Multi-photon detection with superconducting nanowires

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Multi-photon detection with superconducting nanowires

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ter verkrijging van de graad van doctor aan de Technische Universiteit Eindhoven, op gezag van de rector magnificus prof.dr.ir. C.J. van Duijn, voor een commissie aangewezen door het College voor Promoties, in het openbaar te verdedigen op donderdag 30 oktober 2014 om 16:00 uur

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*Multi-photon detection with superconducting nanowires*, by Zili Zhou


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ABSTRACT

Since superconductivity was discovered in 1911 by H. K. Onnes, it has attracted great interest for understanding its underlying mechanism and for its possible applications. Particularly, the optical response of superconductors has been widely studied, driven by the need of high-performance optical detectors. The first nanowire superconducting single-photon detector (SSPD) was reported by Gol’tsman et al. in 2001, based on a ~4-nm thick and ~100-nm wide NbN superconducting film. When a photon is absorbed in a superconducting film, it breaks a number of Cooper pairs and triggers the transition of the superconductor from superconducting to normal state. By properly reading out the large resistance change due to the superconducting-to-normal transition, the superconductor can be operated as a single-photon detector (SPD). Since the photo-detection mechanism in superconducting film is different from that in semiconductors, SSPDs have unique properties and advantages as compared to the other near-infrared SPDs, offering short response time, low timing jitter, low dark count rate and high quantum efficiency.

In the conventional operation scheme, an SSPD is operated as an SPD, and is not capable of detecting multi-photons. However, optical detectors with multi-photon sensitivity or the capability of resolving photon numbers play important roles in many applications. For instance, by measuring the intensity autocorrelations of the input light, nonlinear optical detectors can be used to measure the temporal width of light pulses and to distinguish quantum and classical fields. Photon-number-resolving (PNR) detectors benefit many applications in quantum information processing, such as quantum communication and linear optical quantum computing. Besides, PNR detectors with large dynamic range bridge up the existing gap left between SPDs and linear optical detectors. In the work presented in this thesis, we will show that superconducting nanowires can be used to realize optical autocorrelators, or as PNR detectors. The merits of SSPDs can be preserved in these applications.

We will show that by correctly choosing the bias current, a superconducting nanodetector (an SSPD with novel design), can be operated as a multi-photon detector. The superconducting nanodetector, combined with an interferometer, can be operated as an ultra-sensitive interferometric autocorrelator, capable of measuring high-order autocorrelations with record sensitivity. The nonlinear photo-detection process in the superconducting nanodetector is studied by using short light pulses. The temporal resolution of the nanodetector-based autocorrelator, which is determined by the dynamics
of the photoexcited quasi-particles in the superconducting nanowires, is measured to be \(\sim 20\) ps.

Then we will present a novel PNR detector based on a series array of \(N\) superconducting nanowire elements, named as \(N\)-element series nanowire detector (\(N\)-SND). Each element of the \(N\)-SND is equivalent to an SSPD. The photo-responses of the \(N\) elements are summed up into a main readout, producing a fast output pulse with a height proportional to the detected photon numbers. Particularly, the 12-SND provides a large dynamic range of up to twelve photons. The analysis of the linearity and of the detection noise on the output signals of the SND shows the potential of increasing the already large dynamic range to even larger photon numbers.
CHAPTER 1

Introduction

This chapter gives a brief review of the existing photon detection technologies, including a few important types of photon detectors, multiplexed photon-number-resolving detectors and photon autocorrelators.

1.1. Photon detection techniques

The detection of light can be traced back to the early 19th century. In 1839, Alexandre Edmond Becquerel observed electrical signals from an electrode which was placed in a conductive solution and illuminated by light [1]. This phenomenon is now known as photovoltaic effect and has been applied in solar cells. In 1887, Heinrich Hertz observed that the spark gap (two conducting electrodes separated by a gap) emitted electric sparks more easily when it was illuminated by ultraviolet (UV) light [2]. Hertz’s observation was later explained by the well known theoretical work of Albert Einstein in 1905 [3] and is now known as ‘the photoelectric effect’. According to Einstein’s theory, light consists of discrete packets of energy, namely photons. When an individual photon is absorbed in a material, it will be able to release an electron from the surface of the material if the photon’s energy is higher than the photoelectric workfunction. Based on the photoelectric effect, Johann Elster and Hans Geitel invented the first practical photoelectric tube in 1914 [4], which made it possible to detect light with very low intensity.

Over one century after the early research work on optical detections, the technique of photon detectors has been rapidly developed and widely used for many applications. Nowadays, varieties of photon detectors are developed, including the photomultiplier tubes (PMTs), the photodiodes, the transition edge sensor (TESs), the nanowire superconducting single-photon detectors (SSPDs) and so on. The characteristics of these detectors will be briefly reviewed later in this chapter.

The modern photon detectors are based on the same working principle of the early optical detectors: the optical-to-electrical conversion. The detector absorbs an input photon (or photons) and consequently generates an electrical signal (current or voltage) which can be readout by electrical instruments. This optical-to-electrical process can be either linear or nonlinear, depends on whether the amplitude of the electrical signal is linear to the input light power or not. All the photon detectors discussed in this thesis can be summarized into Fig. 1.1, where the distribution of the electrical signal’s amplitude (usually the output...
voltage $V_{\text{out}}$) is plotted as a function of the input average photon number per light pulse $\mu$ (here we assume the input light is from a pulsed laser with Poissonian photon statistics in order to compare all the detectors on the same platform).

Fig. 1.1. The $V_{\text{out}}$ distribution of different photon detectors is plotted as a function of the input average photon number per light pulse $\mu$. All the photon detectors discussed in this thesis can be described by a certain distribution in this figure. The dark green distribution represents the linear detectors, including PMTs operated in the continuous mode (Section 1.2), photodiodes operated in the photoconductive (Subsection 1.3.2) and linear-avalanche mode (Subsection 1.3.3). The magenta distribution represents the second-order nonlinear detectors based on the combination of the TPA or SHG process and a linear detector. The blue distribution, which is also plotted in Fig. 1.2 (a), represents the SPDs, including PMTs operated in the pulse mode (Section 1.2), photodiodes operated in the Geiger mode (Subsection 1.3.4), and SSPDs operated in the single-photon region (Section 1.5). The red distribution, which is also plotted in Fig. 1.2 (b), represents the PNR detectors, including certain types of PMTs (Section 1.2), photodiodes operated in the Geiger-mode with self-differencing technique (Subsection 1.3.5), TESs (Section 1.4), temporal- (Subsection 1.6.2) and spatial- (Subsection 1.6.3) multiplexed PNR detectors.

1.1.1. Linear detectors

When the requirement of detection sensitivity is not very high, linear detectors are usually employed in applications. As shown in Fig. 1.1, a linear detector (dark green distribution) gives an output $V_{\text{out}}$ proportional to the value of $\mu$ and thus proportional to the input light power. A few types of photon detectors fall into this category, including photodiodes
operated in photoconductive (Subsection 1.3.2) and linear avalanche mode (Subsection 1.3.3), and PMTs operated in continuous mode (Section 1.2). As linear detectors produce an electrical signal proportional to the input light power, they are widely used in optical communication system as signal receivers. In general, they are capable of detecting down to a few hundreds of photons per light pulse. For weaker signals, their sensitivity is limited by detection noises, e.g. thermal noise and short noise. The noise level in Fig. 1.1 is marked by a dashed black line.

1.1.2. Nonlinear detectors

If the amplitude of a detector’s output electrical signal is not linear to the input light power, the detector is called a nonlinear detector. The conventional nonlinear detectors usually involve either a nonlinear optical absorption process in the early phase of the detection or a nonlinear electric process in the readout scheme. The former requires a nonlinear optical absorber in the detection. For instance, a semiconductor which has a band gap \( E_g \) larger than the energy of an incident photon \( h
\nu \) and smaller than the energy of two photons, i.e. \( h \nu < E_g < 2h \nu \), can be employed as a two-photon absorber. This two-photon absorption (TPA) process can used to realize a second-order nonlinear detector. By employing a TPA material as the absorber of a linear detector (Section 1.2 and Subsections 1.3.2 and 1.3.3), its \( V_{out} \) is proportional to the value of \( \mu^2 \), as shown in Fig. 1.1 (magenta distribution). This can also be achieved by combing a second-harmonic-generation (SHG) crystal (where an optical signal is produced at frequency \( 2\nu \) with an intensity proportional to the square of the input intensity at frequency \( \nu \)) together with a linear detector. This kind of detector is widely used to characterize the temporal profile of ultrafast laser pulses [5-7]. Higher-order nonlinear detectors (not shown in Fig. 1.1) ar e also reported in litera tures [8, 9], but since the optical nonlinear processes are much less efficient in the higher orders, the detection sensitivity becomes worse.

The later form of nonlinear detector involves a nonlinear electric process in the detector or readout scheme, whilst the absorption is linear. The single-photon detectors (SPDs) belong to this category, including PMTs operated in pulse mode (Section 1.2), photodiodes operated in Geiger mode (Subsection 1.3.4), and SSPDs operated in single-photon region (Section 1.5). The SPDs have attracted large interest in the recent years as they play an important role in few applications, particularly in photonic quantum information applications, including quantum communication [10] and linear optical quantum computing (LOQC) [11], and also in long-distance laser ranging [12] and fluorescence-lifetime measurement [13]. They also act as an important tool to investigate the fundamental nature of the light [14] (1.7.3). SPDs are much more sensitive than linear detectors as they are capable of detecting single photons in a light pulse. They involve a strong nonlinear electric
multiplication process in the detection, which can amplify the weak signal generated by a single photon by a few orders of magnitude and results in a measurable electric pulse. The electric pulses will then be sent to a fast counter which counts their number and thus the number of incident single photons. SPDs are not able to resolve the number of photons in a light pulse shorter than their response time, instead they give a binary response corresponding to the detection of either ‘0’ or ‘≥ 1’ photons. As shown in Fig. 1.1 (blue distribution) and Fig. 1.2 (a), the $V_{out}$ distribution of an ideal SPD (with unit quantum efficiency), illuminated by a pulsed light, is plotted as a function of $\mu$. In the calculation we assumed the noise level of the SPD to be the same as that of the linear detector, which is feasible if both the SPD and the linear detector work at low temperature; we also assumed that the average amplitude of the SPD’s photo-response is two times higher than the noise so that we can discriminate the photo-response and the noise. Three profiles of the Fig. 1.2 (a), marked by dashed gray lines, are plotted in Fig. 1.2 (c), showing the histograms of $V_{out}$ when $\mu = 0.1$, 1, and 10, respectively. We can see that as the value of $\mu$ increases, the height of the ‘0’ output level in the histogram, which corresponds to the detection of 0 photon, decreases and the height of the ‘1’ level, which corresponds to the detection of ≥1 photon, increases. By setting the trigger level [dashed green line in Fig. 1.2 (a)] of the fast counter between the ‘0’ and ‘1’ output levels, the counter measures the number of the pulses higher than the trigger level in terms of count rate (CR). The CR of the SPD (solid black line) is plotted as a function of $\mu$ to the right axis of Fig. 1.2 (a). We can see that the CR is linear to $\mu$ when $\mu <<1$ and saturates to its maximum value when $\mu >>1$. We should point out that the detection of more than one photon also contributes to the total CR. But since the probability of having $N$ photons in a pulse is $\mu^N N! / N!$ times smaller than that of having a single photon, due to the Poissonian statistics of the source, their contribution is negligible when $\mu <<1$. This enables a true single-photon counting.

The TPA technique can also be combined with SPDs. As we will show later in this chapter, by choosing a right material for the absorber, PMTs operated in pulse mode (Section 1.2) and photodiodes operated in Geiger mode (Subsection 1.3.4) can be operated as two-photon detectors. By lowering the bias current, SSPDs (Section 1.5) can also be operated as two-photon detectors (or even multi-photon detectors, depending on their bias current). The CR of these two-photon detectors is proportional to the value of $\mu^2$, as shown in Fig. 1.2 (a) (dashed black line to the right axis). They can be used for the characterization of the ultrafast laser pulses’ temporal profile and for measuring high-order correlation functions. As will be discussed in later in this thesis, the two-photon detection (or multi-photon detection) process in SSPDs is different from that in photodiodes and PMTs combined with TPA technique. The optical absorption in SSPDs is a linear process and the nonlinear process occurs in the detection after photon absorption. So the sensitivity of the two-photon
detection (or multi-photon detection) in SSPDs is in principle higher than that of the photodiodes and PMTs based on TPA because it does not involve high-order optical nonlinearities.

Fig. 1.2. (a) $V_{out}$ distribution of an ideal SPD, plotted as a function of $\mu$. The CR of SPDs (solid black line) and two-photon detectors (dashed black line) is also plotted as a function of $\mu$ to the right axis. (b) $V_{out}$ distribution of an ideal PNR detector plotted as a function of $\mu$. (c) Three profiles of (a) (marked by dashed gray lines), showing the $V_{out}$ histograms for $\mu = 0.1$, 1, and 10. (d) Two profiles of (b) (marked by dashed gray lines), showing the $V_{out}$ histograms for $\mu = 1$, and 10.

1.1.3. Photon-number-resolving detectors

Photon-number-resolving (PNR) detectors are capable of counting the number of photons in a light pulse. They have applications (or potential applications) in quantum communication [10] and LOQC [11]. PNR detectors can be classified into two categories: those based on a single detecting element and those using multiplexing techniques. As it will be shown later in this chapter, the former include photodiodes operated in the Geiger-mode with self-differencing technique (Subsection 1.3.5), certain types of PMTs (Section 1.2) and TESs (Section 1.4), and the latter include temporal- (Subsection 1.6.2) and spatial- (Subsection 1.6.3) multiplexing techniques based on a group of SPD elements.
The output signal of a PNR detector is proportional to the detected number of photons per pulse $n$. As shown in Fig. 1.1 (red distribution) and Fig. 1.2 (b), the distribution of $V_{out}$ from an ideal PNR detector (with unit quantum efficiency and a perfect linearity), illuminated by a pulsed laser, is plotted as a function of $\mu$. In the calculation, we assumed the noise (independent of $n$) and the single-photon detection amplitude of the PNR detector were the same as those of the SPD as shown in Fig. 1.1 (blue distribution) and Fig. 1.2 (a), in order to distinguish the ‘$n$’ and ‘$n+1$’ output levels. Two profiles of Fig. 1.2 (b), corresponding to $\mu = 1$ and 10, are marked by dashed gray lines and plotted in Fig. 1.2 (d). When the value of $\mu$ increases, the output levels corresponding to the detection of larger photon numbers appear. Indeed, from the $V_{out}$ histogram of the PNR detector, one can reproduce the Poissonian distribution of $n$ in the input laser pulse. It is interesting to see that, the response of the PNR detector converges to the one of a conventional linear detector for large photon numbers, when the standard deviation of the Poisson distribution ($\Delta = \mu^{0.5}$) becomes much smaller than $\mu$, and the histograms of the photon number distribution become a continuous shot noise distribution. Therefore, the PNR detectors fill up the ‘gap’ (marked by dashed gray lines in Fig. 1.1) left between the SPDs and the conventional linear detectors.

1.2. Photomultiplier tubes (PMTs)

A schematic construction of a PMT is shown in Fig. 1.3 [15]. A PMT is a vacuum tube, basically consisting of an input window, a photocathode, several dynodes, an anode and a readout circuit. When an incident photon (or photons) with sufficient energy passes through the input window and impinges on the photocathode, an electron will be emitted from the surface of the photocathode material (known as the photoelectric effect) and accelerated by the focusing electrode to a series of dynodes. The dynodes multiply this early-stage photocurrent by a few orders of magnitude via a series of secondary emission events. The amplified photocurrent is then collected by the anode and directed to the readout circuits.
The gain of the multiplication process can be very large (up to $10^8$) [15], enabling the detection of single photons.

According to their operation methods, PMTs can be generally classified into two categories: the continuous mode and the pulse mode, depending on the input light intensity and the readout circuits. When the input light is weak ($\mu < 1$), the output of PMT will be constituted by discrete electric pulses. Each electric pulse corresponds to a single-photon detection event (or a multi-photon detection event with much smaller probability). By counting the number of these signal pulses one can determine the number of detected single photons. In this case, the PMT works in the pulse mode. When the input light is strong, the density of the discrete electric pulses per unit time largely increases so that they overlap with each other and form a continuous photocurrent signal. In this case, it is not feasible to measure the CR but much more convenient to use the photocurrent as the readout signal. Therefore, the PMT will be operated in the continuous mode where the photocurrent is proportional to the number of detection events.

If $h\nu > E_g + E_{ea}$, where $E_g$ is the bandgap energy of the photocathode material (assumed to be a semiconductor), and $E_{ea}$ is the electron affinity, absorption of a single photon can trigger a detection event, so that the PMT operated in the continuous mode is a linear detector (described by the dark green distribution in Fig. 1.1), and the PMT operated in the pulse mode is an SPD [described by the blue distribution in Fig. 1.1 and the solid black line in Fig. 1.2 (a)].

If $h\nu < E_g$ and $2h\nu > E_g + E_{ea}$, the TPA effect can be used in PMTs. As shown in the inset of Fig. 1.4 (a), the TPA process can be considered as a transition of an electron from valence-band (VB) to a ‘virtual’ state in the energy gap, followed by a transition to a vacuum state in the conduction band (CB) [7, 16]. In this case, the output of the PMT is proportional to the value of $\mu^2$. For instance, as shown in Fig. 1.4 (a) [16], when the PMT using Cs$_3$Sb as its photocathode material was operated in the continuous mode under the illumination of a 0.8-µm laser, its photocurrent is proportional to square of light power (slope=2). The linear response in the low-power range (slope=1) was due to a small contribution of weak single-photon-induced photocurrent. As shown in Fig. 1.4 (b) [6], the CR of a pulsed-mode PMT using GaAs as its photocathode material is proportional to the square of the light power (1.5-µm pulsed laser). The former and the latter are described by the magenta distribution in Fig. 1.1 and by the dashed black line in Fig. 1.2 (a), respectively.
Fig. 1.4 (a) Photocurrent generated by a Cs$_3$Sb PMT, which was illuminated by a 0.8-µm pulsed laser, plotted as a function of the input light power in log-log scale [16]. In the high-power range, the photocurrent is proportional to square of light power (slope=2) due to the TPA effect. The linear response in the low-power range (slope=1) was due to a small contribution of weak single-photon-induced photocurrent. Inset: a schematic diagram of TPA process in the photocathode material of a PMT. (b) Measured CR of a GaAs PMT, which was operated in the pulse mode and illuminated by a 1.5-µm pulsed laser, plotted as a function of the input light power in log-log scale. The CR is proportional to the square of the power (slope=2) [6].

Fig. 1.5. A microchannel-plate PMT resolved up to twelve photons from a pulsed source [17].

Certain types of PMT are able to resolve photon numbers. For instance, microchannel-plate PMTs [17] have a high secondary emission coefficient, which reduces the excess noise in the multiplication process, and thus provides a good PNR capability. As shown in Fig. 1.5 [17], a microchannel-plate PMT resolved up to twelve photons from a pulsed source. The
PMTs with the PNR capability are described by the red distribution in Fig. 1.1 and in Fig. 1.2 (b).

In the telecommunication wavelength range, PMTs can offer an efficiency of a few percent with a dark count rate (DCR) down to a few hundreds of kHz [18]. They have a jitter of down to a few hundreds of ps and their maximum CR can reach up to 10 MHz [18]. Telecom-wavelength PMTs have not been shown to have PNR capability to the best of our knowledge.

1.3. Photodiodes

A photodiode is based on a p-n (or PIN) junction as shown in the inset of Fig. 1.6 (a) and (b). When a photon is absorbed in the depletion area of a photodiode, it creates an electron-hole pair. The electron-hole pair is driven by the built-in electric field so that the electron moves towards the cathode (n-doped region) and the hole towards the anode (p-doped regime). If no bias voltage (or a low reverse bias) is applied to the diode, as shown in Fig. 1.6 (a), the number of detected photons equals to the number of generated charges so there is no internal gain in the detection process. If a high reverse bias is applied to the diode, as shown in Fig. 1.6 (b), the initial electron and hole will be accelerated by the additional electric field with high kinetic energy and create secondary electron-hole pairs via a process called impact ionization, leading to an avalanche multiplication of photocurrent. Since this photon detection technique is based on an avalanche process, the photodiodes operated in this mode are well known as avalanche photodiodes (APDs).

A photodiode can be operated in different modes by choosing the bias [Fig. 1.6 (c)] and readout circuits (Fig. 1.7), respectively, depending on the applications. The current-voltage (IV) characterization of a photodiode under the light illumination with varying powers are shown in Fig. 1.6 (c). The dashed ovals indicate four operation modes of the photodiode: the photovoltaic mode (green), the photoconductive mode (magenta), the linear-avalanche mode (blue) and the Geiger mode (red), which will be introduced in the following.
Fig. 1.6. Schematic diagrams of the photon detections in a photodiode with no bias (a) and a high reverse bias (b). (c) The IV characterization of a photodiode under the illumination of light with varying powers. The dashed ovals indicate four operation modes of the photodiode: the photovoltaic mode (green), the photoconductive mode (magenta), the linear-avalanche mode (blue) and the Geiger mode (red).

1.3.1. Photovoltaic mode

The photodiode works in the photovoltaic mode when no bias voltage is applied, as shown in Fig. 1.6 (c) (dashed green oval). A typical readout circuit in this operation mode is shown in Fig. 1.7 (a). As mentioned, when a photon is absorbed in the depletion area, the generated electron-hole pairs are swept to the two ends of the diode by the intrinsic built-in electric field of the diode, producing a voltage signal to the readout. No internal gain will be introduced in this mode. The operation mode can be used for solar cells, which in principle consist of an array of large area photodiodes.

1.3.2. Photoconductive mode

The photodiode works in photoconductive mode when a reverse bias voltage much higher than the breakdown voltage $V_{BR}$ is applied, as shown in Fig. 1.6 (c) (dashed magenta oval). The readout circuit of this mode is shown in Fig. 1.7 (b). There is no internal gain presents in this detection mode.

When the energy of the incident photon is larger than the band gap of the diode’s material, i.e. $h\nu > E_g$, the photoconductive-diode is operated as a linear detector (described by the dark green distribution in Fig. 1.1), which can have a large range of linearity of several
orders of magnitude. This mode is also faster than the photovoltaic mode since the reverse bias enlarges the depletion layer, which decreases the capacitance of the junction and thus the response time.

As shown in Fig. 1.7 (b), when the condition of the TPA is met, e.g. $h\nu < E_g < 2h\nu$, the absorption of every two photons creates an electron-hole pair, so the photocurrent is proportional to the value of $\mu^2$. This phenomenon was first reported by Takagi et al. [5] in 1992. They observed that the photocurrent generated by a GaAsP photoconductive-diode, illuminated by a 1.06 μm laser, was proportional to the square of the input light power, as shown in Fig. 1.8 (a). The TPA-based photoconductive-diodes are described by the magenta distribution in Fig. 1.1.
1.3.3. Linear-avalanche mode

The photodiode works in the linear-avalanche mode when a reverse bias voltage slightly lower than the $V_{BR}$ is applied, as shown in Fig. 1.6 (c) (dashed blue oval). The readout circuit of this mode is shown in Fig. 1.7 (b). In this mode, the generated electron-hole pairs are accelerated by the applied electric field and create secondary carriers by impact ionization, leading to a non-diverging carrier multiplication process. Internal gain is therefore obtained in this linear avalanche process and the photodiodes operated in this mode are called APDs or linear APDs.

Similar to the photoconductive-diode, when $h\nu > E_g$, the linear APD is operated as a linear detector (described by the dark green distribution in Fig. 1.1). Due to their large internal gain, the linear APDs offer high detection sensitivity and are widely used for many applications such as optical communications where sensitive detectors are needed. When $h\nu < E_g < 2h\nu$, TPA effect can be used in the detection with linear APDs. In this case, the generated photocurrent is proportional to the value of $\mu^2$. The TPA-based linear-APDs are described by the magenta distribution in Fig. 1.1.
1.3.4. Geiger mode

The photodiodes operated in the Geiger mode provides single-photon sensitivity and are used as SPDs. They are therefore named as single-photon avalanche diodes (SPADs). As shown in Fig. 1.6 (c) (dashed red oval), the SPAD works with a reverse bias voltage higher than the $V_{BR}$. The corresponding readout circuit is shown in Fig. 1.7 (c). In this mode, the absorption of a single photon can trigger a diverging avalanche multiplication of the photocurrent and produces a fast electric pulse in the readout. The internal gain of SPADs is much larger than that of linear APDs. By counting the number of the generated pulses, the number of detected single-photons can be determined. Since the avalanche is self-sustaining, the reverse bias must be reduced in order to quench the avalanche current and make the diode ready for the next detection. The quenching can be implemented with a series resistance (passive quenching) or with a circuit which controls the bias current (active quenching). In particular, for InGaAs SPADs used in the telecommunication wavelength range, a gating circuit is used to make the detector active during a short time period, as shown in the upper inset of Fig. 1.7 (c).

Similar to the TPA-based photoconductive-diodes and the TPA-based linear-APDs, the SPADs can also be combined with the TPA. For instance, as shown in the lower inset of Fig. 1.7 (c), when $h\nu < E_g < 2h\nu$, the absorption of two photons can trigger the detection and produce a readout pulse. Therefore, the CR of the signal pulses will be proportional to the value of $\mu^2$. For instance, as shown in Fig. 1.8 (b), it was reported by Roth, et al. [6] that the measured CR from a Si SPAD, which was illuminated by a 1.5-μm laser, was proportional to the square of the input light power. In the low-power region, a small contribution of the single-photon detection presents (slope=1), which was due to the absorption tail in the Si absorber beyond 1.1 μm. The TPA-based SPAD is described by the dashed black line in Fig. 1.2 (a).

1.3.5. SPADs with self-differencing technique

In 2008, Kardynal, et al. [19] reported that an individual photodiode was capable of resolving photon numbers by applying self-differencing technique to its readout signal. As shown in Fig. 1.7 (d), the photodiode is operated in the Geiger mode, but differently from Fig. 1.7 (c), the response of the SPAD, i.e. $V_{out}$, is sent to a self-differencing circuit. In the self-differencing circuit the signal $V_{out}$ is split into two paths, one of which introduces a delay of one period of the alternating bias voltage relative to the other path, and the difference between the signals in the two paths, i.e. $V_{sd}$, is obtained in the output. With the self-differencing technique, the weak signal shortly after the onset of avalanche build-up, which holds the photon number information, is measured by removing the capacitive
response of the diode. As reported in Ref [19], a self-differencing SPAD measured up to four photons as shown in Fig. 1.9, although the discrimination of the photon levels $\geq 2$ is poor due to the low signal-to-noise ratio (SNR). The self-differencing SPADs are described by the red distribution in Fig. 1.1.

![Fig. 1.9. Histogram of the $V_{out}$ of a self-differencing SPAD, which was illuminated by a 1.55-$\mu$m pulsed laser with $\mu=1.49$ [19].](image)

In the telecommunication wavelength range, SPADs typically have an efficiency of 10-30 % with DCR of down to a few tens of Hz, a jitter of a few hundreds of ps and the maximum CR of $\sim$10 kHz [20]. The self-differencing SPAD has an efficiency of $\sim$10 % with DCR of $\sim$16 kHz, a jitter of a few tens of ps and the maximum CR of $\sim$100 MHz [19].

1.4. Superconducting transition edge sensors (TESs)

A superconducting transition edge sensor (TES) is a calorimeter based on a superconducting film operated near its transition temperature $T_c$. As shown in Fig. 1.10 (a), a typical resistance-temperature (RT) characteristic of a superconductor is depicted. When a photon (or photons) is absorbed in the active area of a TES, it slightly changes the temperature in the superconducting film by an amount proportional to the deposited photon energy. Since the TES is biased at the edge of the superconducting-to-normal transition, a small change in its temperature ($\Delta T$) results in an abrupt change in its resistance ($\Delta R$) and thus in the bias current. As shown in Fig. 1.10 (b), a superconducting quantum-interference device (SQUID) array is used to amplify this small change in the current for the readout. The readout voltage $V_{out}$ is proportional to the absorbed photon energy, which enables the TES the intrinsic capability of photon-number-resolving.
Fig. 1.10. (a) The RT characteristic of a TES. The device is biased at the edge of the superconducting-to-normal transition so that a small change in its temperature ($\Delta T$) due to the absorption of photons results in a large change in its resistance ($\Delta R$). (b) A readout circuit for a TES, where a SQUID amplifier array is used for the readout.

Fig. 1.11. Inset: The photo-response curves of a tungsten-TES under the illumination of a pulsed laser with $\mu=4$, recorded by a sampling oscilloscope. The multiple output levels correspond to the detection of up to nine photons [21]. Main panel: histogram of the output from a tungsten-TES at a certain light power. The discrete peaks indicate the detections of up to seven photons [22].

Typical photo-response curves of a tungsten-TES recorded by a sampling oscilloscope are shown in the inset of Fig. 1.11 [21]. Multiple output levels were obtained when the TES was illuminated with a pulsed laser with an average photon number of $\mu = 4$ per pulse, corresponding to the detections of up to nine photons. Another example of the photo-
response of a tungsten-TES is shown in the main panel of Fig. 1.11 [22], where the discrete peaks in the histogram indicate the detections of up to seven photons. As a PNR detector, a TES is described by the red distribution shown in Fig. 1.1.

TESs provide very high efficiency (95%) in the telecommunication wavelength with ultralow DCR [23]. They have relatively poor time responses, including long reset time (in the scale of µs as shown in the inset of Fig. 1.11) and large jitter (~100 ns) [23] due to the slow thermal process. Their requirement of ultralow working temperature (~100 mK) increases the complexity of operation.

