Fatigue failure analysis of stay cables with initial defects: Ewijk bridge case study

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A R T I C L E   I N F O

Article history:
Received 5 April 2013
Received in revised form 19 May 2014
Accepted 20 May 2014

Keywords:
Probabilistic analysis
Bayesian update
Reliability
Inspection
Fatigue
Fracture mechanics
Locked coil cable
Lamination
Bridge

A B S T R A C T

A failure assessment method of stay cables containing fractured wires is presented through an example of an existing bridge. The wires constituting the stay cables of the bridge contained many initial defects that were formed during the rolling process of these wires. Despite moderate loading, fatigue cracks started to grow from these defects, increasing their size, and finally resulting in fracture of the remaining ligament of the wires. A probabilistic model is put forward for assessing the resistance of the stay cable based on a fracture mechanics model of the individual wires. A Bayesian update method is presented for estimating the distribution of initial defects based on the inspection of fractured wires. The calculated reliability of the stay cables of the examined bridge was lower than acceptable.

1. Introduction

1.1. Description of the case study

The bridge at Ewijk, the Netherlands, is a 1055 m long steel box girder bridge containing 5 traffic lanes. The bridge was built in 1972 and is owned by the Dutch Ministry of Transport. Two pylons are located above the piers at either side of the main span, which spans 270 m. Per pylon, 4 locked coil stay cables support the box girder. Two of these cables have a length of 204 m and consist of 5 strands while the other two have a length of 120 m and consist of 3 strands. They are referred to as cables ‘AA’ and ‘BB’, respectively. Fig. 1. Each strand is built up of 235 wires. Layers 1–6 consist of round wires, layer 7 consists of round and half lock wires, and layers 8 and 9 consists of full lock wires. The cables are continuous at the pylons.

A visual inspection of the stay cables carried out in 1990 revealed that a number of these wires were fractured. Only the wires along the cable perimeter, i.e. the wires that are visual from the outside, were inspected for each stay cable. A maximum number of 11 wire fractures were detected in one strand of cable ‘AA’ with inspected length $L_{\text{insp}} = 94.25$ m in 1990. A new inspection in 2012 revealed 29 wire fractures in the same strand. In this manuscript, we further focus on this cable.

Fractured wires have been observed in more bridges, e.g. [1–2]. Models have been put forward in literature for the assessment of parallel wire cables with fractured wires. An especially relevant work in the framework of probabilistic resistance models of cables has been presented by Faber et al. [3]. They presented separate models for the static strength and for the fatigue strength of cables consisting of parallel wires. Their static strength model considers the disadvantageous effect of the length of the cable as well as the beneficial parallel effect of the wires – the so-called Daniels effect, [4]. Considering fatigue they used a Weibull distribution for the fatigue life of individual wires and presented a method to update the probability that a wire is fractured by applying a Bayesian analysis, making use of the inspection result.

The cause of the fractures in the Ewijk bridge is unique and has not been considered before in (probabilistic) failure models of cables. Microscopic and metallurgical examination indicated that the full lock wires had many initial defects (depth ranging from 0.1 to more than 0.5 mm, spaced between 0.5 and 5 millimetres), which is attributed to a manufacturing problem. This unique cause implies that existing models and distributions for the fatigue...
resistance, most of which use a Weibull distribution for the fatigue life – may not be representative for the cables considered. Instead, a fracture mechanics model is applied for the fatigue and fracture behaviour of single wires. Probabilistic fracture mechanics has proven to be an effective method in the failure assessment of bridge structures and components, e.g. [5–8].

In the current work, we present one model that considers the effect of fatigue deterioration and fracture of wires on the static strength of the cable. A Bayesian update procedure has been applied to estimate the distribution of the most uncertain variable – the initial defect size – based on the results of the inspections.

1.2. Motivation for the case study

The fracture cause for the Ewijk stay cable is different than the typical failure causes of stay cables provided in literature. Only a limited number of manufacturers in the 1970’s were able to produce stay cables with similar lengths. For this reason, it is possible that more bridges constructed in the same period contain stay cables with the same manufacturing problem. This study indicates that this potential failure cause requires attention, because it may jeopardize the structural safety of a bridge.

2. Loads on the cable

2.1. Maximum load

The maximum load in one strand in a reference period of 15 years consists of the permanent load, G, and the maximum variable load, Q:

\[ F = G + Q = G_{\text{meas}} + G_{\text{add}} + B_Q Q_{\text{tr}} + \psi Q_{\text{wt}} \]  

(1)

where \( G_{\text{meas}} \) – permanent load in one strand, determined by dynamic measurements in 2010 based on the higher order natural frequencies of the stay cables. The method is explained in [9] and [10]. The expectation is: \( E[G_{\text{meas}}] = 2514 \text{kN} \). \( G_{\text{add}} \) – additional permanent load due to a planned renovation of the bridge deck. Its value is determined by calculation, with an expectation of \( E(G_{\text{add}}) = 866 \text{kN} \). \( Q_{\text{tr}} \) – Maximum traffic load in one strand during a reference period of 15 years. Its value is determined by calculation, using the load model in EN 1991–2 but corrected for trends, reference period and uncertainty. The expectation for the reference period starting in 2013 is \( E(Q_{\text{tr}}) = 735 \text{kN} \). Due to changes in traffic, the expectation of \( Q_{\text{tr}} \) usually increases in time. This is however counterbalanced by a planned second bridge, which is expected to result in a reduction of traffic on the existing bridge. As a consequence, the expectation for \( Q_{\text{tr}} \) at the end of life in 2043 is calculated to be approximately equal to that in 2013. \( B_Q \) – Uncertainty factor that accounts for the assumptions and simplifications in the traffic load model in EN 1991–2. The expectation is \( E(B_Q) = 1 \). \( Q_{\text{wt}} \) – Maximum wind and temperature loads in one strand during a reference period of 15 years. The expectation, determined by calculation with the load models in EN 1991–2, provides \( E(Q_{\text{wt}}) = 160 \text{kN} \). \( \psi \) – Combination factor for wind and temperature loads, with a value of 0.3 according to the Dutch National Annex to the European standard EN 1990 [11].

2.2. Fatigue stress ranges

The stress ranges under traffic load have been measured in 1990, [12]. The strains were measured during 11 h and at the same time the types and numbers of trucks passing the bridge were counted. The division of types and numbers of trucks were in agreement with long-term traffic counting on the highway. Based on this information, it was concluded that the 11 h strain gauge measurement was representative for the stress ranges in 1990.

