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Iterative motion feedforward tuning: A data-driven approach based on instrumental variable identification

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**Abstract**

Feedforward control can significantly enhance the performance of motion systems through compensation of known disturbances. This paper aims to develop a new procedure to tune a feedforward controller based on measured data obtained in finite time tasks. Here, a suitable feedforward parameterization is introduced that provides good extrapolation properties for a class of reference signals. Next, connections with closed-loop system identification are established. In particular, instrumental variables, which have been proven very useful in closed-loop system identification, are selected to tune the feedforward controller. These instrumental variables closely resemble traditional engineering tuning practice. In contrast to pre-existing approaches, the feedforward controller can be updated after each task, irrespective of noise acting on the system. Experimental results confirm the practical relevance of the proposed method.

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**1. Introduction**

Feedforward control is widely used in control systems, since feedforward can effectively reject disturbances before they affect the system. Indeed, many applications to high-performance systems have been reported where feedforward control leads to a significant performance improvement. For servo systems, the main performance improvement is in general obtained by using feedforward to compensate for the reference signal. Relevant examples of feedforward control include model-based feedforward, see, e.g., Zhong, Pao, and de Callafon (2012), Clayton, Tien, Leang, Zou, and Devasia (2009) and Butterworth, Pao, and Abramovitch (2012), and Iterative Learning Control (ILC), see, e.g., Bristow, Tharayil, and Alleyne (2006) and Moore (1993).

On the one hand, model-based feedforward results in general in good performance and provides extrapolation capabilities of tasks. In model-based feedforward, a parametric model is determined that approximates the inverse of the system. The performance improvement induced by model-based feedforward is highly dependent on (i) the model quality of the parametric model of the system and (ii) the accuracy of model-inversion, see, e.g., Devasia (2002). On the other hand, ILC results in superior performance with respect to model-based feedforward. By learning from previous iterations, high performance is obtained for a single, specific task, i.e., at the expense of poor extrapolation capabilities of tasks. In addition, ILC only requires an approximate model of the system.

Recently, an approach is presented in van de Wijdeven and Bosgra (2010) that combines the advantages of model-based feedforward and ILC, resulting in both high performance and good extrapolation capabilities. To this purpose, basis functions are introduced that reflect the dynamical behavior of the system responsible for the dominant contribution to the servo error. In Van der Meulen, Tousain, and Bosgra (2008), the need for an approximate model of the system, as is common in ILC, is eliminated by exploiting concepts from iterative feedback tuning (IFT) (Hjalmarsson, Gevers, Gunnarsson, & Lequin, 1998). This approach is extended to input shaping in Boeren, Bruijnen, van Dijk, and Oomen (2014) and multivariable systems in Heertjes, Hennekens, and Steinbuch (2010), while a comparative study of data-driven feedforward control procedures is reported in Stearns, Yu, Fine, Mishra, and Tomizuka (2008). However, by eliminating the need for an approximate model of the system, the approach presented in Van der Meulen et al. (2008) requires a significantly larger experimental cost to perform an update of the feedforward controller and puts stringent assumptions on noise acting on the system.

Although iterative feedforward tuning is widely successful to improve the performance of motion systems, existing tuning procedures (i) impose stringent requirements on noise acting on the system, (ii) require two tasks for each iterative update of the feedforward controller and (iii) can lead to a bias error. In this paper, it is shown that these deficiencies can be removed by connecting iterative feedforward tuning to system identification, and exploit
concepts from closed-loop system identification in iterative feedforward tuning. In fact, in contrast to pre-existing procedures in Van der Meulen et al. (2008), Boeren, Bruijnen et al. (2014) and Heertjes et al. (2010), the proposed procedure closely resembles manual feedforward tuning procedures for motion systems, see, e.g., Boerlage, Toussain, and Steinbuch (2004). This immediately confirms the practical relevance of the proposed approach for industrial motion systems.

The main contribution of this paper is an iterative feedforward tuning approach that is efficient, i.e., it requires measured data from only a single task, and accurate, i.e., attains optimal performance for feedforward control in the presence of noise. The proposed approach is closely related to Söderström and Stoica (1983), Gilson and Van den Hof (2005), Jung and Enqvist (2013) and Karimi, Butcher, and Longchamp (2008), and extends this work to iterative tuning of feedforward controllers. Furthermore, the motivation for the proposed approach is similar to the approach in Kim and Zou (2013), i.e., combine the advantages of model-based feedforward and ILC without the need for an approximate model of the system. The key difference is that in Kim and Zou (2013) a nonparametric model for the feedforward controller is constructed, while this work aims to determine a parametric model. This paper is an extension of Boeren and Oomen (2013) that includes experimental results, and a complete explanation and analysis.

This paper is organized as follows. In Section 2, the problem formulation is outlined. Then, in Section 3, it is shown that in the presence of noise, existing procedures suffer from a closed-loop identification problem. In Section 4, a new feedforward control procedure is proposed which requires only a single task to update the feedforward controller in the presence of noise. Then, in Section 5 the proposed approach is embedded in the iterative feedforward tuning framework. In Section 6, the experimental results of the proposed approach are presented. Finally, a conclusion is presented in Section 7.

Notation: For a vector \( x \), \( \| x \|^2 = x^T x \). The vector \( u \) is defined as \( u = [u(0), u(1), u(2), \ldots, u(N)]^T \in \mathbb{R}^N \), where \( u(t) \) is a measurement at time instant \( t \) for \( t = 1, 2, \ldots, N \) with \( N \) being the number of samples. The symbol \( q \) denotes the forward shift operator \( q^t u(t) = u(t+1) \). Furthermore, the expected value \( E(x) \) is defined as \( E(x) = \sum_{i=1}^N x_i \), with probability density function \( f(x) \). The correlation function based on a finite number of samples \( N \) is defined as \( R_{yy}(N) = (1/N) \sum_{t=1}^N x(t)y(t) \).

