Multicriteria evaluation on accessibility-based transportation equity in road network design problem

Tao Feng1* and Junyi Zhang2

1Urban Planning Group, Department of the Built Environment, Eindhoven University of Technology, Eindhoven, The Netherlands
2Transportation Engineering Laboratory, Graduate School for International Development and Cooperation, Hiroshima University, Higashi-Hiroshima, Japan

SUMMARY

This paper investigates the performance of accessibility-based equity measurements in transportation and proposes a multiobjective optimization model to simulate the trade-offs between equity maximization and cost minimization of network construction. The equity is defined as the spatial distribution of accessibilities across zone areas. Six representative indicators were formulated, including GINI coefficient, Theil index, mean log deviation, relative mean deviation, coefficient of variation, and Atkinson index, and incorporated into an equity maximization model to evaluate the performance sensitivity. A bilevel multiobjective optimization model was proposed to obtain the Pareto-optimal solutions for link capacity enhancement in a stochastic road network design problem. A numerical analysis using the Sioux Falls data was implemented. Results verified that the equity indicators are quite sensitive to the pattern of network scenarios in the sense that the level of equity varies according to the amount of overall capacity enhancement as well as the assignment of improved link segments. The suggested multiobjective model that enables representing the Pareto-optimal solutions can provide multiple options in the decision making of road network design. Copyright © 2012 John Wiley & Sons, Ltd.

KEY WORDS: accessibility; bilevel programming; equity; multiobjective optimization; network design problem

1. INTRODUCTION

The road network design problems (NDPs) have been addressed in existing literatures from a wide perspective with the emphasis to make an optimal design to mitigate possible externalities, for example, traffic congestion and environmental emissions, in conjunction with road pricing or incentives. However, NDP is inherently multiple objective because of the sensitive characteristics of network performance and the variation of travel behavior. A feasible network design should not only serve the short-term cost–benefit balance but also meet sustainability requirements in which equity issues are regarded as same importance as economic sustainability progress and environmental conservation. In urban area, a nearly equal network cost to different destinations for different people indicates an equitable level of spatial distribution. In recent years, both academic research and practical applications on transportation equity have been put forth. Transportation practitioners in the United States, for example, have been advised to avoid disproportionate adverse impacts on minority and low-income groups and to mitigate such impacts when possible.

In spite that many existing studies address the issue of transportation equity, the concept and/or measurements vary. Yang and Zhang pointed that equity can be in general classified into either social or spatial perspective. The social equity basically refers to the differences of income or social welfare

*Correspondence to: Tao Feng, Urban Planning Group, Department of the Built Environment, Eindhoven University of Technology, The Netherlands. E-mail: t.feng@bwk.tue.nl or ftaocn@hotmail.com

Copyright © 2012 John Wiley & Sons, Ltd.
between individuals or certain population groups. It can be considered as the fairness or justice of the distribution of the impacts (both benefits and costs) of an action on two or more subgroups. The spatial equity commonly indicates the distribution of levels of transportation services (e.g., travel time, cost, distance, and number of transfers) for travelers between different locations by different travel modes. Because the distribution level is dependent on the state of traffic on the road network, the NDP plays a central role on the assessment of spatial equity.

Because of the increasing importance of distributional impact of transportation policies, some researchers (e.g., 4–6) have attempted to apply the formal indicators that are popularly used in evaluating poverty and social welfare to evaluate transportation equity. The primary concern of such a measurement is to represent the differences of impacts by policy interventions across the whole transportation network. Although several different functional forms are available, their performances and implications have not been sufficiently addressed in measuring the spatial equity, especially in the context of NDP. In addition, taking the financial budget on network improvement into account, it is important to provide policy makers with a quantitative decision tool in trading off equity and other policy concerns.

Therefore, the purpose of this study is to investigate the performances of spatial equity measurements in NDP and propose a policy decision tool for trade-offs between equity and construction cost of network improvement. Here, the equity is defined on the basis of accessibilities at zone level and formulated by six representative indicators, including GINI coefficient (GINI), Theil index (THEIL), mean log deviation (LDEV), relative mean deviation (RDEV), coefficient of variation (COV), and Atkinson index (ATK), reflecting different dispersion levels of accessibility distribution across space. An equity maximization model established using bilevel programming approach is adopted specifically to check the different performances of these equity indicators in the context of stochastic NDP (SNDP). This is motivated by the fact that existing studies that only adopted one of these indicators for network analysis (e.g., 7,4) have not clarified the question about whether the derived conclusions are consistent across equity indicators. More specifically, a multiobjective bilevel model with equity maximization and construction cost minimization is proposed to specify the Pareto-optimal solutions of link capacity enhancement. Given the level of improvement link capacities, the lower level problem is formulated as a deterministic user equilibrium model reflecting the mechanism of traffic flow distribution. The link flow obtained from assignment procedure is taken into the upper level problem to find the feasible solution of link capacity enhancement that in turn provides inputs for the lower level problem. Such a modeling approach is capable to incorporate interaction effects between policy goals in the upper level and network performance in the lower level.

The remainder of this paper is organized as follows: Section 2 will briefly review the studies related to transportation equity, paying particular attention to the definition and measurement approach. Afterwards, the formulations and implications of six representative indicators as well as the proposed multiobjective model will be presented in Section 3. A numerical example based on the Sioux Falls data will be introduced in Section 4. Section 5 will present the findings and associated analyses. After some further discussions in Section 6, this study will be summarized and concluded in Section 7, where future research potentials are also indicated.

