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Iterative motion feedforward tuning: A data-driven approach based on instrumental variable identification

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ABSTRACT

Feedforward control can significantly enhance the performance of motion systems through compensation of known disturbances. This paper aims to develop a new procedure to tune a feedforward controller based on measured data obtained in finite time tasks. Hereto, a suitable feedforward parametrization is introduced that provides good extrapolation properties for a class of reference signals. Next, connections with closed-loop system identification are established. In particular, instrumental variables, which have been proven very useful in closed-loop system identification, are selected to tune the feedforward controller. These instrumental variables closely resemble traditional engineering tuning practice. In contrast to pre-existing approaches, the feedforward controller can be updated after each task, irrespective of noise acting on the system. Experimental results confirm the practical relevance of the proposed method.

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1. Introduction

Feedforward control is widely used in control systems, since feedforward can effectively reject disturbances before these affect the system. Indeed, many applications to high-performance systems have been reported where feedforward control leads to a significant performance improvement. For servo systems, the main performance improvement is in general obtained by using feedforward to compensate for the reference signal. Relevant examples of feedforward control include model-based feedforward, see, e.g., Zhong, Pao, and de Callafon (2012), Clayton, Tien, Leang, Zou, and Devasia (2009) and Butterworth, Pao, and Abramovitch (2012), and Iterative Learning Control (ILC), see, e.g., Bristow, Tharayil, and Alleyne (2006) and Moore (1993).

On the one hand, model-based feedforward results in general in good performance and provides extrapolation capabilities of tasks. In model-based feedforward, a parametric model is determined that approximates the inverse of the system. The performance improvement induced by model-based feedforward is highly dependent on (i) the model quality of the parametric model of the system and (ii) the accuracy of model-inversion, see, e.g., Devasia (2002). On the other hand, ILC results in superior performance with respect to model-based feedforward. By learning from previous iterations, high performance is obtained for a single, specific task, i.e., at the expense

of poor extrapolation capabilities of tasks. In addition, ILC only requires an approximate model of the system.

Recently, an approach is presented in van de Wijdeven and Bosgra (2010) that combines the advantages of model-based feedforward and ILC, resulting in both high performance and good extrapolation capabilities. To this purpose, basis functions are introduced that reflect the dynamical behavior of the system responsible for the dominant contribution to the servo error. In Van der Meulen, Tousain, and Bosgra (2008), the need for an approximate model of the system, as is common in ILC, is eliminated by exploiting concepts from iterative feedback tuning (IFT) (Hjalmarsson, Gevers, Gunnarsson, & Lequin, 1998). This approach is extended to input shaping in Boeren, Bruijnen, van Dijk, and Oomen (2014) and multivariable systems in Heertjes, Hennekens, and Steinbuch (2010), while a comparative study of data-driven feedforward control procedures is reported in Stearns, Yu, Fine, Mishra, and Tomizuka (2008). However, by eliminating the need for an approximate model of the system, the approach presented in Van der Meulen et al. (2008) requires a significantly larger experimental cost to perform an update of the feedforward controller and puts stringent assumptions on noise acting on the system.

Although iterative feedforward tuning is widely successful to improve the performance of motion systems, existing tuning procedures (i) impose stringent requirements on noise acting on the system, (ii) require two tasks for each iterative update of the feedforward controller and (iii) can lead to a bias error. In this paper, it is shown that these deficiencies can be removed by connecting iterative feedforward tuning to system identification, and exploit

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concepts from closed-loop system identification in iterative feedforward tuning. In fact, in contrast to pre-existing procedures in Van der Meulen et al. (2008), Boeren, Bruijnen et al. (2014) and Heertjes et al. (2010), the proposed procedure closely resembles manual feedforward tuning procedures for motion systems, see, e.g., Boerlage, Tousain, and Steinbuch (2004). This immediately confirms the practical relevance of the proposed approach for industrial motion systems.

The main contribution of this paper is an iterative feedforward tuning approach that is efficient, i.e., it requires measured data from only a single task, and accurate, i.e., attains optimal performance for feedforward control in the presence of noise. The proposed approach is closely related to Söderström and Stoica (1983), Gilson and Van den Hof (2005), Jung and Enqvist (2013) and Karimi, Butcher, and Longchamp (2008), and extends this work to iterative tuning of feedforward controllers. Furthermore, the motivation for the proposed approach is similar to the approach in Kim and Zou (2013), i.e., combine the advantages of model-based feedforward and ILC without the need for an approximate model of the system. The key difference is that in Kim and Zou (2013) a nonparametric model for the feedforward controller is constructed, while this work aims to determine a parametric model. This paper is an extension of Boeren and Oomen (2013) that includes experimental results, and a complete explanation and analysis.

This paper is organized as follows. In Section 2, the problem formulation is outlined. Then, in Section 3, it is shown that in the presence of noise, existing procedures suffer from a closed-loop identification problem. In Section 4, a new feedforward control procedure is proposed which requires only a single task to update the feedforward controller in the presence of noise. Then, in Section 5 the proposed approach is embedded in the iterative feedforward tuning framework. In Section 6, the experimental results of the proposed approach are presented. Finally, a conclusion is presented in Section 7.

Notation: For a vector x , $\|x\|_2^2 = x^T x$. The vector u is defined as $u = [u(1), u(2), \dots, u(N)]^T \in \mathbb{R}^N$, where $u(t)$ is a measurement at time instant t for $t = 1, 2, \dots, N$ with N being the number of samples. The symbol q denotes the forward shift operator $qu(t) = u(t+1)$. Furthermore, the expected value $\mathbb{E}(x)$ is defined as $\mathbb{E}(x) = \int_{-\infty}^{\infty} xf(x) dx$, with probability density function $f(x)$. The correlation function based on a finite number of samples N is defined as $R_{xy}(N) = (1/N) \sum_{t=1}^N x(t)y(t)$.