2. Problem formulation

2.1. Feedforward control goal

Consider the two degree-of-freedom control configuration as depicted in Fig. 1. The true unknown system is assumed to be discrete-time, single-input single-output and linear time-invariant, and is denoted as \( P(q) \). The control configuration consists of a given stabilizing feedback controller \( C_{fb}(q) \) and feedforward controller \( C_{ff}(q) \). Let \( r \) denote a known \( n \)-th order multi-segment polynomial trajectory with constraints on the first \( n \) derivatives, generated by a trajectory planning algorithm that takes system dynamics into account, see, e.g., Biagiotti and Melchiorri (2012), Lee, Kim, and Choi (2013) and Lambrechts, Boerlage, and Steinbuch (2005). A typical reference \( r \) in a single task is depicted in Fig. 3. Furthermore, \( v \) denotes a disturbance, \( u_{ff} \) the feedforward signal, and \( e \) the servo error. The unknown disturbance \( v \) is assumed to be given by \( v = H(q)e \), where \( H(q) \) is monic and \( e \) is normally distributed white noise with zero mean and variance \( \sigma_e^2 \). Hence, \( v \) and \( e \) are uncorrelated.

The goal in feedforward control is to attain high performance by compensating for known exogeneous input signals that affect the system. The servo error \( e \) in Fig. 1 as given by

\[
e = S(q)(1 - P(q)C_{ff}(q))r - S(q)v
\]

where \( S(q) = (1 + P(q)C_{fb}(q))^{-1} \), reveals that the contribution of \( v \) induced by \( r \) is eliminated if \( C_{ff}(q) = P^{-1}(q) \). For motion systems with dominant rigid-body dynamics, a parametrization for \( C_{ff}(q) \) is proposed in Lambrechts et al. (2005) which compensates for the dominant component of the reference-induced error. The corresponding \( u_{ff} \) is given by

\[
u_{ff} = \theta_a a + \theta_j j + \theta_s
\]

where \( a, j \) and \( s \) correspond to respectively acceleration, jerk and snap, i.e., the 2nd, 3rd and 4th derivative of the multi-segment polynomial trajectory \( r \), and \( \theta_a, \theta_j, \theta_s \) are the corresponding parameters.

To illustrate this parametrization, consider the acceleration profile \( a \) and measured error \( e_m \) as depicted in Fig. 2. In manual tuning of a feedforward controller, the optimal value for \( \theta_a \) is such that the predicted error

\[
\hat{e}(\theta_a) = e_m - S(q)P(q)\theta_d a
\]

and \( a \) are uncorrelated, where \( e_m = S(q)r - S(q)v \). Likewise, the optimal values for \( \theta_j \) and \( \theta_s \) are obtained if \( \hat{e} \), and \( j \) and \( s \) are uncorrelated, respectively. The results in this paper enable an iterative and automated estimation of the optimal values for \( \theta_a, \theta_j, \theta_s \).

2.2. Iterative feedforward control

In iterative feedforward control, measured data is exploited to update \( C_{ff}(q) \) after each task. For the considered class of systems, a sequence of finite time tasks, denoted as \( j = 1, 2, \ldots, \) with length \( N \) samples is executed. In a single task, the system starts at rest in the initial position, followed by a point-to-point motion, before the system comes to a rest in the final position of a task. A typical reference \( r \) in a single task is shown in Fig. 3. A sequence of such tasks is executed during normal operation of the system, where \( r \) is not necessarily identical for each consecutive task.

The measured signals \( e_m \) and \( y_m \) in the \( j \)th task are given by

\[
e_m = e_j - e_j^p, \quad \text{and} \quad y_m = y_j + y_j^p,
\]

where \( e_j^p = S(q)(1 - P(q)C_{ff}(q))r \) and \( e_j^p = S(q)v \), respectively. Note that since \( P(q), S(q) \) and \( v \) are unknown, it is not possible to construct \( e_j \) and \( e_j^p \) from the measured signal \( e_m \). This also holds for \( y_j \). For clarity of exposition, the index \( j \) is omitted if only a single task...
is considered. The data-driven feedforward optimization problem is formulated in Definition 1.

**Definition 1.** Given measured signals $e_m$ and $y_m$ obtained during the $j$th task of the closed-loop system in Fig. 4 with $C_f(q)$ implemented, then,

$$C_{fj}^{i+1}(q) = C_{fj}(q) + C_{fj}^i(q),$$

(2)

where the update $C_{fj}^i(q)$ based on $e_m$ and $y_m$ results from the optimization problem

$$C_{fj}^{\text{opt}}(q) = \arg \min_{C_f} V(C_{fj}^i(q),$$

(3)

with criterion $V(C_{fj}^i)$ and feedforward controller parametrization $C$.

The criterion $V(C_{fj}^i)$ and parametrization $C$ are essential for high performance of the system with $C_{fj}^i(q)$. In this paper, the impact of $V(C_{fj}^i)$ on the performance of the system in Fig. 1 is analyzed for a fixed $C$. To proceed, $C$ is defined in the next section.

### 2.3. Feedforward controller parameterization

In this section, a general polynomial feedforward parametrization is adopted that encompasses common parametrizations in feedforward control for motion systems, including Lambrechts et al. (2005), Van der Meulen et al. (2008), Heertjes et al. (2010), and Boeren, Bruijnen et al. (2014).