1.5. Superconducting single-photon detectors (SSPDs)

Superconducting single-photon detectors [24] are based on superconducting nanowires with a thickness of ~4-5 nm and a width of ~100 nm. The superconducting nanowire (made of NbN, NbTiN, WSi, etc) is kept far below its critical temperature $T_C$ and biased with a current ($I_B$) lower than the critical current $I_C$. As shown in Fig. 1.12 (a), when a photon is absorbed in the wire, it produces a group of quasi-particles (QPs) in the wire [25, 26]. The mechanisms of how the QPs finally trigger the detection are still under debate [27]. Different models are proposed to describe how the presence of QPs triggers the detection, including normal-core hot-spot model [24], QP-diffusion model [28], single vortex crossing model [29], and vortex-antivortex pair (VAP) crossing model[30, 31]. In all the detection models, the presence of QPs suppresses the superconductivity in the wire and creates a
resistive section across the wire. Due to the joule heating, the initial resistive section grows, redirects the bias current to the readout through a bias-T and forms a measurable voltage pulse, as shown in Fig. 1.12 (c). The work presented in Chapter 3 attempts to enable a better understanding of the detection process.

When $I_B$ is close to $I_C$, the absorption of a single-photon is enough to trigger the detection. In this case, the SSPD is operated as an SPD. As shown in Fig. 1.13 [24], the CR measured from an SSPD, which was biased with an $I_B$ of 92% of the $I_C$, is plotted (white squares) as a function of $\mu$ in log-log scale. The fitting slope of 1 indicates the single-photon detection. The SSPDs operated in the single-photon regime are described by the blue distribution in Fig. 1.1 and the solid black line in Fig. 1.2 (a).

![Fig. 1.13. Measured CR of an SSPD plotted as a function of $\mu$ for $I_B = 0.9 I_C$ (white squares) and $I_B = 0.8 I_C$ (black squares), respectively. The fitting slope of 1 and 2 indicates the single-photon and two-photon regions, respectively [24].](image)

It was also found in the initial experiments on SSPDs [24] that when the $I_B$ is reduced, the energy of a single-photon is not enough to trigger the detection, so the absorption of more than one photon is needed. As shown in Fig. 1.12 (b), with a lower $I_B$, the absorption of two photons is needed for a detection event. In this case, one readout pulse corresponds to the detection of two photons, so CR is proportional to $\mu^2$. As shown in Fig. 1.13 (black squares), when $I_B$ was 80% of the $I_C$, the fitting slope equals two, indicating the two-photon detections. SSPDs operated in the two-photon mode are described by the dashed black line in Fig. 1.2 (a). It should be noted that the two-photon detection in an SSPD is based on a linear optical absorption process. The nonlinear part of the two-photon detection process
occurs after the optical absorption. So it is more efficient than the TPA process used in photodiodes and PMTs, which is a nonlinear optical process. The advantage of using linear optical absorption and nonlinear detection in SSPDs will be discussed in the Chapter 4.

SSPDs provide the best comprehensive performance in the telecommunication wavelength range. They can offer a very high efficiency of up to 93% with a negligible DCR of less than 1 Hz [32]. Their jitter can be down to a few tens of ps and their maximum CR can reach up to 1G Hz [33].

1.6. PNR detectors based on multiplexing techniques

PNR detectors can be generally classified into two categories: single-element PNR detectors and multiplexed PNR detectors. The former were already introduced, including TESs, certain types of PMTs and SPADs with self-differencing technique. The latter are realized by multiplexing the input light over a number of SPD elements.

1.6.1. Working principle of multiplexed PNR detectors

![Fig. 1.14. Schematic diagram of a multiplexed PNR detector.](image)

The working principle of multiplexing technique is schematically depicted in Fig. 1.14. When a light pulse with \( n \) photons reaches the multiplexed detector, it is equally split into \( N \) identical elements. In each element there is an SPD which tells if this element detects a photon or not. For an ideal multiplexed PNR detector with an infinite number of elements, by counting the number of the clicking elements, one can count the number of photons in the input light pulse. In the real case, for a multiplexed PNR detector with a limited number of elements, it is possible that more than one photon is coupled into an element, so that the number of clicking elements is smaller than the number of input photons.
According to Fitch, et al. [34], for a multiplexed PNR detector with \( N \)-elements, which is illuminated by a coherent laser source, the probability \( P \) of detecting \( n \) photons can be written as,

\[
P^N_\eta(\eta, \mu) = \sum_{m=n}^\infty \frac{N!}{n!(N-n)!} \frac{(\eta\mu)^\eta e^{-\eta\mu}}{m!} \times \sum_{j=0}^m (-1)^j \frac{n!}{j!(n-j)!} \left[ 1 - \eta + \frac{(n-j)\eta}{N} \right]^m,
\]

(1.1)

Where \( \eta \) is the quantum efficiency and \( \mu \) is the average input photon number per pulse. For instance, when \( \eta = 1 \) and \( \mu = 10 \), the value of \( P^N_\eta(\eta, \mu) \) for different \( N \) values are calculated based on Eq. 1.1 and plotted in Fig. 1.15.

![Fig. 1.15. Calculated distribution of \( P^N_\eta(\eta, \mu) \) for different \( N \) values.](image)

As shown in Fig. 1.15, when \( N = \infty \), the calculated detection probability distribution reproduces the Poissonian statistics of the light source for \( \mu = 10 \); for small \( N \) values, the calculated distribution deviates from Poisson photon number distribution, due to the possibility of several photons absorbed in the same element. Therefore, it is necessary for a multiplexed PNR detector to have as many elements as possible (a large dynamic range) so that the probability of more than one photon enters one element can be minimized.
Multiplexed PNR detectors are described by the red distribution in Fig. 1.1. They can be further classified into two sub-categories: temporal-multiplexed PNR detectors and spatial-multiplexed PNR detectors, as shown in the following.

1.6.2. Temporal-multiplexing

As shown in Fig. 1.16 (a) [22], in the temporal-multiplexing technique, the input light pulse is split equally into $N$ portions separated in time by a series of delay lines and fiber couplers so that they can detected sequentially by two SPDs. Each temporal mode together with the two SPDs is equivalent to one element as shown in Fig. 1.14. Both SPADs [35] and SSPDs [36] were used in the temporal-multiplexing technique. This technique is intrinsically limited by its long response time due to the additional delay intervals, and by the loss introduced in the fibers and couplers.

![Fig. 1.16. (a) Schematic diagram of a temporal-multiplexed PNR detector [22]. (b) An SEM image of a spatial-multiplexed PNR detector with six elements. The six elements are marked by different colors [37].](image)

1.6.3. Spatial-multiplexing

A spatial-multiplexed PNR detector consists of an array of SPDs. The device requires a uniform illumination from the input light beam so that the incident photons have the equal chance to be detected by each element. The photo-responses from different elements can be read out separately [38], which requires a separate amplification and discrimination circuit for each element and is therefore not scalable to tens of photons. Alternatively, the elements can be connected together in such a way that their photo-responses are summed up in a single readout signal with an amplitude proportional to the number of clicking elements and thus to the number of detected photons. For example, a spatial-multiplexed PNR detector with an array of six SSPD elements is shown in Fig. 1.16 (b) [37]. The six elements are connected in parallel and their photo-responses are summed up in the main readout, with the amplitude proportional to the incident photon number. Such device is named as parallel...
nanowire detectors (PND) due to its parallel design. A similar design but with a series connection of $N$ ($N = 4, 8, 12, 24$) SSPD elements, namely series nanowire detectors (SND) will be discussed in Chapter 5. The SPADs were also employed in the spatial-multiplexing technique [39].

1.7. Photon autocorrelators

1.7.1. Introduction

Photon autocorrelators are one of the most common techniques based on multi-photon detection and are used to characterize the coherence properties of a light beam. The light sources with different coherent properties have different temporal characteristics. As shown in the schematic plot of Fig. 1.17 (a), the photons emitted from a chaotic source, e.g. a thermal source (blue dots), tend to bunch together in time; the photons emitted from a single-photon source (red dots) are anti-bunched (i.e. isolated from each other) in time; the photons from a coherent source, e.g. a continuous-wave (CW) laser (green dots), are randomly distributed in time; the photons from a pulsed laser source (black dots) are strongly bunched in time at a certain repetition rate, due to the fact that the light intensity from the source is intentionally modulated in order to create the pulses.

![Fig. 1.17](image)

Fig. 1.17. (a) Schematic plot of the temporal characteristics of photons emitted from a thermal source (blue dots), a single-photon source (red dots), a CW laser (green dots), and a pulsed laser (black dots). (b) Calculated second-order correlation function, i.e. $g^2(\tau_d)$, of a thermal source (blue line), a single-photon source (red line), a CW laser (green line) and a pulsed laser (black line), plotted as a function of $\tau_d$. 

Optical autocorrelation measures the temporal characteristics of the input light and thus determines the coherence properties of the source. The first-order (field) correlation function is defined as,

$$g^{(1)}(\tau_d) = \frac{\langle E^*(t)E(t+\tau_d) \rangle}{\langle |E(t)|^2 \rangle}, \quad (1.2)$$

Where $\tau_d$ is a time delay and the symbol $\langle \rangle$ denotes a temporal average over a long time interval. The $g^{(1)}(\tau_d)$ characterizes the temporal coherence of a source and can be measured using an interferometer together with a conventional linear detector or an SPD.

The second-order (intensity) correlation function is defined as,

$$g^{(2)}(\tau_d) = \frac{\langle I(t)I(t+\tau_d) \rangle}{\langle I(t) \rangle^2}, \quad (1.3)$$

The $g^{(2)}(\tau_d)$ characterizes the temporal duration of pulsed sources and can be used to distinguish quantum and classical fields. For instance, as shown in Fig. 1.17 (b), the calculated values of $g^{(2)}(\tau_d)$ of a thermal source (blue line), a single-photon source (red line), a CW laser (green line) and a pulsed laser (black line) are depicted as a function of $\tau_d$. The $g^{(2)}(\tau_d)$ of the thermal source has the maximum value of 2 at zero delay, representing the temporal characteristic of bunching. The $g^{(2)}(\tau_d)$ of the single-photon source has the minimum value of 0 at zero delay, representing anti-bunching. The $g^{(2)}(\tau_d)$ of the CW laser equals 1 due to the fact that the photons are randomly distributed in time and uncorrelated with each other. The $g^{(2)}(\tau_d)$ of the pulsed laser, as will be shown in Chapter 4, has a maximum value of 2 at the zero delay.

To measure the $g^{(2)}(\tau_d)$, a photon autocorrelator usually needs two key components: a delay line which introduces the time delay $\tau_d$ and a nonlinear element which realizes the multiplication of light intensity. Depending on how these two parts are functionalized, the photon autocorrelators can be classified into two categories: those based on nonlinear optical detectors (Subsection 1.7.2) and Hanbury Brown and Twist (HBT) autocorrelators (Subsection 1.7.3), as shown in the following.

1.7.2. Photon autocorrelators based on nonlinear optical detectors

An autocorrelator based on a second-order nonlinear optical detector, e.g. a TPA-based detector [7, 40], is schematically depicted in Fig. 1.18 (a). The TPA-based detector, as discussed earlier in this chapter, can be a photodiode or a PMT, operated in the two-photon
mode, giving an output signal proportional to the square of the light intensity (or to the value of $\mu^2$). The input light beam is equally split into two paths by a beam-splitter. The light in one of the paths is delayed by a time $\tau_d$ relative to that in the other path. These two paths are then recombined in the beam-splitter and sent into the TPA-based detector which gives a signal proportional to the square of the total intensity. The output of the second-order TPA-based autocorrelator can be written as:

$$\langle I_{\text{tot}}^2 \rangle \propto \langle I^2(t) \rangle + 2 \langle I(t)I(t+\tau_d) \rangle + \text{interference terms}, \quad (1.4)$$

After filtering out the interference terms in Eq. 1.4 using a low-pass filter, the value of $g^{(2)}(\tau_d)$ is obtained from the second term of Eq. 1.4.

![Schematic diagrams of a TPA-based autocorrelator (a) and a SHG-based autocorrelator (b).](image)

An SHG-based autocorrelator [41], as shown in Fig. 1.18 (b), has the same working principle of TPA-based autocorrelators. The only difference between them is that in an SHG-based autocorrelator the TPA-based detector is replaced by an SHG crystal combined with a conventional linear detector or an SPD.

Autocorrelators based on the TPA or SHG technique offer very high temporal resolution (~fs). The TPA process in a semiconductor requires the two photons to be absorbed within the Heisenberg lifetime of the virtual state in the bandgap, i.e. $\hbar/E_g$, which sets the temporal resolution of the TPA-based autocorrelators [7]. The temporal resolution of SHG-based autocorrelators is limited by the bandwidth of the nonlinear crystal and the dispersion of short pulses in the crystal [42]. However, their sensitivity is limited by the low nonlinear susceptibilities involved in the SHG or TPA process. Due to the even lower relevance of higher-order optical nonlinearities, they cannot be effectively applied to the measurement of the autocorrelations of order $N > 2$. 

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1.7.3. Hanbury Brown and Twist autocorrelator

Hanbury Brown and Twiss (HBT) autocorrelators [14] provide an alternative approach to the measurement of second-order correlation function. A second-order HBT autocorrelator is depicted in Fig. 1.19. It consists of two SPDs and an electronic correlation card. The input light is equally split into two channels. Each channel has an SPD which gives an output proportional to the light intensity $I(t)$. The output signals of the two SPDs are then sent to the correlation card, which measures the histogram of delays between the signals from the SPDs. The final output of the HBT autocorrelator is proportional to the value of $I(t)I(t+\tau_d)$ and thus to the value of $g^{(2)}(\tau_d)$.

![Fig. 1.19. Schematic diagram of a HBT autocorrelator](image)

In HBT autocorrelators, SPDs with high sensitivity can be used so that they can offer much higher sensitivity than the autocorrelators based on a nonlinear process. However, their temporal resolution is limited $\geq 50$ ps by the jitter of the SPDs and of the amplification and correlation electronics.

1.8. Scope of the thesis

In this thesis, multi-photon detection processes in the superconducting nanowire detectors are investigated. Two main types of multi-photon detectors are studied: a nonlinear $N$-photon threshold detector ($N = 1, 2, \cdots$), whose response is described by the black lines in Fig. 1.2 (a), and a PNR detector, whose response is described by the red distribution in Fig. 1.1 and Fig. 1.2 (b). As both types of the detectors inherit the merits of SSPDs, they provide better performance as compared to other conventional detectors which belong to the same category. In Chapter 2, the experimental and theoretical methods used for the work of this thesis will be introduced, including the fabrication and characterization of the devices, the details of experimental setups, and the simulation methods. In Chapter 3, measurements of the intrinsic response time of a superconducting nanodetector, which is limited by the QP relaxation process in superconducting nanowires and determines the timescale of the $N$-photon response, will be discussed. Theoretical modes on the multi-photon detections in the detector will be introduced to compare with experimental results.
In Chapter 4, the experimental demonstration of a superconducting nanodetector as an ultrasensitive $N$-photon interferometric autocorrelator will be presented. Theoretical modes on the $N$th-order interferometric autocorrelations will be introduced to compare with the experimental results. Possible improvements on the autocorrelator’s detection sensitivity will be tested by simulations. In Chapter 5, PNR detections realized by a series array of $N$ SSPD elements will be demonstrated ($N = 4, 8, 12, 24$). The simulations which explain the experimental results will also be presented. Finally, Chapter 6 summarizes and concludes the work of the thesis.
References:


CHAPTER 2

Methods

In this chapter the methods used for the fabrication, characterization and simulation of the superconducting nanowire detectors will be introduced.

2.1. Fabrication of the device

The nanofabrication of the devices used for the work of this thesis, including the superconducting nanodetectors (Chapter 3 and Chapter 4) and the SNDs (Chapter 5), was performed by dr. F. Mattioli, dr. A. Gaggero and dr. R. Leoni at the Institute for Photonics and Nanotechnologies (IFN), CNR, Rome. The superconducting NbN films used for these devices were grown by dr. D. Sahin and R. Gaudio at Eindhoven University of Technology (TU/e). The nanofabrication procedures are in principle the same as those used for the fabrication of a standard meander SSPD, and are summarized in the following [1], as shown in Fig. 2.1.

Step (a): A 4–5-nm-thick superconducting NbN film (marked by light gray color) is grown on a GaAs substrate [2], or on a SiO$_2$/Si cavity [3] (marked by purple color), depending on the applications, by reactive magnetron sputtering [4, 5], and coated with a 450-nm-thick positive tone electronic resist polymethyl methacrylate (PMMA, marked by blue color) using a spinner. Then the area of the electric contact pads (patterned as a 50 Ω-coplanar transmission line) and the alignment markers is defined on the PMMA by electron beam lithography (EBL).

Step (b): After the EBL exposure, the PMMA is developed with methyl isobutyl ketone (MIBK) diluted with isopropyl alcohol (IPA) (MIBK:IPA=1:2). The exposed area of PMMA (marked by dark blue color) is removed; the rest part will stay for the next step.

Step (c): The sample is then coated with a Ti/Au film (60-nm-thick Au on 10-nm-thick Ti, marked by gold color), deposited by electron-beam evaporation.

Step (d): The PMMA together with the Ti/Au film deposited on its top is selectively removed by lift-off using acetone. So far, the electric contact pads and the alignment markers are fabricated on the top of the NbN film.
Fig. 2.1. The nanofabrication steps of an SSPD (not to scale) as described in the text. The materials used in the fabrication are marked by different colors in the legend.
Step (e): A layer of positive-tone electronic resist, i.e. hydrogen silsesquioxane (HSQ, FOX-14 diluted in MIBK, marked by green color), with a thickness of ~140 nm, is spun on the sample. The meander pattern of the device (marked by dark green color) is then defined on the HSQ by EBL with the help of the alignment markers.

Step (f): The HSQ pattern is developed in MF322. Only the EBL-exposed area of HSQ remains on the device.

Step (g): The NbN film not covered either by the HSQ pattern or by the Ti/Au is removed by reactive ion etching (RIE, based on CHF$_3$/SF$_6$/Ar gas mixture). So far, the nanofabrication of the SSPD is finished. It is not necessary to remove the HSQ remained on the top of NbN meander wires since it only introduces negligible loss in optical measurement.

The nanofabrication of an SND is in principle the same as that of the SSPD, except that several additional steps need to be added to fabricate the parallel resistors ($R_p$s) and their contact pads:

- After the step (d) as shown in Fig. 2.1, additional steps similar to (a)-(d), including PMMA spinning, EBL pattern defining, developing, electron-beam evaporation and lift off, will be taken to define thin Ti/Au pads [20-nm-thick Au on 5-nm-thick Ti, marked by yellow ovals in Fig. 2.2 (b)] on the NbN, which will be used for the electrical contact of the parallel resistors.

- After the step (g) as shown in Fig. 2.1, additional steps similar to (a)-(d), including PMMA spinning, EBL pattern defining, developing, electron-beam evaporation and lift off, will be taken to define the parallel resistors [40-nm-thick AuPd film, marked by red ovals in Fig. 2.2 (b)] on the contact pads. Each parallel resistor is designed to be 500-nm wide and 3.5-μm long, corresponding to a design value of $R_p = 50 \, \Omega$.

For instance, SEM images of a fabricated superconducting nanodetector and a four-element SND (4-SND) is shown in Fig. 2.2 (a) and (b), respectively. In Fig. 2.2 (a), the nanodetector has a bowtie-shaped constriction with a width of ~150 nm; the constriction is connected to the contact pads (not shown) via meander-shaped wires, which have a width of ~500 nm (marked by white arrows in the lower-right corner) and a length of a few hundred μm. The meander-shaped wires were added on purpose in order avoid latching problem of the device (Section 3.1). In Fig. 2.2 (b), four detection elements of the 4-SND are marked by different colors. The parallel resistors (marked by red ovals) and their
contact pads (marked by yellow ovals) are shown on the left side of the image and are connected to the detection region via 250-nm-wide NbN nanowires. The white bars in Fig. 2.2 (a) and (b) indicate a length of 1 μm and 5 μm, respectively.

![Fig. 2.2. (a) SEM image of a fabricated NbN superconducting nanodetector on GaAs substrate. The nanodetector has a constriction width of ~150 nm; the wider wires with a width of ~500 nm (marked by white arrows in the lower-right corner) and a length of a few hundred μm, were added as the connection between the nanodetector and the contact pads (not shown), in order to avoid latching. (b) SEM image of a fabricated NbN four-element SND (4-SND) on GaAs substrate. The detection areas of the four elements are marked by different colors. The parallel resistors (marked by red ovals) and their contact pads (marked by yellow ovals) are shown on the left side of the image and are connected to the detection region via 250-nm-wide NbN nanowires. The white bars in (a) and (b) indicate a length of 1 μm and 5 μm, respectively.](image)

2.2. Electro-optical characterization of SSPDs

2.2.1. Electro-optical characterization setups

The schematic diagram of the electro-optical setup for the characterization of SSPDs is shown in Fig. 2.3. The SSPD is kept in a cryostat with a working temperature well below its $T_C$ and biased with a current $I_B$ by a bias source (Yokogawa GS200, which can be operated in either the voltage biasing or the current biasing mode) through a 10-Ω bias resistor and the DC arm of a bias-T (mini-circuits ZNBT-60-1W, pass-band: 10 MHz - 6 GHz). In electrical characterization, the RF arm of the bias-T is terminated with a 50-Ω-matched load and the bias-T acts as a low-pass filter which removes the high-frequency electrical noises from the bias source. The $I_B$ is monitored by a voltmeter (Agilent 34410) which measures the voltage across the 10-Ω bias resistor. The value of $I_B$ is calculated by dividing the measured voltage values shown on the voltmeter by the resistance of the bias resistor, i.e. 10 Ω. In optical characterization, the device is illuminated by input light, which is from a laser source and attenuated by a fiber-based variable attenuator (OZ Optics). The
input light power is monitored by a power meter (Ophir nova II), which measures a small portion of the light power, split by a fiber-based beam-splitter (BS). The photon-induced electrical response of the SSPD is collected through the RF arm of the bias-T and amplified by a room-temperature (RT) low-noise amplifier or an amplifier chain (Mini-circuits or MITEQ). The amplitude of the voltage pulses can vary from a few mV to a few V, depending on the amplification and the number of the amplifiers. The amplified voltage pulses are then sent to either a 40-GHz sampling oscilloscope (Agilent 86100) or a 350-MHz fast counter (Agilent 53230A) for analysis.

2.2.2. Current-voltage (IV) characterization

As shown in Fig. 2.4, the current-voltage (IV) characteristic of a superconducting nanodetector was measured in a closed-cycle cryocooler at the temperature of 1.2 K (Subsection 2.3.2). The inset provides an enlarged view of the IV curve near zero bias voltage ($V_B$). This device has the same design as the one shown in Fig. 2.2 (a) but was fabricated with a different NbN film. The IV curve shown in Fig. 2.4 is different from a standard meander SSPD [6]. It can be divided into five regions: the superconducting region (marked by dashed red oval), the first relaxation-oscillation region (dashed blue oval), the ohmic region (dashed green oval), the second relaxation-oscillation region (dashed magenta oval), and the hot-spot plateau region (dashed orange oval), respectively.
Fig. 2.4. The IV characteristic of a superconducting nanodetector measured at a temperature of 1.2 K. The inset provides an enlarged view of the IV curve near zero bias voltage. The IV curve can be divided into five regions: the superconducting region (marked by dashed red oval), the first relaxation-oscillation region (dashed blue oval), the ohmic region (dashed green oval), the second relaxation-oscillation region (dashed magenta oval), and the hot-spot plateau region (dashed orange oval), respectively.

In the superconducting region, an infinite slope on the IV curve is expected since the device is the superconducting state. However, due to the resistance of the cables connected in series to the device, a finite slope corresponding to a resistance of ~15 Ω is observed.

In the first relaxation-oscillation region, \( I_B \) exceeds the critical current of the device \( (I_C = 43.5 \, \mu A) \), which is the critical current of the narrowest section of the constriction. The device enters a meta-stable region where the narrowest section of the constriction oscillates between the superconducting and the normal states, resulting in an oscillation of the current. As the electronics are relatively slower than the oscillation, they only record the average of the oscillating current. The asymmetric shape of the IV curve in the relaxation-oscillation region, which also showed hysteresis, was found to be due to spurious reflections from the amplifiers, which were connected to the device via the bias-T in the IV measurement.

In the ohmic region, where the \( V_B \) is approximately in the range of 6-80 mV, the IV curve shows an ohmic behavior. The slope of the IV curve in this region decreases by about 60 % as the \( V_B \) increases from 6 mV to 80 mV, corresponding to an increase of the device’s
resistance. Indeed, in this region, the $V_B$ is not high enough to make either the meander-shaped wires (500-nm wide) or the wider part of the bowtie-shaped constriction ($\geq$ 500-nm wide) normal. As the $V_B$ increases, only the section of the constriction with a width smaller than the width of the meander-shaped wires, i.e. 500 nm, gradually becomes normal, resulting in an increasing resistance of the device.

When the $V_B$ is high enough, $I_B$ excess the critical current of the 500-nm-wide meander wires ($I_{C2} = 54.1\mu A$). The device enters the second relaxation-oscillation region, where the meander wires oscillate between the superconducting and the normal states, resulting in an oscillation of the $I_B$, similar to the first relaxation-oscillation region.

In the hot-spot plateau region, the $V_B$ is high enough to keep the normal region of the meander wires growing along the wire so that the resistance of the wires increases almost proportionally to the applied $V_B$ and thus the $I_B$ keeps almost constant [7]. By further increasing the $V_B$ (not shown in Fig. 2.4), the total device including the wider section of the constriction will become normal and the IV curve will present another ohmic behavior, which is not shown here in Fig. 2.4.

![Fig. 2.5. Temperature-dependent $I_C$ values (black dots) of a superconducting nanodetector [as shown in Fig. 2.2 (a)], measured in the closed-cycle cryocooler (Subsection 2.3.2). The measurement is fitted by Eq. 2.1 (red line).](image)

A temperature-dependent measurement on the $I_C$ of a superconducting nanodetector [as shown in Fig. 2.2 (a)] was performed in the VeriCold closed-cycle cryocooler during the cooling down (Subsection 2.3.2). The measured $I_C$ is plotted as a function of the varying
temperature \( (T) \) as is shown in Fig. 2.5. The measurement (black dots) is fitted by an empirical formula (red line) written as \[8, 9\],

\[
I_c(T) = I_c(0) \left[ 1 - (T/T_c)^m \right]^n, \tag{2.1}
\]

Where \( I_c(0) \) (\( I_c \) at \( T = 0 \) K), \( m \) and \( n \) are fitting parameters. The best fit to the measurement gives \( I_c(0) = 28.0 \mu A \), \( m = 2.2 \) and \( n = 3.0 \), respectively.

2.2.3. Reset time

![Amplified output voltage pulse](image)

Fig. 2.6. Amplified output voltage pulse (gray dots) of the superconducting nanodetector [as shown in Fig. 2.2 (a)] recorded by a 40-GHz sampling oscilloscope. A fitting (1/e, red line) to the output pulse gives a reset time of ~6.9 ns. Inset: equivalent circuit of the device.

Amplified output voltage pulse of a superconducting nanodetector [as shown in Fig. 2.2 (a)] recorded by the 40-GHz sampling oscilloscope is shown in Fig. 2.6. The temporal characteristics of the voltage pulse can be understood according to the nanodetector’s equivalent circuit as depicted in the inset \[10\]. Before the device absorbs a photon, it is equivalent to a wire with zero resistance and a large kinetic inductance \( L_k \). The absorption of a photon switches the resistance of the wire to its normal resistance \( R_n \), which dynamically changes over time and can be described by an electro-thermal model introduced in Refs \[11, 12\]. Since the value of \( R_n \) is much larger than the load resistance \( R_L \), the \( I_B \) starts to be redirected to the \( R_L \). Consequently, the \( I_B \) in the device decays with a time constant of \( \tau_{\text{dec}} = L_k / (R_L + R_n) \), where the value of \( \tau_{\text{dec}} \) is usually in the scale of \( \approx 10^2 \) ps. At a certain point (within \( \tau_{\text{dec}} \)), the Joule-heating of the nanowire is sufficiently reduced due to
the cooling by the substrate [11], and the device recovers to the superconducting state. Therefore the switch of the device closes and the $I_B$ in the device recovers with a time constant of $\tau_{\text{rec}} = L_k / R_L$. During the period of $\tau_{\text{dec}}$ and $\tau_{\text{rec}}$, as the $I_B$ in the device is sharply decreased, the device is not efficient enough to detect photons. After a time of about $3 \times \tau_{\text{rec}}$, the $I_B$ recovers to $\sim 95\%$ of the original value. The ‘reset time’ of an SSPD is conventionally defined as $\tau_{\text{reset}} = 3 \times \tau_{\text{rec}}$ [13]. In Fig. 2.6, a fitting (1/e, red line) to the output pulse (gray dots) gives a reset time of $\sim 6.9$ ns for the superconducting nanodetector, enabling a repetition rate of $\sim 145$ MHz.

2.2.4. Timing jitter

As illustrated in Fig. 2.7 (a), when a short optical pulse (red) is detected by a detector, the delay $t_d$ between the arriving of the pulse and the generation of the output voltage pulse (gray) varies. The variation of $t_d$ determines the timing jitter of the detector. For instance, the leading edge of the output voltage pulses of a 12-SND (Chapter 5) recorded by a 40-GHz sampling oscilloscope is depicted in Fig. 2.7 (b). The histogram obtained near the center of the leading edge is plotted in the inset of Fig. 2.7 (b), showing a Gaussian distribution of the $t_d$ values for the 12-SND. The full width at half maximum (FWHM) of the leading edge is obtained by fitting the histogram (blue bars) with a Gaussian function (red line), giving a system timing jitter of $\sim 89$ ps. If the timing jitter of the other components in the system (e.g. jitter of the sampling oscilloscope and jitter of the laser source) is relatively small, the system timing jitter can be approximated as the timing jitter.
of the 12-SND. The intrinsic jitter of an SSPD can be as small as \(~30\) ps [14]. The jitter of the 12-SND can be possibly reduced by optimizing the uniformity of the device and the electronics. Another way of measuring the timing jitter is to employ an ultrashort pulsed laser and high-resolution timing electronics with a ‘start-stop’ correlation card, as introduced in Ref [14].

