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Modeling the Influence of Commutation in Voltage Source Inverters on Rotor Losses of Permanent Magnet Machines

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Keywords
Permanent magnet motor, High-speed drive, Voltage Source Inverters (VSI), Pulse Width Modulation (PWM), Harmonics

Abstract
Rotor losses in inverter-fed permanent magnet (PM) machines depend on stator current waveform and consequently on inverter output voltage. These losses can be very harmful due to the vulnerability of magnets to heat. In this study, the rotor losses in high-speed PM machines supplied by voltage-source PWM inverters were analytically correlated with the inverter switching parameters. An analytical model which shows the dependency of the rotor losses on a frequency modulation ratio was derived. The derived model was verified by a 2D transient finite element model.

Introduction
Eddy currents represent an inevitable consequence of time changing magnetic fields in AC electrical machines. In certain applications they can be utilized, like for achieving a damping effect [1]. However, because of additional Joule losses that they cause, eddy currents are usually considered as undesired and measures are taken towards their minimization. Joule losses as consequence of eddy currents in rotors of permanent magnet (PM) machines can have significant influence on the machine operation and are therefore investigated in this paper.

Power dissipation in rotors of PM machines does not usually represent a significant portion of the overall losses. However, due to high sensitivity of magnet materials to the temperature increase [2] and limited possibilities for the rotor cooling, the rotor losses are an important limiting factor of the performance of the PM machines. Rotor eddy currents in slotted PM machines are caused by permeance variation as a result of stator slotting [3], time harmonics in the stator currents [4] and spatial harmonics in a stator winding distribution [5]. In a case of very-high-speed machines consideration of the rotor losses becomes even more important due to a rapid time change in the magnetic field seen by the rotor. Furthermore, the high-speed PM machines usually have a retaining sleeve which causes additional eddy currents to flow if it is made of a conductive material.

For AC machines driven by voltage source inverters (VSI) a compromise needs to be made with respect to the selection of the inverter switching frequency. While a very high switching frequency is desirable for reduction of a harmonic content of the stator currents, it makes switching losses in the inverter quite high. Conversely, a low switching frequency reduces the inverter switching losses, but at the same time deteriorates the current harmonic spectrum. For high-speed machines, a very high switching
frequency might be required. This may represent a challenge, not only due to the high switching losses in the inverter, but often also because of unavailability or high cost of the appropriate hardware which can support such a fast switching [6]. Since the rotor losses in the PM machines are highly dependent on the harmonic content of stator currents, modeling and prediction of these losses in the context of the inverter switching algorithm becomes very important.

There are several techniques that are used to model the armature field and the rotor losses of the permanent magnet machines. In [7] the armature field of slotted machines was obtained using the magnetic scalar potential and based on that in [8] the rotor losses were calculated. Stator windings were represented as an equivalent current sheet distributed over slot openings. However, this approach does not take into account a reaction field of the eddy currents flowing in the rotor, which can significantly overestimate the rotor losses, especially at high speeds [9]. The approach used in [9] and [10] based on the magnetic vector potential takes into account the eddy current reaction field. However, all these techniques assume that the time harmonics in the stator currents are located at frequencies which represent an integer multiple of the fundamental frequency, which may not be the case when a machine is supplied by sinusoidal PWM VSI.

In this paper an analytical technique for the modeling of the armature field and the rotor losses of the high-speed PM machines supplied by the sinusoidal PWM VSI is presented. The analysis presented here does not put any assumptions related to the order of the time harmonics of the stator currents: the armature field and the rotor losses are represented as functions of an arbitrary frequency modulation ratio of the inverter. In applications with strict limitations on the switching frequency, this kind of analysis is of practical importance. The work presented here does not consider influence of the slotting effect on the rotor losses.

As a test case for this study, a high-speed PM machine with a diametrically magnetized magnet is used. The machine is used in a micro-CHP (Combined Heat and Power) system, where it is directly coupled with a micro-gas turbine. A cross section of the machine is shown in Fig. 1, where arrows show the direction of the magnetization and letters in the slots show a disposition of the windings. Parameters of the considered machine are listed in Table I.

