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A Virtual Rider for Motorcycles: Maneuver Regulation of a Multi-Body Vehicle Model

Alessandro Saccon, John Hauser, and Alessandro Beghi

Abstract—This work develops a virtual rider that can be used to make a multi-body two-wheeled vehicle follow a specified ground path with a prescribed velocity profile. The virtual rider system is based on a simplified motorcycle model, the sliding plane motorcycle, which is composed of a single rigid body with two ground contact points. This reduced order nonlinear system was presented in an earlier work, together with a dynamic inversion procedure for computing a state-control trajectory corresponding to the desired task. This dynamic inversion procedure is combined in this work with a maneuver regulation controller to yield a nonlinear feedback control strategy. A transverse coordinate system that is consistent with the mechanical symmetries of ground vehicles is constructed and used in the development of the maneuver regulation controller. An inverse optimal control strategy, which also exploits the mechanical symmetries, is developed to shape the dynamic response of the closed loop system. Numerical results with the virtual rider driving a multi-body vehicle through a demanding maneuver with lateral accelerations reaching 1 g are presented.

Index Terms—Dynamic inversion, inverse optimal control, maneuver regulation, multi-body motorcycle, reduced order model, virtual rider.

I. INTRODUCTION

DURING the last decade, a strong effort has been conducted by researchers to obtain ever more accurate and detailed mathematical models of two-wheeled vehicles. For handling and maneuverability studies, a key role is played by multi-body modeling [1]–[5].

Multi-body system software is used to perform virtual testing of new designs. Virtual testing helps in the evaluation of handling and ride qualities as well as safety and performance issues, and is often conducted in parallel with real tests on hardware prototypes [6]. To enable simulated closed-loop tests, modeling

those in which a real test driver would follow a prescribed road or trajectory, taking corners at different speeds, a *virtual driver* for the multi-body vehicle model is required.

Controlling a two-wheeled vehicle is intrinsically more difficult than a four-wheeled one. The difficulty of controlling a two-wheeled vehicle arises primarily from the fact that vehicle balance and path following are somewhat conflicting objectives—a two-wheeled vehicle can easily fall to the ground while negotiating a corner if steering and speed controls are not combined appropriately.

Early theoretical studies for tracking a desired ground path with a (simplified) nonholonomic riderless bicycle can be found in [7]–[9]. This inspired an approach for controlling a multi-body motorcycle model along a specified ground path [10]. The control strategy, detailed in [11], employs the simplified bicycle model from [7] and uses a model-based predictive control (MPC) paradigm. The roll angle is used as a virtual control input in place of the steering angle. The strategy has been applied to control various multi-body models of scooters and motorcycles [12].

Recent works on the control of multi-body motorcycle may also be found in [13] and [14]. In the former, a look ahead strategy is adopted and two separate control loops are employed to regulate speed and lateral deviation errors. In the latter, the proposed rider model is composed of two parts. A path planning procedure, the optimal maneuver method [15], is used to compute the reference ground path and speed to be followed. The reference optimal trajectory is then stabilized using independent proportional-integral-derivative (PID) loops to control speed and lateral deviation.

A discrete-time optimal linear quadratic regulator with preview approach is adopted in [16]. The reference road information is transformed into a vehicle-based reference frame, so that they represent the rider's view of the road. The control problem is then expressed with respect to this reference frame. Speed control is independent of steering and body lean control and is achieved using a PI regulation. Of interest is the addition of rider upper body lean as a control input together with the usual throttle/brake and steering torque controls.

A model-based predictive control is also employed in [17]. Arc length and lateral deviation from the desired path provide a notion of tracking error, much as what we propose in this work. There are, however, significant differences. The desired state-control curve (the tracking reference) is specified by trimming the vehicle pointwise along the desired ground path as if it were performing a constant speed, constant radius turn at each point. The resulting curve is *not* a system trajectory, in contrast to that provided by the dynamic inversion procedure developed in [18]. Note also that the control law in [17] is obtained using

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A. Saccon is with the Associate Laboratory of Robotics and Systems in Engineering and Science (LaRSyS), Instituto Superior Técnico (IST), Technical University of Lisbon (UTL), Lisbon 1049-001, Portugal (e-mail: asaccon@isr.ist.utl.pt).

A. Beghi is with the Department of Information Engineering, University of Padova, Padova 35131, Italy (e-mail: beghi@dei.unipd.it).

J. Hauser is with the Department of Electrical, Computer, and Energy Engineering, University of Colorado at Boulder, Boulder, CO 80309-0425 USA (e-mail: john.hauser@colorado.edu).

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the linearization about this non-trajectory. This approach appears to work well for a class of smooth maneuvers for which that curvature does not vary too rapidly.

This work presents the development of a nonlinear control system with the capability of driving a multi-body two-wheeled vehicle along a general user-specified ground path with a desired velocity profile. This control system (and strategy), which will be referred to as a *virtual rider*, rests on three main pillars: (a) a *dynamic inversion procedure* for computing a state-control trajectory corresponding to a desired maneuvering task; (b) an *inverse optimal control strategy* for shaping the closed loop dynamics; and (c) a *maneuver regulation controller* charged with executing the maneuver planned in step (a) using the results of step (b) in the computation of the feedback controller. The numerical strategy for accomplishing (a) was developed in [18]. This paper details parts (b) and (c), and presents simulation results of the virtual rider driving a multi-body motorcycle model, developed using a commercial multi-body software package. Preliminary versions of the present work have been presented in [19] and [20].

The virtual rider is based on a simplified motorcycle model, the *sliding plane motorcycle* (SPM) model, which was developed in [18]. The dynamic inversion procedure, inverse optimal control strategy, and maneuver regulation controller are all based on this simplified model. The complete multi-body motorcycle model, with state dimension typically greater than twenty, is accessed through an interface that makes it look like the SPM model from an input-output perspective. The maneuver regulation controller for the SPM model is connected directly to this interface, allowing for closed loop simulations. The SPM model can be viewed as a nonlinear reduced order model of the multi-body motorcycle, with the virtual rider being a reduced order controller.

This paper is organized as follows. In Section II, we review the basic theory of *maneuver regulation* and, in particular, the notions of longitudinal and transverse coordinates and transverse linearization. In Section III, we tailor the general theory of maneuver regulation to the specific case of motorcycle control, detailing the computation of the transverse linearization for the SPM model and the strategy for shaping the closed loop response. In Section IV, we discuss the multi-body motorcycle model employed and the interface to the SPM controller that provides the virtual rider. Finally, in Section V, we present numerical results obtained using the virtual rider to control the multi-body motorcycle model. The conclusion is drawn in Section VI.

II. MANEUVER REGULATION

Consider the time-invariant nonlinear control system

$$\dot{x} = f(x, u) \quad (1)$$

$$y = h(x) \quad (2)$$

with state $x \in \mathbb{R}^n$, control $u \in \mathbb{R}^m$, and output $y \in \mathbb{R}^p$, $p \leq n$. The output map $h(\cdot)$ and the system vector field $f(\cdot, \cdot)$ are both taken to be C^2 so that, for example, local existence and uniqueness of trajectories is ensured for reasonable classes of bounded inputs. The output map $h(\cdot)$ should have the further property

that there exists a C^2 map $r : \mathbb{R}^n \rightarrow \mathbb{R}^{n-p}$ that complements $h(\cdot)$ in the sense that

$$(y, \eta) = (h(x), r(x)) \quad (3)$$

is a global diffeomorphism of $\mathbb{R}^n$ into itself. (Note that this implies that the Jacobian $Dh(x)$ has full rank at every x .)

In our context, f describes the dynamics of the SPM model while, as we shall see, the map h is just a projection onto the two state components that specify the position of the rear wheel contact point. The map r in this case is trivial, being a projection onto the remaining state components.

Our goal is to develop a control system that enables an appropriate execution of a desired task $y_\xi(t)$, $t \geq 0$. We will use the subscript ξ to indicate that an object is associated with the desired task.

A given task $y_\xi(\cdot)$ is *admissible* if $\|\dot{y}_\xi(t)\|$ is bounded away from zero, $\|\ddot{y}_\xi(t)\|$ is bounded, and there is a state-control trajectory $\xi(\cdot) = (x_\xi(\cdot), u_\xi(\cdot))$ such that

$$\dot{x}_\xi(t) = f(x_\xi(t), u_\xi(t)) \quad y_\xi(t) = h(x_\xi(t)) \quad (4)$$

for all $t \geq 0$. Here, we assume that the class of tasks of interest is admissible and that there is a method for computing the solution of the dynamic inversion problem. For the SPM model, the state-control task $\xi(\cdot)$ corresponding to a given task $y_\xi(\cdot)$ can be found using the dynamic inversion procedure detailed in [18].