2. Problem formulation

2.1. Feedforward control goal

Consider the two degree-of-freedom control configuration as depicted in Fig. 1. The true unknown system is assumed to be discrete-time, single-input single-output and linear time-invariant, and is denoted as $P(q)$. The control configuration consists of a given stabilizing feedback controller $C_b(q)$ and feedforward controller $C_{ff}(q)$. Let r denote a known n th-order multi-segment polynomial trajectory with constraints on the first n derivatives, generated by a trajectory planning algorithm that takes system dynamics into account, see, e.g., Biagiotti and Melchiorri (2012), Lee, Kim, and Choi (2013) and Lambrechts, Boerlage, and Steinbuch (2005). A typical reference r in a single task is depicted in Fig. 3. Furthermore, v denotes a disturbance, u_{ff} the feedforward signal,

Fig. 1. Two degree-of-freedom control configuration.

and e the servo error. The unknown disturbance v is assumed to be given by $v = H(q)\epsilon$, where $H(q)$ is monic and ϵ is normally distributed white noise with zero mean and variance λ_ϵ^2 . Hence, v and r are uncorrelated.

The goal in feedforward control is to attain high performance by compensating for known exogenous input signals that affect the system. The servo error e in Fig. 1 as given by

$$e = S(q)(1 - P(q)C_{ff}(q))r - S(q)v,$$

where $S(q) = (1 + P(q)C_b(q))^{-1}$, reveals that the contribution of e induced by r is eliminated if $C_{ff}(q) = P^{-1}(q)$. For motion systems with dominant rigid-body dynamics, a parametrization for $C_{ff}(q)$ is proposed in Lambrechts et al. (2005) which compensates for the dominant component of the reference-induced error. The corresponding u_{ff} is given by

$$u_{ff} = \theta_a a + \theta_j j + \theta_s s, \quad (1)$$

where a, j and s correspond to respectively acceleration, jerk and snap, i.e., the 2nd, 3rd and 4th derivative of the multi-segment polynomial trajectory r , and $\theta_a, \theta_j, \theta_s$ are the corresponding parameters.

To illustrate this parametrization, consider the acceleration profile a and measured error e_m as depicted in Fig. 2. In manual tuning of a feedforward controller, the optimal value for θ_a is such that the predicted error

$$\hat{e}(\theta_a) = e_m - S(q)P(q)\theta_a a,$$

and a are uncorrelated, where $e_m = S(q)r - S(q)v$. Likewise, the optimal values for θ_j and θ_s are obtained if $\hat{e}$, and j and s are uncorrelated, respectively. The results in this paper enable an iterative and automated estimation of the optimal values for θ_a, θ_j and θ_s .

2.2. Iterative feedforward control

In iterative feedforward control, measured data is exploited to update $C_{ff}(q)$ after each task. For the considered class of systems, a sequence of finite time tasks, denoted as $j = 1, 2, \dots$, with length N samples is executed. In a single task, the system starts at rest in the initial position, followed by a point-to-point motion, before the system comes to a rest in the final position of a task. A typical reference r in a single task is shown in Fig. 3. A sequence of such tasks is executed during normal operation of the system, where r is not necessarily identical for each consecutive task.

The measured signals e_m^j and y_m^j in the j th task are given by $e_m^j = e_r^j - e_v^j$, where $e_r^j = S(q)(1 - P(q)C_{ff}^j(q))r$ and $e_v^j = S(q)v^j$, and $y_m^j = y_r^j + y_v^j$, where $y_r^j = S(q)P(q)(C_b(q) + C_{ff}^j(q))r$ and $y_v^j = S(q)v^j$. Note that since $P(q)$, $S(q)$ and v^j are unknown, it is not possible to construct e_r^j and e_v^j from the measured signal e_m^j . This also holds for y_m^j . For clarity of exposition, the index j is omitted if only a single task

Fig. 2. Manual tuning of feedforward parameters—The normalized acceleration profile a (dashed black) and normalized error e_m (red) obtained in the previous task are used to determine θ_a such that the predicted error $\hat{e} = e_m - S(q)P(q)\theta_a a$ and a are uncorrelated. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

Fig. 3. Typical task during normal operation of the considered class of motion systems.

is considered. The data-driven feedforward optimization problem is formulated in Definition 1.

Definition 1. Given measured signals e_m^j and y_m^j obtained during the j th task of the closed-loop system in Fig. 4 with $C_{ff}^j(q)$ implemented. Then,

$$C_{ff}^{j+1}(q) = C_{ff}^j(q) + C_{ff}^{\Delta}(q), \quad (2)$$

where the update $C_{ff}^{\Delta}(q)$ based on e_m^j and y_m^j results from the optimization problem

$$C_{ff}^{\Delta, opt}(q) = \arg \min_{C_{ff}^{\Delta} \in \mathcal{C}} V(C_{ff}^{\Delta}), \quad (3)$$

with criterion $V(C_{ff}^{\Delta})$ and feedforward controller parametrization $\mathcal{C}$.

The criterion $V(C_{ff}^{\Delta})$ and parametrization $\mathcal{C}$ are essential for high performance of the system with $C_{ff}^{j+1}(q)$. In this paper, the impact of $V(C_{ff}^{\Delta})$ on the performance of the system in Fig. 1 is analyzed for a fixed $\mathcal{C}$. To proceed, $\mathcal{C}$ is defined in the next section.

2.3. Feedforward controller parameterization

In this section, a general polynomial feedforward parametrization is adopted that encompasses common parametrizations in feedforward control for motion systems, including Lambrechts et al. (2005), Van der Meulen et al. (2008), Heertjes et al. (2010) and Boeren, Bruijnen et al. (2014).

Definition 2. The feedforward controller C_{ff} is parametrized as

$$\mathcal{C} = \left\{ C_{ff}(q, \theta) \mid C_{ff}(q, \theta) = \sum_{i=1}^{n_{\theta}} \psi_i(q) \theta_i, \psi_i \in \mathbb{R}[q], \theta_i \in \mathbb{R} \right\}$$

with parameter vector

$$\theta = [\theta_1, \theta_2, \dots, \theta_{n_{\theta}}]^T \in \mathbb{R}^{n_{\theta} \times 1}, \quad (4)$$

and polynomial basis functions

$$\Psi(q) = [\psi_1(q), \psi_2(q), \dots, \psi_{n_{\theta}}(q)] \in \mathbb{R}[q]^{1 \times n_{\theta}}. \quad (5)$$

The order n_{θ} of C_{ff} can be determined by means of a model order selection procedure, see, e.g., Ljung (1999, chap. 6). In terms of Definition 2, the feedforward signal u_{ff} in (1) is given by a parametrization with $n_{\theta} = 3$ and basis functions $\psi(q)$ representing acceleration, jerk and snap feedforward, i.e., the 2nd, 3rd and 4th derivative of r .

