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Rotating turbulent Rayleigh–Bénard convection subject to harmonically forced flow reversals

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The characteristics of turbulent flow in a cylindrical Rayleigh–Bénard convection cell which can be modified considerably in case rotation is included in the dynamics. By incorporating the additional effects of an Euler force, i.e., effects induced by non-constant rotation rates, a remarkably strong intensification of the heat transfer efficiency can be achieved. We consider turbulent convection at Rayleigh number $Ra = 10^9$ and Prandtl number $\sigma = 6.4$ under a harmonically varying rotation, allowing complete reversals of the direction of the externally imposed rotation in the course of time. The dimensionless amplitude of the oscillation is taken as $1/Ro^* = 1$ while various modulation frequencies $0.1 \leq Ro_\omega \leq 1$ are applied. Both slow and fast flow-structuring and heat transfer intensification are induced due to the forced flow reversals. Depending on the magnitude of the Euler force, increases in the Nusselt number of up to 400\% were observed, compared to the case of constant or no rotation. It is shown that a large thermal flow structure accumulates all along the centreline of the cylinder, which is responsible for the strongly increased heat transfer. This dynamic thermal flow structure develops quite gradually, requiring many periods of modulated flow reversals. In the course of time, the Nusselt number increases in an oscillatory fashion up to a point of global instability, after which a very rapid and striking collapse of the thermal columnar structure is seen. Following such a collapse is another, quite similar episode of gradual accumulation of the next thermal column. We perform direct numerical simulation of the incompressible Navier–Stokes equations to study this system. Both the flow structures and the corresponding heat transfer characteristics are discussed at a range of modulation frequencies. We give an overview of typical time scales of the system response.

Keywords: Rayleigh–Bénard convection; direct numerical simulation; modulated rotation; forced flow reversals; Nusselt number; turbulence

1. Introduction

Many flows in nature and technology are simultaneously driven by buoyant convection as well as influenced by rotation. Understanding these is essential for predicting heat and mass transfer, e.g., in cooling or controlled crystal growth. A simple model that captures the dynamic consequences arising from interactions between these central mechanisms is found in rotating Rayleigh–Bénard convection: a fluid layer enclosed vertically between parallel rotating walls is heated from below and cooled from above [1]. Much research has been directed to identify conditions that would lead to enhancing the efficiency of heat transfer.

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between the hot and the cold walls. This research established the influence on the Nusselt number due to properties of the fluid expressed by the Prandtl number, but also affects of the strength of the thermal forcing in terms of the Rayleigh number. Additionally, the consequences of rotation as quantified by the Rossby number [1–4] have been investigated. While remarkable insights have been gathered through physical experiments, theory and numerical simulation, the overall changes that were observed in the Nusselt number are quite modest. At constant rotation rate, a maximum increase of about 15% compared to the Nusselt number in the non-rotating case was reported in case water was used [5] while a maximal value of up to 40% was found in case the Prandtl number of the working liquid was varied [6]. This is small compared to heat transfer intensification associated with phase transitions [7]. In the latter, the transport of heat can be strengthened via evaporation and condensation, bringing latent heat contributions into play. This may increase the Nusselt number by a factor of 2–3 as observed in case of water droplets dispersed in a turbulent channel flow of air between two heated walls [8].

In this paper, we focus on the question to what extent time-modulated rotation affects the heat transport properties and flow structuring in a cylindrical Rayleigh–Bénard system. This study is motivated by investigations into ‘resonant turbulence’ in which one considers whether external forcing at specific frequencies and length-scales may induce an ‘optimal’ response of the flow system [9–13]. We present a novel approach to enhance heat transfer in which modulated rotation is employed to induce large-scale coherent flow structures in an overall turbulent flow. It is shown that modulated rotation allows to accumulate over time a domain-size coherency that supports much higher Nusselt numbers than reported until now. This may lead to Nusselt numbers comparable to values achieved using additional effects of phase transition. A first experiment carried out on a rotating Rayleigh–Bénard system subjected to modulated rotation has produced promising results [14]. Significant transient changes in the Nusselt number were observed upon periodically changing the rotation rate. The experimental set-up allowed to measure the heat transfer but further information regarding flow structuring is not available.

The inclusion of a time-dependent rotation rate gives rise to the so-called Euler force density in the governing equations. This additional force may qualitatively alter the flow. Instead of either a domain filling large-scale circulation (LSC; occurring at low constant rotation rates) or dispersed local thermal plumes (at high constant rotation rates) [1,5,15–17], we observe a qualitatively different situation: a pronounced thermal column forms along the centreline of the domain and highly sheared structures appear in the boundary layers near the vertical sidewalls. The ability to manipulate these flow structures allows to influence small-scale mixing [18] and heat transfer characteristics. A dominant Euler force yields very complex multi-scale flow dynamics: a long-time build-up of domain-sized thermal structures arises in an oscillating manner, interspersed by events of very abrupt and considerable collapse of these thermal structures. Associated with this collapse is a strong reduction of the thermal transport efficiency, as quantified by the Nusselt number $\text{Nu}$. This presents an interesting challenge in physical control of such turbulent flow [19], aimed at building up high-$\text{Nu}$ flow structures by modulated rotation, but avoiding the $\text{Nu}$-collapse.