![Cross section of the considered machine](image)

**Table I: Dimensions and parameters of the considered machine**

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fundamental frequency [kHz]</td>
<td>4</td>
<td>$f_0$</td>
</tr>
<tr>
<td>Pole number</td>
<td>2</td>
<td>$p$</td>
</tr>
<tr>
<td>Stack length [mm]</td>
<td>26</td>
<td>$h$</td>
</tr>
<tr>
<td>Magnet radius [mm]</td>
<td>5.5</td>
<td>$r_I$</td>
</tr>
<tr>
<td>Sleeve outer radius [mm]</td>
<td>7.5</td>
<td>$r_{II}$</td>
</tr>
<tr>
<td>Stator inner radius [mm]</td>
<td>9</td>
<td>$r_{III}$</td>
</tr>
<tr>
<td>Magnet recoil permeability</td>
<td>1.035</td>
<td>$\mu_I$</td>
</tr>
<tr>
<td>Sleeve relative permeability</td>
<td>1</td>
<td>$\mu_{II}$</td>
</tr>
<tr>
<td>Magnet conductivity [S/m]</td>
<td>$6.25 \cdot 10^5$</td>
<td>$\sigma_I$</td>
</tr>
<tr>
<td>Sleeve conductivity [S/m]</td>
<td>$8.3 \cdot 10^5$</td>
<td>$\sigma_{II}$</td>
</tr>
</tbody>
</table>

**Harmonic analysis of stator voltages and currents**

For PM machines with diametrically magnetized magnets, which have a sinusoidal back EMF, the harmonic content of the stator currents depends only on the harmonic content of the inverter output voltage. Therefore, an accurate analytical representation of the inverter output voltage waveform is crucial. Existing models of the PM machines make an assumption that only odd non-triplen harmonics are present in the current waveform. This assumption might be valid in a case of BLDC mode of operation [7, 9], machines loaded with diode rectifiers [11] or a current source inverter used in [10]. If a sinusoidal PWM VSI is used, the harmonic content depends on the frequency modulation ratio.

It is known that harmonic components in the sinusoidal PWM VSI output voltage appear at frequencies around multiples of a switching frequency [12, 13]. These components are called sideband harmonics. Modulation strategies based on sampling of reference sinusoidal signals, so-called regular sampled PWM, introduce additional low frequency harmonic components. These components are called baseband harmonics and they are located around the fundamental component, however, only at frequencies higher than the fundamental.
Table II: Parameters of the inverter output voltage

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{dc}$</td>
<td>DC bus voltage</td>
</tr>
<tr>
<td>$M$</td>
<td>Amplitude modulation ratio</td>
</tr>
<tr>
<td>$m$</td>
<td>Index of a sideband group ($m = 0$ indicates the baseband group)</td>
</tr>
<tr>
<td>$n$</td>
<td>Position of a component in a sideband</td>
</tr>
<tr>
<td>$m_f$</td>
<td>Frequency modulation ratio</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>Fundamental angular frequency</td>
</tr>
<tr>
<td>$\theta_0$</td>
<td>Phase shift (0 for $\psi_{ab}$, $-\frac{2\pi}{3}$ for $\psi_{bc}$, $\frac{2\pi}{3}$ for $\psi_{ca}$)</td>
</tr>
<tr>
<td>$J_n$</td>
<td>Bessel function of first kind of order $n$</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
</tr>
</tbody>
</table>

As a test case for this analysis, asymmetrical regular sampled PWM will be considered. Out of different regular sampled PWM which are used in digital modulation systems, this modulation strategy results with the lowest harmonic content [13]. An analytical expression for a line-to-line voltage of the VSI modulated by asymmetrical regular sampled PWM can be written as [13]:

$$v_{LL}(t) = \frac{4V_{dc}}{\pi} \sum_{m=0}^{\infty} \sum_{n=-\infty}^{\infty} m_f \sum_{m_f} m J_n \left( \frac{(m m_f + n) \pi M}{2 m_f} \right) \sin \left( [m + n] \frac{\pi}{2} \right) \sin^2 \left( n \frac{\pi}{3} \right) \times$$

$$\cos \left( (m m_f + n) \omega_0 t + n \left( \theta_0 - \frac{\pi}{3} \right) + \frac{\pi}{2} \right)$$