2.2.5. Photon counts and dark counts

![Graph showing photon CR (red dots) and DCR (black dots) as a function of \(I_B\).](image)

As shown in Fig. 2.6, by setting the trigger level (dashed blue line) of the counter above the electrical noise level, the counter counts the number of detected voltage pulses. If the voltage pulses are generated by photon absorption events, they are called photon counts. When an SSPD is operated in the single-photon regime, the generation of a voltage pulse corresponds to the detection of a single photon. The quantum efficiency (QE) of the SSPD is in principle defined as the count rate (CR) divided by the photon flux (number of photons per second). Depending on how the latter is considered, the QE can be specified as system QE (SQE or \(\eta_{\text{sys}}\)), device QE (DQE or \(\eta_{\text{dev}}\)) or internal QE (IQE or \(\eta_{\text{int}}\)). If one considers the number of photons sent into the first optical access of the whole detection system, the SQE is obtained. The SQE includes all the external and internal optical losses in the system. If one considers input photons those which arrive on the active area of the detector, the
DQE is obtained. The DQE excludes the external optical losses, e.g. the loss in the fiber, and only includes the loss due to the mismatch between the light spot and the active area of the device. If one only considers the input photons absorbed by the detector, the IQE is obtained. The IQE further excludes the limited absorptance of the detector, e.g. the loss due to reflection and transmission. The measured photon CR (red dots) of a superconducting nanodetector [as shown in Fig. 2.2 (a)], which worked at 1.2K and was illuminated by a 1.54-µm laser, is plotted as a function of the \( I_B \), as shown in Fig. 2.8. The corresponding SQE is calculated as shown to the right axis of Fig. 2.8. The SQE for the superconducting nanodetector is relatively low (the highest SQE is \(~0.02\%\)) due to the mismatch between the laser spot (\( e^2 \) diameter: \(~5\ µm\)) and the small detective area (\(~150×150\ nm^2\)) of the device.

SSPDs can generate output voltage pulses even without any input signal light. These false detection events will also contribute to the counts and are named as dark counts (DCs). The origins of DC in an SSPD can be attributed to two main reasons. The first is due to the external radiations, either the stray light from outside of the cryostat or the thermal infrared photons emitted by the surrounding materials. This type of DC can be suppressed by shielding the device from the external radiation. The other type of DC is intrinsic. It is due to the spontaneous superconducting-to-normal transitions in the nanowires, depending on the bias current, the temperature, the geometry and the material of the nanowires. The reason for the intrinsic DC is still under debate, and the processes of vortex-antivortex-pair unbinding, single-vortex crossing and phase slips [15-17] have been suggested. In principle, the intrinsic DC event happens more frequently at higher temperatures, with higher bias currents, and in the narrow and longer devices. Dark count rate (DCR) of the nanodetector was measured and plotted (black dots) as a function of the \( I_B \), as shown in Fig. 2.8. The DC measurement was performed after careful shielding of the thermal radiation from the surroundings (Subsection 2.3.2) so that only the intrinsic DCs were measured. As shown in Fig. 2.8, the measured DCR only appears at high bias currents and it is relatively smaller than that of a standard meander SSPD due to its small active area and the low temperature.

2.3. Cryogenic setups

Two cryogenic setups were employed in the experimental work of this thesis: a cryogenic probe station and a closed-cycle cryocooler. They are introduced in the following.

2.3.1. Cryogenic probe station

The schematic of the cryogenic probe station is shown in Fig. 2.9. It is a continuous flow cryostat (Janis ST-500) which provides a stable cryogenic temperature and the access for
the electrical and optical characterization. This setup was initially designed and built by dr. David Bitauld.

In the measurement, the cryostat is pumped (down to $\sim 10^{-5}$ mbar at RT and $\sim 10^{-7}$ mbar at a few K) using a turbo-molecular pump (connected to the cryostat by a stainless steel flexible pumping tube) and is cooled down by liquid helium (LHe) flux. The LHe from a LH vessel is pumped into the cryostat through a high efficiency transfer line by a He pump. The flux of the LHe can be controlled by an adjustable needle valve integrated on the transfer line in order to optimize the temperature of the cryostat. The LHe pumped in the cryostat cools down a copper rod via a heat exchanger. A silicon diode thermometer and a heater are installed inside the copper rod in order to monitor and adjust the temperature of the copper rod in the range of 3.5–450 K. The sample holder is mounted on the top of the copper rod and cooled down by the rod. The temperature of the sample holder can reach...
down to ~4 K, a bit higher that the rod. The heat exchanger, the copper rod and the sample holder are shielded from the surrounding thermal radiation to ensure a low thermal load and to reduce the DCR of the detectors. The top window of the shield is a cold optical band-pass filter which only transmits the visible and near-infrared light and thus eliminates the stray light and far-infrared black body radiation. The sample is glued on the sampler holder using a cryogenic varnish (Lakeshore VGE-7031). Gluing the sample to the holder is critical for the thermal contact between them. A poor thermal contact results in a high working temperature of the device and thus dramatically reduces the $I_c$ and the efficiency of the device. A tight contact between the sample and the holder is always preferred. Two copper clamps are usually used to help the mounting. After the measurement, the sample can be unmounted from the holder by dipping them into acetone solution for a few minutes to dissolve the glue.

The electrical access to the sample is provided by a 40 GHz radio frequency (RF) probe in the ground-signal-ground (GSG) configuration. The GSG fingers of the probe can be contacted to the ground or signal Au/Ti pads of the device by controlling the position of the probe’s arm mounted on a XYZ-movable stage. The GSG fingers are connected to a semi-rigid coaxial cable with an SMA connector on the other end of the probe’s arm, where the electrical response of the device can be readout. The probe is cooled by copper braids connected to the 10 K stage of the probe station so that it can reach a temperature of 20-25 K [18].

The optical access to the sample is realized from the top of the cryostat. The input light coming from a fiber is coupled into free space via a coupler and collimated by a lens. The parallel light beam is directed to a long-working-distance ($f = 24$ mm) and high numerical aperture (NA = 0.4) reflective objective by a beam-splitter (55% transmission and 45% reflection) and then focused to the sample through the access window and the cold filter. The FWHM of the focused spot on the sample is ~2 $\mu$m for a 1.3-$\mu$m laser. The back-reflection of the light from the sample surface is collected by the objective, passes through the beam-splitter and is then detected by a CCD camera, which provides the real-time view of the sample, the laser spot and the RF-probe. All the free-space optical components are supported by a XYZ-movable stage (Thorlabs motorized servo actuator). By controlling the position of the optical components according to the view of the CCD, the laser spot can be roughly aligned to the device. The fine alignment between the spot and the device’s active area is achieved by optimizing the photon counts via adjusting the focusing of the spot and via scanning the spot in the X and Y directions [19]. One of the advantages of using the cryogenic probe station is that many devices can be characterized in one day, suitable for
quick pre-tests of a sample to get an overview of the devices. The best devices are chosen and tested in a closed-cycle cryocooler as described in the following.

2.3.2. Closed-cycle cryocooler

A lower temperature is required for a better performance of SSPDs. The experiments which will be discussed in Chapter 3, Chapter 4 and Chapter 5 were performed in a closed-cycle cryocooler (VeriCold-V04, as shown in Fig. 2.10), which provides a temperature of 1.2 K [20]. The optical setup in the cryocooler was initially designed and maintained by dr. Giulia Frucci.

The VeriCold cryocooler consists of a Pulse Tube Cooler (PTC) and an additional Joule-Thompson (JT) closed cycle integrated in a Dewar. Before cooling down, the Dewar needs to be pumped (<10⁻² mbar at RT). Then the PTC is started and cools down the 70 K- and the 4 K-plates to the temperatures of ~70 K and ~4 K, respectively. Meanwhile, the RT He gas stored in an external tank is injected into a dedicated circuit which provides a thermal link between the 4 K- and the 1 K-plates. Through this thermal link, the 1 K-plate will be cooled down to almost the same temperature of the 4 K-plate when the thermal equilibrium between these two plates is established. Then this thermal link will be switched off and the JT cycle can be started. When the JT cycle starts, the pre-cooled He (5~6 K, prepared by
the 4 K-plate) is forced to the 1 K-plate where it experiences an adiabatic expansion with an associated temperature reduction so that He gas gets liquefied. A quota of the LHe is then evaporated via an external oil-free pump, resulting in an effective cooling of the 1 K-plate to a temperature of less than 1.5 K. When the thermal equilibrium is reached, the 1 K-plate finally obtains a temperature of ~1.2K, depending on the load of the system. Since the cryocooler is a closed-cycle system, it doesn’t require the refilling of He so it is able to keep the working temperature for a long time (a few days).

A zoom-in view of the 1 K-plate is depicted on the right side of Fig. 2.10. The sample is mounted on the 1 K-plate via a sample holder and illuminated by a laser beam through a polarization-maintaining single-mode lensed fiber mounted on a XYZ-piezo stage (Attocube). The back-reflected light from the sample surface is collected by the fiber tip. As shown in Fig. 2.11, by using a fiber-based circulator, the back-reflected power can be sent to either a power meter or an optical spectrum analyzer (OSA) for analysis. The photon-induced voltage pulses of the device are sent to the readout circuits at RT via bonding wires and semi-rigid coaxial cables with SMA connections mounted in the cryostat.

![Schematic diagram of the circuits and optical components used for the VeriCold closed-cycle cryocooler.](image)

To maximize the coupling efficiency, the lensed fiber tip needs to be aligned close to the sample so that the tip is positioned right above the center active area of the detector and the distance between the sample and the fiber $D_{SF}$ (as shown in a zoomed picture on the right)
side of Fig. 2.10) should be equal to a few tens of μm (depending on the wavelength), which is the focal length of the lensed fiber. The alignment of the fiber tip to the device can be realized by controlling the piezos while monitoring of the back-reflected light power and the CR as described in the following.

In a normal procedure, the fiber tip first needs to be positioned at a focal distance from the sample in order to obtain the best focusing and the minimum spot. As shown in Fig. 2.12 (a), the back-reflected light power collected by the fiber tip is plotted as a function of the Z-piezo steps. The back-reflected power reaches its maximum when the fiber is positioned at the focus. By monitoring the back-reflected power, we can position the fiber to the focal plane while approaching the fiber to the sample surface. On the other hand, the value of $D_{SF}$ can be calculated by analyzing the spectrum of the back-reflected light recorded by the OSA [21]. The light from a broadband LED source (WT&T LE-4, center wavelength $\lambda_C = 1.525$ μm) was sent into the fiber and its back-reflected spectrum was measured when the fiber tip was placed at different distances from the sample surface. As shown in Fig. 2.12 (b), the back-reflected spectra for different $D_{SF}$ values, normalized by the original spectrum of the LED, are plotted as a function of wavelength. The periodic fringes in the spectrum result from the interference between the reflection from the fiber tip and the reflection from the sample surface. The period of the fringes ($\Delta\lambda$) is related to the optical path difference between the fiber tip and the sample surface ($D_{SF}$), so we have:

$$D_{SF} = \frac{\lambda_C^2}{2\Delta\lambda}, \quad (2.2)$$
According to Eq. 2.2, the $D_{SF}$ for each spectrum was calculated and marked on the right side of Fig. 2.12 (b). The spectrum obtained when the fiber was at focus indicates that the focus length is $\sim 58 \, \mu m$ for the 1.525-\( \mu \)m light.

After the fiber tip is aligned to its focal plane, it needs to be aligned to the center of the detection area. This can be achieved by scanning the fiber in X and Y directions while recording the back-reflected power and the CR. For example, XY-mappings of the back-reflected power and of the CR generated by a superconducting nanodetector [as shown in Fig. 2.2 (a)] are shown in the Fig. 2.13 (a) and (b), respectively. Both the back-reflected power and the CR are plotted in log scale and the X and Y axes are plotted in linear scale. A monochromatic CW laser source with $\lambda = 1.525 \, \mu m$ (Thorlabs, LPS-PM1550-FC) was employed in the mapping. As shown in Fig. 2.13 (a), the region of stronger reflections on the left side corresponds to the edge of an Au/Ti contact pad. The remaining part corresponds to either the GaAs substrate or the patterned NbN nanowires, which both give much weaker reflection due to their different refractive index. The X and Y axes are calibrated by comparing the size of the features in the power mapping to that in the design of the sample. The CR mapping was recorded simultaneously in the same region and plotted in Fig. 2.13 (b). Since the superconducting nanodetector has a spatial resolution close to its constriction size $\sim 150$ nm [19], which is much smaller than the spot size, the CR mapping reproduces the intensity distribution of the Gaussian light spot. The measured light intensity of the spot has an ellipse distribution, with an $e^{-2}$ transverse diameter of $\sim 5.8 \, \mu m$, and an $e^{-2}$ conjugate diameter of $\sim 4.4 \, \mu m$, as marked in Fig. 2.13 (b).

Fig. 2.13. XY-mapping of the back-reflected power (a) and of the CR (b), plotted in log scale. The X and Y axes are calibrated by comparing the size of the features in (a) to that in the design of the sample.
2.4. Fiber-based Michelson interferometer

A fiber-based Michelson interferometer, used for the measurements presented in Chapter 3 and Chapter 4, is introduced in this section.

The schematic diagram of the fiber-based Michelson interferometer is depicted in Fig. 2.14. When a free-space pulsed light is sent to the interferometer, it is first coupled into the fiber-based system via a coupler. Then, the input light pulse is split into two arms of the interferometer by a 50:50 beam-splitter. In each arm, the pulse is reflected back by a faraday mirror and introduced with a time delay by a delay line. In the output of the interferometer, the two reflected pulses are combined and have a variable time delay $\tau_d$ between each other. The $\tau_d$ is introduced by a motorized delay line [General Photonics (MDL-002), the coarse control], together with a fiber stretcher (Evanescent Optics, Model 915) that changes the length of a fiber section and introduces the fine delays, needed to observe the interference fringes in the interferometric autocorrelations (Chapter 4). For a fiber-based Michelson interferometer, random birefringence in the fibers may cause polarization rotation of the light and result in reduction in the visibility of the interference fringes [22]. This problem is solved by using Faraday mirrors, which reflect the input pulse
optical signal and rotate its polarization state by 90 degrees. In each arm of the interferometer, as the back-reflected light field sees the birefringence of the fiber in the perpendicular polarization compared to the forward propagating field, the birefringence-induced polarization rotation will be canceled and the two reflected pulses will have the same polarization direction. A driving voltage $V_{FS}$ (zigzag wave, amplitude: ±5V, frequency: 20 mHz), provided by a function generator, was applied on the fiber stretcher to change the optical path of one arm. As an example, a measured interferogram is plotted as a function of $V_{FS}$ as shown in the lower-right inset of Fig. 2.14.

2.5. FDTD Simulation

2.5.1. Introduction

One of the key parameters of an SSPD is QE, which critically depends on the optical absorptance in the active area of the superconducting films. The optical absorptance in a certain area is determined by the electromagnetic (EM) field distribution in that area, which is closely related to its refractive index, geometry and environment. Therefore, it is important to understand the EM field distribution in the device so that we can estimate and improve the absorptance of the detector. For this goal, a finite-difference time-domain (FDTD) software (Lumerical Solutions [23]) was used for the work of this thesis (Chapter 4).

In FDTD simulation, the EM fields and the materials of interest are discretized into a mesh which is based on the so-called Yee cells [23], as shown in Fig. 2.15. The electric field vector components ($E_x, E_y, E_z$) in a cell at a certain time step are calculated based on the magnetic field at the previous time step using Maxwell equations. Then the magnetic field vector components ($H_x, H_y, H_z$) in the same cell are solved at the next time step. This process will be repeated until the calculation is terminated. The FDTD software allows
users to choose properties of light source, refractive index of the materials, and monitors to record the calculated EM fields and both the time- and frequency-domain information. The time needed for the calculation depends on the number of the spatial meshes, the time steps and the number of dimensions considered in the simulation.

2.5.2. FDTD simulation for a NbN SSPD on GaAs substrate

As an example, the two-dimensional (2D) intensity distribution of the electric field (i.e. $|E|^2$, in log scale) in a standard NbN meander SSPD (width = 100 nm, thickness = 4 nm, filling factor = 50%) based on GaAs substrate was calculated by the FDTD software, and plotted in Fig. 2.16 (a). The NbN nanowires have a width of 100 nm, a thickness of 4 nm, and a filling factor of 50%. As shown in Fig. 2.16 (a), the simulation was performed based on a unit cell with periodic boundary conditions at $X = -100$ nm and $X = 100$ nm, and perfect absorbing boundary conditions at $Y = -20$ nm and $Y = 20$ nm. The input light was considered to be a plane wave propagating downwards and had a polarization direction perpendicular to the wire.

The optical absorptance of the NbN films can be obtained by calculating the power absorbed in the NbN area, normalized to the total input power in the unit cell. We calculated the absorptance of the NbN nanowires for the case of Fig. 2.16 (a) and plotted...
the result as a function of wavelength $\lambda$, as shown (blue dashed line) in Fig. 2.16 (b). The absorptance in a few other configurations with the wire width of 100 nm, 150 nm or 200 nm [keeping the filling factor and other parameters the same as those in Fig. 2.16 (a)] for the polarization perpendicular ($\perp$) and parallel ($\parallel$) to the wire, were also calculated and plotted as a function of $\lambda$ as shown in Fig. 2.16 (b). We can see that, in general, the perpendicular polarization corresponds to a lower optical absorptance than the parallel; and the difference of absorptance between different polarizations is larger for the narrower wires. This phenomenon has been observed in the experiment of Ref [24] and it is due the low value of $|E|^2$ in the edge of the NbN film when the polarization is perpendicular to the NbN-vacuum interface.

For the FDTD simulations presented in this thesis, the refractive index of the NbN film (wavelength-dependent) is obtained from Ref [24], and the refractive index for the other materials is provided by the Lumerical FDTD software [23].
References:
CHAPTER 3

Probing the nonlinear response of SSPD

In this chapter the experimental and theoretical study of probing the nonlinear response of the SSPD is presented. A superconducting nanodetector is chosen as the candidate of this study as it provides an easier access to multi-photon detections than meander SSPDs. The time scale of the nonlinear response in the superconducting nanodetector is directly measured, which is related to the relaxation dynamics of photon-induced quasi-particles (QPs) in superconducting nanowires. Part of this chapter has been published by Z. Zhou et al. in “Ultrasensitive N-photon interferometric autocorrelator,” Phys. Rev. Lett., vol. 110, p. 133605, 2013.

3.1. Superconducting nanodetector

As introduced in Section 1.5, it is possible to operate an SSPD as an SPD or a multi-photon detector by varying its bias current $I_B$. When $I_B$ is close to $I_C$, the superconducting state of the SSPD is close to the critical point so it is easily disrupted when photons are absorbed. In this case, absorption of a single photon can break the superconducting state in the wire and results in a detection event. In this case, the SSPD works in single-photon regime (or one-photon regime) where the CR is proportional to the absorbed photon numbers. When lowering $I_B$ well below $I_C$, more than one photon is needed for a detection event. In this case, the SSPD works in the multi-photon regime and its response becomes nonlinear to the incident light power.

The nonlinearity in SSPDs was first reported in Ref [1], where the two-photon response was observed when $I_B$ was set to ~80 % of $I_C$ as shown in Fig. 1.13. However, it is difficult to observe ≥ 3rd-order nonlinearities in meander SSPDs [2]. This is due to the fact that in the multi-photon detection the multiple photons need to be absorbed in the nanowire within a certain distance related to the diffusion length of photon-induced QPs, which was estimated to be few tens of nm [3]. However, since SSPDs are usually fabricated in a meander shape with a total wire length of few hundreds of μm in order to maximize the spatial coupling between the light beam and the detection area, the probability of absorbing multiple photons within such a small distance in the wire is small. The multiple photons, which are not absorbed within the distance required for a multi-photon detection, may still trigger single-photon detections in the wire, producing a linear-detection background on the nonlinear responses.
In 2010, a novel SSPD named superconducting nanodetector (ND) was reported by Bitauld, et al. [4], providing an easier access to high-order multi-photon detections. As shown in the SEM image in Fig. 3.1 (a), the nanodetector is based on a superconducting NbN nanowire (thickness = 4.3 nm, \( T_c = 10.2 \) K) fabricated with a nanoscale constriction (~150 nm) on GaAs substrate [5]. Since the constriction area has the highest bias current density in the wire, it has much higher detection efficiency than the rest of the wire. Therefore, only the photons absorbed in the constriction area will contribute to the detection event. As the kinetic inductance \( L_k \) of the constriction is small, if the nanodetector is connected directly to the contact pads, the \( I_b \) in the device after detecting a photon recovers too fast while the wire is still resistive (\( \tau_{\text{rec}}=L_k/R_L \), Subsection 2.2.3), so that the device will sustain a resistive state, i.e. latch [4]. Wider NbN wires with a width of ~500 nm [marked by white arrows in the lower-right corner of Fig. 3.1 (a)] and a length of few hundreds of \( \mu m \), were added as the connection between the nanodetector and its electrical contact pads, in order to avoid the latching problem.

Similarly to meander SSPDs, absorption of one or more photons produces a non-equilibrium population of QPs (i.e. a hot-spot) in the nanodetector which exists for a certain lifetime. The QP population locally suppresses the superconductivity and increases the probability of vortex crossing [6, 7], which creates a resistive barrier across the constriction area and finally results in a measurable voltage pulse in the readout circuit. As this
probability also depends on the bias current, the number of QPs needed for a detection event can be controlled by setting the bias current. As shown in Fig. 3.1 (b), if $I_B$ is set close to $I_C$, absorption of a single photon with energy $h\nu$ in the constriction can trigger a detection event, so the nanodetector works in the one-photon regime. When $I_B$ is lowered, the QP population produced by a single photon is not sufficient, so more photons are needed for the detection. As shown in Fig. 3.1 (c), $I_B$ can be set so that the absorption of at least two photons is required for a detection event. In other words, the detection will be triggered only if the second photon is absorbed within the lifetime of the QPs produced by the first one. In this case, the nanodetector works in the two-photon regime. When $I_B$ is further lowered, the absorption of three or four (or even more) photons is necessary for the detection so that the detector works in the three-photon and the four-photon regime, as shown in Fig. 3.1 (d) and (e), respectively.

As discussed in Ref [4, 8], a nanodetector working in the $N$-photon regime acts as a threshold detector, which detects $\geq N$ photons. When the nanodetector is under the illumination of a pulsed laser source, the probability of detecting $n$ photons, i.e. $P_n$, is written as:

$$p_n \propto \frac{(\eta_{\text{abs}}\mu)^n}{n!} e^{-(\eta_{\text{abs}}\mu)} , \ (3.1)$$

Where $\mu$ is the average input photon number per pulse, $\eta_{\text{abs}}$ is the optical absorptance (defined as the number of photons absorbed in the detector divided by the number of photons coming at the input of detection system) and thus the value of $\eta_{\text{abs}}\mu$ corresponds to the mean absorbed photon number per pulse. As $P_{n+1} << P_n$ for $\eta_{\text{abs}}\mu << 1$, we expect that a threshold detector working in the $N$-photon regime will mostly click upon absorption of $N$ photons, so that its CR should be proportional to,

$$P_N \propto \frac{(\eta_{\text{abs}}\mu)^N}{N!} e^{-(\eta_{\text{abs}}\mu)} , \ (3.2)$$

In other words, in the $N$-photon regime, the photon-induced response (i.e. the CR) of the nanodetector should be proportional to the input photon number to the power of $N$, given that the light power is low so that the condition of $\eta_{\text{abs}}\mu << 1$ is satisfied. This is indeed observed experimentally as shown in Fig. 3.2, where the measured CR of the nanodetector under the illumination of 1.13-µm laser pulses [generated by an optical parametric oscillator (OPO) and has a time width of ~1.6 ps] is plotted as a function of the light power (measured at the input of the cryostat) at different bias currents. In the low power range, the solid lines with slopes of 1.01, 2.06, 3.06 and 3.99 are plotted to fit the measured points in log-log scale, indicating where we can approximately find the one-, two-, three- and four-
photon regimes by choosing the $I_B$ of 18.0, 12.5, 9.7 and 8.4 $\mu$A, respectively. A full tomographic characterization of the multi-photon response of the nanodetector can be found in Ref [8] and will be discussed in Section 3.4.

Fig. 3.2. Measured CR plotted as a function of the light power (measured at the input of the cryostat) for different $I_B$ values. The fitting lines with slope of 1.04, 2.06, 3.06, and 3.99 indicate the one-, two-, three-, and four-photon regimes, respectively.

3.2. Probing the nonlinear response - theory

In order to describe the nonlinear response of a superconducting nanodetector, we established a semi-classical detection model, which considers the temporal evolution of QPs and the detection mechanism based on the vortex-antivortex pair (VAP) unbinding and the single-vortex crossing (SVC) model. In general, the absorption of a photon (or multiple photons) creates a group of QPs with a concentration varying in time. The presence of QPs locally weakens the superconductivity of the device by reducing the superconducting energy gap. The reduction of the superconducting energy gap increases the probability of VAP unbinding (or SVC), which releases energy sufficient to trigger the detection. The vortex-induced detection theories are chosen here because they support many experimental observations [6, 9, 10] and have a good agreement with the experiments presented in this chapter.

3.2.1. Temporal evolution of QP concentration

According to Ref [11], the temporal and spatial evolution of the photon-induced QP concentration in a superconducting nanowire can be written as:

$$C_{Qp}(t,r) = \frac{\hbar \nu e^2}{4\pi D \Delta dt} \left( 1 - e^{-i/\ell} \right) e^{-r^2/4\Delta d} e^{-i/\ell^2} + C_{Qp_{eq}} \quad (3.3)$$
Where $t$ is time, $r$ is the distance to the photon absorption site, $h\nu$ is the photon energy, $0 < \xi < 1$ is the conversion efficiency, $D$ is the diffusion coefficient, $\Delta$ is the superconducting energy gap, $d$ is the thickness of the wire, $\tau_1$ is the thermalization time, and $\tau_2$ is the QP decay time determined by the time of the electron-phonon interaction and the time of phonons escaping to the substrate. The $C_{QPeq}$ is the concentration of equilibrium QP, which is negligible as it is much smaller than the photon-induced QP concentration [6]. To simplify the model, we neglect the spatial evolution of the QPs. Instead, we assume that the photon-induced QPs are uniformly distributed in a volume of interest $V$, which is comparable to the dimension of the constriction area where the evolution of QPs takes place. Within the volume of $V$, the QP has a uniform concentration $C_{QP}$ varying as a function of time, written as:

$$C_{QP}(t) = \frac{h\nu\xi}{\Delta
u} \left(1 - e^{-t/\tau_1}\right)e^{-t/\tau_2}, \quad (3.4)$$

Where the value of $h\nu\xi/\Delta V$ corresponds to the maximum concentration of QPs induced by a photon with energy $h\nu$. For instance, when $\tau_1 = 10$ ps and $\tau_2 = 80$ ps, the time evolution of the QP concentration induced by absorbing a single photon is depicted in Fig. 3.3 (a), where the photon is absorbed at $t = 0$. 

Fig. 3.3. (a) Time evolution of the QP concentration induced by absorbing a single photon for $\tau_1 = 10$ ps and $\tau_2 = 80$ ps. The photon is absorbed at $t = 0$. (b) Time evolution of the QP concentration induced by the absorption of two photons for $\tau_1 = 10$ ps and $\tau_2 = 80$ ps. The $C_{QP1}$ (dark) and $C_{QP2}$ (blue) are the QP concentration induced by the first and the second photon, respectively. The $C_{QPtot}$ (green) is the total QP concentration. The first photon is absorbed at $t = 0$. The $\tau_{12} = 80$ ps is the time difference between the absorption of the first and the second photon.
In the case more than one photon is absorbed, we assume that the $C_{QP}$ induced by each photon is independent to the others, so the total concentration $C_{QPtot}$ is obtained by summing up the $C_{QP}$ induced by all the photons as a function of time. For instance, when two photons are absorbed, the time evolution of the $C_{QPtot}$ is written as,

$$C_{QPtot}(t, \tau_{12}) = \frac{h\nu}{\Delta V} \left( 1 - e^{-t/\tau_1} \right) e^{-t/\tau_2} + \frac{h\nu}{\Delta V} \left[ 1 - e^{-[(t+\tau_{12})/\tau_1]} \right] e^{-(t+\tau_{12})/\tau_2},$$

(3.5)

Where $\tau_{12}$ is the time difference between the absorption of the first and the second photon. The value of $C_{QPtot}$ calculated based on Eq. 3.5 is shown in Fig. 3.3 (b) for $\tau_1 = 10$ ps, $\tau_2 = 80$ ps and $\tau_{12} = 80$ ps as an example.

3.2.2. Nonlinear response of the detector

3.2.2.1. QP-induced reduction of superconducting energy gap

The click probability of a superconducting nanodetector is dependent on the photon-induced QP concentrations $C_{QPtot}$. In principle, a higher $C_{QPtot}$ leads to a higher detection probability. As introduced in Subsection 3.2.1, the value of $C_{QPtot}$ varies as a function of the time delay between the absorptions of multiple photons, e.g. $\tau_{12}$ for the two-photon regime. Therefore, in the multi-photon regime, it is in principle possible to probe the temporal evolution of $C_{QP}$ by analyzing the photon response of the nanodetector while controlling the time delay between the photon absorptions. We will present how this is achieved in the following.

The presence of photon-induced QPs locally weakens the superconductivity of the device by reducing the superconducting energy gap $\Delta$. Assuming the reduction of the superconducting energy gap, i.e $\delta\Delta$, is much smaller than the value of $\Delta$, we have [12, 13],

$$\delta\Delta = \frac{C_{QP}k_B}{N_0} = K_1 \cdot C_{QP},$$

(3.6)

Where $k_B$ is the Boltzmann constant, $N_0$ is the electronic density of states at the Fermi energy, and $K_1$ is a constant written as,

$$K_1 = \frac{k_B}{N_0},$$

(3.7)

Although different models [1, 6, 7, 14, 15] were proposed to describe how the detection is triggered due to the presence of photon-induced QPs, recent studies [9, 10, 16] showed that magnetic vortices play a key role in the detection process. In general, due to the reduction of superconducting energy gap, i.e $\delta\Delta$, the probability of either VAP unbinding in the wire, or single-vortex crossing (SVC) from one side to the other side of the wire, is increased.
During these actions of vortex, energy is released in the wire and triggers the detection. In the following, we will use the VAP unbinding and the SVC model to investigate the nonlinear detection process in a nanodetector.