2. EQUITY ISSUES AND MEASUREMENT APPROACHES

Equity issues in transportation field have been attracting more and more attentions in recent years from both theoretical studies and practical applications. Discussions cover a wide range of policy concerns, such as demographics, income class, geographic location, travel mode, vehicle type, industry, and trip type and cost. Regarding the methodology in measuring equity, existing indicators have different functional forms and implications. Researchers or planners may adopt one or two more indicators per analysis unit (e.g., per vehicle kilometer, per trip, and per dollar paid) to evaluate the equity. However, there is still a possibility that a particular decision may seem equitable when evaluated in one way but inequitable when evaluated in another. Selection of a feasible indicator is generally context specific such that the methods or conclusions are hard to be generalized.

In the context of transport NDP, the equity issue is generally simulated as the impacts on different individuals or groups caused by changes in transportation services. The impacts on travel time or travel...
cost by policies such as network improvement or road pricing have been discussed in a number of studies. For instance, Yang and Zhang 2 discussed the multiclass network toll design problem by incorporating social and spatial equity constraints. It was found that for the congestion pricing problem, there were significant differences of the benefits between some origin–destination (O–D) pairs. Meng and Yang 8 discussed equity issues gained from the benefit of continuous NDP (CNDP), where a bilevel model incorporating equity as a constraint was proposed. The equity was defined as the ratio of travel time before and after the link capacity enhancement. Results demonstrated the benefits of a capacity improvement in some selected links could lead to an increase in travel costs for other O–D pairs. Further, Chen and Yang 9 discussed two stochastic models with both travel demand uncertainty and equity.Significance of equity issues and demand uncertainty was confirmed in the numerical experiment. Using the similar concept of equity measure, Lee et al. 10 proposed a model representing an equity-based land-use and transportation problem. The model can specify the benefit distribution among network users and the resulting equity associated with land-use development problem in terms of the change of O–D travel cost in the state of equilibrium. Furthermore, Szeto and Lo 11,12 and Lo and Szeto 13 discussed the time-dependent NDP from the perspective of consumer surplus by taking into account the cost recovery and spatial equity issues, which is defined by the difference between total travel cost and willingness to pay.

The levels of equity discussed earlier are mostly measured by the use of travel time, travel distance, or travel cost directly. A different way in accessing transportation equity is the introduction of formal indicators, such as GINI and THEIL. One of the advantages of the formal indicators is its capability in representing the level of distributional differences of transportation resources and services. For instance, Connors et al. 6 used THEIL to examine transportation equity in the context of a SNDP. The equity indicator was formulated by users’ utilities from different user classes with different values of time and link-specific tolls. Zhang et al. 5 conducted a comparative analysis on transportation accessibility in developing cities from equity perspective. The levels of equity of nine developing cities were calculated by use of five accessibility-based measurements. The applicability of these measures was confirmed on the basis of the data collected by Japan International Cooperation Agency. Further, Santos et al. 4 incorporated the formal equity indicators into SNDP by proposing an optimization model that was built in the context of national road planning. The model’s objective was integrated by three elements reflecting different equity perspectives: the accessibility to low-accessibility (regional) centers, the dispersion of accessibility across all the centers defined using GINI index, and the dispersion of accessibility across all the regional centers and across the centers within the same region measured using THEIL.

In spite of the increasing number of equity evaluation adopting formal indicators, there are few research concerning on their performances in response to policy interventions. Further, most NDP-related studies treat equity as an element of constraint conditions. However, like other system targets, equity can be also considered as a goal of policy decision, which is especially important to consensus building when multiple stakeholders are involved in the decision-making processes. Therefore, this paper will investigate the sensitive performances of different equity indicators and propose a model for policy decision making by incorporating equity into NDPs. Here, we define the equity on the basis of sensitivities at zones for the sake that accessibility has been convinced to be an important evaluation indicator of network performance (e.g., 14). Performance sensitivities of six frequently used equity indicators are specifically analyzed and simulated on the basis of an equity maximization model. Then, a multiobjective model aiming at minimizing both inequity and construction cost of link capacity enhancement is proposed to identify the Pareto-optimal solutions of network improvement.

3. MODELING FRAMEWORK

Because the spatial equity issue has not been sufficiently addressed in literatures, we deal with the accessibility-based equity in the context of SNDP. To find the optimal solution of link capacity enhancement, we propose a multiobjective model using bilevel programming approach. In the subsequent sections, we will first introduce the conception and the formulation about accessibility-based equity measurement using formal indicators. Afterwards, the multiobjective model and the solution algorithm will be described accordingly.
3.1. Accessibility-based equity measurements

Equity can be measured in various ways, such as the quality of services perceived by different groups 3, the benefit distribution from road improvement 8, and the degradation of urban quality of life for environmental equity 15,7. Looking at spatial equity specifically, there is a need to measure equity by reflecting the distributional extent of policy impacts. Regarding the practical popularity in different fields, we adopt six representative equity measures that are formulated on the basis of zonal accessibility, as shown in the following.

