Formal specification of a generic separation kernel

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# D31.1
Formal Specification of a Generic Separation Kernel

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**Abstract:**
We introduce a theory of intransitive non-interference for separation kernels with control. We show that it can be instantiated for a simple API consisting of IPC and events.

**Keywords:**
separation kernel with control, formal model, instantiation, IPC, events, Isabelle/HOL
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Executive Summary

Intransitive noninterference has been a widely studied topic in the last few decades. Several well-established methodologies apply interactive theorem proving to formulate a noninterference theorem over abstract academic models. In joint work with several industrial and academic partners throughout Europe, we are helping in the certification process of PikeOS, an industrial separation kernel developed at SYSGO. In this process, established theories could not be applied. We present a new generic model of separation kernels and a new theory of intransitive noninterference. The model is rich in detail, making it suitable for formal verification of realistic and industrial systems such as PikeOS. Using a refinement-based theorem proving approach, we ensure that proofs remain manageable.

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1 Introduction

Separation kernels are at the heart of many modern security-critical systems [23]. With next generation technology in cars, aircrafts and medical devices becoming more and more interconnected, a platform that offers secure decomposition of embedded systems becomes crucial for safe and secure performance. PikeOS, a separation kernel developed at SYSGO, is an operating system providing such an environment [12, 2]. A consortium of several European partners from industry and academia works on the certification of PikeOS up to at least Common Criteria EAL5+, with ”+” being applying formal methods compliant to EAL7. Our aim is to derive a precise model of PikeOS and a precise formulation of the PikeOS security policy.

A crucial security property of separation kernels is intransitive noninterference. This property is typically required for systems with multiple independent levels of security (MILS) such as PikeOS. It ensures that a given security policy over different subjects of the system is obeyed. Such a security policy dictates which subjects may flow information to which other subjects.

Intransitive noninterference has been an active research field for the last three decades. Several papers have been published on defining intransitive noninterference and on unwinding methodologies that enable the proof of intransitive noninterference from local proof obligations. However, in the certification process of PikeOS these existing methodologies could not be directly applied. Generally, the methodologies are based on highly abstract generic models of computation. The gap between such an abstract model and the reality of PikeOS is large, making application of the methodologies tedious and cumbersome.

This paper presents a new generic model for separation kernels called CISK (for: Controlled Interruptible Separation Kernel). This model is richer in details and contains several facets present in many separation kernels, such as interrupts, context switches between domains and a notion of control. Regarding the latter, this concerns the fact that the kernel exercises control over the executions as performed by the domains. The kernel can, e.g., decide to skip actions of the domains, or abort them halfway. We prove that any instantiation of the model provides intransitive noninterference. The model and proofs have been formalized in Isabelle/HOL [21] which are included in the subsequent sections of this document.

We have adopted Rushby’s definition of intransitive noninterference [24]. We first present an overview of our approach and then discuss the relation between our approach and existing methodologies in the next section.

Overview

Generally, there are two conflicting interests when using a generic model. On the one hand the model must be sufficiently abstract to ensure that theorems and proofs remain manageable. On the other hand, the model must be rich enough and must contain sufficient domain-knowledge to allow easy instantiation. Rushby’s model, for example, is on one end of the spectrum: it is basically a Mealy machine, which is a highly abstract notion of computation, consisting only of state, inputs and outputs [24]. The model and its proofs are manageable, but making a realistic instantiation is tedious and requires complicated proofs.

We aim at the other side of the spectrum by having a generic model that is rich in detail. As a result, instantiating the model with, e.g., a model of PikeOS can be done easily. To ensure maintainability of the theorems and proofs, we have applied a highly modularized theorem proving technique.

Figure 1 shows an overview. The initial module “Kernel” is close to a Mealy machine, but has several facets added, including interrupts, context switches and control. New modules are added in such a way that each new module basically inserts an adjective before “Kernel”. The use of modules allows us to prove, e.g., a separation theorem in module “Separation Kernel” and subsequently to reuse this theorem later on when details on control or interrupts are added.

The second module adds a notion of separation, yielding a module of a Separation Kernel (SK). A security policy is added that dictates which domains may flow information to each other. Local proof
obligations are added from which a global theorem of noninterference is proven. This global theorem is the *unwinding* of the local proof obligations.

In the third module calls to the kernel are no longer considered atomic, yielding an Interruptible Separation Kernel (ISK). In this model, one call to the kernel is represented by an *action sequence*. Consider, for example, an IPC call (for: Inter Process Communication). From the point of view of the programmer this is one kernel call. From the point of view of the kernel it is an action sequence consisting of three stages IPC\_PREP, IPC\_WAIT, and IPC\_SEND. During the PREP stage, it is checked whether the IPC is allowed by the security policy. The WAIT stage is entered if a thread needs to wait for its communication partner. The SEND stage is data transmission. After each stage, an interrupt may occur that switches the current context. A consequence of allowing interruptible action sequences is that it is no longer the case that any execution, i.e., any combination of atomic kernel actions, is realistic. We formulate a definition of *realistic execution* and weaken the proof obligations of the model to apply only to realistic executions.

The final module provides an interpretation of control that allows atomic kernel actions to be aborted or delayed. Additional proof obligations are required to ensure that noninterference is still provided. This yields a Controlled Interruptible Separation Kernel (CISK). When sequences of kernel actions are aborted, error codes can be transmitted to other domains. Revisiting our IPC example, after the PREP stage the kernel can decide to abort the action. The IPC action sequence will not be continued and error codes may be sent out. At the WAIT stage, the kernel can delay the action sequence until the communication partner of the IPC call is ready to receive.

In Section 3 we introduce a theory of intransitive non-interference for separation kernels with control, based on [31]. We show that it can be instantiated for a simple API consisting of IPC and events (Section 4). The rest of this section gives some auxiliary theories used for Section 3.

## 2 Preliminaries

### 2.1 Binders for the option type

```
theory Option-Binders
imports Option
begin

The following functions are used as binders in the theorems that are proven. At all times, when a
```
result is None, the theorem becomes vacuously true. The expression “$m \rightarrow \alpha$” means “First compute $m$, if it is None then return True, otherwise pass the result to $\alpha$”. B2 is a short hand for sequentially doing two independent computations. The following syntax is associated to B2: “$m_1 || m_2 \rightarrow \alpha$” represents “First compute $m_1$ and $m_2$, if one of them is None then return True, otherwise pass the result to $\alpha$”.

**definition** B2 :: 'a option ⇒ ('a ⇒ bool) ⇒ bool (infixl → 65)
where B m α ≡ case m of None ⇒ True | (Some a) ⇒ α

**definition** B2 :: 'a option ⇒ 'a option ⇒ ('a ⇒ 'a ⇒ bool) ⇒ bool
where B2 m1 m2 α ≡ m1 → (λ a . m2 ⇒ (λ b . α a b))

**syntax** B2 [(a option, a option, (a ⇒ a ⇒ bool)] ⇒ bool ((− ⋅ − ·) [0, 0, 10] 10)

Some rewriting rules for the binders

**lemma** rewrite-B2-to-cases[simp]:
shows B2 s t f = (case s of None ⇒ True | (Some s1) ⇒ (case t of None ⇒ True | (Some t1) ⇒ f s t))
using assms unfolding B2-def B-def by(cases s,cases t,simp+)

**lemma** rewrite-B-None[simp]:
shows None → α = True

unfolding B-def by(auto)

**lemma** rewrite-B-m-True[simp]:
shows m → (λ a . True) = True

unfolding B-Def by(cases m,simp+)

**lemma** rewrite-B2-cases:
shows (case a of None ⇒ True | (Some s) ⇒ (case b of None ⇒ True | (Some t) ⇒ f t s))
= (∀ s t . a = (Some s) ∧ b = (Some t) → f t s)
by(cases a,simp,cases b,simp+)

**definition** strict-equal :: 'a option ⇒ 'a ⇒ bool
where strict-equal m a ≡ case m of None ⇒ False | (Some a′) ⇒ a′ = a

2.2 Theorems on lists

**theory** List-Theorems

imports List

begin

**definition** lastn :: nat ⇒ 'a list ⇒ 'a list
where lastn n x = drop ((length x) − n) x

**definition** is-sub-seq :: 'a ⇒ 'a ⇒ 'a list ⇒ bool
where is-sub-seq a b x ≡ ∃ n . Suc n < length x ∧ x!n = a ∧ x!(Suc n) = b

**definition** prefixes :: 'a list set ⇒ 'a list set
where prefixes s ≡ { x . ∃ n y . n > 0 ∧ y ∈ s ∧ take n y = x }

**lemma** drop-one[simp]:
shows drop (Suc 0) x = tl x by(induct x,auto)

**lemma** length-ge-one:
shows x ≠ [] → length x ≥ 1 by(induct x,auto)

**lemma** take-but-one[simp]:
shows x ≠ [] → lastn (length x − 1) x = tl x unfolding lastn-def

using length-ge-one[where x=x] by auto

**lemma** Suc-m-minus-n[simp]:
shows m ≥ n → Suc m − n = Suc (m − n) by auto
lemma lastn-one-less:
shows \( n > 0 \land n \leq \text{length } x \land \text{lastn } n \ x = (a \# y) \implies \text{lastn } (n - 1) \ x = y \) unfolding lastn-def
using drop-Suc[where \( n = \text{length } x - n \) and \( \text{x} = x \)] drop-tl[where \( n = \text{length } x - n \) and \( \text{x} = x \)]
by(auto)

lemma list-sub-implies-member:
shows \( \forall \ a \ x . \ 
\text{set} (a \# x) \subseteq Z \implies a \in Z \)
by(auto)

lemma subset-smaller-list:
shows \( \forall \ a \ x . \ 
\text{set} (a \# x) \subseteq Z \implies \text{set } x \subseteq Z \)
by(auto)

lemma second-elt-is-hd-tl:
shows \( \text{tl } x = (a \# x') \implies a = x!1 \)
by(cases \( x \),auto)

lemma length-ge-2-implies-tl-not-empty:
shows \( \text{length } x \geq 2 \implies \text{tl } x \neq [] \)
by(cases \( x \),auto)

lemma length-lt-2-implies-tl-empty:
shows \( \text{length } x < 2 \implies \text{tl } x = [] \)
by(cases \( x \),auto)

lemma first-second-is-sub-seq:
shows \( \text{length } x \geq 2 \implies \text{is-sub-seq} (\text{hd } x) (x!1) x \)
proof-
assume \( \text{length } x \geq 2 \)
hence \( 1 \) by(auto)
hence \( x!0 = \text{hd } x \) by(cases \( x \),auto)
from this \( 1 \) show \( \text{is-sub-seq} (\text{hd } x) (x!1) x \)
unfolding is-sub-seq-def by(auto)
qed

lemma hd-drop-is-nth:
shows \( n < \text{length } x \implies \text{hd} (\text{drop } n \ x) = x!n \)
proof(induct \( x \) arbitrary: \( n \))
case Nil
  thus \( ?\text{thesis} \) by simp
next
case \( \text{Cons } a \ x \)
  have \( \text{hd} (\text{drop } n (a \# x)) = (a \# x)!n \)
  proof(cases \( n \))
case 0
  thus \( ?\text{thesis} \) by simp
next
case \( \text{Suc } m \)
  from Suc Cons show \( ?\text{thesis} \) by(auto)
qed

lemma def-of-hd:
shows \( y = a \# x \implies \text{hd } y = a \) simp
lemma def-of-tl:
shows \( y = a \# x \implies \text{tl } y = x \) by simp
lemma drop-yields-results-implies-nbound:
shows \( \text{drop } n \ x \neq [] \implies n < \text{length } x \)
by(induct \( x \),auto)
lemma hd-take[simp]:
shows \( n > 0 \implies \text{hd} (\text{take } n \ x) = \text{hd } x \)
by(cases \( x \),simp,cases \( n \),auto)
lemma consecutive-is-sub-seq:
shows \( a \# (b \# x) = \text{lastn } n y \implies \text{is-sub-seq } a \ b \ y \)
3 A generic model for separation kernels

This section defines a detailed generic model of separation kernels called CISK (Controlled Interruptible Separation Kernel). It contains a generic functional model of the behaviour of a separation kernel as a transition system,
definitions of the security property and proofs that the functional model satisfies security properties. It is based on Rushby’s approach [25] for noninterference. For an explanation of the model, its structure and an overview of the proofs, we refer to the document entitled “A New Theory of Intransitive Noninterference for Separation Kernels with Control” [31].

The structure of the model is based on locales and refinement:

- locale “Kernel” defines a highly generic model for a kernel, with execution semantics. It defines a state transition system with some extensions to the one used in [25]. The transition system defined here stores the currently active domain in the state, and has transitions for explicit context switches and interrupts and provides a notion of control. As each operation of the system will be split into atomic actions in our model, only certain sequences of actions will correspond to a run on a real system. Therefore, the function \texttt{run}, which applies an execution on a state and computes the resulting new state, is partial and defined for realistic traces only. Later, but not in this locale, we will define a predicate to distinguish realistic traces from other traces. Security properties are also not part of this locale, but will be introduced in the locales to be described next.

- locale “Separation_Kernel” extends ”Kernel” with constraints concerning non-interference. The theorem is only sensical for realistic traces; for unrealistic trace it will hold vacuously.

- locale “Interruptible_Separation_Kernel” refines “Separation_Kernel” with interruptible action sequences. It defines function “realistic_trace” based on these action sequences. Therefore, we can formulate a total run function.

- locale “Controlled_Interruptible_Separation_Kernel” refines “Interruptible_Separation_Kernel” with abortable action sequences. It refines function “control” which now uses a generic predicate “aborting” and a generic function “set_error_code” to manage aborting of action sequences.

3.1 K (Kernel)

theory \texttt{K}

imports \texttt{Main List Set Transitive-Closure List-Theorems Option-Binders}

begin

The model makes use of the following types:

- \texttt{'state_t} A state contains information about the resources of the system, as well as which domain is currently active. We decided that a state does not need to include a program stack, as in this model the actions that are executed are modelled separately.

- \texttt{'dom_t} A domain is an entity executing actions and making calls to the kernel. This type represents the names of all domains. Later on, we define security policies in terms of domains.

- \texttt{'action_t} Actions of type \texttt{'action_t} represent atomic instructions that are executed by the kernel. As kernel actions are assumed to be atomic, we assume that after each kernel action an interrupt point can occur.

- \texttt{'action_t execution} An execution of some domain is the code or the program that is executed by the domain. One call from a domain to the kernel will typically trigger a succession of one or more kernel actions. Therefore, an execution is represented as a list of sequences of kernel actions. Non-kernel actions are not take into account.

- \texttt{'output_t} Given the current state and an action an output can be computed deterministically.

- \texttt{time_t} Time is modelled using natural numbers. Each atomic kernel action can be executed within one time unit.

\texttt{type-synonym \{'action-t\} execution = \{'action-t\} list list}

\texttt{type-synonym time-t = nat}
Function \textit{kstep} (for kernel step) computes the next state based on the current state \textit{s} and a given action \textit{a}. It may assume that it makes sense to perform this action, i.e., that any precondition that is necessary for execution of action \textit{a} in state \textit{s} is met. If not, it may return any result. This precondition is represented by generic predicate \textit{kprecondition} (for kernel precondition). Only realistic traces are considered. Predicate \textit{realistic-execution} decides whether a given execution is realistic.

Function \textit{current} returns given the state the domain that is currently executing actions. The model assumes a single-core setting, i.e., at all times only one domain is active. Interrupt behavior is modelled using functions \textit{interrupt} and \textit{cswitch} (for context switch) that dictate respectively when interrupts occur and how interrupts occur. Interrupts are solely time-based, meaning that there is an at beforehand fixed schedule dictating which domain is active at which time.