One approach to the execution of the task $\xi(\cdot)$ is to use trajectory tracking where the goal is to ensure that $x(t) \rightarrow x_\xi(t)$ as $t \rightarrow \infty$. This might be accomplished by using the linear time varying control law

$$u = k(t, x) = u_\xi(t) + K(t)(x_\xi(t) - x)$$

where $K(\cdot)$ is a bounded feedback gain matrix that provides *exponential* stability for the linearization

$$\dot{z} = \mathbb{A}(t)z + \mathbb{B}(t)v$$

of (1) about $\xi(\cdot)$, where $\mathbb{A}(t)z = D_1 f(x_\xi(t), u_\xi(t)) \cdot z$ and $\mathbb{B}(t)v = D_2 f(x_\xi(t), u_\xi(t)) \cdot v$. In many cases, a suitable feedback gain $K(\cdot)$ can be determined using an LQR strategy.

Note that $D_1 f$ and $D_2 f$ are partial derivatives of f with respect to the first and second arguments respectively. We refer to [21] and [22, Ch. 2] for an introduction to this modern and powerful derivative notation, including definitions of derivative expressions like $DF(x) \cdot z$ and $D^2 F(x) \cdot (z_1, z_2)$, used in this work. In this notation, a Taylor expansion looks like $F(x+z) = F(x) + DF(x) \cdot z + (1/2)D^2 F(x) \cdot (z, z) + o(\|z\|^2)$. When convenient, we also make use of the often seen linearization notation, $f_x(x, u)$ and $f_u(x, u)$.

Exponentially stable trajectory tracking ensures that the tracking error will satisfy a bound of the form $\|x(t) - x_\xi(t)\| < \epsilon$, $t \geq 0$, provided that model errors and disturbances are sufficiently small. For vehicles and other maneuvering systems, it may be unreasonable to operate under such a restriction on the disturbances (and model errors). Consider, for example, the following scenario. A vehicle with a limited (longitudinal) acceleration capability is operating in a hilly environment with

many curves. As the vehicle ascends a hill, it is unable to keep up with desired position trajectory due to a higher than expected acceleration requirement. This might be due to a steeper than expected road grade or even a vehicle mass greater than the modeled value. After cresting the hill and heading downhill, the vehicle accelerates in an attempt to catch up with the desired position, but runs off the road going around a tight curve with too much speed. Informally, we refer to this situation as a *speed-up* phenomenon. The point here is that, while the vehicle remained relatively close to the desired *curve* in terms of lateral position, heading, and velocity, it started to lag behind the desired longitudinal position *trajectory*, leading to a vain attempt to catch up with the desired trajectory by increasing its velocity (speeding up) beyond what is prudent in tight curves. We note that similar difficulties can be experienced when maneuvering a motorcycle close to performance limits, for example, when braking hard to enter a turn [23].

To circumvent difficulties such as the speed-up problem, an alternative control strategy called *maneuver regulation* can be used [24], [25]. With a maneuver regulation controller, the maneuvering error is defined using an appropriate “distance” between the current state $x(t)$ and the *entire* desired state curve $x_\xi(\cdot)$, rather than just the desired state $x_\xi(t)$ at time t . As a consequence, in the case that $x(t) = x_\xi(t - \tau)$, for some $\tau > 0$, the controller simply uses the delayed version of the desired input $u_\xi(t - \tau)$, avoiding the potentially dangerous speed-up phenomenon. In the following, we present some important aspects of this theory, referring the reader to [24]–[27] and [19] for further details and results.

In the control of nonlinear systems, it is common and useful to seek out a coordinate system in which the nature of the problem is clarified and for which the problem is, perhaps, somewhat easier to solve. For maneuver regulation, we will make use of a *transverse* coordinate system that is adapted to the desired maneuvering task.

The first step is to specify a *longitudinal* parametrization for the desired task. Recall that the task $y_\xi(\cdot)$ is a time-parametrized curve with a nonzero tangent at each point. To reparametrize this curve in a regular manner, let $s_\xi : \mathbb{R} \rightarrow \mathbb{R}$ be a strictly increasing function of time and let $\bar{t}_\xi : \mathbb{R} \rightarrow \mathbb{R}$ denote its inverse (which clearly exists) so that $\bar{t}_\xi(s_\xi(t)) = t$ for all t in the domain of $s_\xi(\cdot)$. It is natural to require that $s_\xi(\cdot)$ (and hence $\bar{t}_\xi(\cdot)$) be as smooth as $y_\xi(\cdot)$ so that the reparametrized task

$$\bar{y}_\xi(s) = y_\xi(\bar{t}_\xi(s))$$

is in the same smoothness class. Furthermore, to preserve regularity, we require that $s_\xi(\cdot)$ be at least C^2 with $\dot{s}_\xi(\cdot)$ bounded and bounded away from zero and $\ddot{s}_\xi(\cdot)$ bounded. It is easy to check that $\bar{t}_\xi(\cdot)$ will then possess the same properties. The well known *arc length* parametrization (which will be used below), determined by using $s_\xi(t) = \int_0^t \|\dot{y}_\xi(\tau)\| d\tau$, clearly belongs to the same smoothness class as $y_\xi(\cdot)$.

In the same way, we can write the state-control trajectory as s -parametrized curves

$$\bar{x}_\xi(s) = x_\xi(\bar{t}_\xi(s)) \quad \bar{u}_\xi(s) = u_\xi(\bar{t}_\xi(s))$$

where the *bar* indicates that a quantity is expressed as a function of the longitudinal parameter s (rather than t). Using (4) together with the chain rule, we see that these quantities are related according to

$$\bar{x}'_\xi(s) \bar{\nu}_\xi(s) = f(\bar{x}_\xi(s), \bar{u}_\xi(s)) \quad \bar{y}_\xi(s) = h(\bar{x}_\xi(s)) \quad (5)$$

where $\bar{\nu}_\xi(s) = \nu_\xi(\bar{t}_\xi(s))$ and $\nu_\xi(t) = \dot{s}_\xi(t)$, and the *prime* in (5) indicates differentiation with respect to s .

By regularity, we can parametrize points in a (tubular) neighborhood of $\bar{y}_\xi(\cdot)$ using $(s, \rho) \in \mathbb{R} \times \mathbb{R}^{p-1}$. For instance, suppose that $Y(s)$ is a smoothly varying $p \times (p-1)$ matrix with orthonormal columns and such that $Y(s)^T \bar{y}'_\xi(s) = 0$ so that the columns of $Y(s)$ together with $\bar{y}'_\xi(s)$ provide a basis for $\mathbb{R}^p$ at each value of s . Then, there is a $r_0 > 0$ such that the mapping

$$(s, \rho) \mapsto \Omega(s, \rho) = \bar{y}_\xi(s) + Y(s)\rho$$

is locally invertible whenever $\|\rho\| < r_0$ (choose r_0 so that the Jacobian of the map is invertible at all such points and use the Inverse Function Theorem [21]). Many other choices for $\Omega(\cdot, \cdot)$ are possible, preserving the property that $\Omega(s, 0) = \bar{y}_\xi(s)$ for all s .

For simplicity, we may suppose that $\bar{y}_\xi(\cdot)$ has no self intersections and that the nonlocal “miss” distance is bounded away from zero. In that case, the radius $r_0 > 0$ can be chosen so that $\Omega(\cdot, \cdot)$ is one-to-one on the tube with $\|\rho\| < r_0$, allowing one to define the inverse map

$$(s, \rho) = \Phi(y)$$

satisfying $\Phi(\Omega(s, \rho)) = (s, \rho)$ and in particular $\Phi(\bar{y}_\xi(s)) = (s, 0)$. Note that, in practice, only a local one-to-one property is needed and this is ensured by the properties of $\bar{y}_\xi(\cdot)$.

Given the global diffeomorphism (3), we can extend Φ to obtain a change of coordinates

$$x \mapsto \Psi(x) = (s, w_1, \dots, w_{n-1})$$

that is valid on a neighborhood of the state space curve $\bar{x}_\xi(\cdot)$ and that satisfies $\Psi(\bar{x}_\xi(s)) = (s, 0, \dots, 0)$. Indeed, letting $\pi(x) = \Phi_1(h(x))$ denote the projection of x onto the maneuver (at $\bar{x}_\xi(\pi(x))$), we simply choose

$$s = \pi(x) \quad (6)$$

$$w_i = \Phi_{i+1}(h(x)), \quad i = 1, \dots, p-1 \quad (7)$$

$$w_i = r_{i-p+1}(x) - r_{i-p+1}(\bar{x}_\xi(\pi(x))), \quad i = p, \dots, n-1. \quad (8)$$

Here $\Phi_j(x)$ denotes the j th component of $\Phi(x)$, and similarly for $r(x)$.