We consider modulated rotating Rayleigh–Bénard convection in which the rotation rate is varied harmonically around zero. In other words, the cylinder changes rotation about its vertical axis from clockwise to counter-clockwise and back, and does so periodically with a pre-specified frequency. This extends a previous study in which effects of modulated rotation were investigated in situations in which the direction of the rotation vector was kept the same, i.e., the rotation direction was always kept counter-clockwise [20].
We observe striking flow structuring on a relatively long time scale, combined with more rapid oscillations that occur on the scale of the imposed external flow forcing. For the case considered, i.e., at a Rayleigh number \( Ra = 10^9 \) and Prandtl number \( \sigma = 6.4 \), we notice a ratio of about 10 between the time scale at which the global flow structuring accumulates to full strength and the period of the harmonic modulation of the Euler force. Specifically, at the highest Euler forcing considered in this paper, an increase in the average Nusselt number of up to a factor of 4 (compared to the non-rotating situation) builds up in about 10 periods of the modulated rotation. In the regime of strong Euler forces, additional dynamics develop associated with nonlinear self-interactions of the induced turbulence. Each period of strong Nusselt build-up is followed by a very rapid collapse of the coherent thermal structure, initiating the next cycle of gradual build-up of the thermal column and associated high levels of heat transfer efficiency.

The organisation of this paper is as follows. We first present the governing equations and discuss the numerical method and resolution requirements in Section 2. Subsequently, in Section 3, the effect of time-modulated rotation is shown in terms of the changes in the turbulent flow structures that arise. The consequences for the transport of heat are discussed afterwards in Section 4, and the paper is completed with concluding remarks in Section 5.

2. Governing equations and numerical method

In this section, we first detail the mathematical model used to formulate turbulent flow in a rotating Rayleigh–Bénard configuration (Section 2.1, closely following [5,20]). Subsequently, we specify the numerical set-up (Section 2.2).

2.1. Governing equations for modified rotating Rayleigh–Bénard turbulence

The main aspects governing flow in a rotating Rayleigh–Bénard cylinder are (1) the effect of buoyancy associated with differences in the mass density arising from differences in the temperature and (2) the rotation itself, which will be expressed in terms of the commonly included Coriolis force, which is proportional to the instantaneous value of the rotation rate, and the less familiar Euler force, which arises due to temporal variations in the rotation rate. In this section, we outline the governing equations and describe the mathematical model, including the domain and boundary conditions.

We consider cylindrical domains filled with water, of height \( \tilde{H} \) and diameter \( \tilde{D} = \tilde{H} \), i.e., the aspect ratio \( \Gamma = \tilde{D} / \tilde{H} = 1 \). Here and in the sequel, we denote dimensional variables with a tilde, e.g., \( \tilde{H} \), and dimensionless combinations without the tilde, e.g., \( \Gamma \). The domain is allowed to rotate about the vertical \( \hat{z} \) axis with rotation rate \( \tilde{\omega}(\tilde{t}) = \tilde{\omega}(\tilde{t}) \hat{z} \) where \( \hat{z} \) denotes the unit vector in the \( z \)-direction and \( \tilde{t} \) the time. Gravity is assumed to work in the negative \( z \)-direction. We consider the flow in a co-rotating coordinate frame \( (\hat{x}, \hat{y}, \hat{z}) \) rotating with \( \tilde{\Omega}(\tilde{t}) \) relative to the inertial frame \( (\hat{X}, \hat{Y}, \hat{Z}) \). The relation between the velocity in the inertial reference frame (subscript ‘I’) and that in the rotating frame (subscript ‘R’) can be expressed as

\[
\left( \frac{d\tilde{r}}{d\tilde{t}} \right)_I = \left( \frac{d\tilde{r}}{d\tilde{t}} \right)_R + \tilde{\Omega}(\tilde{t}) \times \tilde{r} \tag{1}
\]
where the position vector is expressed as \( \vec{r} = \vec{r}_x \hat{x} + \vec{r}_y \hat{y} + \vec{r}_z \hat{z} \). For the acceleration, one may derive

\[
\left( \frac{d^2 \vec{r}}{dt^2} \right)_I = \left( \frac{d^2 \vec{r}}{dt^2} \right)_R + 2\Omega(\vec{r}) \times \left( \frac{d\vec{r}}{dt} \right)_R + \vec{\Omega}(\vec{r}) \times \vec{\Omega}(\vec{r}) \times \vec{r} + \frac{d\vec{\Omega}(\vec{r})}{dt} \times \vec{r}
\]

(2)

in which we explicitly distinguished the Coriolis, centrifugal and Euler forces, respectively. In incompressible flow, it is common to absorb the centrifugal force into the pressure term. For rotation about the z-axis, the Euler force can be shown to yield a contribution to the circumferential momentum transport equation only, while for turbulent three-dimensional flow, the Coriolis force is making itself felt in both horizontal directions.

In the Boussinesq approximation, the governing equations that describe incompressible, buoyant flow in a co-rotating frame of reference include conservation of mass and momentum which can be written as

\[
\vec{\nabla} \cdot \vec{u} = 0
\]

\[
\partial_t \vec{u} + (\vec{u} \cdot \vec{\nabla}) \vec{u} = -\vec{\nabla} \vec{p} + g\alpha \vec{T} + \nu \nabla^2 \vec{u} - 2\Omega(\vec{r}) \times \vec{u} - \vec{r} \frac{d\Omega(\vec{r})}{dt} \hat{\phi}
\]

(3)

where \( \vec{u} \) denotes the velocity field, \( \vec{p} \) the pressure and \( \vec{T} \) the temperature; \( \hat{\phi} \) is the unit vector in the azimuthal direction. The variable \( \vec{r} \) denotes the shortest distance to the axis of rotation. The evolution of the temperature field is characterised by an advection–diffusion equation, given by

\[
\partial_t \vec{T} + (\vec{u} \cdot \vec{\nabla}) \vec{T} = \kappa \nabla^2 \vec{T}
\]

(4)

In this formulation, we absorbed the centrifugal contribution into \( \vec{\nabla} \vec{p} \). Moreover, \( \vec{g} \) denotes the gravitational acceleration, \( \alpha \) the thermal expansion coefficient, \( \nu \) the kinematic viscosity and \( \kappa \) the thermal diffusivity.