A stress range histogram is deduced from the measurements by rainflow analysis as described e.g. in [13]. Large stress ranges with low frequencies – which are not included in the measurements – are obtained by fitting the measured data with a distribution function. A lognormal distribution appeared to give the best fit to the measured data. The resulting annual histogram for 1990 is provided in Fig. 2a. The annual number of trucks per traffic direction is determined by measurements until 2010 and estimated by the national highway authority (Rijkswaterstaat) for the years after 2010, Fig. 2b. The tension proportion is approximately 90% of the stress range. Thus the maximum variable tensile stress in 1990 is approximately 81 MPa. This is equivalent with a load on one strand of \( Q_{\text{tr,meas}} = 559 \text{kN} \). The difference between the expectations of \( Q_{\text{tr,meas}} \) and \( Q_{\text{tr}} \) in Eq. (1) is attributed to the difference in the reference period of 1 year versus 15 year and to trends in the vehicle weight and the number of heavy vehicles between 1990 and 2013.

3. Fracture cause

At the time of bridge construction, an extra strand with a length of 4 m was delivered together with the stay cables. A fatigue test
was carried out on this part, described in [14]. The test showed no wire fractures after applying a variable amplitude spectrum with stress ranges and number of cycles representing the traffic between 1972 and 1990. The test was then continued with a constant amplitude range of 200 N/mm² and terminated after 1.25 \times 10^8 cycles. At the end of the test, 39 wires were fractured or came loose from the supports. Only two of these fractures occurred in a half lock wire and one in a round wire. All other fractures occurred in full lock wires. This leads to the conclusion that the full-lock wires are more sensitive to fatigue failure compared to the other types of wire.

Various causes for the observed wire fractures have been considered:

- The typical failure mode of these types of strands is fatigue cracking by fretting, [15–16]. The European standard EN 1993-1-11 [17] provides an S–N curve that is based on this failure mode, with a characteristic fatigue reference strength of 150 MPa (at 2 \times 10^6 cycles). Various literature sources indicate that this fatigue strength is related to the 1st or 2nd visible wire fracture, e.g. [18]. A calculation using the stress spectrum of Fig. 2 together with the S–N curve in [17] provides an infinite fatigue life. This is not in agreement with the observed fractures in 1990. In order to explain the fractures in 1990, the fatigue reference strength should be lowered to 21 N/mm². This is lower than any test result in literature. The conclusion is that fretting is not a critical mechanism for this specific cable.
- Corrosion fatigue or stress corrosion may cause fractures. However, there were no indications of damage to the coating of the stay cables. In addition, the observed wire fractures occurred more or less randomly around the cable perimeter, not at locations where moisture could accumulate. Hence this cause of wire fracture is unlikely for the Ewijk bridge.
- Failure due to static overloading is considered. The expectation of the maximum static load is 4160 kN, with a standard deviation of approximately 5 to 10 %. The manufacturer has tested 10 strands and reported an average breaking load of 9400 kN, with standard deviation of 100 kN. The conclusion is that failure due to static loading cannot be the cause.
- Macroscopic and metallurgical examination of a number of small samples from fractured wires in the actual stay cables and in the tested part indicated that surface defects exist in the wires with a depth of approximately 0.5 mm, Fig. 3. The defects were spaced between 0.5 and 5 mm. Because only a few small samples of a few millimetres in length have been examined, the maximum defect size is expected to be substantially larger than 0.5 mm. The defects are so-called laminations, resulting from wrinkles on the wire surface that are caused by the rolling process and subsequently flattened into the wire. Some of the wires in the actual stay cable were cut at these laminations, and striations were visible. Based on this observation, it is concluded that the most likely cause for the fractures is fatigue starting at the laminations, followed by fracture of the remaining ligament.

4. Failure model for the stay cable

4.1. Fatigue and fracture model for a single wire

The effect of (initial) defects on the fatigue and failure behaviour of a component can be assessed with the theory of fracture mechanics (FM), [19]. This section provides a brief summary of the FM model used in the assessment of a single wire. The governing equations are provided in this section whereas the complete set of equations is given in the annex.

The basis of the model is formed by the stress intensity factor, \( K \). This stress intensity factor for a certain applied stress, \( S \), and a certain defect depth, \( a \), is given as:

\[
K = B_Y SY \sqrt{\pi a}
\]  

(2)

where \( B_Y \) is an uncertainty factor with expectation of 1 and \( Y \) is the geometric correction factor that depends on the defect depth, \( a \), and the area of the wire, \( A_{wire} \).

The fatigue crack extension per stress cycle, \( da/dN \), is a function of the stress intensities \( K(S_{max}) \) and \( K(S_{min}) \), where \( S_{max} \) and \( S_{min} \) are the maximum (peak) and minimum (valley) stresses of a stress cycle, respectively, and \( K \) is determined with Eq. (2). In addition \( da/dN \) is a function of a number of material parameters of which the most important ones are \( \Delta K_{0s} \), \( C \), \( m \):

\[
\frac{da}{dN} = f(K(S_{max}), K(S_{min}), \Delta K_{0s}, C, m)
\]  

(3)

The relationship of Eq. (3) is given in Fig. 4a. Failure occurs as a result of (a combination of) fracture and/or plastic failure of the remaining ligament. These failure modes are expressed with the fracture ratio, \( K_f \), and the plasticity ratio, \( L_p \), respectively:

\[
K_f = f(K(S_{max}), K_f, T, A_{wire})
\]  

(4)
where $K_c$ is the fracture toughness at a certain reference temperature, $T$ is the actual service temperature, $f_s$ is the yield stress, and $f_u$ is the tensile strength.

Interaction between fracture and plastic failure is described with the assessment line in Fig. 4b. In this paper the fracture line is denoted with symbol $\Omega$. Consider a wire with a certain initial defect size, $a_0$. As the number of stress cycles increases, the defect size increases (Eq. (3)) and as a consequence the fracture and plasticity ratios increase Eqs. (4) and (5). This process may continue until the combination of $K_i$ and $L_f$ exceeds $\Omega$ and failure occurs.

4.2. Resistance of a part of one strand

The fracture mechanics model of Eqs. ((1)–(5)) can be used to assess the fatigue life and failure of an individual wire. The assessment of an entire cable, however, requires extension of this model. Because of the unique failure case of the cable, standards and guidelines do not provide an assessment method. Therefore a fracture mechanics model is applied to assess the fatigue life and failure of an individual wire. The assessment diagram).