We call s the *longitudinal coordinate* and $w = (w_1, \dots, w_{n-1})$ the *transverse coordinates*. It is also useful to partition the coordinate change accordingly, defining the mapping $W : \mathbb{R}^n \rightarrow \mathbb{R}^{n-1}$ so that

$$\begin{bmatrix} s \\ w \end{bmatrix} = \begin{bmatrix} \pi(x) \\ W(x) \end{bmatrix} = \Psi(x). \quad (9)$$

Let $x = \Gamma(s, w)$ denote the inverse of $(s, w) = \Psi(x)$.

Using the state-dependent input transformation $u = \bar{u}_\xi(s) + v$, the nonlinear control system (1) can be written in (s, w) coordinates as (cf. [26])

$$\begin{aligned}\dot{s} &= \bar{v}_\xi(s) + f_1(s, w, v) \\ \dot{w} &= A(s)w + B(s)v + f_2(s, w, v)\end{aligned}\quad (10)$$

where $f_1(s, 0, 0) \equiv 0$ and $f_2(s, w, v)$ is higher order in (w, v) . We call (10) the *transverse form* of the dynamics of the nonlinear system (1). The second equation in (10) (for $\dot{w}$) follows easily by noting that $w = 0$ if and only if $x \in \bar{x}_\xi(\cdot)$ and that the state space curve $w = 0$ is invariant (giving $\dot{w} = 0$) under the flow of the system when $v = 0$ (i.e., we stay on $\bar{x}_\xi(\cdot)$ by using $\bar{u}_\xi(s)$ when at $\bar{x}_\xi(s)$). For the first equation, differentiate $s = \pi(x)$ to get $\dot{s} = D\pi(x) \cdot \dot{x} = D\pi(x) \cdot f(x, u)$ and evaluate with $(w, v) = (0, 0)$ to get $\dot{s} = D\pi(\bar{x}_\xi(s)) \cdot f(\bar{x}_\xi(s), \bar{u}_\xi(s)) = \bar{v}_\xi(s)$; the last equality can be obtained by differentiating the identity $s = \pi(\bar{x}_\xi(s))$ with respect to s and using (5).

A closer look reveals some of the structure of the transverse form. Using $\dot{w} = DW(x) \cdot \dot{x}$, the nonlinear $\dot{w}$ dynamics is seen to be

$$\dot{w} = DW(\Gamma(s, w)) \cdot f(\Gamma(s, w), \bar{u}_\xi(s) + w)$$

so that [19]

$$A(s)w = DW(\bar{x}_\xi(s)) \cdot D_1 f(\bar{x}_\xi(s), \bar{u}_\xi(s)) \cdot Z(s)w + D^2 W(\bar{x}_\xi(s)) \cdot (f(\bar{x}_\xi(s), \bar{u}_\xi(s)), Z(s)w) \quad (11)$$

$$B(s)v = DW(\bar{x}_\xi(s)) \cdot D_2 f(\bar{x}_\xi(s), \bar{u}_\xi(s)) \cdot v \quad (12)$$

where $Z(s)w = D_2 \Gamma(s, 0) \cdot w$. Note the requirement for the *second* derivative of the coordinate change. The terms $D_1 f(\bar{x}_\xi(s), \bar{u}_\xi(s))$ and $D_2 f(\bar{x}_\xi(s), \bar{u}_\xi(s))$ in the above expression are those occurring in the standard linearization of the system, evaluated at $(x, u) = (\bar{x}_\xi(s), \bar{u}_\xi(s))$. In the next section, we will see that $A(s)$ and $B(s)$ for the SPM model (and other planar vehicles) can be computed in a much simpler way than the indicated expressions.

Restricting ourselves to inputs that do not depend explicitly on time, we can eliminate t (in a neighborhood of the task in state space) to obtain

$$\bar{w}' = A_T(s)\bar{w} + B_T(s)\bar{v} + f_T(s, \bar{w}, \bar{v}) \quad (13)$$

where $A_T(s) = A(s)/\bar{v}_\xi(s)$, $B_T(s) = B(s)/\bar{v}_\xi(s)$, and $f_T(s, \bar{w}, \bar{v})$ is higher order in $(\bar{w}, \bar{v})$. The systems (10) and (13) are *trajectory equivalent* in the sense that relevant trajectories of each system can be mapped to trajectories of the other. Indeed, each bounded trajectory $(\bar{w}(s), \bar{v}(s))$, $s \geq s_0$, of (13) gives rise to a (10) trajectory $(s(t), w(t), v(t))$, $t \geq 0$, with $(s(0), w(0)) = (s_0, \bar{w}(s_0))$; just integrate the scalar differential equation $\dot{s} = \bar{v}_\xi(s) + f_1(s, \bar{w}(s), \bar{v}(s))$ to determine the strictly increasing s trajectory $s(t)$, $t \geq 0$, and write $w(t) = \bar{w}(s(t))$ and $v(t) = \bar{v}(s(t))$. Conversely, every bounded trajectory $(s(t), w(t), v(t))$, $t \geq 0$, of (10) with strictly positive and bounded $\dot{s}(\cdot)$ (and regardless of how $v(\cdot)$ was determined!) gives rise to the (13) trajectory $(\bar{w}(s), \bar{v}(s)) = (w(\bar{t}(s)), v(\bar{t}(s)))$ where $\bar{t}(s)$ is the inverse function of $s(t)$: $\bar{t}(s(t)) = t$, $t \geq 0$.

Fig. 1. Sliding plane motorcycle model. The position of the rear wheel contact point is (x_r, y_r) . The roll angle is φ , and the vehicle heading (or yaw angle) is ψ . The effective steering angle is δ_f . The inertial reference frame is chosen with the z -axis pointing down.

Expressing the transverse dynamics as a differential equation where the longitudinal state s becomes the independent variable enables the development of a *time-invariant* control law for regulating the transverse states w to zero. If, for example, the *transverse linearization*

$$\bar{w}' = A_T(s)\bar{w} + B_T(s)\bar{v} \quad (14)$$

is exponentially stabilized by a $(s$ -varying) linear state feedback $\bar{v} = -K(s)\bar{w}$, then the time-invariant *nonlinear* state feedback $v = -K(s)w$ will exponentially stabilize the maneuver $\bar{x}_\xi(\cdot)$ for (10), cf. [26]. Written in the original coordinates, the nonlinear feedback

$$u = k(x) = \bar{u}_\xi(\pi(x)) - K(\pi(x))W(x) \quad (15)$$

exponentially stabilizes the maneuver $\bar{x}_\xi(\cdot)$ for (1). Such a $K(\cdot)$ might be developed using an LQR strategy if the transverse linearization is, e.g., uniformly or instantaneously controllable.

III. MANEUVER REGULATION CONTROL DESIGN

The *sliding plane motorcycle* (SPM) model, which we introduced in [18], is a simplified motorcycle model that captures many important aspects of real motorcycle dynamics including sliding and load transfer. The SPM model is a mechanical system consisting of a single rigid body making contact with the ground at two points which are an idealization of the front and rear wheel contacts that a real motorcycle makes with the ground. A graphical representation of the SPM is given in Fig. 1. The generalized coordinates describing the configuration of this vehicle are the position of the point of contact of rear wheel (x_r, y_r) and the roll φ and yaw ψ angles. The vehicle velocity vector is expressed in the body frame using the rear-wheel sideslip angle β and the rear contact point velocity $v = \sqrt{\dot{x}_r^2 + \dot{y}_r^2}$, giving the kinematic relations $\dot{x}_r = v \cos \chi$ and $\dot{y}_r = v \sin \chi$, where $\chi = \psi + \beta$ is the course heading of the vehicle, see Fig. 2. The vehicle acceleration portion of the dynamics, which is *independent* of both the position (x_r, y_r) and the heading ψ , has the form

$$\frac{d}{dt}(v, \beta, \dot{\varphi}, \dot{\psi})^T = f_v(\varphi, (v, \beta, \dot{\varphi}, \dot{\psi})^T, (F_x, \delta_f)^T) \quad (16)$$

Fig. 2. Vehicle velocity kinematics. The vehicle velocity vector is expressed in the body frame using the sideslip angle β and in the spatial frame using the course heading $\chi = \psi + \beta$.

where the control inputs are the thrust force F_x and the effective steering angle δ_f . As discussed in [18], this *symmetry* implies that the *trim* trajectories are precisely those for which the vehicle traverses a path of constant curvature (circles and straights) with constant speed. The trim trajectories are such that the inputs and all states except for (x_r, y_r) and ψ are *constant* which implies that the linearization $(\mathbf{A}(t), \mathbf{B}(t))$ is *constant* except for the $\psi(t)$ dependent terms in $\mathbf{A}(t)$ deriving from the above mentioned kinematic relations.