3.2.2.2. VAP unbinding model

In the VAP unbinding theory, the minimum binding energy of the VAP, i.e. $U_{VAP}$, is dependent on the superconducting energy gap $\Delta$, and can be written as [6, 9, 16],

$$U_{VAP} = \frac{\Delta \tanh(\Delta/2k_BT)}{4e^2R_{SN}/\pi h} \left\{ 2\gamma + \frac{1}{\epsilon} \left[ \ln \left( \frac{2.6I_c}{I_B} \right) + \frac{I_B}{2.6I_c} - 1 \right] \right\}, \quad (3.8)$$

Where $h$ is the Planck constant, $e$ the electric charge of the electron, $R_{SN}$ is the sheet resistance of normal NbN film, $\gamma$ is a temperature-independent parameter, $\epsilon$ is a temperature-dependent parameter corresponding to the averaged polarizability of a VAP within the entire VAP population [16, 17], and the temperature-dependent gap $\Delta$ is written as [11],

$$\Delta(T) = 2.15k_BT \left[ 1 - (T/T_c)^2 \right], \quad (3.9)$$

The probability of VAP unbinding $P_{VAP}$ is proportional to the clicking probability of the detector $P_{\text{click}}$, and can be written as [6, 9, 16],

$$P_{VAP} = \Gamma_{VAP} \exp \left[ -U_{VAP}(\Delta)/k_BT \right], \quad (3.10)$$

Where $\Gamma_{VAP}$ is the VAP attempt rate.

When no photon is absorbed in the detector, $P_{VAP}$ is proportional to the DCR. As shown in Fig. 3.4, measured DCRs of a superconducting nanodetector working at the temperature of 1.2 K (blue circles), 1.9 K (red circles), 3.1 K (dark circles), and 4.5 K (green circles), are plotted as a function of $I_B/I_C$, respectively. The measured DCRs are fitted (by solid lines) based on Eq. 3.10 using the relevant experimental parameters listed in the Appendix and three free fitting parameters of $\epsilon$, $\gamma$ and $\Gamma_{VAP}$. The fitting shows a good agreement with the experimental data and supports the VAP unbinding model. The values of $\epsilon$ and $\gamma$, determined from the fitting, are shown on the top of Fig. 3.4 (a).

When a photon (or photons) is absorbed in the detector, the photon-induced $C_{QP}$ reduces the superconducting energy gap by $\delta\Delta$, consequently reduces the value of $U_{VAP}$ and thus increases the probability of VAP unbinding. In this case, the $P_{VAP}$ is proportional to the photon CR plus the DCR. According to Eq. 3.6-3.10, the VAP unbinding probability in the detector under illumination can be written as,
\[ P_{\text{VAP}} = \Gamma_{\text{VAP}} \exp\left[-U_{\text{VAP}}(\Delta - K_1 \cdot C_{\text{QP}})/k_B T\right], \quad (3.11) \]

Fig. 3.4. Measured DCRs of a superconducting nanodetector working at the temperatures of 1.2 K (blue circles), 1.9 K (red circles), 3.1 K (dark circles), and 4.5 K (green circles), respectively, plotted as a function of \( I_B/I_C \). (a) The measured DCRs are fitted (by solid lines) based on the VAP unbinding model (Eq. 3.10). The values of \( \varepsilon \) and \( \gamma \) determined from the fitting are listed on the top of (a). (b) The measured DCRs are fitted (by solid lines) based on the SVC model (Eq. 3.17).

### 3.2.2.3. SVC model

Similarly, in the SVC theory, the energy barrier for a single vortex to cross the wire, i.e. \( U_{\text{SVC}} \), can be written as [16],

\[ U_{\text{SVC}} = E_S \left[ \ln\left( \frac{2w}{\pi \xi_{\text{GL}} \sqrt{1 + (I_B/I_S)^2}} \right) - \frac{I_B}{I_S} \left( \arctan\left( \frac{I_S}{I_B} \right) - \frac{\pi \xi_{\text{GL}} T}{2w} \right) \right], \quad (3.12) \]

With,

\[ E_S = \frac{h^2}{4\pi \mu_0 e^2 \Lambda}, \quad (3.13) \]

\[ I_S = \frac{h}{4e \mu_0 \Lambda}, \quad (3.14) \]

\[ \xi_{\text{GL}}(T) = \frac{\xi_{\text{GL}}(0)}{\sqrt{1 - T/T_c}}, \quad (3.15) \]
Where $\mu_0$ is the vacuum permeability, $\xi_{GL}$ is the temperature-dependent Ginzburg-Landau (GL) coherence length, and $\Lambda$ is the temperature-dependent effective penetration depth in thin films, written as,

$$
\Lambda(T)=\frac{2}{d} \lambda(0) \left[ \frac{\Delta(T)}{\Delta(0)} \tanh \left( \frac{\Delta(T)}{2k_B T} \right) \right]^{-1}, (3.16)
$$

Where $\lambda(0)$ is the magnetic penetration depth at $T = 0$ K.

Similar to Eq. 3.10, the probability of SVC, i.e. $P_{SVC}$, is proportional to the clicking probability of the detector $P_{click}$, and can be written as,

$$
P_{SVC} = \Gamma_{SVC} \exp \left[ -U_{SVC}(\Delta)/k_B T \right], (3.17)
$$

Where $\Gamma_{SVC}$ is the SVC attempt rate.

According to Eq. 3.12-3.17, if the value of $\Delta$ is reduced (e.g. due to the presence of QPs, or due to the change of temperature or bias current), the value of $U_{SVC}$ will decrease, which leads to a larger value of $P_{SVC}$ and thus a higher clicking probability.

When no photon is absorbed in the detector, $P_{SVC}$ is proportional to the DCR. As shown in Fig. 3.4 (b), DCRs measured at the temperature of 1.2 K (blue circles), 1.9 K (red circles), 3.1 K (dark circles), and 4.5 K (green circles) [the same as that in Fig. 3.4 (a)] are plotted as a function of $I_B/I_C$, respectively. The measured DCRs are fitted based on Eq. 3.17 using the relevant experimental parameters listed in the Appendix and a free fitting parameter of $\Gamma_{SVC}$. As shown in Fig. 3.4 (b), the SVC model does not fit to the experimental data at all temperatures, only the DCR measured at 3.1 K has a good agreement with the model. This result may indicate that the VAP unbinding mechanism is more dominant than the SVC in the origin of DCs under our experimental condition. A similar observation was reported in the work of Ref [9]. However, it should be pointed out that more fitting parameters are used to interpret the data within the VAP unbinding model, so that a firm conclusion cannot be made.

Similar to Eq. 3.11, when a photon (or photons) is absorbed in the superconducting nanodetector, the SVC probability is proportional to the clicking probability of the detector and can be written as,

$$
P_{SVC} = \Gamma_{SVC} \exp \left[ -U_{SVC}(\Delta - K_1 \cdot C_{QP})/k_B T \right], (3.18)
$$
3.2.2.4. Nonlinear response based on the VAP and SVC model

According to Eq. 3.5, Eq. 3.11 and Eq. 3.18, it is possible to calculate the dependence of the nanodetector’s click probability $p_{\text{click}}$ as a function of $\tau_{12}$ for the two-photon regime.

Based on the VAP unbinding model, $p_{\text{click}}(\tau_{12})$ can be written as,

$$p_{\text{click}}(\tau_{12}) \propto \int_{0}^{\tau} \exp\left[-U_{VAP}[\Delta - K_1 \cdot C_{\text{QPtot}}(t, \tau_{12})]/k_B T\right] \cdot dt, \quad (3.19)$$

Similarly, $p_{\text{click}}(\tau_{12})$ based on the SVC model is written as,

$$p_{\text{click}}(\tau_{12}) \propto \int_{0}^{\tau} \exp\left[-U_{SVC}[\Delta - K_1 \cdot C_{\text{QPtot}}(t, \tau_{12})]/k_B T\right] \cdot dt, \quad (3.20)$$

For instance, assuming $\tau_1 = 10$ ps and $\tau_2 = 80$ ps, $h\nu = 1$ eV, $\xi = 0.2$, $I_B/I_C = 0.65$, the volume of interest $V = w^2d$, where $w = 150$ nm is the width of the constriction and $d = 4.8$ nm is the thickness of the NbN film, and using the relevant parameters in the Appendix for $T = 1.9$ K and $T = 3.1$ K, we calculated $p_{\text{click}}(\tau_{12})$ using the VAP unbinding model (Eq. 3.19) and the SVC model (Eq. 3.20). As shown in Fig. 3.5 (a), for $T = 3.1$ K, the calculated $p_{\text{click}}(\tau_{12})$ based on the VAP unbinding model (solid light green line) overlaps with the one calculated based on SVC model (dashed red line), showing an excellent agreement between these two models. Indeed, this agreement is also found in the DCR measurements at $T = 3.1$ K as shown in Fig. 3.4 (a) and (b), where the DCR calculation based on these two models both fit the measurements well. For $T = 1.9$ K, the calculated $p_{\text{click}}(\tau_{12})$ based on the VAP unbinding model (solid blue line) shows the same shape but has a slightly different width as compared to the one calculated using the SVC model (dashed dark green line). This is due to the fact that the fitting parameters used for the calculations were extracted based on Fig. 3.4, where the SVC model deviates from the DCR experiments at $T = 1.9$ K and also from the VAP unbinding model. In general, the good agreement achieved between the VAP unbinding model and the SVC model shows that they offer similar explanations on the nonlinear detection process, and that the VAP unbinding model is more applicable for a larger temperature range as it fits well with the set of the temperature-dependent DCR measurements [Fig. 3.4 (a)]. Since in this chapter we will investigate the nonlinear response at different temperatures, we will only use the VAP unbinding model in the following.

In the VAP unbinding (and SVC) model, $p_{\text{click}}(\tau_{12})$ is a strong function of the photon-induced QP number $h\nu \xi / \Delta V$ and involves many microscopic parameters. For simplification, we introduce an empirical Gaussian nonlinear response function (NRF), i.e. $\eta(\tau_{12})$, to fit the $p_{\text{click}}(\tau_{12})$ dependence. The NRF is defined as,
\[ \eta(\tau_{12}) = \eta(0) \exp\left(-\left(\frac{\tau_{12}}{\tau_{ND}}\right)^2\right), \quad (3.21) \]

Where \( \eta(0) \) is the value of \( \eta(\tau_{12}) \) for \( \tau_{12} = 0 \), and \( \tau_{ND} \) is the intrinsic response time of the nanodetector, which depends on the value of \( \tau_1 \) and \( \tau_2 \) and sets the time scale of the nonlinear photon response in the detector, i.e. the minimum time separation between two photons for which they can contribute to a two-photon detection process. In the ideal two-photon regime, the value of \( \eta(\tau_{12}) \) is expected to decrease from the maximum of \( \eta(0) \) to zero for \( \tau_{12} \gg \tau_{ND} \). Assuming \( \tau_1 = 10 \) ps and \( \tau_2 = 80 \) ps, \( \xi = 0.2 \), and using the experimental parameters for the measurement as it will be shown in Fig. 3.9 (b), including \( h\nu = 1.08 \) eV, \( I_B/I_C = 0.47 \), \( w = 150 \) nm, \( d = 4.8 \) nm, and the relevant parameters in the Appendix for \( T = 1.2 \) K, we calculated the \( P_{\text{click}}(\tau_{12}) \) dependence using the VAP unbinding model and plotted it in Fig. 3.5 (b) (solid red line). A fit to the calculated \( P_{\text{click}}(\tau_{12}) \) using Eq. 3.21 is plotted as a dashed black line in Fig. 3.5 (b), giving \( \tau_{ND} = 20.1 \) ps.

![Graph](image)

**Fig. 3.5.** (a) Calculated \( P_{\text{click}}(\tau_{12}) \) based on the VAP unbinding model (Eq. 3.19) and the SVC model (Eq. 3.20), assuming \( \tau_1 = 10 \) ps and \( \tau_2 = 80 \) ps, \( h\nu = 1 \) eV, \( \xi = 0.2 \), \( I_B/I_C = 0.65 \), \( w = 150 \), \( d = 4.8 \) nm, and using the relevant parameters in the Appendix for \( T = 1.9 \) K and \( T = 3.1 \) K. (b) Calculated \( P_{\text{click}}(\tau_{12}) \) based on the VAP unbinding model (Eq. 3.19), assuming \( \tau_1 = 10 \) ps and \( \tau_2 = 80 \) ps, \( \xi = 0.2 \), and using the experimental parameters for the measurement as shown in Fig. 3.9 (b). A fit to the calculated \( P_{\text{click}}(\tau_{12}) \) using Eq. 3.21 is also plotted (dashed black line), giving \( \tau_{ND} = 20.1 \) ps.

### 3.2.3. Probing the nonlinear response by two pulses

Based on the discussion in Subsection 3.2.2, the NRF can be measured by using a two-pulse probing configuration where \( \tau_{12} \) can be indirectly controlled by varying the delay between the two probing pulses. In this subsection, we will prove how this is theoretically feasible. The experimental results will be presented in Section 3.3.
We assume that a superconducting nanodetector is operated in a regime where the $N$-photon response ($N = 1, 2, \cdots$) is dominant for the wavelength of interest (so this assumption is only applicable when $\eta_{\text{abs}} \mu << 1$). As we focus here on the application to the characterization of classical light sources, we describe the detection process as a sequence of single-photon absorption events using the semi-classical photo-detection theory. As shown in Fig. 3.6, assuming a light pulse with cycle-average intensity $I_{\text{in}}(t)$ incident on the detector, the probability of absorbing a photon within a time interval $[t, t + dt]$ is equal to $\zeta S I_{\text{in}}(t) dt$, where $S$ is the active area and $\zeta = \eta_{\text{abs}} / h \nu$. In the two-photon regime, as shown in Fig. 3.6 (a) for example, the click probability $P_{\text{click}}$ (assume $P_{\text{click}} << 1$) is equal to the probability that two photons are absorbed, multiplied by the probability $P_{\text{click}}(\tau_{12})$ which we approximate as $\eta(\tau_{12})$ [as shown in Fig. 3.5(b) for $\tau_{\text{ND}} = 20.1$ ps and plotted in Fig. 3.7 (a) for a general case],

$$P_{\text{click}} = \int_{-\infty}^{\infty} \eta(t_2 - t_1) S I_{\text{in}}(t_1) S I_{\text{in}}(t_2) dt_1 dt_2 = \zeta^2 S^2 \int_{-\infty}^{\infty} \eta(\tau_{12}) \int_{-\infty}^{\infty} I_{\text{in}}(t) I_{\text{in}}(t + \tau_{12}) dt \, d\tau_{12}, \quad (3.22)$$

Where $\tau_{12}$ is the time difference between absorption of the first photon (at the time of $t_1$) and the second photon (at the time of $t_2$).

Fig. 3.6. Semi-classical photo-detection model in the two-photon regime (a) and in the three-photon regime (b), as described in the text.

Similarly, in the three-photon regime as depicted in Fig. 3.6 (b), the detection probability is written as,

$$P_{\text{click}} = \zeta^3 S^3 \int_{-\infty}^{\infty} \eta(\tau_{13}, \tau_{23}) \int_{-\infty}^{\infty} I_{\text{in}}(t) I_{\text{in}}(t + \tau_{13}) I_{\text{in}}(t + \tau_{23}) \times dt \, d\tau_{13} \, d\tau_{23}, \quad (3.23)$$
Where \( \tau_{13} = t_3 - t_1 \) is the time difference between the absorption of the first and the third photon, \( \tau_{23} = t_3 - t_2 \) is the time difference between the absorption of the second and the third photon. We assume that the NRF of the three-photon regime can be approximated as \( \eta_3(\tau_{13}, \tau_{23}) = \eta(\tau_{13})\eta(\tau_{23}) \). As shown in Fig. 3.7 (b) for an example, the calculated \( \eta_3 \) is plotted as a function of \( \tau_{13} \) and \( \tau_{23} \).

In general, the detection probability in the \( N \)-photon regime is written as,

\[
P_{\text{click}} = \zeta^N S^N \int_{-\infty}^{\infty} \eta(\tau_{1N}, \tau_{2N}, \ldots, \tau_{N-1,N}) I_{in}(t) I_{in}(t + \tau_{1N}) I_{in}(t + \tau_{2N}) \cdots I_{in}(t + \tau_{N-1,N}) \, d\tau_{1N} \, d\tau_{2N} \cdots d\tau_{N-1,N},
\]

(3.24)

Where \( \tau_{iN} = t_N - t_i \) is the time difference between the absorption of the \( i \)th and the last photon, and \( \eta(\tau_{1N}, \tau_{2N}, \ldots, \tau_{N-1,N}) = \eta(\tau_{1N})\eta(\tau_{2N}) \cdots \eta(\tau_{N-1,N}) \) is the NRF of the \( N \)-photon regime.

![Fig. 3.7. NRF of the two-photon regime (a) and of the three-photon regime (b).](image)

Assume the nanodetector is set in the two-photon regime and illuminated by two light pulses, each having a time-dependent Gaussian electric field \( E(t) \) defined as,

\[
E(t) = E_0 \exp[-4 \ln(2) t^2 / \tau_p^2],
\]

(3.25)
Where $E_0$ is the field amplitude and $\tau_f$ is the FWHM of the pulse, and the two pulses have a delay $\tau_d$ between each other. In this case, the response of the nanodetector is found using Eq. 3.22 and reads

$$P_{\text{click}}(\tau_d) \propto 2 \int_{-\infty}^{+\infty} \eta(\tau_{12}) G^{(2)}(\tau_{12}) \cdot d\tau_{12} + \int_{-\infty}^{+\infty} \eta(\tau_{12}) G^{(2)}(\tau_{12} + \tau_d) \cdot d\tau_{12} + \int_{-\infty}^{+\infty} \eta(\tau_{12}) G^{(2)}(\tau_{12} - \tau_d) \cdot d\tau_{12}$$

$$+ 2 \text{Re} \left[ \int_{-\infty}^{+\infty} \eta(\tau_{12}) \int_{-\infty}^{+\infty} E^*(t)E(t + \tau_{12})E(t + \tau_d)E^*(t + \tau_{12} + \tau_d) \cdot dt \cdot d\tau_{12} \right]$$

$$+ \text{interf.}, \quad (3.26)$$

The first term on the right side of Eq. 3.26 is independent of $\tau_d$. The second and the third terms represent the convolution of the NRF with the second-order correlation function of the input pulse defined as $G^{(2)}(\tau) = \langle I(t)I(t + \tau) \rangle$, where the symbol $\langle \rangle$ denotes a temporal average over a long time interval. The fourth term is sensitive to the phase-coherence of the input beam and gives rise to an additional peak in the filtered autocorrelation signal at small delays. The last term on the right side of Eq. 3.26 corresponds to the interference terms which vary with $\omega \tau_d$ or $2\omega \tau_d$. Since they average to zero they can be easily filtered out using a low-pass filter.

In the case where the pulse is much shorter than $\tau_{ND}$, Eq. 3.26 is approximated as,

$$P_{\text{click}}(\tau_d) \approx 2\eta(0) \int_{-\infty}^{+\infty} G^{(2)}(\tau_{12}) \cdot d\tau_{12} + \eta(\tau_d) \int_{-\infty}^{+\infty} G^{(2)}(\tau_{12}) \cdot d\tau_{12} + \eta(-\tau_d) \int_{-\infty}^{+\infty} G^{(2)}(\tau_{12}) \cdot d\tau_{12}$$

$$+ 2\eta(0) G^{(1)}(\tau_d) + \text{interf.}, \quad (3.27)$$

Where $G^{(1)}(\tau_d) = \langle E^*(t)E(t + \tau_d) \rangle$ is the first-order correlation function of the electric filed. Assuming $\eta(\tau_d) = \eta(-\tau_d)$, Eq. 3.27 can be written as,

$$P_{\text{click}}(\tau_d) \propto \eta(0) + \eta(\tau_d) + \frac{\eta(0) G^{(1)}(\tau_d)^2}{\int_{-\infty}^{+\infty} G^{(2)}(\tau_{12}) \cdot d\tau_{12}} + \text{interf.}^*, \quad (3.28)$$

The third term on the right side of Eq. 3.28, which comes from the fourth term in Eq. 3.26, depends on the first-order coherence properties of the input light and produces a peak near zero delay. This coherent peak has a narrow width close to the coherence time of the probe.
light, so it can be neglected in the analysis. More details on measurement of the coherent peak will be discussed in Section 3.5.

For longer delays, the $P_{\text{click}}(\tau_d)$, normalized by its value at $\tau_d \gg \tau_{\text{ND}}$, varies as,

$$P_{\text{click}}(\tau_d)/P_{\text{click}}(\tau_d \gg \tau_{\text{ND}}) = 1 + \eta(\tau_d)/\eta(0), \quad (3.29)$$

Where the click probability, as expected, is the maximum when the two pulses arrive simultaneously, as in this case the probability of absorbing two photons is maximized. Therefore, the NRF and thus the intrinsic response time of the nanodetector, i.e. $\tau_{\text{ND}}$, can be determined.

When the duration of the incident light pulse is much larger than the value of $\tau_{\text{ND}}$, the click probability of the nanodetector in $N$-photon regime is proportional to the $N$th-order interferometric autocorrelation, as it will be discussed later in Chapter 4.

3.3. Probing the nonlinear response - experiment

![Diagram](image)

Fig. 3.8. Schematic diagram of the experimental setup used for the nonlinear response measurement.

The NRF of the nanodetector was measured using the experimental setup as shown in Fig. 3.8. Short pulses with a pulse duration $\tau_p = 1.6$ ps and a wavelength of 1.13 $\mu$m, generated
by an OPO (pumped by a Ti:sapphire laser), were employed as the probe light and sent into a fiber-based Michelson interferometer (Section 2.4). The interferometer split the incident pulse into its two arms and generated two pulses with the same power and with a variable delay $\tau_d$ at the output. The $\tau_d$ was generated by a motorized delay line (the coarse control) together with a fiber stretcher that changes the length of a fiber section and thus introduces the fine delays. The two pulses were then sent through a lensed fiber to the nanodetector which was mounted in the VeriCold cryostat (Subsection 2.3.2) with a temperature of 1.2 K during the measurement. The light spot produced by the lensed fiber had a Gaussian profile with an $e^{-2}$ diameter of ~5 $\mu$m. The $I_C$ of the device was ~26.8 $\mu$A under this condition. The response signal of the nanodetector was measured by a 50-$\Omega$ matched readout circuit at room temperature. The signal pulses were first fed into the RF arm of a bias-T, amplified by a chain of low noise amplifiers, and finally recorded by the 350-MHz counter in terms of CR, as introduced in Chapter 2.

The nanodetector was set in different photon regimes by choosing an $I_B$ of 18.0, 12.5, 9.7 or 8.4 $\mu$A, as shown in the insets of Fig. 3.9 (a)-(d), respectively (and also shown in Fig. 3.2). The CR is plotted as a function of input light power in log-log scale. The solid green lines with slopes of 1.04, 2.06, 3.06 and 3.99 indicate that the nanodetector was working the one-, two-, three- and four-photon regime, respectively. We chose one point in each of the photon regimes (marked by green arrows in the insets) and measured the CR as a function of $\tau_d$, normalized by its values at long delays ($\tau_d \sim 80$ ps), as shown in the main panel of Fig. 3.9 (a)-(d). The data points near the zero delay (-1ps $\leq \tau_d \leq$ 1ps), corresponding to the coherence peak, are not shown in the plot and were not considered in the fit as they are only sensitive to the first-order coherence and introduce additional fitting error.

In the one-photon regime [Fig. 3.9 (a)], the CR is independent of the delay because no nonlinear response is expected in this linear regime. In other words, the detector responds to each photon individually. In the higher-photon regimes [Fig. 3.9 (b)-(d)], a peak is observed near the zero delay due to the nonlinear response of the nanodetector. By fitting the measurements in Fig. 3.9 (b) with the calculation based on Eq. 3.21 and 3.22, the value of $\tau_{ND}$ was determined to be 20.4±0.8ps using least-squares method. With the determined $\tau_{ND}$, the value of $P_{click}(\tau_d)$ in the three- and four-photon regimes were calculated according to Eq. 3.24 without additional fitting parameters, showing an excellent agreement with the experiments as plotted in Fig. 3.9 (c) and (d), respectively, and providing strong experimental support to our model.
Fig. 3.9. Normalized CR (black dots) as a function of \( \tau_d \) measured when the nanodetector was working in the one- (a), two- (b), three- (c) and four-photon (d) regimes, respectively. The solid red lines are calculation results based on Eq. 3.24, showing a good agreement with the measurements. Insets: measured CR plotted as a function of the input light power at different bias currents. The solid green lines with the slopes of 1.04, 2.06, 3.06 and 3.99 are the fittings to the measured points plotted in log-log scale, indicating the one-, two-, three-, and four-photon regimes, respectively. The green arrows indicate the points chosen to perform the measurements in the main panel.

We note that our measurements probed the QP relaxation dynamics upon single-photon absorption in a nanoscale superconducting film, i.e. in operating conditions very similar to those encountered in the meander SSPDs, and thus provided a relevant physical picture of the operation of these detectors as well. The measured \( \tau_{ND} \) of \(~20\) ps [Fig. 3.9 (b)] agrees well with the calculated \( \tau_{ND} \) value as shown in Fig. 3.5 (b), where the calculation was performed using the experimental parameters of Fig. 3.9 (b) and assuming that the thermalization time \( \tau_1 = 10 \) ps, the QP decay time \( \tau_2 = 80 \) ps, and the quantum yield \( \xi = 0.2 \). The values of \( \tau_1 \) and \( \tau_2 \) are longer than the hot-electron energy life time in a NbN...
microbridge measured by electro-optic sampling [18] and but shorter than the one measured by terahertz spectroscopy as reported in [13]. This difference may be related to the difference in the microscopic parameters, e.g. film properties, temperatures, biasing conditions and so on. As the NRF strongly depends on the microscopic parameters, it is not feasible to determine the values of $\tau_1$, $\tau_2$ and $\xi$ by a single fit to the measured NRF. However, our approach provides a powerful tool to directly investigate the nonlinear response in SSPDs. Particularly, the value of $\tau_{ND}$ reasonably sets the time scale of the nonlinear response and therefore the intrinsic response time of SSPDs.

3.4. Probing the nonlinear response - power dependence

3.4.1. Power-dependent measurement of $P_{\text{click}}(\tau_d)$

Power-dependent measurements on the CR of the superconducting nanodetector as a function of $\tau_d$, which is proportional to the value of $P_{\text{click}}(\tau_d)$, were performed at the $I_B$ of 14.5, 13.0, 11.0 and 9.3 $\mu$A. The results are plotted in Fig 3.10 (a), (b), (c) and (d), respectively. It is interesting to see that the height and width of the nonlinear response peaks located near $\tau_d = 0$ vary depending on the light power and the $I_B$. The variation on the height-to-baseline ratio ($R_{PB}$) of the nonlinear response peaks can be explained by using a quantum tomography technique and will be investigated in Subsection 3.4.2; the variation on the width of the nonlinear response peaks will be in discussed in Subsection 3.4.3.
Fig 3.10. CR of the nanodetector plotted as a function of $\tau_d$ and of light power, measured at the $I_b$ of 14.5 $\mu$A (a), 13.0$\mu$A (b), 11.0$\mu$A (c) and 9.3 $\mu$A (d), respectively.

3.4.2. Variation on the $R_{PB}$ of the nonlinear response peak

As discussed in Section 3.1, if $\eta_{abs}\mu << 1$, the click probability of the nanodetector working in the $N$-photon regime can be described by Eq. 3.2. When $\tau_d = 0$, the two probing pulses (each has an input average photon number per pulse $\mu$) arrive within the QP lifetime, so they will contribute to a single $N$-photon detection event. Therefore, the click probability at zero delay, i.e. the probability at the peak, namely $P_P$, can be written as,
When $\tau_d$ becomes much larger than the QP lifetime and larger than the $\tau_{\text{rec}}$ (typically equals a few ns as introduced in Subsection 2.2.3), i.e. $\tau_{\text{ND}} << \tau_{\text{rec}} < \tau_d$, the two input pulses are detected as separated detection events so the click probability at large delays is written as,

$$P(\tau_d > \tau_{\text{rec}}) \propto 2 \left( \frac{\eta_{\text{abs}} \mu}{N!} \right)^N e^{-\left( \frac{\eta_{\text{abs}} \mu}{N!} \right)} e^{-\left( \frac{\eta_{\text{abs}} \mu}{N!} \right)} , (3.31)$$

When the $\tau_d$ is much larger than the QP relaxation time but smaller than the $\tau_{\text{rec}}$, i.e. $\tau_{\text{ND}} << \tau_d < \tau_{\text{rec}}$, the probability of having both of the two pulses detected should be subtracted from Eq. 3.31. This is because after the detection of the first pulse and within $\tau_{\text{rec}}$, the detector is not yet recovered to the superconducting state so it is blind to the second pulse (or the second pulse will be detected with a negligible probability). As shown in Fig. 3.10, the value of $\tau_d$ on the baseline of the CR ($\tau_d$) curves is approximately in the range of 100-400 ps, which belongs to this case. Therefore the click probability on the baselines reads,

$$P_{\text{B}} \propto 2 \left( \frac{\eta_{\text{abs}} \mu}{N!} \right)^N e^{-\left( \frac{\eta_{\text{abs}} \mu}{N!} \right)} - \left[ \left( \frac{\eta_{\text{abs}} \mu}{N!} \right)^N e^{-\left( \frac{\eta_{\text{abs}} \mu}{N!} \right)} \right]^2 , (3.32)$$

We define the ratio between the click probability on the peak and the one on the baseline of the CR ($\tau_d$) curve, i.e. $R_{\text{PB}}$, as,

$$R_{\text{PB}} = \frac{P_p}{P_{\text{B}}} = \frac{(2 \eta_{\text{abs}} \mu)^N}{N!} e^{-\left( \frac{2 \eta_{\text{abs}} \mu}{N!} \right)} - \left[ \frac{(\eta_{\text{abs}} \mu)^N}{N!} e^{-\left( \frac{\eta_{\text{abs}} \mu}{N!} \right)} \right]^2 , (3.33)$$

When $\eta_{\text{abs}} \mu << 1$, Eq.3.33 is approximated as,

$$R_{\text{PB}} = \frac{2^N (\eta_{\text{abs}} \mu)^N}{N!} = 2^{N-1} , (3.34)$$

For instance, in the one-, two-, three- and four-photon regime, the $R_{\text{PB}}$ equals to 1, 2, 4 and 8, agreeing well with the measured values of $R_{\text{PB}}$ as shown Fig 3.9 (a), (b), (c) and (d), respectively.
As discussed in Section 3.1, when $\eta_{\text{abs}} \mu \ll 1$, the $N$-photon regime can be found by fitting the power dependence CR curves in log-log scale as shown in Fig. 3.2. However, if the value of $\eta_{\text{abs}} \mu$ is slightly lower than 1 or larger than 1, this method will not apply. In this case a quantum tomographic characterization for the nanodetector is required.

