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MIMO equalization with adaptive step size for few-mode fiber transmission systems

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Abstract: Optical multiple-input multiple-output (MIMO) transmission systems generally employ minimum mean squared error time or frequency domain equalizers. Using an experimental 3-mode dual polarization coherent transmission setup, we show that the convergence time of the MMSE time domain equalizer (TDE) and frequency domain equalizer (FDE) can be reduced by approximately 50% and 30%, respectively. The criterion used to estimate the system convergence time is the time it takes for the MIMO equalizer to reach an average output error which is within a margin of 5% of the average output error after 50,000 symbols. The convergence reduction difference between the TDE and FDE is attributed to the limited maximum step size for stable convergence of the frequency domain equalizer. The adaptive step size requires a small overhead in the form of a lookup table. It is highlighted that the convergence time reduction is achieved without sacrificing optical signal-to-noise ratio performance.

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References and links


1. Introduction

Recently, spatial division multiplexing (SDM) enabled by multiple-input multiple-output (MIMO) digital signal processing (DSP) has been established as the method to increase the available capacity in a single fiber [1–3]. SDM is achieved through transmission over multi-core fibers, multimode fibers, or a combination of the two. In this work, we focus on a special case of the multimode fibers, the few-mode fiber (FMF). The used 3 mode FMF employs the linearly polarized (LP) LP01, LP11a, and LP11b modes as transmission channels [4].

To unravel the mixed channels at the receiver side, a MIMO weight matrix is used to invert the transmission channel using an adaptive minimum mean squared error (MMSE) algorithm. During convergence, the least mean squares (LMS) algorithm is employed. However, for data transmission, the decision-directed least mean squares (DD-LMS) algorithm is used. In this work, two MMSE MIMO equalizers, the time domain equalizer (TDE) and the frequency domain equalizer (FDE), are compared. Both equalizers heuristically adapt a weight matrix to minimize the final output error. Hence, a convergence time to optimize the weights is required. In [5], only the TDE was investigated. However, the FDE has a lower computational complexity and is therefore of interest for further comparative analysis. For any MIMO equalizer, channel estimation, and hence convergence, should be short to minimize overhead. A key parameter for the convergence time and final error floor of the MMSE MIMO equalizer is the MIMO adaptation factor, commonly known as the MIMO equalizer’s step size \( \mu \). A large step size ensures fast convergence, and a small step size lowers the error floor. Note that, the choice of the step size is critical as making it too small may lead to insufficient channel tracking capabilities, resulting in larger bit error rate or can significantly affect the convergence time.

In comparison to optical transmission systems, the use of an adaptive step size is more common in other fields of research, such as wireless communications. Previously proposed adaptive schemes have been based on the use of linear step sizes based on the squared error [6], the sliding window [7], and by implementing three weight matrices [8]. However, these have only been analyzed in simulations. Here, we implement an adaptive step size algorithm in an experimental optical transmission setup.

In this work, the convergence time and bit error rate (BER) of a lookup table (LUT) based adaptive step size TDE and FDE are investigated and compared with their respective fixed step size equalizers. For the analysis a 28Gb/s 3 mode dual polarization (DP) quadrature phase shift keying (QPSK) coherent transmission over 80km of few mode fiber is used [9]. We show that the convergence time can be greatly reduced for the FMF transmission, without an optical signal to noise ratio (OSNR) penalty in comparison to the fixed step size method.
Reducing the convergence time ensures that the required training sequence length is shorter, which minimizes overhead.