The feedforward controller parametrization $\mathcal{C}$ in Definition 2 has two important advantages. First, this parametrization is linear in θ . Hence, for a quadratic criterion, (4) has an analytic solution. Second, $\mathcal{C}$ is a generalization of a FIR basis, thereby enforcing stability of C_{ff} since all poles are located in the origin. As a result, internal stability of the system in Fig. 1 is guaranteed when $C_{ff}^{j+1}(q)$ is implemented on the system.

Fig. 4. The feedforward controller $C_{ff}^{j+1}(q)$ is constructed based on the known reference r , and measured signals e_m^j and y_m^j in the j th task.

2.4. Problem formulation and outline

In iterative feedforward tuning, the performance of the system in Fig. 1 is improved by exploiting measurements from the previous tasks to iteratively update $C_{ff}(q)$. As stated in (3), the update $C_{ff}^{\Delta}(q) \in \mathcal{C}$ is determined by $V(C_{ff}^{\Delta})$. Two key requirements for $V(C_{ff}^{\Delta})$ in iterative feedforward tuning are imposed:

- R1. Minimization of the experimental cost in terms of the number of tasks required to update $C_{ff}(q)$.
- R2. Estimation of (4) in $C_{ff}^{\Delta}(q)$ such that e is minimized, despite the presence of v in the performed task.

In view of the identified requirements R1–R2 for $V(C_{ff}^{\Delta})$, the contributions of this paper are twofold. First, it is shown that in existing procedures in iterative feedforward control, R1 and R2 are conflicting in the presence of v . Then, it is shown that the underlying problem can be interpreted as a closed-loop identification problem. Second, a novel criterion $V(C_{ff}^{\Delta})$ is proposed that attains requirements R1–R2. This is achieved by establishing a connection to closed-loop identification techniques. Furthermore, it is shown that the proposed $V(C_{ff}^{\Delta})$ has strong similarities with classical manual feedforward tuning.

3. Analysis of existing procedures in the presence of noise

In this section, it is shown that for the iterative feedforward control approach proposed in Van der Meulen et al. (2008), Heertjes et al. (2010), and Boeren, Bruijnen et al. (2014), requirements R1–R2 are conflicting in the presence of v . For clarity of exposition, it is assumed that $C_{ff}^{j+1}(q)$ in (2) is equal to $C_{ff}^{\Delta}(q)$, i.e., $C_{ff}^{j+1}(q)$ is determined based on a single task without prior feedforward controller.

Definition 3. The criterion in (3) is defined as

$$V_2(\theta) = \|\hat{e}(\theta)\|_2^2, \quad (6)$$

where $\hat{e}(\theta)$ is given by

$$\hat{e}(\theta) = e_m - S(q)P(q)C_{ff}(q, \theta)r, \quad (7)$$

with e_m as defined in Section 2.2.

Similar to the analysis provided in Section 2.1, Definition 3 implies that the optimal feedforward controller, i.e., $C_{ff}(q)$ such that $V_2(\theta) = 0$, is given by $C_{ff}(q) = P^{-1}(q)$.

Crucially, θ should be estimated based on measurement data only in a data-driven approach, i.e., without explicitly constructing parametric or nonparametric models of closed-loop transfer functions, e.g., as in Hjalmarsson et al. (1998). The following result derived in Van der Meulen et al. (2008) is essential for subsequent derivations.

Lemma 1. Assume that $v=0$. Then, for $C_{ff}^j(q) = 0$,

$$y_m = S(q)P(q)C_{fb}(q)r + S(q)v,$$

is equivalent to

$$S(q)P(q)r = C_{fb}^{-1}(q)y_r. \quad (8)$$

with y_r as defined in Section 2.2.

This auxiliary result enables the estimation of θ based on the known feedback controller $C_{fb}(q)$ and measured signal y_m , without modelling $S(q)$ and $P(q)$.

Remark 1. An approach to deal with possible instability of $C_{fb}^{-1}(q)$ in computing $C_{fb}^{-1}(q)y_r$ is presented in Appendix A.

In practice, the measured output y_m is always contaminated by the unknown disturbance v . Recall from Section 2.1 that v is assumed to be given by $v = H(q)\epsilon$, where $H(q)$ is monic and ϵ is normally distributed white noise with zero mean and variance λ_ϵ^2 . Following a similar reasoning as in Lemma 1, (8) is for nonzero v given by

$$(S(q)P(q)r)_{\text{est}} = C_{fb}^{-1}(q)y_m. \quad (9)$$

By evaluating the expected value of (9),

$$\begin{aligned} \mathbb{E}\{S(q)P(q)r\}_{\text{est}} &= \mathbb{E}\{C_{fb}^{-1}(q)[y_r + y_v]\} \\ &= S(q)P(q)r, \end{aligned}$$

it follows that the approximation of $S(q)P(q)r$ is unbiased and hence seems suitable. The resulting data-driven optimization problem with respect to θ is stated in the following definition.

Definition 4. Given measured signals e_m, y_m . Then, for $C_{ff}(q, \theta) \in \mathcal{C}$, minimization of (6) with respect to θ

$$\hat{\theta}_N = \arg \min_{\theta} V_2(C_{ff}(q, \theta)),$$

is equivalent to the least squares solution to

$$\Phi\theta = e_m, \quad (10)$$

where $\Phi = \Psi(q)C_{fb}^{-1}(q)y_m \in \mathbb{R}^{N \times n_\theta}$, and e_m as defined in Section 2.2.