(1)

Parameters used in (1) are listed in Table II. Double lower limits in the summation terms in (1) distinguish between baseband and sideband harmonics. Validity of (1) has been confirmed by simulation models created by the authors and this will not be shown in this paper. By using (1) an analytical expression for phase voltages appearing over phases of a 3-phase machine in Y connection supplied by the inverter can be written as:

$$v_p(t) = \frac{8V_{dc}}{3\pi} \sum_{m=0}^{\infty} \sum_{n=-\infty}^{\infty} m_f \sum_{m_f} m J_n \left( \frac{(m m_f + n) \pi M}{2 m_f} \right) \sin \left( [m + n] \frac{\pi}{2} \right) \sin^2 \left( n \frac{\pi}{3} \right) \times$$

$$\cos \left( (m m_f + n) \omega_0 t + n \theta_0 \right); \quad p = \begin{cases} a \iff \theta_0 = 0 \\ b \iff \theta_0 = -\frac{2\pi}{3} \\ c \iff \theta_0 = \frac{2\pi}{3} \end{cases}$$

(2)

If the machine resistance and inductance are known for all frequencies at which the voltage harmonic components are present, a module of the machine impedance and a phase shift introduced by it can be written as:

$$|Z| = \sqrt{R_{mmm}^2 + (m m_f + n)^2 \omega_0^2 L_{mmm}^2} \quad ; \quad \theta_i = \arctan \left( \frac{(m m_f + n) \omega_0 L_{mmm}}{R_{mmm}} \right)$$

(3)

In (3) $R_{mmm}$ and $L_{mmm}$ are the resistance and the synchronous inductance at a given frequency. By using (2) and (3) an analytical expression for phase currents can be written as:

$$i_p(t) = \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} I_{max} \cos \left( (m m_f + n) \omega_0 t + n \theta_0 - \theta_i \right)$$

(4)

where

$$I_{max} = \frac{8V_{dc} m_f}{3\pi (m m_f + n) |Z|} J_n \left( \frac{(m m_f + n) \pi M}{2 m_f} \right) \sin \left( [m + n] \frac{\pi}{2} \right) \sin^2 \left( n \frac{\pi}{3} \right)$$

(5)
**Surface current density**

Influence of the stator windings is modeled through an equivalent surface current density distributed over slot openings, like in [7]. Previously derived stator currents, together with the winding distribution are used to obtain the surface current density. The winding distribution can be described in the following way using the Fourier decomposition:

\[
n_p(\varphi_s) = \frac{2N}{\pi} \sum_{k=1}^{\infty} K_{w,k} \cos(k(\varphi_s + \varphi_p)); \quad p = \begin{cases} 
\alpha & \varphi_p = 0 \\
\beta & \varphi_p = -\frac{2\pi}{3} \\
\gamma & \varphi_p = \frac{2\pi}{3}
\end{cases}
\] (6)

In (6) \(N\) is the number of turns per phase, \(K_{w,k}\) is a harmonic winding factor consisting of distribution, pitch and slot-opening factor while \(\varphi_s\) represents the azimuthal coordinate in the stator reference frame. Index \(k\) represents a harmonic order in the windings distribution. The surface current density for the each phase can be obtained as:

\[
J_p(\varphi_s,t) = \frac{i_p(t)}{r_{111}} n_p(\varphi_s)
\] (7)

The total stator surface current density can be calculated as:

\[
J(\varphi_s, t) = J_a(\varphi_s, t) + J_b(\varphi_s, t) + J_c(\varphi_s, t)
\] (8)