**Definition 2.** The feedforward controller $C_f$ is parametrized as

$$C = \left\{ C_f(q, \theta) | C_f(q, \theta) = \sum_{i=1}^{n_\theta} \psi_i(q) \theta_i, \psi_i \in \mathbb{R}[q], \theta_i \in \mathbb{R} \right\}$$

with parameter vector

$$\theta = [\theta_1, \theta_2, \ldots, \theta_{n_\theta}]^T \in \mathbb{R}^{n_\theta \times 1},$$

(4)

and polynomial basis functions

$$\psi(q) = [\psi_1(q), \psi_2(q), \ldots, \psi_{n_\theta}(q)] \in \mathbb{R}[q]^{1 \times n_\theta}.$$

(5)

The order $n_\theta$ of $C_f$ can be determined by means of a model order selection procedure, see, e.g., Ljung (1999, chap. 6). In terms of Definition 2, the feedforward signal $u_f \theta$ in (1) is given by a parametrization with $n_\theta = 3$ and basis functions $\psi(q)$ representing acceleration, jerk and snap feedforward, i.e., the 2nd, 3rd and 4th derivative of $r$.

The feedforward controller parametrization $C$ in Definition 2 has two important advantages. First, this parametrization is linear in $\theta$. Hence, for a quadratic criterion, (4) has an analytic solution. Second, $C$ is a generalization of a FIR basis, thereby enforcing stability of $C_f$ since all poles are located in the origin. As a result, internal stability of the system in Fig. 1 is guaranteed when $C_{fj}^{i+1}(q)$ is implemented on the system.

### 2.4. Problem formulation and outline

In iterative feedforward tuning, the performance of the system in Fig. 1 is improved by exploiting measurements from the previous tasks to iteratively update $C_f(q)$. As stated in (3), the update $C_{fj}^i(q)$ is determined by $V(C_{fj}^i)$. Two key requirements for $V(C_{fj}^i)$ in iterative feedforward tuning are imposed:

R1. Minimization of the experimental cost in terms of the number of tasks required to update $C_f(q)$.

R2. Estimation of (4) in $C_f(q)$ such that $\epsilon$ is minimized, despite the presence of $v$ in the performed task.

In view of the identified requirements R1–R2 for $V(C_{fj}^i)$, the contributions of this paper are twofold. First, it is shown that in existing procedures in iterative feedforward control, R1 and R2 are conflicting in the presence of $v$. Then, it is shown that the underlying problem can be interpreted as a closed-loop identification problem. Second, a novel criterion $V(C_{fj}^i)$ is proposed that attains requirements R1–R2. This is achieved by establishing a connection to closed-loop identification techniques. Furthermore, it is shown that the proposed $V(C_{fj}^i)$ has strong similarities with classical manual feedforward tuning.

### 3. Analysis of existing procedures in the presence of noise

In this section, it is shown that for the iterative feedforward control approach proposed in Van der Meulen et al. (2008), Heertjes et al. (2010), and Boeren, Bruijnen et al. (2014), requirements R1–R2 are conflicting in the presence of $v$. For clarity of exposition, it is assumed that $C_{fj}^{i-1}(q)$ in (2) is equal to $C_{fj}^i(q)$, i.e., $C_{fj}^{i-1}(q)$ is determined based on a single task without prior feedforward controller.

**Definition 3.** The criterion in (3) is defined as

$$V_2(\theta) = ||\dot{\epsilon}(\theta)||^2_2,$$

(6)

where $\dot{\epsilon}(\theta)$ is given by

$$\dot{\epsilon}(\theta) = e_m - S(q)P(q)C_f(q, \theta)r,$$

(7)

with $e_m$ as defined in Section 2.2.

Similar to the analysis provided in Section 2.1, Definition 3 implies that the optimal feedforward controller, i.e., $C_f(q)$ such that $V_2(\theta) = 0$, is given by $C_f(q) = P^{-1}(q)$.

Crucially, $\dot{\theta}$ should be estimated based on measurement data only in a data-driven approach, i.e., without explicitly constructing parametric or nonparametric models of closed-loop transfer functions, e.g., as in Hjalmarsson et al. (1998). The following result derived in Van der Meulen et al. (2008) is essential for subsequent derivations.

**Lemma 1.** Assume that $v = 0$. Then, for $C_f(q) = 0$,

$$y_m = S(q)P(q)C_f(q)r + S(q)v,$$

where $y_m$ is the measured output.

### 4. The feedforward controller $C_{fj}^{i+1}(q)$ is constructed based on the known reference $r$, and measured signals $e_m$ and $y_m$ in the $j$th task.
is equivalent to
\[ S(q)P(q)r = C_{b_0}^{-1}(q)y_r, \quad (8) \]
with \( y_r \) as defined in Section 2.2.

This auxiliary result enables the estimation of \( \theta \) based on the known feedback controller \( C_{b}(q) \) and measured signal \( y_m \) without modelling \( S(q) \) and \( P(q) \).

**Remark 1.** An approach to deal with possible instability of \( C_{b_0}^{-1}(q) \) in computing \( C_{b_0}^{-1}(q)y_r \) is presented in Appendix A.

In practice, the measured output \( y_m \) is always contaminated by the unknown disturbance \( v \). Recall from Section 2.1 that \( v \) is assumed to be given by \( v = H(q)e \), where \( H(q) \) is monic and \( e \) is normally distributed white noise with zero mean and variance \( \lambda^2 e \). Following a similar reasoning as in Lemma 1, (8) is for non-zero \( v \) given by
\[ (S(q)P(q)r)_{\text{est}} = C_{b_0}^{-1}(q)y_m. \quad (9) \]
By evaluating the expected value of (9),
\[ E\{S(q)P(q)r\}_{\text{est}} = E\left\{ C_{b_0}^{-1}(q)[y_r + y_v]\right\} = S(q)P(q)r, \]
it follows that the approximation of \( S(q)P(q)r \) is unbiased and hence seems suitable. The resulting data-driven optimization problem with respect to \( \theta \) is stated in the following definition.