As introduced in Ref [8, 19], the click probability of the nanodetector under illumination of a coherent laser source can be written as,

$$P_{\text{click}} = \sum_{N}^{\infty} N_p \frac{(\eta_{abs} \mu)^N}{N!} e^{-(\eta_{abs} \mu)}, \ (3.35)$$

Where $p_N$ is the POVM (positive-operator-valued measure) parameter, referring to the probability of a click induced by the detection of $N$ photons. By measuring the $P_{\text{click}}$ as a function of $\eta_{\text{abs}} \mu$, we can reconstruct the set of $p_N$ and thus obtain a full quantum tomographic picture of the nanodetector. It should be note that when the nanodetector works in the single-photon regime, the value of $p_1$ is equal to the internal QE $\eta_{\text{int}}$ as defined in Subsection 2.2.5.

Based on Eq. 3.32 and Eq. 3.35, the click probability on the base lines can be written as,

$$P_{B} = \sum_{N}^{\infty} N_p \left[ 2 \frac{(\eta_{abs} \mu)^N}{N!} e^{-(\eta_{abs} \mu)} - \left( \frac{(\eta_{abs} \mu)^N}{N!} e^{-(\eta_{abs} \mu)} \right)^2 \right], \ (3.36)$$

In order to determine the set of $p_N$, the CR at the baselines (obtained from the curves of Fig 3.10 near $\tau_d = 350$ ps) plotted as a function of the light power (which is proportional to the value of $\mu$) is fitted by Eq. 3.36, as shown in Fig. 3.11.
The sets of $p_N$ and the corresponding values of $\eta_{\text{abs}}$ determined in the fitting are listed in Table 3.1 for different $I_B$ values. Based on the calculations in Fig. 2.16 (b) and Subsection 4.5.1, we roughly estimated an optical absorptance of the nanodetector’s active, i.e. $\eta_{\text{abs}}$, to be $\sim 6.4 \times 10^{-4}$. This value is close but larger than the $\eta_{\text{abs}}$ values extracted from the tomography, which fluctuate in a range of $\sim 5.0-9.5 \times 10^{-5}$ as listed in the second column of Table 3.1. The difference may be due to the overestimation on the volume of the nanodetector’s active area.

<table>
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<th>$I_B$ ($\mu A$)</th>
<th>$\eta_{\text{abs}}$</th>
<th>$p_0$</th>
<th>$p_1$</th>
<th>$p_2$</th>
<th>$p_3$</th>
<th>$p_4$</th>
<th>$p_{N \geq 5}$</th>
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<td>1</td>
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<td>13.0</td>
<td>6.51E-5</td>
<td>0</td>
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<td>1</td>
<td>1</td>
</tr>
<tr>
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<td>6.51E-5</td>
<td>0</td>
<td>0</td>
<td>6.8E-3</td>
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</tr>
<tr>
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<td>4.97E-5</td>
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<td>0</td>
<td>8E-5</td>
<td>8E-2</td>
<td>0.956</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3.1. The parameters determined from the tomography for different $I_B$ values.

According to Eq. 3.33 and Eq. 3.35, the peak-to-baseline ratio reads,
The measured values of $R_{PB}$ extracted from the curves in Fig 3.10 are plotted (circles) as a function of the light power for the $I_B$ of 14.5, 13.0, 11.0 and 9.3 μA, as shown in Fig 3.12 (a), (b), (c) and (d), respectively. Using the parameters listed in Table 3.1, the value of $R_{PB}$ is calculated based on Eq. 3.37 and plotted in Fig 3.12 as solid lines. The calculation shows a good agreement with the measurement.
3.4.3. Variation on the width of the nonlinear response peak

Since the nonlinear detection model introduced in Section 3.2 is based on the assumption of detecting less than one photon per pulse on average, i.e. $\eta_{\text{abs}} \mu \ll 1$, it does not apply to the case of higher light powers where the value of $\eta_{\text{abs}} \mu$ becomes close to 1 or even larger than 1. So it is not feasible to extract $\tau_{\text{ND}}$ for the whole power range of Fig. 3.10. Therefore, instead of extracting $\tau_{\text{ND}}$ from the CR($\tau$) curves as we did in Section 3.3, here we extracted the FWHM of the nonlinear response peaks, i.e. $w_P$, from the curves of Fig. 3.10. The $w_P$ scales with $\tau_{\text{ND}}$, providing a measure of the QP lifetime in the superconducting nanowires. The $w_P$ of the nonlinear response peaks for the $I_B$ of 14.5, 13.0, 11.0 and 9.3 $\mu$A were extracted from Fig. 3.10 and plotted as a function of light power in Fig. 3.13.

![Fig 3.13. The value of $w_P$ extracted from Fig. 3.10, plotted as a function of the light power for the $I_B$ of 14.5 $\mu$A (black), 13.0 $\mu$A (blue), 11.0 $\mu$A (red) and 9.3 $\mu$A (green), respectively.](image)

As shown in Fig. 3.13, the value of $w_P$ generally increases as a function of light power. The $w_P$ slowly increases at low powers, and rapidly increases by almost an order of magnitude as the power exceeds the value of ~100 nW. The growth of $w_P$ indicates a longer nonlinear response time of the nanodetector under a stronger illumination, possibly suggesting a heating effect induced by the input light, which may change the microscopic parameters in the film and thus the value of $w_P$. The heating effect is a common phenomenon observed in the characterization of SSPDs and has been reported in the literature [20-22]. The device can be heated up in two possible ways. On the one hand, the input photons absorbed in the nanowire trigger the detections by switching the wire to the resistive state. The Joule heating due to the resistive section formed in the wire warms up the device for a few ns. As
the light power increases, this happens more often so that there is not enough time for the heat to dissipate through the substrate and the temperature is raised. On the other hand, the photons absorbed by the substrate can heat the substrate when the light power is high. Consequently, the heated substrate increases the temperature of the nanowire through heat exchange.

As shown in Fig. 2.5, the value of $I_C$ decreases when the temperature of the device is increased. Therefore, by monitoring the $I_C$ during the measurement, the temperature of the device can be estimated following Eq. 2.1. As shown in Fig. 3.14, the $I_C$ of the nanodetector during the power-dependent measurements shown in Fig. 3.10 were measured and plotted (black) as a function of light power. When the light power grows to be above ~0.8 nW, the value of $I_C$ starts to decrease and finally drops to ~15 μA at a few μW. The corresponding effective temperature of the device $T_{eff}$ was calculated based on Eq. 2.1 and Fig. 2.5, and plotted in Fig. 3.14 to the right axis (blue) as a function of light power. The $T_{eff}$ increases from the base temperature of ~1.2 K to nearly 5 K as the light power is increased from ~0.8 nW to a few μW.

![Fig 3.14. The $I_C$ (black) measured during the power-dependent measurement of Fig. 3.10, the corresponding $T_{eff}$ (blue) calculated based on Eq. 2.1 and Fig. 2.5, and the FWHM of the NRF (red), $w_{cal}$, calculated based on Eq. 3.19, plotted as a function of the light power.](image)

The consequence of the increase of the device’s temperature is that the value of $w_P$ increases. We found that even if the lifetime of QPs keeps unchanged, the temperature will still increase the width of the nonlinear response peak. For instance, assuming $\tau_1 = 10$ ps, $\tau_2 = 80$ ps, $\xi = 0.2$, and using the experimental parameters in the measurement as shown in
Fig. 3.10, including $h\nu = 1.08$ eV, $I_B/I_C = 0.5$, $w = 150$ nm, $d = 4.8$ nm, and the relevant parameters in the Appendix, we calculated the NRF according to the VAP unbinding model for the temperatures of 1.2 K, 1.9 K, 3.1 K and 4.5 K. The FWHM of the calculated NRF, i.e. $w_{\text{cal}}$, was extracted and plotted (red) as a function of the corresponding light power (based on the dependence of $T_{\text{eff}}$ on light power as shown in Fig. 3.14) as shown in Fig. 3.14 to the right axis. The $w_{\text{cal}}$ increases by almost a factor of two for the whole power range. The increase of $w$ value is not due to the increase of the QP lifetime since the value of $\tau_1$ and $\tau_2$ were kept constant in the calculation; instead, the temperature-dependent parameters used in the calculation changes at higher temperatures, consequently resulting in a larger $w_{\text{cal}}$. Therefore, even when the QP relaxation time keeps constant, the width of the measured nonlinear response peak can vary due to the change in the temperature-dependent parameters of the film. This increase, however, is insufficient to explain the experimental data in Fig. 3.13, where an even larger increase on the width is observed. The difference may possibly due to dependence of $\tau_1$ and $\tau_2$ on the temperature. For instance, as reported in Ref [18], the measured value of $\tau_2$ is larger at higher temperature.

3.5. The coherence peak

![Coherence peak](image)

Fig. 3.15 (a) Measured CR plotted as a function of the $\tau_d$, normalized by its value at long delays. (b) Simulation based on Eq. 3.26. An enlarged view at small delays is shown in the insets.

In order to experimentally investigate the coherence peak as discussed in Subsection 3.2.3, we measured the CR of the nanodetector working in the two-photon regime in a short delay window using OPO pulses. To this aim, the $\tau_d$ was controlled by a motorized delay line for coarse control and by a fiber stretcher for fine control. A driving voltage $V_{\text{FS}}$ (zigzag wave,
±5 V, 20 mHz) was applied on the FS to change the optical path difference between the two arms (Section 2.4). At each coarse delay, the counter recorded the CR as a function of the $V_{FS}$. Due to the short time width of the OPO pulse, the interference fringes were observed at small coarse delays and an additional peak appears in a small window near zero delay [blue points as shown in Fig. 3.15 (a), also shown in an enlarged view in the inset], on top of the broad peak related to the detector’s NRF, as observed in Fig. 3.9 (b) and replotted here as green circles in Fig. 3.15 (a). It gives the information related to the coherence time of the OPO pulse according to the third term in Eq. 3.28. A theoretical comparison based on Eq. 3.26 is shown in Fig. 3.10 (b). In the calculation, $\tau_{ND} = 20.4$ ps and the probe pulse has a temporal width of 1.6 ps (assumed to be Fourier-transform limited) and a wavelength of 1.13 $\mu$m, the same as those used in the experiments. The calculation shows a good agreement with the experiment. We note that this narrow peak also appears in the filtered autocorrelation, shown as a red line in the insets of Fig. 3.15. However, as indicated by Eq. 3.28, its width relates to the first-order correlation function, and therefore to the coherence time, and not to the duration of the probe pulse.
References:


CHAPTER 4

Ultrasensitive $N$-photon interferometric autocorrelator based on superconducting nanowires

In this chapter an ultrasensitive $N$th-order ($N = 1, 2, 3, 4, 5, 6$) interferometric autocorrelator based on a superconducting nanodetector is presented. The nanodetector-based autocorrelator provides much higher sensitivity as compared to the conventional autocorrelators based on all-optical nonlinearities. It has a temporal resolution of ~20 ps, limited by the QP relaxation time in the superconducting films which was measured as demonstrated in Chapter 3. A comparison of the sensitivity to conventional autocorrelators and possible improvements will be discussed. Part of this chapter has been published by Z. Zhou et al., in “Ultrasensitive $N$-photon interferometric autocorrelator,” Phys. Rev. Lett., vol. 110, p. 133605, 2013 [1].

4.1. Introduction

As introduced in Section 1.7, the temporal correlation functions of various orders are of fundamental importance in the classical and quantum description of optical fields. The first-order (field) correlation function, i.e. $G^{(1)}(\tau_d) = \langle E^*(t)E(t + \tau_d) \rangle$, where the symbol $\langle \rangle$ denotes a temporal average over a long time interval, describes temporal coherence and therefore spectral line-width of a light source. The second-order (intensity) correlation function, i.e. $G^{(2)}(\tau_d) = \langle I(t)I(t + \tau_d) \rangle$, is used to measure the temporal properties of pulsed sources and to distinguish quantum and classical fields. The measurement of higher-order autocorrelation is more sensitive to coherence features (e.g. photon bunching) of the light field [2] and can be used to determine the asymmetry of light pulses [3].

Whereas the first-order correlation function is easily measured using an interferometer and a linear detector [4], the measurement of higher-order correlation functions requires a process that is nonlinear in the intensity $I(t)$. As discussed in Subsection 1.7.2, in interferometric autocorrelators, the normalized second-order correlation function $g^{(2)}(\tau_d) = \langle I(t)I(t + \tau_d) \rangle/\langle I(t) \rangle^2$ is usually measured by using nonlinear detection techniques, e.g. SHG in a non-linear crystal, followed by a linear detector [5], or TPA in the detector itself [6, 7]. In both cases, the detector measures the square of the total intensity at the output of the interferometer,
\[ \langle I_{\text{out}}^2(t) \rangle \propto \langle I^2(t) \rangle + 2\langle I(t)I(t + \tau_d) \rangle, \quad (4.1) \]

Where \( \tau_d \) is the delay between the two arms of the interferometer, together with interference terms which are sensitive to the phase properties of the input light source. While these approaches offer very high temporal resolution, since the related nonlinear optical processes are nearly instantaneous, their sensitivity is limited by the low nonlinear susceptibilities involved in the SHG or TPA processes. Because of the even lower relevance of higher-order optical nonlinearities, measurement of the autocorrelations of order \( N > 2 \) requires high input powers [3, 8, 9].

As introduced in Subsection 1.7.3, an alternative approach consists of combining linear optical detection with nonlinear processing in the electrical readout, e.g. in a correlation card for HBT autocorrelators [10]. In this case, sensitive SPDs can be used. However, the temporal resolution of HBT autocorrelators is limited to \( \geq 50 \) ps [11] by the jitter of the detector output and the amplification and correlation electronics. In addition, differently from interferometric autocorrelation, these approaches do not provide any information on the phase properties.

In this Chapter, we present a novel approach to the measurement of interferometric autocorrelations of up to 6th-order, which is based on the combination of an interferometer and a superconducting nanodetector. Based on the theoretical and experimental study in Chapter 3, we will show that the superconducting nanodetector can be applied as an interferometric autocorrelator with high detection sensitivity and temporal resolution, when the temporal width of the input light is longer than the intrinsic response time \( \tau_{\text{ND}} \). On the one hand, as compared to the conventional autocorrelators based on all-optical nonlinearities [12-14], it provides much higher sensitivity, due to the combination of linear optical absorption and nonlinear detection, and due to the low detector noise. A detailed comparison to all-optical conventional autocorrelators will be presented. On the other hand, the nanodetector-based autocorrelator has a better temporal resolution as compared to the HBT autocorrelators [10, 11]. The nanodetector-based autocorrelator’s temporal resolution, limited by QP energy relaxation time, was directly measured to be \( \sim 20 \) ps as shown in Chapter 3; while the temporal resolution of the HBT autocorrelators, limited by the jitter in the two linear detectors and in the electronics, is \( \geq 50 \) ps [11] as introduced in Subsection 1.7.3. In addition, the nanodetector-based autocorrelator uses an easier electrical readout circuit than the HBT autocorrelators and provides interferometric phase information.
4.2. $N$-photon interferometric autocorrelator - theory

As introduced in Chapter 3, by choosing the bias current, the nanodetector can be operated in the multi-photon regime. The multi-photon detection in a nanodetector is realized when the multiple photons are absorbed with the lifetime of QPs and can be described by a nonlinear response function (NRF, Eq. 3.21), which is governed by the intrinsic response time of the nanodetector, i.e. $\tau_{ND}$. By using two short pulses, each with a time width shorter than the value of $\tau_{ND}$, the nonlinear response of the nanodetector was probed and the $\tau_{ND}$ was measured to be $\sim20$ ps.

According to the semi-classical photo-detection theory introduced in Section 3.2, if the temporal width of the input pulse is much larger than the value of $\tau_{ND}$, the intensity autocorrelation function which appears in the integral in Eq. 3.24 can be approximated by its value at zero delays, so that Eq. 3.24 becomes,

$$P_{\text{click}} \approx \xi^N S^N \int_{-\infty}^{\infty} \eta_1(t_1) \cdots \eta_N(t_N) dt_1 \cdots dt_N,$$

$$= \int_{-\infty}^{\infty} I_{\text{in}}(t) I_{\text{in}}(t+\tau_{\text{ND}}) \cdots I_{\text{in}}(t+N\tau_{\text{ND}}) dt,$$

$$\propto \int_{-\infty}^{\infty} I_{\text{in}}^N(t) dt,$$

(4.2)

When the nanodetector is placed at the output of a Michelson interferometer, as shown in Fig. 4.1, the input light intensity is given by $I_{\text{in}}(t) \propto |E(t)+E(t+\tau_d)|^2$. In this case, the click probability in the $N$-photon regime is proportional to the $N$th-order interferometric autocorrelations given by,

$$P_{\text{click}}(\tau_d) \propto \int_{-\infty}^{\infty} |E(t)+E(t+\tau_d)|^2 dt,$$

(4.3)

For instance, in the two-photon regime, by filtering out the high-frequency coherence terms we obtain,

$$P_{\text{click}}(\tau_d) \propto \left\langle I^2(t) \right\rangle + 2\left\langle I(t)I(t+\tau_d) \right\rangle,$$

(4.4)

By normalizing Eq. 4.4 over $\left\langle I^2(t) \right\rangle$, we have,

$$P_{\text{click}}(\tau_d) \propto 1 + \frac{2\left\langle I(t)I(t+\tau_d) \right\rangle}{\left\langle I^2(t) \right\rangle} = 1 + g^{(2)}(\tau_d),$$

(4.5)

Where the normalized second-order correlation function $g^{(2)}(\tau_d)$ is obtained.
Therefore, a nanodetector operated in the N-photon regime and combined with a Michelson interferometer enables Nth-order interferometric autocorrelation measurements in the limit of $\tau_{ND} \ll \tau_p$, where $\tau_p$ is the input pulse width. In the case where the value of $\tau_{ND}$ is close to the value of $\tau_p$, the $P_{\text{click}}(\tau_d)$ must be calculated using Eq. 3.24. In the following, for a case corresponding to the experiment to be described in Section 4.3, we assume that the input light pulse was a Gaussian pulse with a linear chirp, so the time-dependent electric field of the light $E(t)$, is defined as,

$$E(t) = E_0 \exp[-4 \ln(2)(1+iA)t^2/\tau_p^2], \quad (4.6)$$

Where $E_0$ is the field amplitude, $A$ is the linear chirp parameter [15] and $\tau_p$ is the FWHM of the pulse.

Based on Eq. 4.3 and Eq. 4.6, we calculated the Nth-order interferometric autocorrelations of the input light, using the parameters of $A = 5.3$ and $\tau_p = 70.6$ ps, which were determined by fitting the measurements shown later in Fig. 4.5 (a) and (b). The NRF $\eta(\tau)$ (Eq. 3.21) with $\tau_{ND} = 20.4$ ps, determined by using short pulses as shown in Section 3.3, was used for the calculation.
Fig. 4.2. $N$th-order interferometric autocorrelations calculated based on Eq. 4.3 and Eq. 4.6 for $N = 1$ (a), 2 (b), 3 (c), 4 (d), 5 (e) and 6 (f), respectively. The $|g^{(1)}(\tau_d)|$ [dashed blue lines in (a) plotted to the right axis] was obtained by calculating the visibility of the interference fringes using based on Eq. 4.7. Intensity autocorrelation traces [dashed blue lines in (b)-(f)] were obtained by applying a low-pass filter to the interferometric fringes. Upper-right insets: an expanded view of the interference fringes for $-4.5 \text{ fs} \leq \tau_d \leq 4.5 \text{ fs}$. 

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The calculated $N$th-order interferometric autocorrelations for $N = 1, 2, 3, 4, 5, 6$ are in Fig. 4.2 (a)-(f), respectively. The calculation was performed with large time step, i.e. 4 ps, in order to reduce the calculation time. Upon each large time step, small time steps, i.e. 0.2 fs, were added to resolve the interference fringes, covering at least one temporal period of the input light. The upper-right insets of Fig. 4.2 show an enlarged view of the interferograms for $-4.5$ fs $\leq \tau_0 \leq 4.5$ fs. The fringe contrast ratio is observed to increase with $N$, in a good agreement with the theoretical values of 2, 8, 32, 128, 516 and 2048 (marked by green dashed lines in the insets) for $N = 1, 2, 3, 4, 5$ and 6, respectively. In the one-photon regime [Fig. 4.2 (a)], the absolute value of the first order correlation function $|g^{(1)}(\tau_0)|$ is obtained (dashed blue line plotted to the right axis) by calculating the visibility of the interference fringes using [4],

$$
|g^{(1)}(\tau_0)| = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}, \quad (4.7)
$$

Where $I_{\text{max}}$ and $I_{\text{min}}$ are the maximum and minimum of the intensity of the fringes, respectively. In the higher-photon regimes [Fig. 4.2 (b)-(f)], the intensity autocorrelation traces (dashed blue lines) were obtained by applying a low-pass filter to the interferometric fringes. The FWHM of the calculated fringes, extracted from the insets of Fig. 4.2 and normalized by their period, is plotted as a function of $N$ in Fig. 4.3. A fit to the normalized FWHM shows a dependence of approximately $N^{-0.5}$, which is a characteristic of multi-photon interferometry [16] and confirms the reliability of our simulation.

![Fig. 4.3. FWHM of the calculated interference fringes, normalized by their period, plotted as a function of $N$, shows a dependence of approximately $N^{-0.5}$.](image-url)
4.3. \(N\)-photon interferometric autocorrelator - experiment

To demonstrate the application of a nanodetector as an interferometric autocorrelator, a 1.3-\(\mu\)m gain-switched diode laser with a repetition rate of 10 MHz and pulse width \(\sim 70\) ps (larger than the autocorrelator’s temporal resolution \(\tau_{\text{ND}}\)) was employed (Fig. 4.1). The \(N\)-photon \((N = 1, 2, 3, 4, 5, 6)\) regimes were found by choosing a proper bias current \(I_B\). As shown in Fig. 4.4, the CR was measured as a function of input light power, plotted in log-log scale. The experimental data obtained at the \(I_B\) of 18.0, 14.0, 11.5, 9.6, 8.4 and 7.7 \(\mu\)A, fits the solid lines with the slopes of 1, 2, 3, 4, 5 and 6 well, corresponding to the one-, two-, three-, four-, five- and six-photon regimes, respectively.

![Graph showing CR vs. Power](image)

Figure 4.4. CR measured at the \(I_B\) of 18.0, 14.0, 11.5, 9.6, 8.4 and 7.7 \(\mu\)A, plotted as a function of incident light power in log-log scale. The solid lines, which have the slopes of 1, 2, 3, 4, 5 and 6, fit the experiments well, corresponding to the one-, two-, three-, four-, five- and six-photon regimes, respectively. In each photon regime, one point was chosen (marked by green arrows) to perform the autocorrelation measurements as shown in Fig. 4.5.
Fig. 4.5. Normalized CR as a function of $\tau_d$ measured at the chosen points in Fig. 4.4, in the one (a), two (b), three (c), four (d), five (e) and six-photon (f) regimes, respectively. The $|g^{(1)}(\tau_d)|$ [solid red line in (a)] and the low-pass curves [solid red lines in (b)-(f)] are plotted together with the calculations (dashed blue lines). Upper-right insets: an expanded view of the interference fringes at small delays, plotted as a function of the voltage $V_{FS}$ applied to the fiber stretcher.
In each photon regime, one point was chosen (marked by green arrows in Fig. 4.4) to perform the autocorrelation measurements. As shown in Fig. 4.5 (a)-(f), the CR (proportional to $P_{\text{click}}$) recorded in the one-, two-, three-, four-, five- and six-photon regimes, respectively, was normalized by its value at long delays and plotted as a function of $\tau_d$. The measured $P_{\text{click}}(\tau_d)$ shows a good agreement with the calculation as shown in Fig. 4.2. Similarly to the calculation, the measurement was performed with large time step, i.e. 2 ps; upon each large time step, small time steps (< fs) are added to resolve the interference fringes, covering at least one temporal period of the input light. The upper-right insets of Fig. 4.5 show an enlarged view of the interferograms near zero delay. The fringe contrast ratio agrees well with the theoretical values of 2, 8, 32, 128, 516 and 2048 for $N = 1, 2, 3, 4, 5$ and 6, respectively. The $|g^{(1)}(\tau_d)|$ (solid red line) was obtained by calculating the visibility of the interference fringes in the one-photon regime [Fig. 4.5 (a)] based on Eq. 4.7. The intensity autocorrelation traces (solid red line) in the higher-photon regimes were obtained by applying a low-pass filter. By fitting the measured $|g^{(1)}(\tau_d)|$ in the one-photon regime in Fig. 4.5 (a) together with the second-order intensity autocorrelation traces in Fig. 4.5 (b), the values of $A$ and $\tau_p$ were determined to be 5.3 and 70.6 ps, respectively, and applied to the calculation in Fig. 4.2 without additional fitting parameters. The calculated low-pass traces in Fig. 4.2 are replotted in Fig. 4.5 (c)-(f) as dashed blue lines. In Fig. 4.5 (c) and (d), the calculations show a good agreement with the experiments in the three and four-photon regimes; however, in the five and six-photon regimes in Fig. 4.5 (e) and (f), the calculated autocorrelation peaks are wider than the experiments. This difference might result from the simple assumption of a linear chirp of the diode laser in our calculation.

Fig. 4.6. FWHM of the measured interference fringes, normalized by their period, plotted as a function of $N$, shows a dependence of approximately $N^{-0.5}$. 

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The FWHM of the measured fringes, extracted from the insets of Fig. 4.5 and normalized by their period, is plotted as a function of $N$ in Fig. 4.6. A fit to the normalized FWHM shows a dependence of approximately $N^{-0.5}$, agreeing well with our calculation.

4.4. Comparison with conventional autocorrelators

4.4.1. Comparing the sensitivity

A comparison of the sensitivity of nanodetector-based autocorrelators to conventional autocorrelators based on all-optical nonlinearities is presented in the following, providing a useful insight.

In principle, for input pulses longer than the nanodetector's intrinsic response time $\tau_{ND}$, the two-photon click probability according to Eq. 3.22 is written as,

$$P_{\text{click}} = C_{ND} \int_{-\infty}^{\infty} P_{\text{in}}^2(t) \cdot dt , \quad (4.8)$$

Where $P_{\text{in}} = I_{\text{in}} S$ is the power incident on the detector and $C_{ND}$ represents a nonlinear response efficiency for the sensitivity of the autocorrelator, reads,

$$C_{ND} = \sqrt{\pi \eta_{\text{abs}}^2(0) \tau_{ND} / (\hbar \nu)^2} , \quad (4.9)$$

A similar expression of $P_{\text{click}}$ can be derived for nonlinear detectors based on TPA or SHG. For a TPA detector based on a photomultiplier with quantum efficiency $\eta_{PM}$ and TPA coefficient $\beta_2$, the nonlinear response efficiency is easily derived as [1],

$$C_{\text{TPA}} = \eta_{PM} \beta_2 d / h \nu S , \quad (4.10)$$

Where $d$ is the active thickness and $S$ the illuminated area.

For an SHG crystal followed by an SPD with quantum efficiency $\eta_{SPD}$ (at the second-harmonic frequency $2\nu$), $C_{\text{SHG}}$ is given by [1],

$$C_{\text{SHG}} = \eta_{\text{SPD}} \eta_{\text{SHG}} / (2h \nu) , \quad (4.11)$$

Where $\eta_{\text{SHG}} = P_{\text{out},2\nu} / P_{\text{in},\nu}^2$ is the nonlinear conversion efficiency.

Interestingly, in an absorption thickness of 4 nm, the nanodetector reaches a nonlinear efficiency of $C_{ND} = 1.7 \times 10^{19} \text{ W}^{-1}\text{J}^{-1}$, several orders of magnitude higher than $\mu$m-thick TPA absorbers and comparable to cm-long SHG crystals, showing the giant effective nonlinearity achieved through the combination of linear absorption and nonlinear detection.
In all three cases, the sensitivity can be defined by imposing that the CR is equal to the DCR \( R_{\text{dark}} \) in the detector. For a periodic sequence of square pulses with repetition rate \( f_{\text{rep}} \), the condition \( f_{\text{rep}} P_{\text{click}} \geq R_{\text{dark}} \) leads to \( P_{\text{pk},\text{min}} P_{\text{av},\text{min}} \geq R_{\text{dark}} / C \) (where \( P_{\text{pk},\text{min}} \) and \( P_{\text{av},\text{min}} \) are the minimum peak and average light powers, respectively). Using the measured values of \( \eta_{\text{abs}} = 1.5 \times 10^{-4} \) and \( \eta(0) \approx 0.5 \) reported in Ref. [17] where the same device was used under the same experimental condition of Fig. 4.5 (b), and \( R_{\text{dark}} = 1 \) Hz, we derive a sensitivity of \( P_{\text{pk},\text{min}} P_{\text{av},\text{min}} \approx 5.8 \times 10^{-20} \text{ W}^2 \), corresponding to \( \sim 4 \) photons per pulse in the experiment.

