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Robust time-domain source stepping for DC-solution of circuit equations

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Summary. Most analyses of circuit equations start with solving the steady-state (DC) solution. In several cases this can be very hard. We present a novel time domain source stepping procedure to obtain a DC solution of circuit equations. The source stepping procedure is automatically adaptive. Controlled sources can be elegantly dealt with. The method can easily be combined with existing pseudo-transient procedures. The method is robust and efficient.

1 Introduction

The circuit equations can be written as [5, 10]

\[ \frac{d}{dt} q(x) + j(x) + s(t, x) = 0 \]  \hspace{1cm} (1)

Here \( s(t, x) \) represents the specifications of the sources. The unknown \( x = x(t) \) consists of nodal voltages and of currents through voltage defined elements. We assume that \( q(0) = 0 \), and \( j(0) = 0 \). The steady state solution, which is called DC-solution (Direct Current solution), \( x_{DC} \), satisfies

\[ j(x_{DC}) + s(0, x_{DC}) = 0. \]  \hspace{1cm} (2)

Usually, and already hinted by setting \( t = 0 \) in [2], the DC-solution provides the initial value for the transient problem [1]. In general, the problem [2] is non-linear. How to solve this problem is the subject of this note. The importance of the DC-problem lies in the fact that the DC-solution is crucial as starting solution for a number of next analyses (transient analysis, AC analysis, Harmonic Balance analysis, Periodic Steady-State analysis). In general, [1] forms a system of Differential-Algebraic Equations (DAEs). With \( G = \frac{d q(x)}{dx} \) \hspace{1cm} and \hspace{1cm} \( f = \frac{d j(x)}{dx} \) \hspace{1cm} \hspace{1cm} [\begin{array}{l} \lambda \end{array} \ G + \ f] \), we assume that \( \lambda \ G + \ f \) is non-singular for \( \lambda \) in some neighbourhood of 0 (may be excluding \( \lambda = 0 \)). To solve the equations Newton’s method, or variants, may be applied [3, 5, 8], which can be combined with \( g_{\min} \)-stepping, in which linear conductors \( g \) are placed parallel to the non-linear part inside each transistor (device). Iteratively \( g \downarrow g_{\min} \) after which the Newton counter is increased.

Another approach is Pseudo-Transient [2]. In Pseudo-Transient (PT) one can use relaxed tolerances for the Newton process and for the time step control procedure. Also this can be combined with \( g_{\min} \)-stepping during each time step. In PT one has to provide a non-trivial initial solution. A new procedure is described in the next section. Other methods are: temperature stepping, source stepping (the sources are iteratively increased to their final value), homotopy methods, or optimization [1, 4, 7, 9–12].

2 Time-domain Source Stepping

Usually, in Source Stepping one introduces a parameter \( \lambda \) and considers the problem

\[ j(x(\lambda)) + \lambda s(0, x(\lambda)) = 0. \]  \hspace{1cm} (3)

In this case it is assumed that for \( \lambda = 0 \) the problem [3] is easily solved so that in the end the original problem is solved. The same parameter \( \lambda \) is applied to all sources \( s \) in the circuit. In general, for each value of \( \lambda \) a nonlinear problem has to be solved.

We introduce a time-domain variant (SSPT) that offers an automatic continuation process, based on PT and adapting the transient stepsize and the \( \lambda \) stepsize at the same time.

We define a time \( t = T \) at which we want to have solved the original DC-problem. We also introduce a time \( T_{\alpha} = \alpha T \) (by default \( \alpha = 0.5 \)) at which ordinary PT will start simulation using the sources as in the original DC-problem, i.e. using \( \lambda = 1 \) and where PT integrates from \( T_{\alpha} \) to \( T' \), where \( T' \leq T \) is the point where all transient effects have become negligible (see also Fig. [1]).

On the interval \([0, T_{\alpha}]\), a special PT integration is performed with the function \( \lambda(t) = t / T_{\alpha} \). Hence, at each time step, also the actual applied source values change. The interval \([0, T_{\alpha}]\) is the switch-on interval, the interval \([T_{\alpha}, T]\) is the interval to damp-out transient effects. On both intervals PT uses an automatic time step determination procedure. On the interval \([T_{\alpha}, T]\) an ordinary PT procedure is executed. Hence,
if, at some time point, the Newton iterative process does not converge, a re-integration will be done with a smaller stepsize. Recursion in controlled sources asks for a modification in \( \psi \). An expression for a controlled voltage source \( E_I(0,1) \) may look like

\[
V(E_I) = 5 + 4(E_I) + 6V(R_I) + 7I(E_I) + 12^2
\]

It is controlled by the controlling "ev’s" (electrical variables) \( I(E_I), V(R_I), \) and \( I(E_2) \). We write the expression for the applied value \( V(E_I) \) as

\[
V(E_I) = \psi(ev_1, ev_2, \ldots, ev_n)
\]

As value during the source stepping at time \( t \) on \([0,T_a]\) we propose to take

\[
\psi(ev_1, \ldots, ev_n) = \psi(ev_1, \ldots, ev_n) + \lambda (t) - 1 \psi(0, \ldots, 0).
\]

Note that in (4), \( \psi(0, \ldots, 0) = 149 \). This value has to be calculated once. When in \( E_2 \) is a controlled voltage source too, contributions to the Jacobian matrix are calculated by \( \frac{\partial \psi}{\partial x} = \frac{\partial \psi}{\partial ev} \frac{\partial ev}{\partial x} \), which gives recursion. Note that \( \lambda \) does not occur in the matrix. Clearly, for \( \lambda = 0 \) the applied voltage is zero (assuming starting from the zero solution, which implies that all ev’s are zero), which makes the zero solution the exact solution. When \( \lambda = 1 \) the original voltage expression is used. Since our equations (1) are DAEs we remark that for all \( t \) the generated solution is consistent for the problem at hand. Because of the switch-on and the ramp-out phase the process mimics a real physical process.

### 3 Results

We tested the SSPT on a set of difficult problems where parameters were swept (temperature, and statistics). The SSPT was always convergent (without needing \( g_{min} \)-iteration). It was 1-13 times faster than Newton-Raphson (that sometimes needed internal \( g_{min} \)-iteration). Normal PT was less robust than SSPT. Further improvements in the time-domain integrations, after starting with a proper \( x_{DC} \), have been tuned to fault analysis [3].

### References