Theoretical Study of Colliding Pulse Passively Mode-Locked Semiconductor Ring Lasers With an Intracavity Mach–Zehnder Modulator

Valentina Moskalenko, Julien Javaloyes, Member, IEEE, Salvador Balle, Member, IEEE, Meint K. Smit, Fellow, IEEE, and Erwin A. J. M. Bente, Member, IEEE

Abstract—In this paper, we theoretically study the impact of an intracavity filter based on a Mach–Zehnder interferometer (MZI) on the pulses emitted by InGaAsP/InP passively mode-locked quantum well ring lasers. The filter allows to control the net gain curvature in the device, hereby providing for control over the modes that participate in the dynamics. Simulations of a traveling-wave model indicate that the pulsewidth can be controlled and reduced down to 500 fs. We present and verify a simple algorithm which can be used for calculating the optimum values of the MZI parameters. The optimum parameters are then used in the study of an MZI passive-mode-locked laser under various operating conditions.

Index Terms—Semiconductor laser, mode-locked laser, quantum well laser, short pulse generation.

I. INTRODUCTION

A LARGE number of applications in areas such as high speed optical communications [1], optical clock recovery [2] and high-resolution imaging [3] require short, stable and tunable optical pulses. However, the source and related optical systems involved in these applications can be still quite bulky, vulnerable and costly. The use of optical systems realized as an indium phosphide based photonic integrated circuit (PIC) can overcome these issues and bring additional advantages in terms of stability, footprint, and power consumption. In such a photonic integration platform short optical pulses can be generated using passive mode-locked lasers (PMLLs). The most simple of such PMLLs formed by a semiconductor optical amplifier (SOA) and a saturable absorber (SA) in a Fabry-Perot waveguide resonator. However, their use is often limited either by their average output power, with typical values below 5 mW, relatively long pulse durations of 2-3 ps at repetition rates ranging from 1 GHz to 100 GHz [4] or a small coherent output bandwidth. There are several highly relevant developments that are helping to bring this area forward. For instance, it was shown that the pulse width can be reduced by the increasing the field intensity profile in the SA [5] thereby modifying the effective saturation energies of the absorber.

On the other hand, the PMLL performance could be improved by including passive components into the laser cavity. Active-passive integration technology allows one to combine optical active and passive components in the same monolithic chip. In [6] it was shown that amplitude and self-phase modulations can be reduced by decoupling of the amplifier length and the resonator length which determines the repetition rate. In [7] it was demonstrated that the optical bandwidth of a ring mode-locked laser can be significantly broadened by including an intra-cavity integrated frequency dependent filter. This filter was an asymmetrical Mach-Zehnder interferometer (MZI) and its purpose was to flatten the gain spectrum. The gain curvature flattening can be obtained by judiciously tuning both the amplitude and the free spectral range of the MZI. In [8] by the same group it was experimentally demonstrated that also the pulse characteristics of a PMLL can be significantly improved by this gain spectrum flattening technique. A bandwidth of over 2 THz at the -10 dB power level and a 620 fs pulse width were demonstrated. The studies in [7] and [8] were focused on the experimental characteristics such as the optical output spectrum and the pulse duration obtained at a single fixed SOA injection current level and applied SA voltage. However PMLL output properties vary significantly over its operating conditions. A theoretical steady state analysis of the effect of the MZI in the laser cavity was presented. Such an analysis does not include any laser dynamics.

In this paper we present a theoretical study using traveling wave simulations of how the dynamic regimes of a PMLL with an intracavity asymmetrical MZI depend on the MZI parameters. The MZI will be described by three parameters. The period is determined by the fixed path length difference between the two arms in the MZI. The modulation depth is determined by the fixed path length difference between the two arms in the MZI. The modulation depth is

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J. Javaloyes and S. Balle are with the Departament de Física, Universitat de les Illes Balears, Palma de Mallorca E-07122, Spain (e-mail: julien.javaloyes@uib.es; salvador@imedea.uib-csic.es).