The time modulation of the rotation rate is taken in this paper as

\[
\vec{\Omega}(\vec{r}) = \vec{\Omega}_0 + \Delta\Omega \sin(\omega \vec{r})
\]

(5)

where we denote the mean rotation rate by \( \vec{\Omega}_0 \), the depth of modulation by \( \Delta\Omega \) and the frequency of modulation by \( \omega \). This harmonic variation of the rotation rate is the first choice of perturbation protocol. More general, acceleration procedures can be selected in order to introduce specific modulation patterns and indirectly achieve some control over the transport properties in the flow. This is a subject of future research, associated with the development of physical control strategies tailored for heat transfer optimisation in modulated rotating Rayleigh–Bénard convection.

The formulation as given in Equation (5) can equivalently be expressed in terms of three ‘Rossby numbers’. In fact, multiplying Equation (5) by two times the time scale \( \bar{H}/\bar{U} \) in terms of a velocity scale \( \bar{U} \), we find

\[
\frac{1}{Ro}(t) = \frac{1}{Ro_0} + \frac{1}{Ro^*} \sin\left( \frac{t}{2Ro_0} \right)
\]

(6)
where \( t = \tilde{t}/(\tilde{H}/\tilde{U}) \), and we relate the total Rossby number \( Ro \) to
\[
Ro_0 = \frac{\tilde{U}}{2H\Omega_0}; \quad Ro^* = \frac{\tilde{U}}{2H\Delta\Omega}; \quad Ro_\omega = \frac{\tilde{U}}{2H\omega}
\] (7)

These Rossby numbers characterise the mean rotation rate through \( Ro_0 \), the 'modulation depth' through \( Ro^* \) and the 'modulation frequency' through \( Ro_\omega \). The factor '2' in the definition of these Rossby numbers is chosen to correspond to tradition in atmospheric boundary layer literature. These can be varied independently, giving detailed control over the precise flow regime that dominates the turbulent transport. In this paper, the primary interest lies with the dynamics induced by the Euler force in case of externally imposed flow reversals. We specifically consider cases in which the mean rotation is absent, i.e., \( Ro_0 \to \infty \) and the modulation depth is taken such that \( Ro^* = 1 \), providing a characteristic case of reference. Investigating the effects of the Euler force, we will systematically vary \( Ro_\omega \).

It is convenient to present the final computational model in non-dimensional form. We use the convective velocity scale \( \tilde{U} = \sqrt{g\alpha \Delta T \tilde{H}} \), the temperature scale \( \tilde{\Delta T} \), which is the temperature difference between the bottom and the top walls, and the length scale \( \tilde{H} \). The following dimensionless groups can be used to characterise the flow: \( Ra = (\tilde{g}\alpha \Delta T \tilde{H}^3)/(\tilde{\nu}\tilde{\kappa}) \) the Rayleigh number, \( \sigma = \tilde{\nu}/\tilde{\kappa} \) the Prandtl number and \( Ro_\omega \), the Rossby number of the modulation frequency, since we selected \( Ro_0 \to \infty \) and \( Ro^* = 1 \). The corresponding dimensionless form of the governing equations can be expressed as
\[
\nabla \cdot u = 0
\]
\[
\partial_t u + (u \cdot \nabla) u = -\nabla p + \tilde{T} \mathbf{\hat{z}} + \sqrt{\frac{\sigma}{Ra}} \nabla^2 u - \sin \left( \frac{t}{2Ro_\omega} \right) \mathbf{\hat{z}} \times u - \frac{1}{4Ro_\omega} \cos \left( \frac{t}{2Ro_\omega} \right) r \mathbf{\hat{\phi}}
\]
\[
\partial_t T + (u \cdot \nabla) T = \frac{1}{\sqrt{\sigma Ra}} \nabla^2 T
\] (8)

We dropped all tildes over the variables and differential operators to denote the final dimensionless formulation. As an example, this implies \( u = \tilde{u}/\tilde{U} \) and similarly for other flow components and variables with respect to their reference scales. These equations are defined in a cylindrical domain of equal height and diameter \( \tilde{H} \). Note that for the remainder of this paper, all variables are dimensionless. The boundary conditions for the velocity are taken as no-slip \( u = 0 \) at all the walls of the domain. For the temperature, we adopt Dirichlet conditions \( T = 1 \) at the bottom plate and \( T = 0 \) at the top plate, while at the sidewall of the cylinder, adiabatic conditions are expressed by the homogeneous Neumann boundary \( \partial_r T = 0 \). We consider pure water with Prandtl number \( \sigma = 6.4 \) as working fluid and investigate turbulent flow at \( Ra = 10^9 \), identical to the value used in the study of [15,16], in order to facilitate comparison with the constant rotation case.

