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Economic and environmental performance of buyer-supplier coordination

Yann BOUCHERY, Asma GHAFFARI, Zied JEMAÏ, Yves DALLERY
Economic and environmental performance of buyer-supplier coordination

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Abstract: This paper aims at studying the economic and environmental performance of buyer-supplier ordering policies according to different coordination relationships. Several models illustrating different outcomes of the buyer-supplier negotiation are proposed. Among them, a new model of a supplier leader supply chain is introduced and discussed. The impact of buyer-supplier coordination on the supply chain environmental performance is challenged in this paper. It is namely shown that the total supply chain carbon emissions may be greater when buyer and supplier ordering policies are fully coordinated. The setting of a carbon price may also lead to similar outcome.

Keywords: Inventory control, sustainable supply chain, carbon emissions, buyer-supplier coordination.
1 Introduction

Supply chains are generally composed of several independent entities aiming at optimizing their individual performance. In this situation, the companies should try to coordinate their operations in order to optimize the system performance instead of their individual one (Li and Wang, 2007). In practice, the buyer-supplier negotiation may lead to several outcomes depending on the respective bargaining power and willingness to collaborate of the different entities. This paper aims at exploring the economic and environmental performance of buyer-supplier supply chain according to different coordination relationships.

Sustainability concerns are increasingly shaping customers’ behavior as well as companies’ strategy since the Brundtland’s report publication (WCED, 1987). Sustainable supply chain management has thus received considerable attention in the last decade. We refer to Linton et al. (2007), Seuring and Müller (2008), Carter and Rogers (2008), Kleindorfer et al. (2009) and Dekker et al. (2012) for reviews. An emerging idea presented in this literature states that sustainability concerns may foster coordination. “In this context, ecological sustainability becomes one of the driving forces for a more cooperative business environment in terms of vertical cooperation between customers, suppliers and service providers, as well as horizontal cooperation between industrial companies” (DHL, 2010). This paper aims at analyzing this new trend in buyer-supplier relationships.

The literature dealing with buyer-supplier relationship and sustainability has rapidly grown. In this review, we restrict our attention to papers including sustainability concerns into single-buyer single-supplier models. To the best of our knowledge, the first paper analyzing buyer-supplier relationships by taking sustainability concerns into account is Corbett and DeCroix (2001). In this paper, the authors assess indirect material consumption in a single-buyer single-supplier supply chain. They prove that a well designed “shared-savings” contract can allow both parties to benefit from a consumption reduction. Vachon and Klassen (2008) examine the impact of environmental collaboration on manufacturing performance based on a survey of North American manufacturers. They highlight that green collaboration with suppliers generally leads to superior delivery and flexibility performance. On the other hand, they found that green collaboration with customers generally leads to better quality performance. Ni et al. (2010) include Corporate Social Responsibility (CSR) into a single-buyer single-supplier model. They study how CSR
should be allocated by using game-theoretical analysis on six different games. They prove that economic performance is not aligned with CSR performance and propose an optimal allocation scheme. Benjaafar et al. (2010) include carbon emission constraints on a multi-stage lot-sizing model with a cost minimization objective. The impact of collaboration is numerically studied under several carbon regulatory policies. Among others, they observe that the presence of carbon constraints may increase the value of supply chain collaboration. El Saadany et al. (2011) focus on a Joint Economic Lot Size (JELS) problem where the demand is assumed to be a function of product’s price and environmental quality. Analytical results and numerical examples are provided. In Ghosh and Shah (2012), the buyer-supplier relationship is analyzed by including green investment in a game-theoretical framework. They find that collaboration leads to higher greening level and higher retail price. Jaber et al. (2012) include carbon emissions into a JELS problem by considering different emissions trading schemes. Carbon emissions are assumed to be a function of the production rate. Their numerical study proves that coordination minimizes the total system cost without automatically reducing carbon emissions. Finally, Bouchery et al. (2012) include sustainability criteria into a single-buyer single-supplier inventory model by using multiobjective optimization. The efficient frontier is analytically characterized and an interactive procedure allowing the company to quickly identify the preferred option is proposed.

Combining the numerical observations of Benjaafar et al. (2010) and Jaber et al. (2012) may apparently provide contradictory results. The presence of carbon constraints may thus foster collaboration that would minimize the total system cost without automatically reducing carbon emissions. In this paper, the link between buyer-supplier coordination and carbon emissions is formally analyzed by focusing on simple inventory models. We prove that the total supply chain carbon emissions may be increased when companies coordinate their operations. We also prove that a higher carbon price can lead to higher total carbon emissions in non-coordinated situations. This paper thus demonstrates that even if sustainability seems to be an incentive to increase collaborative behaviors, collaboration may have a negative impact on sustainability.

The paper is organized as follows. Several models illustrating different outcomes of the buyer-supplier negotiation are presented in section 2. First, the centralized case is analyzed. In a second model, the buyer is placed in the position of the supply chain leader. Finally, a new
model that enables the supplier to act as the supply chain leader is presented. In section 3, the models presented in section 2 are compared both in terms of cost and carbon emissions. Several insights are given. Finally, section 4 is devoted to the conclusion and to future research directions.

2 Different outcomes in buyer-supplier relationships

In this section, a brief overview of the buyer-supplier literature is given before stating the assumptions of the present paper. Then, different outcomes of the buyer-supplier negotiation are illustrated by three models.

2.1 Literature review

The operations management literature dealing with buyer-supplier relationships is very vast. In this paper, we restrict our attention to single-buyer single-supplier situations with deterministic constant demand. Schwarz (1973) derives the centralized problem optimal solution. In this case, a single decision maker controls the entire supply chain. This centralized solution may be seen as a benchmark for independent firms aiming at coordinating their operations. The first paper dealing with this problem known as the Joint Economic Lot Size (JELS) problem is Goyal (1977). Several papers refine Goyal’s model by taking more realistic assumptions into account. The optimal solution for a general shipment policy with finite production rate and lot streaming is derived in Hill (1999). We refer to Goyal and Gupta (1989) for a review on early works on the JELS problem and to Ben-Daya et al. (2008) for a review on recent extensions of the JELS problem.

Even if coordination’s benefits are extensively recognized, non-coordinated supply chains are still very common in practice. In non-coordinated supply chains, an entity often acts independently so as to minimize its individual cost. In what follows, this entity is called the supply chain leader. Goyal (1977) presents a model where the buyer is the supply chain leader. Contrarily, in Lu (1995), the supplier seeks to minimize his total cost subject to the maximum cost the buyer is willing to incur. In these two situations, a side-payment contract can be designed so as to entice the leader to modify his behavior to achieve coordination. We refer to Cachon (2003), Sarmah et al. (2006) and Leng and Zhu (2009) for reviews on
coordination under side-payment contracts. Note that game theory is often used in such situations to find an equilibrium solution.

### 2.2 Assumptions and preliminary results

In this paper, the considered supply chain is composed of a single supplier (vendor) delivering a single product to a single buyer (customer). Figure 1 describes the supply chain under consideration.

![The supply chain structure](image)

The supplier produces the item with an infinite production rate. The product is then sent in batch to the buyer who faces a constant continuous demand. We assume that the entire batch is delivered to the buyer at the same time. Leadtimes are assumed to be zero for clarity (fixed leadtimes can be easily handled) and no shortage is allowed. Moreover, initial inventories are assumed to be zero. Fixed ordering costs and linear holding costs are supported by both the supplier and the buyer. Finally, we consider an infinite time horizon.

Let $Q_B$ be the batch quantity ordered by the buyer and $Q_S$ be the production lot size at the supplier. The following preliminary results were first derived by Schwarz (1973).