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determined by the power ratio in the arms and the precise tuning of the MZI is determined by an additional variable phase difference of $0 - 2\pi$ between the two arms. In order to improve the characteristics of a PMLL, the optimal parameters of the MZI should be determined. E.g. using a large modulation depth and a large modulation period introduces large additional losses in the cavity. These losses will reduce the energy of the field in the laser and this will adversely affect the pulse width if they are too high. Too fast a modulation period of the MZI leads to the appearance of ripples on the gain profile which will prevent proper modelocked operation. We study the performance of a self-colliding ring InP/InGaAsP mode-locked laser with an intra-cavity MZI filter as a function of the various MZI parameters and present and validate a mode-locked laser with an intra-cavity MZI filter as a function We study the performance of a self-colliding ring InP/InGaAsP gain profile which will prevent proper modelocked operation. We study the performance of a self-colliding ring InP/InGaAsP mode-locked laser with an intra-cavity MZI filter as a function of the various MZI parameters and present and validate a simple method for determining the optimum MZI parameters.

The article is organized as follows: in section II, we briefly summarize our spatially resolved Traveling Wave Model [9] and we detail the device design. In Section III, we explain our methodology for optimizing the MZI. We also present major features of the PML dynamics by performing simulations over a wide range of bias currents. We point out the limitations of our approach and we will present the algorithm for calculating optimized design parameters of the MZI. Using these calculated parameters we show PMLL performance dependency in Section IV.

II. MODEL AND SIMULATED DEVICE

In this section, we will first discuss the outline of the main hypothesis of the model we used for the simulation and the device geometry.

A. Model

We assume single-transverse mode waveguides, and we use the traveling wave model (TWM) developed in [9] —which we summarize below—for determining the dynamics and spatial distribution of the slowly varying amplitudes [10] of the Clockwise and Counter-Clockwise fields, $E_{\pm}(z, t)$, of a quasi-monochromatic TE field. By performing the slowly varying approximation around the optical carrier $(\omega_0, q_0)$ one obtain the following model equation

$$\begin{align*}
(c_\ell \pm \bar{c}_\ell) E_{\pm}(z, t) &= i P_{\pm}(z, t) - \alpha_i E_{\pm}(z, t), \\
\bar{c}_\ell D_0(z, t) &= J - R(D_0) \\
&- i (P_+ E_+^* + P_- E_-^* - c.c.), \\
\bar{c}_\ell D_{\pm 2}(z, t) &= - \left[ R'(D_0) + 4D_0^2 \right] D_{\pm 2} \\
&- i (P_+ E_+^* - E_- P_-^*), \\
R(D) &= AD + BD^2 + CD^3.
\end{align*}$$

For convenience purposes we have scaled space and time to the cavity length $L$ and to the photon transit time $t_c = L/v_p$, respectively. $P_{\pm}$ are the projections of the total polarization at $(z, t)$ onto the forward and backward propagation directions. They are obtained by a coarse graining procedure that consists in averaging the polarization over a few wavelength [10]. We use the natural convention in which the gain is given by $-\text{Im}\{P_{\pm}/E_{\pm}\}$ and the index of refraction by $\text{Re}\{P_{\pm}/E_{\pm}\}$.

The scaled internal losses for the field amplitudes are $\alpha$. The total carrier density is normalized to the transparency $N_t$ and decomposed as

$$D(z, t) = D_0(z, t) + \left[ D_2(z, t)e^{2i\Omega_0z} + D_{-2}(z, t)e^{-2i\Omega_0z} \right],$$

where $D_0(z, t)$ is the quasi-homogeneous component and $D_{\pm 2}(z, t) = D_{-\pm 2}(z, t)$ is the weak grating component arising from the standing wave effects in the system, the so-called spatial hole burning. $J$ is the current density injected per unit time normalized to $N_t$ and the recombination term includes the usual non radiative (A), bi-molecular (B) and Auger (C) recombination terms. The coefficient $B$ and $C$ are normalized to $N_t$ and $N_t^2$, respectively. The differential carrier recovery rate is $R'(D) = dR/dD$. The ambipolar diffusion coefficient is $\bar{D}$ and it is assumed sufficiently large to ensure that $|D_3| \ll D_0$ to justify the perturbative treatment of the standing wave population grating, i.e. the neglecting of the higher order harmonics in $D_3$.