$\text{R}_{\text{f}} = \text{A}_{\text{f}} \text{S}_{\text{f}} + \frac{\text{A}_{\text{fl}}}{\text{N}_{\text{w}}} \sum_{i=1}^{\text{N}_{\text{w}}} \text{S}_{\text{fl}}$  \hspace{1cm} (6)

$\text{S}_{\text{f}} = \text{min}\left(\varepsilon, \text{f}_{\text{yr}}\right)$  \hspace{1cm} (7)

$\text{S}_{\text{fl}} = \text{min}\left(\varepsilon, \text{f}_{\text{yr}}\right)$  \hspace{1cm} (8)

$\text{S}_{\text{fl}} = \begin{cases} 0 & \text{if } (L_i, K_{i,1}) > \Omega \\ \text{min}\left(\varepsilon + \varepsilon_0, E\right) & \text{if } (L_i, K_{i,1}) \leq \Omega \end{cases}$  \hspace{1cm} (9)

where $\text{R}_{\text{f}}$ – Resistance of the part of the strand at strain $\varepsilon$. $\text{A}_{\text{f}}$ – Total cross-sectional area of all full lock wires. $\text{A}_{\text{fl}}$ – Cross-sectional area of all half lock wires. $\text{A}_{\text{w}}$ – Cross-sectional area of all full lock wires. $\varepsilon$ – Strain applied to the strand. $\varepsilon_0$ – Initial strain in a wire for an unloaded strand. $E$ – Young’s modulus. $f_{\text{yr}}$ – Yield stress of round wires. $f_{\text{fl}}$ – Yield stress of half lock wires. $f_{\text{yr}}$ – Yield stress of full lock wires. $L_i$ – Plasticity ratio for wire $i$ at stress $(\varepsilon + \varepsilon_0, E)$ (Eq. (5)). $K_{i,1}$ – Fracture ratio for wire $i$ at stress $(\varepsilon + \varepsilon_0, E)$ (Eq. (4)). $\Omega$ – Assessment line (Fig. 4b). $n_w$ – Number of full-lock wires in one strand.

The resistance $R$ is the maximum value of $R_{\text{f}}$:

$R = \text{max}\left\{R_{\text{f}}\right\}$  \hspace{1cm} (10)

An approximation in this procedure is that the stress in the wire is considered as a step function, with zero stress along the recovery length of a broken wire and full stress $(\varepsilon + \varepsilon_0, E)$ outside the recovery length. Note that the value of the Young’s modulus is not important for the result of this iterative procedure.

4.3. Failure probability

The limit state function, $g$, and failure probability, $P_{\text{frec}}$, of a part of a strand with length $2\ell$ are a function of the random variables, $X$, and the time, $t$:

$g(X, t) = R - F$  \hspace{1cm} (11)

$P_{\text{frec}} = P[g(X, t) \leq 0]$  \hspace{1cm} (12)

with $R$ and $F$ according to Eq. (10) and (1), respectively.

The stay cable consists of a combination of a parallel system – the strands – and a series system – the division of each strand into parts with length $2\ell$. It is possible to determine the failure probability of this system. However, this requires analysis of all parts with length $2\ell$ of all strands in the total system in one calculation. It requires considerable computation time to determine the failure probability of such a system. Instead, we have determined an upper- and lower bound of the failure probability, which require the analysis of a single part with length $2\ell$. The upper bound solution assumes a series system for all strand parts. The upper bound failure probability is equal to:

$P_{\text{fr}} = 1 - \left(1 - P_{\text{frec}}(\text{N}_{\text{i}})\right)^{\frac{L}{\ell}}$  \hspace{1cm} (13)

where $L$ is the length of the stay cable and $N_i$ is the number of strands per stay cable = 5.
A lower limit of the failure probability is determined by assuming that the stay cable consists of one artificial strand with a number of wires equal to $N_t$ times the number of wires in a real strand. Eq. (6), (10), (11), (12) and (13) are then replaced by:

$$R_{\text{d}} = N_t A_t C_1 + N_s A_s C_2 + \frac{A_f}{n_0 N_t} \sum_{i=1}^{n_0} C_{i}$$  \hspace{1cm} (14)

$$R_{t} = \max (R_{\text{d}})$$  \hspace{1cm} (15)

$$g_{\text{r}}(X, t) = R_t - N_t F$$  \hspace{1cm} (16)

$$P_{\text{free}} = P[g_{\text{r}}(X, t) \leq 0]$$  \hspace{1cm} (17)

$$P_{\beta} = 1 - (1 - P_{\text{free}})^{(k)}$$  \hspace{1cm} (18)

The reliability index, $\beta$, is used as acceptance criterion. It is related to the failure probability by:

$$P_{\beta} = \Phi(-\beta)$$  \hspace{1cm} (19)

where $\Phi$ is the cumulative distribution function of the standard normal distribution.

5. Bayesian interference procedure for the initial crack depth

Updating procedures based on Bayes’ theorem have proven to be effective in accounting for results of inspections or monitoring information on the fatigue failure probability. Faber et al. [3] have used this procedure for the fatigue life assessment of cables. Examples on other (bridge) components are presented e.g. in [22–24]. In this paper, the Bayesian update procedure is used to estimate the distribution of the initial crack depth, $a_0$. For this variable we are unable to rely on other research with respect to fatigue because the cause of the initial cracks in the wires differs from the usual cause of weld defects or surface irregularities.

5.1. Selection of the distribution function

The sharp laminate defects can be regarded as independent and identically distributed random variables because they are generated by the same rolling process. Variable $a_0$ is the maximum of all sharp laminate defects within a wire length of $2z$. and thus an extreme value distribution should be considered for $a_0$. In addition the distribution should have a lower bound because $a_0$ is always positive. The Frechet distribution matches both requirements and is therefore used in this paper. The relatively heavy upper tail of this distribution reflects the uncertainty with respect to the distribution. The probability density function of the Frechet distribution is:

$$f_{a_0}(a_0) = \frac{k}{a_0} \left( \frac{u}{a_0} \right)^k \exp \left( -\left( \frac{u}{a_0} \right)^k \right)$$  \hspace{1cm} (20)

The parameters $u$ and $k$ are determined with a Bayesian interference procedure using the information from the inspections in 1990 and 2012. The equation used in the paper for this purpose reads:

$$P(u, k | \text{inspection data}) = K \cdot P(\text{inspection data}|u, k) \cdot P(u, k)$$

$$= K \Lambda W$$  \hspace{1cm} (21)

$P(u, k | \text{inspection data}) = \text{posterior two dimensional probability density function for } u \text{ and } k$, i.e. the degree of belief in the combination of $u$ and $k$ having accounted for the inspection data.

$K = \text{constant tuned in such a way that the total probability is equal to 1.}$

$\lambda = P(\text{inspection data} | u, k) = \text{likelihood of obtaining the inspection result for given } u \text{ and } k.$

$W = P(u, k) = \text{prior two dimensional probability density function of } u \text{ and } k$, i.e. the initial degree of belief in the combination of $u$ and $k$.

The prior and likelihood are elaborated hereafter.