**Preliminary Results.** An optimal policy is stationary-nested and respects the zero-inventory condition i.e.:

- $Q_B$ and $Q_S$ are time invariant,
- $Q_S = k Q_B$, with $k \in \mathbb{R}^+$.

The buyer orders only if its inventory level is null,

The supplier orders when both the buyer and the supplier have no inventory.
In the following notations, B and S represent the buyer and the supplier respectively. C is used to identify the cost parameters (in opposition to E that identifies carbon emissions parameters):

\[ Q_B = \text{ordering quantity at the buyer (first decision variable)}, \]
\[ Q_S = \text{production lot size at the supplier}, \]
\[ k = \text{strictly positive integer such that } Q_S = kQ_B \text{ (second decision variable)}, \]
\[ D = \text{demand per time unit at the buyer}, \]
\[ h_{CB} = \text{constant inventory holding cost per product unit and time unit at the buyer}, \]
\[ h_{CS} = \text{constant inventory holding cost per product unit and time unit at the supplier}, \]
\[ O_{CB} = \text{fixed ordering cost at the buyer}, \]
\[ O_{CS} = \text{fixed production cost at the supplier}. \]

The purchasing cost at the buyer is assumed to be linear in the ordered quantity. It does not affect the purchasing decision and is indeed omitted in what follows. \( O_{CB} \) accounts for the fixed part of the ordering cost at the buyer. A linear part may also be considered. However, it would not affect the purchasing decisions and is thus omitted. The same observation holds for the production costs at the supplier.

Even if sustainable development is a vast concept that embraces economic, environmental and social aspects, global warming problem seems to overwhelm other concerns. Carbon footprint is now extensively adopted as an indicator of environmental friendly supply chains activities. We thus focus on carbon emissions in this paper. Including carbon emissions into inventory models is a new challenge that triggers more and more research. In this paper, we propose to model carbon emissions as in Benjaafar et al. (2010) and Bouchery et al. (2012). Indeed, a fixed amount of carbon emissions is associated to each order at the buyer. It represents the emissions related to order processing and transportation. A fixed amount of carbon emissions is also associated to each production run at the supplier due to production set up and waste. A linear emissions parameter is associated to the storage of the items at both locations. The carbon emissions structure is thus similar to the cost structure. As for the cost, the linear parts of ordering and production emissions (if they exist) are not explicitly considered as they do not affect the purchasing decisions.
In the following notations, E identifies carbon emissions parameters:

\[ h_{EB} = \text{constant inventory holding emissions per product unit and time unit at the buyer}, \]
\[ h_{ES} = \text{constant inventory holding emissions per product unit and time unit at the supplier}, \]
\[ O_{EB} = \text{fixed ordering emissions at the buyer}, \]
\[ O_{ES} = \text{fixed production emissions at the supplier}. \]

This carbon emissions structure takes into account emissions from production, transportation and storage. As the production rate at the supplier is not considered as a decision variable, any linear part of production emissions, as considered in Jaber et al. (2012), does not affect the supply chain decisions.

### 2.3 The centralized model: Model (c)

In the centralized model, the buyer and the supplier coordinate their operations in order to improve the system performance. Buyer’s and supplier’s operations performance is then jointly optimized. The cooperation mechanism that enables the distribution of coordination’s benefits among both parts is not made explicit. We refer to this model as Model (c).

With the assumptions presented in section 2.2, the total supply chain cost \( Z_C \) can be expressed as a function of \( Q_B \) and \( k \):

\[
Z_C(k, Q_B) = (h_{CB} + (k - 1)h_{CS}) \frac{Q_B}{2} + (O_{CB} + \frac{O_{CS}}{k}) \frac{D}{Q_B}.
\]  

(1)

The total supply chain carbon emission function \( Z_E \) has the following expression:

\[
Z_E(k, Q_B) = (h_{EB} + (k - 1)h_{ES}) \frac{Q_B}{2} + (O_{EB} + \frac{O_{ES}}{k}) \frac{D}{Q_B}.
\]  

(2)

When coordinating their operations, the buyer and the supplier may aim at optimizing their economic and / or their environmental performance. In the present framework, optimizing the economic (respectively the environmental) performance of the system corresponds to minimizing the total cost function \( Z_C \) (respectively the total carbon emission function \( Z_E \)). Note that a multiobjective formulation of the model may also be found in
Bouchery et al. (2012). In the present paper, only single objective optimization is considered. The aim of the model is thus to minimize \( Z_i, i \in \{C; E\} \).

The optimal values of \( Q_B \) and \( k \) noted respectively \( Q_B^{\ast(c)} \) and \( k_i^{\ast(c)} \), can be calculated as follows:

If \( h_{ib} < h_{is} \), the minimum of \( Z_i \) is found for \( k_i^{\ast(c)} = 1 \). Else, let \( k_i^{inf} = \sqrt{\frac{O_{is} (h_{ib} - h_{is})}{O_{ib} h_{is}}} \).

\( k_i^{\ast(c)} \) is a strictly positive integer that can be found by using the following rule:

If \( k_i^{inf} < 1 \), it is optimal to choose \( k_i^{\ast(c)} = 1 \). Else, let \( k' \leq k_i^{inf} \leq k' + 1 \) with \( k' \in \mathbb{R}^* \).

If \( \frac{k_i^{inf}}{k'} \leq k' + 1 \) then it is optimal to choose \( k_i^{\ast(c)} = k' \). Otherwise, \( k_i^{\ast(c)} = k' + 1 \) (Axsäter, 2006).

It follows that:

\[
Q_B^{\ast(c)} = \sqrt{\frac{2(O_{ib} + \frac{O_{is}}{k_i^{\ast(c)}})D}{h_{ib} + (k_i^{\ast(c)} - 1)h_{is}}},
\]

(3)

Model (c) can be interpreted as a perfect buyer-supplier coordination situation (see Figure 2).

Figure 2
Illustration of model (c)

In the next sections, some non-coordinated situations are studied.
2.4 Some decentralized models

In this section, two different non-coordinated situations are considered. First, we assume that the buyer is the supply chain leader. This situation is referred as Model (b). Second, the supplier is assumed to be the supply chain leader. A new model referred as Model (s) is proposed.

2.4.1 The buyer is the supply chain leader: Model (b)

In this model, we consider that the buyer has the strongest bargaining power and so is acting as the supply chain leader. The buyer thus optimizes its operations without taking the whole supply chain performance into account. The supplier then reacts by optimizing its operations. We refer to this model as Model (b).

In this case, the buyer would be better ordering the quantity that minimizes the following function:

\[
Z_{ib}(Q_b) = h_{ib} \frac{Q_b}{2} + O_{ib} \frac{D}{Q_b},
\]

with \(i \in \{C; E\}\).

The minimum of Formula 4 is the economic (respectively environmental) order quantity (Harris, 1913):

\[
Q^{*b}_{ib} = \sqrt{\frac{2O_{ib}D}{h_{ib}}}.
\]

The supplier then chooses the optimal value \(k^{*b}_i\) minimizing the following function:

\[
Z_{is}(k) = h_{is} (k-1) \frac{Q^{*b}_{ib}}{2} + O_{is} \frac{D}{kQ^{*b}_{ib}}.
\]

Let \(k^{\text{ind}} = \sqrt{\frac{O_{is}h_{ib}}{O_{ib}h_{is}}} \). \(k^{*b}_i\) is a strictly positive integer that can be found by using the rounding rule described in section 2.3.
Figure 3 illustrates the decision process of Model (b).

Models’ comparison and insights may be found in section 3.1.