a) GINI coefficient (GINI)

\[
Z_{\text{gini}} = \frac{1}{2N^2A} \sum_{j \in N} \sum_{k \in N} |A_j - A_k| \tag{1}
\]

b) Theil index (THEIL)

\[
Z_{\text{theil}} = \frac{1}{N} \sum_{k \in N} \frac{A_k}{\bar{A}} \log \left( \frac{A_k}{\bar{A}} \right) \tag{2}
\]

c) Mean log deviation (LDEV)

\[
Z_{\text{ldev}} = \frac{1}{N} \sum_{k \in N} |\log(\bar{A}) - \log(A_k)| \tag{3}
\]

d) Relative mean deviation (RDEV)

\[
Z_{\text{rdev}} = \frac{1}{NA} \sum_{k \in N} |\bar{A} - A_k| \tag{4}
\]

e) Coefficient of variation (COV)

\[
Z_{\text{cov}} = \frac{1}{\bar{A}} \left[ \frac{1}{N} \sum_{k \in N} (A_k - \bar{A})^2 \right]^{1/2} \tag{5}
\]

f) Atkinson index (ATK)

\[
Z_{\text{atk}} = 1 - \frac{1}{\bar{A}} \left[ \left( \frac{1}{N} \sum_{i=1}^{N} (A_i)^{1-\varepsilon} \right)^{1/(1-\varepsilon)} \right], \text{ when } \varepsilon \neq 1 \tag{6}
\]

\[
Z_{\text{atk}} = 1 - \frac{1}{\bar{A}} \left( \prod_{k \in N} A_k \right)^{1/\varepsilon}, \text{ when } \varepsilon = 1 \tag{7}
\]

where \(Z\) is the equity indicator, \(A_k\) is the accessibility at zone \(k\), \(\bar{A}\) is the average accessibility across the whole network, \(N\) is the total number of zones, and \(\varepsilon\) is a parameter to reflect decision-making concerns about the distribution of accessibility benefits across zones.

The ATK is a measure proposed by Atkinson 16. The distinguishing feature of the ATK is its ability to gauge movements or emphasis changes in different segments of the distribution, here specifically...
referring to the accessibility distribution. Depending on the values of $\varepsilon$, various types of decision-making concerns can be reflected. When $\varepsilon$ is positive, the equity level will be largely influenced by lower zonal accessibility, meaning that those zones with lower accessibility will be given a higher weight in policy decision. When $\varepsilon$ is zero, trade-offs between lower and higher accessibilities will be completely compensated with a result of equity as 0. When $\varepsilon$ equals 1, Equation (7) can be derived, which means that a Nash-type equity can be reached. In short, the closer to zero the parameter $\varepsilon$ is, the higher the compensation of trade-offs can be reached and vice versa. In reality, the problem becomes how to select the value of $\varepsilon$. This should be determined by considering the preferences of various stakeholders in practice for the sake of consensus building. To meet the requirements in economic theory and keep the consistency with previous research 17, we set $\varepsilon$ within $[0, 1]$ in the numerical study.

Among all the indexes listed earlier, GINI is probably the most popular one used to measure dispersion of distribution of income and wealth. It is defined as a ratio with values ranging between 0 and 1. Similar to the inherent meaning in evaluating social welfare, a lower GINI indicates a more equal accessibility distribution. The value “0” corresponds to perfect equity, meaning that each zone has the same level of accessibility, and “1” corresponds to perfect inequity, indicating that only one zone obtains the whole accessibility, whereas other zones all have a zero level of accessibility. The values of 0 and 1 are theoretically two extreme cases that are impossible to happen in real situations.

Theil index is also popular in measuring income equity. Actually, THEIL and mean LDEV are two special cases of the general entropy (GE) model proposed by Theil in 18. The GE model can be formulated as follows:

$$GE(\rho) = \frac{1}{\rho^2 - \rho} \left[ \frac{1}{N} \sum_{k \in N} \left( \frac{A_k}{A} \right)^\rho - 1 \right]$$  \hspace{1cm} (8)

where $\rho$ is a parameter to represent distribution variations. The values 1 (equally sensitive to changes across distribution) and 0 (sensitive to low end of the distribution) correspond to THEIL and mean LDEV, respectively. The values of both indicators range from 0, when all zones have the same level of accessibility, to $\log N$, when one zone has the total of accessibilities.

The RDEV and COV are two statistical evaluation indicators, which are generally used to clarify distributional dispersion (represented as the distance from the average level). They are both within the range from 0 to 1, with 0 as the situation without any difference across distributions. High values of those indicators indicate low level of equity. It should be mentioned that the GE model turns to one-half of the COV when $\rho$ equals to 2.

The concept of accessibility-based equity means that the dispersion of accessibility across zones should be in fairness. Without considering the differences of distributional emphases, the most ideal state is that the accessibilities at all zones are equally same, indicating that people in any of the zones can access to their destinations with the same level of transport service (e.g., travel cost and/or time). The completely equal accessibility is hard to achieve and such an extreme case is normally irrelevant to practice; however, the concept of accessibility-based equity in measuring spatial dispersion can be easily generalized in other situations. For instance, the popularity of compact city planning and transit-oriented development (TOD) in practice suggests the rationality of spatially skewed distribution of accessibility. The compact city concept attaches the most important to the accessibility at central urban areas, whereas the TOD imposes to areas along transit lines. To accommodate such spatially skewed accessibility, the distributional variations need to be captured with weighted importance. The areas with high residential density should be given more emphasis on accessibility than scattered areas. In reality, we can define the weight parameters additionally in terms of regional characteristics, such as population, average income level, network density, and size of the area. In the current study, we assume that the accessibility at each of the zones has a same weighted importance, leaving the spatially skewed accessibility as a future research issue.