Expression (8) can be expanded using (7) taking into account (4) and (6). After a transformation of the cosine products appearing in (8) and rearranging, it can be written as:

\[
J(\varphi_s, t) = \frac{N}{\pi r_{111}} \sum_{m=0}^{\infty} \sum_{m>0}^{\infty} \sum_{n=-\infty}^{\infty} \sum_{k=1}^{\infty} I_{max} K_{w,k} \left\{ \begin{array}{l}
\cos(X) + \cos\left(X - \frac{2\pi}{3}(n+k)\right) + \\
\cos\left(X + \frac{2\pi}{3}(n+k)\right) + \cos(Y) + \\
\cos\left(Y - \frac{2\pi}{3}(n-k)\right) + \cos\left(Y + \frac{2\pi}{3}(n-k)\right)
\end{array} \right\}
\] (9)

where

\[
X = (mm_f + n)\omega_0 t - \theta_i + k\varphi_s \quad ; \quad Y = (mm_f + n)\omega_0 t - \theta_i - k\varphi_s
\] (10)

Result of the summation of the first 3 terms in the brackets differs from zero only under the following condition:

\[
n + k = 3L \Rightarrow k = 3L - n, \quad L \in \mathbb{Z}
\] (11)

and equals to the threefold value of the first term. On the other hand, result of the summation of the last 3 terms in the brackets differs from zero only when the following condition is met:

\[
n - k = 3L \Rightarrow k = n - 3L, \quad L \in \mathbb{Z}
\] (12)

and equals to the threefold value of the forth term. Having in mind that index \(n\) is never divisible by 3 or equal to 0 (currents do not have the same phase, but their sum is always equal to 0), conditions (11) and (12) express the fact that spatial harmonics of an order which is a multiple of 3 do not create the net armature field. After taking into account (11) and (12) and having in mind that \(k\) is a positive integer, the surface current density can be written as:

\[
J(\varphi_s, t) = \frac{3N}{\pi r_{111}} \sum_{m=0}^{\infty} \sum_{m>0}^{\infty} \sum_{n=-\infty}^{\infty} I_{max} \left\{ \begin{array}{l}
\sum_{L,3L>n} K_{w,3L-n} \cos((mm_f + n)\omega_0 t - \theta_i + (3L-n)\varphi_s) + \\
\sum_{L,3L<n} K_{w,3L-n} \cos((mm_f + n)\omega_0 t - \theta_i - (n-3L)\varphi_s)
\end{array} \right\}
\] (13)
In (13) the harmonic order number $k$ is substituted following the conditions (11) and (12) within $K_{wk}$ (further on referred to as $K_{w,3L-n}$). Finally, after merging two terms in (13) it can be written:

$$J(\phi_s, t) = \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \sum_{L=-\infty}^{\infty} J_{\text{max}} \cos \left( (mm_f + n)\omega_0 t - \theta_i + (3L - n)\phi_s \right)$$  \hspace{1cm} (14)

where

$$J_{\text{max}} = \frac{3N_s}{\pi f_{11}} I_{\text{max}} K_{w,3L-n}$$  \hspace{1cm} (15)

The surface current density consists of an infinite number of rotating waves, which rotate in different directions with different speeds. The speed of the rotation is directly proportional to the order of the time harmonics and inversely proportional to the order of the spatial harmonics. The direction of the rotation can be determined in the following way:

Direction of waves rotation: \hspace{1cm}

- in the rotor direction $\leftrightarrow$ \hspace{0.5cm} $\text{sgn} (3L - n) \neq \text{sgn} (mm_f + n)$
- opposite of the rotor direction $\leftrightarrow$ \hspace{0.5cm} $\text{sgn} (3L - n) = \text{sgn} (mm_f + n)$

$$\text{(16)}$$

The azimuthal coordinate expressed in the stator reference frame can be expressed through the azimuthal coordinate expressed in the rotor reference frame $\phi_r$ and the initial rotor position $\phi_0$ (expressed in the stator reference frame) as:

$$\phi_s = \phi_r + \omega_0 t + \phi_0$$  \hspace{1cm} (17)

By inserting (17) into (14) the surface current density can be expressed in the rotor reference frame, which is necessary for getting insight in the field seen by the rotor:

$$J(\phi_r, t) = \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \sum_{L=-\infty}^{\infty} J_{\text{max}} \cos \left( (mm_f + 3L)\omega_0 t - \theta_i + (3L - n)(\phi_r + \phi_0) \right)$$  \hspace{1cm} (18)

Finally, the surface current density can be written in a complex harmonic form:

$$J(\phi_r, t) = \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \sum_{L=-\infty}^{\infty} J_{\text{max}} e^{-j(3L)\omega_0 t - \theta_i + (3L - n)(\phi_r + \phi_0)}$$  \hspace{1cm} (19)

To calculate the rotor losses by use of the Poynting vector [10], the electromagnetic field in the machine has to be solved in the complex domain. Since the field solution has the same form as the field source (the surface current density) the previous transition to the complex harmonic form is necessary.