2.4.2 The supplier is the supply chain leader: Model (s)

In this section, we consider that the supplier has an advantage over the buyer in the purchasing negotiation. As stated in Lu (1995), this situation can be encountered when the supplier is the sole vendor of an item and the buyer lacks of bargaining power to ask for a price discount. As shown in Formula 7, the supplier objective function $Z_{s}^{i}, i \in \{C; E\}$ depends on both $Q_B$ and $k$:

$$Z_{s}^{i} (Q_B, k) = h_{s} (k - 1) \frac{Q_B}{2} + O_{s} \frac{D}{kQ_B}.$$  \hspace{1cm} (7)

Formally, this objective function may be reduced to zero if the supplier requires a very large order quantity $Q_B \rightarrow \infty$ and chooses $k = 1$. However, this may not be possible in practice as the buyer may not accept such situation ($Z_{s}^{i}$ as defined in Formula 4 tends to infinity). Lu (1995) thus proposes to minimize $Z_{s}^{i}$ subject to the maximum increase in the objective function that the buyer is prepared to incur. To our knowledge, this is the only single-buyer single-supplier deterministic model that assumes that the supplier is the supply chain leader.

In what follows, we propose a new model that addresses such situation. This model has several advantages over that studied in Lu (1995) as shown hereafter.

Based on the preliminary results stated in section 2.2, it is interesting for the supplier to meet up orders that are synchronized with its production pattern. This synchronization may reduce supplier’s inventory as some items can be sent to the buyer as soon as produced avoiding the warehousing operations (Wang, 2004). To achieve such synchronization, the supplier may
require that the buyer orders with a minimal frequency \( N \). More frequent orders may also be accepted given that the buyer’s ordering frequency is a multiple of \( N \). Based on the chosen frequency \( N \), the supplier decides on the production lot size \( Q_s = \frac{D}{N} \). The buyer then decides on its ordering quantity \( Q_b = \frac{D}{kN} \) by choosing \( k \in \mathbb{R}^+ \). This negotiation process leads to stationary-nested ordering policies and is thus consistent with the preliminary results stated in section 2.2. We refer to this model as Model (s).

Mathematical derivations of Model (s) can be found in Appendix A. Theorem 1 states that the supplier may decide on the production lot size \( Q_s \) that will minimize \( Z_{is} \) (as defined by Formula 7) by using the following rule:

\[
\text{Theorem 1.} \quad \text{There exists } (k_{i1}, k_{i2}) \in \mathbb{R}^+ \times \mathbb{R}^+ \text{ such that:}
\]

\[
1 < k_{i1} \leq k_{i2},
\]

\[
Q_{is}(k) = \sqrt{k(k+1)Q_{ib}^{(b)}} \text{ for all } k < k_{i1},
\]

\[
Q_{is}(k) = \sqrt{\frac{k}{k-1} \frac{2Q_{is}D}{h_{is}}} \text{ for all } k \text{ such that } k_{i1} \leq k < k_{i2},
\]

\[
Q_{is}(k) = \sqrt{(k-1)kQ_{ib}^{(b)} + \varepsilon} \text{ for all } k \geq k_{i2}, \text{ with } \varepsilon \text{ being a small positive number.}
\]

Then we prove that:

\[
k_{i}^{(s)} \leq k_{i1} \leq k_{i}^{(b)} + 2,
\]

and that:

\[
Q_{ib}^{(s)} = \begin{cases} 
\frac{k_{i}^{(s)} + 1}{k_{i}^{(s)}} Q_{i}^{(b)} & \text{if } k_{i}^{(s)} < k_{i1}, \\
\frac{1}{k_{i}^{(s)}(k_{i}^{(s)} - 1)} \frac{2Q_{is}D}{h_{is}} & \text{else.}
\end{cases}
\]

(8)
Model (s) has several advantages. First, the supplier would share a part of his savings with the buyer even if he perfectly dominates the buyer. This feature is common with the model developed in Lu (1995). Second, we prove that the maximal buyer’s increase in objective function is limited to 6.1% in comparison to Model (b). Indeed, the buyer order quantity cannot exceed $\sqrt{2Q^{(b)}_{ib}}$ due to the negotiation process described above. Finally, this negotiation process may favor horizontal cooperation between several buyers as they are required to pass their orders at given time intervals. In this setting, it may be possible to consolidate shipments (Minner, 2007).

In order to implement Model (s), an important practical issue should be considered. The supplier indeed need numerical estimates of $k_i^{(b)}$, $Q^{(b)}_{ib}$ and $D$ to determine his optimal inventory policy. To estimate these parameters, the supplier only needs to know buyer’s demand and previous order frequency as in Lu (1995). These parameters may be inferred from buyer’s past ordering behavior. Other assumptions are proposed in the literature. For instance, Li et al. (2012) propose a single-supplier single-buyer inventory model where the buyer’s cost information is private. The same assumption is taken in Ha (2001).

In the next section, the centralized and decentralized models presented above are compared both in terms of costs and carbon emissions. The case of a carbon price setup is also studied.
3 Economic and environmental performance of buyer-supplier coordination

In this section, the effect of supply chain coordination on costs and carbon emissions is analyzed. We prove that supply chain coordination can have a negative impact on the total amount of carbon emissions. Finally, we focus on situations where a tax is associated to carbon emissions.

3.1 Economic performance of coordinated versus non-coordinated models

We first focus on the economic performance of the buyer-supplier coordination. Some typical situations are illustrated by the following numerical examples. The data related to Example 1 taken from Goyal (1977) are presented in Table 1.

Table 1
Example 1 data set

<table>
<thead>
<tr>
<th>demand rate (D)</th>
<th>12 000</th>
</tr>
</thead>
<tbody>
<tr>
<td>buyer inventory holding cost (h_{CB})</td>
<td>0.30</td>
</tr>
<tr>
<td>Supplier inventory holding cost (h_{CS})</td>
<td>0.24</td>
</tr>
<tr>
<td>buyer ordering cost (O_{CB})</td>
<td>10</td>
</tr>
<tr>
<td>supplier ordering cost (O_{CS})</td>
<td>100</td>
</tr>
</tbody>
</table>

The related optimal values and resulting costs are presented in Table 2.

Table 2
Example 1 results

<table>
<thead>
<tr>
<th></th>
<th>Q^*_{CB}</th>
<th>k^*_C</th>
<th>Z_{CB}</th>
<th>Z_{CS}</th>
<th>Z_C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model (b)</td>
<td>894</td>
<td>4</td>
<td>268.33</td>
<td>657.40</td>
<td>925.73</td>
</tr>
<tr>
<td>Model (s)</td>
<td>1 033</td>
<td>3</td>
<td>271.11</td>
<td>635.17</td>
<td>906.28</td>
</tr>
<tr>
<td>Model (c)</td>
<td>1 633</td>
<td>2</td>
<td>318.43</td>
<td>563.38</td>
<td>881.82</td>
</tr>
</tbody>
</table>

As shown in Table 2, we obtain that Q^*_{CB} < Q^*_{CB}. Moreover, k^*_C > k^*_C. In general, the results of Formula 10 hold:

\[
\begin{cases}
Q^*_{CB} < Q^*_{CB} \\
k^*_C \geq k^*_C
\end{cases}
\]  

(10)
Formula 10 is proven in Appendix B. By comparing Model (b) to Model (c), we notice that the buyer has to increase its ordering quantity in order to achieve coordination. Hence, the buyer may allow the supplier to produce lots with larger size while reducing its inventory. This trend is often observed in multi-echelon inventory systems. In coordinated supply chains, the buyers often increase their average inventory levels in order to reduce the inventory level at the supplier, but the increase in buyer’s supply chain cost is less than the decrease in supplier cost. Quantity discounts are thus often proposed by the supplier to foster independent buyers to increase their ordering quantities (Li and Wang, 2007). This type of side payment is extensively studied in the literature (Sarmah et al., 2006).