### 2.2. Numerical set-up and spatial resolution

The dynamics of turbulent flow in a rotating Rayleigh–Bénard cylinder under modulated rotation conditions are investigated using direct numerical simulation (DNS) of Equation (8). The DNS is based on an extension of the method by [21–23] to also include the Euler force.
In cylindrical coordinates \((r, \phi, z)\) with \(r\) and \(\phi\) denoting the radial and circumferential coordinates, respectively, the governing equations (8) possess terms that include a factor \(1/r\). These need special treatment to be evaluated at the cylinder axis \(r = 0\). Verzicco & Orlandi [21] propose to rewrite the equations using \(q_r = ru_r\), \(q_\phi = u_\phi\) and \(q_z = u_z\) in terms of the three velocity components \((u_r, u_\phi, u_z)\). This, in combination with the use of a staggered grid (Figure 1), alleviates the problems near \(r = 0\). In fact, \(q_r(r \to 0) = 0\), and due to the staggered grid this is the only velocity component that needs to be evaluated at \(r = 0\).

The equations are discretised on the staggered grid by central finite-difference formulations of second-order accuracy. The solution method uses a fractional step procedure with the elliptic Poisson equation for the pressure inverted using trigonometric expansions in the circumferential direction and a direct solver for the other two directions. The discretisation method numerically preserves kinetic energy for inviscid conditions. The method was parallelised using OpenMP. Time integration is done with a third-order Runge–Kutta scheme with adaptive time step, consistent with the stability condition. Specifically, this implies a CFL number such that \(\text{CFL} = 1.5 < \sqrt{3}\) in which the latter is exactly at the stability boundary. Further details can be found in [21].

The current numerical code has been validated and used for many studies into turbulent convection. Hence, there is also a wealth of information on resolution requirements. Here we follow the resolution settings as used before in [16], with \((n_r, n_\phi, n_z) = (193, 385, 385)\), the number of grid points in the radial, azimuthal and vertical directions, respectively. The distribution of grid points in the radial and vertical directions was not uniform: close to the walls a denser grid is applied than in the centre. The maximal grid spacing in the centre is \(\Delta z = 0.0048\). For most of the (constant) rotation rates, this proved adequate to fully resolve the Batchelor scale \(\eta_B = \eta \sigma^{-1/2}\), which is the smallest lengthscale of temperature structures, which is smaller than the Kolmogorov lengthscale \(\eta\). For the boundary layers near bottom and top, a minimum of six grid points have been advised [22], which also follows from the criteria set forth in [24]. The current grid, designed for rapidly rotating convection with very thin Ekman boundary layers, actually provides 13 grid points within the thermal boundary layer. For reference, at the highest (constant) rotation rate considered in [16], the viscous boundary layer is thinner than the thermal layer and actually still contains 10 grid points.

Information on the current runs in terms of spatial resolution is gathered in Table 1. The Nusselt numbers are averaged in time; the standard deviation is also included, which is considerably larger than at constant rotation rates due to the quasi-periodic cycle of heat-flux rise and collapse found in this case. Still, the time-averaged Nusselt numbers
Table 1. Runs considered in this paper all adopt a resolution of 193 × 385 × 385 in the radial, azimuthal and axial directions, respectively. The oscillation frequency is quantified with the modulation Rossby number $Ro_{\omega}$ and the dimensionless modulation frequency $f = \tilde{\omega} H / (2\pi \bar{U})$. We report the resulting time-averaged Nusselt numbers $\langle Nu_w \rangle_t$ (evaluated at the bottom and top plates) and $\langle Nu_b \rangle_t$ (evaluated over the entire domain), as well as mean thicknesses of the thermal ($\delta_T$) and viscous ($\delta_v$) boundary layers, inferred from the location of the peak root-mean-square temperature and velocity, with corresponding number of grid points ($n_T$, $n_v$) contained to capture these boundary layers. The global estimates of the dissipation scales are also included: Kolmogorov scale $\eta = \sigma^{1/2} / (Ra Nu)^{1/4}$ and Batchelor scale $\eta_B = \eta / \sigma^{1/2}$.

<table>
<thead>
<tr>
<th>$Ro_{\omega}$</th>
<th>$f$</th>
<th>$\langle Nu_w \rangle_t$</th>
<th>$\langle Nu_b \rangle_t$</th>
<th>$\delta_T$</th>
<th>$n_T$</th>
<th>$\delta_v$</th>
<th>$n_v$</th>
<th>$\eta$</th>
<th>$\eta_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.796</td>
<td>187 ± 52</td>
<td>179 ± 49</td>
<td>0.0017</td>
<td>3</td>
<td>0.0032</td>
<td>6</td>
<td>0.0039</td>
<td>0.0015</td>
</tr>
<tr>
<td>0.2</td>
<td>0.398</td>
<td>129 ± 32</td>
<td>133 ± 33</td>
<td>0.0041</td>
<td>7</td>
<td>0.0033</td>
<td>6</td>
<td>0.0042</td>
<td>0.0017</td>
</tr>
<tr>
<td>0.5</td>
<td>0.159</td>
<td>74.3 ± 10.2</td>
<td>73.9 ± 25.4</td>
<td>0.0044</td>
<td>7</td>
<td>0.0069</td>
<td>11</td>
<td>0.0048</td>
<td>0.0019</td>
</tr>
<tr>
<td>1.0</td>
<td>0.0796</td>
<td>61.2 ± 9.7</td>
<td>59.6 ± 1.9</td>
<td>0.0076</td>
<td>12</td>
<td>0.031</td>
<td>40</td>
<td>0.0051</td>
<td>0.0020</td>
</tr>
</tbody>
</table>

evaluated at the bottom and top plates and the domain-averaged values are in satisfactory agreement. Due to the strong fluctuations in time, the boundary-layer scales are also to be interpreted as indicative values for the entire cycle. The enormous magnitude of the heat transfer enhancement reported here puts great demands on resolution throughout the domain, although the sheer size of the dominant flow structure (e.g. Figure 4) appears to be more easily resolved than the tiny plumes of ‘traditional’ Rayleigh–Bénard convection. These demands are most stringent at $Ro_{\omega} = 0.1$. A careful examination of the resolution is underway and will be reported elsewhere, related to the development of an algorithm for space–time parallel flow simulation.