5.2. Prior

For reasons of convenience we replace the dependent parameters $u$ and $k$ by the independent parameters $e_u$ and $v$ with the following relationships:

$$u = Ae_u$$  \hspace{1cm} (22)

$$k = B + D e_k$$  \hspace{1cm} (23)

$$e_k = e_u + G \nu$$  \hspace{1cm} (24)

In this system $e_u$ is a random variable with a standard exponential distribution and $\nu$ is a random variable with a standard normal distribution. One may prove that $e_k$ is approximately standard exponentially distributed. So this way we have defined $e_u$ and $e_k$ as two correlated exponentially distributed variables. The degree of correlation can be influenced by the parameter $G$. The initial estimate of the probability of the combination of $e_u$ and $\nu$ is:

$$W = P(e_u, \nu) = f_{e_u}(e_u) \cdot f_{\nu}(\nu) = \exp(-e_u) \cdot \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\nu^2}{2}\right)$$  \hspace{1cm} (25)

The value of $A$ chosen as 0.3 mm so that the probability of $u > 1.5 \text{ mm}$ is small: $P(u > 1.5 \text{ mm}) = P(e_u > 1.5 \text{ mm}/A) = \exp(-1.5 \text{ mm} / A) = 0.007$. The values of $B$ and $D$ are chosen as 1.0 and 1.5, respectively, so that the probability of $k > 8.5$ is small: $P(k > 8.5) = P(e_k > (8.5 - B)/D) \approx \exp(-(8.5 - B)/D) = 0.007$. Parameter $G$ is chosen as 0.25. Note that other values could have been selected for $A, B, D$ and $G$ that also result in the desired small probabilities of large values of $u$ and $k$. However, the influence of the prior is small if the number of ruptured wires is small in comparison with the total number of wires, [3], so that the values for $A, B, D$ and $G$ are unimportant.

5.3. Likelihood

For each (reasonable) combination of $e_u$ and $\nu$, the probability of fracture in a single wire section with length equal to the recovery length must be determined at the time of inspection, $t_{\text{insp}}$. This conditional probability is denoted as $p$. The maximum number of detected wire fractures within one recovery length of one strand is 2. For such a small number of fractured wires the load on individual wires of the strand is not significantly influenced by fractures of other wires. Hence at the time of inspection $p$ given $e_u$ and $\nu$ can be approximated by:

$$p = P\{[l_{r1}(S_i, t_{\text{insp}}), K_{r1}(S_i, t_{\text{insp}})] > \Omega | e_u, \nu\}$$  \hspace{1cm} (26)

$$S_i \approx \varepsilon_0 E + \frac{F}{A_t + A_\nu + A_t}$$  \hspace{1cm} (27)

Evaluation of Eq. (26) for each combination of $e_u$ and $\nu$ is time consuming. In order to reduce the computational time, the probability of fracture per wire can be determined for a given initial crack depth, $a_0$, and then be combined with the probability of that crack depth given $e_u$ and $\nu$:

$$p = \int_0^\infty (f_{a_0} \cdot p_{a_0}) da_0$$  \hspace{1cm} (28)

$$p_{a_0} = P\{[l_{r1}(S_i, t_{\text{insp}}), K_{r1}(S_i, t_{\text{insp}})] > \Omega | a_0\}$$  \hspace{1cm} (29)
Consider a number of detected fractures \( n_{\text{det}} \) out of a total of \( n_{\text{insp}} \) wires sections with length \( 2 l \) at the inspection at time \( t_{\text{insp}} \). The number of fractures follows a binary process, with average \( \gamma_{\text{insp}} \) and standard deviation \( \sqrt{n_{\text{insp}}} \bar{p} - p^2 \). Based on the central limit theory, we may assume a normal distribution for the number of fractures in \( n_{\text{insp}} \) wires sections. For each combination of \( e_x \) and \( v \), the probability of \( n_{\text{det}} \) can be determined:

\[
P(n_{\text{det}}|e_x, v) = \Phi \left( \frac{n_{\text{det}} - 0.5}{\sqrt{\bar{p} - p^2}} \right) - \Phi \left( \frac{n_{\text{det}} - 0.5}{\sqrt{\bar{p} - p^2}} \right)
\]

\[
= \left. \frac{1}{\sqrt{\pi \bar{p} - p^2}} \exp \left( - \frac{x^2}{2(\bar{p} - p^2)} \right) \right|_{x = 0.5}
\]

where \( x \) is the probability of fracture detection of the visual inspection. For a second inspection, a similar set of equations applies to the additional number of failures observed between the two inspections. The likelihood may be written as:

\[
L = P(n_{\text{det}}|e_x, v, t_1, t_2)
\]

where \( n_{\text{det},t1} \) and \( n_{\text{det},t2} \) are the number of detected wire fractures at inspection times \( t_1 \) and \( t_2 \), respectively, and \( \Delta n_{\text{det}} = n_{\text{det},t2} - n_{\text{det},t1} \). In case of negligible correlation of the number of fractures at times \( t_1 \) and \( t_2 \), Eq. (31) can be approximated by:

\[
L = P(n_{\text{det},t1}|e_x, v, t_1) \cdot P(n_{\text{det},t2}|e_x, v, t_2)
\]

where we can make use of Eq. (30) for evaluating the individual probabilities. Negligible correlation of the number of fractures at times \( t_1 \) and \( t_2 \) can be demonstrated relatively easily. Consider a single wire element of length \( 2 l \) at times \( t_1 \) and \( t_2 \). At both times the wire may have \( n_I \) fractures, with possibilities \( n_I = 0 \) (no failure) or \( n_I = 1 \) (failure). Let \( p_I \) be the probability of a fracture at \( t_I \) and \( p_2 > p_1 \) the probability of a fracture \( t_2 \):

\[
p_1 = P(n_I = 1)
\]

\[
p_2 = P(n_I = 0)
\]

where \( n_I \) and \( n_2 \) are \( n_I \) at times \( t_1 \) and \( t_2 \), respectively. We further introduce the transition probability \( p^* \), being the probability that failure occurs between \( t_1 \) and \( t_2 \):

\[
p^* = P(\Delta n_I = 1) = P(n_2 = 1 | n_I = 0)
\]

where \( \Delta n_I = n_2 - n_I \). We consider the fact that probabilities \( p_1 \) and \( p_2 \) are small. The following relations apply:

\[
p_2 = p_1 + (1 - p_1) p^*
\]

\[
p^* \approx p_2 - p_1
\]

The possible combinations of \( n_I \) and \( \Delta n_I \) are listed in Table 1. The expectation of \( n_I \) is \( \mu(n_I) = p_1 \) and the variation of \( n_I \) is \( \sigma^2(n_I) = p_1 - p^2 \approx p_1 \). In a similar way \( \mu(\Delta n_I) = p^* \approx (p_2 - p_1) \) and \( \sigma^2(\Delta n_I) = (p_2 - p_1)^2 \approx p_2 - p_1 \). Finally we may calculate the covariance:

\[
\text{Cov}(n_I, \Delta n_I) = \mu(n_I) \Delta n_I - \mu(n_I) \mu(\Delta n_I) = 0 - p_1 (p_2 - p_1) \approx 0
\]

The neglected terms in the covariance are of the same order of magnitude as the neglected terms in the variances. From the (almost) zero covariance we conclude that \( n_I \) and \( \Delta n_I \) are (almost) uncorrelated for one wire and thus also for the sum of \( n_{\text{insp}} \) wires.