The nonlinear control system $\dot{x} = f(x, u)$ is now taken to be the SPM model with state $x = (x_r, y_r, \varphi, \psi, v, \beta, \dot{\varphi}, \dot{\psi})^T$ and input $u = (F_x, \delta_f)$, while the task output is taken to be the position of the rear wheel contact point, $y = h(x) = (x_r, y_r)^T$. Please note that x and y are the system's state and task, while x_r and y_r are the coordinates of the rear wheel contact point. While some confusion may be possible when using the standard notations for state and output along with that for Cartesian coordinates, the subscript r (denoting *rear*) should help the reader keep them straight.

In [18], we developed a dynamic inversion procedure for computing a state-input trajectory of the SPM model that is consistent with the desired ground trajectory. Such a trajectory will be called the *lifted* trajectory, as the procedure *lifts* a planar ground trajectory up to a full state-input trajectory.

A. Longitudinal and Transverse Coordinates

Given a desired maneuver regulation task

$$y_\xi(t) = (x_{r\xi}(t), y_{r\xi}(t))$$

$t \geq 0$, the lifted state-control trajectory $\xi(t) = (x_\xi(t), u_\xi(t))$, $t \geq 0$, for the SPM model will be written in expanded form as

$$\begin{aligned} x_\xi(t) &= (x_{r\xi}, y_{r\xi}, \varphi_\xi, \psi_\xi, v_\xi, \beta_\xi, \dot{\varphi}_\xi, \dot{\psi}_\xi)^T(t) \\ u_\xi(t) &= (F_\xi, \delta_\xi)^T(t). \end{aligned}$$

For vehicles, it is natural to use an *arc length* parametrization

$$s_\xi(t) = \int_0^t \sqrt{\dot{x}_{r\xi}^2(\tau) + \dot{y}_{r\xi}^2(\tau)} d\tau.$$

For admissible tasks, the desired velocity of the vehicle is bounded away from zero and the mapping $t \mapsto s_\xi(t)$ is invertible. Using the inverse $\bar{t}_\xi(s)$, we obtain the s parametrized

Fig. 3. Local coordinates around the path. The arclength s of the point having minimum distance from (x_r, y_r) and the (signed) distance w_1 of the point $P = (x_r, y_r)$ from the path can be used as coordinates to indicate the position of rear wheel contact point about the desired path $\bar{y}_\xi(\cdot)$.

curves for the state $\bar{x}_\xi(s) = x_\xi \bar{t}_\xi(s)$, input $\bar{u}_\xi(s) = u_\xi \bar{t}_\xi(s)$, and output (task) $\bar{y}_\xi(s) = y_\xi \bar{t}_\xi(s)$. Also, since we are using an arc length parametrization, the desired path velocity $\bar{v}_\xi(s)$ is equal to the vehicle velocity component $\bar{v}_\xi(s)$ of the desired state task $\bar{x}_\xi(s)$. Exploiting this apparent coincidence and with the aim of reducing the possibility of confusion that may arise from the use of v as the transverse form control input (from $u = \bar{u}(s) + v$), we will use ν , $\bar{v}_\xi(s)$, etc., to refer to vehicle velocity objects below.

In a neighborhood of the desired output maneuver $\bar{y}_\xi(\cdot)$, we can parametrize points using

$$\begin{bmatrix} x_r \\ y_r \end{bmatrix} = \begin{bmatrix} \bar{x}_{r\xi}(s) \\ \bar{y}_{r\xi}(s) \end{bmatrix} + \begin{bmatrix} -\sin \bar{\chi}_\xi(s) \\ \cos \bar{\chi}_\xi(s) \end{bmatrix} w_1 = \Omega(s, w_1) \quad (17)$$

as depicted in Fig. 3.

Here, as in the previous section, we have taken the (transverse) displacement to be orthogonal to the (unit) tangent vector $(\bar{x}'_\xi(s), \bar{y}'_\xi(s)) = (\cos \bar{\chi}_\xi(s), \sin \bar{\chi}_\xi(s))$ with course heading $\bar{\chi}_\xi(s) = \bar{\psi}_\xi(s) + \bar{\beta}_\xi(s)$. The inverse $(s, w_1) = \Phi(x_r, y_r)$ of this map is locally well defined around points (s, w_1) satisfying $w_1 \bar{\sigma}_\xi(s) < 1$ where $\bar{\sigma}_\xi(s) = \bar{\chi}'_\xi(s)$ is the *curvature* of the path at $(\bar{x}_{r\xi}(s), \bar{y}_{r\xi}(s))$. Note that w_1 is the signed distance from (x_r, y_r) to the (locally) *nearest* point (which occurs at location s along the maneuver) and represents the lateral displacement of the rear wheel contact point from the desired ground path.

Using (6)–(8) with $\tau(x) = (\varphi, \psi, v, \beta, \dot{\varphi}, \dot{\psi})$ to define the mapping $(s, w) = \Psi(x)$, we see that the remaining components of the transverse coordinates are given by

$$\begin{aligned} w_2 &= \varphi - \bar{\varphi}_\xi(s), & w_5 &= \beta - \bar{\beta}_\xi(s) \\ w_3 &= \psi - \bar{\psi}_\xi(s), & w_6 &= \dot{\varphi} - \dot{\bar{\varphi}}_\xi(s) \\ w_4 &= \nu - \bar{v}_\xi(s), & w_7 &= \dot{\psi} - \dot{\bar{\psi}}_\xi(s) \end{aligned} \quad (18)$$

with $s = \pi(x)$. The transverse coordinates measure the difference between the current state and desired state along the desired path at position $s = \pi(x)$, recalling that $\pi(x) = \Phi_1(h(x))$ is the first component of the diffeomorphism $(s, w) = \Psi(x)$.

B. Rotational Invariance of the Transverse Form of the SPM Model

When driving a motorcycle, a rider has little or no concern for the orientation (with respect to north) of the track—his/her

actions are primarily driven by the local curvature of the track. Here, we show that the transverse form of the SPM model dynamics is similarly *rotationally invariant* in the sense that a re-orientation (by planar rotation) of the task $\bar{y}_\xi(\cdot)$ results in *no* change in the transverse form.

To get expressions for $\dot{s}$ and $\dot{w}_1$, we work implicitly and differentiate (17) with respect to time to get

$$\begin{bmatrix} \cos \chi & -\sin \chi \\ \sin \chi & \cos \chi \end{bmatrix} \begin{bmatrix} \nu \\ 0 \end{bmatrix} = \begin{bmatrix} \cos \bar{\chi}_\xi(s) & -\sin \bar{\chi}_\xi(s) \\ \sin \bar{\chi}_\xi(s) & \cos \bar{\chi}_\xi(s) \end{bmatrix} \times \begin{bmatrix} (1 - w_1 \bar{\sigma}_\xi(s)) \dot{s} \\ \dot{w}_1 \end{bmatrix}$$

which, recognizing the planar rotation matrices, leads to

$$\begin{aligned} \dot{s} &= \nu \cos(\chi - \bar{\chi}_\xi(s)) / (1 - w_1 \bar{\sigma}_\xi(s)) \\ \dot{w}_1 &= \nu \sin(\chi - \bar{\chi}_\xi(s)) \end{aligned}$$

and finally, substituting for χ and ν , to

$$\dot{s} = (\bar{\nu}_\xi(s) + w_4) \cos(w_3 + w_5) / (1 - w_1 \bar{\sigma}_\xi(s)) \quad (19)$$

$$\dot{w}_1 = (\bar{\nu}_\xi(s) + w_4) \sin(w_3 + w_5). \quad (20)$$

Here, we see that the rate of change of the local kinematic variables (s, w_1) is independent of the task course heading $\bar{\chi}_\xi(s)$ (and hence of the task heading $\bar{\psi}_\xi(s)$); in contrast, the rate of change of the (globally expressed) kinematic variables (x_r, y_r) depends explicitly on the course heading χ . In place of the course heading χ , the (s, w_1) kinematics makes use of what might be called the *differential* course heading $w_3 + w_5 (= \chi - \bar{\chi}_\xi(s))$, a quantity that is independent of planar rotations of the desired task.