Finally, we add function \textit{control}. This function represents control of the kernel over the execution as performed by the domains. Given the current state \textit{s}, the currently active domain \textit{d} and the execution \textit{α} of that domain, it returns three objects. First, it returns the next action that domain \textit{d} will perform. Commonly, this is the next action in execution \textit{α}. It may also return None, indicating that no action is done. Secondly, it returns the updated execution. When executing action \textit{α}, typically, this action will be removed from the current execution (i.e., updating the program stack). Thirdly, it can update the state to set, e.g., error codes.

```
locale Kernel =

  fixes kstep :: 'state-t ⇒ 'action-t ⇒ 'state-t
  and output-f :: 'state-t ⇒ 'action-t ⇒ 'output-t
  and s0 :: 'state-t
  and current :: 'state-t ⇒ 'dom-t
  and cswitch :: time-t ⇒ 'state-t ⇒ 'state-t
  and interrupt :: time-t ⇒ bool
  and kprecondition :: 'state-t ⇒ 'action-t ⇒ bool
  and realistic-execution :: 'action-t execution ⇒ bool
  and control :: 'state-t ⇒ 'dom-t ⇒ 'action-t execution ⇒
    (('action-t option) × 'action-t execution × 'state-t)
  and kinvolved :: 'action-t ⇒ 'dom-t set

begin

3.1.1 Execution semantics

Short hand notations for using function control.

attribute predefinition

```

\[\text{next-action s execs} = \text{fst (control s (current s) } (\text{execs (current s)})\]
\[\text{next-execs s execs} = (\text{fun-upd execs (current s)} (\text{fst (snd (control s (current s) } (\text{execs (current s))))}))\]
\[\text{next-state s execs = snd (snd (control s (current s) } (\text{execs (current s))))}\]

A thread is empty iff either it has no further action sequences to execute, or when the current action sequence is finished and there are no further action sequences to execute.

```

abbreviation thread-empty::'action-t execution ⇒ bool
where thread-empty exec = \[\] ∨ exec = [\[]

```

Wrappers for function kstep and kprecondition that deal with the case where the given action is None.

```

definition step where step s oa ≡ case oa of None ⇒ s | (Some a) ⇒ kstep s a
definition precondition :: 'state-t ⇒ 'action-t option ⇒ bool
where precondition s a ≡ a ≈ kprecondition s
definition involved
where involved oa ≡ case oa of None ⇒ \{\} | (Some a) ⇒ kinvolved a

```

Execution semantics are defined as follows: a run consists of consecutively running sequences of actions. These sequences are interruptable. Run first checks whether an interrupt occurs. When this
happens, function cswitch may switch the context. Otherwise, function control is used to determine the next action \( a \), which also yields a new state \( s' \). Action \( a \) is executed by executing \((\text{step } s')\). The current execution of the current domain is updated.

Note that run is a partial function, i.e., it computes results only when at all times the preconditions hold. Such runs are the realistic ones. For other runs, we do not need to – and cannot – prove security. All the theorems are formulated in such a way that they hold vacuously for unrealistic runs.

\[
\begin{align*}
\text{function } \text{run} : \text{time-t} \Rightarrow '\text{state-t option}' \Rightarrow ('\text{dom-t} \Rightarrow '\text{action-t execution}') \Rightarrow '\text{state-t option}' \\
\text{where run } 0 \text{ s execs } s = s \\
\text{interrupt } (\text{Suc } n) \Rightarrow \text{run } (\text{Suc } n) (\text{Some } s) \text{ execs } = \text{run } n (\text{Some } (\text{cswitch } (\text{Suc } n) s)) \text{ execs} \\
\neg \text{interrupt } (\text{Suc } n) \Rightarrow \text{thread-empty}(\text{execs } (\text{current } s)) \Rightarrow \text{run } (\text{Suc } n) (\text{Some } s) \text{ execs } = \text{run } n (\text{Some } s) \text{ execs} \\
\neg \text{interrupt } (\text{Suc } n) \Rightarrow \neg \text{thread-empty}(\text{execs } (\text{current } s)) \Rightarrow \neg \text{precondition } (\text{next-state } s \text{ execs}) (\text{next-action } s \text{ execs}) \Rightarrow \text{run } (\text{Suc } n) (\text{Some } s) \text{ execs } = \text{None} \\
\neg \text{interrupt } (\text{Suc } n) \Rightarrow \neg \text{thread-empty}(\text{execs } (\text{current } s)) \Rightarrow \text{precondition } (\text{next-state } s \text{ execs}) (\text{next-action } s \text{ execs}) \Rightarrow \text{run } (\text{Suc } n) (\text{Some } s) \text{ execs } = \text{run } n (\text{Some } (\text{step } (\text{next-state } s \text{ execs}) (\text{next-action } s \text{ execs}))) (\text{next-execs } s \text{ execs}) \\
\text{using not0-implies-Suc by (metis option.exhaust prod-cases3,auto)} \\
\text{termination by lexicographic-order} \\
\end{align*}
\]

3.2 SK (Separation Kernel)

Locale Kernel is now refined to a generic model of a separation kernel. The security policy is represented using function ia. Function \text{vpeq} is adopted from Rushby and is an equivalence relation representing whether two states are equivalent from the point of view of the given domain.

We assume constraints similar to Rushby, i.e., weak step consistency, locally respects, and output consistency. Additional assumptions are:

**Step Atomicity** Each atomic kernel step can be executed within one time slot. Therefore, the domain that is currently active does not change by executing one action.

**Time-based Interrupts** As interrupts occur according to a prefixed time-based schedule, the domain that is active after a call of switch depends on the currently active domain only (\text{cswitch_consistency}). Also, cswitch can only change which domain is currently active (\text{cswitch_consistency}).

**Control Consistency** States that are equivalent yield the same control. That is, the next action and the updated execution depend on the currently active domain only (\text{next_action_consistent}, \text{next_execs_consistent}), the state as updated by the control function remains in \text{vpeq} (\text{next_state_consistent}, locally_respects_next_state). Finally, function control cannot change which domain is active (\text{current_next_state}).

\[
\begin{align*}
\text{definition actions-in-execution: } '\text{action-t execution} \Rightarrow '\text{action-t set}' \\
\text{where actions-in-execution exec} \equiv \{ a \cdot \exists aseq \in \text{set exec} . a \in \text{set aseq} \} \\
\end{align*}
\]

Locale Separation-Kernel = Kernel kstep output-f s0 current cswitch interrupt kprecondition realistic-execution control kinvolved

for kstep :: 'state-t \Rightarrow 'action-t \Rightarrow 'state-t \\
and output-f :: 'state-t \Rightarrow 'action-t \Rightarrow 'output-t
and s0 :: 'state-t
and current :: 'state-t => 'dom-t — Returns the currently active domain
and cswitch :: time-t => 'state-t => 'state-t — Switches the current domain
and interrupt :: time-t => bool — Returns t iff an interrupt occurs in the given state at the given time
and kprecondition :: 'state-t => 'action-t => bool — Returns t if an precondition holds that relates the current action to the state
and realistic-execution :: 'action-t execution => bool — In this locale, this function is completely unconstrained.
and control :: 'state-t => 'dom-t => 'action-t execution => ((('action-t option) × 'action-t execution × 'state-t)
and kinvolved :: 'action-t => 'dom-t set
+
fixes ifp :: 'dom-t => 'dom-t => bool
and vpeq :: 'dom-t => 'state-t => bool
assumes vpeq-transitive: ∀ a b c u. (vpeq u a b ∧ vpeq u b c) → vpeq u a c
and vpeq-symmetric: ∀ a b u. vpeq u a b → vpeq u b a
and vpeq-reflexive: ∀ a u. vpeq u a a
and ifp-reflexive: ∀ u. ifp u u
and weakly-step-consistent: ∀ s t u a. vpeq u s t ∧ vpeq (current s) s t ∧ kprecondition s a ∧ kprecondition t a ∧ current s = current t → vpeq u (kstep s a) (kstep t a)
and locally-respects: ∀ a s u. ¬ifp (current s) u ∧ kprecondition s a → vpeq u s (kstep s a)
and output-consistent: ∀ a s t. vpeq (current s) s t ∧ current s = current t → (output-f s a) = (output-f t a)
and step-atomicity: ∀ a s u. current (kstep s a) = current s
and cswitch-independent-of-state: ∀ n s t . current s = current t → current (cswitch n s) = current (cswitch n t)

and cswitch-consistency: ∀ u s t n . vpeq u s t → vpeq u (cswitch n s) (cswitch n t)
and next-action-consistent: ∀ s t execs . vpeq (current s) s t ∧ (∀ d ∈ involved (next-action s execs) . vpeq d s t) ∧ current s = current t → next-action s execs = next-action t execs
and next-execs-consistent: ∀ s t execs . vpeq (current s) s t ∧ (∀ d ∈ involved (next-action s execs) . vpeq d s t) ∧ current s = current t → fst (snd (control s (current s) (execs (current s)))) = fst (snd (control t (current s) (execs (current s))))
and next-state-consistent: ∀ s t u execs . vpeq (current s) s t ∧ vpeq u s t ∧ current s = current t → vpeq u (next-state s execs) (next-state t execs)
and current-next-state: ∀ s execs . current (next-state s execs) = current s
and locally-respects-next-state: ∀ s u execs. ¬ifp (current s) u → vpeq u s (next-state s execs)
and involved-ifp: ∀ s a. ∀ d ∈ (involved a) . kprecondition s (the a) → ifp d (current s)
and next-action-from-exec: ∀ s execs . next-action s execs → (λ a . a ∈ actions-in-execution (execs (current s)))
and next-execs-subset: ∀ s execs u . actions-in-execution (next-execs s execs u) ⊆ actions-in-execution (execs u)
begin

Note that there are no proof obligations on function “interrupt”. Its typing enforces the assumptions that switching is based on time and not on state. This assumption is sufficient for these proofs, i.e., no further assumptions are required.

3.2.1 Security for non-interfering domains

We define security for domains that are completely non-interfering. That is, for all domains u and v such that v may not interfere in any way with domain u, we prove that the behavior of domain u is independent of the actions performed by v. In other words, the output of domain u in some run is at all times equivalent to the output of domain u when the actions of domain v are replaced by some other set actions.

A domain is unrelated to u if and only if the security policy dictates that there is no path from the domain to u.

abbreviation unrelated : 'dom-t => 'dom-t => bool
where unrelated d u ≡ ¬ifp"++" d u
To formulate the new theorem to prove, we redefine purging: all domains that may not influence domain \( u \) are replaced by arbitrary action sequences.

**Definition** purge

\[
\text{purge} \doteq \begin{cases}
  \text{dom} \Rightarrow '\text{action-t execution}' \Rightarrow \text{dom} \Rightarrow ('\text{dom} \Rightarrow '\text{action-t execution}') \\
  \text{where} \text{purge execs } u \equiv \lambda d . (\text{if unrelated } d u \text{ then} \\
  (\text{SOME} \text{ alpha} . \text{realistic-execution alpha}) \\
  \text{else execs } d)
\end{cases}
\]

A normal run from initial state \( s_0 \) ending in state \( s_f \) is equivalent to a run purged for domain \((\text{currents}_f)\).

**Definition** NI-unrelated

\[\forall \text{ execs } a n . \text{ run } n \text{ (Some } s_0) \text{ execs} \Rightarrow \]

\[
(\lambda s-f . \text{ run } n \text{ (Some } s_0) \text{ (purge execs (current } s-f)) \Rightarrow \]

\[
(\lambda s-f2 . \text{ output-f } s-f a = \text{ output-f } s-f2 a \land \text{ current } s-f = \text{ current } s-f2)
\]

The following properties are proven inductive over states \( s \) and \( t \):

1. Invariably, states \( s \) and \( t \) are equivalent for any domain \( v \) that may influence the purged domain \( u \). This is more general than proving that "\( \text{vpeq } u s t \)" is inductive. The reason we need to prove equivalence over all domains \( v \) is so that we can use weak step consistency.

2. Invariably, states \( s \) and \( t \) have the same active domain.

**Abbreviation** equivalent-states :: 'state-t option \Rightarrow 'state-t option \Rightarrow 'dom \Rightarrow bool

**Where** equivalent-states \( s t u \equiv s \parallel t \rightarrow \left( (\forall v . \text{ifp}^* v u \rightarrow \text{vpeq } v s t) \land \text{ current } s = \text{ current } t \right) \)

Rushby’s view partitioning is redefined. Two states that are initially \( u \)-equivalent are \( u \)-equivalent after performing respectively a realistic run and a realistic purged run.

**Definition** view-partitioned::bool

\[\forall \text{ execs } a n . \text{ run } n \text{ (Some } s_0) \text{ execs} \rightarrow \]

\[
(\text{run n ms execs} \parallel \text{run n mt (purge execs u)} \rightarrow \]

\[
(\lambda rs rt . \text{ vpeq } u rs rt \land \text{ current } rs = \text{ current } rt)
\]

We formulate a version of predicate view-partitioned that is on one hand more general, but on the other hand easier to prove inductive over function run. Instead of reasoning over execs and (purge execs u), we reason over any two executions execs1 and execs2 for which the following relation holds:

**Definition** purged-relation :: 'dom \rightarrow ('dom \Rightarrow 'action-t execution) \Rightarrow ('dom \Rightarrow 'action-t execution) \Rightarrow bool

**Where** purged-relation u execs1 execs2 \equiv \forall d . \text{ifp}^* d u \rightarrow \text{execs1 } d = \text{execs2 } d

The inductive version of view-partitioning says that runs on two states that are \( u \)-equivalent and on two executions that are purged-related yield \( u \)-equivalent states.

**Definition** view-partitioned-ind::bool

\[\forall \text{ execs1 execs2 } s t n u . \text{ equivalent-states } s t u \land \text{ purged-relation } u \text{ execs1 execs2} \rightarrow \text{ equivalent-states } (\text{run n s execs1}) (\text{run n t execs2})\]

A proof that when state \( t \) performs a step but state \( s \) not, the states remain equivalent for any domain \( v \) that may interfere with \( u \).

**Lemma** vpeq-s-nt:

**Assumes** prec-t: precondition (next-state t execs2) (next-action t execs2)

**Assumes** not-ifp-curr-u: ¬ \text{ifp}^* (current t) u

**Assumes** vpeq-s-t: \( \forall v . \text{ifp}^* v u \rightarrow \text{vpeq } v s t \)

**Shows** \( \forall v . \text{ifp}^* v u \rightarrow \text{vpeq } v s \left( \text{step } \left( \text{next-state t execs2} \right) \left( \text{next-action t execs2} \right) \right) \)

**Proof**

\{ fix v \}
assume ifp-v-uc ifp** v u

from ifp-v-uc not-ifp-curr-u have unrelated: ~ifp** (current t) v using rtranclp-trans by metis
from this current-next-state[THEN spec,THEN spec,where x1=t] locally-respects[THEN spec,THEN spec,THEN spec,where x1=next-state t execs2] vpeq-reflexive prec-s have vpeq v (next-state t execs2) (step (next-state t execs2) (next-action t execs2)) unfolding step-def precondition-def B-def by (cases next-action t execs2,auto)
from unrelated this locally-respects-next-state vpeq-transitive have vpeq v t (step (next-state t execs2) (next-action t execs2)) by blast
from this and ifp-v-u and vpeq-s-t and vpeq-symmetric and vpeq-transitive have vpeq v s (step (next-state t execs2) (next-action t execs2)) by metis
thus ?thesis by auto
qed

A proof that when state s performs a step but state t not, the states remain equivalent for any domain v that may interfere with u.

lemma vpeq-ns-t:
assumes prec-s: precondition (next-state s execs) (next-action s execs)
assumes not-ifp-curr-u: ~ifp** (current s) u
assumes vpeq-s-t: ∀ v. ifp** v u → vpeq v s t
shows ∀ v. ifp** v u → vpeq v (step (next-state s execs) (next-action s execs)) t
proof-

{ fix v
assume ifp-v-uc ifp** v u

from ifp-v-uc and not-ifp-curr-u have unrelated: ~ifp** (current s) v using rtranclp-trans by metis
from this current-next-state[THEN spec,THEN spec,where x1=s] vpeq-reflexive unrelated locally-respects[THEN spec,THEN spec,THEN spec,where x1=next-state s execs and x=v and x2=the (next-action s execs)] prec-s have vpeq v (next-state s execs) (step (next-state s execs) (next-action s execs)) unfolding step-def precondition-def B-def by (cases next-action s execs,auto)
from unrelated this locally-respects-next-state vpeq-transitive have vpeq v s (step (next-state s execs) (next-action s execs)) by blast
from this and ifp-v-u and vpeq-s-t and vpeq-symmetric and vpeq-transitive have vpeq v (step (next-state s execs) (next-action s execs)) t by metis
thus ?thesis by auto
qed

A proof that when both states s and t perform a step, the states remain equivalent for any domain v that may interfere with u. It assumes that the current domain can interact with u (the domain for which is purged).

lemma vpeq-ns-nt-ifp-u:
assumes vpeq-s-t: ∀ v. ifp** v u → vpeq v s t'
and current-s-t: current s = current t'
sshows precondition (next-state s execs) a ∧ precondition (next-state t' execs) a → ((ifp** (current s) u → (∀ v. ifp** v u → vpeq v (step (next-state s execs) a) (step (next-state t' execs) a))))
proof-

fix a
assume prec-s: precondition (next-state s execs) a ∧ precondition (next-state t' execs) a
assume ifp-curr: ifp** (current s) u
from vpeq-s-t have vpeq-curr-s-t: ifp** (current s) u → vpeq (current s) s t' by auto
from ifp-curr precs
next-state-consistent[\textit{THEN} spec,\textit{THEN} spec, where \(x \downarrow = s\) \textbf{and} \(x = t\)] vpeq-curr-s-t vpeq-s-t 
\textbf{show} \(\forall v. \textit{ifp}^{**} v u \rightarrow vpeq v (\text{step (next-state s execs)} a) (\text{step (next-state t execs)} a)\)

\textbf{proof}\ 

\begin{align*}
\textsf{execs} &= \textsf{execs2} \\
\textsf{current} &\textsf{-next-state (current-s-t weakly-step-consistent[\textit{THEN} spec,\textit{THEN} spec,\textit{THEN} spec,\textit{THEN} spec, where }x_3=\textsf{next-state s execs} \textbf{and} x_2=\textsf{next-state t execs} \textbf{and} x=\textit{the a}] \\
\textsf{show} \(\forall v. \textit{ifp}^{**} v u \rightarrow vpeq v (\text{step (next-state s execs)} a) (\text{step (next-state t execs)} a)\) \\
\textbf{unfolding} &\text{step-def precondition-def B-def} \\
\textbf{by} &\text{(cases a,auto)}
\end{align*}

\textbf{qed}

A proof that when both states \(s\) and \(t\) perform a step, the states remain equivalent for any domain \(v\) that may interfere with \(u\). It assumes that the current domain \textit{cannot} interact with \(u\) (the domain for which is purged).