### 3.2. An optimization model to minimize inequity and construction costs

To incorporate equity issue into the policy-making process, we propose an additional modeling approach in the context of CNDP. It is fundamentally a multiobjective optimization model built with
the bilevel programming approach. The two targets, maximization of accessibility-based equity and construction cost of link capacity enhancement, are formulated as the objective functions in the upper level problem. The mathematical formulations are shown as follows:

**Upper level problem:**

Minimize:

\[ Z(A_k(y_i | i \in I_1) | k \in N) \]  

Minimize:

\[ \sum_{i \in I} g_i(y_i) \]  

Subject to:

\[ 0 \leq y_i \leq y_{0i} \]  

\[ A_k(y_i | i \in I_1) = \sum_{j \neq k} [P_j / t_{kj}(y_i | i \in I_1)]^0, \forall k, j \in N \]  

where

- \( Z \): the value of equity
- \( A_k \): the accessibility at zone \( k \)
- \( y_i \): capacity enhancement on link \( i \)
- \( y_{0i} \): the upper bound of capacity enhancement on link \( i \)
- \( P_j \): population in zone \( j \)
- \( g_i(y_i) \): construction cost function on link \( i \)
- \( t_{kj} \): average travel time from zone \( k \) to zone \( j \)
- \( \theta \): interzonal travel impedance parameter, and
- \( I_1 \): set of links with capacity enhancement.

The value of equity (\( Z \)) indicates the level of spatial differences in accessibilities across zones in the sense that smaller \( Z \) value means a more equal distribution of accessibility. The equity can be formulated as a function of zonal accessibility (\( A_k \)) as shown in Equations (1)–(7).

The accessibility in general can be defined at either individual level or zone level. The individual-based accessibility concerns about the opportunity issue that an individual at a given location possesses to participate in a particular activity or a set of activities (e.g., 19). Effects of spatial, temporal, and interpersonal constraints on the accessibility can be well evaluated on the basis of the individual-based accessibility, and as a result, the accessibility can be applied to evaluate a wide range of policies. However, the individual-based accessibility suffers from the disadvantage that it is data intensive. For the current study, it would be more operational to adopt the conventional location-based accessibility measure. Such type of accessibility has been popularly applied in zone-based travel demand analysis based on gravity-type trip distribution models. Hence, without loss of generality, we define the accessibility as a function of zonal population \( P_j \) and interzonal travel time, \( t_{kj} \), as represented in Equation (12).

The decision variable of link capacity enhancement (\( y_i \)) on the selected link \( i \) can be specified within the range of \([0, y_{0i}]\). The upper bound of the constraint (\( y_{0i} \)) is given by, for example, taking into account the actual limitations of road space. This could be understandable especially in high density urban areas where space for any expansion of road segment is substantially limited.

The objective function in the upper level problem consists of two parts: minimization of inequity and construction cost of network improvement. This is a typical multiobjective optimization problem, where policy makers want to maximize the benefits of network improvement (here, refers to the
minimization of distribution differences of accessibility across zones) on the one hand and to minimize
the associated construction cost on the other hand. The trade-off effects between two objectives can be
measured by the set of Pareto-optimal solutions, suggesting that there is no absolute optimal solution
for both of the objectives simultaneously. Policy makers can trade off between equity and construction
cost in terms of their specific contexts.

Given the link capacity enhancement, it is necessary to capture the travel choice behavior to
calculate the network performance correctly. For simplicity, we adopt the standard user equilibrium
model with fixed demand in lower level to accommodate the effects of link capacity enhancement
on travel choice. The model is shown as follow.

Lower level problem:

Minimize:

\[ \sum_i \int_0^{v_i} t_i(x) \, dx \quad (13) \]

Subject to

\[ q_{jk} = \sum_r f_{jk}^r, j, k \in N, r \in R_{jk} \quad (14) \]

\[ v_i = \sum_r \sum_{jk} f_{jk}^r \delta_{jk}^r, i \in (I_1, I_2), r \in R_{jk}, j, k \in N \quad (15) \]

\[ t_i(v_i, y_i) = t_i(0) \cdot \left[ 1 + \alpha (v_i / (S_i + y_i))^{\beta} \right], \forall i \in I_1 \quad (16) \]

\[ t_i(v_i) = t_i(0) \cdot \left[ 1 + \alpha (v_i / S_i)^{\beta} \right], \forall i \in I_2 \quad (17) \]

\[ f_{jk}^r \geq 0, r \in R_{jk}, j, k \in N \quad (18) \]

where

- \( t_i \) : actual travel time on link \( i \)
- \( t_i(0) \) : travel time under free flow on link \( i \)
- \( v_i \) : traffic flow on link \( i \)
- \( f_{jk}^r \) : traffic flow on path \( r \) between zone \( j \) to zone \( k \)
- \( q_{jk} \) : travel demand (traffic flow) between zone \( j \) and zone \( k \)
- \( \delta_{jk}^r \) : link-path incidence
- \( S_i \) : capacity of link \( i \)
- \( R_{jk} \) : set of paths between zone \( j \) and zone \( k \)
- \( I_2 \) : set of links without capacity enhancement, and
- \( \alpha, \beta \) : impedance parameters.

The traffic assignment problem in lower level formulations can be solved on the basis of the traditional
Frank–Wolfe algorithm where users choose their routes with the shortest travel time. The link impedance
functions of CNDP are defined using the Bureau of Public Roads function. The denominators in the Bureau
of Public Roads functions are different in terms of the links with and without capacity enhancement, as
shown in Equations (16) and (17), respectively.

The nature of the bilevel model reflects the feedback mechanism between upper and lower level
problems. Planners determine the optimum link capacity enhancement by taking into account the
accessibility distributions across zones and relevant construction costs of network improvement at upper level, and the improved link capacity is taken into the lower level problem that consequently affects accessibility and equity at upper level. In this sense, the optimal solutions of network improvement are obtained as an equilibrium state between the two-level problems.

3.3. Solution method

In the field of transportation, several operational algorithms have been designed and applied in dealing with the bilevel programming problem. However, the theoretical global optimum cannot be guaranteed because of the inherent nonconvex characteristics and complexity of mathematical calculations. In practice, one of the efficient options is genetic algorithm (GA), which looks for the optimal solution by iterated progress in a given space in terms of fitness function values. The effectiveness of GA has been verified in many studies addressing bilevel programming problems (e.g., 20, 21, 10, 22).