The macroscopic polarization $P$ after approximations described in [11] and [12] is given by the convolution of the response kernel with the optical field as:

$$P(t) = \frac{-i d^2}{\pi W h^2} \int_0^\infty ds R(s, t - s) E(t - s),$$

where $d$ is dipole moment and $W$ is QW width. The convolution kernel $R(s, t)$ is composed of three contributions

$$R(s, t) = I_0(s, t) + I_0(s, t) - I_0(s).$$

The term $I_0(s)$ represents the contribution of the empty bands and reads as:

$$I_0(s) = \frac{m}{\hbar} \frac{1 - e^{-i\Omega_T s}}{is},$$

where $\Omega_T$ is the maximal energy of the allowed transitions within the band. The first and second term in the Eq. 7 describe the contributions of the electrons and holes to the optical response at finite temperature and are given by

$$I_{c,v} = \int_0^K dk \frac{\exp(i k^2 s)}{1 + \exp(\beta[E_{c,v}(k) - F_{N,H}(t)])},$$

where $\beta$ is inverse of the thermal energy $k_B T$ and $F_N$ and $F_H$ correspond to the time-dependent quasi-Fermi levels for the electrons and holes respectively, which are related to the instantaneous densities of electrons and holes by

$$N_{n,h} = N_t^0 \frac{\gamma_{c,v}}{\gamma} \ln\left( \frac{1 + \exp(-\beta F_{N,H}(t))}{\exp(-\beta F_{c,v}(K)) + \exp(-\beta F_{N,H}(t))} \right),$$

where

$$\gamma_{c,v} = \frac{k_B}{T} m_{c,v}, \quad N_t^0 = \frac{m^3}{\pi W h}.$$

$\gamma_{c,v}$ represent the effective thermal broadening of the electrons and holes, which depend on the electron and hole masses $m_{c,v}$ and on the temperature $T$. $N_t^0$ denotes the transparency carrier density at zero temperature.

Our parameters are summarized in Table. 1. The parameters of the SOA section $(\alpha, \gamma, \chi_0, N_t)$ were confirmed by gain
The recombination coefficients measurements of an InGaAsP/InP quantum well based SOA. MOSKALENKO et al.

τslow saturation, thus the relaxation time InGaAsP/InP material [13]. The SA works in the regime of SOA/SA interfaces have a residual field reflectivity. Any residual reflectivity, we assume that both left and right would occur stable. Since this regime is very unstable against be irrealistic in the sense that only unidirectional mode-locking the facets. However, such a perfectly transmitting device would sake of simplicity we assume perfect transmission at each of conditions, that connect active sections with the MZI. For the time. The SA used in the simulation has 10 ps recovery time.

The cavity of the simulated ring mode-locked laser has a length 2.08 mm with two 500 μm SOAs, one 30 μm and C represent the case of SA of the SA must be taken longer than the pulse duration and shorter than roundtrip time. The SA used in the simulation has 10 ps recovery time.

Eqs. (1–4) have to be solved with the appropriate boundary conditions, that connect active sections with the MZI. For the sake of simplicity we assume perfect transmission at each of the facets. However, such a perfectly transmitting device would be unrealistic in the sense that only unidirectional mode-locking would occur stable. Since this regime is very unstable against any residual reflectivity, we assume that both left and right SOA/SA interfaces have a residual field reflectivity R.

**B. Device Geometry**

The self-colliding ring mode-locked laser configuration is of interest for several reasons. First the cavity length of such a laser can be controlled more accurately than that of a cleaved facet linear mode-locked laser. Secondly, since there is no need of facet mirrors the PMLL can be freely located in any position of a semiconductor chip for integration with other optical components [14]. Another reason is that the operation of the laser in a self-colliding regime provides more stable mode-locking mainly due to a more deeply saturated absorber. The colliding pulse mode-locked laser (CPML) design has been known for allowing extremely short pulses at repetition rates as high as 350 GHz in the 1550 nm window as discussed in [15]. Here, we present simulations of CPML at 20 GHz with pulse-widths as short as 499 fs under the best mode locking conditions with optimized device parameters.