Practically, the \( N \)th-order \((N = 1, 2, 3, 4, 5 \text{ and } 6) \) interferometric autocorrelations as shown in Fig. 4.5 were obtained with ultralow powers, ranging from pW to nW. Comparing to the conventional autocorrelators based on all-optical nonlinearities, e.g. TPA or SHG, the sensitivity of our nanodetector-based autocorrelator is much higher. The two-photon autocorrelation trace in Fig. 4.5 (b) was taken with \( P_{\text{pk}} P_{\text{av}} = 5.6 \times 10^{-17} \text{ W}^2 \) (where \( P_{\text{pk}} \) and \( P_{\text{av}} \) are the peak and average light powers, respectively), about seven orders of magnitude lower than the minimum reported value of \( P_{\text{pk}} P_{\text{av}} \) for TPA technique [12], and about two orders of magnitude lower than the lowest \( P_{\text{pk}} P_{\text{av}} \) value reported in the previous second-order interferometric correlations based on SHG [13]. The advantage of using linear absorption is even larger when \( N > 2 \). For instance, in Fig. 4.5 (c), the three-photon autocorrelation was performed at an average power six orders of magnitude lower than the best previous record with third-order interferometric correlations [14], corresponding to \( (P_{\text{pk}})^2 P_{\text{av}} = 2.0 \times 10^{-21} \text{ W}^3 \), an improvement of about twenty-one orders of magnitude over Ref. [14]. To the best of our knowledge, \( N \)-photon interferometric autocorrelation for \( N > 3 \) has not been reported before.

4.4.2. Defining a figure of merit

In nanodetector-based autocorrelators a compromise exists between the \( C_{\text{ND}} \) and the \( \tau_{\text{ND}} \). For instance, the two-photon detection will be triggered only if the two photons arrive within the \( \tau_{\text{ND}} \), which is determined to be \( \sim 20 \) ps in our work. Higher autocorrelation sensitivity is expected if the \( \tau_{\text{ND}} \) increases since \( C_{\text{ND}} \) is proportional to the value of \( \tau_{\text{ND}} \); however, this will lower its temporal resolution, which is limited by the \( \tau_{\text{ND}} \).

A similar compromise can be seen in the TPA- and SHG-based autocorrelators. The temporal resolution of TPA-based autocorrelators, limited by the lifetime of the virtual states associated to the TPA transition, is in the order of fs [7], comparably shorter than other autocorrelators. However, this is obtained at the expense of the sensitivity of TPA-based autocorrelators. In the SHG-based autocorrelators, higher conversion efficiency can be achieved using a longer SHG crystal, which results in a smaller phase-matching bandwidth and thus a lower temporal resolution [18, 19].
In order to take into account both the detection sensitivity and the temporal resolution of the autocorrelators, we define a figure of merit (FOM) as the product of $P_{pk}P_{av}$ and $\tau_{res}$ for second-order interferometric autocorrelators, written as,

$$\text{FOM} = P_{pk}P_{av}\tau_{res}, \quad (4.12)$$

We compare the FOM of three different types of autocorrelators based on: nanodetector, TPA, and SHG [including Periodically Poled Lithium Niobate (PPLN)], as shown in Table 4.1.

<table>
<thead>
<tr>
<th>Type</th>
<th>Description</th>
<th>$P_{pk}P_{av}$ (W²)</th>
<th>$\tau_{res}$</th>
<th>FOM (W²·s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nanodetector</td>
<td>Fig.4.4 (b)</td>
<td>$5.6 \times 10^{-17}$</td>
<td>~20.4 ps</td>
<td>~$1.1 \times 10^{-27}$</td>
</tr>
<tr>
<td></td>
<td>Estimated in main text</td>
<td>$5.8 \times 10^{-20}$</td>
<td>~20.4 ps</td>
<td>~$1.2 \times 10^{-30}$</td>
</tr>
<tr>
<td>TPA</td>
<td>GaAs PM tube [20]</td>
<td>$1.7 \times 10^{-10}$</td>
<td>~fs</td>
<td>~$1.7 \times 10^{-25}$</td>
</tr>
<tr>
<td></td>
<td>InGaAsP diode [11]</td>
<td>$1.5 \times 10^{-10}$</td>
<td>~fs</td>
<td>~$1.5 \times 10^{-25}$</td>
</tr>
<tr>
<td>SHG (PPLN)</td>
<td>Bulk SHG crystal [19]</td>
<td>$1 \times 10^{-6}$</td>
<td>~fs</td>
<td>~$1 \times 10^{-21}$</td>
</tr>
<tr>
<td></td>
<td>PPLN [19]</td>
<td>$3.2 \times 10^{-13}$*</td>
<td>~22 ps</td>
<td>~$7 \times 10^{-24}$</td>
</tr>
<tr>
<td></td>
<td>A-PPLN [19]</td>
<td>$3.2 \times 10^{-13}$</td>
<td>~100 fs</td>
<td>~$3.2 \times 10^{-26}$</td>
</tr>
<tr>
<td></td>
<td>MIFA+PPLN [21]</td>
<td>$1.1 \times 10^{-13}$</td>
<td>~200 fs</td>
<td>~$2.2 \times 10^{-26}$</td>
</tr>
<tr>
<td></td>
<td>MIFA+PPLN+Pulse-shaper [13]</td>
<td>$2.7 \times 10^{-15}$</td>
<td>~200 fs</td>
<td>~$5.4 \times 10^{-28}$</td>
</tr>
</tbody>
</table>

Table 4.1. FOM of three different types of autocorrelators based on: nanodetector, TPA, and SHG [including PPLN]. *Assuming the PPLN in Ref [19] obtained the same efficiency as the A-PPLN in Ref [19].

Our nanodetector-autocorrelator outstands in this table for its FOM as compared to most of the reported second-order interferometric autocorrelators. The FOM of our present nanodetector [Fig 4.5 (b)] is about two order of magnitude better than the record obtained using TPA autocorrelator [12], six order of magnitude better than the bulk SHG crystal [19], four order of magnitude better than the record based on PPLN technique [19], one order of magnitude better than the record using A-PPLN [19], and comparable with the measurement reported in Ref [13] where many complex techniques (MIFA, PPLN, Pulse-shaper, etc.) were combined together.

In addition, our nanodetector is suitable for a large wavelength range, typical from visible to near-infrared without changing its design, which makes it much more convenient than conventional autocorrelators, particularly those based on SHG in long crystals, which limits the operating bandwidth [18, 19]. In higher-photon regime its advantage is even more
obvious. As compared to the complete rebuilding of the experimental setup needed to go to higher harmonics in autocorrelations based on nonlinear frequency conversion, a simple change of the bias current can easily set our nanodetector in higher-photon regimes and allow the measurement of higher-order autocorrelations.

4.5. Possible improvements on the sensitivity
Due to the small active area and the small thickness of the nanodetector, the sensitivity of the present nanodetector-based autocorrelator is limited by the low optical absorptance $\eta_{abs}$ under far-field illumination. In the following we will discuss several possible approaches which can increase the $\eta_{abs}$ and therefore the sensitivity of the autocorrelator.

4.5.1. Reducing the beam spot size
Reducing the size of the input beam spot can increase the value of $\eta_{abs}$. In the current experiment as shown in Fig. 4.5 (b), the $e^2$ diameter of the Gaussian beam spot is ~5 $\mu$m. According to Fig. 2.16 (b) and assuming the active area of the nanodetector is 150×150 nm$^2$, only ~0.94 % of the light power reaches the active area, giving an $\eta_{abs}$ value of ~0.073 % at 1.3 $\mu$m. By using a high-numerical aperture lens, and/or a solid-immersion lens, it is possible to reduce the spot $e^2$ diameter to ~0.5 $\mu$m [22]. In this case, ~6.2 % of the light power reaches the active area, leading to an $\eta_{abs}$ value of ~0.48 % at 1.3 $\mu$m. Consequently, the value of $\eta_{abs}$ can be increased by nearly two orders of magnitude.

4.5.2. Increasing the active area
Increasing the active area of the nanodetector may increase the value of $\eta_{abs}$. We assume that the active area of the nanodetector approximately equals to the area of its constriction, i.e. $A_C = w_C L_C$, where $w_C$ and $L_C$ are the width and length of the constriction as indicated in Fig. 4.7 (a), respectively. Further assuming the size of the input laser beam is kept constant, a larger portion of input light can reach the active area if either the $w_C$ or the $L_C$ is increased. In principle we have,

$$\eta_{abs} \propto w_C L_C, \quad (4.13)$$

However, increasing the size of the active area may introduce side effects. On the one hand, increasing the value of $w_C$ (while keeping $L_C$ constant) may cause a reduction in the detector’s internal efficiency [23], and thus reduce the sensitivity. Therefore, a compromise exists between the $\eta_{abs}$ or the $\eta_{int}$ when one tries to improve the sensitivity by changing the $w_C$. On the other hand, increasing the value of $L_C$ (while keeping $w_C$ constant) may introduce an additional linear background on the multi-photon detections (Section 3.1), which reduces the signal-to-noise ratio (SNR) and the sensitivity in the autocorrelation
measurements. This is because in a multi-photon detection the multiple photons need to be absorbed within the area of the distributed QPs (i.e. hot-spot area) in the superconducting nanowire. If the value of $L_C$ is larger than the size of the hot-spot, the multiple photons which are not absorbed close enough to each other may individually trigger single-photon detections and consequently creates a linear-detection background on the multi-photon responses. We point out that the optimum value of $L_C$ should be close to the size of the hot-spot.

4.5.3. Using bottom mirrors and plasmonic nanoantennas

It has been demonstrated that by integrating a meander SSPD on top of a distributed Bragg reflector (DBR) [24], or by integrating plasmonic nanostructures close to an SSPD [25, 26], the optical absorptance $\eta_{\text{abs}}$ of the device can be increased. In both of these approaches, the spatial distribution of the EM field near the SSPD’s active area is modified by the introduced nanostructure so that more light power flows to the active area. In the following, we will theoretically investigate the enhancement of $\eta_{\text{abs}}$ by integrating an AlAs/GaAs DBR and/or a plasmonic nanoantenna pair to a superconducting nanodetector, using FDTD simulation (Section 2.5).

We assume that a nanodetector has a thickness of 4 nm and a constriction with the geometry of $w_C = L_C = 100$ nm. An AlAs/GaAs DBR [24] is designed to be resonant at $\lambda = 1530$ nm, consisting of 14.5 periods of (131.1-nm AlAs, 113.0-nm GaAs), capped by a 226.0-nm thick GaAs layer as a $\lambda/2$ spacer between the top of the DBR and the GaAs air interface. The nanoantenna pair is made of gold and has a bow-tie shape. The geometry of the nanoantenna is marked in Fig. 4.7 (c) and 4.7 (d). Each triangular prism of the pair has thickness ($t$) of 152 nm, edge length ($L$) of 200 nm, and height ($h$) of 170 nm. The distance ($d$) between the tips of the nanoantennas is 150 nm. Such geometry of the nanoantenna is chosen as it provides a plasmonic resonance wavelength at $\sim 1530$ nm, which matches the resonance of the DBR. A plane wave is sent to the device with a polarization direction either along the X- or Y-axis. We assume an active volume of $100 \times 100 \times 4$ nm$^3$ and define the optical absorptance as ratio between the net power flowing into this volume (detected by a $100 \times 100 \times 4$ nm$^3$ monitor box as marked by green square in Fig. 4.7) and the total power incident into the illuminated area (dashed red line, 1$\times$1 $\mu$m$^2$). A schematic drawing of the superconducting nanodetector, the substrate and the plasmonic nanoantennas used in the simulation is depicted in Fig. 4.7. Four configurations were simulated, including a nanodetector on GaAs substrate [Fig. 4.7 (a)], a nanodetector on the DBR [Fig. 4.7 (b)], a nanodetector integrated with a plasmonic nanoantenna pair on GaAs substrate [Fig. 4.7 (c)], and on the DBR [Fig. 4.7 (d)], respectively.
Fig. 4.7. Schematic drawing of the superconducting nanodetector, the substrate and the plasmonic nanoantenna used in the FDTD simulation. Four configurations are simulated, including a nanodetector on GaAs substrate (a), a nanodetector on DBR (b), a nanodetector integrated with a plasmonic nanoantenna pair on GaAs substrate (c) and on DBR (d), respectively.

The calculated optical absorptance for the configurations of Fig. 4.7 (a)-(d) with the light polarization direction along both the X-axis (solid lines) and Y-axis (dashed lines) is plotted as a function of wavelength as shown in Fig. 4.8. For the configuration of Fig. 4.7 (a) where the nanodetector is placed on GaAs substrate, the optical absorptance is about \(2 \times 3 \times 10^{-4}\), shown as green lines in Fig. 4.8. When the DBR is applied as the substrate of the nanodetector as depicted in Fig. 4.7 (b), the absorptance at the resonance wavelength of \(\approx 1530\) nm increases to about \(2 \times 3 \times 10^{-3}\), as shown in Fig. 4.8 (black lines), giving an enhancement of about one order of magnitude compared to the configuration of Fig. 4.7 (a). For the configuration of Fig. 4.7 (c) where the nanodetector is integrated with the nanoantennas and placed on the GaAs substrate, the absorptance at the resonance wavelength (\(\approx 1530\) nm) is \(\approx 1.3 \times 10^{-3}\) for the light polarization along X-axis (solid blue line in Fig. 4.8), giving an enhancement of about five times; for the light polarization along Y-axis (dashed blue line in Fig. 4.8), a suppression of absorptance centered at \(\approx 1380\) nm is observed. When combining the DBR and the nanoantenna approaches, i.e., the nanodetector is integrated with the nanoantennas and placed on the DBR as shown Fig. 4.7 (d), an even
larger absorptance enhancement is observed. As shown in Fig. 4.8, for the polarization along X-axis, the absorption resonance shifts to ~1550 nm and increases to be ~6.8×10^{-3}, giving an enhancement of about twenty-five times as compared to the configuration of Fig. 4.7 (a).

![Fig. 4.8. The calculated optical absorptance for the configurations of Fig. 4.6 (a)-(d) with the light polarization direction along both the X-axis (solid lines) and Y-axis (dashed lines), plotted as a function of wavelength.](image)

As an example, the electric field intensity distribution calculated for the configuration of Fig. 4.7 (d) with the light polarization along X-axis at the wavelength of 1550 nm is plotted in Fig. 4.9 in log scale as a function of position. We can see that the electric field is strongly modified due to the presence of the nanoantennas and the field in the active area of the nanodetector is consequently enhanced. As the most intense electric field presents near the tips of the nanoantenna, a much larger enhancement will be achieved if the nanoantennas can be fabricated closer to the nanodetector. However, this requires high precision in aligning the nanoantenna to the detector’s active area, which will introduce complexity in the fabrication process. We also found that by modifying the geometry of the nanoantennas, the absorption resonance peak can be tuned in a large range from visible to infrared [27].
Fig. 4.9. The distribution of electric field intensity calculated for the configuration of Fig. 4.7 (d) with the light polarization along X-axis at the wavelength of 1550 nm, plotted in log scale as a function of position.

Considering the approaches introduced in 4.5.1, 4.5.2 and 4.5.3, it is possible to increase the value of $\eta_{\text{abs}}$ to a fraction of percent, which is lower than standard meander SSPDs but would still allow increasing the FOM for the use in interferometric autocorrelators by orders of magnitude, opening the way to the characterization of $N$th-order correlation functions of quantum light sources with unprecedented sensitivity.
References:


CHAPTER 5

Superconducting \( N \)-element series nanowire detectors

In this chapter, we demonstrate a novel PNR detector based on a series array of \( N \) superconducting NbN nanowire elements, named as \( N \)-element series nanowire detectors (\( N \)-SND). Experimental characterization and theoretical analysis on the SNDs with \( N = 4, 8, 12, \) and 24 elements are presented in Section 5.2, Section 5.3, Section 5.4 and Section 5.5, respectively. The 4-SND, 8-SND and 12-SND are capable of resolving up to four, eight and twelve photons, respectively. The device quantum efficiency and the temporal properties of the devices were measured. Photon statistics were performed on the experimental data, showing a good agreement with the theory. A detailed analysis of the SND’s linearity and detection noises will be also presented, providing valuable information on the SND operation and on the potential for further scaling the dynamic range. Part of this chapter has been published by Z. Zhou, et al. in “Superconducting series nanowire detector counting up to twelve photons,” Opt. Express, vol. 22, p. 3475, 2014.

5.1. Introduction

As introduced in Chapter 1, PNR detectors have attracted a large interest in the last decade, potentially playing a key role in many fields such as linear-optics quantum computing [1] and quantum communication [2]. Ultimately, a PNR detector with a large dynamic range would represent an ideal photon detector combining single-photon sensitivity with a linear response. However, making a PNR detector that meets the requirements of these applications is challenging. It should have high efficiency, high speed, low jitter, low noise, high sensitivity at telecommunication wavelengths and the ability of resolving photon numbers with a large dynamic range. None of the existing PNR detectors meet all of these standards. For instances, TES [3], charge integrated photon detectors [4] and PNR detectors based on time-multiplexing [5] are limited by low maximum counting rate, arrays of InGaAs single-photon avalanche detectors (SPADs) [6, 7] and arrays of silicon photomultipliers [8] have high dark count rates (DCRs), visible light photon counters [9] are not sensitive at telecommunication wavelengths, and InGaAs SPADs with self-differencing technique have a limited PNR dynamic range [10]. PNR detectors based on SSPDs are promising to outpace the other existing PNR techniques as they benefit from the advantages of SSPDs, including short response time, low timing jitter, low DCR and high quantum efficiency at telecommunication wavelengths [11].
SSPDs operated with the conventional readout scheme, i.e. by using an amplifier chain with 50-Ω input impedance, do not provide PNR functionality. Since the resistance of the NbN nanowire after photon absorption (~10^3 Ω) is much larger than the load resistance (50 Ω), the absorption of more than one photon in the wire results in a readout pulse with nearly the same height as the one produced by the absorption of a single photon. Therefore, the SSPDs give a binary response to the number of incident photons (either ‘0 photon’ or ‘≥ 1 photon’). As introduced in Section 1.5 and Section 3.1, lowering the bias current of an SSPD will bring it to the multi-photon regime [11, 13-15]. In this mode, the SSPD is operated as a threshold detector (responding to ≥ n photons) but not as a PNR detector. Integrating an SSPD with a high-impedance cryogenic pre-amplifier may in principle enable the SSPD to have the PNR functionality [16]. However, the PNR resolution with this method was limited due to the latching problems [17].

As introduced in Subsection 1.6.3, spatial multiplexed SSPD arrays provide the PNR functionality [18, 19]. In this case, a number of SSPD elements are arranged in an array, equally illuminated under the input light beam, and each element acts as an SPD. Their responses are read either separately [18] or together [19] in the output. The former technique, i.e. reading the responses from a number of SSPD elements separately, requires the same number of readout electronic sets as the number of elements in the array. Therefore it is not scalable to large photon numbers due to the correspondingly increasing experimental complexity. In contrast, the latter approach, i.e. reading the responses from different elements summed up together as a single readout signal, can avoid this problem. In this case, the detected photon number is measured by resolving the height of the single readout signal. As reported in [19], an array of six superconducting nanowire elements connected in parallel, named as parallel nanowire detector (PND), was able to resolve up to five photons with a record PNR performance. However, the PND has a drawback of current redistribution problems due to its parallel design. The bias current from the firing elements is partially redirected to the unfiring elements. This phenomenon introduces false photon detections and consequently limits the dynamic range [20].

In order to solve the current redistribution problem of the PND, Jahanmirinejad, et al. [21] proposed a novel design of series connection of \( N \) superconducting nanowires, i.e. \( N \)-SND. In each element of the \( N \)-SND, a superconducting nanowire is connected in parallel with an integrated resistor and biased with an electric current slightly below its critical current. When one of the elements absorbs a photon, the superconducting nanowire becomes resistive and shunts the bias current to its corresponding parallel resistor, producing a voltage pulse in the readout. The photon-induced voltage pulses from the parallel resistors are summed up in a single readout pulse with a height proportional to the number of the
firing elements and thus to the detected photon number. According to [21], the design of series connections circumvents the current redistribution problems inherent to the parallel design. A large array of series nanowires may resolve up to a few tens of photons, providing a large dynamic range for PNR measurement. In this chapter, after reviewing the working principle and first experimental results [22] on a four-element SND (4-SND), SNDs with record dynamic range of 8 and 12 photons are described.

5.2. \( N \)-SND working principle and 4-SND

In this section, the general working principle of \( N \)-SNDs and the previous experimental results of a 4-SND will be presented.

5.2.1. \( N \)-SND working principle

A schematic diagram of an \( N \)-SND is shown in Fig. 5.1 (a). In the device, \( N \) elements are connected in series and each of them (marked by a dashed blue box) consists of a section of superconducting nanowire, shunted with a parallel resistor \( R_p \). The working principle of each element is similar to a standard SSPD [12]. Each element is biased with a current \( I_B \), slightly lower than the critical current \( I_C \), using a current source. In the absence of photons, the nanowire is in the superconducting state and \( I_B \) flows through the nanowire. When a photon is absorbed, its energy suppresses the superconductivity in the nanowire and triggers the transition to the normal state. Since the resistance of the nanowire after photon absorption is much larger than the value of \( R_p \), \( I_B \) is diverted to the \( R_p \) and builds a voltage pulse across it. The photo-induced voltages of different elements are summed up in the readout resistor \( R_L \), producing a single output voltage pulse with a height proportional to the number of firing elements, and therefore to the number of detected photons. For
instance, the output voltage pulse generated by a 4-SND [22] was calculated based on the electro-thermal model reported in Ref [21] and plotted as shown in Fig. 5.1 (b). The calculated output pulses with different height correspond to the detection of \( n = 1 \)–4 photons. As demonstrated in Ref [21], an \( N \)-SND with a large array of series nanowires (i.e. a large \( N \)) can offer a large dynamic range from one up to a few tens of photons.

5.2.2. 4-SND

The first experimental demonstration of the \( N \)-SNDs, as a proof of principle, was reported by Jahanmininejad, et al. in [22] with four elements (4-SND), providing a dynamic range of up to four photons. The photon responses of a 4-SND, illuminated by a 1.3-\( \mu \)m pulsed laser with an input power of \(~2.1\) nW, was measured using the 40-GHz sampling oscilloscope as shown in Fig. 5.2 (a) [22]. The corresponding histogram, recorded within a narrow time window as marked by a blue square in Fig. 5.2 (a), is plotted as a function of the output voltage \( V_{\text{out}} \) in Fig. 5.2 (b). Five distinct output levels corresponding to the detections of \( n = 0 \)–4 photons were obtained as marked by red numbers.

![Fig. 5.2. (a) Photon responses of a 4-SND under the illumination of a 1.3-\( \mu \)m pulsed laser with an input power of \(~2.1\) nW, measured by using a 40-GHz sampling oscilloscope [22]. (b) The corresponding histogram recorded within a narrow time window as marked by the blue square in (a), plotted as a function of \( V_{\text{out}} \). Five output levels corresponding to the detections of \( n = 0 \)–4 photons are marked by red numbers in both (a) and (b).](image)

The \( V_{\text{out}} \) distribution measured in a power range of 0.5–5 nW is plotted as shown in Fig. 5.3. The profile at \(~2.1\) nW on the dashed red line of Fig. 5.3 corresponds to the histogram as shown in Fig. 5.2 (b). A detailed analysis of linearity, noise, and photon statistics on the \( V_{\text{out}} \) distribution will be presented for the 12-SND in Section 5.4. The 4-SND reported in Ref [22] has a reset time of \(~10\) ns range and an SQE of \(~2.6\)% for the wavelength of 1.3 \( \mu \)m with negligible DCR.
As discussed in Subsection 1.6.7, it is of importance for an $N$-element multiplexed PNR detector to have a large dynamic range, as the large dynamic range is beneficial for PNR measurements where the photon statistics of the input light source needs to be characterized with high fidelity. On the other hand, as discussed in Section 1.1, a PNR detector with a large dynamic range can bridge up the gap left between SPDs and conventional linear optical detectors. For this purpose, the SNDs with more elements were fabricated and characterized. As it will be shown in the following sections, the characterization of the 8-SND, 12-SND and 24-SND were performed, revealing useful information for the fabrication and operation of the SNDs. Fabrication of the 8-, 12-, and 24-SNDs was performed by dr. F. Mattioli, dr. A. Gaggero and dr. R. Leoni at the Institute for Photonics and Nanotechnologies (IFN), CNR, Rome, following the fabrication procedure described in Section 2.1.

5.3. 8-SND

In this section, the characterization of an 8-SND is presented. The PNR functionality of the device is demonstrated. The photo-response of the 8-SND measured while scanning the light spot on the device is analyzed, providing information on the inhomogeneities of the device.
5.3.1. SEM and IV characterization

Fig. 5.4. (a) SEM image of an 8-SND. The eight active nanowire sections are highlighted in colors. The eight \( R_p \)s, the signal (S) and ground (G) contact pads, and the direction of \( I_B \) are marked. The white scale bar in the upper-right corner of the image indicates a length of 10 \( \mu \)m.

(b) IV characterization of the 8-SND at \( T = 1.2 \) K with the amplifier connected to the device. The value of \( 8 \times R_p \) is determined by calculating the reciprocal of the slope using a linear fit (dashed blue line). The \( I_C \) of the device was \( \sim 16.6 \) \( \mu \)A.

An SEM image of the 8-SND is shown in Fig. 5.4 (a). The \( I_B \) flows through contact pads, as marked by ‘G’ (ground) and ‘S’ (signal), into the device. The eight active NbN nanowire sections (highlighted by different colors) were patterned in a 12 \( \mu \)m \( \times \) 12 \( \mu \)m array with a filling factor of 40% on GaAs substrate. Eight parallel resistors (\( R_p \))s are located on the sides of the nanowire array; each of them has a design value of \( \sim 50 \) \( \Omega \) and is connected to the array through the 250 nm-wide NbN nanowires. The nominal width of the NbN nanowires in the array is 100 nm. However, using the SEM with high magnification, the width was observed to vary from 75 nm to 105 nm, depending on the position in the array. Particularly, we found that the wires in the center of the array are slightly wider than those on the edge. This phenomenon is due to the proximity effect of the electron beam lithography (EBL) [23]. When defining the meander pattern of the NbN nanowires [step (e) of the nanofabrication process as shown in Fig. 2.1], the electrons scattered from the resist (HSQ) and the substrate (GaAs) may cause a non-zero dose outside of the pattern, which results in a wider pattern than the design. As the structures in the center of the whole pattern have a higher probability to be illuminated by the electrons scattered from the neighboring, the nanowires in the center of the array are more susceptible to the proximity effect and thus have a larger width than those on the edge of the array. The EBL proximity effect can be corrected by accordingly varying the dose in space. Due to the slight variation of the width, the bias current density is lower in the center of the array. As a result, the
device is expected to be less efficient in the center than in the edge, as it will be shown in Subsection 5.3.3.

In the electrical and optical characterization, the device was kept at a temperature of 1.2 K in the VeriCold cryostat (Subsection 2.3.2). According to the IV characterization (solid red line) of the 8-SND as shown in Fig. 5.4 (b), the $I_C$ of the device was $\approx 16.6 \, \mu A$ at 1.2 K. The typical relaxation-oscillation regime is not observed on the IV curve due to the presence of $R_p$s. When $I_B$ exceeds $I_C$, the entire nanowire becomes resistive, so the measured differential resistance ($dV_B/dI_B$) equals to the parallel equivalent of the nanowire’s normal resistance and the value of $8 \times R_p$. Since the resistance of the normal nanowire is much larger than the value of $8 \times R_p$, the value of $dV_B/dI_B$ can be approximated to be $8 \times R_p$ [22]. As shown in Fig. 5.4 (b), by calculating the reciprocal of the slope (dashed blue line) on the IV curve, the value of $8 \times R_p$ was determined to be $\approx 398 \, \Omega$, and thus the average value of $R_p$ was $\approx 49.8 \, \Omega$, which agrees well with the design value of 50 $\Omega$. The asymmetric shape of the IV curve in the normal region, which also showed hysteresis, was checked and found to be attributed to spurious reflections from the amplifier, which was connected to the device via the bias-T in the IV measurement and in the optical characterization presented below.

5.3.2. Characterization of multi-photon response

A 1.31-$\mu$m gain-switched diode laser with a pulse width ($\tau_p$) of $\approx 100$ ps was used of the optical characterization for the 8-SND and the 12-SND (Section 5.4). The current source of the laser was triggered externally by a function generator with a repetition rate of 14 MHz. The 8-SND was illuminated by the laser through a polarization-maintaining single-mode lensed fiber mounted on a XYZ-piezo stage in the closed-cycle cryocooler (Subsection 2.3.2). The light spot was aligned to the center of the 8-SND and the lensed fiber tip was lifted up from its optimal focusing position to achieve a uniform illumination on the active area of the device. The FWHM of the Gaussian spot was measured to be $\approx 12 \, \mu m$ using the knife-edge method [24], which was performed by measuring the power of the light reflected back through the fiber tip while scanning the spot across the edge of the contact pads (Subsection 2.3.2). Considering the size of the light spot, a ratio of $\approx 33$ % of the light power from the fiber tip flowed into the 12 $\mu m \times 12 \, \mu m$ active area in the measurements. The optical response signal of the device was collected through the RF arm of the bias-T at room temperature, then amplified by a low-noise amplifier, and finally sent to either a 40-GHz sampling oscilloscope or a 350-MHz counter for analysis.

The optical characterization of the 8-SND was first performed with the 40-GHz sampling oscilloscope. The device was biased with an $I_B$ of 15.5 $\mu A$. The sampling oscilloscope was synchronized with the laser’s trigger signal and measured the amplified output voltage.
signals ($V_{\text{out}}$) from the device. A low-pass filter with a cutoff wavelength of 80 MHz was added in the readout circuit to improve the signal-to-noise ratio (SNR) of the output signals by removing the high frequency noise. A MITEQ low-noise amplifier with 51 dB amplification, a 1.1 dB noise figure, and a passband of from 0.5 to 500 MHz, was used in this measurement.

The $V_{\text{out}}$ distribution of the output signals were measured in a power range of 3-580 nW and plotted in Fig. 5.5 (a). Nine distinct output levels corresponding to the detections of $n = 0-8$ photons were obtained, as marked by black numbers in Fig. 5.5 (a). As an example, the histogram obtained at the power of $\sim78$ nW is plotted in Fig. 5.5 (b) as gray dots, which corresponds to the profile in Fig. 5.5 (a) marked by a dashed red line.