The dynamics of the rest of the transverse variables is also independent of the task heading $\bar{\psi}_\xi(s)$ (and hence invariant under planar rotation). To see this (and in preparation for linearization), let Π_w be the projection taking w to $(w_2, \dots, w_7)^T$, Π_x be the projection taking x to $(x_3, \dots, x_8)^T$, and Λ_w be the mapping taking w to $(0, 0, w_2, \dots, w_7)^T$, so that (18) can be written as

$$\Pi_w w = \Pi_x (x - \bar{x}_\xi(s)) \quad (21)$$

which further implies that $\Pi_x x = \Pi_x (\bar{x}_\xi(s) + \Lambda_w w)$. Differentiating (21) with respect to time and using the fact that $f(x, u)$ is independent of the first two components of x , we find that

$$\begin{aligned} \Pi_w \dot{w} &= \Pi_x f(\bar{x}_\xi(s) + \Lambda_w w, \bar{u}_\xi(s) + v) - \Pi_x \bar{x}'_\xi(s) \dot{s} \\ &= \Pi_x f(\bar{x}_\xi(s) + \Lambda_w w, \bar{u}_\xi(s) + v) \\ &\quad - \Pi_x \bar{x}'_\xi(s) (\bar{\nu}_\xi(s) + w_4) \cos(w_3 + w_5) / (1 - w_1 \bar{\sigma}_\xi(s)). \end{aligned} \quad (22)$$

Furthermore, since $\Pi_x f(x, u)$ is independent of the fourth component of x , namely ψ , we see that (all of) $\dot{w}$ is independent of $\bar{\psi}_\xi(s)$ which implies that the transverse form of the SPM is rotational invariant.

C. Transverse Linearization of the SPM Model

The computation of the $A(s)$ and $B(s)$ matrices in the transverse form (10) of the SPM model is now straightforward. Differentiating the right-hand side of (20) with respect to w , we find that the first row of $A(s)$ is given by

$$[0 \quad 0 \quad \bar{\nu}_\xi(s) \quad 0 \quad \bar{\nu}_\xi(s) \quad 0 \quad 0].$$

Differentiating the right-hand side of (22) with respect to w , we see that the remaining rows of $A(s)$ satisfy

$$\begin{aligned} \Pi_w A(s) w &= \Pi_x f_x(\bar{x}_\xi(s), \bar{u}_\xi(s)) \Lambda_w w \\ &\quad - \Pi_x \bar{x}'_\xi(s) (\bar{\sigma}_\xi(s) \bar{\nu}_\xi(s) w_1 + w_4). \end{aligned}$$

In the same way, we see that the last six rows of $B(s)$ are determined by

$$\Pi_w B(s) = \Pi_x f_u(\bar{x}_\xi(s), \bar{u}_\xi(s))$$

while the first row of $B(s)$ is zero since the input v does not appear in (20). Finally, from (13), we note that the transverse linearization is given by

$$A_T(s) = A(s) / \bar{\nu}_\xi(s) \quad B_T(s) = B(s) / \bar{\nu}_\xi(s)$$

where $\bar{\nu}_\xi(s)$ is the desired velocity along the curve. Once again, we emphasize that $A(s)$ and $B(s)$ (and $A_T(s)$ and $B_T(s)$) are independent of $\bar{x}_{r\xi}(s)$, $\bar{y}_{r\xi}(s)$, and $\bar{\psi}_\xi(s)$.

Recall that each SPM trim trajectory corresponds to a task $\bar{y}_\xi(\cdot)$ with constant velocity $\bar{\nu}_\xi(s) \equiv \nu$ and constant curvature $\bar{\sigma}_\xi(s) \equiv \sigma$. For such tasks, it is easy to see that $\bar{u}_\xi(s)$ and $\Pi_x \bar{x}'_\xi(s)$ are constant and that, with the exception of the ψ component, $\Pi_x \bar{x}_\xi(s)$ is also constant. Since, as noted above, $\Pi_x f(x, u)$ is independent of ψ , it follows that the linearization matrices are constant and parametrized by the trim task curvature and velocity

$$A(s) \equiv A^{(\sigma, \nu)} \text{ and } B(s) \equiv B^{(\sigma, \nu)}$$

and similarly for $A_T^{(\sigma, \nu)}$ and $B_T^{(\sigma, \nu)}$.

Within the *regular* portion of the equilibrium manifold [18], the transverse linearization $(A_T^{(\sigma, \nu)}, B_T^{(\sigma, \nu)})$ is controllable. The regular region includes the set of trim conditions of interest. It is interesting to note that controllability of the system is actually what ensures the (local) regularity of the equilibrium manifold.

D. Maneuver Regulation for the SPM Model

As discussed in Section II, a maneuver regulation control law can be obtained if a linear feedback $\bar{v}(s) = -K(s)\bar{w}(s)$ that exponentially stabilizes the transverse linearization (13) can be found. Generally speaking, stability concepts are not applicable on finite intervals, as is the case here where the path to be followed has length $L < \infty$. However, if we extend the path using constant curvature and the task using constant velocity to one of infinite extent, then a stabilizing controller can be designed. In practice, it is sufficient that the constant-velocity, constant-curvature tail of the task be long enough that the components (neglecting (x_r, y_r) and ψ) of the lifted state-control task

$(\bar{x}_\xi(\cdot), \bar{u}_\xi(\cdot))$ exhibit near steady state values as s approaches L . In that case, $A_T(L) \approx A_T^{(\sigma, \nu)}$ and $B_T(L) \approx B_T^{(\sigma, \nu)}$, where $\sigma = \bar{\sigma}_\xi(L)$ and $\nu = \bar{\nu}_\xi(L)$ are the constant terminal curvature and velocity.

For such a task, we can obtain the desired stabilizing $K(\cdot)$ by solving the finite horizon, linear quadratic optimal control problem

$$\begin{aligned} & \text{minimize } \frac{1}{2} \int_{s_0}^L \bar{w}(s)^T Q \bar{w}(s) + \bar{v}(s)^T R \bar{v}(s) ds \\ & \quad + \frac{1}{2} \bar{w}(L)^T P_L \bar{w}(L) \\ & \text{subject to } \bar{w}' = A_T(s) \bar{w} + B_T(s) \bar{v}, \quad \bar{w}(s_0) = w_0 \end{aligned} \quad (23)$$

for (the limiting case) $s_0 = 0$. Here $Q = Q^T$ and $R = R^T$ are arbitrary positive definite matrices (and possibly time-varying) and P_L is the positive definite algebraic Riccati equation solution associated with the time-invariant LQR problem for $A_T^{(\sigma, \nu)}$, $B_T^{(\sigma, \nu)}$, Q , R . The feedback solution to (23) provides the (space-varying) gain matrix

$$K(s) = R^{-1} B_T^T(s) P(s) \quad (24)$$

where $P(s)$, $s \in [0, L]$, is the solution of the differential Riccati equation (DRE)

$$\begin{aligned} -P' &= A_T^T(s)P + P A_T(s) - P B_T(s) R^{-1} B_T^T(s) P + Q \\ P(L) &= P_L. \end{aligned} \quad (25)$$

The minimum value (or cost to go) [28] of (23) is $V(s_0, w_0) = w_0^T P(s_0) w_0 / 2$. Since the cost in (23) is strongly positive definite on the linear space of homogenous trajectories of the linear dynamics, it is clear that the Riccati solution $P(s)$ is positive definite for each $s \in [0, L]$. Extending $P(\cdot)$ to the infinite horizon using $P(s) = P_L$, $s > L$, we obtain a $K(\cdot)$ that exponentially stabilizes (the extended version of) the transverse linearization (13) as well as the maneuvering task $w \equiv 0$ for the nonlinear system in transverse form (10).

We emphasize again that the exponential stability property is obtained for every bounded, uniformly positive definite choice of $Q(\cdot)$ and $R(\cdot)$. In this fashion, one obtains a feedback controller where the gain is *scheduled* (according to the longitudinal state of the system) and *stability is guaranteed*. Note, however, that the *quality* of regulation for the obtained $K(\cdot)$ is strongly affected by the choice of Q and R .

E. Shaping the Closed-Loop Response

Our task then is to choose the linear quadratic regulator weights (or weighting functions) Q and R in such a manner that the (space varying) closed-loop system

$$\bar{w}' = [A_T(s) - B_T(s)K(s)]\bar{w} \quad (26)$$

has a satisfactory dynamic response (for a desired class of admissible tasks). For instance, it is essential that the response from reasonable initial conditions $\bar{w}(s_0) = w_0$ converges to zero in a manner that is neither too fast nor too slow and that does not result in too large a response in, e.g., the roll φ . Many of these (and other) considerations are driven by the fact that

the system is a mechanical system that cannot be made to move rapidly without exerting forces that are larger than what can be produced by tire-road interactions. The roll (lean) angle is limited by similar considerations. Finally, since we are interested in controlling a more complex multi-body motorcycle model, the feedback $\bar{v} = -K(s)\bar{w}$ should provide a *regulator* that is somewhat robust to modeling errors including parameter mismatch and unmodeled dynamics such as the intentionally neglected suspension.