2. Experimental setup

The experimental setup depicted in Fig. 1 is used to investigate the convergence time reduction. Two cases, namely back-to-back (BTB) and 80 km transmission are investigated. A free running external cavity laser (ECL) at 1550.112 nm with a linewidth of <100kHz is used and the output is equally split into 4 outputs. Three are used as coherent receiver local oscillators (LOs). The remaining ECL output is used as the transmission signal going into an IQ-modulator. The IQ-modulator is driven by two $2^{15}$ pseudo random bit sequences (PRBSs), where one is delayed by $2^{14}$ symbols with respect to the other, to generate a 28GBaud QPSK output. This output is split into two equal outputs. Both outputs go into the polarization multiplexing stage, where one is delayed by 312 symbols for decorrelation purposes before recombinging them. The dual polarization signal is then split into three outputs. For additional decorrelation purposes, two outputs are delayed by 2413 and 3847 symbols, respectively. Each output is separately amplified before going into the mode multiplexer (MUX). Note that the MUX employed in the experiment is a phase-plate based mode multiplexer. Two MUX inputs are converted to the spatial linearly polarized LP$_{11a}$ and LP$_{11b}$ modes, and the remaining input is used as the fundamental LP$_{01}$ mode. The combined output is transmitted over the FMF, which allows three spatial modes to propagate simultaneously. In the transmission line, a few-mode erbium doped fiber amplifier is employed [10]. The FMF is partially differential mode delay (DMD) compensated with a residual DMD of 1.5 ns after 80 km transmission [11]. The DMD compensation is achieved by concatenating fibers which have respective positive and negative DMDs. DMD is defined as the LP$_{01}$ – LP$_{11}$ group velocity difference, multiplied by the fiber length. After the fiber transmission a phase-plate based mode demultiplexer (DMUX) is used to perform the inverse operation of the mode multiplexer. The properties of the mode multiplexer, demultiplexer, and fiber have been detailed in [9]. Each of the three DMUX outputs are connected to individual coherent receivers where two synchronized real-time oscilloscopes act as 12 ($=3$ modes $\times 2$ polarizations $\times 2$ real-valued tributaries of a complex symbol) analog to digital converters (ADCs). The used oscilloscopes are a 4-port 50GS/s and an 8-port 40GS/s oscilloscope, where the 50GS/s acts as the master trigger. To ensure correct timing of both oscilloscopes, the optical and synchronization delays were aligned in time in the digital domain. Then, in the digital domain, the digitized signals were up sampled to 56Gsamples/s before digital signal processing was employed. Each data capture consists of 560,000 symbols per polarization, and two data captures were taken for system BER averaging. Staticauly, the first 25,000 symbols are used for equalizer convergence using the LMS algorithm, which results in system BER averaging over a total of 1,070,000 symbols (2,140,000 bits). The digital signal processing steps are further detailed in section 4.
3. MIMO system model

The aforementioned optical transmission system shown in the experimental setup can be described at a sample instance \( k \) by the linear time invariant (LTI) equation

\[
\hat{R}^{(k)} = H^{(k)} S^{(k)} + N^{(k)},
\]

where \( \hat{R}^{(k)} \) is the \([12L \times 1]\) Nyquist (2-fold) oversampled received input vector, \( H \) the \([12L \times 6]\) transmission matrix, \( S \) the \([12 \times 1]\) transmitted signal vector, and \( N \) the \([12L \times 1]\) additive white Gaussian noise (AWGN) vector. Note that all variables are real-valued. \( L \) describes the third dimension in the transmission matrix, namely time. The value \( L \) denotes the number of impulse response samples per input (taps) taken into account at the equalizer. \( L \) can be expressed in the time domain as

\[
\zeta = \frac{LT_s}{2} \text{[seconds]},
\]

where \( T_s \) denotes the symbol time. The factor 2 indicates the 2-fold oversampling of \( \hat{R}^{(k)} \). For time domain equalization \( L \) commonly is an odd number, and has been chosen as 31 and 131, for BTB and 80 km transmission, respectively. For frequency domain equalization \( L \) equals \( 2^\text{round}(\log_2 T_s) \), which is 32 and 256, for BTB and 80 km transmission, respectively. \( \text{round}[] \) denotes rounding up to the nearest integer. This choice is limited by the fast fourier transform (FFT) sizes used in the FDE equalizer. To adaptively equalize and track the transmission matrix \( H^{(k)} \), linear filtering and an adaptive update algorithm are used. The linear filter is a fractionally spaced (2 taps/symbol) weight matrix \( W^{(i)} \). To optimize the \([12 \times 1]\) real-valued output, the LMS algorithm heuristically minimizes the function

\[
\|W^{(i)} \hat{R}^{(i)} - S^{(i)}\|
\]

where \( W^{(i)} \) is \([12 \times 12L]\). As each output is separately optimized by the LMS algorithm, the weight matrix \( W^{(i)} \) can conveniently be rewritten as \( 12 W^{(i)} \) vectors, with size \([1 \times 12L]\). Only the terminology changes, the complexity remains the same. However, each output can now be optimized by a separate step size, denoted as \( \mu_i \).