The following assumption ensures that $\hat{\theta}_N$ can be uniquely determined.

Assumption 1. $\Phi^T\Phi$ is nonsingular.

Assumption 1 imposes a persistence of excitation condition on r . Note that the validity of this assumption is closely related to the selected order n_θ of C_{ff} in Definition 2. For motion systems with dominant rigid-body dynamics, this assumption holds in general for the parametrization proposed in Section 2.3 based on acceleration, jerk and snap feedforward. That is, the inclusion of derivatives of r up to snap in C_{ff} improves the performance in terms of Definition 3, see, e.g., Lambrechts et al. (2005). The solution to (10) is given by

$$\hat{\theta}_N = \left(\frac{1}{N}\Phi^T\Phi\right)^{-1}\frac{1}{N}\Phi^Te_m. \quad (11)$$

Next, it is shown that the requirements R1 and R2 are conflicting in the presence of v . First, consider the following definition of the optimal feedforward controller.

Definition 5. The optimal feedforward controller $C_{ff}(q, \theta_0)$ with true parameter vector θ_0 is defined as $C_{ff}(q, \theta_0) = \Psi(q)\theta_0 = P^{-1}(q)$, and is optimal in the sense that the reference-induced error e_r is eliminated, i.e., $e_r - \Phi\theta_0 = 0$ for $C_{ff}^j(q) = 0$.

For the polynomial parametrization of C_{ff}^{j+1} as defined in Definition 2, a necessary condition for the existence of $C_{ff}(q, \theta_0)$ is that $P(q)$ is restricted to a rational function with unit numerator. Then, Definition 5 implies that the measured error signal e_m is equivalent to

$$e_m = \Phi\theta_0 + e_v. \quad (12)$$

Substitute (12) in (11) to obtain

$$\hat{\theta}_N = \theta_0 + R_{\Phi\Phi}^{-1}(N)R_{\Phi e_v}(N),$$

with $R_{\Phi\Phi}(N)$ an estimate of the autocorrelation matrix $R_{\Phi\Phi}$ and $R_{\Phi e_v}(N)$ an estimate of the cross-correlation vector $R_{\Phi e_v}$ based on N samples given by

$$R_{\Phi\Phi}(N) = \frac{1}{N}\Phi^T\Phi, \quad R_{\Phi e_v}(N) = \frac{1}{N}\Phi^Te_v.$$

In Söderström and Stoica (1989, chap. 7) it is shown that under mild assumptions,

$$R_{\Phi\Phi} = \lim_{N \rightarrow \infty} R_{\Phi\Phi}(N), \quad R_{\Phi e_v} = \lim_{N \rightarrow \infty} R_{\Phi e_v}(N).$$

As a result, $\hat{\theta}_N$ converges if N tends to infinity with probability 1 to

$$\hat{\theta}_N = \theta_0 + R_{\Phi\Phi}^{-1}R_{\Phi e_v}. \quad (13)$$

Expression (13) reveals that $\hat{\theta}_N$ is an unbiased estimate of θ_0 if $R_{\Phi e_v} = 0$. Recall from Definition 4 that Φ is constructed based on y_m , which is contaminated by the unknown v . Hence, e_v and Φ are correlated, and the bias is given by

$$\Delta\hat{\theta}_N = R_{\Phi\Phi}^{-1}R_{\Phi e_v}.$$

Summarizing, the approach presented in Van der Meulen et al. (2008), Heertjes et al. (2010), and Boeren, Bruijnen et al. (2014) results in a biased estimate, i.e., $\mathbb{E}\hat{\theta}_N \neq \theta_0$ for $\lambda_\epsilon > 0$, when measurements from a single task are used. This shows that the requirements R1 and R2 in Section 2.4 are conflicting in the presence of v .

Remark 2. In Van der Meulen et al. (2008, Section 2.4), inspired by a similar approach developed in IFT (Hjalmarsson et al., 1998), a procedure is proposed that results in an unbiased estimate at the expense of measuring two tasks. However, this two-step approach implies that C_{ff} cannot be updated after each task, thereby conflicting requirement R1.

4. Proposed approach based on closed-loop identification

In this section, a new procedure is presented that exploits knowledge of r in the optimization criterion $V(\theta)$ to simultaneously achieve requirements R1–R2 in Section 2.4. To this purpose, a connection is proposed between instrumental variable identification techniques, see, e.g., Söderström and Stoica (1983) and iterative feedforward tuning as in Van der Meulen et al. (2008). The instrumental variable approach is well-established in closed-loop identification, see, e.g., Gilson, Garnier, Young, and Van den Hof (2011) and Söderström and Stoica (1983). For iterative feedforward control, the corresponding criterion is posed in the following definition.

Definition 6. The criterion in (3) is defined as

$$V_z(\theta) = \left\| Z^T \hat{e}(\theta) \right\|_W^2,$$

where $Z \in \mathbb{R}^{N_z \times n_\theta}$ are instrumental variables, W is a positive-definite weighting matrix, $N_z \geq N$, and $\hat{e}(\theta)$ as given in (7).

In this section, the basis instrumental variable approach is pursued, see, e.g., Söderström and Stoica (1983, chap. 3), in which case $Z \in \mathbb{R}^{N \times n_\theta}$ and $W = I$. The parameters θ of $C_{ff}(q, \theta)$ then result from the set of equations

$$Z^T \left[e_m - \Phi \hat{\theta}_N^{IV} \right] = 0. \quad (14)$$

The solution to (14) is given by

$$\hat{\theta}_N^{IV} = \left(\frac{1}{N} Z^T \Phi \right)^{-1} \frac{1}{N} Z^T e_m, \quad (15)$$

where $\Phi = \Psi(q)C_{fb}^{-1}(q)y_m \in \mathbb{R}^{N \times n_\theta}$. The following assumption guarantees that $\hat{\theta}_N^{IV}$ can be uniquely determined.

Assumption 2. $Z^T \Phi$ is nonsingular.