Our approach is to find, if possible, a (single) positive definite pair Q, R that results in a satisfactory (linearized LQR) dynamic response over a desired class of constant operating conditions (trim trajectories). That is, for each constant (σ, ν) in the desired set, the dynamic response of the linear system

$$\bar{w}' = \left[A_T^{(\sigma, \nu)} - B_T^{(\sigma, \nu)} K_{(Q, R)}^{(\sigma, \nu)} \right] \bar{w} \quad (27)$$

satisfies the chosen performance measures, where $K_{(Q, R)}^{(\sigma, \nu)}$ is the constant LQR gain associated with $A_T^{(\sigma, \nu)}$, $B_T^{(\sigma, \nu)}$, Q , R . This Q, R pair is then used to construct $K(s)$ for the chosen task using (24), (25) above. Although we do not have a formal justification, our experience seems to indicate that the s -varying system (26) inherits many of the performance characteristics of the family of constant systems (27).

The design of a suitable Q, R pair is approached using an *inverse optimal control* approach. Roughly speaking, in the inverse optimal control problem, one is given A, B , and K and asks whether there is an appropriate Q, R pair such the K is the LQR gain associated with A, B, Q, R . For our purposes, it will be sufficient to note that K is an optimal gain for the controllable pair (A, B) if and only if there exist $P = P^T > 0$ and $R = R^T > 0$ such that

$$RK = B^T P \quad Q = -(A^T P + P A - K^T R K) > 0. \quad (28)$$

This set of linear matrix inequalities (LMIs) and equalities (LMEs) is equivalent to requiring that there is a positive definite solution $P = P^T$ to algebraic Riccati equation (ARE)

$$A^T P + P A - P B R^{-1} B^T P + Q = 0 \quad (29)$$

where $Q = Q^T > 0$ and $R = R^T > 0$. For more information on the inverse optimal control problem, introduced (and solved in the scalar case) by Kalman [29], see [28] and [30] and for the LMI formulation of the most general case, see [31]. For controlling *nonlinear* systems, we prefer to *fully* penalize state deviations by using a *positive definite* Q .

Noting that the conditions (28) are positive homogeneous (positive multiples of valid P, R are also valid), we formulate the search for P and R (and hence Q) using

$$\begin{aligned} & \text{minimize } \rho \\ & \text{subject to } I \leq P \leq \rho I, \quad RK = B^T P \\ & \quad I \leq -(A^T P + P A - K^T R K) \\ & \quad I \leq R = R^T, \quad P = P^T \end{aligned} \quad (30)$$

which is a convex optimization problem [32] over the symmetric cone of positive semi-definite P and R . This problem is basically a convex feasibility problem; minimization of ρ is

Fig. 4. Open- and closed-loop poles at design point. The motorcycle is trimmed at the velocity $v = 30$ m/s along a turn having curvature $\sigma = 1/180$ m $^{-1}$. Open-loop (no control) transverse poles are reported in part (a), while part (b) shows closed-loop poles. Note the unstable pole in (a) corresponding to the capsized, inverted pendulum like, mode.

used to encourage better conditioning of the solution. We make use of the *SeDuMi* package [33] and the *SeDuMi Interface* [34] in solving this problem, including the determination of infeasibility (K not inverse optimal for (A, B)). This inverse optimal control strategy has also been used in the development of a (on-line optimization based) receding horizon controller that was deployed in flying a full sized jet aircraft as a reliable wingman to another jet [35].

To make the design of K more manageable, we exploit the approximate decoupling between the control of the (longitudinal) vehicle velocity (w_4) by the thrust control (the deviation v_1 of the force control F_x) and the control of the lateral states (the rest of w) by the steering control (the deviation v_2 of the steering control δ_f). The (linearized, transverse and otherwise) lateral and longitudinal dynamics are (exactly) decoupled when running straight ($\sigma = 0$) but not otherwise; indeed, the throttle is an important control for the (human) rider when operating with a significant lean. However, working with a scalar longitudinal system is easy and working with a single input, seven state lateral system (adding the steering angle to the state and using the steering rate as input, see Section IV-A below) allows one to use the desired closed-loop poles to *parametrize* the desired closed-loop lateral dynamics and hence the (potentially inverse-optimal) lateral feedback K_{lat} .

We begin by choosing a design trim condition, e.g., $\nu = 30$ m/s (108 km/h) and $\sigma = 1/180$ m $^{-1}$ (corresponding to a lateral acceleration of approximately 0.5 g) and setting $A_T = A_T^{(\sigma, \nu)}$ and $B_T = B_T^{(\sigma, \nu)}$. Fig. 4 depicts the open loop system poles (together with the [to be designed] closed-loop poles) at the chosen design point. Note the presence of the unstable mode due to an inverted pendulum like dynamics within the SPM model. The SPM model does not admit the kind of self stabilization regions found in multi-body bicycle and motorcycle models with steering dynamics.

The augmented (with a steering angle state) lateral system matrix A_{lat} is obtained by adjoining the second column of B_T

to A_T , adding a row of zeros at the bottom, and then removing the fourth row and column (for w_4 [the ν error]). The lateral control matrix B_{lat} is simply a column vector of six zeros followed by a single one (corresponding to the $\dot{v}_2$ [and δ_f] input). The resulting *lateral dynamics* system $(A_{\text{lat}}, B_{\text{lat}})$ is controllable (for each (σ, ν) in the regular region). For each choice of closed loop poles, the corresponding (uniquely defined) state feedback K_{lat} can be determined and the resulting closed loop system evaluated according to desired performance measures. Of course, not every stabilizing K_{lat} will be inverse-optimal but, with experience, one finds that the addition of inverse-optimality to the control specifications does not significantly increase the difficulty of the feedback control design problem. Once an inverse-optimal K_{lat} has been chosen, the 7×7 weighting matrix Q_{lat} and the scalar weight R_{lat} are then determined by solving (30) using A_{lat} , B_{lat} , and K_{lat} . We have found that the resulting Q_{lat} matrices are *far* from being diagonal. (Our experience suggests that a diagonal Q results in a dynamic response that is both too slow and too fast.)

To build up the overall Q and R matrices, we combine the lateral Q_{lat} and R_{lat} together with scalar weights Q_{long} and R_{long} (to be determined) in a decoupled manner. That is, R is 2×2 with R_{long} and R_{lat} on the diagonal and Q is the 8×8 matrix obtained by placing a row and column of zeros after the third row and column and placing Q_{long} at the (4,4) location (so that Q is a rearranged block diagonal matrix). Note that, for a scalar (single state, single input) linear system, the LQR optimal gain K (and hence the time constant) depends only on the ratio of the scalar weights Q and R and not on their absolute size. The ratio of Q_{long} to R_{long} is thus chosen to set the speed (in spatial terms) of $A_{\text{long}} - B_{\text{long}} K_{\text{long}}$ where, in the decoupled *longitudinal dynamics*, A_{long} is the (4,4) element of A_T and B_{long} is the (4,1) element of B_T . (One may want to take A_{long} and B_{long} from a straight running [$\sigma = 0$] condition with the same velocity as the design condition.) The ratio of Q_{long} to Q_{lat} is then used to tune (iteratively) the amount of v_1 that is

Fig. 5. Open- and closed-loop poles using the same Q and R . Open-loop poles are reported in (a), while closed loop in (b). The velocity is held constant at $v = 30$ m/s while the lateral acceleration is varied from zero to 1.2 g.

used (relative to $\dot{v}_2$) in regulating the lateral dynamics when operating in a leaned condition.

Finally, the quality of the designed Q and R matrices is judged by evaluating the performance of the resulting $K_{(Q,R)}^{(\sigma,\nu)}$ regulators over the desired set of (σ, ν) operating conditions. The procedure is iterated as needed.

It is a pleasant surprise to find that this approach has been successful in designing Q and R matrices that are suitable for a wide range of velocities and lateral accelerations (hence curvatures). For example, in Fig. 5, we report the locus of open- and closed-loop poles as the lateral acceleration is varied from zero to 1.2 g while holding the velocity constant at $v = 30$ m/s. For the entire range of lateral accelerations, the closed-loop eigenvalues remain inside a rather small region of the complex plane, with sufficient damping. A similar root locus plot with varying velocity shows a similar dynamic response grouping. We remark that this design heuristic has been used thus far in the development of maneuver regulation controllers for three racing motorcycles and a couple of sport bikes.

IV. CONTROLLING A MULTI-BODY MOTORCYCLE MODEL

In the previous sections, we have detailed the design of a maneuver regulation controller for the SPM model. This section shows how this controller can be used to control a multi-body model of a fully articulated motorcycle.

To this end, we have used the commercial multi-body code ADAMS/Car together with the VI-motorcycle add-on devoted to motorcycle dynamics simulation (see Fig. 6).