The cavity of the simulated ring mode-locked laser has a length 2.08 mm with two 500 μm SOAs, one 30 μm SA and a passive section of 3.07 mm. The effective group index is set at 3.6, hence the fundamental repetition rate is 20 GHz and the cavity roundtrip τ = 50 ps. These parameters are based on a laser design without intra-cavity filter that has been realized and operated successfully. The cavity length is also close to the shortest cavity that can accommodate a realistic MZI structure. The other parameters that were used in the simulation and their origin can be found in Table I.

One can then replace the long passive section by an intra-cavity MZI filter which has two arms with a fixed path length difference inducing a time delay δ, a power splitting ratio x² over the two arms and a variable phase φ in the lower arm. Describing the filter as a function of these three variables allows for easy manipulation of the period, amplitude and phase of the modulation. The diagram of the simulated ring mode-locked laser with MZI is shown on Fig. 1. The fields from the opposite sites of MZI can be linked as:

\[ E_\pm(t) = \left\{ \begin{array}{ll} E_\pm(t) & \text{for } -\tau_\text{MZ} < t < 0 \\ -x^2 E_\pm(t - \tau_\text{MZ} - \delta)e^{-i\phi} & \text{for } 0 < t < \tau_\text{MZ} \end{array} \right\} e^{-i\varphi}, \]

where τ is the time transit along the shortest branch of MZI and ψ = 2πτ/λ is the global phase acquired while traversing the MZI. E±(t) and E±(t − τ) are fields after and before propagating through the filter.

**III. SIMULATION OF THE RING PMLL**

**A. Mode-Locking Dynamics**

1) Mode-Locking Regimes Without Intra-Cavity MZI Filter: Different operational regimes of the PMLL can be observed when varying the injection current I in the gain section and the reverse bias voltage applied to the SA. In order to analyze the influence of the intra-cavity filter presence on the PMLL performance the parameter boundaries of the various operational regimes without the filter should be determined first. This can be done easily in the model by setting the power splitting x² in the MZI filter to 0. The condition x = 0 makes the pulse propagate only into the one arm which makes the configuration of the laser identical to that without intracavity filter. The performance of the PMLL can be characterized either in time-domain by analyzing the time dependency of the output signal or in the frequency domain e.g. by observing and analyzing optical and RF spectral components. In this work the performance of the PMLL is described using the following parameters: the pulse width τp, the optical spectrum

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**TABLE I**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
<th>Meaning</th>
</tr>
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<tbody>
<tr>
<td>λ₀</td>
<td>1.55</td>
<td>μm</td>
<td>Emission wavelength</td>
</tr>
<tr>
<td>n₂</td>
<td>3.6</td>
<td></td>
<td>Effective group index</td>
</tr>
<tr>
<td>τ₂</td>
<td>6.05</td>
<td>ps</td>
<td>Amplifier transit time</td>
</tr>
<tr>
<td>τₐ</td>
<td>0.40</td>
<td>ps</td>
<td>Absorber transit time</td>
</tr>
<tr>
<td>τₘ₂</td>
<td>37.2</td>
<td>ps</td>
<td>MZI section transit time</td>
</tr>
<tr>
<td>δ</td>
<td>0.1</td>
<td></td>
<td>MZI extra section transit time</td>
</tr>
<tr>
<td>x²</td>
<td>1-15%</td>
<td></td>
<td>MZI power splitting</td>
</tr>
<tr>
<td>R</td>
<td>0.02</td>
<td></td>
<td>SOA/SA interface reflectivity</td>
</tr>
</tbody>
</table>