When considering Model (s), it may be noticed that the negotiation process entices the buyer to reasonably increase its order quantity in order to reduce the supplier cost. In the above example, the buyer is not willing to accept a cost increase leading to the results of Model (c). However, an increase of 15.6% in order quantity is possible in exchange of an increase of 1.04% in buyer’s cost. This result is due to the relative insensitivity of the economic order quantity model to a variation of the ordering quantity in the neighborhood of the optimal value.

A particular case is presented in Example 2. The related data are presented in Table 3. The related optimal values and resulting costs are presented in Table 4.

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Example 2 data set</th>
</tr>
</thead>
<tbody>
<tr>
<td>demand rate (D)</td>
<td>12 000</td>
</tr>
<tr>
<td>buyer inventory holding cost (h_{CB})</td>
<td>1.50</td>
</tr>
<tr>
<td>Supplier inventory holding cost (h_{CS})</td>
<td>0.3975</td>
</tr>
<tr>
<td>buyer ordering cost (O_{CB})</td>
<td>25</td>
</tr>
<tr>
<td>supplier ordering cost (O_{CS})</td>
<td>78</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 4</th>
<th>Example 2 results</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_b^*$</td>
<td>$k^*$</td>
</tr>
<tr>
<td>Model (b)</td>
<td>632</td>
</tr>
<tr>
<td>Model (s)</td>
<td>730</td>
</tr>
<tr>
<td>Model (c)</td>
<td>730</td>
</tr>
</tbody>
</table>
In Example 2, Model (s) leads to the same results as Model (c). The negotiation process of Model (s) may thus imply to reach perfect buyer-supplier coordination without any side payment agreement while considering independent entities. This result strengthens Model (s) as it may be seen as a balanced buyer-supplier relationship without any side-payment contract.

In the next sections, carbon emissions considerations are included into the three presented models and several counter-intuitive insights are highlighted.

3.2 The effect of buyer-supplier coordination on environmental performance

Even if coordination’s financial benefits are extensively recognized, non coordinated supply chains are still very common in practice. Several barriers such as communication, mutual trust or benefit sharing issues may indeed discourage the companies from collaborating. The sustainable supply chain literature often argues that sustainability issues may encourage the firms to coordinate their operations. However, is the buyer-supplier coordination always environmental friendly? To answer to this question, we aim at evaluating the environmental performance of the models defined in Section 2. An illustration is presented in Example 3 with related data provided in Table 5. The related optimal values and resulting costs and carbon emissions are presented in Table 6.

Table 5
Example 3 data set

<table>
<thead>
<tr>
<th>Demand rate (D)</th>
<th>12,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buyer inventory holding cost (h_{CB})</td>
<td>0.50</td>
</tr>
<tr>
<td>Buyer inventory holding emissions (h_{EB})</td>
<td>2.00</td>
</tr>
<tr>
<td>Supplier inventory holding cost (h_{CS})</td>
<td>0.40</td>
</tr>
<tr>
<td>Supplier inventory holding emissions (h_{ES})</td>
<td>3.00</td>
</tr>
<tr>
<td>Buyer ordering cost (O_{CB})</td>
<td>100</td>
</tr>
<tr>
<td>Buyer ordering emissions (O_{EB})</td>
<td>500</td>
</tr>
<tr>
<td>Supplier ordering cost (O_{CS})</td>
<td>150</td>
</tr>
<tr>
<td>Supplier ordering emissions (O_{ES})</td>
<td>75</td>
</tr>
</tbody>
</table>

Table 6
Example 3 results

<table>
<thead>
<tr>
<th>Model</th>
<th>Q_{CB}</th>
<th>k_c</th>
<th>Z_{CB}</th>
<th>Z_{CS}</th>
<th>Z_{C}^*</th>
<th>Z_{EB}</th>
<th>Z_{ES}</th>
<th>Z_{E}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model (b)</td>
<td>2,191</td>
<td>1</td>
<td>1,095.45</td>
<td>821.58</td>
<td>1,917.03</td>
<td>4,929.50</td>
<td>410.79</td>
<td>5,340.29</td>
</tr>
<tr>
<td>Model (s)</td>
<td>3,098</td>
<td>1</td>
<td>1,161.90</td>
<td>580.95</td>
<td>1,742.84</td>
<td>5,034.88</td>
<td>290.47</td>
<td>5,325.35</td>
</tr>
<tr>
<td>Model (c)</td>
<td>3,464</td>
<td>1</td>
<td>1,212.44</td>
<td>519.62</td>
<td>1,732.05</td>
<td>5,196.15</td>
<td>259.81</td>
<td>5,455.96</td>
</tr>
</tbody>
</table>
In Example 3, coordinating operations with a cost minimization objective leads to an increase in the total supply chain carbon emissions comparing to decentralized models. Model (c) leads to an increase in total supply chain emissions in the following conditions stated in Theorem 2 and Theorem 3.

**Theorem 2.** Model (c) leads to an increase in the total supply chain carbon emissions comparing to Model (b) if the following conditions are verified:

\[ k_C^{*(c)} = k_C^{*(b)} = k_E^{*(c)} , \]

\[ Q_{CB}^{*(b)} \geq Q_{EB}^{*(c)} . \]

**Theorem 3.** Model (c) leads to an increase in the total supply chain carbon emissions comparing to Model (s) if the following conditions are verified:

\[ k_C^{*(c)} = k_C^{*(s)} = k_E^{*(c)} , \]

\[ Q_{CB}^{*(s)} \geq Q_{EB}^{*(c)} . \]

These results are proven in Appendix C. Note that the conditions stated in Theorems 2 and 3 are only sufficient ones. In the previous example, these conditions are verified by Model (s).

On the other hand, we can observe that \( Q_{CB}^{*(b)} < Q_{EB}^{*(c)} \) (\( Q_{CB}^{*(b)} = 2191 \) and \( Q_{EB}^{*(c)} = 2627 \)). Nevertheless, Model (b) performs better than Model (c) in terms of carbon emissions.

The next section focuses on situations where carbon emissions are taken into account by using a carbon tax.

### 3.3 The impacts of a carbon tax regulatory policy

The carbon tax is a commonly used regulatory policy to foster companies to reduce their carbon emissions. In this section, we consider that a price is associated to carbon emissions. The notion of carbon price is indeed more general than a carbon tax. For instance, this price can be setup by the company through an internal evaluation, by considering the cost of the energy used or the cost issued from an environmental accounting analysis. Hua et al. (2011) have also proven that emissions levels depend only on the carbon price in the economic order quantity model under a cap and trade regulation. In what follows, we assume that both the
buyer and the supplier are charged with the same carbon price $\alpha \in [0; \infty)$ per unit of carbon emissions.

In this context, the companies aim at minimizing their total cost resulting from both carbon emission cost and supply chain cost. It can be noticed that the results of Section 2 can be directly applied in this context by replacing $h_{ib}$ by $h_{CB} + \alpha h_{EB}$, $O_{ib}$ by $O_{CB} + \alpha O_{EB}$ and by introducing the same modification for the supplier’s parameters. In what follows, the implications of setting up or increasing a carbon price are studied. It is proven that setting up a carbon tax may have a negative impact on total supply chain emissions in certain situations.

**The carbon emissions are non-linear in function of the carbon price:**
Consider Model (c). An increase in $\alpha$ necessarily implies a decrease in total carbon emissions. However, this decrease is non-linear in $\alpha$ and may also be discontinuous. Such situation is illustrated in Example 4 with related data presented in Table 7.