Assumption 2 implies that Z should be correlated with Φ . The freedom that exists in the construction of Z can be used to obtain an unbiased estimate, i.e., $\mathbb{E}\hat{\theta}_N^{IV} = \theta_0$, $\forall \lambda_c \geq 0$. To show this, substitute (12) in (15) to obtain

$$\hat{\theta}_N^{IV} = \theta_0 + R_{Z\Phi}^{-1}(N)R_{Ze_v}(N),$$

with

$$R_{Z\Phi}(N) = \frac{1}{N}Z^T \Phi, \quad R_{Ze_v}(N) = \frac{1}{N}Z^T e_v,$$

an estimate of the cross-correlation matrix $R_{Z\Phi}$ and cross-correlation vector R_{Ze_v} based on N samples, respectively. In Söderström and Stoica (1989, chap. 7) it is shown that under mild assumptions,

$$R_{Z\Phi} = \lim_{N \rightarrow \infty} R_{Z\Phi}(N), \quad R_{Ze_v} = \lim_{N \rightarrow \infty} R_{Ze_v}(N).$$

As a result, $\hat{\theta}_N^{IV}$ converges if N tends to infinity with probability 1 to

$$\hat{\theta}_N^{IV} = \theta_0 + R_{Z\Phi}^{-1}R_{Ze_v}. \quad (16)$$

The estimate $\hat{\theta}_N^{IV}$ is (asymptotically) unbiased if Z is constructed such that Z and e_v are uncorrelated, i.e., $R_{Ze_v} = 0$.

Based on (16) and Assumption 2, the proposed design of instrumental variables is given by $Z = [\psi_1 r, \psi_2 r, \dots, \psi_{n_\theta} r]$ with basis functions $\Psi(q)$ as in Definition 2. There are two key reasons for this design. First, recall that the feedforward control goal is to minimize the servo error $e_m = r - y_m$, which implies that r and y_m are correlated. As a result, the proposed instruments Z and Φ are correlated, and Assumption 2 holds. Second, the known reference r and e_v are uncorrelated. Hence, (16) reveals that an unbiased estimate $\hat{\theta}_N^{IV}$ is obtained with the proposed design of Z . For the feedforward signal u_{ff} in Section 2.1, the proposed instruments Z represent respectively acceleration, jerk and snap, i.e., the 2nd, 3rd and 4th derivative of the multi-segment polynomial trajectory r .

Concluding, a new criterion $V_z(\theta)$ is employed that results in $\mathbb{E}\hat{\theta}_N^{IV} = \theta_0$, $\forall \lambda_c \geq 0$, when measurements from a single task are used. This illustrates that requirements R1–R2 in Section 2.4 are simultaneously attained for $V_z(\theta)$.

Remark 3. As stated in Remark 2, the two-step data-driven approach presented in Van der Meulen et al. (2008, Section 2.4) determines an estimate $\hat{\theta}_N$ based on two tasks. This approach has a clear interpretation in the instrumental variable framework. In particular, the second task is exploited to construct instrumental variables $Z = \hat{\Phi}_2$. This approach imposes the stringent condition on the system that v^1 and v^2 are uncorrelated. If this condition holds, v is eliminated from the optimization problem and consequently requirement R2 is attained. Still, two tasks are required to update the feedforward controller C_{ff} , i.e., the experimental cost is not minimal.

Remark 4. Besides the two-step data-driven approach, Van der Meulen et al. (2008, Section 2.4) also formulated an ILC-related model-based approach. That is, the instrumental variables become $Z = [SP(q)\psi_1(q)r, SP(q)\psi_2(q)r, \dots, SP(q)\psi_{n_\theta}(q)r]$, where it is assumed that a model of $SP(q)$ is available.

5. Iterative tasks

In this section, the instrumental variable method in Section 4 is embedded in the iterative task framework in Section 2. As argued in Gunnarsson and Norrlöf (2006), iterative tasks are used to

minimize the influence of iteration-invariant disturbances, nonlinearities and measurement noise.

The pursued approach to adapt $C_{ff}(q, \theta)$ is to use recursive estimates of θ . Consider the two degree-of-freedom control configuration as depicted in Fig. 4. The feedforward controller $C_{ff}^j(q, \theta_N^j)$ in the j th task is updated by $C_{ff}^j(q, \hat{\theta}_N^j)$. Herein, $\hat{\theta}_N^j$ is determined based on the measured signals e_m^j and y_m^j in the j th task given by

$$e_m^j = S(q)(1 - P(q)C_{ff}^j(q, \theta^j))r - S(q)v, \\ y_m^j = S(q)P(q)(C_{fb}(q) + C_{ff}^j(q, \theta^j))r + S(q)v.$$

Proposition 1. Given e_m^j , y_m^j , and $C_{ff}^j(q, \theta^j) \in \mathcal{C}$ in the j th task. The IV estimate $\hat{\theta}_N^{\Delta, IV}$ is the solution to

$$\hat{\theta}_N^{\Delta, IV} = \left(\frac{1}{N}Z^T \Phi^j \right)^{-1} \frac{1}{N}Z^T e_m^j,$$

where

$$\Phi^j = \Psi(q)(C_{fb}(q) + C_{ff}^j(q, \theta^j))^{-1} y_m^j \in \mathbb{R}^{N \times n_\theta}, \quad (17)$$

and $Z = [\psi_1 r, \psi_2 r, \dots, \psi_{n_\theta} r] \in \mathbb{R}^{N \times n_\theta}$.

The following result enables recursive estimation of θ .

Theorem 1. For $C_{ff}^j, C_{ff}^{\Delta} \in \mathcal{C}$ with identical basis functions $\Psi(q)$, $C_{ff}^{j+1}(q, \theta^{j+1})$ in (2) is given by

$$C_{ff}^{j+1}(q, \theta^{j+1}) = \sum_{i=1}^{n_\theta} \psi_i(q) \theta_i^{j+1},$$

where $\theta_i^{j+1} = \theta_i^j + \theta_i^{\Delta}$.