A modern sport motorcycle is composed of a center subsystem (including frame, engine, gear box, and fuel tank), a rear subsystem (including swing arm and brake calipers), an upper steering subsystem (including handlebars and upper fork), a bottom steering subsystem (including lower fork and brake calipers), and front and rear wheel subsystems (including tires, rims, and brake disks). Tire ground interaction is modeled using Pacejka magic formula tire model (PAC-MC 1.1). The

Fig. 6. (Standard) VI-Motorcycle motorcycle model. The motorcycle is described using different assemblies stored in a database that can be connected to form a complete vehicle, ready for dynamic simulations.

tire model [36] includes relaxation length, providing phase lag between kinematic slip and tire force (for both longitudinal slip and sideslip angle).

The aerodynamics is modeled as a drag force parallel to the ground acting at the center of pressure in the opposite direction of vehicle's motion. The motorcycle model includes a telescopic fork suspension at the front and a monoshock (1 linkage) suspension at the rear. The engine is modeled as a massive disk rotating about the crankshaft axis, which is orthogonal to the vehicle plane of symmetry. Depending on its spinning rate (RPM) and the opening of the (virtual) throttle valve, the engine provides a torque that is counteracted by the frame. A motorcycle gearbox is modeled and connected to the engine. The gearbox includes primary drive gear ratio, first to sixth gear ratios, and secondary drive gear ratio, this last specified by drive and wheel sprocket radii. The secondary drive is composed of a massless chain, modeled as two nonlinear "springs" attached at bottom and top of drive and wheel sprockets. The rider is modeled as a rigid body that is firmly attached to the center assembly: no rider motion is used in this work.

Fig. 7. Front and rear normalized lateral forces. Part (a) shows the front lateral force F_y versus sideslip α for different values of the camber angle γ and for zero longitudinal slip. The force F_y is normalized for a nominal load of $F_z = 1500$ N. Part (b) shows the same information for the rear tire.

Although a complete description of the multi-body motorcycle is beyond the scope of this paper, we provide a representative sample of important parameters. The total mass of the vehicle including the rider is $m = 281.53$ kg. The height of the center of mass is 0.680 m and its longitudinal displacement relative to the rear wheel contact point is 0.69 m. At 1 g, the vehicle wheelbase is 1.340 m, the caster angle is 22 deg, the geometric trail is 0.122 m, and the steering offset is 0.045 m. The drag coefficient is $C_d = 0.32$ with reference area $A = 0.6$ m², the air density is $\rho = 1.225$ kg/m³, and the height of the center of pressure is 0.53 m. From these parameters, one can obtain a rough estimate of the lift limits for the front or rear tire. As an example, at 145 km/h (40 m/s), one obtains that the lift limit for the rear wheel is about 1 g. From the (PAC MC 1.1) tire model used for multi-body simulations, we obtain, at a nominal normal load of 1500 N, cornering and camber stiffness coefficients of $C_{\alpha_f} = 10.90$ rad⁻¹ and $C_{\gamma_f} = -0.3892$ rad⁻¹ for the front tire and $C_{\alpha_r} = 16.22$ rad⁻¹ and $C_{\gamma_r} = -0.9134$ rad⁻¹ for the rear tire (with sign convention according to the SPM model described in [18]). Fig. 7 depicts the lateral forces of the front and rear tires normalized for a nominal normal load of 1500 N. The lateral relaxation length for the two tires varies in the interval 0.16–0.22 for normal tire forces in the range 800–2000 N. Tire adherence limits for front and rear tires are approximately 1.3 longitudinal and 1.2 lateral. Fig. 8 shows, for a camber angle of 50 degrees and a normal load of 1500 N, the envelope of the normalized longitudinal and lateral forces obtained by varying the longitudinal slip ratio and lateral sideslip angle within the interval $[-1, 1]$ and $[-15, 15]$ degrees, respectively.

A. Interfacing the Controller With the Plant

Because the states and inputs of the sliding plane motorcycle (the *control design model*) are different from those of the multi-body motorcycle (the *plant*), one needs to develop a suitable interface between the controller and the plant in order for the former to act on the latter as if it were the control design model.

In the SPM model developed in [18] and described in the first part of this paper, the effective steering angle is taken to be one of the control inputs. In order to obtain a smoother steering control action, we have augmented the dynamics of the SPM model to use the effective steering angle *rate* as the control input, making the effective steering angle δ_f part of the state vector. We remark that the steps for obtaining the maneuver regulation controller and the dynamics inversion procedure remain unchanged. We delayed introducing this *detail* in order to allow a simpler presentation of the main ideas.

The input to the controller is obtained by taking measurements of the multi-body plant: the roll angle and rate, yaw angle and rate, and rear wheel contact point position, and velocity from the multi-body vehicle are passed to the controller without further processing. The rear wheel contact point for the multi-body vehicle is the center of the contact patch that the (torus shaped) rear tire makes with the ground.

The maneuver regulation controller produces, as output, the thrust force and the effective steering angle rate. These are transformed into the plant inputs, steering angle rate (at the handle bar) and throttle and brake commands, as follows. To provide a steering angle rate command, the effective steering angle and rate (output) from the controller are transformed using the approximation $\cos \varphi \tan \delta_f = \tan \delta$, discussed in the Appendix of [18], which is an approximation of the exact motorcycle kinematics [37].

The maximum traction force available depends primarily on the current gear ratio and the engine RPM. During dynamic simulation, the thrust force demand F from the maneuver regulation controller is either positive or negative. If the required amount can be generated by the engine, then the throttle valve is opened accordingly up to the wide open position, at which point it saturates. Saturations may also occur in the other direction (as the throttle command is rolled off) when the desired (negative) braking force F cannot be fully obtained using engine braking torque. In that condition, brakes are used to provide the additional required braking force. The map from the

Fig. 8. Rear tire force envelope for a fixed roll angle. The figure shows the envelope of the rear tire longitudinal and lateral forces, normalized with respect to the normal load, for a camber angle of 50 degrees. The longitudinal slip ratio and the sideslip angle vary within the intervals $[-1, 1]$ and $[-15, 15]$ degrees, respectively. The normal load is 1500 N.

controller output F and the throttle and brake demands is based on the simple longitudinal model

$$m_{eq}\dot{v} = F - F_A$$

where m_{eq} is the equivalent mass (sum of the vehicle mass and inertial contributions of rotating parts), F_A is the aerodynamics drag, and F is the thrust force computed as

$$F = \frac{\rho_{e,rw}\tau_e - \tau_{rb}}{\tau_{rw}} - \frac{\tau_{fb}}{\tau_{fw}} \quad (31)$$

where $\rho_{e,rw}$ is the current gear ratio relating engine RPM to rear wheel angular velocity, and τ_e , τ_{rb} , and τ_{fb} are, respectively, the engine torque, rear, and front wheel torques. For simplicity, the braking torque is always applied on the front wheel (i.e., the rear wheel braking torque τ_{rb} is zero), even if a different solution, for instance, constant or configuration-dependent bias between front and rear wheels, could be devised. Roughly speaking, (31) is inverted computing τ_e and τ_{fb} during the closed loop simulation based on the commanded value of F and the RPM of the multi-body vehicle, the selected gear, and the wheel radii.

V. NUMERICAL EXAMPLE

This section illustrates the numerical results obtained when using the maneuver regulation controller developed in Section III to control the multi-body motorcycle model described in Section IV.

We use the desired path and velocity profile from [18], together with the corresponding *lifted* state-control trajectory for the SPM model. The desired ground path is reported in Fig. 9(a)

while desired velocity profile (as a function of arc length) is shown in Fig. 10(a). Making use of the lifted trajectory, we have computed the s -varying (transverse) state feedback controller according to the procedure developed in Section III-C, integrating (backwards in s) the corresponding s -varying Riccati equation with constant weights Q and R .

Fig. 10(a) shows the desired velocity profile and the closed loop simulation velocity profile. The speed varies between 58 km/h (16 m/s) up to 140 km/h (40 m/s) and the maximum tracking error is less than 0.5 m/s.

Desired and closed-loop roll angles are shown in the same figure, in part (b), the multi-body roll angle is higher at large camber due, in particular, to the toroidal profile of the tires. Part (c) shows the desired and closed-loop lateral accelerations. A g-g diagram of the desired maneuver is presented Fig. 9(b). The diagram shows that the maximum desired longitudinal acceleration is about 0.4 g, maximum deceleration is -0.6 g, and lateral acceleration is 1 g. Fig. 11 shows that the maximum lateral error of closed-loop simulation is about 0.6 m. Maxima are reached in proximity to the apex of each turn where lateral acceleration is 1 g.

In Fig. 12(a), the feedforward thrust signal is compared to the sum of longitudinal rear and front tire forces. As explained in Section IV, the closed-loop thrust signal is divided in two separate control inputs according to, mainly, its sign. Two spikes can be noticed at 100 and 1080 m which correspond to the effect of upshifts. Also, a downshift occurs around 780 m, with less evident effects on the force signals.