<table>
<thead>
<tr>
<th>Table 7</th>
<th>Example 4 data set</th>
</tr>
</thead>
<tbody>
<tr>
<td>demand rate (D)</td>
<td>12 000</td>
</tr>
<tr>
<td>buyer inventory holding cost ($h_{CB}$)</td>
<td>2.50</td>
</tr>
<tr>
<td>buyer inventory holding emissions ($h_{EB}$)</td>
<td>1.00</td>
</tr>
<tr>
<td>Supplier inventory holding cost ($h_{CS}$)</td>
<td>0.50</td>
</tr>
<tr>
<td>Supplier inventory holding emissions ($h_{ES}$)</td>
<td>0.30</td>
</tr>
<tr>
<td>buyer ordering cost ($O_{CB}$)</td>
<td>25</td>
</tr>
<tr>
<td>buyer ordering emissions ($O_{EB}$)</td>
<td>150</td>
</tr>
<tr>
<td>supplier ordering cost ($O_{CS}$)</td>
<td>150</td>
</tr>
<tr>
<td>supplier ordering emissions ($O_{ES}$)</td>
<td>75</td>
</tr>
</tbody>
</table>

Figure 5 illustrates the variation of the total supply chain carbon emissions $Z_E$ in function of the carbon price $\alpha \in [0;25]$. We can observe some discontinuities in $Z_E$. For instance, a slight variation of the carbon price from $\alpha = 1.0215$ to $\alpha = 1.0216$ would imply carbon emissions to decrease from more than 6% (from $Z_E = 2320$ to $Z_E = 2179$). This feature has several implications. First, this would imply that if the carbon price is setup by the company thanks to an internal evaluation, then the precision of this evaluation is of crucial importance. Second, if the company faces a cap and trade regulation, a tiny variation of the carbon price is likely to have major impacts on company’s optimal carbon emissions.
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Figure 5
Carbon emissions in function of the carbon price

\[ Z_E \]

The total supply chain carbon emissions may be increasing in the carbon price:

Consider then Model (b). In this case, the buyer’s emissions are decreasing in \( \alpha \). On the other hand, it may happen that the supplier’s emissions increase in \( \alpha \). The total supply chain carbon emissions may thus be increasing in \( \alpha \). Example 5 illustrates this situation. The related data are presented in Table 8. The related optimal values and resulting costs and carbon emissions are presented in Table 9.

Table 8
Example 5 data set

<table>
<thead>
<tr>
<th>demand rate (D)</th>
<th>10 000</th>
</tr>
</thead>
<tbody>
<tr>
<td>buyer inventory holding cost (h_CB)</td>
<td>0.50</td>
</tr>
<tr>
<td>buyer inventory holding emissions (h_EB)</td>
<td>2.00</td>
</tr>
<tr>
<td>Supplier inventory holding cost (h_CS)</td>
<td>15.0</td>
</tr>
<tr>
<td>Supplier inventory holding emissions (h_ES)</td>
<td>5.00</td>
</tr>
<tr>
<td>buyer ordering cost (O_CB)</td>
<td>15</td>
</tr>
<tr>
<td>buyer ordering emissions (O_EB)</td>
<td>25</td>
</tr>
<tr>
<td>supplier ordering cost (O_CS)</td>
<td>150</td>
</tr>
<tr>
<td>supplier ordering emissions (O_ES)</td>
<td>120</td>
</tr>
</tbody>
</table>

Table 9
Example 5 results

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( Q_{a0}^{(b)} )</th>
<th>( k_a^{(b)} )</th>
<th>( Z_{CB} )</th>
<th>( Z_{CS} )</th>
<th>( Z_C )</th>
<th>( Z_{EB} )</th>
<th>( Z_{ES} )</th>
<th>( Z_E )</th>
<th>( Z_C + \alpha Z_E )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>775</td>
<td>1</td>
<td>387.30</td>
<td>1 936.49</td>
<td>2 323.79</td>
<td>1 097.35</td>
<td>1 549.19</td>
<td>2 646.54</td>
<td>2 323.79</td>
</tr>
<tr>
<td>0.5</td>
<td>606</td>
<td>1</td>
<td>399.10</td>
<td>2 477.17</td>
<td>2 876.27</td>
<td>1 018.39</td>
<td>1 981.73</td>
<td>3 000.13</td>
<td>4 376.33</td>
</tr>
<tr>
<td>1</td>
<td>566</td>
<td>1</td>
<td>406.59</td>
<td>2 651.85</td>
<td>3 058.24</td>
<td>1 007.63</td>
<td>2 121.32</td>
<td>3 128.95</td>
<td>6 187.18</td>
</tr>
<tr>
<td>10</td>
<td>508</td>
<td>1</td>
<td>422.12</td>
<td>2 950.06</td>
<td>3 372.18</td>
<td>1 000.14</td>
<td>2 360.04</td>
<td>3 360.19</td>
<td>36 974.03</td>
</tr>
</tbody>
</table>
In this example, the total supply chain emissions are increased by 27% (from \( Z_E = 2647 \) to \( Z_E = 3360 \)) by setting up a carbon price \( \alpha = 10 \). This surprising result implies that a carbon tax regulatory policy may be ineffective in reducing carbon emissions in certain situations. The same conclusion may be drawn for a cap and trade regulatory policy. Setting up a carbon tax or a carbon price in Model (b) would indeed entice the buyer to reduce its emissions by modifying its ordering quantity. \( Q_{ab}^{(b)} \) is indeed monotonous in \( \alpha \). This change in buyer’s ordering quantity may negatively affect the supplier performances both in terms of cost and carbon emissions. The same analysis can be performed with model (s).

*An increase in the carbon tax may favor coordination without decreasing carbon emissions:* The modification of buyer’s ordering quantity induced by the setup or the increase in carbon price may favor the supplier in some cases. The supplier total cost (operations cost + carbon cost) may indeed be lower than the operations cost before the change in buyer’s ordering quantity. This situation is illustrated for Model (b) in Example 6 with related data presented in Table 10. The related optimal values and resulting costs and carbon emissions are presented in Table 11.

<table>
<thead>
<tr>
<th>Table 10</th>
<th>Example 6 data set</th>
</tr>
</thead>
<tbody>
<tr>
<td>demand rate (D)</td>
<td>2000</td>
</tr>
<tr>
<td>buyer inventory holding cost (h(_{CB}))</td>
<td>5.00</td>
</tr>
<tr>
<td>buyer inventory holding emissions (h(_{EB}))</td>
<td>2.00</td>
</tr>
<tr>
<td>Supplier inventory holding cost (h(_{CS}))</td>
<td>15.0</td>
</tr>
<tr>
<td>Supplier inventory holding emissions (h(_{ES}))</td>
<td>0.10</td>
</tr>
<tr>
<td>buyer ordering cost (O(_{CB}))</td>
<td>50</td>
</tr>
<tr>
<td>buyer ordering emissions (O(_{EB}))</td>
<td>25</td>
</tr>
<tr>
<td>supplier ordering cost (O(_{CS}))</td>
<td>800</td>
</tr>
<tr>
<td>supplier ordering emissions (O(_{ES}))</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 11</th>
<th>Example 6 results for Model (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>( Q_{ab}^{(b)} )</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>0</td>
<td>200</td>
</tr>
<tr>
<td>1</td>
<td>207</td>
</tr>
<tr>
<td>5</td>
<td>216</td>
</tr>
</tbody>
</table>

In this example, the supplier total cost is decreasing for \( \alpha = 1 \) and \( \alpha = 5 \). On the other hand, the buyer faces a huge increase in his own total cost. Assume that coordination was not feasible before setting up a carbon price. For instance, the buyer who is the supply chain leader may not be willing to share the benefit of coordinating operations with the supplier.
The setup of the carbon price may change this situation. The buyer who faces a huge increase in his total cost may be more prone to share coordination’s benefits.