Proof. Since $C_{ff}^j, C_{ff}^{\Delta} \in \mathcal{C}$ have identical $\Psi(q)$ in (5), C_{ff}^{j+1} is given by

$$C_{ff}^{j+1} = C_{ff}^j(q) + C_{ff}^{\Delta}(q) = \sum_{i=1}^{n_\theta} \psi_i(q) \theta_i^j + \sum_{i=1}^{n_\theta} \psi_i(q) \theta_i^{\Delta}.$$

Since C_{ff}^j and C_{ff}^{Δ} are linear in respectively θ^j and θ^{Δ} , superposition implies that

$$C_{ff}^{j+1}(q, \theta^{j+1}) = \sum_{i=1}^{n_\theta} \psi_i(q) (\theta_i^j + \theta_i^{\Delta}) = \sum_{i=1}^{n_\theta} \psi_i(q) \theta_i^{j+1}. \square$$

Theorem 1 shows that the update of θ^{j+1} with respect to θ^j is solely based on measurements from the j th task.

Remark 5. The proposed recursive estimation based on measurements from a single task can be directly generalized to determine $\hat{\theta}_N^{\Delta}$ based on measurement data from multiple iterative tasks. An analysis of iterations in system identification is provided in Rojas, Oomen, Hjalmarsson, and Wahlberg (2012). In this iterative framework, the variance of $\hat{\theta}_N^{\Delta}$ is reduced at the expense of performing multiple tasks.

Combining Proposition 1 and Theorem 1 leads to the following procedure to update C_{ff} based on the j th task, which implements the main contribution of this paper.

Procedure 1. Estimation of $\hat{\theta}_N^{\Delta}$ after the j th task

- (1) Measure e_m^j and y_m^j in the j th task with $C_{ff}^j(q, \theta^j)$ applied to the system.
- (2) Construct $\Phi^j = \Psi(q)(C_{fb}(q) + C_{ff}^j(q))^{-1} y_m^j$.
- (3) Construct instrumental variables $Z = [\psi_1 r, \psi_2 r, \dots, \psi_{n_\theta} r]$.
- (4) Solve $\hat{\theta}_N^{\Delta} = \left(\frac{1}{N}Z^T \Phi^j \right)^{-1} \frac{1}{N}Z^T e_m^j$.
- (5) Construct the new feedforward controller $C_{ff}^{j+1}(q) = \Psi(q)(\theta^j + \hat{\theta}_N^{\Delta})$.
- (6) Set $j \rightarrow j+1$ and go to Step 1.

An approach to deal with possible instability of $(C_{fb}(q) + C_{ff}^j(q, \theta^j))^{-1}$ in computing $\Phi^j = \Psi(q)(C_{fb}(q) + C_{ff}^j(q))^{-1}y_m^j$ is presented in [Appendix A](#). This approach assumes an infinite time horizon and can therefore result in transients at the initial and final position of a finite time task, see, e.g., [Norrlöf and Gunnarsson \(2002\)](#). For the proposed instrumental variable method, $\hat{\theta}_N^A$ is not affected by these transients. To illustrate this statement, recall that for the tasks considered in this paper, the system starts at rest in an initial position and comes to a rest in a final position. Since Z consist of derivatives of r , the instruments Z are equal to zero at the start and the end of such tasks. This implies that in [Procedure 1](#), $(1/N)Z^T \Phi^j$ in $\hat{\theta}_N^A = ((1/N)Z^T \Phi^j)^{-1}(1/N)Z^T e_m^j$ is not influenced by transients that result from computing Φ^j .

Remark 6. Contrary to the instrumental variable method, the least squares method given in (11) suffers from transients that result from computing Φ^j . This is explained by observing that (11) contains the term $(1/N)(\Phi^j)^T \Phi^j$.

[Procedure 1](#) provides a systematic procedure to improve the performance of the system in [Fig. 1](#) by exploiting recursive estimates of θ to update $C_{ff}^j(q, \theta^j)$.

6. Experimental results

In this section, the theoretical results proposed in this paper are validated on an experimental setup. In [Section 6.1](#), the two-mass spring damper setup is described that is used to conduct experiments. In [Section 6.2](#), the control goal is specified. In [Section 6.3](#), the parametrization of the feedforward controller $C_{ff}(q)$ is introduced. In [Section 6.4](#), an adjustable disturbance v_{add} is introduced that acts on the closed-loop system. In [Section 6.5](#), a controlled experiment is performed to analyze the effect of the single independent variable v_{add} on the estimates $\hat{\theta}_N$ and $\hat{\theta}_N^{IV}$, given by respectively (11) and (15).

6.1. Experimental setup

In this section, the theoretical results proposed in this paper are validated on the two-mass spring damper setup depicted in [Fig. 5](#). A schematic illustration of the two-mass spring damper setup is shown in [Fig. 6](#), where the flexible shaft is modelled as a spring and a damper. This experimental setup is used to experimentally validate control strategies for high-precision motion systems. The dynamical behavior of this system contains key aspects in motion control,

Fig. 5. Photograph of the experimental two-mass spring damper setup at the CST group, Department of Mechanical Engineering, Eindhoven University of Technology.

including collocated and non-collocated dynamics, while measurement noise, friction, delay and nonlinearities are small. As a result, a controlled experiment can be performed to analyze the influence of a single parameter on the servo performance of the system. The characteristics make this system also relevant as a benchmark problem in robust control, see, e.g., [Wie and Bernstein \(1992\)](#).

The setup consists of two masses m_1 and m_2 which are connected through a flexible shaft. Furthermore, the inertia of the system is given by $3.7 \times 10^{-4} \text{ kg m}^2$. The angular position of both m_1 and m_2 are measured by means of encoders with a resolution of $1 \times 10^{-3} \cdot \pi \text{ rad}$. However, in the presented experiments only measurements of the angular position of m_1 are exploited to evaluate the servo performance obtained with $C_{fb}(q)$. The setup is equipped with a single DC motor that is rigidly connected to m_1 . All experiments are performed with sample time $T_s = 1/2048 \text{ s}$.