**Definition 4.** Given measured signals \( e_m, y_m \). Then, for \( C_{b}(q, \theta) \in \mathcal{C} \), minimization of (6) with respect to \( \theta \)
\[ \hat{\theta}_N = \arg\min_{\theta} V_2(C_{b}(q, \theta)), \]
is equivalent to the least squares solution to
\[ \Phi\hat{\theta} = e_m, \quad (10) \]
where \( \Phi = \Psi(q)C_{b_0}^{-1}(q)y_m \in R^{N_{\text{est}}} \), and \( e_m \) as defined in Section 2.2.

The foregoing assumption ensures that \( \hat{\theta}_N \) can be uniquely determined.

**Assumption 1.** \( \Phi^T\Phi \) is nonsingular.

Assumption 1 imposes a persistence of excitation condition on \( r \). Note that the validity of this assumption is closely related to the selected order \( n_g \) of \( C_{b} \) in Definition 2. For motion systems with dominant rigid-body dynamics, this assumption holds in general for the parametrization proposed in Section 2.3 based on acceleration, jerk and snap feedforward. That is, the inclusion of derivatives of \( r \) up to snap in \( C_{b} \) improves the performance in terms of Definition 3, see, e.g., Lambrechts et al. (2005). The solution to (10) is given by
\[ \hat{\theta}_N = \left(\frac{1}{N} \Phi^T\Phi\right)^{-1}\frac{1}{N} \Phi^T e_m. \quad (11) \]

Next, it is shown that the requirements R1 and R2 are conflicting in the presence of \( r \). First, consider the following definition of the optimal feedforward controller.

**Definition 5.** The optimal feedforward controller \( C_{b}(q, \theta_0) \) with true parameter vector \( \theta_0 \) is defined as \( C_{b}(q, \theta_0) = \Psi(q)\theta_0 = P^{-1}(q) \), and is optimal in the sense that the reference-induced error \( e_r \) is eliminated, i.e., \( e_r - \Phi\hat{\theta}_0 = 0 \) for \( C_{b}(q) = 0 \).

For the polynomial parametrization of \( C_{b}(q) \) as defined in Definition 2, a necessary condition for the existence of \( C_{b}(q, \theta_0) \) is that \( P(q) \) is restricted to a rational function with unit numerator. Then, Definition 5 implies that the measured error signal \( e_m \) is equivalent to
\[ e_m = \Phi\theta_0 + e_v. \quad (12) \]
Substitute (12) in (11) to obtain
\[ \hat{\theta}_N = \theta_0 + R^{-1}_{\Phi}(N)R_{\Phi e_r}(N), \]
with \( R_{\Phi e_r}(N) \) an estimate of the autocorrelation matrix \( R_{\Phi e_r} \) and \( R_{\Phi e_r}(N) \) an estimate of the cross-correlation vector \( R_{\Phi e_r} \), based on \( N \) samples given by
\[ R_{\Phi e_r}(N) = \frac{1}{N}\Phi^T \Phi, \quad R_{\Phi e_r}(N) = \frac{1}{N}\Phi^T e_v. \]
In Söderström and Stoica (1989, chap. 7) it is shown that under mild assumptions,
\[ R_{\Phi e_r} = \lim_{N \to \infty} R_{\Phi e_r}(N), \quad R_{\Phi e_r} = \lim_{N \to \infty} R_{\Phi e_v}(N). \]
As a result, \( \hat{\theta}_N \) converges if \( N \) tends to infinity with probability 1 to
\[ \hat{\theta}_N = \theta_0 + R^{-1}_{\Phi}(N)R_{\Phi e_r}. \quad (13) \]
Expression (13) reveals that \( \hat{\theta}_N \) is an unbiased estimate of \( \theta_0 \) if \( R_{\Phi e_r} = 0 \). Recall from Definition 4 that \( \Phi \) is constructed based on \( y_m \) which is contaminated by the unknown \( v \). Hence, \( e_v \) and \( \Phi \) are correlated, and the bias is given by
\[ \Delta \theta_N = R^{-1}_{\Phi}(N)R_{\Phi e_r}. \]

Summarizing, the approach presented in Van der Meulen et al. (2008), Heertjes et al. (2010), and Boeren, Bruijnen et al. (2014) results in a biased estimate, i.e., \( \hat{\theta}_N \neq \theta_0 \) for \( \lambda \geq 0 \), when measurements from a single task are used. This shows that the requirements R1 and R2 in Section 2.4 are conflicting in the presence of \( v \).

**Remark 2.** In Van der Meulen et al. (2008, Section 2.4), inspired by a similar approach developed in IFT (Hjalmarsson et al., 1998), a procedure is proposed that results in an unbiased estimate at the expense of measuring two tasks. However, this two-step approach implies that \( C_{b} \) cannot be updated after each task, thereby conflicting requirement R1.

4. Proposed approach based on closed-loop identification

In this section, a new procedure is presented that exploits knowledge of \( r \) in the optimization criterion \( V(\theta) \) to simultaneously achieve requirements R1–R2 in Section 2.4. To this purpose, a connection is proposed between instrumental variable identification techniques, see, e.g., Söderström and Stoica (1983) and iterative feedforward tuning as in Van der Meulen et al. (2008). The instrumental variable approach is well-established in closed-loop identification, see, e.g., Gilson, Garnier, Young, and Van den Hof (2011) and Söderström and Stoica (1983). For iterative feedforward control, the corresponding criterion is posed in the following definition.

**Definition 6.** The criterion in (3) is defined as
\[ V_2(\theta) = \left\| Z^T e(\theta) \right\|_W^2, \]
where \( Z \in R^{N_{\text{est}} \times n_0} \) are instrumental variables, \( W \) is a positive-definite weighting matrix, \( N_z \geq 0, \) and \( e(\theta) \) as given in (7).