Table 12 presents the results obtained with Model (c) for the same parameters.

Table 12
Example 6 results for Model (c)

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$Q^*(c)$</th>
<th>$k^*(c)$</th>
<th>$Z_{CB}$</th>
<th>$Z_{EB}$</th>
<th>$Z_{CB}+\alpha Z_{EB}$</th>
<th>$Z_{CS}$</th>
<th>$Z_{ES}$</th>
<th>$Z_{CS}+\alpha Z_{ES}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>825</td>
<td>1</td>
<td>2 182.82</td>
<td>882.26</td>
<td>2 182.82</td>
<td>1 940.29</td>
<td>3.64</td>
<td>1 940.29</td>
</tr>
<tr>
<td>1</td>
<td>708</td>
<td>1</td>
<td>1 910.58</td>
<td>778.36</td>
<td>2 688.94</td>
<td>2 260.80</td>
<td>4.24</td>
<td>2 265.04</td>
</tr>
<tr>
<td>5</td>
<td>512</td>
<td>1</td>
<td>1 475.01</td>
<td>609.54</td>
<td>4 522.73</td>
<td>3 125.86</td>
<td>5.86</td>
<td>3 155.16</td>
</tr>
</tbody>
</table>

In this case, setting up a carbon price $\alpha = 5$ would favor collaboration, however, the total supply chain carbon emissions increases:

Before the carbon price setup, $Z_E = Z_{EB} + Z_{ES} = 450 + 17.50 = 467.50$ for Model (b) (see Table 11).

After setting up $\alpha = 5$, $Z_E = Z_{EB} + Z_{ES} = 609.54 + 5.86 = 615.40$ for Model (c) (see Table 12).

In this section, it is shown that a carbon price may have several drawbacks in controlling carbon emissions. An increase in $\alpha$ may indeed imply an increase in total carbon emissions. We also demonstrate that a carbon price setup may increase collaborative behaviors without necessarily having a positive effect on carbon emissions due to the results of Section 3.2.

4 Conclusion

This paper investigates the economic and environmental performance of buyer-supplier coordination. The study is based on a single-buyer single-supplier supply chain. Several situations illustrating different outcomes of the buyer-supplier negotiation are presented. We propose a new model that enables the supplier to act as the supply chain leader. This model has several advantages comparing to the existing models. We prove that the maximal cost increase for the buyer is limited to 6.1% comparing to the buyer’s economic order quantity. This model may also be easily implemented in practice.
Sustainability is becoming an essential feature in supply chain management. The sustainable supply chain management literature often argues that sustainability issues may encourage the firms to coordinate their operations. However, we show that coordination may increase the total supply chain carbon emissions. The same result is also established in case of a carbon price setting up. Finally, we show that an increase in carbon price may favor collaborative behaviors without necessarily having a positive effect on carbon emissions. These counterintuitive results may warn both practitioners and policy makers.

Several future research directions may be considered. First, other sustainability criteria may be included into the models. For instance, alternative type of pollutants, resource consumption or social issues may be considered. Second, this paper focuses on a simple supply chain structure with a single buyer. The effect of coordination may perhaps be different in a multi-buyer context. Studying the effect of sustainability considerations in a single-supplier multi-buyer context thus deserves future research. Model(s) may also be extended to a multi-buyer context. This negotiation process may indeed favor horizontal cooperation between buyers as they would be required to pass their orders at given time intervals. In this setting, it may be possible to consolidate shipments in order to improve the sustainable performances of the supply chain.

Finally, introducing stochastic variables into the presented models may also be considered for future research. The proposed models consider that both the demand and the leadtimes are deterministic. These simple assumptions enable providing useful insights but may be relaxed to focus on more realistic situations. The amount of carbon emissions may also be considered as a stochastic variable due to variations in production setup and waste.
Appendix A: Analytical derivations of Model (s)

The buyer-supplier negotiation process may be described as follows:

**Negotiation process:** The supplier first decides on the production lot size \( Q_s \in (0; \infty) \).

This value is then transmitted to the buyer that decides on its order quantity \( Q_B = \frac{Q_s}{k} \) by choosing \( k \in \mathbb{R}^+ \).

The buyer still aims at minimizing its own objective function \( Z_{iB} \) given by Formula 4. The buyer’s decision is made as follows. If \( Q_s \leq Q_{iB}^{*b} \), then it is optimal for the buyer to choose \( k = 1 \), else there exists \( k' \in \mathbb{R}^+ \) such that \( \frac{Q_s}{k'+1} < Q_{iB}^{*b} \leq \frac{Q_s}{k'} \). If \( Z_{iB} \left( \frac{Q_s}{k'+1} \right) < Z_{iB} \left( \frac{Q_s}{k'} \right) \) then it is optimal to choose \( k = k'+1 \), else it is optimal to choose \( k = k' \).

Due to the structure of Formula 4, the interval \((0; \infty)\) can thus be divided into subintervals \( [Q_{iS\min}(k); Q_{iS\max}(k)] \) such that the buyer decides to choose the given integer \( k \) for any proposed value of \( Q_S \in (Q_{iS\min}(k); Q_{iS\max}(k)] \).

**Proposition 1:** For all \( k \in \mathbb{R}^+ \):

\[
Q_{iS\max}(k) = \sqrt{k(k+1)Q_{iB}^{*b}},
\]

\[
Q_{iS\min}(k) = \begin{cases} 
0 & \text{if } k = 1 \\
\sqrt{(k-1)kQ_{iB}^{*b}} & \text{else.}
\end{cases}
\]

**Proof:** The buyer decides to choose the given integer \( k \) for any proposed value of \( Q_S \in (Q_{iS\min}(k); Q_{iS\max}(k)] \) if and only if:

\[
Z_{iB} \left( \frac{Q_s}{k} \right) \leq Z_{iB} \left( \frac{Q_s}{k+1} \right) \iff \frac{Q_s}{k} Z_{iB} \left( Q_{iB}^{*b} \right) \leq \frac{Q_s}{k+1} Z_{iB} \left( Q_{iB}^{*b} \right) \iff \frac{1}{2} \left( \frac{Q_s}{k} - \frac{1}{Q_{iB}^{(b)}} + \frac{1}{Q_{iB}^{(b)}} + \frac{1}{Q_{iB}^{(b)}} \right) \iff \frac{Q_s}{k+1} Z_{iB} \left( Q_{iB}^{*b} \right) \leq \frac{1}{2} \left( \frac{Q_s}{k} + Q_{iB}^{(b)} k + 1 + Q_{iB}^{(b)} k \right)
\]

\[
\iff Q_s \leq k(k+1)Q_{iB}^{*b} \iff Q_s \leq \sqrt{k(k+1)Q_{iB}^{*b}}.
\]

It follows that \( Q_{iS\max}(k) = \sqrt{k(k+1)Q_{iB}^{*b}} \) for all \( k \in \mathbb{R}^+ \).
By using the calculation above, it follows that for all $k > 1$, the buyer decides to choose the given integer $k$ for any proposed value of $Q_S \in \left( Q_{S\min}(k); Q_{S\max}(k) \right)$ if and only if:

$$Z_{ib} \left( \frac{Q_S}{k-1} \right) > Z_{ib} \left( \frac{Q_S}{k} \right) \iff Q_S > \sqrt[2]{k(k-1)Q_{ib}^{(b)}}.$$  

It follows that $Q_{S\min}(k) = \sqrt[2]{k(k-1)Q_{ib}^{(b)}}$ for all $k > 1$. Moreover, $Q_{S\min}(1) = 0$.