**Active Material Parameters**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>n₂</td>
<td>424</td>
<td>cm⁻¹</td>
<td>Saturated gain factor</td>
</tr>
<tr>
<td>κ</td>
<td>400</td>
<td>THz</td>
<td>Empty band contribution</td>
</tr>
<tr>
<td>γ</td>
<td>5</td>
<td>THz</td>
<td>Polarization decay rate</td>
</tr>
<tr>
<td>γₑ</td>
<td>50</td>
<td>THz</td>
<td>Electron thermal broadening</td>
</tr>
<tr>
<td>γₒ</td>
<td>250</td>
<td>THz</td>
<td>Hole thermal broadening</td>
</tr>
<tr>
<td>N₂</td>
<td>1×10⁻¹⁸</td>
<td>cm⁻³</td>
<td>Transparency Carrier Density</td>
</tr>
<tr>
<td>D</td>
<td>12</td>
<td>cm²/s</td>
<td>Ambipolar diffusion coefficient</td>
</tr>
<tr>
<td>A</td>
<td>1×10⁸</td>
<td>s⁻¹</td>
<td>Non radiative recombination</td>
</tr>
<tr>
<td>B</td>
<td>7×10⁻¹⁰</td>
<td>cm²/s</td>
<td>Spontaneous recombination</td>
</tr>
<tr>
<td>C</td>
<td>1×10⁻²⁰</td>
<td>cm²/s</td>
<td>Auger recombination</td>
</tr>
<tr>
<td>J₁</td>
<td>8.1×10⁹</td>
<td>cm²/s</td>
<td>Transparency current</td>
</tr>
<tr>
<td>τₚ</td>
<td>10</td>
<td></td>
<td>Absorber recovery time</td>
</tr>
</tbody>
</table>
Fig. 2. The pulse width (a), width of the optical spectrum (b), the average deviation of the central optical frequency (c), TBP (d), RF peak height (e) and average peak intensity (f) as a function of current for \( x = 0 \).

full-width at the half maximum \( v_{fwhm} \), the central frequency of the optical spectrum with respect to the bandgap of the gain section \( < v > \), the time-bandwidth product (TBP) and the pulse peak power. However, these parameters are mainly suitable for a description of the mode-locking regime in steady situation. Therefore in order to meaningfully describe other regimes of the PMLL as for instance Q-switch and instable PMLL regimes, the height of the RF peak (in dB) normalized to the DC value is also presented. In Fig. 2 the simulation results are presented of the configuration with \( x = 0 \) under fixed reverse bias voltage (represented by a fixed value of the absorber recovery time) and a varying injection current starting from the lasing threshold value \( J_{th} \).

In the range of currents between \( J_{th} \) and 3.1 \( J_{th} \) generation of the stable pulse train is observed in the simulations. Inside this region the quality of the pulses and optical spectrum improves proportionally to the injected current. This improvement is caused by the broadening of the gain spectrum with injected current and thus the introduction of more longitudinal modes which participate in the laser pulse formation. The increase of the injected current also leads to the blue shift of the gain spectrum which results in a gradual shifting of the laser central frequency (Fig. 2(c)). The stability of the optical pulses amplitude is indicated together with the data on the average peak intensity dependency in Fig. 2(f), where dotted lines represent the maximum and minimum values of the peak intensities of the laser pulses. At the currents above 3.1 \( J_{th} \) the cavity gain becomes high enough to provide more than one pulse per round trip and satellite pulses start to appear. The energy of the satellite pulses increases with injected current and leads to destabilization of the pulse train and appearance of the low frequency components in electrical spectrum. Fig. 2(f) and 2(e) show a wide spread for the pulse peak intensities and a corresponding decrease of the height of the fundamental RF peak. A further increase of the injected current leads to the establishment of a harmonic mode-locking regime, where two stable short optical pulses are circulating in the laser cavity.

2) Mode-Locking Regimes With MZI Filter: In this section we examine the influence of the gain-flattening filter on the performance of the mode-locked laser. Simulations of the MLL were performed at the fixed injected current of 2.2 \( J_{th} \). This value was chosen to be well above the threshold in order to compensate the possible losses introduced by the MZI and far away from the current at which instability starts to appear. As mentioned above the improvement in the PMLL performance can be achieved when all three parameters of MZI are properly chosen. In this section we present the simulation results of a PMLL with MZI at various combinations of \( \delta, \phi, \) and \( x \). The typical gain bandwidth of a InGaAsP QW based active section covers around 50 nm, hence, in order to flatten the gain curvature the MZI period should be chosen around this value. However, the PMLL performance is a trade-off between the number of phase-locked modes and the net gain per roundtrip. In this section we will show the results of the PMLL simulations with a relatively large period and a high modulation amplitude of the MZI \( \delta = 250 \text{ fs} \) as well as a relatively small period \( \delta = 1 \text{ ps} \) and lower modulation depth. Depending on the position of the modulation minimum the gain spectrum can be either broadened or narrowed. Fig. 3 shows an example of the PMLL characterstics as a function of the MZI phase, i.e. the spectral position of the minimum transmission of the filter. The shown curves were obtained for the MZI path length difference \( \delta = 250 \text{ fs} \), which corresponds to a modulation period of around 30 nm, and three values of the amplitude balance in the arms \( x = 0.1, 0.2, 0.3 \).