In this section, the basis instrumental variable approach is pursued, see, e.g., Söderström and Stoica (1983, chap. 3), in which case \( Z \in R^{N_{\text{est}} \times n_0} \) and \( W = I \). The parameters \( \theta \) of \( C_{b}(q, \theta) \) then result from the set of equations
\[ Z^T \left[ e_m - \Phi\theta_N \right] = 0. \quad (14) \]
The solution to (14) is given by
\[ \theta_N = \left(\frac{1}{N} Z^T \Phi\right)^{-1}\frac{1}{N} Z^T e_m. \quad (15) \]
where $\Phi = \Psi(q)C_{ff}^{-1}(q)y^*_{m} \in \mathbb{R}^{N \times n_r}$. The following assumption guarantees that $\hat{\theta}_N^{IV}$ can be uniquely determined.

**Assumption 2.** $Z^T \Phi$ is nonsingular.

Assumption 2 implies that $Z$ should be correlated with $\Phi$. The freedom that exists in the construction of $Z$ can be used to obtain an unbiased estimate, i.e., $\hat{\theta}_N^{IV} = \theta_0$, $\forall \lambda_c \geq 0$. To show this, substitute (12) in (15) to obtain

$$
\hat{\theta}_N^{IV} = \theta_0 + R_{Z}\Phi \hat{R}_{Z},
$$

with

$$
R_{Z}(N) = \frac{1}{N}Z^T \Phi, \quad \hat{R}_{Z}(N) = \frac{1}{N}Z^T e_v,
$$

an estimate of the cross-correlation matrix $R_{Z}\Phi$ and cross-correlation vector $\hat{R}_{Z}$ based on $N$ samples, respectively. In Söderström and Stoica (1989, chap. 7) it is shown that under mild assumptions,

$$
R_{Z} = \lim_{N \to \infty} R_{Z}(N), \quad \hat{R}_{Z} = \lim_{N \to \infty} \hat{R}_{Z}(N).
$$

As a result, $\hat{\theta}_N^{IV}$ converges if $N$ tends to infinity with probability 1 to

$$
\hat{\theta}_N^{IV} = \theta_0 + R_{Z}\Phi \hat{R}_{Z}.
$$

The estimate $\hat{\theta}_N^{IV}$ is (asymptotically) unbiased if $Z$ is constructed such that $Z$ and $e_v$ are uncorrelated, i.e., $R_{Z} = 0$.

Based on (16) and Assumption 2, the proposed design of instrumental variables is given by $Z = [\psi_1 r, \psi_2 r, \ldots, \psi_n r]$ with basis functions $\Psi(q)$ as in Definition 2. There are two key reasons for this design. First, recall that the feedforward control goal is to minimize the servo error $e_m = \hat{y} - y_m$, which implies that $r$ and $y_m$ are correlated. As a result, the proposed instruments $Z$ and $\Phi$ are correlated, and Assumption 2 holds. Second, the known reference $r$ and $e_v$ are uncorrelated. Hence, (16) reveals that an unbiased estimate $\hat{\theta}_N^{IV}$ is obtained with the proposed design of $Z$. For the feedforward signal $u_{ff}$ in Section 2.1, the proposed instruments $Z$ represent respectively acceleration, jerk and snap, i.e., the 2nd, 3rd and 4th derivative of the multi-segment polynomial trajectory $r$.

Concluding, a new criterion $V_f(\theta)$ is employed that results in $\hat{\theta}_N^{IV} = \theta_0$, $\forall \lambda_c \geq 0$, when measurements from a single task are used. This illustrates that requirements R1–R2 in Section 2.4 are simultaneously attained for $V_f(\theta)$.

**Remark 3.** As stated in Remark 2, the two-step data-driven approach presented in Van der Meulen et al. (2008, Section 2.4) determines an estimate $\hat{\theta}_N$ based on two tasks. This approach has a clear interpretation in the instrumental variable framework. In particular, the second task is exploited to construct instrumental variables $Z = \hat{\Phi}_2$. This approach imposes the stringent condition on the system that $\nu^1$ and $\nu^2$ are uncorrelated. If this condition holds, $\nu$ is eliminated from the optimization problem and consequently requirement R2 is attained. Still, two tasks are required to update the feedforward controller $C_{ff}$, i.e., the experimental cost is not minimal.

**Remark 4.** Besides the two-step data-driven approach, Van der Meulen et al. (2008, Section 2.4) also formulated an ILC-related model-based approach. That is, the instrumental variables become $Z = [SP(q)\psi_1 q, SP(q)\psi_2 q, \ldots, SP(q)\psi_n q]$, where it is assumed that a model of $SP(q)$ is available.

## 5. Iterative tasks

In this section, the instrumental variable method in Section 4 is embedded in the iterative task framework in Section 2. As argued in Gunnarsson and Norrlöf (2006), iterative tasks are used to minimize the influence of iteration-invariant disturbances, nonlinearities and measurement noise.

The pursued approach to adapt $C_{ff}(q, \theta)$ is to use recursive estimates of $\theta$. Consider the two degree-of-freedom control configuration as depicted in Fig. 4. The feedforward controller $C_{ff}(q, \theta_0)$ in the jth task is updated by $C_{ff}(q, \theta_j)$. Herein, $\Delta \theta_j$ is determined based on the measured signals $e_m^j$ and $y_m^j$ in the jth task given by

$$
\hat{y}_m^j = S(q)(C_{fb}(q) + C_{ff}(q, \theta_j))r + S(q)v,
$$

where $\Phi' = \Psi(q)(C_{fb}(q) + C_{ff}(q, \theta_j))^{-1}\hat{y}_m^j$.

**Proposition 1.** Given $e_m^j, \hat{y}_m^j$, and $C_{ff}(q, \theta_j) \in \mathcal{C}$ in the jth task. The IV estimate $\hat{\theta}_N^{IV}$ is the solution to

$$
\hat{\theta}_N^{IV} = \left(\frac{1}{N}Z^T \Phi\right)^{-1} \frac{1}{N}Z^T e_m^j,
$$

where

$$
\Phi' = \Psi(q)(C_{fb}(q) + C_{ff}(q, \theta_j))^{-1}\hat{y}_m^j \in \mathbb{R}^{N \times n_r},
$$

and $Z = [\psi_1 r, \psi_2 r, \ldots, \psi_n r] \in \mathbb{R}^{n \times N}$.