4. Digital signal processing

After the analog to digital conversion in Fig. 1, digital signal processing is employed. The key processing steps in the receiver DSP have been depicted in Fig. 2. First, the optical front-end is compensated [12]. Then, chromatic dispersion (CD) estimation and compensation is performed [13, 14]. The next step is the heart of the receiver, the MIMO equalizer. As introduced, it unravels the incoming data to separate outputs. The focus of this paper is the...
MIMO equalizer, where the TDE or FDE variants are further investigated as detailed in section 4.1 and 4.2, respectively. For all cases shown in this work, the MIMO equalizer is heuristically updated using the LMS algorithm the first 25,000 symbols, before switching to the DD-LMS algorithm. After the MIMO equalizer, carrier phase estimation (CPE) is employed. CPE performs carrier phase offset estimation between the transmitter laser and local oscillator laser, and phase noise estimation. CPE is performed on the complex output of the MIMO equalizer. To this end, two real-valued outputs of a complex valued polarization mode output are combined. After the carrier offset has been removed, the MIMO output error is determined. A CPE block consists of a phase detector and a digital phase locked loop, as shown in Fig. 1 of [15]. After the frequency offset has been removed, the output constellations are demapped, and the BER is estimated.

4.1 Adaptive step size time domain equalization

Figure 3 depicts the proposed time domain equalizer, which includes the combining of two real-valued outputs to a complex valued output on which carrier phase estimation is performed in the feedback path. In section 3, the weight matrix \( W^{(k)} \) has been introduced. Using a time domain equalizer, \( W_i^{(k)} \) is updated on a symbol basis as

\[
W_i^{(k+1)} = W_i^{(k)} + \mu e_i^{(k)} \hat{R}^{(k)},
\]

where \( e_i^{(k)} = d_i^{(k)} - R_i^{(k)} \). \( e_i^{(k)} \) denotes the \( i \)th entry of the [12 \times 1] real-valued error vector, and \( d_i^{(k)} \) is the \( i \)th entry of the [12 \times 1] real-valued desired signal vector. For LMS, \( d_i^{(k)} \) is the training symbol, and for DD-LMS it is the maximum likelihood symbol.

As introduced previously, the step size \( \mu \) is an important value, as it limits the final error, and determines the convergence time. Note that the convergence time is inversely proportional to the step size and gives insight to the system’s adaptive tracking capabilities. For fast tracking, a high step size is desirable, however for a low final error value, a small step size is preferred. Also note that, for a stable system, the maximum step size has to be constrained [16]. For time domain equalization, the maximum step size \( \mu_{\text{max}} \leq \frac{2}{\lambda_{\text{max}}} \), where \( \lambda_{\text{max}} \) is the maximum eigenvalue of the autocorrelation of the input signal. The BER for various fixed step sizes is shown in Figs. 4(a) and 4(b) for the back-to-back and the 80 km transmission case, respectively. Figure 4 contains OSNR captures per dB. The markers are for
identification only. Having a larger or smaller step size than required causes additional bit errors, because of a noise floor caused by the step size, or improper channel tracking capabilities, respectively. This effect is more pronounced after transmission, as is shown in Fig. 4(b).

In the feedback path in Fig. 3, the error $e$ is multiplied by $\mu$. For both the real and imaginary tributary of the complex output, the same $\mu$ is used. By averaging the error $e$ over 50 symbols as $e_{av}[i] = 1/50 \cdot \sum_{k=0}^{i-1} e_k[i]$, and translating the averaged error $e_{av}$ by a LUT, $\mu$ is adapted. The entries in the lookup table are logarithmically distributed as $\mu_{LUT} = [10^{-3}, 9 \times 10^{-4}, \ldots, 10^{-5}]$, and the respective error levels are chosen as $\sqrt{131.6 \cdot \mu_{LUT}/2}$. These levels have been chosen based on the number of taps and the number of transmitted channels for the 80 km transmission case. Note that, by increasing the number of entries in the LUT, it is possible to optimize the convergence rate further at the cost of additional complexity.