It is of interest to compare the desired front and rear wheel sideslip angles of the SPM model (linear tire model) with those of the closed loop multi-body model (with PAC-MC tire model),

Fig. 9. Desired ground path and g-g diagram of the desired maneuver. Part (a) shows the flatland path (x versus y) chosen for the numerical experiment. The path is traversed clockwise; Part (b) shows the envelope of the desired longitudinal and lateral accelerations of the rear wheel contact point.

Fig. 10. Speed, roll, and lateral acceleration. Part (a) shows the desired velocity profile (solid green line) and closed-loop simulation velocity (dashed blue). Part (b) shows the SPM model roll angle (solid green line) and multi-body model (dashed blue line) obtained in closed-loop simulation. Part (c) shows the desired lateral acceleration (solid green) and closed-loop simulation lateral acceleration (dashed blue line) reaches 1 g.

Fig. 11. Lateral error. Relative to the desired trajectory (green line), the maximum lateral error of closed-loop simulation (dashed blue line) is about 0.6 m. Maxima appear in proximity to the apex of each turn, where lateral acceleration is 1 g.

as done in Figs. 12(b) and (c). The SPM sideslip angles (solid green) are computed via the formulas

$$\alpha_f := \arctan((v_y + p\omega)/v_x) - \delta_f, \quad \alpha_r := \arctan(v_y/v_x)$$

where ω is the yaw rate, δ_f is the effective steering angle, v_x and v_y are the longitudinal and lateral velocities at rear contact point, and p is the vehicle wheelbase. The signals have a good agreement in magnitude despite the huge differences in the steering kinematics, suggesting that the SPM model is capable of capturing the main aspects of the complex motorcycle model (due to, we believe, the use of cornering and camber stiffness values from the full model).

Finally, Fig. 13 shows the closed-loop steering torque applied at the handlebar of the multi-body model by the virtual rider. As the maximum torque applied to the vehicle is approximately 40 Nm, with relatively low frequency content, we conclude that the control action exerted by the virtual rider is quite reasonable and likely to be feasible for a human driver.

Fig. 12. Thrust and sideslip angles. Part (1a) shows desired thrust force for the SPM model (green line) and closed-loop multi-body motorcycle thrust force (dashed blue line). As shown in Part (2a), the closed-loop thrust force is the sum of rear tire (dashed-dotted black line) and front tire (dashed red line) longitudinal forces. Parts (b) shows the SPM rear-tire sideslip angles (solid green line) and closed loop rear-tire sideslip (dashed blue line) obtained during closed-loop simulation. An equivalent plot, but for the front tire, is shown in Part (c).

Fig. 13. Steering torque. The figure shows the closed-loop steering torque applied by the virtual rider to the handlebar of the multi-body motorcycle model.

VI. CONCLUSION

We have proposed a virtual rider system for driving a multi-body motorcycle model along a specified path with a desired velocity profile. The proposed control strategy has been found to perform well in the presence of unmodeled dynamics such as that due to the suspension system, gear shifting, and even short lived wheelies.

This control architecture has been chosen for several reasons. First, the virtual rider had to be interfaced with a commercial multi-body software and needed to be applicable to different types of two-wheeled vehicles (e.g., scooters or motorcycles) with quite different suspension and drive train systems, for which standard parametric models are not available. Second, we are skeptical of the notion that excellent tracking results can only be obtained with a virtual rider based on a full multi-body model. It is our opinion that a human rider bases his/her actions (and predictions) on a mental model (of different kinds, depending on her/his experience and skills) that neglects many model details (including, e.g., tire relaxation length, vehicle mass distribution, or tire profiles). Also, our previous experience in designing a virtual rider suggests that it is possible to

obtain quite remarkable tracking performance, for relative low speed (less than 30 m/s), using only a simple *nonholonomic* vehicle model combined with a simple look ahead strategy [11].

The proposed virtual rider provides quite satisfying tracking results for aggressive maneuvers (such as that experienced, e.g., in racing applications). A detailed discussion of such numerical experiments, however, goes beyond the scope of this work where the aim has been to expose the mathematical and geometric ideas underlying the virtual rider and to show that the feedback strategy has been successfully applied to a multi-body (code) model. We refer the reader to [23] for an accessible discussion on the generation of optimal velocity profiles. In that paper, the authors consider the problem of generating an optimal speed profile, taking into account the constraints imposed to the maximum acceleration and deceleration by the tires, engine, and the possible lifting of the front wheel (wheelie) and rear wheel (stoppie). We plan to address these issues in more depth in a future work which will include detailed simulation results.

As is usual for a reduced order model control design methodology, one might ask whether, and to what extent, stability and performance properties will persist when the maneuver regulation controller is used in a feedback configuration with the complex, multi-body model as opposed to when it is used on the control design model (the SPM). The exponential stability of the design ensures that there is a neighborhood of systems (with respect to parameter variations and unmodeled dynamics) for which exponential stability is retained. It is unlikely that the theoretical estimates provided by theory will be useful in any practical sense. Our approach is to design the maneuver regulation controller so that the convergence rate to the desired trajectory respects the time constants one would expect of a human rider driving a real vehicle. The inverse optimal control strategy that we have proposed helps us to achieve this result. (Further investigation is needed to better understand the capabilities and limitations of this approach.)

As we have seen, the resulting closed-loop control action and, in particular, the steering torque are quite reasonable. In our experience, this design philosophy is able to prevent the emergence of instability phenomena due to the presence of unmodeled dynamics (which are in fact excited if the closed-loop gains

are set unreasonably high) and to obtain quite remarkable performance even when, for instance, the front wheel pops up! We claim that these quite satisfying results can be ascribed to the choice of the good reference control model (the SPM model), careful construction of an SPM *trajectory* in the dynamic inversion step, and to the maneuver regulation control strategy which allows the computation of meaningful feedforward and feedback inputs which are based on the current location s along the path (rather than attempting to synchronize the current state with a desired time-indexed trajectory).

Finally, we are also confident that this control paradigm can be extended to include rider motion (thought of as an input) and paths with banking and elevation changes, which are of interest when simulating high performance motorcycle maneuvers for racing applications.

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Alessandro Saccon received the laurea degree *cum laude* in computer engineering and the Ph.D. degree in control system theory with a scholarship by M.Sc. Software Corporation from the University of Padova, Padova, Italy, in 2002 and 2006, respectively.

From 2006 to 2008, he has been a Research Fellow with the Department of Information Engineering, University of Padova, where he was working on control and optimization methods for the exploration of the dynamics of racing motorcycles for virtual prototyping studies. Currently, he is a postdoctoral

scholar with the Institute for Systems and Robotics, Istituto Superior Tecnico,

Lisbon, Portugal, where he is working on dynamic motion planning algorithms and identification methods for autonomous underwater vehicles (AUV). His research interests include optimal control theory, applied optimization, motorcycle dynamics and control, underwater vehicle dynamics and control, geometric mechanics, variational integrators, maneuver regulation of nonlinear systems, and receding horizon control.

John Hauser received the B.S. from the United States Air Force Academy, Colorado Springs, CO, in 1980 and the M.S. and Ph.D. degrees from the University of California at Berkeley, Berkeley, in 1986 and 1989, respectively.

From 1989 to 1992, he was the Fred O'Green Assistant Professor with the University of Southern California, Los Angeles. Since 1992, he has been with the Department of Electrical, Computer, and Energy Engineering, University of Colorado at Boulder. He has held visiting positions with Instituto Superior Tecnico, University of Stuttgart, University of Padova, Caltech, Lund Institute of Technology, and Ecole Supérieure d'Electricité. His research interests include nonlinear dynamics and control, optimization and optimal control, aggressive maneuvering for high performance motorcycles and aircraft, and dynamic visualization. Recent work has focused on the development of optimization (and optimal control) tools and techniques for trajectory exploration

with an eye toward characterizing the trajectory space of highly maneuverable nonlinear systems.

Prof. Hauser was a recipient of the Presidential Young Investigator Award (NSF) in 1991.

Alessandro Beghi received the laurea degree *cum laude* in electrical engineering and the Ph.D. degree in control systems engineering from the University of Padova, Padova, Italy, in 1989 and 1993, respectively.

In 1994, he joined the Department of Information Engineering, University of Padova, where he is currently an Associate Professor of control systems theory. He held visiting positions at universities and international research centers both in Europe and in the USA. He has worked on a wide range of control applications, from control of fusion devices, guidance algorithms for virtual vehicles, to control of HVAC&R systems, Adaptive Optics systems, and Semiconductor Manufacturing processes. He is the author of over 100 publications in journals, books, and conference proceedings.

Prof. Beghi is a member of the IEEE Control System Society, serving also as Associate Editor for the Conference Editorial Board and co-chair of the Technical Committee on Power Generation.