For all MZI amplitudes (Fig. 3(a)) the pulse width as a function of \( \phi \) has two extreme values: a maximum value occurs when the MZI filter works as a gain-narrowing filter and a minimum occurs when the filter provides gain-flattening. Fig. 3(b) shows that the phase of the minimal pulse width corresponds to the widest frequency comb and vice versa. Notice that the largest changes in the spectral and time characteristics are observed at the highest MZI modulation depth (green curve \( x = 0.3 \)). The MZI phase change leads to the central frequency shift, which is more significant for the higher amplitudes (values of \( x \)). The average output intensity dependencies on \( \phi \) are given in Fig. 3(e). According to this graph the output
intensity is reduced most at the phase which provides the broadest optical spectrum. At this phase the intracavity losses are highest. Notice that the output intensity change is more dramatic for the deepest MZI modulation, e.g. \( x = 0.3 \). The use of an MZI with \( x \) above 0.3 leads to a minimum in the gain spectrum at the point of minimum transmission of the filter. This prevents mode-locking. Fig. 3(d) shows the dependency of the TBP on the phase. Interestingly, the position of the maximum and minimum are slightly shifted with respect to the maximum and minimum of \( \tau_p \) and \( v_{\text{fwhm}} \). The Fig. 3 (f, g, h) displays the RF spectra, pulse shapes and optical spectra for the highest and lowest TBP values. At both conditions the RF spectrum shows a clear peak at 20 GHz which corresponds to the roundtrip time and output signal shows stable mode-locked operation. One can thus consider to use the possibility to vary the pulse and spectral width by tuning the phase of the MZI. The maximum and minimum achieved for the pulse duration \( \tau_p \) was 1.18 and 0.63 ps respectively. The corresponding optical widths were 0.65 THz and 1.2 THz.

Mode-locking characteristics for another set of MZI parameters are shown in Fig. 4. The results represented on these plots correspond to \( \delta = 1 \) ps and \( x = 0.02, 0.1, 0.15 \) and 0.18. The \( x = 0.18 \) was the highest value at which stable mode-locking was possible. As expected, similar shapes as in Fig. 3 were obtained. However, due to the shorter period of modulation the flattened gain region is much smaller than that at \( \delta = 250 \) fs. It leads to less significant changes in the \( \tau_p, v_{\text{fwhm}}, < v > \) and TBP (Fig. 4 (a, b, c, d)). The use of smaller modulation amplitude leads to less significant increase of the internal losses which can be confirmed from Fig. 4 (e). The output intensity decrease is less than 2% when
for the $\delta = 250$ fs it is 6%. The pulse width and optical spectra width were tuned in the range between 0.86 and 1.1 ps and 0.75 and 0.92 THz respectively, much smaller ranges than the ones in the case of $\delta = 250$ fs.

B. Optimization

In the previous section we examined the performance of the laser with various MZI parameters. It was shown that, for example, for a fixed value of $\delta$ there is only one value of $x$ and $\varphi$ that offers the most significant reduction of the gain curvature. Thus, in order to achieve the optimal regime of the MZI a large amount simulations should be performed for all possible variations of the MZI parameters. In this section we will discuss an optimization method that avoids time-consuming simulations. With this method one can calculate the MZI parameters for optimal PMLL performance directly.

