic paradigm amongst didactic theories which professed explicit connections with Anglo-Saxon research on teaching. In addition, Habermas initiated another alternative approach, the critical-communicative didactics, which was based on critical theory. These relatively recent theoretical models have had numerous variations and didactics has kept close contact with teacher education.

The American tradition of research on teaching can be traced back to pragmatism and representatives such as Spencer, James, Dewey and Heard. Its fundamental interest was practical by nature. The predominant approach within research on teaching has been to start from the assumption that there are ‘expert’ teachers and to identify and determine those factors which are crucial in those teachers’ practice. In order to develop theoretical models empirical research (mainly quantitative) has been conducted and models have been tested in classroom situations (mainly based on the process-product paradigm).

Hopmann and Riquarts (1995) argue that the American educationists of the late 19th century cherished Herbart, but not the reform pedagogues who came after him, and therefore ignored the rise of changed didactics developing in Germany. The term didactics disappeared from the terminology, except for the ‘didactical’ teaching of the Prussian, pre-reform pedagogy period. According to Hopmann and Riquarts (1995) educationists like Dewey did not take from Herbart the whole of didactics but only the educational psychology that underpins it. Herbart became one of the founders of American educational psychology. With the psychology of education as the main background discipline, learning problems, for example are of major importance. The subsequent practical approach in American research in education centred around empiric-analytic research on learning, motivation or assessment, for example. It is claimed (Kansanen 1999) that the practical approach has neglected the importance of research into the content of the curriculum and its teaching.

The ‘curriculum’ tradition is often contrasted to German didactics. According to Mitter (1981) the term ‘curriculum’ should be related to two ‘core’ terms used in German educational theory and
practice, namely to *Didaktik* and *Lehrplan* (syllabus). He asserts that didactics is concerned with the pedagogical discipline dealing with the principles of education and subject matter. The original approach to didactics was derived from educational philosophy (as we have already seen) and therefore not necessarily linked to teaching and learning strategies, whereas the contemporary approaches are informed by learning theory, amongst others. The syllabus is the ‘normative framework for what has to be taught and learnt in schools’ (Mitter 1981). It is usually restricted to the definition of aims (in a generalised form in the preambles) and the specification of subject matter. On the other hand the Anglo-American term ‘curriculum’ refers to an integrated model including several or all of the following: the definition of aims and objectives; the selection of subject matter; the choice of adequate organisational forms, media and methods; the implementation of materials; and the evaluation of the implementation process (Mitter 1981). The application of the term ‘curriculum’ was based on the American idea of *Reformpädagogik* by Dewey whose focus was on the individual pupil and his/her learning experiences. The curriculum was, therefore, defined through learning experiences, it focused on the individual pupil and his/her learning experiences at school.

In 1967 Robinsonsohn introduced the term ‘curriculum’ into the German didactics debate and since then the terms ‘curriculum’ and ‘curriculum theory’ have been competing with traditional German terms in educational discussions, with the result that sometimes the term ‘curriculum’ is used for ‘syllabus’ and ‘curriculum theory’ for ‘didactics’ (Mitter 1981). According to Klafki (1974 in Kansanen 1999) it seemed as if *Didaktik* would be included in the more general concept of curriculum. Eventually in the early 1980s didactics and curriculum were accepted as parallel areas (Kansanen 1999). Kansanen (1995b) concludes that ‘the didactic aspects of curriculum have integrated into *Didaktik*, but that there is ‘hardly any evidence of impulses in the opposite direction’.

Kansanen (1995b) summarises the developments in America and Germany in the following way:

... it can be said that the erudition-centred *Didaktik* did not gain footing in the USA in the beginning of this century. Instead, the reflection on teaching continued in psychology of education. In Germany reform pedagogy transformed [the Herbartian didactics] into erudition-centred *Didaktik* which got later some rival directions. The empiric-analytic approach did not succeed in getting a breakthrough in Germany in spite of a good beginning ... It lived some time as descriptive *Didaktik* but did not develop into psychology of education. The latter got its impulses from the USA and has been a separate area alongside *Didaktik* (p.108).

What can be seen from the above developments is that ‘*die Didaktik*’ in Germany has always been a form of philosophical thinking, theorising and the construction of theoretical models’ (Kansanen 1999), whereas American research into teaching has been more empirically based. Comparing the descriptive *Didaktik* and Anglo-American research on teaching, Kansanen (1995a) argues that the difference between the two is ‘in their background or in the purpose of their model building’. He claims that *Didaktik* is of ‘genuine German origin’ and based on ‘philosophical traditions of its own with such names as Kant, Herbart, Schleiermacher, etc’. He asserts:

*Didaktik* is mainly intended for teacher education and the models are based on a philosophical conception of man and on the nature of research concerning his education. The empirical research results are not a prerequisite for model-building, but are used in a corrective way when they are in conflict with the model variables. Research on teaching reflects an empirical tradition and that is why its models are mainly inductive by nature and based directly on research results. Practical conclusions can, of course, be drawn from these models and thus they can also function in teacher education (Kansanen 1995a, p.348).
However, there is an empirical side of research into teaching in Germany, *Unterrichtsforschung*. Kansanen (1995b) claims that Didaktik subsumes *Unterrichtsforschung*, and that it appears that the more empirical elements are in a model of Didaktik, the more references can be found to American research on teaching.

The emergence of research on what constitutes teachers’ knowledge in a particular subject has created the *Fachdidaktik* (subject didactics) which denotes the pedagogical transformation of factual content for the purposes of teaching, taking into consideration all factors of the teaching-learning process. As mentioned earlier, in the French comparison, a comparison between pedagogical content knowledge and *Fachdidaktik* might produce fruitful discussions.

Referring back to the discussion of pedagogy and didactics, education can usefully be split into the two sub-areas of pedagogy and *Didaktik*. According to Shulman (1987) and Simon (1994), the sub-area of didactics appears to be missing. Kansanen (1995b) claims that much of its content belongs to educational psychology, and that in American literature of research on teaching ‘the problems of teaching and learning in general are usually held together without any theoretical model building’. The theoretical background in terms of philosophical questions is discussed when a suitable methodology is selected. Whereas in Germany *Didaktik* and educational psychology are clearly separate fields, in the USA the same people can be working in both areas, which influences the aspects of research chosen in the sense that there is more emphasis on the aspects of learning in the teaching process than in Germany.

**Concluding remarks**
There are at least two conclusions that can be drawn from the comparison of existing representations of knowledge in teaching in the Anglo/American, the French and the German educational scene. Firstly, there appears to be a commonality amongst representations of knowledge in teaching in the sense that it is not seen as static, but as a process of development, that it grows and changes, and that experience in the classroom contributes to its growth and change. Secondly, there seem to be differences in traditions within the research into knowledge of teaching. The German (and French) educational research into teaching appears to be traditionally concerned with philosophical thinking, theorising and the construction of theoretical models, the *Didaktik* (which is nevertheless informed by empirical research). Anglo/American educational research is to a large extent based on empirical studies, in order to identify and be able to determine factors that are influential for teaching (and learning) and to develop an understanding of the processes involved in teaching and learning.
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