6. Distributions of variables and correlations

The average values, \( \mu \), and standard deviations, \( \sigma \), or coefficients of variation, \( V = \sigma / \mu \), of all (random) variables in Eqs. (1)–(32) are listed in Table 2. The values provided in the table apply to one strand. A number of variables is correlated. Spatial correlation between the area of individual wires of one type (full lock, half lock or round) depends on the milling process and the wire length of one mill batch. Information on this is not available for the cable produced. However, the influence of the area on the failure probability appears to be small (Section 7). In order to limit the number of variables, we have assumed full correlation between the variables of one type.

Considering the yield strength, \( f_y \), and tensile strength, \( f_u \), spatial correlation between wires, \( \rho_{fy} \) and \( \rho_{fu} \), and correlation between \( f_y \) and \( f_u \), \( \rho_{fyfu} \), can be distinguished. The distributions of the yield strength, \( f_y \), and tensile strength, \( f_u \), in Table 2 are based on material certificates provided by the cable manufacturer. For this zero spatial correlation should be applied. According to [27] a weak correlation exists between \( f_y \) and \( f_u \) of a mild steel sample. This may also be the case for high strength steels. However, \( f_y \) and \( f_u \) appear to be relatively unimportant for the failure probability. In order to limit the number of variables \( \rho_{fy} = 1 \) and \( \rho_{fu} = 1 \) and \( \rho_{fyfu} = 0 \) in the model.

Many sources indicate that a strong correlation exists between the fatigue crack growth parameters \( C \) and \( m \), [32–33]. In agreement with [27] we have used a deterministic value for \( m \) and have accounted for scatter in the crack growth data in the distribution of variable \( C \). Full negative correlation \( (\rho = -1) \) has been assumed between crack growth parameters \( C \) and \( \Delta K_c \) of each individual wire, so that a large crack growth rate is accompanied by a low threshold value of the crack growth and vice versa. A spatial correlation of the crack growth parameters between the individual wires is not considered. This is an approximation applied because of lack of information on the magnitude of the actual correlation. For the same reason the fracture toughness of the wires is uncorrelated.

In the model, full spatial correlation is considered for the external loads acting at a certain point in time on the individual wires (Eqs. (11) and (16)) because the entire cable is subjected to this load. For the same reason full spatial correlation is also applied for the load uncertainty factor, \( B_L \). The types of load – such as self-weight and traffic load – are uncorrelated. The uncertainty in the stress intensity factors applies to all wires. Thus full spatial correlation is assumed for \( B_L \).

7. Results and discussion

Fig. 6 provides the conditional probability of a fracture in a single wire section of length \( 2 l \times \mu_{\text{hub}} \) as a function of the initial crack size given that (almost) all other wire sections of the strand are intact according to Eq. (29). The probabilities have been determined using the first order reliability method (FORM). Results are provided for the year of installation, \( t_0 = 1972 \), and the years of inspection, \( t_1 = 1990 \) and \( t_2 = 2012 \). The larger failure probabilities at later years for given \( u_0 \) are due to fatigue crack growth in the

| Table 1 |
|-----------------|-----------------|
| \( n_I \)        | \( \Delta n_I \) |
| \( n_I = 0 \)    | \( (1 - p_1) (1 - p^*) \) |
| \( n_I = 1 \)    | \( p_1 \cdot p^* \) |
| \( \Delta n_I = 0 \) | \( (1 - p_1) p^* \) |
| \( \Delta n_I = 1 \) | \( p_1 \) |
Table 2
Distribution functions of (random) variables per strand as applied in the calculations. Units [N, mm, °C].

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Distribution</th>
<th>μ</th>
<th>V</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_{fl} )</td>
<td>Area of all full lock wires</td>
<td>Uniform</td>
<td>3850 (1 ± 0.05)</td>
<td>MB(^a)</td>
<td></td>
</tr>
<tr>
<td>( A_{hl} )</td>
<td>Area of all half lock wires</td>
<td>Uniform</td>
<td>436 (1–0.05) to 436 (1 + 0.21)</td>
<td>MB(^a)</td>
<td></td>
</tr>
<tr>
<td>( L )</td>
<td>Recovery length</td>
<td>Deterministic</td>
<td>2614 (1–0.05) to 2614 (1 + 0.21)</td>
<td>MB(^a)</td>
<td></td>
</tr>
<tr>
<td>( n_{w} )</td>
<td>Number of full-lock wires</td>
<td>Deterministic</td>
<td>77</td>
<td></td>
<td>MB(^a)</td>
</tr>
<tr>
<td>( a_{0i} )</td>
<td>Initial crack depth for each wire ( i )</td>
<td>Frechet</td>
<td></td>
<td>Section 5</td>
<td></td>
</tr>
<tr>
<td>( n_{ins} )</td>
<td>Number of inspected wire sections of length ( 2\lambda )</td>
<td>Deterministic</td>
<td>3991</td>
<td></td>
<td>MB(^b)</td>
</tr>
<tr>
<td>( n_{det,i} )</td>
<td>Number of detected wire fractures</td>
<td>Deterministic</td>
<td>11 at ( t = 1990 )</td>
<td>MB(^b)</td>
<td></td>
</tr>
<tr>
<td>( \xi )</td>
<td>Probability of detection</td>
<td>Deterministic</td>
<td>0.8</td>
<td></td>
<td>estimate</td>
</tr>
<tr>
<td>( f_{k,i} )</td>
<td>Tensile strength of full lock wires</td>
<td>Lognormal</td>
<td>1310</td>
<td>0.02</td>
<td>MB(^a)</td>
</tr>
<tr>
<td>( f_{0.02,r} )</td>
<td>0.2% proof stress full lock wires</td>
<td>Lognormal</td>
<td>1094</td>
<td>0.05</td>
<td>MB(^a)</td>
</tr>
<tr>
<td>( f_{0.02,m} )</td>
<td>0.2% proof stress half lock wires</td>
<td>Lognormal</td>
<td>1203</td>
<td>0.05</td>
<td>MB(^a)</td>
</tr>
<tr>
<td>( f_{0.02,r} )</td>
<td>0.2% proof stress round wires</td>
<td>Lognormal</td>
<td>1251</td>
<td>0.05</td>
<td>MB(^a)</td>
</tr>
<tr>
<td>( K_{IC} )</td>
<td>Fracture toughness at 20 °C for specimen thickness of 25 mm</td>
<td>Lognormal</td>
<td>3800</td>
<td>0.35</td>
<td>[25–26]</td>
</tr>
<tr>
<td>( C )</td>
<td>Paris law constant</td>
<td>Lognormal</td>
<td>1.10 ( ^{11} )</td>
<td>0.30</td>
<td>[25–27]</td>
</tr>
<tr>
<td>( m )</td>
<td>Crack growth law</td>
<td>Deterministic</td>
<td>2.5</td>
<td></td>
<td>[25]</td>
</tr>
<tr>
<td>( \Delta K_0 )</td>
<td>SIF threshold</td>
<td>Lognormal</td>
<td>165</td>
<td>0.15</td>
<td>[25–27]</td>
</tr>
<tr>
<td>( G_{mean} )</td>
<td>Measured part of self-weight</td>
<td>Normal</td>
<td>2514.10 (^3)</td>
<td>0.04</td>
<td>[28]</td>
</tr>
<tr>
<td>( G_{add} )</td>
<td>Additional part of self-weight due to renovation</td>
<td>Normal</td>
<td>866.10 (^3)</td>
<td>0.07</td>
<td>[28]</td>
</tr>
<tr>
<td>( Q_{fr} )</td>
<td>Traffic load Ref. per. = 15 years</td>
<td>Normal</td>
<td>755.10 (^3)</td>
<td>0.07</td>
<td>[28]</td>
</tr>
<tr>
<td>( \psi Q_{wt} )</td>
<td>Combination value of wind and temperature loads</td>
<td>Deterministic</td>
<td>4810 (^3)</td>
<td>0</td>
<td>[11]</td>
</tr>
<tr>
<td>( T )</td>
<td>Operating temperature</td>
<td>Normal</td>
<td>( \mu = 15 \sigma = 10 )</td>
<td>estimate</td>
<td></td>
</tr>
<tr>
<td>( E_{St,i} )</td>
<td>Initial stress for each wire ( i )</td>
<td>Normal</td>
<td>( \mu = 0 \sigma = 20 )</td>
<td>estimate</td>
<td></td>
</tr>
<tr>
<td>( \Delta S )</td>
<td>Fatigue stress spectrum</td>
<td>Normal</td>
<td>Fig. 2+trends</td>
<td>0.04</td>
<td>[28]</td>
</tr>
<tr>
<td>( B_E )</td>
<td>Model uncertainty in the stress intensity factor</td>
<td>Lognormal</td>
<td>1.0</td>
<td>0.2</td>
<td>[29]</td>
</tr>
<tr>
<td>( B_Q )</td>
<td>Model uncertainty in the traffic load model</td>
<td>Normal</td>
<td>1.0</td>
<td>0.14</td>
<td>[28]</td>
</tr>
</tbody>
</table>