### 4.2 Adaptive step size frequency domain equalization

When increasing the number of taps $L$, the complexity of the FDE becomes favorable over the TDE [16,17]. Therefore, it is worth to compare the performance of the two equalizers. The FDE used in this work is depicted in Fig. 5. It is an expanded version of the scheme used in [18], where only complex inputs and outputs were taken into account. Here, by using real-valued inputs and outputs, IQ imbalances and skew issues can be compensated [17]. This however comes at the cost of additional complexity, but performs optimally even when the input alignment is imperfect.

The inputs are first converted from serial to parallel (S/P), before splitting even and odd samples. Each of the even and odd sample blocks are separately transferred to the
frequency domain by an FFT of size $N_{\text{fft}}$. The overlap-save method is used with 50% overlap [18], therefore $N_{\text{fft}}$ equals $L$. In the frequency domain, the weight and the data are multiplied, before being summed. After summation, an inverse fast fourier transform (IFFT) returns the frequency domain multiplication to the time domain. Then, parallel to serial (P/S) conversion is performed before combining the real and imaginary outputs. On the combined output carrier phase estimation is performed. The feedback path is the same as the TDE. The gradient estimation is a multiplication in the frequency domain between the feedback path and the inputs. For the FDE, stable convergence is achieved when $\mu_{\text{max}} = 4/\left(N_{\text{fft}}\lambda_{\text{max}}\right)$ [17].

As a block is processed, the maximum tolerable step size is smaller than the TDE step size. The fixed step size performance for back-to-back and 80 km transmission is shown in Fig. 6. Note that for the 80 km transmission case, the maximum step size was limited to $2.5 \times 10^{-4}$.

![Figure 6](image)

**Fig. 6.** Fixed step size frequency domain equalizer performance for (a) back-to-back and (b) 80 km transmission.

### 5. Results

Through OSNR characterization, the adaptive step size equalizer is compared with the static step size. Figure 7(a) depicts the bit error rate between the best performing fixed step size and adaptive step size for the time domain equalizer. Figure 7(b) depicts the same, but for the frequency domain equalizer. In terms of BER versus OSNR, note that both equalizers perform the same in the optimal case. However, when inspecting the adaptive step size TDE and FDE with respect to the fixed step size TDE and FDE convergence performance, a difference is noticed. Figure 8 depicts the convergence for both equalizers for both the BTB case, as well as the 80 km transmission case. We denote the system convergence time as the time it takes for the MIMO equalizer to reach an average output error which is within a margin of 5% of the average output error after 50,000 symbols. Note that fixed $\mu = 1e-3$ has the shortest convergence time, but a high error floor, and hence an increased BER. Therefore, as the system BER performance of both the fixed $\mu = 1e-4$ and $\mu = 5e-5$ is similar, we compare the convergence time of the adaptive step size with the fixed $\mu = 1e-4$. For the TDE with adaptive step size the convergence time is reduced by approximately 75% and 50% for the back-to-back and 80 km transmission case, respectively. For the FDE, the convergence

![Figure 7](image)

**Fig. 7.** Performance comparison between best fixed step size and adaptive step size for (a) the TDE and (b) the FDE.
time is reduced by approximately 75% and 30% for the back-to-back and 80 km transmission case, respectively. The TDE converges to the minimum error floor faster, as the maximum step size limit for stable convergence is higher with respect to the FDE. The step size limitation of the FDE is caused due to the block processing of the FDE, detailed in section 4.

6. Conclusion
A successful 28GBaud 3 mode QPSK transmission over 80km few-mode fiber is demonstrated using a TDE and FDE with adaptive step size. The best BER performance both MIMO equalizers can achieve is the same, however, the complexity of the FDE for long impulse responses is lower. On the other hand, due to the more stringent maximum step size limit of the FDE, the TDE shows faster convergence. The convergence time is defined as the time it takes for the MIMO equalizer to reach an average output error, which is within a margin of 5% of the average output error after 50,000 symbols. With respect to the static step size equalizers ($\mu = 1 \times 10^{-4}$), the convergence time was reduced by approximately 50% and 30% for the TDE and the FDE, respectively. The FDE step size limitation is caused by processing of the input per block, instead of on a per symbol basis.

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