\textbf{lemma} vpeq-ns-nt-not-ifp-u 
\textbf{assumes} purged-a-a2: purged-relation \(u \textsf{execs execs2}\) 
\textbf{and} \(\textsf{prec-s precondition (next-state s execs) (next-action s execs)}\) 
\textbf{and} \(\textsf{current-s-t current s = current t}\) 
\textbf{and} \(vpeq-s-t: \forall v. \textit{ifp}^{**} v u \rightarrow vpeq v s t\) 
\textbf{shows} \(\neg\textit{ifp}^{**} (current s) \land \textsf{precondition (next-state t execs2)} \rightarrow (\forall v. \textit{ifp}^{**} v u \rightarrow vpeq v (\text{step (next-state s execs)} a) (\text{step (next-state t execs2)} (\text{next-action t execs2})))\)

\textbf{proof--}

\{ 
\textbf{assume} \(\textit{not-ifp}^{**} (current s) u\) 
\textbf{assume} \(\textsf{prec-t precondition (next-state t execs2)} (\text{next-action t execs2})\) 
\textbf{fix} \(a a' v\) 
\textbf{assume} \(\textit{ifp-v-ic}^{**} v u\) 
\textbf{from} \(\textit{not-ifp and purged-a-a2 have} \neg\textit{ifp}^{**} (current s) u\) \textbf{unfolding} \textit{purged-relation-def by auto} 
\textbf{from} \(\textit{this and ifp-v-ic} v u\) \textbf{have} \(\textit{not-ifp-curr-v:} \neg\textit{ifp}^{**} (current s) v\) \textbf{using} \textit{rtranclp-trans by metis} 
\textbf{from} \(\textit{this current-ns-nt} [\textit{THEN spec,THEN spec,THEN spec,THEN spec,where }x_1=\textit{s and} x=\textit{execs2}] \textsf{prec-s vpeq-reflexive}\) 
\textbf{locally-respects[\textit{THEN spec,THEN spec,THEN spec,THEN spec,where }x_1=\textsf{next-state s execs and} x_2=\textsf{the (next-action s execs)} and} x=v\} 
\textbf{have} \(vpeq v (\text{step (next-state s execs)} a) (\text{step (next-state s execs)} (\text{next-action s execs}))\) 
\textbf{proof}\ 

\{ 
\begin{align*}
\textbf{unfolding} &\text{step-def precondition-def B-def} \\
\textbf{by} &\text{(cases next-action s execs,auto)} \\
\textbf{from} &\text{not-ifp-curr-v this locally-respects-next-state vpeq-transitive} \\
\textbf{have} &\text{vpeq-s-ns vpeq v s (step (next-state s execs) (next-action s execs))} \\
\textbf{by} &\text{blast} \\
\textbf{from} &\text{not-ifp-curr-v current-s-t current-ns-nt [\textit{THEN spec,THEN spec,THEN spec,THEN spec,where }x_1=t' \textbf{and} x=\textit{execs2}] \textsf{prec-t}\ } \\
\textbf{locally-respects[\textit{THEN spec,THEN spec,THEN spec,THEN spec,where }x=\textsf{next-state t execs2}] vpeq-reflexive}\) \\
\textbf{have} &\(\theta: vpeq v (\text{step (next-state t execs2)} (\text{step (next-state t execs2)} (\text{next-action t execs2})))\) \\
\textbf{unfolding} &\text{step-def precondition-def B-def} \\
\textbf{by} &\text{(cases next-action t execs2,auto)} \\
\textbf{from} &\text{not-ifp-curr-v current-s-t current-ns-nt have} \(\textit{ifp}^{**} (current t') v\) \\
\textbf{using} &\textit{rtranclp-trans by auto} \\
\textbf{from} &\textit{0 1 locally-respects-next-state vpeq-transitive} \\
\textbf{have} &\(vpeq-t-nt vpeq v t' (\text{step (next-state t execs2)} (\text{next-action t execs2})))\) \\
\textbf{by} &\text{blast} \\
\textbf{from} &\text{vpeq-s-ns and vpeq-t-nt and vpeq-s-t and ifp-v-uc and vpeq-symmetric and vpeq-transitive} \\
\textbf{have} &\(\text{vpeq-s-ns vpeq v s (step (next-state s execs) (next-action s execs)) (step (next-state t execs2) (next-action t execs2)))}\) \\
\textbf{by} &\text{blast} \\
\} 
\textbf{thus} \(\text{thesis by auto}\) 

\textbf{qed}

A run with a purged list of actions appears identical to a run without purging, when starting from two states that appear identical.

\textbf{lemma} unwinding-implies-view-partitioned-ind: 
\textbf{shows} view-partitioned-ind
proof
{
  fix execs execs2 s t u
  have equivalent-states s t u ∧ purged-relation u execs execs2 −→ equivalent-states (run n s execs) (run n t execs2) u
  proof
  (induct n s execs arbitrary: t u execs2 rule: run.induct)
  case (1 s execs t u execs2)
    show ?case by auto
  next
  case (2 n execs t u execs2)
    show ?case by simp
  next
  case (3 n s execs t u execs2)
  assume interrupt-s : interrupt (Suc n)
  assume IH : (equivalent-states (Some (cswitch (Suc n) s)) t u ∧ purged-relation u execs execs2 −→ equivalent-states (run n (Some (cswitch (Suc n) s)) execs) (run n t execs2) u)
  {
    fix t'
    assume t = Some t'
    fix rs
    assume rs : run (Suc n) (Some s) execs = Some rs
    fix rt
    assume rt : run (Suc n) (Some t') execs2 = Some rt
    assume vpeq-s-t : ∀ v . ifpˆ∗∗ v u −→ vpeq v s t'
    assume current-s-t : current s = current t'
    assume purged-a-a2 : purged-relation u execs execs2
  }
  — The following terminology is used: states rs and rt (for: run-s and run-t) are the states after a run. States ns and nt (for: next-s and next-t) are the states after one step.
  — We prove two properties: the states rs and rt have equal active domains (current-rs-rt) and are vpeq for all domains v that may influence u (vpeq-rs-rt). Both are proven using the IH. To use the IH, we have to prove that the properties hold for the next step (in this case, a context switch). Statement current-ns-nt states that after one step states ns and nt have the same active domain. Statement vpeq-ns-nt states that after one step states ns and nt are vpeq for all domains v that may influence u (vpeq-ns-rt).

  from current-s-t cswitch-independent-of-state
  have current-ns-nt: current (cswitch (Suc n) s) = current (cswitch (Suc n) t') by blast
  from cswitch-consistency vpeq-s-t
  have vpeq-ns-nt: ∀ v . ifpˆ∗∗ v u −→ vpeq v (cswitch (Suc n) s) (cswitch (Suc n) t') by auto
  from current-ns-nt vpeq-ns-nt interrupt-s vpeq-reflexive purged-a-a2 current-s-t IH[where u=u and t=Some (cswitch (Suc n) t') and ?execs2.0=execs2]
  have current-rs-rt: current rs = current rt using rs rt by(auto)
  {
    fix v
    assume ia : ifpˆ∗∗ v u
    from current-ns-nt vpeq-ns-nt ia interrupt-s vpeq-reflexive purged-a-a2 IH[where u=u and t=Some (cswitch (Suc n) t') and ?execs2.0=execs2]
    have vpeq-rs-rt: vpeq v rs rt using rs rt by(auto)
  }
  from current-rs-rt and this have equivalent-states (Some rs) (Some rt) u by auto
  }
  thus ?case by(simp add:option.splits,cases t,simp+)
  next
  case (4 n execs s t u execs2)
  assume not-interrupt : ¬interrupt (Suc n)
  assume thread-empty-s : thread-empty(execs (current s))
assumption IH: (∀ u execs2. equivalent-states (Some s) t u ∧ purged-relation u execs execs2 → equivalent-states (run n (Some s) execs) (run n t execs2) u)

\{ 
fix t’
assume t: t = Some t’
fix rs
assume rs: run (Suc n) (Some s) execs = Some rs
fix rt
assume rt: run (Suc n) (Some t’) execs2 = Some rt

assume vpeq-s-t: ∀ v . ifp∗∗ v u → vpeq v s t’
assume current-s-t: current s = current t’
assume purged-a-a2: purged-relation u execs execs2

— The following terminology is used: states rs and rt are the states after one run. States ns and nt (for: next-s and next-t) are the states after one step.

— We prove two properties: the states rs and rt have equal active domains (current-rs-rt) and are vpeq for all domains v that may influence u (vpeq-rs-rt). Both are proven using the IH. To use the IH, we have to prove that the properties hold for the next step (in this case, nothing happens in s as the thread is empty). Statement current-ns-nt states that after one step states ns and nt have the same active domain. Statement vpeq_ns_nt states that after one step states ns and nt are vpeq for all domains v that may influence u (vpeq-rs-rt).

from ifp-reflexive and vpeq-s-t have vpeq-s-t-u. vpeq u s t’ by auto
from thread-empty-s and purged-a-a2 and current-s-t have purged-a-na2: ~ifp∗∗ (current t’) u → purged-relation u execs (next-execs t’ execs2)
by (unfold next-execs-def, unfold purged-relation-def, auto)
from step-atomicity current-next-state current-s-t have current-s-nt: current s = current (step (next-state t’ execs2)) (next-action t’ execs2))
unfolding step-def
by (cases next-action t’ execs2, auto)

— The proof is by case distinction. If the current thread is empty in state t as well (case t-empty), then nothing happens and the proof is trivial. Otherwise (case t-not-empty), since the current thread has different executions in states s and t, we now show that it cannot influence u (statement not-ifp-curr-t). If in state t the precondition holds (case t-prec), locally respects shows that the states remain vpeq. Otherwise, (case t-not-prec), everything holds vacuously.

have current-rs-rt: current rs = current rt
proof (cases thread-empty(execs2 (current t’)) rule : case-split[case-names t-empty t-not-empty])
case t-empty
from purged-a-a2 and vpeq-s-t and current-s-t IH[where t=Some t’ and u=u and ?execs2.0=execs2]
have equivalent-states (run n (Some s) execs) (run n (Some t’) execs2) u using rs rt by(auto)
from this not-interrupt t-empty thread-empty-s
show ?thesis using rs rt by(auto)
next
case t-not-empty
from t-not-empty current-next-state and vpeq-s-t-u and thread-empty-s and purged-a-a2 and current-s-t
have not-ifp-curr-t: ~ifp∗∗ (current (next-state t’ execs2)) u unfolding purged-relation-def by auto
show ?thesis
proof (cases precondition (next-state t’ execs2) (next-action t’ execs2) rule : case-split[case-names t-prec t-not-prec])
case t-prec
from locally-respects-next-current-next-state not-ifp-curr-t vpeq-s-t locally-respects vpeq-s-nt
have vpeq-s-nt: (∀ v . ifp∗∗ v u → vpeq v s (step (next-state t’ execs2) (next-action t’ execs2))) by auto
from vpeq-s-nt purged-a-na2 this current-s-nt not-ifp-curr-t current-next-state
IH[where t=Some (step (next-state t’ execs2) (next-action t’ execs2)) and u=u and ?execs2.0=next-execs t’ execs2]
have equivalent-states (run n (Some s) execs) (run n (Some (step (next-state t’ execs2) (next-action t’ execs2)))) by auto
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execs2)) (next-execs t’ execs2)) u
  using rs rt by auto
  from t-not-empty t-prec vpeq-s-t this thread-empty-s not-interrupt
  show ?thesis using rs rt by auto
next
case t-not-prec
  thus ?thesis using rt t-not-empty not-interrupt by(auto)
qed
qed

{ fix v
  assume ia : ifp∗∗ v u
  have vpeq v rs rt proof (cases thread-empty(execs2 (current t)) rule :case-split[case-names t-empty t-not-empty])
    case t-empty
      from purged-a-a2 and vpeq-s-t and current-s-t IH where t = Some t’ and u = u and execs2.0 = execs2
      have equivalent-states (run n (Some s) execs) (run n (Some t’) execs2) u using rs rt by(auto)
    next
case t-not-empty
    show ?thesis proof (cases precondition (next-state t’ execs2) (next-action t’ execs2) rule :case-split[case-names t-prec t-not-prec])
      case t-prec
        from not-empty current-next-state and vpeq-s-t-u and thread-empty-s and purged-a-a2 and current-s-t
        have not-ifp-curr t : ¬ ifp∗∗ (current (next-state t’ execs2)) u unfolding purged-relation-def
        by auto
        from t-prec current-next-state locally-respects-next-state this and vpeq-s-t and locally-respects and vpeq-s-nt
        have vpeq-s-nt : (v’ : ifp∗∗ v u → vpeq v s (step (next-state t’ execs2)) (next-action t’ execs2))) by auto
        from purged-a-na2 this current-s-nt not-ifp-curr t current-next-state
        IH[where t = Some (step (next-state t’ execs2)) (next-action t’ execs2)) and u = u and execs2.0 = execs execs t’ execs2]
        have equivalent-states (run n (Some s) execs) (run n (Some (step (next-state t’ execs2)) (next-action t’ execs2))) (next-execs t’ execs2) u
        using rs rt by(auto)
        from ia t-not-empty t-prec vpeq-s-nt this thread-empty-s not-interrupt
        show ?thesis using rs rt by auto
      next
case t-not-prec
      thus ?thesis using rt t-not-empty not-interrupt by(auto)
    qed
  qed
}
  from current-rs-rt and this have equivalent-states (Some rs) (Some rt) u by auto
}
  thus ?case by(simp add:option.splits,cases t,simp+)
next
case (5 n execs s t u execs2)
assume not-interrupt : ¬ interrupt (Suc n)
assume thread-not-empty-s : ¬ thread-empty(execs (current s))
assume not-prec-s : ¬ precondition (next-state s execs) (next-action s execs)
— Whenever the precondition does not hold, the entire theorem flattens to True and everything holds vacuously.
hence run (Suc n) (Some s) execs = None using not-interrupt thread-not-empty-s by simp
thus case by(simp add:option.splits)
next
case (6 n execs s t u execs2)
assume not-interrupt ¬interrupt (Suc n)
assume thread-not-empty-s ¬thread-empty(execs (current s))
assume prec-s pre-condition (next-state s execs) (next-action s execs)
 assume IH: (∀ t u execs2.
   equivalent-states (Some (step (next-state s execs) (next-action s execs))) t u ∧
   purged-relation u (next-exec s execs) execs2 →
   equivalent-states
   (run n (Some (step (next-state s execs) (next-action s execs))) (next-exec s execs))
   (run n t execs2) u)
{
  fix t′
  assume t ′ = Some t′
  fix rs
  assume rs: run (Suc n) (Some s execs) = Some rs
  fix rt
  assume rt: run (Suc n) (Some t′) execs2 = Some rt
  assume vpeq-s-s′: ∀ v. ifp∗∗ v u → vpeq v s t′
  assume current-s-t: current s = current t′
  assume purged-a-a2: purged-relation u execs execs2

  — The following terminology is used: states rs and rt (for: run-s and run-t) are the states after a run. States ns
  and nt (for: next-s and next-t) are the states after one step.
  — We prove two properties: the states rs and rt have equal active domains (current-rs-rt) and are vpeq for all
  domains v that may influence u (vpeq-rs-rt). Both are proven using the IH. To use the IH, we have to prove that
  the properties hold for the next step (in this case, state s executes an action). Statement current-ns-nt states that
  after one step states ns and nt have the same active domain. Statement vpeq-ns-nt states that after one step states ns and
  nt are vpeq for all domains v that may influence u (vpeq-ns-rt).