The following result enables recursive estimation of $\theta$.

**Theorem 1.** For $C_{ff}, C_{fb} \in \mathcal{C}$ with identical basis functions $\Psi(q)$, $C_{ff,j+1}^{-1} = (q, \theta_j^{-1})$ in (2) is given by

$$
C_{ff,j+1}(q, \theta_j^{-1}) = \sum_{i=1}^{n_a} \psi_i(q) \theta_i^{j+1},
$$

where $\theta_i^{j+1} = \hat{\theta}_i + \theta_i^j$.

**Proof.** Since $C_{ff}, C_{fb} \in \mathcal{C}$ have identical $\Psi(q)$ in (5), $C_{ff,j+1}$ is given by

$$
C_{ff,j+1} = C_{ff,j}(q) + C_{ff,j}^{-1}(q) = \sum_{i=1}^{n_a} \psi_i(q) \theta_i^{j+1} + \sum_{i=1}^{n_a} \psi_i(q) \theta_i^{j+1}.
$$

Since $C_{fb}$ and $C_{ff,j}$ are linear in respectively $\theta$ and $\theta^j$, superposition implies that

$$
C_{ff,j+1} = \sum_{i=1}^{n_a} \psi_i(q) (\theta_i^j + \theta_i^j) = \sum_{i=1}^{n_a} \psi_i(q) \theta_i^{j+1}. \quad \square
$$

Theorem 1 shows that the update of $\theta^{-1}$ with respect to $\theta$ is solely based on measurements from the jth task.

**Remark 5.** The proposed recursive estimation based on measurements from a single task can be directly generalized to determine $\hat{\theta}_N^{IV}$ based on measurement data from multiple iterative tasks. An analysis of iterations in system identification is provided in Rojas, Oomen, Hjalmarsson, and Wahlberg (2012). In this iterative framework, the variance of $\hat{\theta}_N^{IV}$ is reduced at the expense of performing multiple tasks.

Combining Proposition 1 and Theorem 1 leads to the following procedure to update $C_{ff}$ based on the jth task, which implements the main contribution of this paper.

**Procedure 1.** Estimation of $\hat{\theta}_N^{IV}$ after the jth task

1. Measure $e_m^j$ and $\hat{y}_m^j$ in the jth task with $C_{ff}(q, \theta_j)$ applied to the system.
2. Construct $\Phi' = \Psi(q)(C_{fb}(q) + C_{ff}(q, \theta_j))^{-1}\hat{y}_m^j$.
3. Construct instrumental variables $Z = [\psi_1 r, \psi_2 r, \ldots, \psi_n r]$.
4. Solve $\Delta \theta_j = \left(\frac{1}{N}Z^T \Phi\right)^{-1} \frac{1}{N}Z^T e_m^j$.
5. Construct the new feedforward controller $C_{ff,j+1}(q) = \Psi(q) (\theta^j + \Delta \theta_j)$.
6. Set $j \leftarrow j+1$ and go to Step 1.
An approach to deal with possible instability of $C_{p}(q) + C_{y}^{f}(q, \dot{q})$ in computing $\Phi = \Psi(q)(C_{p}(q) + C_{y}^{f}(q))^{-1}y_{m}^{d}$ is presented in Appendix A. This approach assumes an infinite time horizon and can therefore result in transients at the initial and final position of a finite time task, see, e.g., Norrlöf and Gunnarsson (2002). For the proposed instrumental variable method, $\hat{\theta}_{N}$ is not affected by these transients. To illustrate this statement, recall that for the tasks considered in this paper, the system starts at rest in an initial position and comes to a rest in a final position. Since $Z$ consists of derivatives of $r$, the instruments $Z$ are equal to zero at the start and the end of such tasks. This implies that in Procedure 1, $(1/N)Z^{d}q_{i}^{d}$ in $\hat{\theta}_{N} = ((1/N)Z^{d} \Phi^{d})^{-1}(1/N)Z^{d}v_{m}$ is not influenced by transients that result from computing $\Phi^{d}$.

**Remark 6.** Contrary to the instrumental variable method, the least squares method given in (11) suffers from transients that result from computing $\Phi$. This is explained by observing that (11) contains the term $(1/N)\Phi^{d}q_{i}^{d}$. Procedure 1 provides a systematic procedure to improve the performance of the system in Fig. 1 by exploiting recursive estimates of $\theta$ to update $C_{y}^{f}(q, \dot{q})$.

### 6. Experimental results

In this section, the theoretical results proposed in this paper are validated on an experimental setup. In Section 6.1, the two-mass spring damper setup is described that is used to conduct experiments. In Section 6.2, the control goal is specified. In Section 6.3, the parametrization of the feedforward controller $C_{f}(q)$ is introduced. In Section 6.4, an adjustable disturbance $v_{add}$ is introduced that acts on the closed-loop system. In Section 6.5, a controlled experiment is performed to analyze the effect of the single independent variable $v_{add}$ on the estimates $\hat{\theta}_{N}$ and $\hat{\theta}_{N}^{R}$, given by respectively (11) and (15).

#### 6.1. Experimental setup

In this section, the theoretical results proposed in this paper are validated on the two-mass spring damper setup depicted in Fig. 5. A schematic illustration of the two-mass spring damper setup is shown in Fig. 6, where the flexible shaft is modelled as a spring and a damper. This experimental setup is used to experimentally validate control strategies for high-precision motion systems. The dynamical behavior of this system contains key aspects in motion control, including collocated and non-collocated dynamics, while measurement noise, friction, delay and nonlinearities are small. As a result, a controlled experiment can be performed to analyze the influence of a single parameter on the servo performance of the system. The characteristics make this system also relevant as a benchmark problem in robust control, see, e.g., Wie and Bernstein (1992).