\(^{a}\) MB = Material certificate of manufacturer, or information from bridge drawings & calculations.  
\(^{b}\) In principle, the recovery length should be regarded as a random variable. However, \( \lambda \) is not an explicit parameter in the limit state function, but considered afterwards via Eq. (13) or Eq. (18). Therefore the recovery length is a deterministic parameter in the calculation. Note that \( \lambda \) increases as more wires fracture. This has not been considered in the calculation.  
\(^{c}\) The average value for \( K_{IC} \) originates from [25], and the standard deviation is based on the results of CTOD tests reported in [26], adjusted for the thickness according to the annex. In addition, the relatively large standard deviation is required because of the average value being on the border between the lower shelf and the transition region, as indicated in [30–31].  
\(^{d}\) The average value of \( C \) and \( \Delta K_0 \) originate from [25]. The variations are tuned in such a way that they match the values in [27].  
\(^{e}\) Average values are determined by measurement or calculation, variations are taken from [28].  
\(^{f}\) A deterministic value is selected for \( \psi Q_{wt} \) instead of a distribution function. This is considered justified because the value of \( \psi Q_{wt} \) is only 2% of the expectation of the total load.

- For large values of \( a_0 \) the dominant variables are the self-weight, \( G_{mean} \), the traffic load uncertainty factor, \( B_Q \), and the initial stress, \( E_{St,i} \).

The FORM sensitivity factors of the remaining variables are all lower than 0.1.

The results of Fig. 6 have been used to determine the likelihood and then the posterior of the initial crack depth distribution according to Eqs. 32 and 21, respectively. The posterior is visualized in Fig. 7 in terms of \( e_0 \) and \( v \). Using Eq. (22)-(24) the variables \( e_0 \) and \( v \) are transformed into \( u \) and \( k \) and these have been curve-fitted. A Weibull function has been used to fit the posterior of variable \( u \):

\[
\tilde{f}_d(u) = \frac{K}{\bar{\alpha} - \delta} \left( \frac{u - \delta}{\bar{\alpha} - \delta} \right)^{K-1} \exp \left[ - \left( \frac{u - \delta}{\bar{\alpha} - \delta} \right)^K \right] 
\]  

where the parameter set \( \alpha = 0.95 \text{ mm}, \delta = 0 \text{ mm} \) and \( K = 9.66 \) gives a good fit of the data. The expectation of the posterior of variable \( k \) is fitted as a function of \( u \):
\[ \mu_k = 1.81 \left( \frac{u}{\text{mm}} \right)^2 + 3.44 \left( \frac{u}{\text{mm}} \right) + 0.96 \]  

(41)

The variation of \( k \) around this expectation appears to be largely independent of \( u \) and is fitted with a normal distribution with a coefficient of variation equal to 0.04. The original data and the curve fits of Eqs. (40), (41) are presented in Fig. 8. Single values are drawn from the posterior distributions of \( u \) and \( k \) in each simulation. This sets the Frechet distribution of the initial flaw size, \( a_0 \), Eq. (20). Subsequently initial flaw sizes are drawn from that Frechet distribution for all full lock wires. Together with the distributions of the other variables in Table 2, this completes the model.

The failure probability of the entire system could not be determined with FORM because of the discontinuous – ‘spike’ – resistance curve of Fig. 5a. Crude Monte Carlo (MC) has been used instead. The period up to 2043 has been simulated – i.e. 71 years after erection of the bridge and 31 years after the last inspection – which equals the required service life. The simulations were stopped and the failure probability evaluated after obtaining 50 failed MC samples. A number of randomly selected resistance curves resulting from the MC samples are indicated in Fig. 5b. The curves demonstrate that the deformation at maximum resistance is only slightly larger than the elastic deformation. Note that this applies to the strand section with length \( L \). When considering the entire strand, additional elastic deformations result from the rest of the strand. This further reduces the ratio between the deformations at maximum resistance and the elastic deformations, indicating that the failure mode of the strand is brittle.