  — Some lemma’s used in the remainder of this case.
  from ifp-reflexive and vpeq-s-s′ have vpeq-s-s′-u vpeq u s t′ by auto
  from step-atomicity and current-s-t current-next-state
  have current-ns-nt: current (step (next-state s execs) (next-action s execs)) = current (step (next-state t′
  execs2) (next-action t′ execs2))
  unfolding step-def
  by (cases next-action s execs,cases next-action t′ execs2,simp,simp,cases next-action t′ execs2,simp,simp)
  from vpeq-s-s′ have vpeq-curr-s-t: ifp∗∗ (current s) u → vpeq (current s) s t′ by auto
  from prec-s involved-ifp[THEN spec,THEN spec,where x1=next-state s execs and x=next-action s execs]
  vpeq-s-t have vpeq-involved: ifp∗∗ (current s) u → (∀ d ∈ involved (next-action s execs) . vpeq d s t′)
  using current-next-state
  unfolding involved-def pre-condition-def B-def
  by(cases next-action s execs,simp,auto,metis converse-rtranclp-into-rtranclp)
  from current-s-t next-exec-consistent vpeq-curr-s-t vpeq-involved
  have next-exec-s-t: ifp∗∗ (current s) u → next-exec s t′ execs = next-exec s execs
  unfolding next-exec-def
  by(auto)
  from current-s-t purged-a-a2 thread-not-empty-s next-exec-consistent[THEN spec,THEN spec,where x1=s
  and x=x′] vpeq-curr-s-s′ vpeq-involved
  have next-exec-s-t: ifp∗∗ (current s) u → next-exec s t′ execs2 = next-exec s execs
  by(unfold next-exec-def,unfold purged-relation-def,auto)
  from purged-a-a2 current-s-t next-exec-consistent[THEN spec,THEN spec,THEN spec,where x2=s and x1=t′
  and x=execs]
  vpeq-curr-s-s′ vpeq-involved
  have purged-na-na2: purged-relation u (next-exec s execs) (next-exec t′ execs2)
formal specification of a generic separation kernel

— The proof is by case distinction. If the current domain can interact with u (case curr-ifp-u), then either in state t the precondition holds (case t-prec) or not. If it holds, then lemma vpeq-ns-nt-ifp-u applies. Otherwise, the proof is trivial as the theorem holds vacuously. If the domain cannot interact with u, (case curr-not-ifp-u), then lemma vpeq-ns-nt-not-ifp-u applies.

have current-rs-rt: current rs = current rt

proof (cases ifp^** (current s) u rule :case-split[case-names curr-ifp-u curr-not-ifp-u])

case curr-ifp-u

show ?thesis

proof (cases precondition (next-state t' execs2) (next-action t' execs2) rule :case-split[case-names prec-t prec-not-t])

case prec-t

have thread-not-empty-t: ~thread-empty(execs2 (current t')) using thread-not-empty-t curr-ifp-u by auto

from
current-s-t next-execs-t next-action-s-t curr-ifp-u

have equivalent-states (Some (step (next-state s execs) (next-action s execs))) (Some (step (next-state t' execs2) (next-action t' execs2))) u

by auto

from prec-t thread-not-empty-t prec-s and this and not-interrupt and thread-not-empty-s and next-action-s-t

show ?thesis using rs rt by auto

next
case prec-not-t

from curr-ifp-u prec-not-t thread-not-empty-t not-interrupt show ?thesis using rt by simp

qed

next
case curr-not-ifp-u

show ?thesis

proof (cases thread-empty(execs2 (current t')) rule :case-split[case-names t-empty t-not-empty])

case t-not-empty

show ?thesis

proof (cases precondition (next-state t' execs2) (next-action t' execs2) rule :case-split[case-names t-prec t-not-prec])

case t-prec

from curr-not-ifp-u t-prec IH[where u = u and ?execs2.0 = (next-execs t' execs2) and t = Some (step (next-state t' execs2) (next-action t' execs2))]


current-ns-nt next-execs-t purged-na-na2 vpeq-ns-nt-not-ifp-u current-s-t vpeq-s-t prec-s purged-a-a2
have equivalent-states (run n (Some (step (next-state s execs) (next-action s execs))) (next-exec s execs))
(run n (Some (step (next-state t' execs2) (next-action t' execs2))) (next-exec t' execs2))

u by auto

from this t-prec curr-not-ifp-u t-not-empty prec-s not-interrupt thread-not-empty-s show ?thesis using rs
rt by auto

next case t-not-prec
from t-not-prec t-not-empty not-interrupt show ?thesis using rt by simp
qed

next case t-empty
from curr-not-ifp-u and prec-s and vpeq-s-t and locally-respects and vpeq-ns-t current-next-state

have vpeq-ns-t (∀ v. ifp ∗∗ v u → vpeq v (step (next-state s execs) (next-action s execs)) t')

by blast
from curr-not-ifp-u IH[where t=Some t' and u=u and ?execs2.0=?execs2] and current-ns-t and next-exec-t and
purged-na-a-a2 and vpeq-ns-t and this

have equivalent-states (run n (Some (step (next-state s execs) (next-action s execs))) (next-execs s execs))
(run n (Some (step (next-state t' execs2) (next-action t' execs2))) (next-exec t' execs2))

u by auto from this not-interrupt thread-not-empty-s t-empty prec-s show ?thesis using rs rt by auto

qed

fix v
assume ia: ifp ∗∗ v u

have vpeq v rs rt

proof (cases ifp ∗∗ (current s) u rule :case-split[case-names curr-ifp-u curr-not-ifp-u])
case curr-ifp-u
show ?thesis
proof (cases precondition (next-state t' execs2) (next-action t' execs2) rule :case-split[case-names t-prec t-not-prec])
case t-prec
have thread-not-empty-t ~thread-empty(execs2 (current t')) using thread-not-empty-t curr-ifp-u by auto
from current-ns-nt next-execs-t next-action-s-t purged-a-a2
curr-ifp-u t-prec prec-s vpeq-ns-nt-ifp-u[where a=(next-action s execs)] vpeq-s-t current-s-t
have equivalent-states (Some (step (next-state s execs) (next-action s execs))) (Some (step (next-state t' execs2) (next-action t' execs2)))

unfolding purged-relation-def next-state-def

by auto
from this IH[where u=u and ?execs2.0=(next-execs t' execs2) and t=Some (step (next-state t' execs2) (next-action t' execs2))]
current-ns-nt purged-na-na2
have equivalent-states (run n (Some (step (next-state s execs) (next-action s execs))) (next-execs s execs))
(run n (Some (step (next-state t' execs2) (next-action t' execs2))) (next-exec t' execs2))

by auto
from ia curr-ifp-u t-prec thread-not-empty-t prec-s and this and not-interrupt and thread-not-empty-s

and next-action-s-t
show ?thesis using rs rt by auto

next case t-not-prec
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from curr-ifp-u t-not-prec thread-not-empty-t not-interrupt show \(?thesis using\) rt by simp
qed

next
case curr-not-ifp-u
show \(?thesis

proof (cases thread-empty(execs2 (current t')) rule :case-split[case-names t-empty t-not-empty])
case t-not-empty
show \(?thesis

proof (cases precondition (next-state t' execs2) (next-action t' execs2) rule :case-split[case-names t-prec t-not-prec])
case t-prec

from curr-not-ifp-u t-prec IH
where u = u and \(?execs2.0 = (next-execs t' execs2) and t = Some (step (next-state t' execs2) (next-action t' execs2))\)
current-ns-nt next-execs-t purged-na-na2 vpeq-ns-nt-not-ifp-u current-s-t vpeq-s-t prec-s purged-a-a2
have equivalent-states (run n (Some (step (next-state s execs) (next-action s execs))) (next-execs s execs))

u by auto

from ia this t-prec curr-not-ifp-u t-not-empty prec-s not-interrupt thread-not-empty-s show \(?thesis using\) rs rt by simp
qed

next
case t-not-prec

from t-not-prec t-not-empty not-interrupt show \(?thesis using\) rt by simp
qed

next
case t-empty

from curr-not-ifp-u IH[where u = u and \(?execs2.0 = (next-execs t' execs2) and t = Some (step (next-state t' execs2) (next-action t' execs2))\)]
current-ns-nt next-execs-t purged-na-na2 vpeq-ns-nt-not-ifp-u current-s-t vpeq-s-t prec-s purged-a-a2
have equivalent-states (run n (Some (step (next-state s execs) (next-action s execs))) (next-execs s execs))

u by auto

from ia this t-prec curr-not-ifp-u t-not-empty prec-s not-interrupt thread-not-empty-s show \(?thesis using\) rs rt by auto
rs rt by auto

next
case t-not-prec

from t-not-prec t-not-empty not-interrupt show \(?thesis using\) rt by simp
qed

next
case t-empty

from curr-not-ifp-u IH[where t = Some t' and u = u and \(?execs2.0 = execs2\) and current-ns-t and next-execs-t
and purged-na-a and vpeq-ns-t and this]
have equivalent-states (run n (Some (step (next-state s execs) (next-action s execs))) (next-execs s execs))

(u by auto

from ia this not-interrupt thread-not-empty-s t-empty prec-s show \(?thesis using\) rs rt by auto
qid

qed

}\)
from current-rs-rt and this have equivalent-states (Some rs) (Some rt) u by auto
}
thus \(?case by(simp add:option.splits,cases t,simp+)\)
qed
\}
thus \(?thesis

unfolding view-partitioned-ind-def by auto
qed

From the previous lemma, we can prove that the system is view partitioned. The previous lemma was inductive, this lemma just instantiates the previous lemma replacing s and t by the initial state.

lemma unwinding-implies-view-partitioned:
shows view-partitioned
proof-
from assms unwinding-implies-view-partitioned-ind have view-partitioned-inductive: view-partitioned-ind
by blast
have purged-relation: \(\forall u execs . purged-relation u execs \) (purge execs u)
by(unfold purged-relation-def, unfold purge-def, auto)
{ fix execs s t n u
assume I: equivalent-states s t u
from this view-partitioned-inductive purged-relation
have equivalent-states (run n s execs) (run n t (purge execs u)) u
unfolding view-partitioned-ind-def by auto
from this ifp-reflexive
have run n s execs ∥ run n t (purge execs u) ⇔ (∀ rs rt vpeq u rs rt ∧ current rs = current rt)
unfolding view-partitioned-def by auto
have run n s execs ∥ run n t (purge execs u) ⇔ (∀ rs rt vpeq u rs rt ∧ current rs = current rt)
unfolding view-partitioned-def by auto
from this run n s execs, simp, cases run n t (purge execs u), simp, auto)
thus ?thesis unfolding view-partitioned-def Let-def by auto
qed

Domains that many not interfere with each other, do not interfere with each other.

theorem unwinding-implies-NI-unrelated:
shows NI-unrelated
proof-
{ 
  fix execs a n
  from assms unwinding-implies-view-partitioned
  have vp: view-partitioned by blast
  from vp and vpeq-reflexive
  have I: ∀ u . (run n ( Some s0) execs ∥ run n ( Some s0) (purge execs u)
    ⇔ (∀ rs rt vpeq u rs rt ∧ current rs = current rt))
  unfolding view-partitioned-def by auto
  have run n ( Some s0) execs → (∀ rs-f . run n ( Some s0) (purge execs (current s-f)) → (∀ s-f2 . output-f s-f a = output-f s-f2 a ∧ current s-f = current s-f2))
  proof(cases run n ( Some s0) execs)
  case None
  thus ?thesis unfolding B-def by simp
  next
  case ( Some rs)
  thus ?thesis
  proof(cases run n ( Some s0) (purge execs (current rs)))
  case None
  from Some this show ?thesis unfolding B-def by simp
  next
  case ( Some rt)
  from run n ( Some s0) execs = Some rs Some I[THEN spec ,where x=current rs]
  have vpeq: vpeq (current rs) rs rt ∧ current rs = current rt
  unfolding B-def by auto
  from this output-consistent have output-f rs a = output-f rt a
  by auto
  from this vpeq (run n ( Some s0) execs = Some rs Some
  show ?thesis unfolding B-def by auto
  qed
  qed
}
thus ?thesis unfolding NI-unrelated-def by auto
qed

3.2.2 Security for indirectly interfering domains

Consider the following security policy over three domains A, B and C: A ~ B ~ C, but A / C. The semantics of this policy is that A may communicate with C, but only via B. No direct communication from A to C is allowed. We formalize these semantics as follows: without intermediate domain B, domain A cannot flow information to C. In other words, from the point of view of domain C the run
where domain $B$ is inactive must be equivalent to the run where domain $B$ is inactive and domain $A$ is replaced by an attacker. Domain $C$ must be independent of domain $A$, when domain $B$ is inactive.

The aim of this subsection is to formalize the semantics where $A$ can write to $C$ via $B$ only. We define to two ipurge functions. The first purges all domains $d$ that are intermediary for some other domain $v$. An intermediary for $u$ is defined as a domain $d$ for which there exists an information flow from some domain $v$ to $u$ via $d$, but no direct information flow from $v$ to $u$ is allowed.

definition intermediary \(\equiv \text{dom-t} \Rightarrow \text{dom-t} \Rightarrow \text{bool}\)
where intermediary \(d \equiv v . \text{ifp} \equiv v \land d \land \text{ifp} d u \land \neg \text{ifp} v u \land d \neq u\)
primrec remove-gateway-communications \(\equiv \text{dom-t} \Rightarrow \text{action-t execution} \Rightarrow \text{action-t execution}\)
where remove-gateway-communications \(u \equiv []\)
removel Gateway-communications \(u \equiv \lambda \text{execs} u.\) (remove-gateway-communications \(u \equiv \) involved \(\lambda \text{execs} u\) \(\equiv \) \((\lambda \exists a \in \text{set aseq} \cdot \exists v . \text{intermediary} v u \land v \in \text{involved} (\text{Some} a) \text{ then } []\) \(\lambda \text{else} \) aseq \(\lambda \text{else})\) 

\(\lambda \text{execs} u\)

The second ipurge removes both the intermediaries and the indirect sources. An indirect source for $u$ is defined as a domain that may indirectly flow information to $u$, but not directly.

abbreviation ind-source \(\equiv \text{dom-t} \Rightarrow \text{dom-t} \Rightarrow \text{bool}\)
where ind-source \(d u \equiv \text{ifp} d u \land \neg \text{ifp} d u\)

definition ipurge-l ::
\(\text{dom-t} \Rightarrow \text{action-t execution} \Rightarrow \text{dom-t} \Rightarrow (\text{dom-t} \Rightarrow \text{action-t execution})\)
where
ipurge-l \(\equiv \lambda \text{execs} u. \) if intermediary \(d u\) then
\[
\begin{cases}
\[] & \text{else if } d = u \text{ then} \\
\text{remove-gateway-communications } u (\text{execs} u) & \text{else } \text{execs} d
\end{cases}
\]

The second ipurge removes both the intermediaries and the indirect sources. An indirect source for $u$ is defined as a domain that may indirectly flow information to $u$, but not directly.