On the other hand, the supplier aims at minimizing its total objective function $Z_{iS}$ as given by Formula 7. For any given value of $k \in \mathbb{R}^*$, the minimum of $Z_{iS}$ is obtained in $Q_{iS}^*(k)$ as given in Proposition 2:

**Proposition 2:** For all $k \in \mathbb{R}^*$:

$$Q_{iS}^*(k) = \begin{cases} \infty & \text{if } k = 1 \\ \frac{k}{k-1} \sqrt[2]{2O_{iS}D} & \text{else.} \end{cases}$$

**Proof:** The supplier objective function is expressed as follows:

$$Z_{iS} (Q_S, k) = h_{iS} \left( \frac{k-1}{k} \frac{Q_S}{2} + O_{iS} \frac{D}{Q_S} \right).$$

For $k = 1$, $Z_{iS}$ tends to zero as $Q_S$ tends to infinity.

For any given value of $k > 1$, the minimum of $Z_{iS}$ can be obtained by setting the first derivative of $Z_{iS}$ with respect to $Q_S$ equal to zero.

$$\frac{\partial Z_{iS} \left( Q_S, k \right)}{\partial Q_S} = \frac{h_{iS} (k-1)}{2} \frac{Q_S}{k} + O_{iS} \frac{D}{Q_S^2} = 0 \iff Q_S^* = \frac{k}{k-1} \frac{2O_{iS}D}{h_{iS}} \iff Q_S = \sqrt[2]{k} \frac{2O_{iS}D}{h_{iS}}.$$  

Then, for all $k \in \mathbb{R}^*$,

$$Q_{iS}^*(k) = \begin{cases} \infty & \text{if } k = 1 \\ \frac{k}{k-1} \sqrt[2]{2O_{iS}D} & \text{else.} \end{cases}$$

In other words, the supplier would like to choose $Q_{iS}^*(k)$ but is required to choose $Q_{iS}(k) \in \left( Q_{iS\min}(k); Q_{iS\max}(k) \right)$ due to the proposed negotiation process. The supplier may thus
choose \( Q_{iS}(k) = \max\{Q_{iS,\min}(k) + \varepsilon; \min\{Q_{iS}^*(k); Q_{iS,\max}(k)\}\} \), with \( \varepsilon \) being a small positive number.

**Theorem 1.** There exists \((k_{i1}, k_{i2}) \in \mathbb{R}^* \times \mathbb{R}^*\) such that:

\[
1 < k_{i1} \leq k_{i2},
\]

\[
Q_{iS}(k) = \sqrt{k(k+1)Q_{ib}^*(b)} \quad \text{for all } k < k_{i1},
\]

\[
Q_{iS}(k) = \sqrt{\frac{k}{k-1} \frac{2O_{iS}D}{h_{iS}}} \quad \text{for all } k \text{ such that } k_{i1} \leq k < k_{i2},
\]

\[
Q_{iS}(k) = \sqrt{(k-1)\bar{k}Q_{ib}^*(b) + \varepsilon} \quad \text{for all } k \geq k_{i2}, \text{ with } \varepsilon \text{ being a small positive number.}
\]

**Proof:** \( Q_{iS}(k) = \max\{Q_{iS,\min}(k) + \varepsilon; \min\{Q_{iS}^*(k); Q_{iS,\max}(k)\}\} \). \( Q_{iS,\min}(k) \) and \( Q_{iS,\max}(k) \) are strictly increasing in \( k \). On the opposite, \( Q_{iS}^*(k) \) is strictly decreasing in \( k \). Moreover, \( Q_{iS}(1) = Q_{iS,\max}(1) = \sqrt{2}Q_{ib}^*(b) \). \( \square \)

Proposition 3 gives additional information on \( k_{i1} \) and \( k_{i2} \):

**Proposition 3:** \( k_i^{*(b)} \leq k_{i1} \leq k_{i2} \leq k_i^{*(b)} + 2 \).

**Proof:** By definition of \( k_{i1} \), we obtain that:

\[
\begin{align*}
&Q_{iS}^*(k_{i1}) > Q_{iS}(k_{i1}) - 1 \quad (A) \\
&Q_{iS}(k_{i1}) \leq Q_{iS,\max}(k_{i1}) \quad (B)
\end{align*}
\]

\((A) \iff \sqrt{\frac{k_{i1}-1}{k_{i1}-2}} \frac{2O_{iS}D}{h_{iS}} > \sqrt{k_{i1}(k_{i1}-1)} \frac{2O_{ib}D}{h_{ib}} \iff \sqrt{k_{i1}(k_{i1}-2)} < \sqrt{\frac{O_{iS}h_{ib}}{O_{ib}h_{iS}}} \leq k_i^{*(b)} + 1 \]

\( \iff k_{i1} - 2 < \sqrt{k_{i1}(k_{i1}-2)} \leq k_i^{*(b)} + 1 \iff k_{i1} \leq k_i^{*(b)} + 2 \).

\((B) \iff \sqrt{\frac{k_{i1}}{k_{i1}-1}} \frac{2O_{iS}D}{h_{iS}} \leq \sqrt{k_{i1}(k_{i1}+1)} \frac{2O_{ib}D}{h_{ib}} \iff \sqrt{(k_{i1} + 1)(k_{i1}-1)} \geq \sqrt{\frac{O_{iS}h_{ib}}{O_{ib}h_{iS}}} \geq k_i^{*(b)} - 1 \]

\( \iff k_{i1} > \sqrt{(k_{i1} + 1)(k_{i1}-1)} \geq k_i^{*(b)} - 1 \iff k_{i1} \geq k_i^{*(b)} \).

We thus obtain that \( k_i^{*(b)} \leq k_{i1} \leq k_i^{*(b)} + 2 \).
By definition of $k_{i2}$, we obtain that :

$$
\begin{align*}
\begin{cases}
Q_{is}^*(k_{i2}) - 1 > Q_{is}^*(k_{i2} - 1) & (A) \\
Q_{is}^*(k_{i2}) \leq Q_{is}^*(k_{i2}) & (B)
\end{cases}
\end{align*}
$$

\( (A) \Leftrightarrow \frac{k_{i2} - 1}{k_{i2} - 2} \sqrt{2O_{is}D} > \sqrt{(k_{i2} - 2)(k_{i2} - 3)} \sqrt{\frac{2O_{ib}D}{h_{is}}} \Leftrightarrow k_{i2} - 2 < \sqrt{\frac{O_{is}h_{ib}}{O_{ib}h_{is}}} \leq k_{i1}^{(b)} + 1
\)

\( \Leftrightarrow k_{i2} \leq k_{i1}^{(b)} + 2. \)

\( (B) \Leftrightarrow \frac{k_{i2} - 1}{k_{i2} - 2} \sqrt{2O_{is}D} \leq \sqrt{(k_{i2} - 2)(k_{i2} - 3)} \sqrt{\frac{2O_{ib}D}{h_{is}}} \Leftrightarrow k_{i2} - 1 \geq \sqrt{\frac{O_{is}h_{ib}}{O_{ib}h_{is}}} \geq k_{i1}^{(b)} - 1 \Leftrightarrow k_{i2} \geq k_{i1}^{(b)}
\).

We thus obtain that $k_{i1}^{(b)} \leq k_{i2} \leq k_{i1}^{(b)} + 2$.