In the next section, we consider the flow structure that develops due to a time-dependent rotation rate.

3. Qualitative flow-structure changes under modulated rotation conditions

We first present visualisations of the flow at slow but constant rotation rate, providing a point of reference for the discussion of alterations in the properties of the flow due to time-dependent rotation rates, which will be discussed subsequently.

Simulations at various constant rotation rates show that the organisation of the flow into coherent structures is strongly dependent on rotation [15,16]. Since in this paper we focus on a rotating Rayleigh–Bénard setting in which the mean rotation rate equals zero, we first consider flow in a slowly rotating system. For low rotation rates, the domain-filling LSC is the dominant feature. This is illustrated in Figure 2 in which we observe a dominant overturning of the flow of the size of the entire flow domain (Figure 2(a) and 2(b)). The temperature field does not show striking features at this low rotation rate but mainly displays a thin thermal boundary layer near the cold and the hot walls (Figure 2(c)). At larger rotation rates, an irregular, rapidly changing array of vertically oriented vortices is found. The turbulence intensity is reduced by strong rotation, compared to the non-rotating case, and the vertical inhomogeneity increases, reflecting the consequences of the thermal wind balance [16,25].

The effect of time modulation of the rotation rate depends strongly on the modulation frequency $Ro_{\omega}$. Changing $Ro_{\omega}$, i.e., changing the importance of the Euler force, may induce quantitative changes in the dominant structures of the flow. In Figure 3, we display snapshots of the vertical velocity at $t = 200$ that arise at different values of $Ro_{\omega}$. While at $Ro_{\omega} = 0.5$,
we may observe a domain-sized coherent structure with overturning flow (Figure 3(a)), reminiscent of the LSC seen at low, constant rotation rates, the flow is changed qualitatively as $Ro_\omega$ is reduced to 0.2 and 0.1 (cf. Figure 3(b) and 3(c)). Isosurfaces of the vertical velocity are highly fragmented near the vertical sidewalls. Both positive and negative values of vertical velocity appear quite close to each other, indicating high values of vorticity in the vicinity of the sidewalls, corresponding to qualitative changes in the properties of the transient boundary layers near this wall. The fragmentation is stronger as $Ro_\omega$ decreases further. A more precise comparison could be possible if one would consider snapshots at different moments in time but at comparable ‘phase’ of the external forcing. This was not selected here as mainly the characteristic qualitative impressions were the point of focus – for that purpose the choice $t = 200$ is adequate. We observe that the fragmentation of the velocity contours is more intense compared to the situation we studied in [20]. In that work, the rotation was modulated such that at any point in time the rotation was maintained
in a counter-clockwise sense. Apparently, at the same value of $Ro_\omega$, externally imposed flow reversals are more effective in breaking up the flow near the sidewalls compared to modulated rotation without such reversals. This also suggests stronger small-scale mixing in the boundary layers near the vertical walls. Finally, while relatively high values of vertical velocity are found fragmented in the shear layers near the vertical walls, the inner region of the flow domain is observed to be relatively smooth at time $t = 200$ at which the snapshots were recorded. Modulated rotation with external reversals appears to enhance the radial heterogeneity with distinctive dynamics near the sidewalls and in the inner region of the flow.

In Figure 4, an overview of temperature fields is shown at $t = 200$, characterising the qualitative effects due to changes in $Ro_\omega$. The modulation frequency is seen to have only a modest structural effect on the temperature contours at $Ro_\omega = 0.5$ as can be inferred from Figure 4(a) and 4(b). The flow is mainly in a state reminiscent of the LSC state at constant rotation, although an occasional vortical plume is seen in the temperature field, penetrating into the interior of the flow domain. Increasing the modulation frequency is seen to yield a strong thermal structure centred around the axis of the cylinder. The ‘middle section’ of the flow domain is occupied by a ‘thermal column’ if $Ro_\omega$ is sufficiently low. The width of this column appears to reduce slightly with reducing $Ro_\omega$. This thermal column induces an almost direct contact between hot and cold fluids, thereby creating a basis for an intensified heat transfer to which we return in the next section. The width of the thermal column is comparable to the domain size and likely to be dependent on $Ra$.

The snapshots of the velocity and temperature fields shown in this section yield a first impression of the qualitative changes in the flow and transport properties arising from rapidly modulated external reversals imposed on the Rayleigh–Bénard cell. However, the flow is more dynamic than might be inferred from the snapshots alone. This dynamics is characterised by both slow and rather fast time scales as will be discussed in the next section by concentrating on the Nusselt number.

4. Fast and slow dynamics of heat transfer

In this section, we first present the calculation of the Nusselt number $Nu$, expressing the effectiveness of the heat transfer in the modulated rotating Rayleigh–Bénard configuration. Then, we focus on the case $Ro_\omega = 0.2$ and compare $Nu$ as evaluated at the bottom and top walls with the bulk value. This will show a characteristic dynamics: the imposed fast external modulation frequency is observed next to a much slower phenomenon associated with the gradual accumulation of the thermal column followed by its very rapid collapse. Together, this leads to a characteristic slow ‘pulsation’ of $Nu$ on top of which a rapid oscillatory behaviour may be seen. Subsequently, we present the dependence of the fast and slow dynamics on $Ro_\omega$ and establish the scaling of the main time scales with changing Euler force amplitude.