The net gain per round trip $g_{\text{net}}$ is a combination of the gain of amplifier, absorption, frequency independent internal losses and frequency dependent losses of the MZI:

$$g_{\text{net}}(\omega, D_{\text{SOA}}, D_{\text{SA}}, \varphi, x) =$$

$$-g_{\text{SOA}}(\omega, D_{\text{SOA}})L_{\text{SOA}} - g_{\text{SA}}(\omega, D_{\text{SA}})L_{\text{SA}} - \alpha L_{\text{cavity}} + \ln T(\varphi, x, \delta),$$

(13)

where $\alpha$ is the internal loss and $T$ is the MZI transmission function, which is defined as:

$$T = (1 - x^2) - x^2 e^{-i(\omega t + \varphi)}.$$  

(14)

The gain spectra $g_{\text{SOA}}(\omega, D_{\text{SOA}})$ and $g_{\text{SA}}(\omega, D_{\text{SA}})$ in Eq.(13) are defined as the frequency and carrier dependent imaginary part of electrical susceptibility of the amplifier and saturable absorber, respectively.

The goal of the optimization procedure is to calculate the parameters of the MZI function that provide the lowest curvature of the $g_{\text{net}}$ as a function of optical frequency in the range around the gain peak $\omega_0$. For the sake of simplicity the parameters were optimized for the threshold conditions ($g_{\text{net}} = 0$). Assuming a vanishingly small field it entails that $D_{\text{SA}} = 0$. The position of the gain peak can be determined as the maximum point of the function $g_{\text{net}}$ and this can be obtained by solving the equation $\frac{dg_{\text{net}}}{d\omega} = 0$. The curvature of the function is defined by the second and third derivatives. Setting these to zero at the position of the gain peak ($\frac{d^2g_{\text{net}}}{d\omega^2} = \frac{d^3g_{\text{net}}}{d\omega^3} = 0$) is used to find the optimal gain flattening conditions. Thus solving a system of four equations for a fixed value of the MZI period $\delta$ enables one to calculate four unknown parameters: $\omega_0$, the frequency of the gain peak; threshold current density $D_0$; optimal phase $\varphi$ and amplitude $x$.

Fig. 5 (a-d) shows the resulting spectra of $g_{\text{net}}$ (green), which is a combination of the amplifier gain (blue), the absorber loss (red), the internal losses $\alpha$ and MZI frequency dependent losses (black) at fixed values of $\delta$ and optimized $x$ and $\varphi$. The optimization results are presented for $\delta$ equal to 125 fs, 250 fs, 500 fs and 1 ps. Notice, that the spectral region of flattened gain and minimal amplifier gain which can be achieved at threshold varies with $\delta$. By using a deep and slow modulation in the MZI (Fig. 5(a,b)) the gain spectrum at threshold can be flattened over a wide region (> 2THz), but this introduces large additional losses in the cavity which can increase the threshold amplification too far and adversely affect PMLL performance. On the other hand, too small and fast modulation of MZI (Fig. 5(c, d)) will lead to the appearance of ripples on the gain profile.

Fig. 5(e) shows the optimized MZI amplitude $x$ as a function of the MZI period $\delta$. The blue solid curve was obtained using the optimization algorithm described above. The green dots represent the values of $x$ which produce the broadest optical coherent comb from the laser for a given value of $\delta$ according to the PMLL simulations. Each point corresponds to the $\delta$ shown on Fig. 5(a-d). Notice that there are differences between the optimal value found by simulations of the full model and the values predicted by the optimization algorithm. The optimization method assumes a vanishingly small field and in principle is only valid close to threshold. The carrier density within the gain and the absorber regions are assumed to be $D_0$ and 0, respectively. However above threshold the carrier densities in the two sections oscillate in time, hence a static analysis is not strictly valid. In mode-locked operation the field in the cavity partially bleaches the saturable absorber and the time averaged losses in the laser are reduced. Therefore the optical gain necessary to support a pulse circulating in the cavity is lower and thus the laser operates at a reduced
average carrier density in the SOA. This lower carrier density means a change in gain spectrum and a possible departure from the optimized situation.