For a system with an intransitive policy to be called secure for domain $u$ any indirect source may not flow information towards $u$ when the intermediaries are purged out. This definition of security allows the information flow $A \rightarrow B \rightarrow C$, but prohibits $A \rightarrow C$.

definition NI-indirect-sources :: bool
where NI-indirect-sources
\(\equiv \forall \text{execs} a n . \text{run} n (\text{Some} s0) \text{execs} \rightarrow (\lambda s \cdot \text{run} n (\text{Some} s0) (\text{ipurge-l execs} (\text{current} s-f)) \rightarrow \text{run} n (\text{Some} s0) (\text{ipurge-r execs} (\text{current} s-f)) \rightarrow (\lambda s \cdot \text{output-f} s l a = \text{output-f} s r a))\)

This definition concerns indirect sources only. It does not enforce that an unrelated domain may not flow information to $u$. This is expressed by “secure”.

This allows us to define security over intransitive policies.

definition isecure :: bool
where isecure \(\equiv \) NI-indirect-sources \(\land\) NI-unrelated

abbreviation equivalent-states \(\equiv \text{state-t option} \Rightarrow \text{state-t option} \Rightarrow \text{dom-t} \Rightarrow \text{bool}\)
where equivalent-states \(s t u \equiv s \parallel t \rightarrow (\lambda s t . (\forall v . \text{ifp} v u \land \neg \text{intermediary} v u \Rightarrow \text{vpeq} v s t) \land \text{current} s = \text{current} t)\)
**Definition**  
**does-not-communicate-with-gateway**  

**Where**  
**does-not-communicate-with-gateway** \( u \text{ execs} \equiv \forall a . a \in \text{actions-in-execution} (\text{execs} u) \rightarrow (\forall v . \text{ intermediary } v u \rightarrow v \notin \text{involved} \ (\text{Some } a)) \)

**Definition**  
**iview-partitioned** \( \land \)  

**Where**  
**iview-partitioned** \( \equiv \forall \text{ execs } ms mt n u . \text{iequivalent-states } ms mt u \rightarrow (\forall v . \text{intermediary } v u \rightarrow v \notin \text{involved} (\text{next-action } s \text{ execs})) \)

**Definition**  
**ipurged-relation1** \( \land \)  

**Where**  
**ipurged-relation1** \( u \text{ execs1 execs2} \equiv \forall d . (\text{ifp } d u \rightarrow \text{execs1 } d = \text{execs2 } d) \land (\text{intermediary } d u \rightarrow \text{vpeq } d s t) \)

Proof that if the current is not an intermediary for \( u \), then all domains involved in the next action are \( \text{vpeq} \).

**Lemma**  
**vpeq-involved-domains**  

**Assumes**  
**ifp-curr** : \( \text{ifp } (\text{current } s) u \land \text{not-intermediary-curr} : \neg\text{intermediary } (\text{current } s) u \land \text{no-gateway-comm} : \text{does-not-communicate-with-gateway } u \text{ execs} \land \text{vpeq-s-t} : \forall v . \text{ifp } v u \land \neg\text{intermediary } v u \rightarrow \text{vpeq } v s t' \land \text{prec-s precondition } (\text{next-state } s \text{ execs}) \land (\text{next-action } s \text{ execs}) \)

**Shows**  
\( \forall d \in \text{involved} \ (\text{next-action } s \text{ execs}) \land \text{vpeq } d s t' \)

**Proof**  

- **Fix** \( v \)
  - **Assume** \( \text{involved} : v \in \text{involved} \ (\text{next-action } s \text{ execs}) \)
  - **From** \( \text{this prec-s involved-ifp[THEN spec,THEN spec,where } x1=\text{next-state } s \text{ execs and } x=\text{next-action } s \text{ execs]} \)
    - **Have** \( \text{ifp-v-curr: ifp } v \ (\text{current } s) \land \text{using current-next-state using unfolding involved-def precondition-def B-def by (cases next-action s execs,auto)} \)
    - **Have** \( \text{vpeq } v s t' \)
  - **Proof**  
    - **Assume** \( \text{ifp } v u \land \neg\text{intermediary } v u \)
    - **From** \( \text{this vpeq-s-t} \)
      - **Have** \( \text{vpeq } v s t' \) **by** \( (\text{auto}) \)
    - **Moreover**  
      - **Assume** \( \text{not-intermediary-v: intermediary } v u \)
      - **From** \( \text{ifp-curr not-intermediary-curr ifp-v-curr not-intermediary-v have curr-is-u current } s = u \land \text{using rtrancp-trans r-into-rtrancp by (metis intermediary-def)} \)
        - **From** \( \text{curr-is-u next-action-from-execs[THEN spec,THEN spec,where } x=\text{execs and } x1=s] \text{ not-intermediary-v involved no-gateway-comm[unfolded does-not-communicate-with-gateway-def,THEN spec,where } x=\text{the (next-action s execs)} \)] \)
          - **Have** \( \text{False} \land \text{unfolding involved-def B-def by (cases next-action s execs,auto) hence vpeq } v s t' \) **by** \( (\text{auto}) \)
      - **Moreover**  
        - **{}**
assume intermediary-v \:\{¬ \text{ifp } v \text{ u} \}
from \text{ifp-curr } \text{not-intermediary-curr } \text{ifp-v-curr intermediary-v}
  \text{have False unfolding intermediary-def } \text{by auto}
  \text{hence } vpeq v s t' \text{ by auto}
\}
ultimately
\text{show } vpeq v s t' \text{ unfolding intermediary-def } \text{by auto}
\text{qed}
\}
\text{thus } ?\text{thesis by auto}
\text{qed}

Proof that purging removes communications of the gateway to domain u.

\begin{verbatim}
lemma ipurge-l-removes-gateway-communications:
shows does-not-communicate-with-gateway u (ipurge-l execs u)
proof--
{ fix aseq u execs a v
  assume 1: aseq \in \text{set (remove-gateway-communications u \text{ execs u})}
  assume 2: a \in \text{set aseq}
  assume 3: intermediary v u
  have 4: v \notin \text{involved (Some a)}
  proof--
  { fix a: 'action-t
     fix aseq u exec v
     have aseq \in \text{set (remove-gateway-communications u exec)} \land a \in \text{set aseq} \land intermediary v u \rightarrow v \notin \text{involved}
       (Some a)
       by (induct exec:auto)
  }
  from 1 2 3 this show ?thesis by metis
  qed
}
from this
show ?thesis
\text{unfolding does-not-communicate-with-gateway-def ipurge-l-def actions-in-execution-def}
\text{by auto}
\text{qed}
\end{verbatim}

Proof of view partitioning. The lemma is structured exactly as lemma unwinding_implies_view_partitioned_ind
and uses the same convention for naming.

\begin{verbatim}
lemma iunwinding_implies_view_partitioned1:
shows iview-partitioned
proof--
{ fix u execs execs2 s t n
  have does-not-communicate-with-gateway u execs \land iequivalent-states s t u \land ipurged-relation1 u execs execs2 \rightarrow iequivalent-states (run n s execs) (run n t execs2) u
  proof (induct n s execs arbitrary: t u execs2 rule: run.induct)
  case (1 s execs t u execs2)
  show ?case by auto
  next
  case (2 n execs t u execs2)
  show ?case by simp
  next
  case (3 n s execs t u execs2)
  assume interrupt-s: interrupt (Suc n)
  assume IH: (\land t u execs2, does-not-communicate-with-gateway u execs \land
}\end{verbatim}
\[
\text{iequivalent-states} \ (\text{Some } (\text{cswitch } (\text{Suc } n ) \ s )) \ t \ u \land \text{ipurged-relation1 } u \ \text{execs} \ \text{execs2} \rightarrow \\
\text{iequivalent-states} \ (\text{run } n \ (\text{Some } (\text{cswitch } (\text{Suc } n ) \ s )) \ \text{execs}) \ (\text{run } n \ t \ \text{execs2}) \ u \\
\]

\{ \\
\text{fix } t' = '\text{state-t} \\
\text{assume } t = \text{Some } t' \\
\text{fix } rs \\
\text{assume } rs: \text{run } (\text{Suc } n ) \ (\text{Some } s ) \ \text{execs} = \text{Some } rs \\
\text{fix } rt \\
\text{assume } rt: \text{run } (\text{Suc } n ) \ (\text{Some } t' ) \ \text{execs2} = \text{Some } rt \\
\}

\text{assume } \text{no-gateway-comm: does-not-communicate-with-gateway } u \ \text{execs} \\
\text{assume } \text{vpeq-s-t: } \forall \ v . \ \text{ifp } v \ u \land \neg \text{intermediary } v \ u \rightarrow \ \text{vpeq } v \ s \ t' \\
\text{assume } \text{current-s-t: current } s = \text{current } t' \\
\text{assume } \text{purged-a-a2: ipurged-relation1 } u \ \text{execs} \ \text{execs2} \\
\]

\text{from } \text{current-s-t cswitch-independent-of-state} \\
\text{have } \text{current-ns-nt: current } (\text{cswitch } (\text{Suc } n ) \ s ) = \text{current } (\text{cswitch } (\text{Suc } n ) \ t' ) \\
\text{by blast} \\
\text{from } \text{cswitch-consistency vpeq-s-t} \\
\text{have } \text{vpeq-ns-nt: } \forall \ v . \ \text{ifp } v \ u \land \neg \text{intermediary } v \ u \rightarrow \ \text{vpeq } v \ (\text{cswitch } (\text{Suc } n ) \ s ) \ (\text{cswitch } (\text{Suc } n ) \ t' ) \\
\text{by auto} \\
\text{from } \text{no-gateway-comm current-ns-nt vpeq-ns-nt interrupt-s vpeq-reflexive current-s-t purged-a-a2 IH [where } u=u \ \text{and } t=\text{Some } (\text{cswitch } (\text{Suc } n ) \ t' ) \ \text{and } ?\text{execs2.0=execs2]} \\
\text{have } \text{current-ss-rt: current } rs = \text{current } rt \ \text{using } rs \ rt \text{ by(auto)} \\
\}

\text{fix } v \\
\text{assume } ia: \ \text{ifp } v \ u \land \neg \text{intermediary } v \ u \\
\text{from } \text{no-gateway-comm interrupt-s current-ns-nt vpeq-ns-nt vpeq-reflexive ia current-s-t purged-a-a2 IH [where } u=u \ \text{and } t=\text{Some } (\text{cswitch } (\text{Suc } n ) \ t' ) \ \text{and } ?\text{execs2.0=execs2]} \\
\text{have } \text{vpeq } v \ rs \ rt \ \text{using } rs \ rt \text{ by(auto)} \\
\}

\text{from } \text{current-ss-rt and this have iequivalent-states (Some } rs \) (\text{Some } rt ) \ u \ \text{by auto} \\
\}

\text{thus } ?\text{case by}(\text{simp add:option.splits,cases t,simp+}) \\
\text{next} \\
\text{case } (4 \ n \ \text{execs } s \ u \ \text{execs2}) \\
\text{assume } \text{not-interrupt: } \neg \text{interrupt } (\text{Suc } n ) \\
\text{assume } \text{thread-empty-s: thread-empty(} \text{execs } (\text{current } s )) \\
\]

\text{assume IH: } (\forall \ t \ \text{execs2. does-not-communicate-with-gateway } u \ \text{execs} \land \ \text{iequivalent-states } (\text{Some } s ) \ t \ u \land \ \text{ipurged-relation1 } u \ \text{execs} \ \text{execs2} \rightarrow \ \text{iequivalent-states} \ (\text{run } n \ (\text{Some } s ) \ \text{execs}) \ (\text{run } n \ t \ \text{execs2}) \ u \\
\}

\text{fix } t' \\

\text{assume } t: t = \text{Some } t' \\
\text{fix } rs \\
\text{assume } rs: \text{run } (\text{Suc } n ) \ (\text{Some } s ) \ \text{execs} = \text{Some } rs \\
\text{fix } rt \\
\text{assume } rt: \text{run } (\text{Suc } n ) \ (\text{Some } t' ) \ \text{execs2} = \text{Some } rt \\
\]

\text{assume } \text{no-gateway-comm: does-not-communicate-with-gateway } u \ \text{execs} \\
\text{assume } \text{vpeq-s-t: } \forall \ v . \ \text{ifp } v \ u \land \neg \text{intermediary } v \ u \rightarrow \ \text{vpeq } v \ s \ t' \\
\text{assume } \text{current-s-t: current } s = \text{current } t' \\
\text{assume } \text{purged-a-a2: ipurged-relation1 } u \ \text{execs} \ \text{execs2} \\
\]

\text{from } \text{ifp-reflexive vpeq-s-t have vpeq-u-s-t: vpeq } u \ s \ t' \text{ unfolding intermediary-def by auto} \\
\text{from } \text{step-atomicity current-next-state current-s-t have current-s-nt: current } s = \text{current } (\text{step } (\text{next-state } t' 

execs2) (next-action t' execs2))

unfolding step-def
   by (cases next-action s execs, cases next-action t' execs2, simp simp, cases next-action t' execs2, simp simp)
from vpeq-s-t have vpeq-curr-s-t: ifp (current s) u \and \neg-intermediary (current s) u \rightarrow vpeq (current s) s t' by auto
   have iequivalent-states (run (Suc n) (Some s) execs) (run (Suc n) (Some t') execs2) u
   proof (cases thread-empty (execs2 (current t'))) 
   case True
      from purged-a-a2 and vpeq-s-t and current-s-t IH [where t=Some t' and u=u and ?execs2.0=execs2] 
      no-gateway-comm
      have iequivalent-states (run n (Some s) execs) (run n (Some t') execs2) u using rs rt by (auto)
      from this not-interrupt True thread-empty-s
      show ?thesis using rs rt by (simp add:option.splits)
     qed

next
   case False
   have prec-t precondition (next-state t' execs2) (next-action t' execs2)
   proof
      { 
      assume not-prec-t: \neg-precondition (next-state t' execs2) (next-action t' execs2)
      hence run (Suc n) (Some t') execs2 = None using not-interrupt False not-prec-t by (simp)
      from this have False using rt by (simp add: option.splits) 
      }
      thus ?thesis by auto
     qed

from False purged-a-a2 thread-empty-s current-s-t
have I: ind-source (current t') u \or unrelated (current t') u unfolding ipurged-relation1-def intermediary-def 
by auto 
   
   { 
   fix v
   assume ifp-v: ifp v u
   assume v-not-intermediary: \neg-intermediary v u

   from I ifp-v v-not-intermediary have not-ifp-curr-v: \neg-ifp (current t') v unfolding intermediary-def by auto
   from not-ifp-curr-v prec-t locally-respects[THEN spec, THEN spec, THEN spec, where x1=next-state t' execs2 and x=v and x2=the (next-action t' execs2)] current-next-state vpeq-reflexive
   have vpeq v (next-state t' execs2) (step (next-state t' execs2) (next-action t' execs2))
   unfolding step-def precondition-def B-def
   by (cases next-action t' execs2, auto)
   from this vpeq-transitive not-ifp-curr-v locally-respects next-state
   have vpeq-t-nt: vpeq v t' (step (next-state t' execs2) (next-action t' execs2))
   by blast
   from vpeq-s-t ifp v v-not-intermediary vpeq-t-nt vpeq-transitive vpeq-symmetric vpeq-reflexive
   have vpeq v s (step (next-state t' execs2) (next-action t' execs2))
   by (metis)
   }
   hence vpeq-ns-nt: \forall v . ifp v u \and \neg-intermediary v u \rightarrow vpeq v s (step (next-state t' execs2) (next-action t' execs2)) by auto
   from False purged-a-a2 current-s-t thread-empty-s have purged-a-na2: ipurged-relation1 u execs (next-execs t' execs2)
   unfolding ipurged-relation1-def next-execs-def by (auto)
   from vpeq-ns-nt no-gateway-comm
   and IH [where t=Some (step (next-state t' execs2) (next-action t' execs2)) and ?execs2.0=(next-execs t' execs2) and u=u]
   and current-s-nt purged-a-na2
   have eq-ns-nt: iequivalent-states (run n (Some s) execs)
\begin{align*}
\text{(run } n \ (\text{Some } \text{(step } (\text{next-state } t' \ \text{execs2}) \ (\text{next-action } t' \ \text{execs2})\text{)} \ (\text{next-exec } t' \ \text{execs2})) \text{)} \ \text{by auto}
\end{align*}