In order to quantify the consequences of time-modulated rotation on the efficiency with which heat can be transferred from the hot bottom plate to the colder top plate, we concentrate on the Nusselt number $Nu$. We may evaluate the Nusselt number in different ways. A well-known convenient expression with which the time-dependent $Nu$ can be obtained from the gradient of the temperature at the bottom and top walls is

$$Nu_w(t) = \langle \partial_z T \rangle_w$$  \hspace{1cm} (9)
Figure 4. Snapshots of isosurfaces of the temperature at $t = 200$ displaying the qualitative effect of modulation of the rotation rate using $Ro^* = 1$ and $Ro_{\omega} = 0.5$ (a, b), $Ro_{\omega} = 0.2$ (c, d) and $Ro_{\omega} = 0.1$ (e, f), showing a perspective and a side view. Labelling of the contours is as in Figure 2.
where we added the subscript ‘\(w\)’ to emphasise that the Nusselt number is found at the hot and cold walls. In this expression, \(\langle \cdot \rangle_w\) denotes averaging over the bottom or the top wall. An alternative expression for the Nusselt number is obtained by also incorporating the vertical convective transport \(\langle wT \rangle_z\), averaged over the radial \(r\) and circumferential \(\phi\) directions at any height \(z\) in the domain. Integration over the entire domain then yields the ‘bulk’ value \(N_{ub}\) given by

\[
N_{ub}(t) = \int_0^1 dz \left( \langle \partial_z T \rangle_z + \sqrt{\sigma Ra} \langle wT \rangle_z \right) = 1 + \sqrt{\sigma Ra} \langle wT \rangle
\]  

(10)

where the use was made of the fact that the temperature difference between the top and bottom walls is equal to unity and the fact that integration over \(z\) and over \((r, \phi)\) may be interchanged. Averaging \(\langle \cdot \rangle\) without a subscript implies integration over the entire domain.

The dependence of the Nusselt number on the Rossby number was investigated earlier for the case of constant rotation in [15]. It was shown that for water with \(\sigma = 6.4\), a maximum arises at \(Ro_0 \approx 2.5\) at which \(N_{uw}(Ro_0)/N_{uw}(\infty) \approx 1.15\), comparing the Nusselt number in the rotating system to the Nusselt number without rotation, \(N_{uw}(\infty)\). Moreover, if the rotation rate is increased further, corresponding to \(Ro_0 < 2.5\), a monotonously decreasing \(N_{uw}\) is found which is approximately equal to \(N_{uw}(\infty)\) as \(Ro_0 \approx 0.1\). The long-time averaged value of \(N_{uw}(\infty)\) in the current system was found to be around 66 [5,15].

The time dependence of the Nusselt number for the modulated rotation case at \(Ro_0 = 0.2\) is completely different from that seen at constant or no rotation. The ‘bulk’ and the ‘wall’ values are shown in Figure 5(a), together with an impression of the external forcing. Following an initial transient of about 30 dimensionless time units, we observe a more or less regularly repeating slow ‘pulsation’ in both \(N_{uw}\) and \(N_{ub}\). Qualitatively, both measures for the Nusselt number appear comparable, although the fluctuations about this slow pulsation in \(N_{uw}\) are much smaller than in \(N_{ub}\). In fact, \(N_{ub}\) displays very rapid oscillations about the global pulsatile behaviour, while \(N_{uw}\) appears much closer to a simpler periodic behaviour.

Figure 5. Comparison between \(N_{uw}(t)\) (black solid line) and \(N_{ub}(t)\) (red line) at \(Ro_0 = 0.2\), for a long time interval (a) and zoomed in on a shorter time interval (b). In (b) we also added dashed vertical lines at \(t = 130, 140, 150, 160\) which denote instants at which snapshots of the temperature are presented in Figure 6, respectively, in (a, b), (c, d), (e, f) and (g, h).
at one frequency. We notice, in addition, periods of remarkably high values of the heat transfer efficiency, up to \( \approx 170 \) for \( \text{Nu}_w \) and even occasionally up to \( \approx 200 \) for \( \text{Nu}_b \). Next to these episodes with quite high instantaneous values, we observe regularly a rather dramatic collapse of the heat transfer, as measured by \( \text{Nu}_b \), dropping down to values on the order of that seen in the non-rotating case. In Figure 5(b), we show a zoomed-in view of one such pulsatile episode. This zoomed-in view shows that the signal for \( \text{Nu}_w \) precedes that of \( \text{Nu}_b \) in the sense that growth (decrease) in \( \text{Nu}_w \) arises earlier than in \( \text{Nu}_b \). A closer graphical inspection of \( \text{Nu}_w \) displays small oscillations about the main more-or-less periodic behaviour, at the same frequency as given to the external forcing. This direct connection to the external forcing is not seen as clearly in \( \text{Nu}_b \), for which we also notice faster time scales than those introduced by the external forcing. We return to the frequency composition of the response in the Nusselt number momentarily. An additional interesting observation is the lag of \( \text{Nu}_b \) (solid red curve) with respect to \( \text{Nu}_w \) (dash-dotted black curve); apparently, the flow structure that enhances the heat flux first forms near the bottom and top plates and subsequently enters the bulk, before destabilising. This is in line with the visualisations in Figures 4 and 6.