C. Laser Operation With Optimized MZI Parameters

Using the optimization algorithm discussed in the previous section we will examine and compare the PMLL performance for various periods of the MZI and corresponding optimal values of the MZI phase and amplitude. The injected current values were taken the same as those in Section 3.1.2. In Fig. 6 the simulated laser output characteristics as a function of $\delta$ are shown as black circles. The simulations of the PMLL were performed for $\delta$ values from 0.054 ps to 1 ps. For each value of $\delta$ the $x(\delta)$ and $\phi(\delta)$ were calculated using the optimization algorithm. The characteristics of the PMLL without the MZI filter under the same operating conditions are presented as a red dashed line for comparison. As one might expect, the obtained results show that the PMLL performance can be improved in a finite range of the MZI parameters where the modulation period is of the same order of magnitude as the width of the gain spectrum. Within this range an optimal relation between the increase in cavity losses and flattened gain can be achieved. The minimal pulse width ($\tau = 0.55$ ps) and maximum spectral width ($\nu = 1.3$ THz) were obtained at $\delta = 0.1$ ps and $x = 0.48$ which corresponds to an almost symmetrically balanced MZI. The minimal time-bandwidth product (TBP = 0.69) was obtained at $\delta = 0.308$ ps and $x = 0.27$.

As the value of $\delta$ is increased and therefore the value of $x$ is decreased, the period of the MZI induced ripples in the gain spectrum become faster and smaller, and thus the influence of the MZI becomes less and less important. This explains the asymptotic behavior of the graphs in Fig. 6 where the laser characteristics approach the PMLL regime without MZI for large values of $\delta$.

In the previous sections, we discussed the features of the PML regime with the MZI filter present for a fixed value of the injected current. However, as already mentioned, the mode-locking regime dramatically evolves with the injected current. Fig. 7 shows the output laser characteristics as a function of bias current for the laser with MZI (black curves) and without (red curves). The black curves represent the results of simulations for the MZI with $\delta = 250$ fs, $x = 0.31$ and $\phi = 1.43$. These values of $x$ and $\phi$ are chosen optimal for this value of $\delta$.
characteristics shown in Fig. 7 indicate the improvement of the MZI PMLL performance comparing with the one without MZI. The minimum calculated pulse width for the PMLL with MZI is 490 fs and 680 fs for PMLL without MZI. The maximal spectral width was improved from 1.079 THz to 1.652 THz by using an MZI. At the injected currents above 3 $J_{th}$ relatively slow pulse amplitude oscillations start to take place in both cases. By increasing the pump current in PMLL without MZI it shows harmonic mode-locking behavior at the currents above 5.7 $J_{th}$. The MZI PML essentially stops emitting pulses due to detuning of the gain section with respect to the MZI due to the increased current and enters a regime of CW operation.

IV. Summary

We have presented a study of simulations of a colliding pulse semiconductor PMLL with an intracavity MZI based filter using a travelling wave model (FreeTWM [9]). We have developed an algorithm which enables us to calculate the power ratio and phase of MZI that provides a widest optical bandwidth for a given value of the delay between the two MZI arms. Despite the fact that the presented algorithm predicts optimal parameters of the MZI at the condition of a small field, i.e. at threshold conditions, the values of $\chi$ and $\phi$ optimized using the complete PMLL simulations are in good agreement with those predicted by the algorithm. An improvement in pulse width and spectral bandwidth of the PMLL output caused by the presence of the MZI over the wide range of operating injected currents was shown. The narrowest achieved pulse width for the PMLL with an MZI was 490 fs and 680 ps for PMLL without MZI. The maximal optical bandwidths were 1.652 THz and 1.079 THz for the laser with and without MZI respectively. Notice, that the TBP of both cases is larger than the transform limit. This is caused by the self-phase modulation in the amplifier and absorber as well as other sources of dispersion which can not be compensated by a linear intracavity filter. Further studies of the ultimate limits of such a CPML setup would consider the effects of spectral hole burning and carrier heating. Such ultrafast non linearity that correspond to a local depopulation of the available carriers within the valence and conduction(s) bands could in principle play a role for short pulses. It is however difficult to infer if such additional effects would lead to an increased bandwidth beyond the transform limit or if such amplitude-phase modulation would further compress the pulses. Such ultra-fast non linearity could have different effects in the saturable absorber and in the gain sections. Additionally, several other sources of dispersion could contribute to further spreading of the pulse like e.g. the group velocity dispersion stemming from the background index and the vertical and transversal confinement of the field.

REFERENCES