Considering the characteristics of multiobjective bilevel optimization, we adopt a multiobjective GA to deal with the proposed model. Even though there is no single best solution with respect to both objectives, the nondominated solutions or Pareto-optimal solutions can be obtained. More specifically, we use the nonsorting GA (NSGA-II) as the main calculation engine. The elitism in NSGA-II can speed up the performance of the GA significantly and can also prevent the loss of good solutions once they are found 23. In addition, it can provide various Pareto-optimal solutions in a single run and consequently reduce the burden of performing multiple runs for various values of weights. NSGA-II also has the capability of constraint handling, which is useful to meet our calculation requirements. In the application of numerical example, a traffic assignment modular is embedded in the GA program to calculate the fitness value. The optimal results of decision variable of link capacity enhancement \( y_i \) are searched through a pre-given solution space by evaluating the fitness values. Only solutions that make the fitness value change along the direction of optimizing the objective function can be reproduced into the next generation.

4. A NUMERICAL EXAMPLE

To investigate the performance of accessibility-based equity indicators and the efficacy of the proposed model, we carry out a numerical analysis by using the Sioux Falls data, which was firstly built by Leblanc 20. The data consists of 24 zones, 24 nodes, and 76 links. The network depiction prepared by Meng and Yang 8 is shown in Figure 1.

The improved links are arbitrarily selected in advance along the horizontal line from nodes 12 to 18, passing through the links of 27, 29, 32, 33, 36, 48, 50, and 55. Because no population data are available in original dataset, zonal population is approximated in advance on the basis of trip generation from the O–D table. For the purpose of simulation, we assume the links can be either improved or not improved, labeled as 1 and 0, respectively. The link with improvement has a fixed level of capacity enhancement, 1000 pcu/h; as a result, the \( y_0 \) in Equation (10) equals to either 0 or 1000. Other parameters of \( \alpha \), \( \beta \), and \( \theta \) are given as 0.15, 4, and 1 for all the links, respectively. To be simple, an assumption is given for construction cost calculation that one unit of construction cost equals to one unit of link capacity enhancement. Then, the total of construction cost equals to the total of link capacity enhancement in values.

Here, the probabilities of crossover and mutation in the GA are set as 0.7 and 0.9, respectively. Because the improved links are selected in advance, the length of each chromosome in the GA process equals the number of the selected links. Regarding the efficient searching for optimal solutions with the GA, we set the population size as 200 by following the suggestion by Goldberg 24 that at least 200 of the population size of chromosomes can be convinced to be sufficient to find out optimal solutions. The maximum number of generations is set as 250.

5. RESULTS

To investigate the sensitive performances of the proposed equity measurement as well as the multiobjective model, all indicators discussed earlier are examined in terms of different link capacity improvement scenarios, and a vector of possible solutions is used to clarify the trade-offs between equity and network construction cost.
5.1. Levels of equity without network improvement

To preview the levels of these equity measurements, an evaluation process is implemented for all indicators in case of the current situation without network improvement. Concretely speaking, the accessibilities at zones are obtained through a standard traffic assignment procedure with fixed demand, and the equity values are calculated using Equations (1)–(7), as shown in Table 1.

It is found that the levels of different equity indicators are different while with all values ranging between 0 and 1. In case of the Atkinson indicators specifically, the values change increasingly from 0.00159 to 0.01083 when \( e \) changes from 0.1 to 0.7. Here, a higher value of \( e \) denotes a higher degree of inequity, implying that higher weights are attached to the zones with low accessibility. For purpose of comparative analysis in the rest of this paper, we arbitrarily set \( e \) as 0.5 for convenience.

The differences among all indicators are significant because these indicators are formulated in different ways in which different sets of weights are inherently attached to transfer at various points in a distribution. To avoid any contradictory conclusion by directly comparing these indicators, we investigate their performance sensitivity upon varied network improvement scenarios.

<table>
<thead>
<tr>
<th>Indicators</th>
<th>Equity values</th>
</tr>
</thead>
<tbody>
<tr>
<td>GINI</td>
<td>0.09666</td>
</tr>
<tr>
<td>THEIL</td>
<td>0.01602</td>
</tr>
<tr>
<td>LDEV</td>
<td>0.13810</td>
</tr>
<tr>
<td>RDEV</td>
<td>0.13978</td>
</tr>
<tr>
<td>COV</td>
<td>0.18447</td>
</tr>
<tr>
<td>ATK (( e = 0.1 ))</td>
<td>0.00159</td>
</tr>
<tr>
<td>ATK (( e = 0.3 ))</td>
<td>0.00473</td>
</tr>
<tr>
<td>ATK (( e = 0.5 ))</td>
<td>0.00781</td>
</tr>
<tr>
<td>ATK (( e = 0.7 ))</td>
<td>0.01083</td>
</tr>
</tbody>
</table>

Figure 1. Sioux Falls network.
5.2. Performances of the equity indicators: equity maximization

Here, we adopt the single-objective optimization model based on equity maximization only, where the construction cost is excluded from the objective function. The optimization procedure is implemented for each of the six indicators. Because the solution space is limited in the sense that in total 256 possible sets of solutions that exist for each case of network improvement scenarios, we present the descriptive statistics on equity values.

The values in Table 2 are calculated from the optimal equities that are based on different network scenarios. It shows that the values of equity vary in a small numerical scale (GINI, for example, changes on the interval of 0.00827). This probably means that a small variation of equity value might indicate a significant difference of accessibility distribution. Comparing these results with the average values in Table 1, the equities after network improvement in some cases are lower than those without improvement. This means that the link capacity enhancement would not definitely lead to equity increase.