\text{from prect-nt eq-ns-nt not-interrupt False thread-empty-s}
\text{show ?thesis using } t \ rs \ rt \ \text{by(auto)}
\text{qed}
\text{thus ?case by(simp add:option.splits,cases } t,\text{simp+)}
\text{next}
\text{case } (5 \ n \ \text{execs } s \ t \ u \ \text{execs2})
\text{ assume not-interrupt: } \neg\text{-interrupt } (\text{Suc } n)
\text{ assume thread-not-empty-s: } \neg\text{-thread-empty} (\text{execs } (\text{current } s))
\text{ assume not-prec-s: } \neg\text{-precondition } (\text{next-state } s \ \text{execs}) \ (\text{next-action } s \ \text{execs})
\text{ hence run } (\text{Suc } n) \ (\text{Some } s) \ \text{execs } = \text{None using not-interrupt thread-not-empty-s by simp}
\text{thus ?case by(simp add:option.splits)}
\text{next}
\text{case } (6 \ n \ \text{execs } s \ t \ u \ \text{execs2})
\text{ assume not-interrupt: } \neg\text{-interrupt } (\text{Suc } n)
\text{ assume thread-not-empty-s: } \neg\text{-thread-empty} (\text{execs } (\text{current } s))
\text{ assume prec-s: precondition } (\text{next-state } s \ \text{execs}) \ (\text{next-action } s \ \text{execs})
\text{ assume IH: } (\forall u \ \text{execs2}, \text{does-not-communicate-with-gateway } u \ (\text{next-exec } s \ \text{execs}) \ \land
\text{iequivalent-states } (\text{Some } (\text{step } (\text{next-state } s \ \text{execs}) \ (\text{next-action } s \ \text{execs}))) \ t \ u \ \land
\text{ipurged-relation1 } u \ (\text{next-exec } s \ \text{execs}) \ \text{execs2 } \ \rightarrow
\text{iequivalent-states}
\text{(run } n \ (\text{Some } \text{(step } (\text{next-state } s \ \text{execs}) \ (\text{next-action } s \ \text{execs})) \ (\text{next-exec } s \ \text{execs})) \ (\text{next-exec } t \ \text{execs2}) \ u)
\text{\{}
\text{fix } t'
\text{ assume } t: t' = \text{Some } t'
\text{ fix } rs
\text{ assume } rs: \text{run } (\text{Suc } n) \ (\text{Some } s) \ \text{execs } = \text{Some } rs
\text{ fix } rt
\text{ assume } rt: \text{run } (\text{Suc } n) \ (\text{Some } t') \ \text{execs2 } = \text{Some } rt
\text{ assume no-gateway-comm: } \text{does-not-communicate-with-gateway } u \ \text{execs}
\text{ assume vpeq-s-t: } \forall v. \ \text{ifp } v \ u \ \land \neg\text{-intermediary } v \ u \ \rightarrow \ vpeq v s t'
\text{ assume current-s-t: current } s = \text{current } t'
\text{ assume purged-a-a2: ipurged-relation1 } u \ \text{execs} \ \text{execs2}
\text{ from ifp-reflexive vpeq-s-t have vpeq-u-s-t: vpeq } u \ s t' \ \text{unfolding intermediary-def by auto}
\text{ from step-atomicity and current-s-t current-next-state}
\text{ have current-ns-nt: current } (\text{step } (\text{next-state } s \ \text{execs}) \ (\text{next-action } s \ \text{execs})) = \text{current } (\text{step } (\text{next-state } t' \ \text{execs2}) \ (\text{next-action } t' \ \text{execs2}))
\text{ unfolding step-def}
\text{ by } (\text{cases next-action } s \ \text{execs}, \text{cases next-action } t' \ \text{execs2}, \text{simp}, \text{simp}, \text{cases next-action } t' \ \text{execs2}, \text{simp}, \text{simp})
\text{ from step-atomicity current-next-state current-s-t have current-ns-t: current } (\text{step } (\text{next-state } s \ \text{execs}) \ (\text{next-action } s \ \text{execs})) = \text{current } t'
\text{ unfolding step-def}
\text{ by } (\text{cases next-action } s \ \text{execs}, \text{auto})
\text{ from vpeq-s-t have vpeq-curr-s-t: ifp } (\text{current } s) \ u \ \land \neg\text{-intermediary } (\text{current } s) \ u \ \rightarrow \ vpeq (\text{current } s) s t'
\text{ unfolding intermediary-def by auto}
\text{ from current-s-t purged-a-a2}
\text{ have eq-execs ifp } (\text{current } s) \ u \ \land \neg\text{-intermediary } (\text{current } s) \ u \ \rightarrow \ \text{execs } (\text{current } s) = \text{execs2 } (\text{current } s)
\text{ by(auto simp add: ipurged-relation1-def)}
\text{ from vpeq-involved-domains no-gateway-comm vpeq-s-t vpeq-involved-domains prec-s}
\text{ have vpeq-involved: ifp } (\text{current } s) \ u \ \land \neg\text{-intermediary } (\text{current } s) \ u \ \rightarrow \ (\forall d \ \text{involved } (\text{next-action } s \ \text{execs}) \cdot \ vpeq d s t')
by blast
from current-s-t next-execs-consistent[THEN spec, THEN spec, THEN spec, where \( s2 = s \) and \( x1 = t' \) and \( x = \text{execs} \)]
vpeq-curr-s-t vpeq-involved
have next-execs-t: \( \text{ifp} \) (current s) \( u \land \neg \text{intermediary} \) (current s) \( u \longrightarrow \text{next-execs} \ t' \) execs = next-execs s execs
by (auto simp add: next-execs-def)
from current-s-t and purged-a-a2 and thread-not-empty-s next-action-consistent[THEN spec, THEN spec, where \( x1 = s \) and \( x = t' \)]
vpeq-curr-s-t vpeq-involved
have next-action-s-t: \( \text{ifp} \) (current s) \( u \land \neg \text{intermediary} \) (current s) \( u \longrightarrow \text{next-action} \ t' \) execs2 = next-action s execs
by (unfold next-action-def, unfold ipurged-relation1-def, auto)
from purged-a-a2 and thread-not-empty-s and current-s-t
have thread-not-empty-t: \( \text{ifp} \) (current s) \( u \land \neg \text{intermediary} \) (current s) \( u \longrightarrow \neg \text{thread-empty}(\text{execs}2 \ (\text{current} \ t'))
unfolding ipurged-relation1-def by auto
have vpeq-ns-nt-1: \( \land a \). precondition (next-state s execs) \( a \land \text{precondition} \) (next-state t' execs) \( a \longrightarrow \text{ifp} \) (current s) \( u \land \neg \text{intermediary} \) (current s) \( u \longrightarrow (\forall v. \text{ifp} \ v u \land \neg \text{intermediary} \ v u \longrightarrow \text{vpeq} v \ (\text{step} \ \text{next-state} \ s \ \text{execs}) \ a) \) (step \ (next-state t' execs) a)
proof
fix a
assume prec: precondition (next-state s execs) \( a \land \text{precondition} \) (next-state t' execs) \( a \)
assume ifp-curr: \( \text{ifp} \) (current s) \( u \land \neg \text{intermediary} \) (current s) \( u \)
from ifp-curr
next-state-consistent[THEN spec, THEN spec, where \( x1 = s \) and \( x = t' \)]
vpeq-curr-s-t vpeq-s-t
current-next-state current-s-t weakly-step-consistent[THEN spec, THEN spec, THEN spec, THEN spec, where \( x3 = \text{next-state} \ s \ \text{execs} \) and \( x2 = \text{next-state} \ t' \ \text{execs} \) and \( x = \text{the} \ a \)]
show \( \forall v. \text{ifp} \ v u \land \neg \text{intermediary} \ v u \longrightarrow \text{vpeq} v \ (\text{step} \ \text{next-state} \ s \ \text{execs}) \ a) \) (step \ (next-state t' execs) a)
unfolding step-def precondition-def B-def
by (cases a auto)
qed
have no-gateway-comm-na: does-not-communicate-with-gateway u (next-execs s execs)
proof
{ 
  fix a
  assume a ∈ actions-in-execution (next-execs s execs u)
  from this no-gateway-comm[unfolded does-not-communicate-with-gateway-def, THEN spec, where \( x = a \)]
  next-execs-subset[THEN spec, THEN spec, THEN spec, where \( x2 = s \) and \( x1 = \text{execs} \) and \( x0 = u \)]
  have \( \forall v. \text{intermediary} \ v u \longrightarrow v \notin \text{involved} \ (\text{Some} \ a) \)
  unfolding actions-in-execution-def
  by (auto)
}
thus \( \text{thesis} \) unfolding does-not-communicate-with-gateway-def by auto
qed
have iequivalent-states (run (Suc n) (Some s) execs) (run (Suc n) (Some t') execs2) u
proof (cases \( \text{ifp} \) (current s) \( u \land \neg \text{intermediary} \) (current s) \( u \) rule : case-split[case-names T F])
case T
  show \( \text{thesis} \)
  proof (cases thread-empty(execs2 \ (current \ t')) rule : case-split[case-names T2 F2])
case F2
    show \( \text{thesis} \)
    proof (cases precondition (next-state t' execs2) \ (next-action t' execs2) rule : case-split[case-names T3 F3])
case T3
  from T purged-a-a2 current-s-t
  next-execs-consistent[THEN spec, THEN spec, where \( x1 = s \) and \( x = t' \)]
vpeq-curr-s-t vpeq-involved
  have purged-na-na2: ipurged-relation1 u (next-execs s execs) (next-execs t' execs2)
  unfolding ipurged-relation1-def next-execs-def
  by auto
from IH \[ where \quad t = \text{Some} \left( \text{step} \left( \text{next-state} \ t' \ \text{execs2} \right) \ (\text{next-action} \ t' \ \text{execs2}) \right) \quad \text{and} \quad \text{?execs2}.0 = \text{next-exec} \ t' \ \text{execs2} \quad \text{and} \quad \text{u} = u \] 
purged-na-na2 current-ns-nt vpeq-ns-nt-1 \[ where \quad a = (\text{next-action} \ s \ \text{execs}) \] 
next-action-s-t eq-execs current-s-t no-gateway-comm-na 
have eq-ns-nt \equiv \text{equivalent-states} \ (\text{run} \ n \ (\text{Some} \ (\text{step} \ (\text{next-state} \ s \ \text{execs}) \ (\text{next-action} \ s \ \text{execs}))) \ (\text{next-exec} \ s \ \text{execs}) \ (\text{run} \ n \ (\text{Some} \ (\text{step} \ (\text{next-state} \ t' \ \text{execs2}) \ (\text{next-action} \ t' \ \text{execs2}))) \ (\text{next-exec} \ t' \ \text{execs2}))) \ u 
unfolding next-state-def 
by (auto, metis) 
from this \text{not-interrupt} \ \text{thread-not-empty-s} \ \text{prec-s} \ F2 \ T3 
have current-rs-rt \equiv \text{current} \ rs = \text{current} \ rt \ \text{using} \ rs \ rt \ \text{by auto} 
\{ 
\text{fix v} 
assume ia : \text{ifp} \ v \ \text{u} \ \wedge \ \neg \text{intermediary} \ v \ \text{u} 
from this eq-ns-nt \ \text{not-interrupt} \ \text{thread-not-empty-s} \ \text{prec-s} \ F2 \ T3 
\text{have vpeq v rt using rs rt by auto} 
\} 
from this and current-rs-rt \ ?\text{thesis} \ \text{using} \ rs \ rt \ \text{by auto} 
next case \ F3 
from \ F3 \ F2 \ \text{not-interrupt} \ \text{show} \ ?\text{thesis} \ \text{using} \ rt \ \text{by simp} 
qed 
next case \ T2 
from T2 \ \text{purged-a-a2} \ \text{thread-not-empty-s} \ \text{current-s-t} \ \text{prec-s} \ \text{next-action-s-t} \ \text{vpeq-u-s-t} 
\text{have ind-source} \ \text{False} \ \text{unfolding} \ \text{ipurged-relation1-def} \ \text{by auto} 
\text{thus} \ ?\text{thesis} \ \text{by auto} 
qed 
next case \ F 
\text{hence} \ 1: \ \text{ind-source} \ (\text{current} \ s) \ \text{u} \ \lor \ \text{unrelated} \ (\text{current} \ s) \ \text{u} \ \lor \ \text{intermediary} \ (\text{current} \ s) \ \text{u} 
\text{unfolding} \ \text{intermediary-def} 
\text{by auto} 
\text{from} \ \text{purged-a-a2} \ \text{and} \ \text{thread-not-empty-s} 
\text{have} \ 2: \ \neg \text{intermediary} \ (\text{current} \ s) \ \text{u} \ \text{unfolding} \ \text{ipurged-relation1-def} \ \text{by auto} 
\text{let} \ ?nt = \text{if thread-empty} \ (\text{execs2} \ (\text{current} \ t')) \ \text{then} \ t' \ \text{else} \ \text{step} \ (\text{next-state} \ t' \ \text{execs2}) \ (\text{next-action} \ t' \ \text{execs2}) 
\text{let} \ ?na2 = \text{if thread-empty} \ (\text{execs2} \ (\text{current} \ t')) \ \text{then} \ \text{execs2} \ \text{else} \ \text{next-exec} \ t' \ \text{execs2} 
\text{have prec-t:} \ \neg \text{thread-empty} \ (\text{execs2} \ (\text{current} \ t')) \ \Longrightarrow \ \text{precondition} \ (\text{next-state} \ t' \ \text{execs2}) \ (\text{next-action} \ t' \ \text{execs2}) 
proof 
\text{assume} \ \text{thread-not-empty-t:} \ \neg \text{thread-empty} \ (\text{execs2} \ (\text{current} \ t')) 
\{ 
\text{assume} \ \text{not-prec-t:} \ \neg \text{precondition} \ (\text{next-state} \ t' \ \text{execs2}) \ (\text{next-action} \ t' \ \text{execs2}) 
\text{hence} \ \text{run} \ (\text{Suc} \ n) \ (\text{Some} \ t') \ \text{execs2} = \text{None} \ \text{using} \ \text{not-interrupt} \ \text{thread-not-empty-t} \ \text{not-prec-t by (simp)} 
\text{from this have False using rt by (simp add: option.splits)} 
\} 
\text{thus} \ ?\text{thesis} \ \text{by auto} 
qed 

show ?\text{thesis} 
proof 
\{ 
\text{fix v} 
\text{assume ifp-v: ifp} \ v \ \text{u}
assume v-not-intermediary: \( \neg \text{intermediary } v u \)

have not-ifp-curr-v: \( \neg \text{ifp (current } s \text{) } v \)

proof
  assume ifp-curr-v: \( \text{ifp (current } s \text{) } v \)
  thus False
  proof
  \{ 
  assume ind-source (current } s \text{) } u 
  from this ifp-curr-v ifp-v have intermediary \( v u \) unfolding intermediary-def by auto 
  from this v-not-intermediary have False unfolding intermediary-def by auto 
  \}
  moreover 
  \{ 
  assume unrelated: unrelated (current } s \text{) } u 
  from this ifp-v ifp-curr-v have False using rtranclp-trans r-into-rtranclp by metis 
  \}
  ultimately show \( ?\text{thesis using } 1 \ 2 \) by auto 
qed

qed

from this current-next-state[THEN spec,THEN spec,where } x1=s \text{ and } x=execs] \text{locally-respects[THEN spec,THEN spec,where } x=\text{next-state } s \text{ execs}] \text{vpeq-reflexive}

have vpeq v (next-state } s \text{ execs) (step (next-state } s \text{ execs) (next-action } s \text{ execs))}

unfolding step-def precondition-def B-def by (cases next-action } s \text{ execs,auto)}

from not-ifp-curr-v this locally-respects-next-state vpeq-transitive

have vpeq-s-ns: vpeq v s (step (next-state } s \text{ execs) (next-action } s \text{ execs)}

by blast

from not-ifp-curr-v current-s-t current-next-state[THEN spec,THEN spec,where } x1=t' \text{ and } x=execs2] \text{prec-t}

locally-respects[THEN spec,THEN spec,where } x=\text{next-state } t' \text{ execs2}] 

F vpeq-reflexive

have \( \theta : \neg \text{thread-empty (execs2 (current } t') \text{)} \rightarrow \text{vpeq v (next-state } t' \text{ execs2) (step (next-state } t' \text{ execs2) (next-action } t' \text{ execs2))} \) by metis

from this vpeq-reflexive

have vpeq-t-nt: vpeq v t' ?nt by auto

from vpeq-s-t ifp-v v-not-intermediary

have vpeq v s t' by auto

from this vpeq-s-ns vpeq-t-nt vpeq-transitive vpeq-symmetric vpeq-reflexive

have vpeq v (step (next-state s execs) (next-action s execs)) ?nt by (metis (hide-lams, no-types))

\}

hence vpeq-ns-nt: \( \forall v. \text{ifp } v u \land \neg \text{intermediary } v u \rightarrow \text{vpeq } v (\text{step (next-state } s \text{ execs) (next-action } s \text{ execs)}) \) ?nt by auto

from vpeq-s-t 2 F purged-a-a2 current-s-t thread-not-empty-s have purged-na-na2: ipurged-relation1 u (next-execs s execs) ?na2

unfolding ipurged-relation1-def next-execs-def intermediary-def by(auto)

from current-ns-nt current-ns-t current-next-state have current-ns-nt: current (step (next-state s execs) (next-action s execs)) = current ?nt by auto
Returns True iff and only if the two states have the same active domain, or if one of the states is None.