Moreover, $1 < k_{i1} \leq k_{i2}$, thus :

$k_{i1}^{(b)} \leq k_{i1} \leq k_{i1}^{(b)} + 2.$

The supplier may thus aim at finding $k_{i1}^{(s)}$ that minimizes $Z_{is}(Q_{is}(k), k)$. Proposition 4 enables restricting the search space for $k_{i1}^{(s)}$:

**Proposition 4:**

$k_{i1}^{(s)} \leq k_{i1} \leq k_{i1}^{(b)} + 2.$

**Proof:** For all $k > 1$,

$$Z_{is}(Q_{is}^*(k)) = \frac{h_{is}}{2k} (k - 1) k \sqrt{\frac{k}{k - 1} \sqrt{2O_{is}D} h_{is} + O_{is}D} \sqrt{\frac{k - 1}{k} \sqrt{2O_{is}D} h_{is}} = \frac{k - 1}{k} \sqrt{2h_{is}O_{is}D}.$$ 

In addition, $Z_{is}(Q_{is}^*(1)) = 0$. $Z_{is}(Q_{is}^*(k))$ is thus strictly increasing in $k$.

Moreover, for all $k \in \mathbb{R}^+$, $Z_{is}(Q_{is}(k))$ is thus strictly increasing in $k$.

If $k_{i1} < k_{i2}$, then $Q_{is}(k_{i1}) = Q_{is}^*(k_{i1})$.

For all $k > k_{i1}$:

- either $Q_{is}(k) = Q_{is}^*(k)$, then $Z_{is}(Q_{is}(k)) > Z_{is}(Q_{is}^*(k))$ as $Z_{is}(Q_{is}^*(k))$ is thus strictly increasing in $k$,
- or $Q_{is}(k) = Q_{is}(k_{i1})$, then $Z_{is}(Q_{is}(k)) > Z_{is}(Q_{is}^*(k)) > Z_{is}(Q_{is}(k_{i1}))$ as $Z_{is}(Q_{is}(k))$ is thus strictly increasing in $k$.

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Else $Q_{iS}(k_{1i}) = Q_{iS}\min(k_{1i}) + \varepsilon$.

In this case, $Q_{iS}(k_{1i} - 1) = Q_{iS}\max(k_{1i} - 1)$. It follows that $Z_{iS}(Q_{iS}(k_{1i})) > Z_{iS}(Q_{iS}(k_{1i} - 1))$ as $Z_{iS}(Q_{iS}\min(k + 1) + \varepsilon) > Z_{iS}(Q_{iS}\max(k))$ for all $k \in \mathbb{R}^*$.

Moreover, for all $k \geq k_{1i}$, $Z_{iS}(Q_{iS}\max(k)) > Z_{iS}(Q_{iS}\min(k))$ as $Z_{iS}$ is convex in $Q_{iS}$ and $Q_{iS}^*(k) < Q_{iS}\min(k) < Q_{iS}\max(k)$. Then $Z_{iS}(Q_{iS}(k + 1)) > Z_{iS}(Q_{iS}\max(k)) > Z_{iS}(Q_{iS}(k))$. By induction, we obtain that $k_{1i}^* \leq k_{1i} - 1$.

It follows that $k_{1i}^* \leq k_{1i} \leq k_{1i}^* + 2$.

It is thus possible to assess $Z_{iS}(Q_{iS}(k), k)$ for all $k \in [1; k_{1i}^* + 2]$ and to deduce $k_{1i}^*$.

The following algorithm can be used to determine the optimal ordering policy for Model (s):

Step 1: Estimate $k_{1i}^*$.

Step 2: For all $k \in [1; k_{1i}^* + 2]$, compute $Q_{iS}\max(k)$, $Q_{iS}\min(k)$ and $Q_{iS}^*(k)$ by using Propositions 1 and 2.

Step 3: Obtain $k_{1i}$ and $k_{1i}$ by using Theorem 1.

Step 4: For all $k < k_{1i}$, compute $Z_{iS}(Q_{iS}\max(k), k)$.

Step 5: If $k_{1i} \neq k_{1i}$, compute $Z_{iS}(Q_{iS}^*(k_{1i}), k_{1i})$.

Step 6: Obtain $k_{1i}^*$ and $Q_{iS}(k_{1i}^*)$.

Then we have thus proven that:

$$k_{1i}^* \leq k_{1i} \leq k_{1i}^* + 2,$$

$$Q_{iS}^*(x) = \begin{cases} \frac{k_{1i}^* + 1}{\frac{k_{1i}^* + 1}{k_{1i}^* + 2} - 1}, & \text{if } k_{1i}^* < k_{1i} \ , \\ \frac{1}{k_{1i}^*(k_{1i}^* - 1)} \sqrt{\frac{2O_{iS}D}{h_{iS}}} & \text{else.} \end{cases}$$
Appendix B: Proof of Formula 10

Comparison of Model (b) and Model (c):

\[ Q_{CB}^{(b)} = \sqrt{\frac{2O_{CB}D}{h_{CB}}} \] is independent of \( k \).

\[ Q_{CB}^{(c)}(k) = \sqrt{\frac{2(O_{CB} + \frac{O_{CS}}{k})D}{h_{CB} + (k-1)h_{CS}}} \] is strictly decreasing in \( k \) and tends to \( Q_{CB}^{(b)} \) as \( k \) tends to infinity, thus \( Q_{CB}^{(c)}(k) > Q_{CB}^{(b)} \) for all \( k \in \mathbb{R}^+ \).

It follows that \( Q_{CB}^{(c)} > Q_{CB}^{(b)} \).

For Model (c), \( k^{inf} = \sqrt{\frac{O_{CS} (h_{CB} - h_{CS})}{O_{CB} h_{CS}}} \) is rounded by using the rule presented in Section 2.3 to obtain \( k_c^{(c)} \). For Model (b), \( k^{inf} = \sqrt{\frac{O_{CS} h_{CB}}{O_{CB} h_{CS}}} > \sqrt{\frac{O_{CS} (h_{CB} - h_{CS})}{O_{CB} h_{CS}}} \) is rounded by using the same rule to obtain \( k_c^{(b)} \) thus \( k_c^{(b)} \geq k_c^{(c)} \).

Appendix C: Proofs of Theorem 2 and 3

Proof of Theorem 2:

Assume that \( k_c^{(c)} = k_c^{(b)} = k_E^{(c)} \) and \( Q_{CB}^{(b)} \geq Q_{EB}^{(c)} \).

By applying Theorem 4 of Bouchery et al. (2012), we obtain that the efficient frontier (the set of Pareto optimal solutions) of the two-echelon serial SOQ problem restricted to \( k = k_c^{(c)} \) is \( E_{k_c^{(c)}} = [\min(Q_{CB}^{(c)}, Q_{EB}^{(c)}), \max(Q_{CB}^{(c)}, Q_{EB}^{(c)})] = [Q_{CB}^{(c)}, Q_{CB}^{(c)}] \) as \( Q_{CB}^{(c)} > Q_{CB}^{(b)} \geq Q_{EB}^{(c)} \) by applying Formula 10.

It follows that \( Q_{CB}^{(b)} \in E_{k_c^{(c)}} \). By definition of an efficient solution, we finally obtain that:

\[ Z_E(k_c^{(b)}, Q_{CB}^{(b)}) < Z_E(k_c^{(c)}, Q_{CB}^{(c)}) \] as \( Z_c(k_c^{(b)}, Q_{CB}^{(b)}) > Z_c(k_c^{(c)}, Q_{CB}^{(c)}) \).

Proof of Theorem 3:

Idem as Theorem 2.
References