Table 3 shows the optimized equity values and other associated evaluation indicators for each of the six measurements. The titles in the first column relate to the model names with equity optimization (i.e., the M1 means the equity maximization model using GINI). Values in diagonal cells with bold characters correspond to the optimal equity with respect to the six indicators (called optimal values). Others are nonoptimal values that are indirectly calculated using the optimal network solutions associated with the optimized equity. As expected, the optimal equity values for each of the models are no larger than the nonoptimal ones that are calculated according to the derived optimal link capacity improvement. This is the case that, for example, all nonoptimal values of THEIL (in the second column) are larger than the optimal level of equity (0.01554), indicating the less equities.

The optimal solutions of network improvement scenarios with respect to the six indicators differ in specific links and the total amount of improvement. One can find that the optimal solution calculated with respect to one indicator would not always be the optimal for others. In the cases of the GINI and ATK05, capacity improvements at Links 5 and 8 lead to the optimal equity values, whereas the LDEV and RDEV models have the optimal solutions at Links 5 and 7. In addition, COV and THEIL, respectively, require the largest (4000 pcu/h) and the second largest (3000 pcu/h) amounts of capacity improvement.

In response to choose the most feasible indicators among different measurements in practice, the results of total network time cost and average accessibility are calculated, as shown in Table 3. The total time cost indicates the network congestion efficiency at equilibrium. It is consistent with what is popularly adopted in network analysis by summing the product between link volume and travel time. Results show that COV has the minimum time cost and the lowest average accessibility and THEIL relates to the highest level of average accessibility and the second smallest value of total time cost. Nevertheless, to compare the results from models using different equity indicators is difficult. In reality, decision makers may need to specify the most suitable indicator according to their specific emphasis. One may evaluate the indicator performance in terms of one or several possible policy concerns, that is, level of equity, average accessibility, system travel time, investment for capacity improvement, and so on. As in the case here, THEIL will outperform other indicators according to the average accessibility, whereas COV will be the most feasible option if considering the total network cost specifically.

Table II. Descriptive statistics on different equity indicators.

<table>
<thead>
<tr>
<th>Equity indicators</th>
<th>Average</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>GINI</td>
<td>0.09811</td>
<td>0.09468</td>
<td>0.10295</td>
</tr>
<tr>
<td>THEIL</td>
<td>0.01655</td>
<td>0.01554</td>
<td>0.01808</td>
</tr>
<tr>
<td>LDEV</td>
<td>0.13837</td>
<td>0.13131</td>
<td>0.14654</td>
</tr>
<tr>
<td>RDEV</td>
<td>0.14043</td>
<td>0.13368</td>
<td>0.14839</td>
</tr>
<tr>
<td>COV</td>
<td>0.18789</td>
<td>0.18172</td>
<td>0.19643</td>
</tr>
<tr>
<td>ATK05</td>
<td>0.00488</td>
<td>0.00459</td>
<td>0.00534</td>
</tr>
</tbody>
</table>
5.3. Sensitivity analysis

The formulations of accessibility-based equity indicate that they might be sensitive to the patterns of network improvement that indirectly influence accessibility levels at zones. Here, we check the sensitivity of these indicators from two perspectives: (i) assuming the same set of links but different amount of capacity enhancement for one of the links (Case I) and (ii) assuming different set of links but the same total amount of capacity enhancement (Case II).

In Case I, we set two contexts that vary in the assigned links (here, Link 27 and Link 55) for network improvement scenarios. The links of 32, 33 and 48 obtain 1000 pcu/h of capacity enhancement, whereas keeping other links without improvement. As show in Table 4, the two contexts demonstrated consistent performances according to different indicators. The values of all equity indicators, except for LDEV and RDEV, change decreasingly with respect to the increase of link capacity enhancement. In addition, LDEV has the same performance as RDEV, resulting in a sudden increase when the link capacity changes from 0 to 1000 pcu/h.

In Case II, which has the same total amount of network improvement but different set of links, is designed by considering the issue of budget allocation for network improvement. An example of calculated equities by THEIL indicator is presented in Figure 2. It is obvious that, even with the same total amount of capacity enhancement, the resulted equities are different. Values are sensitive to which links observe the capacity improvement. This is understandable from the perspective of NDP where network efficiency in general depends on the efficacy of link capacity improvement scenarios. It further enhances our understanding that a feasible network design scenario should not only serve to the congestion alleviation but also the balance of spatial differences.

Table III. Calculation results with equity maximization only.

<table>
<thead>
<tr>
<th>Models</th>
<th>GINI</th>
<th>THEIL</th>
<th>LDEV</th>
<th>RDEV</th>
<th>COV</th>
<th>ATK05</th>
<th>Total time cost</th>
<th>Average accessibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1: GINI</td>
<td>0.09468</td>
<td>0.01555</td>
<td>0.13163</td>
<td>0.1396</td>
<td>0.18247</td>
<td>0.00756</td>
<td>8198539</td>
<td>883</td>
</tr>
<tr>
<td>M2: THEIL</td>
<td>0.09515</td>
<td>0.01554</td>
<td>0.13397</td>
<td>0.13604</td>
<td>0.18196</td>
<td>0.00756</td>
<td>8124333</td>
<td>1022</td>
</tr>
<tr>
<td>M3: LDEV</td>
<td>0.09481</td>
<td>0.01565</td>
<td>0.13368</td>
<td>0.18304</td>
<td>0.00760</td>
<td>8225280</td>
<td>1009</td>
<td></td>
</tr>
<tr>
<td>M4: RDEV</td>
<td>0.09481</td>
<td>0.01565</td>
<td>0.13131</td>
<td>0.13368</td>
<td>0.18198</td>
<td>0.00756</td>
<td>8225280</td>
<td>1009</td>
</tr>
<tr>
<td>M5: COV</td>
<td>0.09534</td>
<td>0.01557</td>
<td>0.13597</td>
<td>0.13763</td>
<td>0.18172</td>
<td>0.00756</td>
<td>7847980</td>
<td>659</td>
</tr>
<tr>
<td>M6: ATK05</td>
<td>0.09468</td>
<td>0.01555</td>
<td>0.13163</td>
<td>0.1396</td>
<td>0.18247</td>
<td>0.00756</td>
<td>8198539</td>
<td>883</td>
</tr>
</tbody>
</table>