**Definition**: \( mc\text{currents} :: \text{state-t option} \Rightarrow \text{state-t option} \Rightarrow \text{bool} \)

where \( mc\text{currents} m1 m2 \equiv m1 \parallel m2 \rightarrow (\lambda s t . \text{current } s = \text{current } t) \)

Proof that switching/interrupts are purely time-based and happen independent of the actions done by the domains. As all theorems in this locale, it holds vacuously whenever one of the states is None, i.e., whenever at some point a precondition does not hold.

**Lemma**: \( current\text{-independent-of-domain-actions} \)

assumes \( current\text{-s-t}: mc\text{currents} s t \)
shows mcurrents (run n s execs) (run n t execs2)
proof
{
  fix n s execs t execs2
  have mcurrents s t \rightarrow mcurrents (run n s execs) (run n t execs2)
  proof (induct n s execs arbitrary: t execs2 rule: run.induct)
  case (next execs t execs2)
    from this show ?case using current-s-t unfolding B-def by auto
  next
  case (2 n execs t execs2)
    show ?case unfolding mcurrents-def by(auto)
  next
  case (3 n s execs t execs2)
    assume interrupt: interrupt (Suc n)
    assume IH: (\forall t execs2. mcurrents (Some (cswitch (Suc n) s)) t \rightarrow mcurrents (run n (Some (cswitch (Suc n) s)) execs) (run n t execs2))
    {
      fix t'
      assume t: t = (Some t')
      assume curr: mcurrents (Some s) t
      from t curr cswitch-independent-of-state[THEN spec,THEN spec,THEN spec,where sI=s] have current-s-n: current (cswitch (Suc n) s) = current (cswitch (Suc n) t')
      unfolding mcurrents-def by simp
      from current-s-n IH[where t=Some (cswitch (Suc n) t') and ?execs2.0=?execs2] have mcurrents-s-n: mcurrents (run n (Some (cswitch (Suc n) s)) execs) (run n (Some (cswitch (Suc n) t')) execs2)
      unfolding mcurrents-def by(auto)
      from mcurrents-s-n interrupt t have mcurrents (run (Suc n) (Some s) execs) (run (Suc n) t execs2)
      unfolding mcurrents-def B2-def B-def by(cases run n (Some (cswitch (Suc n)) execs), cases run (Suc n) t execs2,auto)
    }
    thus ?case unfolding mcurrents-def B2-def by(cases t,auto)
  next
  case (4 n execs s t execs2)
    assume not-interrupt: ~interrupt (Suc n)
    assume thread-empty-s: thread-empty(execs (current s))
    assume IH: (\forall t execs2. mcurrents (Some s) t \rightarrow mcurrents (run n (Some s) execs) (run n t execs2))
    {
      fix t'
      assume t: t = (Some t')
      assume curr: mcurrents (Some s) t
      {
        assume thread-empty-t: thread-empty(execs2 (current t'))
        from t curr not-interrupt thread-empty-s this IH[where ?execs2.0=?execs2 and t=Some t'] have mcurrents (run (Suc n) (Some s) execs) (run (Suc n) t execs2)
        by auto
      }
      moreover
      {
        assume not-prec-t: ~thread-empty(execs2 (current t')) \land ~precondition (next-state t' execs2) (next-action t' execs2)
        from t this not-interrupt have mcurrents (run (Suc n) (Some s) execs) (run (Suc n) t execs2)
        unfolding mcurrents-def by (simp add: rewrite-B2-cases)
      }
      moreover
    }
  }

\[
\{ \\
\text{assume } \text{step-t: } \neg \text{thread-empty}(\text{execs2} (\text{current } t')) \land \text{precondition} (\text{next-state } t' \text{ execs2}) (\text{next-action } t' \text{ execs2}) \\
\text{have mcurrents } (\text{Some } s) (\text{Some } (\text{step } \text{next-state } \text{execs2}) (\text{next-action } t' \text{ execs2})) \\
\text{using step-atomicity } \text{curr } t \text{ current-next-state unfolding mcurrents-def} \\
\text{unfolding step-def} \\
\text{by } (\text{cases } \text{next-action } t' \text{ execs2,auto}) \\
\text{from } t \text{ step-t curr not-interrupt thread-empty-s this } \text{IH[where } \text{execs2.0=next-execs } t' \text{ execs2 and } t=\text{Some} \\
(\text{step } (\text{next-state } t' \text{ execs2}) (\text{next-action } t' \text{ execs2}))) \\
\text{have mcurrents } (\text{run } (\text{Suc } n) (\text{Some } s) \text{ execs}) (\text{run } (\text{Suc } n) t \text{ execs2}) \\
\text{by auto} \\
\} \\
\text{ultimately have mcurrents } (\text{run } (\text{Suc } n) (\text{Some } s) \text{ execs}) (\text{run } (\text{Suc } n) t \text{ execs2}) \text{ by blast} \\
\} \\
\text{thus } ?\text{case unfolding mcurrents-def B2-def by(cases } t,\text{auto) next} \\
\text{case } (5 \text{n execs } s \text{ execs2}) \\
\text{assume not-interrupt-s: } \neg \text{interrupt } (\text{Suc } n) \\
\text{assume thread-not-empty-s: } \neg \text{thread-empty}(\text{execs } (\text{current } s)) \\
\text{assume not-prec-s: } \neg \text{precondition } (\text{next-state } s \text{ execs}) (\text{next-action } s \text{ execs}) \\
\text{hence run } (\text{Suc } n) (\text{Some } s) \text{ execs } = \text{None using not-interrupt-s thread-not-empty-s by simp} \\
\text{thus } ?\text{case unfolding mcurrents-def by(simp add-option.splits) next} \\
\text{case } (6 \text{n execs } s \text{ execs2}) \\
\text{assume not-interrupt: } \neg \text{interrupt } (\text{Suc } n) \\
\text{assume thread-not-empty-s: } \neg \text{thread-empty}(\text{execs } (\text{current } s)) \\
\text{assume prec-s: } \text{precondition } (\text{next-state } s \text{ execs}) (\text{next-action } s \text{ execs}) \\
\text{assume IH: } (\neg \exists \text{execs2}. \\
(\text{mcurrents } (\text{Some } (\text{step } \text{next-state } \text{execs2}) (\text{next-action } s \text{ execs}))) t \rightarrow \\
(\text{mcurrents } (\text{run } (\text{Suc } n) (\text{Some } (\text{step } \text{next-state } \text{execs2}) (\text{next-action } s \text{ execs}))) (\text{run } n t \text{ execs2}))) (\text{run } n t \text{ execs2})) \\
\} \\
\text{fix } t' \\
\text{assume } t: t = (\text{Some } t') \\
\text{assume curr: mcurrents } (\text{Some } s) \text{ } t \\
\} \\
\text{assume thread-empty-t: } \neg \text{thread-empty}(\text{execs2 } (\text{current } t')) \\
\text{have mcurrents } (\text{Some } (\text{step } \text{next-state } \text{execs2}) (\text{next-action } s \text{ execs}))) (\text{Some } t') \\
\text{using step-atomicity } \text{curr } t \text{ current-next-state unfolding mcurrents-def} \\
\text{unfolding step-def} \\
\text{by } (\text{cases } \text{next-action } s \text{ execs,auto}) \\
\text{from } t \text{ curr not-interrupt thread-not-empty-s prec-s thread-empty-t this } \text{IH[where } \text{execs2.0=execs2 and } t=\text{Some } t'] \\
\text{have mcurrents } (\text{run } (\text{Suc } n) (\text{Some } s) \text{ execs}) (\text{run } (\text{Suc } n) t \text{ execs2}) \\
\text{by auto} \\
\} \\
\text{moreover} \\
\} \\
\text{assume not-prec-t: } \neg \text{thread-empty}(\text{execs2 } (\text{current } t')) \land \neg \text{precondition } (\text{next-state } t' \text{ execs2}) (\text{next-action } t' \text{ execs2}) \\
\text{from } t \text{ this not-interrupt} \\
\text{have mcurrents } (\text{run } (\text{Suc } n) (\text{Some } s) \text{ execs}) (\text{run } (\text{Suc } n) t \text{ execs2}) \\
\text{unfolding mcurrents-def B2-def by (auto) } \\
\} \\
\text{moreover} \\
\} \\
\text{assume step-t: } \neg \text{thread-empty}(\text{execs2 } (\text{current } t')) \land \text{precondition } (\text{next-state } t' \text{ execs2}) (\text{next-action } t' \text{ execs2}) \\
\text{by (auto) }
execs2)
    have mcurrents (Some (step (next-state s execs) (next-action s execs))) (Some (step (next-state t' execs2) (next-action t' execs2)))
    using step-atomicity curr t current-next-state unfolding mcurrents-def unfolding step-def
    by (cases next-action s execs, simp, cases next-action t' execs2, simp, simp, cases next-action t' execs2, simp, simp)
    from current-next-state t step-t curr not-interrupt thread-not-empty-s prec-s this IH \[\text{where } ?execs2.0=\text{next-execs} t' \text{ execs2} \]
    have mcurrents (run (Suc n) (Some s) execs) (run (Suc n) t execs2)
    by auto
  }
ultimately have mcurrents (run (Suc n) (Some s) execs) (run (Suc n) t execs2) by blast
}
thus ?thesis using current-s-t by auto
qed

theorem unwinding-implies-NI-indirect-sources:
shows NI-indirect-sources
proof -
  { fix execs a n
    from assms iunwinding-implies-view-partitioned1
    have vp: iview-partitioned by blast
    from vp and vpeq-reflexive
    have 1: \(\forall u . \text{run } n (\text{Some } s0) (\text{ipurge-l execs } u) || \text{run } n (\text{Some } s0) (\text{ipurge-r execs } u) \rightarrow (\lambda rs rt. vpeq u rs rt \land current rs = current rt)\)
      unfolding iview-partitioned-def by auto
    have run n (Some s0) execs \rightarrow (\lambda s-f. run n (Some s0) (ipurge-l execs (current s-f)) ||
      run n (Some s0) (ipurge-r execs (current s-f)) \rightarrow
      (\lambda s-l s-r. output-f s-l a = output-f s-r a))
      proof (cases run n (Some s0) execs)
      case None
      thus ?thesis unfolding B-def by simp
      next
case (Some s-f)
      thus ?thesis
      proof (cases run n (Some s0) (ipurge-l execs (current s-f)))
      case None
      from Some this show ?thesis unfolding B-def by simp
      next
case (Some s-ipurge-l)
      show ?thesis
      proof (cases run n (Some s0) (ipurge-r execs (current s-f)))
      case None
      from \(\text{run } n (\text{Some } s0) \text{ execs } = \text{Some } s-f\) \(\text{Some this show } ?\text{thesis } \text{unfolding } B-def \text{ by simp}\)
      next
case (Some s-ipurge-r)
    from csswitch-independent-of-state
    \(\text{run } n (\text{Some } s0) \text{ execs } = \text{Some } s-f\) \(\text{run } n (\text{Some } s0) (\text{ipurge-l execs (current s-f)}) = \text{Some } s-ipurge-l:\text{current-independent-of-domain-actions}[\text{where } n=n \text{ and } s=\text{Some } s0 \text{ and } t=\text{Some } s0 \text{ and } \text{execs} = \text{execs and } ?\text{execs2.0=(ipurge-l execs (current s-f))}]\)
    have 2: current s-ipurge-l = current s-f
    unfolding mcurrents-def B-def by auto
from \( \text{run } n \begin{pmatrix} \text{Some } s_0 \end{pmatrix} \text{execs } = \text{Some } s-f \begin{pmatrix} \text{ipurge-l execs } (\text{current } s-f) \end{pmatrix} = \text{Some } s-ipurge-b \begin{pmatrix} \text{I[THEN spec, where } x=\text{current } s-f \end{pmatrix} \)

have \( vpeq \begin{pmatrix} \text{current } s-f \end{pmatrix} \text{s-ipurge-l } s-ipurge-r \land \text{current } s-ipurge-l = \text{current } s-ipurge-r \)

unfolding B-def by auto
from this 2 have \( \text{output-f s-ipurge-l } a = \text{output-f s-ipurge-r } a \)
using output-consistent by auto
from \( \text{run } n \begin{pmatrix} \text{Some } s_0 \end{pmatrix} \text{execs } = \text{Some } s-f \begin{pmatrix} \text{ipurge-l execs } (\text{current } s-f) \end{pmatrix} = \text{Some } s-ipurge-b \begin{pmatrix} \text{I[THEN spec, } \text{where } x=\text{current } s-f \end{pmatrix} \)
this Some
show ?thesis unfolding B-def by auto
qed
qed
qed
}

thus ?thesis unfolding NI-indirect-sources-def by auto
qed

theorem unwinding-implies-secure:
shows secure
using unwinding-implies-NI-indirect-sources unwinding-implies-NI-unrelated assms unfolding secure-def by(auto)
end
end

3.3 ISK (Interruptible Separation Kernel)

theory ISK
  imports SK
begin

At this point, the precondition linking action to state is generic and highly unconstrained. We refine
the previous locale by given generic functions “precondition” and “realistic_trace” a definiton. This
yields a total run function, instead of the partial one of locale Separation_Kernel.

This definition is based on a set of valid action sequences AS_set. Consider for example the following
action sequence:

\[ \gamma = [COPY\_INIT, COPY\_CHECK, COPY\_COPY] \]

If action sequence \( \gamma \) is a member of AS_set, this means that the attack surface contains an action COPY,
which consists of three consecutive atomic kernel actions. Interrupts can occur anywhere between these
atomic actions.

Given a set of valid action sequences such as \( \gamma \), generic function precondition can be defined. It now
consists of 1.) a generic invariant and 2.) more refined preconditions for the current action.

These preconditions need to be proven inductive only according to action sequences. Assume, e.g.,
that \( \gamma \in \text{AS_set} \) and that \( d \) is the currently active domain in state \( s \). The following constraints are assumed
and must therefore be proven for the instantiation:

- “AS_precondition s d COPY\_INIT”
  since COPY\_INIT is the start of an action sequence.

- “AS_precondition (step s COPY\_INIT) d COPY\_CHECK”
  since (COPY\_INIT, COPY\_CHECK) is a sub sequence.

- “AS_precondition (step s COPY\_CHECK) d COPY\_COPY”
  since (COPY\_CHECK, COPY\_COPY) is a sub sequence.