The reliability index associated with the upper limit of the failure probability (Eqs. (13) and (19)) resulting from the MC simulations is \( \beta_u = 2.4 \). The reliability index associated with the lower limit of the failure probability (Eqs. (18) and (19)) is \( \beta_l = 2.6 \). The minimum required – or target – reliability for new structures and for a reference period of 50 years resulting from [11] is \( \beta_l = 4.3 \). According to Dutch legislation this value may be lowered to \( \beta_l = 3.6 \) for existing structures. The calculated reliabilities of the cable – \( \beta_u \) and \( \beta_l \) – are lower than the target reliability \( \beta_t \). Thus, measures need to be taken in order to obtain a sufficient reliability level. For this purpose permanent monitoring methods of cables in bridges have been developed and applied [34–35]. The owner of the Ewijk bridge has decided to replace the stay cables during the already planned renovation of the bridge.

The study carried out contains some assumptions and simplifications. Examples are:

- the geometric correction factor \( Y \) in Eq. (2) is an approximation, refer to the annex;
- assuming that the number of cycles to crack initiation \( N_c = 0 \);
- assuming unrestrained bending of the full lock wires with defects;
- assuming zero load transfer in a fractured wire at a distance smaller than the recovery length and full load transfer at a distance larger than the recovery length away from the fracture;
- selection of a deterministic value of the recovery length, and;
- ignoring the fact that larger stresses than considered in the study develop in the curved parts of the stay cables near the anchorages and at the pylon saddle.

Some of these assumptions are conservative, some are unconservative. However, considering the large difference between the calculated reliability index of the cables and the target reliability index, it is unlikely that the cables would satisfy the target reliability index in case of a more refined calculation.

8. Conclusions

The fractures found in a number of wires in the stay cables of the Ewijk bridge are due to large initial defects caused by laminations that were formed during the rolling process of the full lock wires. Despite moderate loading, fatigue crack growth starting from these defects increased the defect size, finally resulting in fracture of the remaining ligament of the wires. Because the number of manufacturers that were able to produce stay cables with long lengths in the 1970’s is small, more cable stayed bridges constructed in the same period may have the same manufacturing problem.

A method is presented to determine the failure probability of stay cables consisting of wires with initial defects. The method is based on a probabilistic model of the cable with an underlying fracture mechanics model for the individual wires. Further, this paper provides a Bayesian update method for updating the distribution functions of those random variables that have significant uncertainty, making use of inspection results.

The results of the simulations demonstrate a brittle failure mode of the stay cable. The upper bound and lower bound values of the failure probabilities, determined by assuming a parallel and a series system for the strands of one stay cable, respectively, differ with a factor of 2. The calculated reliability of the stay cables was lower than the target value, so that measures need to be taken.
Acknowledgements

The authors would like to thank the highway authority of the Dutch Ministry of Transport (Rijkswaterstaat) for their support and permission for publishing this study. Further, the authors would like to thank prof. dr. Daniël Straub and prof. dr. Bertram Kühn for their advice to this study and the engineering company Arup for developing the structural model of the bridge.

Appendix A. Annex – Fracture mechanics model for a single wire

A fatigue and failure assessment of a member with an initial defect can be carried out using the fracture mechanics (FM) theory. The basic parameter in a FM analysis is the stress intensity factor K. This stress intensity factor for a certain applied stress, S, and a certain defect depth, a, is given as:

\[
K = B_a SY \sqrt{\pi a}
\]  
(A.1)

where Y is the geometric correction factor. Various reference books, such as [36], provide equations that give an approximation of Y for standard details, but equations are not available for the full lock wire. Therefore we have used an approximation where the wire is modelled as two cylinders with a total cross-sectional area equal to the cross-section of the wire, Fig. A1. Failure of the wire occurs if one of the two cylinders fails. Because of the approximation a model uncertainty factor, \(B_a\), is applied in Eq. (A.1) with an expectation equal to 1. We have used the expression for the geometrical correction factor of a semi-circular flaw in a cylinder as provided in [37]:

\[
Y = \frac{1.84}{\pi} \sqrt{\sin \left(\frac{\theta}{2}\right) / (\theta / 2)} \left(0.752 + 2.02 \left(\frac{a}{2r}\right) + 0.37 \left(1 - \sin \left(\frac{\pi a}{4r}\right)\right)^2\right)
\]  
(A.2)

\[
r = \sqrt{\frac{A_w}{2\pi}}
\]  
(A.3)

where \(r\) – radius of the cross-section of the artificial cylinder.

\(A_w\) – area of one full lock wire. The driving parameter for fatigue crack growth is the stress intensity factor range \(\Delta K\):

\[
\Delta K = K_{\text{max}} - K_{\text{min}}
\]  
(A.4)

where \(K_{\text{max}}\) and \(K_{\text{min}}\) are the stress intensity factor at stress \(S_{\text{max}}\) or \(S_{\text{min}}\), respectively (Eq. (A.1)). The relationship between the crack extension per cycle, \(da/dN\), and the stress intensity factor range is described by the crack growth law. We have used the Forman-Mettu crack growth law, [38]:

\[
\frac{da}{dN} = \begin{cases} 
C \cdot \left(1 - \frac{\Delta K}{\Delta K_{\text{th}}}\right)^m \left(\frac{1 - \Delta K}{\Delta K_{\text{th}}}\right)^n & \text{for } \Delta K > \Delta K_{\text{th}} \\
0 & \text{for } \Delta K \leq \Delta K_{\text{th}} 
\end{cases}
\]  
(A.5)

\[
\Delta K_{\text{th}} = \begin{cases} 
\Delta K_0 \sqrt{\frac{a}{2r}} \left(1 - \frac{1}{(1 - a/2r)^{1 - \gamma_C}}\right)^{-1} & \gamma \leq \gamma_C \\
\Delta K_0 \sqrt{\frac{a}{2r}} \left(1 - \frac{1}{(1 - a/2r)^{1 - \gamma_C}}\right)^{-1} & \gamma > \gamma_C
\end{cases}
\]  
(A.6)

\[
f = \frac{K_{\text{op}}}{K_{\text{max}}} = \begin{cases} 
\max \{\gamma; A_0 + A_1 \gamma + A_2 \gamma^2 + A_3 \gamma^3\} & \gamma \geq 0 \\
\frac{A_0 + A_1 \gamma}{-2 \leq \gamma < 0}
\end{cases}
\]  
(A.7)

\[
f_\delta = \max \left\{\gamma_{\text{ef}}; A_0 + A_1 \gamma_{\text{ef}} + A_2 \gamma_{\text{ef}}^2 + A_3 \gamma_{\text{ef}}^3\right\}
\]  
(A.8)

\[
A_0 = (0.825 - 0.34x + 0.05x^2) \left(\cos \left(\pi \frac{S_{\text{max}}}{f_0}\right)\right)^{4/3}
\]  
(A.9)