Table IV. Results of sensitivity analyses across different equity indicators (Case I).

<table>
<thead>
<tr>
<th>Case I</th>
<th>GINI</th>
<th>THEIL</th>
<th>LDEV</th>
<th>RDEV</th>
<th>COV</th>
<th>ATK05</th>
</tr>
</thead>
<tbody>
<tr>
<td>Link 27</td>
<td>+0</td>
<td>0.09696</td>
<td>0.01595</td>
<td>0.13281</td>
<td>0.13495</td>
<td>0.18456</td>
</tr>
<tr>
<td>+1000</td>
<td>0.09573</td>
<td>0.13483</td>
<td>0.13693</td>
<td>0.18434</td>
<td>0.00767</td>
<td></td>
</tr>
<tr>
<td>+3000</td>
<td>0.09534</td>
<td>0.13319</td>
<td>0.13534</td>
<td>0.18394</td>
<td>0.00760</td>
<td></td>
</tr>
<tr>
<td>+5000</td>
<td>0.09432</td>
<td>0.13053</td>
<td>0.13236</td>
<td>0.18147</td>
<td>0.00753</td>
<td></td>
</tr>
<tr>
<td>Link 55</td>
<td>+0</td>
<td>0.09696</td>
<td>0.01595</td>
<td>0.13281</td>
<td>0.13495</td>
<td>0.18456</td>
</tr>
<tr>
<td>+1000</td>
<td>0.09573</td>
<td>0.13483</td>
<td>0.13693</td>
<td>0.18434</td>
<td>0.00767</td>
<td></td>
</tr>
<tr>
<td>+3000</td>
<td>0.09534</td>
<td>0.13319</td>
<td>0.13534</td>
<td>0.18394</td>
<td>0.00760</td>
<td></td>
</tr>
<tr>
<td>+5000</td>
<td>0.09432</td>
<td>0.13053</td>
<td>0.13236</td>
<td>0.18147</td>
<td>0.00753</td>
<td></td>
</tr>
</tbody>
</table>
5.4. Trade-offs between equity and the construction cost

Considering that solutions for equity maximization only may deviate from other policy concerns, it is necessary to further investigate the trade-offs between equity and the associated construction cost. As presented in the proposed multiobjective model, a vector of Pareto-optimal solution can provide multiple options for decision makers especially when there are budget limitations on network investment. The trade-offs between the total construction cost and the level of equity by different measurement approaches are plotted in Figure 3.

In general, the Pareto-optimal solutions are not unique, and they cannot be improved with respect to any objective without worsening the other objective. Considering the number of possible solutions is countable, we plot the whole solutions in Figure 3. The Pareto-optimal solutions can be obtained by proposed GA algorithm in the case of more complicated applications. As represented in Figure 3, the scenario with the highest total construction cost (8000) does not result in the highest level of equity for each of the indicators. This means higher amount of capacity enhancement would not always result into higher level of equity. There are multiple options for network improvement that can lead to the same level of equity. The best scenario depends on the number of improved links and which links are assigned for capacity enhancement.

6. DISCUSSIONS

The aforementioned simulation results demonstrated that the accessibility-based equity indicators have very sensitive performances in measuring the distributional difference across zones. Taking the results in Table 3 as an example, the optimal solution of link capacity enhancement calculated on the basis of one indicator may not be the minimum for others. This is mainly attributed to the nature of different equity formulations in which different weight schemes are inherently imposed. In response to the selection of most feasible indicator, decision makers should be aware the difference among these indicators. As revealed in the numerical example, LDEV and RDEV may have the same performance, and THEIL may relate to a high average accessibility when considering equity maximization only. Indeed, the attempt to rank these indicators, which are defined differently, seems to be tricky and out of the concern of this paper. A thorough theoretical induction and comprehensive numerical analysis might be contributable to this issue in future.

Considering that the equity in this paper is defined by accessibility at zone level, factors affecting zonal accessibility could consequently influence the level of equity. Here, the accessibility is treated as the function of interzonal travel time and population at zones. It is evident that other accessibility formulations will also affect the equity in the sense that accessibility at individual level can enhance its ability in representing the vertical equity with more social emphases. In the proposed bilevel model, the accessibility is calculated through a traffic assignment process, and as a result, the level of equity would be sensitive to the distribution of traffic flow. In case of long-term policy decision making, an average level of accessibility can be adopted, whereas for short-term applications, accessing

![Figure 2. Theil index by same capacity improvement but different link segments (Case II).](image)
accessibility-based equity needs to take the dynamics of network congestion into account. For example, a hierarchy scheme may be necessary according to peak-hour and off-hour evaluations. Although the proposed indicator is capable to measure transport equity, it does not indicate the extent of how equitable of the system is. In practice, it might be desirable to identify a feasible threshold for providing references in equity evaluations. For example, the society associated with income having a GINI that is no larger than 0.2 is empirically treated as be equitable. The distributional differences across zonal accessibilities may have specific characteristics that differ from the criteria used for income evaluation. However, it would be always true that the closer to zero of the value, the more equitable of the accessibility distribution.