**Field and losses modeling**

The armature field is solved by means of the magnetic vector potential. The problem is treated as 2D, with the surface current density, the magnetic vector potential and the induced electric field having only $z$ component. The stator inner surface is treated as smooth with the surface current density at the boundary between the air-gap and the infinitely permeable iron, like shown in Fig. 2. In conducting regions (the magnet and the sleeve) the Helmholtz equation (20) has been solved for the field components which rotate asynchronously with respect to the rotor, while in the air gap the Laplace equation (21) has been solved. In (20) $\mu_r$ is the relative permeability and $\sigma$ is the conductivity of the conducting regions.

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial A_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 A_z}{\partial \phi_r^2} - \mu_0 \mu_r \sigma \frac{\partial A_z}{\partial t} = 0$$  \hspace{1cm} (20)

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial A_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 A_z}{\partial \phi_r^2} = 0$$  \hspace{1cm} (21)
If the following notation is introduced:

\[ \beta = (m m_f + 3L) \omega_0 t - \theta_r + (3L - n)(\phi_r + \phi_0) \]  

(22)

a solution for the magnetic vector potential in all three considered regions can be written as:

\[ A_{I I I}(r, \phi_r, t) = \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \sum_{L=-\infty}^{\infty} C_I I_{3L-n}(p r f) e^{-j \beta} \]  

(23)

\[ A_{I I I}(r, \phi_r, t) = \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \sum_{L=-\infty}^{\infty} (C_{I I} I_{3L-n}(p r f) + D_{I I} K_{3L-n}(p r f)) e^{-j \beta} \]  

(24)

\[ A_{I I I}(r, \phi_r, t) = \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \sum_{L=-\infty}^{\infty} (C_{I I I} e^{3L-n} + D_{I I I} e^{-3L-n}) e^{-j \beta} \]  

(25)

\[ p_{II} = j^2 \sqrt{(m m_f + 3L) \omega_0 \mu_0 \mu_r \sigma_{I I}} \quad p_I = j^2 \sqrt{(m m_f + 3L) \omega_0 \mu_0 \mu_r \sigma_I} \]  

(26)

All field components of interest in all the regions can be calculated in the complex domain from the magnetic vector potential using the constitutive relations:

\[ E_z = -\frac{\partial A_z}{\partial t} \quad B_r = \frac{1}{r} \frac{\partial A_z}{\partial \phi_r} \quad H_\phi = -\frac{1}{\mu_0 \mu_r} \frac{\partial A_z}{\partial r} \]  

(27)

For the armature field components which are in the synchronism with the rotor, the Laplace equation (21) is valid also in the conducting regions. However, these components do not cause rotor losses and their solution is necessary only to obtain the total field.

Analytical expressions for the eddy current losses in the sleeve and in the magnet can be calculated by the use of the Poynting vector [10]. In 2D cylindrical problems, like the considered one, these expressions are obtained from the following equation:

\[ P_{\text{loss}} = \frac{1}{2} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \sum_{L=-\infty}^{\infty} \text{Re} \oint_S \left( \hat{E}_z(r, \phi_r) \hat{H}_\phi^*(r, \phi_r) \right) dS \]  

(28)