The setup consists of two masses $m_{1}$ and $m_{2}$ which are connected through a flexible shaft. Furthermore, the inertia of the system is given by $3.7 \times 10^{-4}$ kg m$^{2}$. The angular position of both $m_{1}$ and $m_{2}$ are measured by means of encoders with a resolution of $1 \times 10^{-3}$ rad. However, in the presented experiments only measurements of the angular position of $m_{1}$ are exploited to evaluate the servo performance obtained with $C_{y}(q)$. The setup is equipped with a single DC motor that is rigidly connected to $m_{1}$. All experiments are performed with sample time $T_{s} = 1/2048$ s.

The frequency response function of the considered two-mass spring damper setup is depicted in Fig. 7. Inspection reveals rigid-body behavior below 25 Hz, while the resonance phenomenon related to the flexible dynamics of the system appears at 55 Hz. The feedback controller designed to stabilize the system $P$, as depicted in Fig. 8, is given by

$$C_{p}(q) = \frac{0.04578q^{2} - 0.09119q + 0.04541}{q^{2} - 2.91q^{2} + 2.822q - 0.9121}$$

which results in a bandwidth $f_{bw} = 5$ Hz.

#### 6.2. Control goal

The control goal defined for the experimental setup in Fig. 7 is to minimize the servo error $e_{m} = r - y_{m}$, where $r$ is a 3rd order servo task, designed according to the procedure proposed in Lambrechts et al. (2005), and $y_{m}$ is the measured angular position of the mass $m_{1}$. In Fig. 9, $r$ is depicted together with the corresponding velocity $\dot{r}$ and acceleration $\ddot{r}$.

#### 6.3. Parametrization feedforward controller

In this section, a parametrization for $C_{f}$ is proposed for the two-mass spring damper setup. As in Lambrechts et al. (2005), the

![Fig. 5. Photograph of the experimental two-mass spring damper setup at the CST group, Department of Mechanical Engineering, Eindhoven University of Technology.](image)

![Fig. 6. Schematic illustration of the two-mass spring damper setup.](image)

![Fig. 7. Frequency response function of the two-mass spring damper setup.](image)
feedforward controller is parametrized as $C_f(q, \theta) = \psi(q)\theta_{acc}$, where $\psi(q) = \frac{q^2 - 2q + 1}{T_s^2q^2}$ with initial value $\theta_{acc}^{init} = 1 \times 10^{-4}$. This parameterization of $C_f(q)$ consists of acceleration feedforward to compensate for the rigid-body dynamics of $P$ in 0–20 Hz.

6.4. Disturbance design

In this section, an adjustable disturbance term $v_{add}$ is defined that is applied as a disturbance to the two-mass spring damper setup in Fig. 5. As a result, a controlled experiment is created to analyze the influence of the single independent disturbance variable $v_{add}$ on the estimates $\dot{\theta}_N$ and $\bar{\theta}_N$, given by respectively (11) and (15). Similar to Section 2.1, $v_{add}$ is modeled as $v_{add} = H(q)e$, where $H(q)$ is given by

$$H(q) = \frac{0.8048q^2 - 1.61q + 0.8048}{q^2 - 1.571q + 0.6481}.$$

and $e$ is normally distributed white noise with zero mean and variance $\lambda^2_e$. The disturbance $v_{add}$ represents high-frequent measurement noise, which is a typical disturbance in motion systems. Here, the standard deviation $\lambda_e$ constitutes a design parameter that is exploited in the next section to analyze the influence of $v_{add}$ on the parameter estimation.

6.5. Experimental results

In this section, the iterative task framework established in Section 5 is used to determine $\theta_{acc}$ such that the reference-induced error $e_r$ is minimized, despite the presence of $v_{add}$. To this purpose, $j$ tasks are executed. After each task, $\theta_{acc}$ is updated. For the proposed instrumental variable approach in Procedure 1, based on minimization of $V_z(\theta)$, this gives the estimator $\bar{\theta}^{IV}_j$ after the $j$th iteration. The instrumental variables $Z$ are given by $Z = \psi(q)r$, with $\psi(q)$ as defined in Section 6.3.

To compare the results with existing iterative feedforward tuning procedures, the estimator $\bar{\theta}^{IV}_j$ is included which is obtained based on minimization of $V_z(\theta)$ in Definition 3. The complete experiment is repeated $m=5$ times. The sample means corresponding to the $j$th task are given by

$$\bar{\theta}^{IV}_j = \frac{1}{m} \sum_{i=1}^{m} \bar{\theta}^{IV}_{ij}, \quad \bar{\theta} = \frac{1}{m} \sum_{i=1}^{m} \bar{\theta}_{ij},$$

where the number of samples is given by $N=1630$.

Two cases are experimentally illustrated. First, the sample mean for both approaches in the final task, i.e., $j=10$, is analyzed as a function of $\lambda_e$. Second, recursive estimation of $\bar{\theta}^{IV}_j$ and $\bar{\theta}$ as a function of $j$ is considered for a fixed standard deviation $\lambda_e$. The presented cases combined illustrate the advantages the instrumental variable approach proposed in this paper has to offer compared to pre-existing approaches.

The sample means $\bar{\theta}^{10,IV}$ and $\bar{\theta}^{10}$ in the 10th task are depicted in Fig. 10 as a function of $\lambda_e$. The following observations are made:

1. For $\lambda_e = 0$, $\bar{\theta}^{10,IV} \neq \bar{\theta}^{10}$. As discussed in Section 6.3, $C_f$ consist solely of acceleration feedforward. This results in undermodeling of $P^{-1}$, which is known to shape the bias in the frequency domain, see, e.g., Ljung (1999, Eq. 8.71). The difference between $\bar{\theta}^{10,IV}$ and $\bar{\theta}^{10}$ is explained by a different shape of the bias due to undermodelling.