7. SUMMARY AND CONCLUSIONS

The policy decision making issue in NDP involves in general multiple concerns (e.g., financial investments, economic, and environmental sustainability) from different stakeholders. Emphasizing the associated
construction costs of network improvement is of utmost importance for urban and transportation planners. The investment on road construction will benefit to increasing the network performance especially in urban areas with low density and improving local accessibilities. However, it may not automatically lead to an optimal balance of zonal accessibilities as an equitable distribution. Although a few existing studies have used formal indicators to measure the spatial equity, discussions on the definitions and the sensitive performances are scarce in previous literatures, especially in the context of NDP.

Therefore, in this paper, we investigated the performances of the spatial equity measurements that are formulated by six representative indicators and proposed a multiobjective optimization model to specify the trade-offs between spatial inequity and the associated construction cost of network improvement. The equity was defined by zonal accessibility and be represented by indicators of GINI, THEIL, mean LDEV, RDEV, COV, and ATK.

The performances of different equity measures were investigated on the basis of equity maximization model using the Sioux Falls network data. Results demonstrated that the accessibility-based equity measurements are very sensitive to the link capacity enhancement. The values of the six indicators have different scales in terms of their natures of formulations. The sensitivity analyses based on different scenarios found that the level of equity depends on the extent of network improvement as well as which links are assigned for capacity enhancement. A larger amount of network improvement would not always result in a higher level of equity. The scenarios having the same investment but different assigned links can have different level of equities, suggesting the necessity to carefully select equity indicators in policy decision-making processes. Regarding the trade-offs between network construction cost and the equity, a multiobjective model was proposed. The plot figures represented a clear paradigm on how much to invest and which link segments should be improved. This would provide meaningful references for decision makers to identify the feasible scenario from the vector of Pareto-optimal solutions.

For the purpose of simplicity, we assumed in this paper an equally weighted importance for each of the zones. However, policy decision makers may have different emphases in reality. For instance, the promotion of compact city design requires policies supporting the growth at central urban areas, indicating that higher weights to central urban areas should be set in the optimization processes. In addition, a policy of TOD strategy may require high weights for the areas along transit lines. In this sense, effects of unequal weighting schemes on network improvement should be examined in future. Nevertheless, it will be necessary to extend the proposed modeling approach to cover travel mode choices and transit network analyses when dealing with the TOD policy. Because the network improvement might result in different levels of accessibility for different population groups, the accessibility measurement adopted in this study should be further extended to reflect the heterogeneous responses across population, by taking into account the influences of not only individual attributes (e.g., age, gender, and income) but also the availability of different travel modes at individual levels, at different time periods of a day, at different days of a week, and so on. More importantly, it will be necessary to clarify which equity indicator should be used in practice considering the different consequences. Therefore, both theoretical studies and applications should be further evaluated in future in a more general sense of sustainability requirement as well as simulations based on actual network data.

8. LIST OF SYMBOLS AND ABBREVIATIONS

8.1. Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{A} )</td>
<td>average accessibility across the whole network</td>
</tr>
<tr>
<td>( N )</td>
<td>total number of zones</td>
</tr>
<tr>
<td>( E )</td>
<td>parameter to reflect decision-making concerns about accessibility distribution</td>
</tr>
<tr>
<td>( P )</td>
<td>a parameter to represent distribution variations</td>
</tr>
<tr>
<td>( Z )</td>
<td>the value of equity</td>
</tr>
<tr>
<td>( A_k )</td>
<td>the accessibility at zone ( k )</td>
</tr>
<tr>
<td>( y_i )</td>
<td>capacity enhancement on link ( i )</td>
</tr>
<tr>
<td>( y_{0i} )</td>
<td>the upper bound of capacity enhancement on link ( i )</td>
</tr>
<tr>
<td>( P_j )</td>
<td>population in zone ( j )</td>
</tr>
<tr>
<td>( g(y_i) )</td>
<td>construction cost function on link ( i )</td>
</tr>
</tbody>
</table>
\( t_{kj} \)  
average travel time from zone \( k \) to zone \( j \)

\( \theta \)  
inter-zonal travel impedance parameter

\( I_1 \)  
set of links with capacity enhancement

\( t_i \)  
actual travel time on link \( i \)

\( t_i(0) \)  
travel time under free flow on link \( i \)

\( v_i \)  
traffic flow on link \( i \)

\( f_{jk}^r \)  
traffic flow on path \( r \) between zone \( j \) to zone \( k \)

\( q_{jk} \)  
travel demand (traffic flow) between zone \( j \) and zone \( k \)

\( \gamma_{jk}^i \)  
link-path incidence

\( S_i \)  
capacity of link \( i \)

\( R_{jk} \)  
set of paths between zone \( j \) and zone \( k \)

\( I_2 \)  
set of links without capacity enhancement

\( \alpha, \beta \)  
impedance parameters

8.2. Abbreviations

GINI GINI coefficient
THEIL Theil index
LDEV mean log deviation
RDEV relative mean deviation
COV coefficient of variation
ATK Atkinson index
NDP network design problems
SNDP stochastic network design problem
CNDP continuous network design problem
O-D origin-destination

ACKNOWLEDGEMENTS

Authors give their great thanks to the anonymous referees for their valuable comments and suggestions on the improvement of this paper.

REFERENCES