To further confirm the PNR functionality of the 8-SND, we studied the CR dependence on light power, using the 350-MHz counter. As shown in Fig. 5.6, the CR was measured as a function of the input light power by setting different trigger levels of the counter (the DCR was subtracted from the data). The trigger level was chosen between the $n$th and the $(n+1)$th output levels in Fig. 5.5 (a) so that the counter selectively recorded the ‘$\geq n$-photon’ responses. According to [22], when the light power is low so that $\eta_{\text{dev}}\mu \ll 1$ ($\mu$ is the average photon number per light pulse incident on the active area of the device and $\eta_{\text{dev}}$ is the device quantum efficiency), the CR should be approximately proportional to the value of $(\eta_{\text{dev}}\mu)^n$ and thus proportional to the $n$th-order of the light power. In this case, the
recorded response of the 8-SND is equivalent to a threshold detector which detects $\geq n$ photons in the $n$-photon regime, similar to the superconducting nanodetector as shown in Section 3.1. This is confirmed by our results as shown in Fig. 5.6. In the power range of up to $\sim 10$ nW, which approximately corresponds to $\eta_{\text{dev}} < 1$, the solid black lines with the slope values of 1, 2, 3, 4, 5, 6 and 7, fit well with the experiments, corresponding to the one-, two-, three-, four-, five-, six- and seven-photon regimes, respectively.

Fig. 5.6. Measured CR of the 8-SND plotted as a function of light power by setting different trigger levels of the counter, corresponding to the detections of $\geq n$-photons ($n=1$-8). The slopes of the curves at low powers agree well with the $(\eta_{\text{dev}})^n$ dependence (indicated by solid black lines), providing a proof for the 8-SND’s PNR functionality.

5.3.3. Mapping of back-reflected power and CR

The light power back-reflected from the sample surface and the CR of the 8-SND were measured in the one-, two-, three, and four-photon regimes (Fig. 5.6), while scanning the light spot on the device using the same method introduced in Subsection 2.3.2. In order to obtain the best spatial resolution, the lensed fiber is positioned at its focal plane and thus the minimum spot with an $e^{-2}$ diameter of $\sim 5$ $\mu$m was obtained and applied during the scan. The measured back-reflected power and the CR are plotted as a function of the fiber tip’s position as shown in Fig. 5.7 (a) and (b) for the one-photon regime, in Fig. 5.7 (c) and (d) for the two-photon regime, in Fig. 5.7 (e) and (f) for the three-photon regime, and in Fig. 5.7 (g) and (h) for the four-photon regime, respectively. The active area of the eight elements is marked by dashed black lines in Fig. 5.7. The back-reflected power maps [Fig. 5.7 (a), (c), (e) and (g)] are reproducible. The eight $R_p$s and the contact pads, which show
higher reflectivity than the NbN and the GaAs substrate, can be found on the sides of the active area.

Fig.5.7. Measured back-reflected power (a, c, e, and g) and CR (b, d, f, and h), plotted as a function of the fiber tip’s position in (a) and (b) for the one-photon regime, in (c) and (d) for the two-photon regime, in (e) and (f) for the three-photon regime, and in (g) and (h) for the four-photon regime, respectively. The active area of the eight elements is marked by dashed black lines.
Two main features are found in the CR maps of Fig. 5.7. Firstly, the distribution of the CR in the active area is not uniform due to inhomogeneities of the device. Besides the small variations on the CR distribution, CR in the center of the active area is generally lower than that on the edge. This agrees with the variation of the nanowires’ width as observed in the SEM. As the wire is wider in the center area, one expects a lower efficiency and thus a lower CR from the center than that the edge. Secondly, the CR areas tend to shrink in higher-photon regimes, similar to the characteristic of multi-photon interferometry [25]. In principle, in the $n$-photon regime, the device gives a click when the spot covers $n$ elements; as the chance for the spot to cover $n$ elements becomes smaller when $n$ increases, the feature of the CR areas becomes finer in the higher-photon regime.

In order to investigate the observation, we established a model to calculate the response of the $N$-SND in the $n$-photon regime, which is dependent on the position of a light spot scanning on the device. The model is presented in the following.

We assume that for an $N$-SND without inhomogeneities, it provides uniform unit efficiency within its active area. As shown in Fig. 5.8, the spatial-dependent efficiency of an 8-SND, i.e. $\eta_{\text{SND}}(x, y)$, is plotted as a function of the space coordinate of $(x, y)$. The active area of the eight elements is marked by dashed black lines.

![Fig. 5.8. Spatial-dependent efficiency of an 8-SND without inhomogeneities, plotted as a function of the spatial coordinates $(x, y)$.](image)

When a light spot with Gaussian profile with $e^2$ diameter $w_L = 5 \, \mu m$ is introduced and locates at the point of $(X, Y)$, where $X$ and $Y$ are coordinates of the center of the spot relative to the origin $(x = 0, y = 0)$, the probability that the $m$th element ($m=1, 2, \cdots, 8$) clicks can be written as,
\[
\Pi_{mth}(X,Y) \propto \int_{y_{min}}^{y_{max}} \int_{x_{min}}^{x_{max}} \eta_{SND}(x,y) \exp\left[-\frac{(x-X)^2 + (y-Y)^2}{w^2/8}\right] \, dx \cdot dy, \quad (5.1)
\]

Where \( x_{max}, x_{min}, y_{max}, \) and \( y_{min} \) are the upper and lower spatial limits of the \( m \)th element on the \( x \) and \( y \) axis, respectively.

As discussed in Subsection 5.3.2, in the \( n \)-photon regime, the counter selectively measures the \( \geq n \)-photon responses of the \( N \)-SND. Since the probability of detecting \( > n \) photons is negligible as compared to that of detecting \( n \) photons when \( \eta_{dev,\mu} << 1 \), the measured response of the \( N \)-SND in the \( n \)-photon regime, i.e. \( R_{N-SND}^{n \text{ph}} \), is approximated to the \( n \)-photon responses of the \( N \)-SND.

In the one-photon regime \((n = 1)\), the measured response of the \( N \)-SND, i.e. \( R_{N-SND}^{1 \text{ph}} \), equals to the sum of the probabilities that any one of the \( N \) elements clicks, written as,

\[
R_{N-SND}^{1 \text{ph}}(X,Y) = \sum_{i=1}^{N} \Pi_{i\text{th}}(X,Y), \quad (5.2)
\]

In the \( n \)-photon regime \((1 < n \leq N)\), a detection event corresponds a combination of \( n \) clicking elements chosen from the set of \( N \) elements. In this case, the measured response of the \( N \)-SND equals to the probability of having all of such combinations. For instance, the measured response of the \( N \)-SND in the two-, three- and four-photon regime, i.e. \( R_{N-SND}^{2 \text{ph}} \), \( R_{N-SND}^{3 \text{ph}} \), and \( R_{N-SND}^{4 \text{ph}} \), respectively, can be written as,

\[
R_{N-SND}^{2 \text{ph}}(X,Y) = \sum_{i=1,j=1,i\neq j}^{N} \Pi_{i\text{th}}(X,Y) \cdot \Pi_{j\text{th}}(X,Y), \quad (5.3)
\]

\[
R_{N-SND}^{3 \text{ph}}(X,Y) = \sum_{i=1,j=1,k=1,i\neq j\neq k}^{N} \Pi_{i\text{th}}(X,Y) \cdot \Pi_{j\text{th}}(X,Y) \cdot \Pi_{k\text{th}}(X,Y), \quad (5.4)
\]

\[
R_{N-SND}^{4 \text{ph}}(X,Y) = \sum_{i=1,j=1,k=1,l=1,i\neq j\neq k\neq l}^{N} \Pi_{i\text{th}}(X,Y) \cdot \Pi_{j\text{th}}(X,Y) \cdot \Pi_{k\text{th}}(X,Y) \cdot \Pi_{l\text{th}}(X,Y), \quad (5.5)
\]

Based on Fig. 5.8 and Eq. 5.1-5.5, the value of \( R_{N-SND}^{n \text{ph}}(X,Y) \) is calculated for \( n = 1, 2, 3 \) and 4, normalized and plotted in Fig. 5.9 (a), (b), (c), and (d), respectively. The active area of the eight elements is marked by dashed black lines. In the one-photon regime [Fig. 5.9(a)], the calculated \( R_{N-SND}^{1 \text{ph}}(X,Y) \) represents a spatial convolution of the light spot and the 8-SND’s active area. In the two-, three-, and four-photon regimes [Fig. 5.9 (b)-(d), respectively], the calculated distribution of \( R_{N-SND}^{n \text{ph}}(X,Y) \) shrinks to the center of the array,
showing a feature similar to the characteristic of multi-photon interferometry [25]. This is because when the spot is aligned to the center of the array, it has a larger chance to trigger more than one element, consequently benefiting the multi-photon detections which require multiple elements to participate. It is clear that these calculated response distributions are different from the experimental ones as shown in Fig. 5.7 (b), (d), (f) and (h). We attribute this difference to the effect of inhomogeneities in the nanowires’ width.

Fig. 5.9. Normalized value of \( R_{\text{N-SND}}^{\text{inh}(n)}(X,Y) \) of an 8-SND without inhomogeneities, calculated based on Fig. 5.8 and Eq. 5.1-5.5, plotted as a function of the light spot’s position for the one- (a), two- (b), three- (c), and four-photon regime (d), respectively. The active area of the eight elements is marked by dashed black lines.

For an N-SND with inhomogeneities, its spatial-dependent efficiency is not uniform. In our case as shown in Fig. 5.7 (b), (d), (f) and (h), the efficiency of the array is lower in the center area, which agrees with the observed variation in the nanowires’ width due to the EBL proximity effect as discussed in Subsection 5.3.1. According to Ref [23], the proximity effect introduced in the fabrication of 8-SND can be described by a function \( f_{PE}(x,y) \), written as,
\[
    f_{PE}(x, y) = \int_{-7.5 \mu m}^{7.5 \mu m} \int_{-6 \mu m}^{6 \mu m} \exp\left[-\frac{(x-x')^2 + (y-y')^2}{w_{PE}^2 / 8}\right] \, dx' \, dy', \quad (5.6)
\]

Where \(x'\) and \(y'\) are the coordinates of the center point of the electron beam relative to the origin \((x = 0, y = 0)\), \(w_{PE} = 17.8 \mu m\) is the \(e^2\) diameter of the Gaussian distribution of the electron scattering region, and the integration range of \((-6 \mu m, 6 \mu m)\) for \(dx'\) and \((-7.5 \mu m, 7.5 \mu m)\) for \(dy'\), respectively, is determined according to Fig. 5.4 (a).

As the proximity effect increases the width of the wire, the efficiency of the wire will be decreased in the presence of proximity effect. As the functional dependence of the efficiency on the width depends on the type of photo-detection process [26] and is not known for the present structure, we approximate it with linear dependence. As shown in Fig. 5.7 (b), the CR in the center of the array drops to zero, we thus assume the efficiency in the center of the array is zero under the experimental condition. Therefore, considering the proximity effect, we define the spatial-dependent efficiency of the 8-SND as,

\[
    \eta_{SND}(x, y) = 1 - \frac{f_{PE}(x, y)}{f_{PE}(x = 0, y = 0)}, \quad (5.7)
\]

According to Eq. 5.7, when there is no proximity effect, i.e. \(f_{PE}(x, y) = 0\), a uniform unit efficiency \(\eta_{SND}(x, y) = 1\) is found within the active area, the same as shown in Fig. 5.8. When the proximity effect presents, i.e. \(f_{PE}(x, y) > 0\), the value of \(\eta_{SND}(x, y)\) becomes smaller than 1. At the center of the array \((x = 0\) and \(y = 0)\), we have \(\eta_{SND}(x = 0, y = 0) = 0\), so the efficiency drops to zero. The calculated \(\eta_{SND}(x, y)\) based on Eq. 5.6 and 5.7 is plotted in Fig. 5.10, where the active area of the eight elements is marked by dashed black lines.

![Fig. 5.10. Spatial-dependent efficiency of an 8-SND with inhomogeneities due to the EBL proximity effect, calculated based on Eq. 5.6 and 5.7, plotted as a function of the space coordinate of \((x, y)\).](image-url)
Based on Fig. 5.10 and Eq. 5.1-5.5, the value of \( R_{N,SND}^{nph}(X,Y) \) is calculated for \( n = 1, 2, 3 \) and 4, normalized and plotted in Fig. 5.11 (a), (b), (c), and (d), respectively. The active area of the eight elements is marked by dashed black lines. Comparing Fig. 5.11 (a)-(d) to Fig. 5.7 (b), (d), (f) and (h), the calculation shows a good qualitative agreement with the experiment, which supports our explanations on the CR distributions measured in the one-, two-, three- and four photon regimes. As will be seen in the following section, the analysis suggests that the inhomogeneities will play a role also in the 12-SND.

5.4. 12-SND

In this section, the characterization of a 12-SND is presented. The PNR functionality of the device is demonstrated, showing a large dynamic range of up to twelve photons. The linearity and detection noise of the output signals of the 12-SND are also analyzed.
5.4.1. SEM and IV characterization

An SEM image of the 12-SND is shown in Fig. 5.12 (a). The active area of the 12-SND is kept the same as that of the 8-SND. Twelve active NbN nanowire sections (highlighted by different colors) were patterned in a 12 μm × 12 μm array with a filling factor of 40% on GaAs substrate. Twelve $R_p$s are fabricated on the sides of the array and each $R_p$ is connected to the array through the 250 nm-wide NbN nanowires. Similarly to the 8-SNDs, the variation in the width of the wire due to the EBL proximity effect was also observed in the 12-SNDs, since the 8-SNDs and 12-SNDs were fabricated under the same condition. As will be discussed later in this section, the inhomogeneities of the wire introduce nonlinearity and noises on the electrical output signals of the 12-SND. The IV characterization (solid red line) of the 12-SND kept at 1.2 K is shown in Fig. 5.12 (b). The $I_C$ of this particular device was measured to be $\sim 13.4$ μA. By calculating the reciprocal of the fitting slope (dashed blue line in Fig. 5.12 (b)) on the IV curve, the value of $12R_p$ was determined to be $\sim 542$ Ω, and thus the average value of $R_p$ was $\sim 45.2$ Ω, in good agreement with the design value of 50 Ω.

![Fig. 5.12. (a) SEM image of a 12-SND. The twelve active nanowire sections are highlighted in colors. The twelve $R_p$s, the signal (S) and ground (G) contact pads, and the direction of $I_B$ are marked. The white scale bar in the upper-left corner of the image indicates a length of 10 μm. (b) IV characterization of the 12-SND at $T = 1.2$ K with the amplifier connected to the device. The value of $12R_p$ is determined by calculating the reciprocal of the slope using a linear fit (dashed blue line). The inset presents an enlarged view on the IV curve. The $I_C$ of the device was $\sim 13.4$ μA.](image)

5.4.2. Characterization of multi-photon response

The experimental setup and readout circuits for the optical characterization of the 12-SND [27] were the same as those used for the 8-SND, as introduced in Section 5.3. The 1.31-μm
diode laser, triggered externally by a function generator with a repetition rate of 1 MHz, was used in the optical characterization. The 12-SND was biased with an $I_B$ of 13.0 $\mu$A at the working temperature of 1.2 K. As shown in Fig. 5.13, The $V_{out}$ distribution of the 12-SND were obtained in a light power range of 0-64 nW using the 40-GHz sampling oscilloscope. Thirteen distinct output levels corresponding to the detections of $n = 0$-12 photons (marked by black numbers) were obtained, showing a large dynamic range of the 12-SND.

![Fig. 5.13. $V_{out}$ distribution of the 12-SND’s output signal measured in a power range of 0-64 nW. Thirteen distinct output levels corresponding to the detections of 0-12 photons (marked by black numbers) were obtained.](image)

Similar to the methods used in Fig. 5.6, the PNR functionality of the 12-SND was further confirmed by studying the CR dependence on light power, using the 350-MHz counter. As shown in Fig. 5.14, the CR was measured as a function of the input light power by setting different trigger levels of the counter (the DCR was subtracted). The trigger level was chosen between the $n$th and the $(n+1)$th output levels in Fig. 5.13 so that the counter selectively recorded the ‘$\geq n$-photon’ responses. In the power range of up to $\sim 4$ nW, which approximately corresponds to $\eta_{dev,\mu} < 1$, the solid black lines with the slope values of 1, 2, 3, 4, 5, 6 and 7, fit the experiments well, corresponding to the one-, two-, three-, four-, five-, six- and seven-photon regimes, respectively. Although the fitting cannot be done for $n \geq 8$ due the fact that the value of $\eta_{dev,\mu}$ becomes close and larger than 1, the statistical analysis as it will be shown in Subsection 5.4.3 confirms the PNR operation also for $n = 8$-12.
Fig. 5.14. Measured CR of the 12-SND plotted as a function of light power by setting different trigger levels of the counter, corresponding to the detections of \(\geq n\)-photons \((n = 1-12)\). The slopes of the curves at lower powers agree well with the \((\eta_{\text{dev}}\mu)^n\) dependence (indicated by solid black lines), providing a proof for the 12-SND’s PNR functionality.

After aligning the light’s polarization direction parallel to the nanowires [28], the device quantum efficiency \(\eta_{\text{dev}}\) was obtained by measuring the CR with the counter whose trigger level is set between the 0th and 1st output levels. As shown in Fig. 5.15, the value of \(\eta_{\text{dev}}\) is plotted as a function of \(I_B\), reaching a maximum value of \(~0.17\%\) at \(I_B = 13.2\ \mu A\). The low efficiency of the present device is due to the low optical absorptance (in the order of few percent [29] as shown in Fig. 2.16) of NbN nanowires on GaAs substrates, and also due to the inhomogeneities of the NbN nanowires which effectively reduced the device’s active area as discussed above [30, 31]. The efficiency can be improved by changing the substrate [28], integrating a micro-cavity [29, 32] or an optical waveguide [33], and by optimizing the quality of the NbN film and the fabrication process.

The temporal response of the 12-SND was also characterized. As shown in the inset of Fig. 5.15, the photo-response pulse for the 12-photon detection was recorded by the 40-GHz sampling oscilloscope. A chain of two Mini-circuit low noise amplifiers, both of which had 13 dB amplification, 2.6 dB noise figure, and 0.02-3 GHz passband, was used in this measurement for a better temporal resolution. The 80 MHz low-pass filter was removed from the readout circuit in this case. As shown in the inset of Fig. 5.15, we calculated the temporal photo-response (dashed blue line) for the 12-photon detection based on the
electro-thermal model reported in [21, 34] using the parameters of the present 12-SND. After applying a band-pass filter (0.02-3 GHz, corresponding to the passband of the amplifiers) to the dashed blue line, the result (solid red line) presents a good agreement with the measurement, giving a reset time of ~33.9 ns, which enables a maximum repetition rate of ~30 MHz. As already shown in Fig. 2.7(b), the system time jitter was measured at the leading edge of the photo-response pulse to be ~89 ps, including the jitter of the 12-SND, of the laser and of the amplifiers.

Fig. 5.15. Device quantum efficiency plotted as a function of $I_B$, reaching a maximum value of ~0.17% at $I_B = 13.2 \mu A$. Inset: photo-response pulse for the 12-photon detection recorded by a 40-GHz sampling oscilloscope. The dashed blue line is the calculated photo-response pulse for the 12-photon detection event, giving a reset time of ~33.9 ns. After applying a band-pass filter (0.02-3 GHz, corresponding to the passband of the amplifiers) to the simulation, the result (solid red line) shows a good agreement with the measurement.

We note that with the present SNDs, a trade-off exists between the temporal performance and the SNR in the PNR operation, depending on whether the low-pass filter is used in the circuit. However, this will not intrinsically limit the performance of the SNDs. We can increase the SNR by using cryogenic pre-amplifiers and by improving the uniformity of the device, releasing the need for a low-pass filter.

5.4.3. Photon number statistics

In the following, the capability of the 12-SND in measuring photon number statistics is assessed, by comparing the photon number distribution measured in Fig. 5.13 with the
expected distribution of a Poissonian source. The histograms of Fig. 5.13 at different input light powers were fitted by a sum of Gaussian peaks, using the amplitude, position and width of the peaks as fitting parameters for each power. For instance, Fig. 5.16 (a) and (b) show two profiles (gray dots) of Fig. 5.13 at two different powers of 5.3 nW and 20.6 nW together with their fittings (dashed red lines), corresponding to the detections of $n = 0$-$6$ and $n = 3$-$10$ photons, respectively.

Fig. 5.16. Two profiles (gray dots) of Fig. 5.13 at two different light powers of 5.3 nW (a) and 20.6 nW (b). They are fitted using a sum of Gaussian peaks, corresponding to the detections of $n=0$-$6$ (a) and $n=3$-$10$ (b) photons, respectively. The single Gaussian fits (Fit_n) are shown as solid blue lines and their sum (Fit_tot) is depicted as a dashed red line.

The normalized area of each fitting Gaussian peak in the $n$th output level corresponds to the probability $P$ of detecting $n$ photons. This detection probability distribution deviates from the simple Poissonian statistics of the light source due to the possibility that several photons are absorbed in the same element, which is intrinsic to all multiplexed PNR detectors with a finite number of elements, as discussed in Subsection 1.6.1. The value of $P_{\eta_{\text{dev}}}(n|\mu)$ obtained at different light powers in the experiment are extracted from Fig. 5.13 and are presented in Fig. 5.17 as black dots. The measured value of $P_{\eta_{\text{dev}}}(n|\mu)$ was then fitted by calculations (red stars in Fig. 5.17) based on Eq. 1.1, using a single fitting parameter $\eta_{\text{dev}}$ for each power. A good agreement between the measurements and the calculations is achieved over the entire power range. The fitted values of $\eta$ are lower than the value reported in Fig. 5.15 for the $I_B$ of 13.0 $\mu$A, since in this case the polarization was not aligned along the wires.
Fig. 5.17. The value of $P_{\text{dev}}(\mu | \nu)$ (black dots) at different light powers extracted from Fig. 5.13 and the fitting (red stars) based on Eq. 1.1 using a single fitting parameter $\eta_{\text{dev}}$. Inset: the value of $\eta_{\text{dev}}$ obtained from the fitting is plotted as a function of the light power.

We notice that when the light power increases, $\eta_{\text{dev}}$ first decreases and then maintains the minimum value at higher powers, as shown in the inset of Fig. 5.17. We attribute this observation to the non-uniformity of the elements’ efficiency of the 12-SND. With low powers (so low $\mu$), only the most efficient elements click so we see the highest value of $\eta_{\text{dev}}$ in the low power range; when $\mu$ increases, the less efficient elements also participate in the detection so that the average efficiency drops. There are three possible reasons for the non-uniformity of the elements’ efficiency. The first is due to the fact that the illumination was not perfectly uniform on the twelve elements. The illumination uniformity can be improved by either further increasing the Gaussian spot size or changing the spatial arrangement of the elements [35]. The second is due to the imperfections in the nanowires from the fabrication. Particularly the observed variation in wire width results in a variation of the efficiency among the different elements as discussed above. This can be avoided by further optimizing the fabrication process. The third reason is due to the decreasing of the bias.
current in the unfiring elements \((I_{uf})\) when other elements fired. As shown in Fig. 5.18 (a), we calculated the value of \(I_{uf}\) as a function of time based on the electro-thermal model [21, 34]. The value of \(I_{uf}\) obviously decreases in the case of detecting \(n=1-11\) photons due to the partial redistribution of the \(I_B\) to the 50 \(\Omega\) load [21]. The inset provides an enlarged view of the \(I_{uf}\), where the temporal profile of the incident pulse (\(\tau_p=100\) ps) is also indicated. Since the value of \(I_{uf}\) decreases in the time window where the detection events take place, the efficiency of the unfiring elements will decrease. Although this is not desirable for PNR detection, it can be avoided by using a much shorter light pulse or using a larger load resistance (e.g. a cryogenic pre-amplifier with high input impedance [21]). As an example, the calculation of \(I_{uf}\) for \(R_L=1\ M\Omega\) was performed and is presented in Fig. 5.18 (b), keeping the other parameters unchanged. In this case, the value of \(I_{uf}\) remains constant so the efficiency of the unfiring elements will not decrease as a function of time.

Fig. 5.18. The calculated value of \(I_{uf}\) (solid blue lines) as a function of time in the cases of detecting \(n=1-11\) photons, for \(R_L=50\ \Omega\) (a) and \(R_L=1\ M\Omega\) (b), respectively. The inset provides an enlarged view of \(I_{uf}\) where the temporal profile of the incident pulse (\(\tau_p=100\) ps, dashed red line) is also indicated.

5.4.4. Linearity of the 12-SND

As introduced in Section 1.1, for an optical detector with perfect linearity, its response signal \(V_{out}\) should be proportional to the input average photon number \(\mu\) and thus to the input light power. According to this criterion, a nonlinearity is found in the photo-responses of the 12-SND as shown in Fig. 5.13, where the output signal amplitude of 12-SND \(V_{out}\) does not scale linearly to the input light power. As it will be presented in the following two paragraphs, we attribute the nonlinearity of the 12-SND to two main reasons: limited number of pixels and inhomogeneities of the device.
5.4.4.1. Nonlinearity due to limited number of pixels

The first reason of the nonlinearity is the limited number of pixels. As discussed in Subsection 1.6.1, for a multiplexed PNR detector, when the input average photon number $\mu$ is much smaller than the number of elements $N$, the probability that more than one photon is absorbed in one element is negligible. In this case, the distribution of $V_{out}$ reproduces the number distribution of the input photons, which is determined by $\mu$ for a Poissonian source, and the dependence of $V_{out}(\mu)$ is linear. When the value of $\mu$ becomes comparable to the number of elements $N$, the probability that more than one photon are absorbed in one element is not negligible, which reduces the detected photon number and effectively reduces the detected value of $\mu$. As a result, nonlinearity is introduced in the dependence of $V_{out}(\mu)$.

![Graph](image)

**Fig. 5.19.** (a) Calculated $\mu$-dependent histograms of a 12-SND in the range of $\mu = 0-35$ for a SNR of 2. Thirteen distinct output levels corresponding to the detections of 0-12 photons are marked by black numbers. (b) The calculated histogram (solid red line) at $\mu = 15$, plotted as a function of $V_{out}$, corresponding to a profile of (a) as marked by the dashed red line. The $n$th Gaussian peaks are depicted as solid blue lines.
As shown in Fig. 5.19 (a), we calculated the \( \mu \)-dependent histogram of an ideal 12-SND (with homogeneous response of the elements) for \( \mu = 0\text{--}35 \). The calculated histogram for \( \mu = 15 \) is depicted in Fig. 5.19 (b) as a solid red line, corresponding to a profile of 5.19 (a) as marked by a dashed red line. For a certain \( \mu \), the probability of seeing the \( n \)th \( V_{\text{out}} \) level equals to the area of the \( n \)th Gaussian peak (solid blue lines) as shown in Fig. 5.19 (b), and is given by Eq. 1.1, which considers a Poissonian distribution of the input photon number and takes into account the probability that more than one photon enters one element. In the calculation, we assumed that the voltage difference between the amplitude of two neighboring output levels is two times larger than the FWHM of an individual level, so that the photon levels are clearly resolved. This is equivalent to the case of choosing the signal-to-noise ratio (SNR) of 2 as it will be shown later in Subsection 5.4.6. The calculation result is shown in Fig. 5.19 (a), where the nonlinearity is observed in the calculated \( V_{\text{out}} (\mu) \) dependence. The nonlinearity turns to be stronger as the value of \( \mu \) becomes comparable to the limit of the dynamic range, i.e. \( N = 12 \). The calculation confirms that the nonlinearity due to the limited number of pixels is intrinsic for the any multiplexed PNR detectors, and the linearity can be improved by increasing the dynamic range. On the other hand, comparing the calculation [Fig. 5.19(a)] with the experimental data (5.13), we found that the limited number of elements is not the only source of nonlinearity. Indeed, as shown in Fig. 5.19, the voltage difference between two neighboring output levels keeps constant; while this voltage difference in Fig. 5.13 decreases as \( n \) increases, suggesting that there are other types of nonlinearities in the experiment.

### 5.4.4.2. Nonlinearity due to inhomogeneities of the device

In the previous paragraph, we discussed the nonlinearity due to having limited number of elements, which effectively decreases the detected value of \( \mu \), but does not affect the height of the output levels. In this paragraph, we show another type of nonlinearity of the 12-SND, which causes decrease in the amplitude of the \( n \)-photon output levels, due to the inhomogeneities of the device.

We first investigate the dependence of the amplitude of the output levels on the input light power. The amplitude of the output voltage levels, defined as the height \( (H) \) of the fitting Gaussian peaks extracted from Fig. 5.13, is plotted as a function of light power in the range of 1.95-30.11 nW for \( n = 0\text{--}11 \) in Fig. 5.20 (a) \( (H \) of the \( n = 12 \) peak is not shown here since the \( n = 12 \) peak is only observed when the power is larger than 30.11 nW, where the fitting becomes unreliable due to the noise at high powers). As shown in Fig. 5.20 (a), for each \( n \), the value of \( H \) is almost independent of the power when the power is low; as the power increases, the value of \( H \) starts to slightly decrease. To visualize the small amount of the change of \( H \), we define \( \Delta H \) as the value of \( H \) at different powers subtracted by the value of
$H$ at the lowest power for the same $n$ as shown in Fig. 5.20 (a). The value of $\Delta H$ is plotted in Fig. 5.20 (b) as a function of the power for different $n$, showing a decrease of $H$ by up to a few percent as the power increases. Since the value of $\Delta H$ is small, the increase of light power does not have a significant influence on the detection linearity.

It is interesting to understand why $H$ decreases as the power increases. Indeed we have also observed that the height of the response pulses from a standard SSPD decreases with the light power. A similar observation was reported in [17]. We attribute this phenomenon to the variation of the film’s local characteristics or geometry (i.e. width and thickness) along the nanowires. In general, the narrower sections of the wire have higher current density and are more easily triggered by single photons. Thus, at low light powers, the output pulse height will be determined by the normal resistance of these narrower sections. When the power increases, the wider sections of the wire will be triggered by multi-photons, contributing to the CR. Since the wider sections have a lower normal resistance, the height of the photo-response pulses will decrease at higher powers, and the distribution of the height will be broadened [as shown below in Fig. 5.22 (a)].