The frequency response function of the considered two-mass spring damper setup is depicted in [Fig. 7](#). Inspection reveals rigid-body behavior below 25 Hz, while the resonance phenomenon related to the flexible dynamics of the system appears at 55 Hz. The feedback controller designed to stabilize the system P , as depicted in [Fig. 8](#), is given by

$$C_{fb}(q) = \frac{0.04578q^2 - 0.09119q + 0.04541}{q^3 - 2.91q^2 + 2.822q - 0.9121},$$

which results in a bandwidth $f_{bw} = 5 \text{ Hz}$.

6.2. Control goal

The control goal defined for the experimental setup in [Fig. 7](#) is to minimize the servo error $e_m = r - y_{m1}$, where r is a 3rd order servo task r , designed according to the procedure proposed in [Lambrechts et al. \(2005\)](#), and y_{m1} is the measured angular position of the mass m_1 . In [Fig. 9](#), r is depicted together with the corresponding velocity v and acceleration a .

6.3. Parametrization feedforward controller

In this section, a parametrization for C_{ff} is proposed for the two-mass spring damper setup. As in [Lambrechts et al. \(2005\)](#), the

Fig. 6. Schematic illustration of the two-mass spring damper setup.

Fig. 7. Frequency response function of the two-mass spring damper setup.

Fig. 8. Bode diagram of feedback controller $C_{fb}(q)$.

Fig. 9. Position r , velocity vel and acceleration a of the servo task.

feedforward controller is parametrized as $C_{ff}(q, \theta) = \psi(q)\theta_{acc}$, where

$$\psi(q) = \frac{q^2 - 2q + 1}{T_s^2 q^2},$$

with initial value $\theta_{acc}^{init} = 1 \times 10^{-4}$. This parameterization of $C_{ff}(q)$ consists of acceleration feedforward to compensate for the rigid-body dynamics of P in 0–20 Hz.

6.4. Disturbance design

In this section, an adjustable disturbance term v_{add} is defined that is applied as a disturbance to the two-mass spring damper setup in Fig. 5. As a result, a controlled experiment is created to analyze the influence of the single independent disturbance variable v_{add} on the estimates $\hat{\theta}_N$ and $\hat{\theta}_N^{IV}$, given by respectively (11) and (15). Similar to Section 2.1, v_{add} is modelled as $v_{add} = H(q)\epsilon$, where $H(q)$ is given by

$$H(q) = \frac{0.8048q^2 - 1.61q + 0.8048}{q^2 - 1.571q + 0.6481},$$

and ϵ is normally distributed white noise with zero mean and variance λ_ϵ^2 . The disturbance v_{add} represents high-frequency measurement noise, which is a typical disturbance in motion systems. Here, the standard deviation λ_ϵ constitutes a design parameter that is exploited in the next section to analyze the influence of v_{add} on the parameter estimation.

6.5. Experimental results

In this section, the iterative task framework established in Section 5 is used to determine θ_{acc} such that the reference-induced error e_r is minimized, despite the presence of v_{add} . To this purpose, $j = 10$ tasks are executed. After each task, θ_{acc} is updated. For the proposed instrumental variable approach in Procedure 1, based on minimization of $V_z(\theta)$, this gives the estimator $\hat{\theta}_N^{j,IV}$ after the j th iteration. The instrumental variables Z are given by $Z = \psi(q)r$, with $\psi(q)$ as defined in Section 6.3.

To compare the results with existing iterative feedforward tuning procedures, the estimator $\hat{\theta}_N^j$ is included which is obtained based on minimization of $V_2(\theta)$ in Definition 3. The complete experiment is repeated $m = 5$ times. The sample means corresponding to the j th task are given by

$$\bar{\theta}^{j,IV} = \frac{1}{m} \sum_{l=1}^m \hat{\theta}_{N,l}^{j,IV}, \quad \bar{\theta}^j = \frac{1}{m} \sum_{l=1}^m \hat{\theta}_{N,l}^j,$$

where the number of samples is given by $N = 1630$.

Two cases are experimentally illustrated. First, the sample mean for both approaches in the final task, i.e., $j = 10$, is analyzed as a function of λ_ϵ . Second, recursive estimation of $\bar{\theta}^{j,IV}$ and $\bar{\theta}^j$ as a function of j is considered for a fixed standard deviation λ_ϵ . The presented cases combined illustrate the advantages the instrumental variable approach proposed in this paper has to offer compared to pre-existing approaches.

The sample means $\bar{\theta}^{10,IV}$ and $\bar{\theta}^{10}$ in the 10th task are depicted in Fig. 10 as a function of λ_ϵ . The following observations are made:

- (1) For $\lambda_\epsilon = 0$, $\bar{\theta}^{10,IV} \neq \bar{\theta}^{10}$. As discussed in Section 6.3, C_{ff} consist solely of acceleration feedforward. This results in undermodelling of P^{-1} , which is known to shape the bias in the frequency domain, see, e.g., Ljung (1999, Eq. 8.71). The difference between $\bar{\theta}^{10,IV}$ and $\bar{\theta}^{10}$ is explained by a different shape of the bias due to undermodelling.
- (2) The sample mean $\bar{\theta}^{10,IV}$ is independent of λ_ϵ . That is, the estimator based on instrumental variables compensates for the dominant contribution of e_r , irrespective of v .
- (3) The sample mean $\bar{\theta}^{10}$ depends on λ_ϵ . As a result, pre-existing approaches for iterative feedforward control result in a $C_{ff}(q, \theta)$ which depends on v . This implies that e_r is not minimal, and as a result servo performance is compromised.
- (4) The sample mean $\bar{\theta}^{10,IV}$ is equal to the inertia of the system as specified in Section 6.1.