The episodes of significantly increased heat transfer are supported by specific thermal structures in the flow. In order to illustrate such an episode and its collapse, we display in Figure 6 a sequence of snapshots of the temperature at \( t = 130, 140, 150 \) and 160, in which instants were selected in view of the time dependence in the Nusselt number as shown in Figure 5(b). Since the overall period of the pulsatile behaviour is \( \approx 30 \), the conditions at \( t = 130 \) and \( t = 160 \) are expected to be quite comparable. This is indeed also observed in the instantaneous solution, comparing Figure 6(a) and 6(b) with 6(g) and 6(h), which show a good general agreement. The maximal Nusselt number is attained at \( t \approx 140 \), at which we do see a thermal column which displays temperature isosurfaces at \( T = 0.05 \) and \( T = 0.95 \) that are remarkably close to each other near the middle of the flow domain (cf. Figure 6(c) and 6(d). In addition, around \( t = 150 \), we expect a much reduced thermal column corresponding to the stage of collapse of the Nusselt number that is seen in Figure 5(b). This is readily confirmed from the visualisation. We also considered changes in the structure of the velocity field, corresponding to the observed changes in Nusselt number. During periods of a still developing pronounced thermal column, high values of the vertical velocity are concentrated in fragmented patterns near the vertical walls as was shown before in Figure 3. Conversely, at stages close to a rapidly collapsing Nusselt number, high turbulent fluctuation levels are seen throughout the entire flow domain and not just in the shear layers. This was also observed in case of modulated rotation without imposed large-scale flow reversals in [20].

The influence of \( \text{Ro}_\omega \) on the dynamics of the bulk Nusselt number is quantified in Figure 7. We observe that the value of \( \text{Nu}_w \) generally increases with decreasing \( \text{Ro}_\omega \), i.e., in case the Euler force becomes more important in the flow. While the results for the Nusselt number at \( \text{Ro}_\omega = 1 \) are virtually indistinguishable from those of non-rotating flow, we notice a significantly higher variability at \( \text{Ro}_\omega = 0.5 \), while not yet increasing the average much. Some signs of sharp events of collapsing \( \text{Nu}_w \) can already be appreciated. This trend is further established when considering \( \text{Ro}_\omega = 0.2 \), at which the time-averaged Nusselt number is also seen to increase and events of collapse of \( \text{Nu}_w \) become more clear. We observe in addition that the period of the long-time pulsatile variation of \( \text{Nu}_w \) increases, i.e., the heat transfer is maintained at a high level for a longer time, while the period of the superimposed short-time oscillations appears to decrease. This trend is continued when changing to \( \text{Ro}_\omega = 0.1 \) at which peak values of \( \text{Nu}_w \approx 350–400 \) may be realised. This is almost 5–6 times larger than observed in the non-rotating case.
Figure 6. Snapshots of the temperature field in perspective and side views at $t = 130$ (a, b), $t = 140$ (c, d), $t = 150$ (e, f) and $t = 160$ (g, h) using $Ro_\omega = 0.2$. 
Figure 7. Time-dependent bulk Nusselt number $N_{ub}$ obtained for time-modulated rotation at $Ro^* = 1$ and $Ro_\omega = 0.1$ (red), 0.2 (black), 0.5 (blue) and 1.0 (magenta). In (a), a global view is displayed while in (b) the rapid oscillatory behaviour is illustrated in a zoomed view.
For an easier comparison of the dynamic response in case of strong Euler forces, i.e., $Ro_\omega = 0.1$ and $Ro_\omega = 0.2$, we display $Nu_b$ in rescaled time in Figure 8. Time is shown in terms of $\alpha_\omega t/(2Ro_\omega)$ in which the parameter $\alpha_\omega$ is introduced to allow an additional scaling of the time axis. We show approximately four repetitions of the build-up and collapse cycles of $Nu_b$ at $Ro_\omega = 0.1$, using $\alpha_\omega = 1$, i.e., directly displaying the results in terms of the frequency as used in the imposed rotation rate (6). This setting will be adopted as point of reference. In order to create a close correspondence in case $Ro_\omega = 0.2$, an additional scaling of the time axis with a factor $\alpha_\omega = 3.5$ was selected. The value $\alpha_\omega = 3.5$ does not yield a full collapse of the dynamic response of $Nu_b$ but a striking similarity is nevertheless observed. This indicates that under the selected flow conditions a full cycle of high $Nu_b$ build-up and collapse takes roughly $\alpha_\omega \Delta t_{Nu} / (2Ro_\omega) \approx 325$. Specifically, this implies that

$$\left(\frac{\alpha_\omega \Delta t_{Nu}}{2Ro_\omega}\right) / \left(\frac{\Delta t_{Ro}}{2Ro_\omega}\right) \approx \frac{325}{2\pi} \Rightarrow \frac{\alpha_\omega \Delta t_{Nu}}{\Delta t_{Ro}} \approx 50$$

for the ratio between the duration of the $Nu_b$ cycle, $\Delta t_{Nu}$, and the forcing period $\Delta t_{Ro}$. So, in the high Euler force regime it takes at $Ro_\omega = 0.1$, (0.2) roughly 50 (respectively 15) forcing periods for the Nusselt number to build up high values and subsequently collapse.

The response of the wall Nusselt number $Nu_w$ at various values of $Ro_\omega$ is collected in Figure 9(a). The general trends observed for $Nu_b$ in Figure 7 are also clearly expressed in
this figure. We observe that the peak Nusselt numbers strongly increase with decreasing \( Ro_\omega \). Moreover, the frequency with which the Nusselt cycle of high and low values takes place is seen to decrease considerably with decreasing \( Ro_\omega \). To facilitate a comparison of the dynamic response at different values of \( Ro_\omega \), we also included \( Nu_w \) in rescaled time in Figure 9(b). The inclusion of an additional rescaling of time through the introduction of \( \alpha_\omega \) is seen to yield a clear general correspondence in terms of observed frequency in rescaled time. It is of interest to further investigate the dependence of the time-scaling factor \( \alpha_\omega \) on \( Ro_\omega \). The degree of agreement observed in Figure 9(b) was obtained using \( \alpha_\omega = (1, 3.7, 21) \) for \( Ro_\omega = (0.1, 0.2, 0.5) \). Although the correspondence is not entirely strict, this hints at an approximately quadratic dependence of \( \alpha_\omega \) on \( Ro_\omega \).