\[
A_1 = (0.415 - 0.071x) \frac{S_{\text{max}}}{f_0}
\]  
(A.10)

\[
A_2 = 1 - A_0 - A_1 - A_3
\]  
(A.11)

\[
A_3 = 2A_0 + A_1 - 1
\]  
(A.12)

where \(da/dN\) – crack extension per stress cycle. \(\gamma\) – stress intensity ratio: \(\gamma = K_{\text{max}} / K_{\text{min}}\). \(\Delta K_0\) – stress intensity threshold at stress ratio \(R = 0\). \(f_\delta\) – crack opening functions. \(K_{\text{op}}\) – minimum stress intensity required to re-open the crack. \(x\) – constraint factor, taken as 2.5. \(f_0\) – flow stress. \(f_\epsilon\) – ultimate tensile strength. \(K_{IC,T}\) – plane strain fracture toughness (linear elastic), corrected for wire radius and temperature according to Eqs. (A.13), (A.14). \(C, m, \Delta K_0, p, q, a, c, C_{th}, \gamma_{\text{ef}}, \gamma_{\text{ef}}\) – material parameters according to Table A1. In the main manuscript, distributions are considered for the dominant parameters \(C, m\) and \(\Delta K_0\) and expectations are considered for \(p, q, a, C_{th}, \gamma_{\text{ef}}\).

Values for \(K_{IC}\) of various high strength steels are provided in [25]. The average value for \(K_{IC}\) at room temperature, for a standardized thickness of 25 mm and for alloys with a tensile strength that agrees with the tensile strength of the wire material, is approximately \(K_{IC} = 3800\) N/mm\(^2\) [25]. Data in [30–31] indicates that this value corresponds approximately with the border between the lower shelf and the transition zone of the temperature-dependent fracture toughness – although the chemical composition and grain structure may be different from the material used for the stay cables. For a thickness other than 25 mm, the fracture toughness at 20 °C can be obtained through [25]:

\[
K_{IC,T} = \frac{K_{IC} - 632}{(2r/25)^{0.25}} + 632
\]  
(A.13)

with the wire radius, r, in [mm] and the fracture toughness, \(K_{IC}\), in [N/mm\(^2\)]. For a number of high-strength alloys, [25] provides \(K_{IC}\) at room temperature and at temperatures of approximately -45 °C. From these data, we have deduced the following, approximate relationship for the temperature dependency of \(K_{IC}\):

\[
K_{IC,T} = K_{IC,T} \cdot \left(1 - 5 \cdot 10^{-3} (20 - T)\right)
\]  
(A.14)

where \(T\) is the operating temperature in [°C].

Eq. (A.5) is evaluated through an incremental numerical procedure, resulting in the relationship between the number of cycles, N, and the defect size, a. Starting values are the initial defect size.

Table A1

<table>
<thead>
<tr>
<th>C</th>
<th>m</th>
<th>(\Delta K_0)</th>
<th>p</th>
<th>q</th>
<th>a</th>
<th>C_{th}</th>
<th>(\gamma_{\text{ef}})</th>
<th>(K_{IC})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 \times 10^{-11}</td>
<td>2.5</td>
<td>165</td>
<td>0.25</td>
<td>2.5</td>
<td>0.038</td>
<td>1.25</td>
<td>0.7</td>
<td>3800</td>
</tr>
</tbody>
</table>
dimensions, \( a_0 \) and the number of cycles up to crack initiation, \( N_0 \). Since \( N_0 \) is unknown, we have set its value to 0. This is an important conservatism in the calculation procedure. Due to fatigue, the crack grows from initial depth \( a_i \) to its final depth \( a_f \), where \( a_i \) is associated with fracture of the remaining ligament.

The final crack depth, \( a_f \), depends on the flow strength, \( f_f \) and fracture toughness, \( K_{0c} \), of the material. In this work, we have applied the so-called failure assessment curve of option 1 in [27]. Failure occurs when a load cycle causes \( K_r > \Omega \):

\[
\Omega = \left\{ \begin{array}{ll}
0 & \text{for } L_r > L_{r,\text{max}} \\
1 - 0.14L_r^2 \left( 0.3 + 0.7 \exp \left( -0.65L_r^2 \right) \right) & \text{for } L_r \leq L_{r,\text{max}}
\end{array} \right.
\]

(A.15)

\[
K_r = K_{r,\text{app}} \frac{\rho}{\sqrt{a_i}} \quad \text{and} \quad L_r = \frac{S_{\text{ref}}}{f_f} \quad \text{and} \quad L_{r,\text{max}} = \frac{f_f}{2f_f^2}
\]

(A.16)

where \( K_r \) – fracture ratio; \( \Omega \) – assessment line; \( L_r \) – plasticity ratio; \( L_{r,\text{max}} \) – plasticity ratio at plastic failure of an unnotched cross-section; \( \rho \) – plasticity correction factor (provided in [27]). \( S_{\text{ref}} \) – reference stress, which is a function of the maximum tensile stress, \( S_{\text{mm}} \), and the crack size, \( a \) (provided in [27]).

In order to check the model, it is used to simulate the fatigue test mentioned in Section 3. The test was stopped after 1.25 x 10^6 cycles with \( S_{\text{mm}} = 341 \text{ N/mm}^2 \) and \( S_{\text{max}} = 200 \text{ N/mm}^2 \). At that moment, 30% of the full lock wires were fractured. The initial defects observed through microscope had a depth ranging between approximately 0.05 mm and 0.5 mm. The examined fracture surfaces showed that the initial defect depths that finally resulted in fatigue crack growth and fracture were \( a_0 = 0.1 \) – 0.2 mm. The crack depth just before fracture was \( a_f = 1 \) to 3 mm.

The fracture mechanics calculation is carried out with average values for the crack growth variables according to Table A1. The relative number of fractured wires, 30 %, is close enough to 50 % to justify this. The final crack depth resulting from the simulations was \( a_f = 2.9 \) mm. This agrees reasonably well with the test \( (a_f = 1 \) to 3 mm). Table A2 provides the results of the simulations in terms of the crack depth just before fracture and the number of cycles until fracture for various initial crack depths. The number of cycles until failure agree reasonably between the simulations and the test for an initial defect depth of 0.2 mm. Note that the number of cycles until failure for an initial defect depth of 0.5 mm is significantly smaller than that of 0.1 or 0.2 mm, whereas the test showed that most fractures occurred at initial defect depths of 0.1 to 0.2 mm. The difference is attributed to the number of cycles before crack initiation \( N_0 \); since we do not have information on the sharpness of the initial defect, the value of \( N_0 \) is set to 0 in the model, which corresponds with a theoretical, infinitely sharp crack tip. In reality, the crack tip of some initial defects may not be this sharp.

## References