2. The sample mean $\bar{\theta}^{10,IV}$ is independent of $\lambda_e$. That is, the estimator based on instrumental variables compensates for the dominant contribution of $e_r$, irrespective of $\nu$.

3. The sample mean $\bar{\theta}^{10}$ depends on $\lambda_e$. As a result, pre-existing approaches for iterative feedforward control result in a $C_f(q, \theta)$ which depends on $\nu$. This implies that $e_r$ is not minimal, and as a result servo performance is compromised.

4. The sample mean $\bar{\theta}^{10,IV}$ is equal to the inertia of the system as specified in Section 6.1.
In the second experimental case, recursive estimation of $\overrightarrow{\theta}^{IV}$ and $\overrightarrow{\theta}$ as a function of $j$ is considered for $\lambda_c = 0$ and $\lambda_c = 8 \times 10^{-2}$. In Fig. 11, the sample mean $\overrightarrow{\theta}$ based on minimizing $V_2(\theta)$ is depicted, while $\overrightarrow{\theta}^{IV}$ is shown in Fig. 12. The following observations are made:

1. For $\lambda_c = 0$, measured data from a single task is sufficient for both approaches to reach a converged value.
2. The sample mean $\overrightarrow{\theta}^{IV}$ converges to an identical value after a single task for $\lambda_c = 0$ and $\lambda_c = 8 \times 10^{-2}$. This illustrates that $\overrightarrow{\theta}^{IV}$ is independent of $\lambda_c$, $\forall j$.

Finally, the error signal $e_m$, averaged over $m=5$ realizations, is depicted in Fig. 13 for $\lambda_c = 8 \times 10^{-2}$ in the 10th task. The results confirm that the proposed approach enhances performance compared to pre-existing approaches.

7. Conclusions

In this paper, a new approach for iterative feedforward control is presented based on closed-loop identification techniques, which significantly enhances existing feedforward control algorithms. In Section 3, it is shown that a trade-off exists in pre-existing iterative feedforward tuning procedures between (i) the number of tasks required to update the feedforward controller and (ii) the servo performance of the system. The main contribution of this paper is a new iterative feedforward tuning procedure that is efficient, i.e., requires measured data from only a single task to update the feedforward controller, and accurate, i.e., minimizes the servo error induced by the reference signal, independent of an unknown disturbance $v$. Experimental results provided in Section 6 confirm that the proposed approach is superior with respect to pre-existing approaches.

The proposed approach can be straightforwardly extended to other optimization problems in a closed-loop configuration. Furthermore, the proposed IV procedure directly generalizes to closely related projection methods, see, e.g., Van den Hof and Schrama (1995) and Forssell and Ljung (2000). Finally, ongoing research focuses on accuracy properties of the estimates as in Boeren, Oomen, and Steinbuch (2014), extensions to recursive estimation which exploit measured data from multiple tasks, a rational basis as in Bolder and Oomen (in press), parametrized friction feedforward, stochastic approximation and multivariable systems.

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Appendix A. Stable inversion

In this section, a stable inversion procedure is described to determine (17) if $(C_{j_{\text{ff}}} + C_j^f)$ is unstable.

1. Let the discrete-time system $R(z) = (C_{j_{\text{ff}}} + C_j^f(z))^{-1}$ have a state-space realization given by

\[ x(t + 1) = Ax(t) + By_m(t), \]
\[ u(t) = Cx(t) + Dy_m(t). \]
(2) Transform this system under similarity to
\[
\begin{bmatrix}
    x_{st}(t+1) \\
    x_{un}(t+1)
\end{bmatrix}
= \begin{bmatrix}
    A_{st} & 0 \\
    0 & A_{un}
\end{bmatrix}
\begin{bmatrix}
    x_{st}(t) \\
    x_{un}(t)
\end{bmatrix}
+ \begin{bmatrix}
    B_{st} \\
    B_{un}
\end{bmatrix} y_{in}(t),
\]
\[
u(t) = \begin{bmatrix}
    C_{st} & C_{un}
\end{bmatrix}
\begin{bmatrix}
    x_{st}(t) \\
    x_{un}(t)
\end{bmatrix} + D y_{in}(t),
\]
where \begin{bmatrix}
    \lambda_1 \\
    \lambda_2
\end{bmatrix} is the Jordan canonical form of A. Furthermore, the eigenvalues of \(A_{st}\), denoted as \(\lambda(A_{st})\), satisfy \(\lambda(A_{st}) < 1, \forall i\), while \(\lambda(A_{un}) > 1, \forall k\). That is, \(R\) is decomposed into a stable \((A_{st}, B_{st}, C_{st}, D)\) and an unstable \((A_{un}, B_{un}, C_{un}, 0)\) subsystem, with corresponding state variables \(x_{st}\) and \(x_{un}\), respectively.

(3) Solve forward in time with initial condition \(x_{st}(0) = 0\):
\[
x_{st}(t+1) = A_{st} x_{st}(t) + B_{st} y_{in}(t),
\]
and backward in time with initial condition \(x_{un}(N) = 0\):
\[
x_{un}(t+1) = A_{un} x_{un}(t) + B_{un} y_{in}(t).
\]

(4) Determine
\[
u(t) = \begin{bmatrix}
    C_{st} & C_{un}
\end{bmatrix}
\begin{bmatrix}
    x_{st}(t) \\
    x_{un}(t)
\end{bmatrix} + D y_{in}(t).
\]

Remark 7. A state-space realization of \(R\) exists if \(R\) is proper. If this condition is not valid, preview-based techniques are required to determine a state-space realization of \(R\), see, e.g., Zou and Devasia (1999).

References