The total rotor losses have been obtained by calculating the previous integral over the sleeve outer surface, while the magnet loss has been obtained by calculating the same integral over the magnet outer surface.
Difference of these 2 results gives the sleeve loss. In (28) \( \hat{E}_z (r, \varphi_r) \) and \( \hat{H}^* (r, \varphi_r) \) represent a complex amplitude of the induced electric field and a complex conjugated amplitude of the azimuthal component of the magnetic field, respectively, which are only position dependent. By combining (28) with (27) and (23) the following expressions for the magnet loss can be obtained:

\[
P_{\text{lossI}} = \frac{\pi \mu_0 \omega_0}{\mu l} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \sum_{L=-\infty}^{\infty} (mm + 3L) \times \\
\times \Re \left( j C_I I_{3L-n}(p_I r_I) \left[ C_I \left((n-3L)I_{3L-n}(p_I r_I) - p_I r_I I_{3L-n+1}(p_I r_I) \right) \right]^* \right)
\]

Similarly, the sleeve loss can be calculated by combining (28) with (27) and (25), which gives the following expression:

\[
P_{\text{lossII}} = \frac{\pi \mu_0 \omega_0}{\mu l} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \sum_{L=-\infty}^{\infty} (mm + 3L)(n-3L)r_{11}^{-(3L+n)+1} \times \\
\times \Re \left( j \left( C_{II1} r_{11}^{6L} + D_{II1} r_{11}^{2n} \right) \left[ C_{II1} r_{11}^{3L-n-1} - D_{II1} r_{11}^{-(3L-n)-1} \right]^* - P_{\text{lossI}} \right)
\]

Expressions (29) and (30) were obtained using the field solution in the magnet and the air-gap, respectively. Equivalently, these expressions could be derived using the analytical solution for the field in the sleeve. However, this would result with more complex analytical expressions, having in mind a form of the solution for the magnetic vector potential in the sleeve (24).

By observing derived expressions it can be seen that as long as term \((mm + 3L)\) differs from 0, the rotor will experience time changing electromagnetic field. Components of the armature field for which this term equals to zero rotate synchronously with the rotor and therefore do not induce the eddy currents.

If the frequency modulation ratio is an integer not divisible by 3, then in both every sideband group and in the baseband group there will be harmonic components which will result in the field components rotating synchronously with the rotor. If \(m_f\) is an integer divisible by 3, these harmonic components will exist both in every sideband group and in the baseband group. If \(m_f\) is not an integer, synchronously rotating field components can result only from the baseband harmonics.

**Results**

Analytical models developed in the previous sections were verified with 2D transient finite element analysis. In order to minimize the effect of slotting, remanent flux density of the magnet was set to 0. Only harmonic components of the stator currents which cause non-negligible values of the rotor losses according to the analytical model were taken into account and included in the finite element model. The machine impedance, necessary for calculating the current harmonic amplitudes, was measured with an impedance analyzer. The result comparison was done for the case of asynchronous PWM with the frequency modulation ratio value of \(m_f = 15.625\), as currently used in the considered application.

In Fig. 3 a comparison of the radial and tangential flux density distribution at two different radii in the rotor reference frame obtained by finite element analysis and analytically is shown. It can be seen that a good matching exists between two methods. Table III gives a comparison of the rotor losses per harmonic calculated analytically and with the finite element model. Again, a very good matching between two methods is achieved, with the difference less than 2 percent.

To get a satisfactory accuracy of the finite element model, a sufficiently small simulation step had to be chosen. During the analysis of the analytical and finite element model results, it was concluded that 100 steps per an electric period is necessary in order to get a good matching. Having in mind this and the fact that order of the highest harmonic included in the simulation is above 40, it is clear that finite element simulation becomes extremely time consuming. Therefore, results shown further on have been obtained only analytically, since validity of the analytical method has been already confirmed.

Based on the developed analytical model, the rotor losses in the analyzed machine have been calculated for a range of the frequency modulation ratio from 8 to 15. The results are shown in Fig. 4. It can be seen that the rotor losses decrease consistently with the increase of the frequency modulation ratio. The same trend is maintained regardless of an integer or non-integer value of \(m_f\) (synchronous or asynchronous PWM).
(a) Radial flux density in the middle of the air gap (b) Tangential flux density at the sleeve surface

Figure 3: Comparison of the field distribution obtained analytically and with 2D FEM