Fig. 10. Sample mean $\bar{\theta}^{10,IV}$ (dashed green) and $\bar{\theta}^{10}$ (red) as a function of λ_ϵ illustrate that $\bar{\theta}^{10,IV}$ is unbiased for $\lambda_\epsilon > 0$, while $\bar{\theta}^{10}$ is a biased estimator if $\lambda_\epsilon > 0$. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

In the second experimental case, recursive estimation of $\bar{\theta}^{j,IV}$ and $\bar{\theta}^j$ as a function of j is considered for $\lambda_e = 0$ and $\lambda_e = 8 \times 10^{-2}$. In Fig. 11, the sample mean $\bar{\theta}^j$ based on minimizing $V_2(\theta)$ is depicted, while $\bar{\theta}^{j,IV}$ is shown in Fig. 12. The following observations are made:

- (1) For $\lambda_e = 0$, measured data from a single task is sufficient for both approaches to reach a converged value.
- (2) The sample mean $\bar{\theta}^{j,IV}$ converges to an identical value after a single task for $\lambda_e = 0$ and $\lambda_e = 8 \times 10^{-2}$. This illustrates that $\bar{\theta}^{j,IV}$ is independent of λ_e , $\forall j$.

Fig. 11. Pre-existing approach: The sample mean $\bar{\theta}^j$ as a function of tasks for $\lambda_e = 0$ (blue) and $\lambda_e = 8 \times 10^{-2}$ (dashed red) illustrates that (i) multiple iterations are required to reach a converged value for $\bar{\theta}^j$ if $\lambda_e = 8 \times 10^{-2}$ and (ii) the converged value for $\bar{\theta}^j$ depends on λ_e . (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

Fig. 12. Proposed approach: In contrast to the pre-existing approach in Fig. 11, the sample mean $\bar{\theta}^{j,IV}$ as a function of tasks for $\lambda_e = 0$ (blue) and $\lambda_e = 8 \times 10^{-2}$ (dashed green) confirms that $\bar{\theta}^{j,IV}$ is independent of λ_e , $\forall j$. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

Fig. 13. The servo performance is significantly improved by means of the proposed approach (green) compared to pre-existing approaches (red). (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

- (3) For $\lambda_e = 8 \times 10^{-2}$, measured data from multiple tasks is required for the sample mean $\bar{\theta}^j$ to reach a converged value. This shows that the convergence rate of $\bar{\theta}^j$ is deteriorated if $\lambda_e = 8 \times 10^{-2}$.
- (4) As already illustrated in Fig. 10, the converged value of $\bar{\theta}^{j,IV}$ is identical for all λ_e , while the converged value of $\bar{\theta}^j$ depends on λ_e . This shows that iterating does not diminish the bias of $\bar{\theta}^j$ for $\lambda_e > 0$.

Finally, the error signal e_m , averaged over $m=5$ realizations, is depicted in Fig. 13 for $\lambda_e = 8 \times 10^{-2}$ in the 10th task. The results confirm that the proposed approach enhances performance compared to pre-existing approaches.

7. Conclusions

In this paper, a new approach for iterative feedforward control is presented based on closed-loop identification techniques, which significantly enhances existing feedforward control algorithms. In Section 3, it is shown that a trade-off exists in pre-existing iterative feedforward tuning procedures between (i) the number of tasks required to update the feedforward controller and (ii) the servo performance of the system. The main contribution of this paper is a new iterative feedforward tuning procedure that is efficient, i.e., requires measured data from only a single task to update the feedforward controller, and accurate, i.e., minimizes the servo error induced by the reference signal, independent of an unknown disturbance v . Experimental results provided in Section 6 confirm that the proposed approach is superior with respect to pre-existing approaches.

The proposed approach can be straightforwardly extended to other optimization problems in a closed-loop configuration. Furthermore, the proposed IV procedure directly generalizes to closely related projection methods, see, e.g., Van den Hof and Schrama (1995) and Forssell and Ljung (2000). Finally, ongoing research focuses on accuracy properties of the estimates as in Boeren, Oomen, and Steinbuch (2014), extensions to recursive estimation which exploit measured data from multiple tasks, a rational basis as in Bolder and Oomen (in press), parametrized friction feedforward, stochastic approximation and multivariable systems.

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Appendix A. Stable inversion

In this section, a stable inversion procedure is described to determine (17) if $(C_b + C_{ff}^j)^{-1}$ is unstable.

- (1) Let the discrete-time system $R(z) = (C_b(z) + C_{ff}^j(z))^{-1}$ have a state-space realization given by

$$\begin{aligned} x(t+1) &= Ax(t) + By_m^j(t), \\ u(t) &= Cx(t) + Dy_m^j(t). \end{aligned}$$

(2) Transform this system under similarity to

$$\begin{bmatrix} x_{st}(t+1) \\ x_{un}(t+1) \end{bmatrix} = \begin{bmatrix} A_{st} & 0 \\ 0 & A_{un} \end{bmatrix} \begin{bmatrix} x_{st}(t) \\ x_{un}(t) \end{bmatrix} + \begin{bmatrix} B_{st} \\ B_{un} \end{bmatrix} y_m^j(t),$$

$$u(t) = [C_{st} \ C_{un}] \begin{bmatrix} x_{st}(t) \\ x_{un}(t) \end{bmatrix} + D y_m^j(t),$$

where $\begin{bmatrix} A_{st} & 0 \\ 0 & A_{un} \end{bmatrix}$ is the Jordan canonical form of A . Furthermore, the eigenvalues of A_{st} , denoted as $\lambda(A_{st})$, satisfy $|\lambda_i(A_{st})| < 1, \forall i$, while $|\lambda_k(A_{un})| > 1, \forall k$. That is, R is decomposed into a stable (A_{st} , B_{st} , C_{st} , D) and an unstable (A_{un} , B_{un} , C_{un} , 0) subsystem, with corresponding state variables x_{st} and x_{un} , respectively.

(3) Solve forward in time with initial condition $x_{st}(0) = 0$:

$$x_{st}(t+1) = A_{st}x_{st}(t) + B_{st}y_m^j(t),$$

and backward in time with initial condition $x_{un}(N) = 0$:

$$x_{un}(t+1) = A_{un}x_{un}(t) + B_{un}y_m^j(t).$$

(4) Determine

$$u(t) = [C_{st} \ C_{un}] \begin{bmatrix} x_{st}(t) \\ x_{un}(t) \end{bmatrix} + D y_m^j(t).$$

Remark 7. A state-space realization of R exists if R is proper. If this condition is not valid, preview-based techniques are required to determine a state-space realization of R , see, e.g., Zou and Devasia (1999).

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