![Fig. 5.20. (a) $H$ values extracted from Fig. 5.13 plotted as a function of light power for different $n$. (b) $\Delta H$, defined as the value of $H$ at different powers subtracted by the value of $H$ at the lowest power for the same $n$, plotted as a function of power for a few $n$ in the 0-10 range.](image)

We then investigate the dependence of $H$ on $n$. Since the change of $H$ is relatively small as the power changes, we calculated the value of $\overline{H}$ (i.e. $H$ averaged for different powers) according to Fig. 5.20 (a), and plotted it as a function of $n$ in Fig. 5.21 (a). A power-law fit to $\overline{H}$, defined as $\overline{H} = A \cdot n^\alpha$ ($A$ and $\alpha$ are fitting parameters), is also plotted, giving $\alpha = 0.81$. The $\Delta \overline{H}$, defined as $\Delta \overline{H} = \overline{H}(n) - \overline{H}(n-1)$, is plotted as a function of $n$ in the inset of Fig. 5.21 (a), showing that the value of $\overline{H}$ increases nonlinearly with $n$. We calculated the
$V_{\text{out}}$ as a function of time for $n = 1$-12 using the electro-thermal model [21, 34] and plotted the results in the inset of Fig. 5.21 (b). The calculated $H$, i.e. the maximum $V_{\text{out}}$ of these pulses, is extracted and plotted as a function of $n$ in Fig. 5.21 (b). By fitting the calculated value of $H$, we obtained $\alpha = 0.98$, showing a better linearity than that in the experiment.

![Graphs showing $\overline{H}$ and $H$ vs. $n$](image)

Fig. 5.21. (a) The value of $H$ averaged for different powers (namely $\overline{H}$) plotted as a function of $n$ (black circles). A power-law fit (red line) to $\overline{H}$, defined as $\overline{H} = A \cdot n^{\alpha}$, is also plotted, giving $\alpha = 0.81$. Inset of (a): The value of $\Delta \overline{H}$, defined as $\Delta \overline{H} = \overline{H}(n) - \overline{H}(n-1)$, plotted as a function of $n$. Inset of (b): the calculated $V_{\text{out}}$ plotted as a function of time for $n=1$-12 using the electro-thermal model [21, 34]. Main panel of (b): the value of $H$ (black circles) extracted from the inset of (b), plotted as a function of $n$ together with a power-law fit (red line), giving $\alpha = 0.98$.

We attribute the difference of the $\alpha$ values between the experiment and the simulation to the inhomogeneities among different elements, e.g. variation in the wire’s cross-section between different elements. In principle, the elements with narrower wires have higher efficiency, so with small $n$ only they will contribute to the readout voltage pulse; the wider elements will participate in detections as $n$ increases. Since the voltage pulse produced by the wider elements is lower than that of the narrower elements, the height of total voltage pulse scales nonlinearly with the $n$. As shown in the discussions below, this variation in the voltage amplitudes of different elements has major consequences also in the noise.

5.4.5. Detection noise of the 12-SND

5.4.5.1. Dependence of noise on light power and photon number

We investigate the dependence of the detection noise on the light power and on $n$. The noise $V_N$ on the output levels, defined as the FWHM of the fitting Gaussian peak, is extracted from Fig. 5.13 for $n = 0$-11 in the power range of 1.95-30.11 nW.
On the one hand, as shown in Fig. 5.22 (a), the value of $V_N$ for the $n$th detection increases with the light power. This phenomenon is related to the observation that the value of $H$ decreases as the light power increases as shown in Fig. 5.20. As discussed above, it can be attributed to the triggering of wire’s wider sections by multi-photon events at high powers, which in principle results in a smaller pulse height and at the same time a broadened distribution of height. Since this type of noise critically depends on the uniformity of the film along the wire, it can be suppressed by optimizing the processes of film deposition and of device fabrication.

![Fig. 5.22. (a) The value of $V_N$ plotted as a function of the light power for a few $n$ in the 0-10 range. (b) The value of $V_{EN}$ plotted as a function of $n$ for different light powers.](image)

On the other hand, the excess noise $V_{EN}$, defined as $V_{EN} = \sqrt{V^2_N(n) - V^2_N(n = 0)}$, is plotted as a function of $n$ at fixed light powers as shown in Fig. 5.22 (b). For all powers, the value of $V_{EN}$ first increases with $n$, reaching a maximum for $n \approx 4-6$, depending on the power, then decreases. This dependence indicates a surprising suppression of the excess noise, which is beneficial for the measurement of large photon numbers.

### 5.4.5.2. Possible origins of the excess noise

We attribute the excess noise to three possible origins. The first is due to the thermal noise of the amplifier. The thermal noise produced by the equivalent noise current generator of the amplifier depends on the detector’s impedance, which dynamically varies during the detection events and depends on the number of absorbed photons [22].

The second source of noise is due to the accumulation of the intrinsic noises from each of the $n$ detection events. As discussed above, the intrinsic noise from each element is related
to the inhomogeneities of the nanowires and increases with the light power. With a fixed light power, the noises in distinct wires are uncorrelated; the variance of their sum scales as $n$ and their total contribution to $V_{EN}$ is expected to scale as $n^{0.5}$. Both the first and the second noise sources are expected to result in a monotonic dependence of $V_{EN}$ on $n$, which is not observed in the experiment.

The third source of noise results from the variation of the height of the output pulses produced by different elements, which is due to the variation of the nanowire’s normal resistance in different elements [36]. Based on our explanation to Fig. 5.21 (a), we assume that at low power, only the most efficient element, which has the narrowest wire and produces the highest photo-response pulse, participates in the detection; while at high powers, the least efficient element starts to participate. In this case, according to the inset of Fig. 5.21 (a), the most efficient element produces a pulse height of $\sim 12.8$ mV; likewise, the element with the lowest efficiency produces a pulse height of $\sim 7.5$ mV, and the other elements produce pulse heights distributed around a mean value of $\sim 10$ mV. In the following, we model the noise performance assuming this third noise sources, i.e. the distribution of the pulse heights, is the only contribution to the noise.

Using the above assumptions, we calculate the expected excess noise for the $n$-photon detection. We assume an arbitrary distribution $N_{\text{element}}(H_1)$ of the number of elements producing a peak height $H_1$ for the one-photon detection ($n=1$) [gray bars in Fig. 5.23 (a), right axis], chosen so that the highest and the smallest pulse heights have values of 12.8 mV and 7.5 mV, and the width is $\sim 2$ mV [using a Gaussian fit (red line)], which corresponds to the experimental excess noise for $n=1$ [Fig. 5.22 (b)]. Based on the distribution of $N_{\text{element}}(H_1)$, we are able to calculate the distribution of $P_n(H_n)$ considering a uniform efficiency, where $P_n$ is the probability of detecting a pulse with the height of $H_n$ in the $n$-photon detection event ($n = 0 - 12$). When $n = 0$, $H_0 = 0$ and the width of the distribution equals zero. When $n = 1$, the distribution of $P_1(H_1)$ [Fig. 5.23 (a), left axis] reproduces the distribution of $N_{\text{element}}(H_1)$. When $n > 1$, each $n$-photon detection is a combination of $n$ one-photon detections corresponding to any $n$ of the twelve elements. The value of $P_n(H_n)$ equals to the sum of the probabilities for all the possible combinations which produce $H_n$.

Four examples of $P_n(H_n)$ for $n = 1, 4, 6$ and 11 are plotted as histograms (gray bars) in Figs. 5.23 (a)-(d), respectively. Each calculated $P_n(H_n)$ distribution is fitted by a Gaussian peak [red lines in Figs. 5.23 (a)-(d)]. The FWHM of the fitting Gaussian peak, which represents the calculated $V_{EN}$, is plotted as a function of $n$ in Fig. 5.23 (e). Interestingly, the shape of the $P_n(H_n)$ distribution for the $n$-photon event is identical to that of the $(12-n)$-photon
event. For instance, the one-photon event as shown in Fig. 5.23 (a) reproduces the distribution of the eleven-photon event as shown in Fig. 5.23 (d), despite of the shift of the $H_n$ values. It is also interesting to see that the excess noise drops to zero when $n = 12$ as shown in Fig. 5.23 (e). Indeed, if all the twelve elements are triggered, only a single value of $H_{12}$ is possible, so no excess noise appears for $n=12$. The calculated $n$-dependence of $V_{EN}$ reproduces some key features of the experimental data in Fig. 5.22 (b), particularly the decrease of the excess noise at large $n$. The calculation [Fig. 5.23 (e)] gives the highest noise at $n = 6$, while in the experiment [Fig. 5.22 (b)] the highest noises take place in the range of $n = 4-6$. The agreement clearly indicates that the statistical distribution of the nanowire’s normal resistance plays an important role in the observed excess noise.

5.4.6. PNR capability of the 12-SND

Based on the discussions of the linearity and the detection noise of the 12-SND, we take the SNR, i.e. $\Delta H/V_n$, as a measure of the 12-SND’s PNR capability. Indeed, the fidelity of discriminating an $n$-photon detection from $(n-1)$ or $(n+1)$ directly depends on the ratio between the peak spacing and the peak width, i.e. the noise. For instance, for equally spaced Gaussian peaks with constant width, the probability of correctly measuring an $n$-photon peak with an optimal discrimination threshold [37] is 76.09 %, 98.15 % and 99.96 %.
for $\Delta H/V_N = 1, 2$ and $3$, respectively. In order to distinguish an output level from its neighboring levels, the value of $\Delta H/V_N$ should be in principle larger than 1.

The measured ratio of $\Delta H/V_N$, which was calculated based on the data of the inset of Fig. 5.21 (a) and of Fig. 5.22 (a), is plotted in Fig. 5.24 as a function of $n$ for different powers. As shown in Fig. 5.24, the increase of light power results in a decrease of the PNR capability, this is mainly due to the fact that the noise, i.e. $V_N$, is increased as the power increases. Besides, the PNR capability decreases as $n$ increases when $n$ is small and then saturates when $n$ becomes larger. This is due to the saturation of $V_{EN}$ as discussed above in paragraph 5.4.5.2, which shows a potential of scaling to a larger dynamic range.

5.5. 24-SND

In this section, characterization of a 24-SND are presented. The optical measurement showed limited SNR in the 24-SND’s output signal with the present readout scheme. Possible improvements on the SNR are discussed.

5.5.1. SEM and IV characterization

An SEM image of a 24-SND is shown in Fig. 5.25 (a). The active area of the 24-SND is kept the same as the 8-SND and the 12-SND. Twenty-four active NbN nanowire sections (highlighted by different colors) were patterned in a $12 \mu m \times 12 \mu m$ array with a filling factor of 40%. Twenty-four $R_P$s are located on the sides of the array and are connected to the corresponding nanowire sections through the 250 nm-wide NbN nanowires. In order to
Improve the detection efficiency, the 24-SND was fabricated on a micro-cavity, which consists of a $\lambda/4$-thickness (225 nm) SiO$_2$ layer on top of Si substrate. The cavity is designed to enhance the absorptance near the resonance wavelength of ~1310 nm.

**Fig. 5.25.** (a) SEM image of the 24-SND. The twenty-four active nanowire sections are highlighted in colors. The twenty-four $R_p$, the signal (S) and ground (G) contact pads, and the direction of $I_B$ are marked. The blue scale bar indicates a length of 10 $\mu$m. (b) IV characterization of the 24-SND at $T = 4.2$ K with amplifiers connected to the device. The value of $24 \times R_p$ is determined by calculating the reciprocal of the slope using a linear fit (dashed blue line). The $I_C$ of the device was ~10.9 $\mu$A.

IV characterization of the 24-SND was performed at the temperature of ~4.2 K in a cryogenic probe station (Subsection 2.3.1). The measured IV curve (solid red line) is shown in Fig. 5.25 (b). The $I_C$ of the device was ~10.9 $\mu$A in the experimental condition. By calculating the reciprocal of the fitting slope (dashed blue line) on the IV curve, the value of $24 \times R_p$ was determined to be ~1798 $\Omega$, corresponding to an average $R_p$ value of ~74.9 $\Omega$.

5.5.2. Optical characterization

Optical characterization of the 24-SND was performed in the cryogenic probe station (Subsection 2.3.1) using a 9.7-$\mu$m pulsed laser, which had a duration of ~70 ps and was triggered externally by a function generator with a repetition rate of 10 MHz. The 24-SND was biased with an $I_B$ of 10.1 $\mu$A. The readout circuit was kept the same as that used for the optical characterization of the 8-SND and of the 12-SND as shown in Fig. 5.5 and Fig. 5.13, respectively.
The $V_{\text{out}}$ distribution of the 24-SND at a limited number of light powers in the range of 0-500 pW were obtained using a 40-GHz sampling oscilloscope, as plotted Fig. 5.26 (a). In the figure, the area in the power range of about 200-300 pW shows artificial features since no data was not recorded in this range. The histograms obtained at the power of 22 pW, 115 pW and 408 pW are plotted in Fig. 5.26 (b) as solid green, solid red and solid black lines, corresponding to the profiles in 5.26 (a) marked by the dashed lines with the same color, respectively. As shown in Fig. 5.26, individual output levels are not resolved; instead, the histograms present a broad peak with the center position shifting to higher $V_{\text{out}}$ values as the light power increases. We attribute this phenomenon to the limited SNR in the 24-SND’s output signal under the present experimental condition. Assume that the 24 output levels of the device are equally spaced in the $V_{\text{out}}$ range of 0-80 mV as shown in Fig. 5.26 (a), we have $\Delta \bar{H} = 80 \text{mV}/24 \approx 3.3 \text{mV}$ for the 24-SND. Further assuming the noise width of each output level is similar to that of the 12-SND, i.e. $V_N \approx 6 \text{mV}$ according to Fig. 5.22 (a), we estimate the SNR in the measurement of the present 24-SND to be $\Delta \bar{H}/V_N \approx 0.55$. Since the estimated value of $\Delta \bar{H}/V_N$ is smaller than 1, the output levels cannot be resolved.
Fig. 5.27. Calculated $\mu$-dependent histograms of a 24-SND in the range of $\mu = 0-80$ for a SNR of 2 (a) and of 0.55 (b). The calculated histogram (solid red line) at $\mu = 30$, corresponding to a profile of (a) and of (b) as marked by the dashed red lines, are plotted as a function of $V_{\text{out}}$, in (c) and (d), respectively. The $n$th Gaussian peaks are depicted as solid blue lines in (c) and (d). Using the same methods as applied in Fig. 5.19, we calculated the $\mu$-dependent histogram of an ideal 24-SND with homogeneous elements in range of $\mu = 0-80$ for the cases of $\Delta H/V_N \approx 2$ and $\Delta H/V_N \approx 0.55$, and plotted the results in Fig. 5.27 (a) and (b), respectively. The calculated histograms for $\mu = 30$ are depicted in Fig. 5.27 (c) and (d) as solid red lines, corresponding to the profiles in 5.27 (a) and (b) marked by dashed red lines, respectively. The $n$th Gaussian peaks are plotted as solid blue lines in Fig. 5.27 (c) and (d). For the SNR of 2 as shown in Fig. 5.27 (a) and (c), the multiple output levels are clearly resolved. However, for the SNR of 0.55 as shown in Fig. 5.27 (b) and (d), since the spacing between two neighboring output levels is about two times smaller than the width of an individual level, the levels overlap with each other, therefore summed as a broad peak, which is similar to the observation in the experiment as shown in Fig. 5. 26. We notice that the value of $\Delta H$ for the 8-, 12- and 24-SNDs as shown in Fig. 5.5, Fig. 5.13 and Fig. 5.26, respectively, decreases as the number of elements $N$ increases. This is due to the fact that as the value of load resistance $R_L$ (50 $\Omega$) is similar to the value of the $R_p$s, only a fraction of the voltages produced across the $R_p$s can be read at the $R_L$. By introducing a cryogenic pre-amplifier with high input impedance, the value of $\Delta H$ can be increased [21].
5.5.3. Possible improvements on the SNR

In the following, we briefly propose three possible approaches which can in principle improve the SNR of the SNDs, by either increasing the amplitude of the output levels or reducing the noise on the levels.

Firstly, as discussed in Subsection 5.4.4 and 5.4.5, the inhomogeneities in the device introduce nonlinearity and excess noise to the output levels. By fabricating nanowires with a better uniformity, the value of $\Delta H$ can be increased whilst the value of $V_N$ can be decreased, thus improving the SNR.

Secondly, introducing a cryogenic pre-amplifier with high input impedance to the readout circuit can improve the SNR. As shown in Fig. 5.18, a cryogenic pre-amplifier with high input impedance can avoid the decrease of the unfiring elements’ efficiency and thus improve the homogeneity in the response of the device. On the other hand, as discussed in Ref [21], the pre-amplifier can increase the amplitude of SND’s response signal and ensures a large dynamic range with a good linearity, which can boost the value of $\Delta H$. Besides, as thermal noise will be reduced at a lower temperature, the cryogenic pre-amplifier introduces less thermal noise than the room-temperature and therefore reduces the value of $V_N$. During the period of this PhD work, there was no cryogenic pre-amplifier ready for the SND experiments. A preliminary test on a prototype of cryogenic pre-amplifier combined with SSPD devices can be found in Ref [39].

Thirdly, as reported in Ref [38], SSPDs based on a parallel connection of NbN nanowires, so called the cascade switching SSPD (CS-SSPD) can largely increase the amplitude of the output signals without introducing additional noise. In our case, for example, if we divide each element of the SND into several parallel connected wires so that each element acts as a CS-SSPD, the amplitude of the photo-response from each element will be increased. As a result, the value of $\Delta H$ and thus the SNR can be increased with the help of the cascade design.
References:


[36] The variation of $R_p$ may also change the pulse height, but it is not considered in the discussion for simplicity.
CHAPTER 6

Conclusions and outlook

This chapter summarizes the most relevant results achieved in this PhD work and gives an outlook of the future work.

6.1. Conclusions

In this thesis, we demonstrated two types of multi-photon detectors, including the superconducting nanodetector and the N-element series nanowire detector (N-SND). As a nonlinear threshold detector, the nanodetector can be used to measure interferometric autocorrelations; the N-SNDs are capable of resolving the number of photons in the input light pulse. Both of the detectors inherit the advantages of the SSPDs and therefore outperform the conventional multi-photon detectors. Based on the work presented in this thesis, the following conclusions are drawn:

- The nonlinear response of superconducting nanowires was investigated. By measuring the click probability of a superconducting nanodetector operated in the multi-photon regime while varying the time delay between two short probing pulses, the time scale of the nonlinear response was determined to be ~20 ps. A semi-classical model was established, showing that the time scale is determined by the QP relaxation dynamics in the superconducting nanowires, depending on the microscopic parameters. The characterization of the nonlinear response was performed while varying the bias current and the light power. The results were well explained by applying a detector tomography technique to the response of the device.

- A novel N-photon interferometric autocorrelator based on a superconducting nanodetector was demonstrated. Up to 6th-order interferometric autocorrelations were measured with high detection sensitivity and temporal resolution. A detailed comparison of the sensitivity to the all-optical conventional autocorrelators was discussed, showing an advantage of the nanodetector due to the combination of linear optical absorption and nonlinear detection. The temporal resolution of the nanodetector-based autocorrelator, related to the QP relaxation dynamics, was directly measured to be ~20 ps, better than that of the HBT autocorrelator. In addition, the nanodetector-based autocorrelator has an easier electrical read out than the HBT technique and provides interferometric phase information.
A PNR detector with large dynamic range, based on a series connection of \(N\) superconducting NbN nanowires, was investigated. The detector, named as \(N\)-SND, provides a single output pulse with a height proportional to the detected photon number. The characterization of the 4-SND, 8-SND, 12-SND and 24-SND was performed. Particularly, the 12-SND resolved up to twelve photons, providing a record dynamic range among the existing fast telecom-wavelength PNR detectors. Nonlinearity was found in the response of the SNDs, due to the limited number of elements and the inhomogeneities in the device. The correct PNR operation was verified by studying the count rate and the distribution of the output levels as a function of the photon number and of the light power. The noise analysis revealed a surprising suppression of the excess noise at high photon numbers, which was attributed to the statistical distribution in the normal resistance of the nanowires. This indicates a potential of SNDs of scaling to even higher photon numbers, especially if the uniformity of the device is improved.

6.2. Outlook

There are many approaches in which the work presented in this PhD thesis can be improved. In the future, the following developments can be envisaged in this research area:

- The efficiency of superconducting nanodetectors can be increased by enhancing the optical coupling between the light fields and the devices. For the nanodetectors, reducing the spot size, increasing the length of active area, or integrating with plasmonic nanostructures can help to increase the optical absorptance. These methods do not apply for the \(N\)-SNDs as they will reduce the uniformity of the illumination and thus introduce inhomogeneities in the \(N\)-SNDs’ multi-photon responses. Both the nanodetectors and the \(N\)-SNDs can be integrated with micro-cavities or waveguides to increase the absorptance. Besides, the efficiency can also be increased by using different superconducting materials, e.g. WSi, which are reported to offer 93% system efficiency.

- The temporal resolution of the present nanodetector-based autocorrelator is related to the timescale of QP relaxation dynamics in the NbN film and could be much improved by using different superconducting materials, such as high-\(T_c\) YBaCuO films, where relaxation times as short as \(\sim 1\) ps have been observed, opening the way to the characterization of \(N\)th-order correlation functions in the few-ps range with unprecedented sensitivity.
Based on the improvement on their quantum efficiency and temporal resolution, \(N\)th-order autocorrelators based on superconducting nanodetector can be applied to the characterization of quantum light sources (e.g. single-photon sources), providing a better performance than standard HBT autocorrelators.

The SNR of the present \(N\)-SNDs can be improved by increasing the number of elements. To achieve this goal, fabrication-induced inhomogeneities need to be minimized as they introduce nonlinearities and additional noise in the device’s photo-response and limit the dynamic range.

Another type of inhomogeneities, induced by the nonuniform illumination of light beam on the elements, should be minimized. A uniform illumination can be achieved by changing the spatial arrangement of all the elements or by modifying the light field distribution on the device.

Using a cryogenic pre-amplifier with high input impedance in the readout circuit can improve the SNR. The pre-amplifier can improve the linearity in the photo-response of the device, increase the amplitude of the SND’s output signal and reduce the thermal noise.

Based on the improvement of SNDs, i.e. efficiency, homogeneities and SNR, an SND with high efficiency, fast response and a large dynamic range of a few tens of photons should be achieved. Such PNR detector will provide a strong tool in the application in photonic quantum information processing, e.g. quantum computing, quantum communication, etc.

The intrinsic working mechanisms of the SSPDs are still under active discussion. A better understanding on how the SSPDs work should be achieved as it is of large importance in this field.
APPENDIX

The parameters used for the calculations in Chapter 3 are listed in the following table.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c )</td>
<td>Speed of light</td>
<td>( 3 \times 10^8 \text{ m/s} )</td>
</tr>
<tr>
<td>( k_B )</td>
<td>Boltzmann constant</td>
<td>( 1.381 \times 10^{-23} \text{ m}^2\text{kg}\text{s}^{-2}\text{K}^{-1} )</td>
</tr>
<tr>
<td>( e )</td>
<td>Elementary charge</td>
<td>( 1.602 \times 10^{-19} \text{ C} )</td>
</tr>
<tr>
<td>( h )</td>
<td>Planck's constant</td>
<td>( 6.626 \times 10^{-34} \text{ m}^2\text{kg}\text{s}^{-1} )</td>
</tr>
<tr>
<td>( \mu_0 )</td>
<td>Vacuum permeability</td>
<td>( 1.257 \times 10^{-6} \text{ Hm}^{-1} )</td>
</tr>
<tr>
<td>( R_{SN} )</td>
<td>NbN normal resistance per film squared</td>
<td>( 700 \text{ \Omega} ) [1]</td>
</tr>
<tr>
<td>( N_0 )</td>
<td>Electronic density of states at Fermi energy</td>
<td>( 2.2 \times 10^{24} \text{ m}^{-3}\text{K}^{-1} ) [2]</td>
</tr>
<tr>
<td>( \xi_{GL}(0) )</td>
<td>Ginzburg-Landau coherence length at ( T = 0 \text{K} )</td>
<td>( 4.2 \text{ nm} ) [2]</td>
</tr>
<tr>
<td>( \lambda(0) )</td>
<td>Magnetic penetration depth at ( T = 0 \text{K} )</td>
<td>( \lambda(0) = \sqrt{\frac{hR_{SN}d}{2\pi^2\mu_0\Delta(0)}} ) [3]</td>
</tr>
</tbody>
</table>

References:
SUMMARY

Multi-photon detection with superconducting nanowires

In this thesis, the multi-photon detection processes in superconducting nanowires are investigated. Two types of photon detectors based on superconducting nanowires, i.e. the superconducting nanodetector and the $N$-element series nanowire detector ($N$-SND), are investigated, capable of detecting multiple photons and resolving photon numbers, respectively. The nanodetector, combined with an interferometer, is employed as an $N$-photon interferometric autocorrelator and measures high-order autocorrelations with ultra-high sensitivity. Its nonlinear response is probed by short pulses, providing useful information on the operation. The $N$-SND, which consists of a series array of $N$ superconducting nanowire elements, shows a large dynamic range of up to twelve photons. Their performances are compared with conventional optical detectors with similar functionality, showing large advantages in terms of sensitivity, response time, and dynamic range.

In Chapter 1, a brief review of the existing photon detection technologies is given, including photomultiplier tubes (PMTs), photodiodes, transition edge sensors (TESs) and superconducting single-photon detectors (SSPDs). These detectors operated in different modes are classified into the categories of linear detectors, nonlinear detectors and photon-number-resolving (PNR) detectors, according to the dependence of their photo-responses on the input photon number. The superconducting nanodetectors and the $N$-SNDs, which are investigated in this thesis, can also be found in the classification.

In Chapter 2, the experimental and simulation methods used in the work of this thesis are introduced, including the steps used for the device fabrication, the experimental setups used for the electrical and optical characterization, and the finite-difference time-domain (FDTD) simulation software used to calculate the optical absorptance.

In Chapter 3, experimental and theoretical studies of the nonlinear response of the superconducting nanodetector are presented. The time scale of the nonlinear response in the nanodetector, which is related to the relaxation dynamics of the photon-induced quasiparticles, is measured using short pulses. The nonlinear response is found to be dependent on the bias current and on the light power. The dependence is explained by applying a detector tomography technique to the experimental results.
In Chapter 4, an ultrasensitive $N$-photon ($N = 1, 2, 3, 4, 5, 6$) interferometric autocorrelator based on the superconducting nanodetector is demonstrated. The autocorrelator provides much higher sensitivity than the conventional autocorrelators using all-optical nonlinearities. The temporal resolution of the autocorrelator is measured to be $\sim 20$ ps. A new figure of merit for nonlinear detectors is introduced and a comparison of nanodetectors and conventional detectors is provided. Possible improvements on the sensitivity are proposed, including reducing the beam spot size, increasing the active area of the nanodetector, and integrating bottom mirrors and plasmonic nanoantennas with the nanodetector.

In Chapter 5, the $N$-SND, i.e. a PNR detector based on a series array of $N$ superconducting NbN nanowire elements, is investigated. The SNDs with $N = 4, 8, 12, 24$ elements are experimentally characterized. Particularly, the 12-SND provides a dynamic range of up to twelve photons, which is a record for fast PNR detectors working at telecommunication wavelengths. Characterization of the 8-SND showed the evidence of inhomogeneities in the device’s active area. Such inhomogeneities are also found in the 12-SNDs, introducing nonlinearities and excess noise in output signal. A detailed analysis of the 12-SND’s linearity and noise shows that the dynamic range of the present $N$-SNDs can be further improved by improving the uniformity of the device and/or using a cryogenic pre-amplifier in the readout circuit. The device quantum efficiency and temporal properties of the 12-SND are measured. The measured photo-detection statistics is also compared with the theoretical one, showing a good agreement.

Finally, Chapter 6 summarizes the most relevant results achieved and gives an outlook of the future work.
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LIST OF PUBLICATIONS

Peer-reviewed Journals


- **Optics Express**, 22, 3475 (2014)


- **Optics Express**, 20, 2806 (2012)
  J. Renema, G. Frucci, Zili Zhou, F. Mattioli, A. Gaggero, R. Leoni, M. de Dood, A. Fiore, M. van Exter, “Modified detector tomography technique applied to a superconducting multiphoton nanodetector”

Major Conference Contributions

- **SPIE Photonics West**, San Francisco, USA (2014). *(oral presentation)*

• **Van der Waals symposium**, Eindhoven University of Technology, Eindhoven, The Netherlands (2013). *(Award for the best oral presentation)*  
  Zili Zhou, “Superconducting nanowire array counting photons”

• **CLEO (QELS)**, San Jose, USA (2012). *(oral presentation)*  
  Zili Zhou, G. Frucci, S. Jahanmirinejad, F. Mattioli, A. Gaggero, R. Leoni, A. Fiore,  
  “Picosecond N-photon autocorrelator based on superconducting nanodetectors”

• **Quantum Information and Measurement**, Berlin, Germany (2012). *(oral presentation)*  
  Zili Zhou, G. Frucci, S. Jahanmirinejad, F. Mattioli, A. Gaggero, R. Leoni, A. Fiore,  
  “N-photon autocorrelator with picosecond temporal resolution”

• **Annual Meeting NNV AMO**, Lunteren, the Netherlands (2012). *(oral presentation)*  
  Zili Zhou, G. Frucci, S. Jahanmirinejad, F. Mattioli, A. Gaggero, R. Leoni, A. Fiore,  
  “Picosecond N-photon autocorrelator based on superconducting nanodetectors”

• **NanoNextNL theme 6 meeting**, Utrecht, The Netherlands (2013). *(Award for the best poster presentation)*  
  Zili Zhou, S. Jahanmirinejad, F. Mattioli, D. Sahin, G. Frucci, A. Gaggero, R. Leoni, A. Fiore,  
  “Resolving up to twelve photons with superconducting nanowires”

• **Physics@FOM Veldhoven conference**, Veldhoven, The Netherlands (2013). *(poster presentation)*  
  Zili Zhou, G. Frucci, F. Mattioli, A. Gaggero, R. Leoni, S. Jahanmirinejad, T. B. Hoang, A. Fiore,  
  “Ultrsensitive N-photon interferometric autocorrelator based on superconducting nanodetectors”

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