Table III: Comparison of the rotor losses calculated analytically and by 2D FEM

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Fundamental</td>
<td>1.2</td>
<td>1.18</td>
</tr>
<tr>
<td>m = 1, n = 2</td>
<td>3.77</td>
<td>3.71</td>
</tr>
<tr>
<td>m = 1, n = -2</td>
<td>3.87</td>
<td>3.83</td>
</tr>
<tr>
<td>m = 2, n = 1</td>
<td>0.572</td>
<td>0.564</td>
</tr>
<tr>
<td>m = 2, n = -1</td>
<td>1.1</td>
<td>1.07</td>
</tr>
<tr>
<td>m = 3, n = -4</td>
<td>0.184</td>
<td>0.181</td>
</tr>
<tr>
<td>Total</td>
<td>10.7</td>
<td>10.5</td>
</tr>
</tbody>
</table>

Figure 4: Change of the rotor losses with the frequency modulation ratio

Discussion

The results presented in Fig. 4 suggest that the rotor losses decrease with the increase in the frequency modulation ratio. For a given fundamental frequency, the increase in the frequency modulation ratio
shifts the voltage harmonic components towards higher frequencies. This affects the armature field in two ways. Firstly, the amplitudes of the field components are reduced since the amplitudes of the harmonic components of the stator currents are reduced due to the inductive nature of the machine. Secondly, the relative speeds of the field components (which do not rotate at the synchronous speed) with respect to the rotor increase.

As a consequence of the first effect, it is expected that amplitudes of the rotor eddy current density components will tend to decrease. Conversely, an expected consequence of the second effect is an increase in the rotor eddy current density amplitudes, since the frequency of the armature magnetic field in the rotor reference frame, which equals \((mm_f + 3L)\omega_0\), increases. Although these two effects tend to influence the rotor eddy current density amplitudes in opposite ways, the rotor losses are reduced significantly.

In addition to the previously mentioned effects, the frequency modulation ratio also influences the penetration depth in the rotor, which is expressed by (31).

\[
\delta = \frac{1}{\sqrt{(mm_f + 3L)\pi f_0\mu_0\mu_r\sigma}}
\]  

Finally, it is necessary to comment on the procedure for measuring the machine impedance. As previously mentioned, the machine impedance was measured with an impedance analyzer. This measurement had to be carried out at very high frequencies (far beyond the fundamental frequency of 4 kHz). The impedance was measured for a number of frequencies. Based on that, a curve fitting procedure was used to get a continuous impedance function, as shown in Fig. 5. Since the impedance analyzer which was used in the measurement could supply the windings with a limited voltage, the measurement was carried out with a limited current, especially at higher frequencies.

Additionally, the measurement produced unreliable results at frequencies above 160 kHz. Because of that, the current harmonic components at frequencies above 160 kHz were not calculated or included in the model. However, due to the high value of the impedance and the small penetration depth, it is expected that the influence of the current components on the rotor losses at such high frequencies will not be significant. This claim can be supported by the last harmonic component shown in Table III. It is located at 171.5 kHz, and the impedance at this frequency is obtained by extrapolating the curve displayed in Fig. 5. This harmonic component causes less than 2 percent of the overall losses. For lower values of \(m_f\) (where the rotor losses are higher) the relative influence of the components at very high frequencies is expected to be at a similar level.

**Conclusions**

This paper proposes an analytical technique that can be used to calculate rotor losses in permanent magnet machines supplied by voltage-source inverters. The technique described here is universal in the sense that it does not make any assumptions related to the order of the time harmonics in the stator currents.
It facilitates an analysis of the armature reaction field and the rotor losses as a function of the inverter frequency modulation ratio. The results obtained by the analytical model were verified by 2D transient finite element analysis and a very good agreement was achieved, with the conclusion that the proposed analytical method is much less time consuming. This makes the proposed technique very useful as an optimization tool for use in system design. A potential limitation of the method is the fact that it requires values of the machine impedance for every relevant harmonic frequency of the inverter voltage, which is very difficult to either model or measure.

The results shown in this paper reveal that the rotor losses in the permanent magnet machine decrease consistently with the increase in the frequency modulation ratio. This effect is attributed to a reduction in the penetration depth in the rotor. The penetration depth is reduced as a consequence of the increase in the frequency of the armature field in the rotor reference frame due to the increase in the frequency modulation ratio.

References