Interpreting the response of \( Nu_w \) as a simple periodic function of time, we observe that the dominant period of the response as well as its amplitude is roughly proportional to \( \sim 1/Ro_\omega \). In fact, from Figure 9(a) we count about 3.5 cycles of variation in the Nusselt number at \( Ro_\omega = 0.1 \), about 6.5 cycles at \( Ro_\omega = 0.2 \) and about 16 such cycles at \( Ro_\omega = 0.5 \) between \( t = 0 \) and \( t = 200 \). This corresponds quite well with proportionality with \( 1/Ro_\omega \). We may also address the amplitude of the oscillations in \( Nu_w \). A closer inspection of Figure 9(a) reveals, for example, at \( Ro_\omega = 0.1 \), a maximum value of about 251 and a minimum of about 65 for \( 0 \leq t \leq 200 \), implying an amplitude of oscillations of about 93 about a mean of 157. Likewise, we find at \( Ro_\omega = 0.2 \) an amplitude of about 58 and a mean of about 120. Moreover, at \( Ro_\omega = 0.5 \), we observe an amplitude of about 18 and a mean of about 75. As before, although the correspondence is not strictly following \( 1/Ro_\omega \) there is a fair agreement, also for the amplitude of the oscillations. Finally, in Figure 10, we collect the dependence of the amplitude of the global oscillations in the wall Nusselt number \( \Delta Nu_w \) and the mean wall Nusselt number \( \bar{Nu}_w \) as function of \( Ro_\omega \). We observe a good agreement of \( \Delta Nu_w \) with \( 1/Ro_\omega \), as also discussed quantitatively above. Moreover, we observe a remarkably close agreement with a scaling \( \sim 1/\sqrt{Ro_\omega} \) for \( \bar{Nu}_w \). Further simulations at different \( Ro_\omega \) are planned to further underpin this dependence.

The spectrum of the response of the wall Nusselt number \( Nu_w \) at various values of \( Ro_\omega \) is collected in Figure 11. The values of the dominant frequencies establish once more that the main response in the Nusselt cycle is much slower than the time scale of the
Figure 10. Amplitude of global oscillations in the wall Nusselt number $\Delta Nu_w$ (a) and mean wall Nusselt number $\bar{Nu}_w$ (b) as function of $Ro_\omega$ (black) compared to a scaling $\sim 1/Ro_\omega$ in (a) and $\sim 1/\sqrt{Ro_\omega}$ in (b) shown as (red, dashed).

Figure 11. Amplitude spectrum $|A_f|^2$ of the response of the wall Nusselt number $Nu_w$ at $Ro_\omega = 0.1$ (red), $Ro_\omega = 0.2$ (black) and $Ro_\omega = 0.5$ (blue). Vertical dashed lines in the right-hand side of the figure correspond to the dimensionless forcing frequency as taken from Table 1.

external oscillations. This is particularly true in the case that $Ro_\omega$ is further reduced as seen from the distance between the maximum in the spectrum and the imposed forcing frequency. The latter is indicated in corresponding colours with vertical dashed lines in the figure.
5. Concluding remarks

In this paper, we presented DNS results of time-modulated rotating Rayleigh–Bénard convection in a cylindrical domain of unit aspect ratio. The inclusion of a time-dependent rotation rate introduces an additional term in the equations which represents the so-called Euler force. This force – which acts in the circumferential direction only – may qualitatively alter the flow: instead of a domain-filling LSC (at low constant rotation rates) or dispersed local thermal plumes (at high constant rotation rates), we observe a more or less segregated situation in which a pronounced thermal column forms along the centreline of the domain and highly sheared structures appear in the boundary layer near the vertical sidewalls. The flow structuring was observed during episodes of high Nusselt number, while events of strong collapse were found associated with velocity fields that displayed high turbulence levels throughout the entire domain. This suggests that the collapse of a thermal column originates from flow instabilities developing near the walls of the domain. Further research into this phenomenon is required in order to link the slow and fast dynamics to qualitatively different features of the flow. We studied the situation in which the external forcing corresponds to imposed large-scale flow reversals with varying frequency.

The peak Nusselt numbers observed are considerably larger than in cases without rotation. This is potentially of interest to new technological applications requiring rapid ‘dumping’ of excess heat without invoking heat transfer and associated explosive formation of vapour, for example, from evaporation of dispersed liquid droplets. The Nusselt number was found to increase in amplitude and period of the large-scale pulsation approximately as $\sim 1/R_\omega$.

A dominant Euler force was shown to yield very complex flow dynamics in which a long-time build-up of thermal structures arises in an oscillatory manner, interspersed by events of very abrupt and considerable collapse with associated strong reduction of the thermal transport efficiency, as quantified by the Nusselt number. This presents an interesting challenge in physical control of such turbulent flow, aimed at building up high-$Nu$ flow structures by modulated rotation. This is subject of ongoing research and new agitation procedures will be developed that prevent too high levels of turbulent velocity fluctuations near the centreline of the cylinder.

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References

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