Additionally, the precondition for domain \( d \) must be consistent when a context switch occurs, or when
ever some other domain \( d' \) performs an action.
Locale Interruptible_Separation_Kernel refines locale Separation_Kernel in two ways. First, there is a definition of realistic executions. A realistic trace consists of action sequences from AS_set.

Secondly, the generic control function has been refined by additional assumptions. It is now assumed that control conforms to one of four possibilities:

1. The execution of the currently active domain is empty and the control function returns no action.
2. The currently active domain is executing the action sequence at the head of the execution. It returns the next kernel action of this sequence and updates the execution accordingly.
3. The action sequence is delayed.
4. The action sequence that is at the head of the execution is skipped and the execution is updated accordingly.

As for the state update, this is still completely unconstrained and generic as long as it respects the generic invariant and the precondition.

```plaintext
locale Interruptible_Separation_Kernel = Separation_Kernel kstep output-f s0 current cswitch interrupt kprecondition realistic-execution-control kinvolved ifp vpeq

for kstep :: 'state-t \Rightarrow 'action-t \Rightarrow 'state-t
and output-f :: 'state-t \Rightarrow 'action-t \Rightarrow 'output-t
and s0 :: 'state-t
and current :: 'state-t \Rightarrow 'dom-t — Returns the currently active domain
and cswitch :: 'time-t \Rightarrow 'state-t \Rightarrow 'state-t — Switches the current domain
and interrupt :: 'time-t \Rightarrow bool — Returns true iff an interrupt occurs in the given state at the given time
and kprecondition :: 'state-t \Rightarrow 'action-t \Rightarrow bool — Returns true if an precondition holds that relates the current action to the state

and realistic-execution :: 'action-t execution \Rightarrow bool — In this locale, this function is completely unconstrained.
and control :: 'state-t \Rightarrow 'dom-t \Rightarrow ('action-t execution \times 'state-t)
and kinvolved :: 'action-t \Rightarrow 'dom-t set
and ifp :: 'dom-t \Rightarrow 'dom-t = bool
and vpeq :: 'dom-t \Rightarrow 'state-t \Rightarrow 'state-t = bool
+ fixes AS-set :: ('action-t list) set — Returns a set of valid action sequences, i.e., the attack surface
and invariant :: 'state-t \Rightarrow bool
and AS-precondition :: 'state-t \Rightarrow 'dom-t \Rightarrow 'action-t \Rightarrow bool
and aborting :: 'state-t \Rightarrow 'dom-t \Rightarrow 'action-t \Rightarrow bool
and waiting :: 'state-t \Rightarrow 'dom-t \Rightarrow 'action-t \Rightarrow bool

assumes empty-in-AS-set: [] \in AS-set
and invariant-s0: invariant s0
and invariant-after-cswitch \forall s n . invariant s \Rightarrow invariant (cswitch n s)
and preconditions-after-cswitch \forall s d n a . AS-precondition s d a \Rightarrow AS-precondition (cswitch n s) d a
and AS-prec-first-action: \forall s d aseq . invariant s \& aseq \in AS-set \& aseq \# [] \Rightarrow AS-precondition s d (hd aseq)
and AS-prec-after-step: \forall s a a' . (\exists aseq \in AS-set . is-sub-seq a a' aseq) \& invariant s \& AS-precondition s (current s) a \& ~ aborting s (current s) a \& ~ waiting s (current s) a \Rightarrow AS-precondition (kstep s a) (current s) a'
and AS-prec-dom-independent: \forall s d a a' . current s \& d AS-precondition s d a \Rightarrow AS-precondition (kstep s a a') d a
and spec-of-invariant: \forall s a . invariant s \Rightarrow invariant (kstep s a)

and kprecondition-def: kprecondition s a \equiv invariant s \& AS-precondition s (current s) a
and realistic-execution-def: realistic-execution aseq \equiv set aseq \subseteq AS-set
and control-spec: \forall s d aseqs . case control s d aseqs of (a, aseqs', s') \Rightarrow
(thread-empty aseqs \& (a, aseqs') = (None, [])) \lor (* Nothing happens *)
(aeqs \# [] \& hd aseqs \# [] \& ~ aborting s' d (the a) \& ~ waiting s' d (the a) \& (a, aseqs') = (Some (hd (hd aseqs)), (tl (hd aseqs)) \# (tl aseqs))) \lor (* Execute the first action of the current action sequence *)
```
We can now formulate a total run function, since based on the new assumptions the case where the precondition does not hold, will never occur.

The major part of the proofs in this locale consist of proving that function run_total is equivalent to function run, i.e., that the precondition does always hold. This assumes that the executions are realistic. This means that the execution of each domain contains action sequences that are from AS_set. This ensures, e.g., that a COPY_CHECK is always preceded by a COPY_INIT.

Lemma run_total_equal_to_run is proven by doing induction. It is however not inductive and can therefore not be proven directly: a realistic execution is not necessarily realistic after performing one action. We generalize to do induction. Predicate realistic_executions_ind is the inductive version of realistic_executions. All action sequences in the tail of the executions must be complete action sequences (i.e., they must be from AS_set). The first action sequence, however, is being executed and is therefore not necessarily an action sequence from AS_set, but it is the last part of some action sequence from AS_set.

We need to know that invariably, the precondition holds. As this precondition consists of 1.) a generic invariant and 2.) more refined preconditions for the current action, we have to know that these two are invariably true.

Proof that “execution is realistic” is inductive, i.e., assuming the current execution is realistic, the execution returned by the control mechanism is realistic.
shows \text{realistic-AS}\text{-partial aseq} \land \text{set aseqs} \subseteq \text{AS-set}

\begin{proof}
\begin{align*}
\text{let } & \ ?c = \text{control s (current s)} \ (\text{execs (current s)}) \\
\{ \\
\text{assume } &c\text{-empty: let } (a,\text{aseqs}',s') = ?c \text{ in} \\
& (a,\text{aseqs}') = (\text{None},[]) \\
\text{from } &\text{na-def d-is-curr c-empty} \\
\text{have } &?\text{thesis} \\
\text{unfolding } &\text{realistic-executions-ind-def next-exec-def by (auto)} \\
\}
\end{align*}
\end{proof}

moreover
\begin{align*}
\{ \\
\text{let } & ?\text{ct= execs (current s)} \\
\text{let } & ?\text{execs}' = (\text{tl (hd ?ct)}) \#(\text{tl ?ct}) \\
\text{let } & ?a' = \text{Some (hd (hd ?ct))} \\
\text{assume } &hd\text{-thread-not-empty: hd} (\text{execs (current s)}) \neq [] \\
\text{assume } &c\text{-executing: let } (a,\text{aseqs}',s') = ?c \text{ in} \\
&(a,\text{aseqs}') = (?a', ?\text{execs}') \\
\text{from } &\text{na-def c-executing d-is-curr} \\
\text{have } &\text{as-defs: aseq = tl (hd ?ct) \land aseqs = tl ?ct} \\
\text{unfolding } &\text{next-exec-def by (auto)} \\
\text{from } &\text{realistic\text{[unfolded realistic-executions-ind-def\text{,THEN spec\text{,where } x=d\text{]} d-is-curr}} \\
\text{have } &\text{subset: set (tl ?execs') \subseteq AS-set} \\
\text{unfolding } &\text{Let-def realistic-AS\text{-partial-def}} \\
\text{by } &\text{(cases execs d,auto)} \\
\text{from } &\text{d-is-curr thread-not-empty hd\text{-thread-not-empty realistic\text{[unfolded realistic-executions-ind-def\text{,THEN spec\text{,where } x=d\text{]} d-is-curr}} \\
\text{obtain } &n \text{ aseq' where } n\text{-aseq': } n \leq \text{length aseq'} \land \text{aseq' } \in \text{AS-set} \land \text{hd ?ct = lastn n aseq'} \\
\text{unfolding } &\text{realistic-AS\text{-partial-def}} \\
\text{by } &\text{(cases execs d,auto)} \\
\text{from } &\text{this hd\text{-thread-not-empty have } n > 0 \text{ unfolding lastn-def by (cases n,auto)} \\
\text{from } &\text{this n-aseq' lastn-one-less\text{[where } n=n \text{ and } x=aseq' \text{ and } a=hd (hd ?ct) \text{ and } y=tl (hd ?ct) \text{]} hd\text{-thread-not-empty} \\
\text{have } &n - 1 \leq \text{length aseq'} \land \text{aseq' } \in \text{AS-set} \land \text{tl (hd ?ct) = lastn (n - 1) aseq'} \\
\text{by } &\text{auto} \\
\text{from } &\text{this as-defs subset have } ?\text{thesis} \\
\text{unfolding } &\text{realistic-AS\text{-partial-def}} \\
\text{by } &\text{auto} \\
\}
\end{align*}

moreover
\begin{align*}
\{ \\
\text{let } & ?\text{ct= execs (current s)} \\
\text{let } & ?\text{execs}' = ?ct \\
\text{let } & ?a' = \text{Some (hd (hd ?ct))} \\
\text{assume } &c\text{-waiting: let } (a,\text{aseqs}',s') = ?c \text{ in} \\
&(a,\text{aseqs}') = (?a', ?\text{execs}') \\
\text{from } &\text{na-def c\text{-waiting d-is-curr}} \\
\text{have } &\text{as-defs: aseq = hd ?execs' \land aseqs = tl ?execs'} \\
\text{unfolding } &\text{next-exec-def by (auto)} \\
\text{from } &\text{realistic\text{[unfolded realistic-executions-ind-def\text{,THEN spec\text{,where } x=d\text{]} d-is-curr set-tl-is-subset[where } x=?\text{execs}]}} \\
\text{have } &\text{subset: set (tl ?execs') \subseteq AS-set} \\
\text{unfolding } &\text{Let-def realistic-AS\text{-partial-def}} \\
\text{by } &\text{(cases execs d,auto)} \\
\text{from } &\text{na-def c\text{-waiting d-is-curr}} \\
\text{have } &?\text{execs' } \neq [] \text{ unfolding next-exec-def by auto} \\
\text{from } &\text{realistic\text{[unfolded realistic-executions-ind-def\text{,THEN spec\text{,where } x=d\text{]} d-is-curr thread-not-empty}} \\
\text{obtain } &n \text{ aseq' where } \text{witness: } n \leq \text{length aseq'} \land \text{aseq' } \in \text{AS-set} \land \text{hd(execs d) = lastn n aseq'}
\end{align*}
The lemma that proves that the total run function is equivalent to the partial run function, i.e., that in this refinement the case of the run function where the precondition is False will never occur.

**Lemma run-total-equals-run:**

**Assumes** realistic-exec: realistic-executions execs

and invariant: invariant s

**Shows** strict-equal (run n (Some s) execs) (run-total n s execs)

**Proof**

{  
  fix n ms s execs

  have strict-equal ms s ∧ realistic-executions-ind execs ∧ precondition-ind s execs —> strict-equal (run n ms execs) (run-total n s execs)

  proof (induct n ms execs arbitrary: s rule: run.induct)

  case (1 s execs sa)

  show ?case by auto

  next

  case (2 n execs s)

  show ?case unfolding strict-equal-def by auto

  next

  case (3 n s execs sa)
}
assume interrupt: interrupt (Suc n)
assume IH: (\(\forall\) sa. strict-equal (Some (cswitch (Suc n) s)) sa ∧ realistic-executions-ind execs ∧ precondition-ind sa execs) \(\rightarrow\) strict-equal (run n (Some (cswitch (Suc n) s)) execs) (run-total n sa execs)

\[
\begin{aligned}
\{ \\
\text{assume equal-s-sa: strict-equal (Some s) sa} \\
\text{assume realistic: realistic-executions-ind execs} \\
\text{assume inv-sa: precondition-ind sa execs} \\
\text{have inv-nsa: precondition-ind (cswitch (Suc n) sa) execs} \\
\text{proof--} \\
\{ \\
\text{fix d} \\
\text{have fst (control (cswitch (Suc n) sa) d (execs d)) \(\rightarrow\) AS-precondition (cswitch (Suc n) sa) d} \\
\text{using next-action-after-cswitch inv-sa unfolded precondition-ind-def,THEN conjunct2,THEN spec,where} \\
x=d] \\
\text{precondition-after-cswitch} \\
\text{unfolding Let-def B-def precondition-ind-def} \\
\text{by(cases fst (control (cswitch (Suc n) sa) d (execs d)),auto)} \\
\} \\
\text{thus ?thesis using inv-sa invariant-after-cswitch unfolding precondition-ind-def by auto} \\
\text{qed} \\
\text{from equal-s-sa realistic inv-nsa inv-H[where sa=cswitch (Suc n) sa]} \\
\text{have equal-ns-nt: strict-equal (run n (Some (cswitch (Suc n) s)) execs) (run-total n (cswitch (Suc n) sa) execs)} \\
\text{unfolding strict-equal-def by(auto)} \\
\} \\
\text{from this interrupt show ?case by auto} \\
\text{next} \\
\text{case (4 n execs s sa)} \\
\text{assume not-interrupt: \(\neg\)interrupt (Suc n)} \\
\text{assume thread-empty: thread-empty(execs (current s))} \\
\text{assume IH: (\(\forall\) sa. strict-equal (Some s) sa ∧ realistic-executions-ind execs ∧ precondition-ind sa execs) \(\rightarrow\) strict-equal (run n (Some s) execs) (run-total n sa execs))} \\
\text{have current-s-sa: strict-equal (Some s) sa \(\rightarrow\) current s = current sa unfolding strict-equal-def by auto} \\
\{ \\
\text{assume equal-s-sa: strict-equal (Some s) sa} \\
\text{assume realistic: realistic-executions-ind execs} \\
\text{assume inv-sa: precondition-ind sa execs} \\
\text{from equal-s-sa realistic inv-sa IH[where sa=sa]} \\
\text{have equal-ns-nt: strict-equal (run n (Some s) execs) (run-total n sa execs)} \\
\text{unfolding strict-equal-def by(auto)} \\
\} \\
\text{from this current-s-sa thread-empty not-interrupt show ?case by auto} \\
\text{next} \\
\text{case (5 n execs s sa)} \\
\text{assume not-interrupt: \(\neg\)interrupt (Suc n)} \\
\text{assume thread-not-empty: \(\neg\)thread-empty(execs (current s))} \\
\text{assume not-prec \(\neg\)precondition (next-state s execs) (next-action s execs)} \\
\text{— In locale ISK, the precondition can be proven to hold at all times. This case cannot happen, and we can prove False.} \\
\{ \\
\text{assume equal-s-sa: strict-equal (Some s) sa} \\
\text{assume realistic: realistic-executions-ind execs} \\
\text{assume inv-sa: precondition-ind sa execs} \\
\text{from equal-s-sa have s-sa s = sa unfolding strict-equal-def by auto} \\
\text{from inv-sa have} \\
\text{next-action sa execs \(\rightarrow\) AS-precondition sa (current sa)}
\end{aligned}
\]
unfolding precondition-ind-def B-def next-action-def
by (cases next-action sa execs,auto)
from this next-state-precondition
have next-action sa execs \rightarrow \text{AS-precondition (next-state sa execs)} (current sa)
unfolding precondition-ind-def B-def
by (cases next-action sa execs,auto)
from inv-sa this s-sa next-state-invariant current-next-state
have \text{prec-s} : \text{precondition (next-state s execs)} (next-action s execs)
unfolding precondition-ind-def kprecondition-def precondition-def B-def
by (cases next-action sa execs,auto)
from this not-prec have False by auto
}
thus \text{?case} by auto
next
\begin{itemize}
  \item \text{case (6 n execs s sa)}
    \begin{itemize}
      \item assume not-interrupt : \text{\neg interrupt (Suc n)}
      \item assume thread-not-empty: \text{\neg thread-empty (execs (current s))}
      \item assume \text{prec: precondition (next-state s execs)} (next-action s execs)
      \item assume IH: \text{(\forall s. \text{strict-equal (Some (step (next-state s execs)) (next-action s execs))} s \land}
                   \text{realistic-executions-ind (next-execs s execs) \land precondition-ind sa (next-execs s execs) \rightarrow}
                   \text{strict-equal (run n (Some (step (next-state s execs)) (next-action s execs)) (next-execs s execs)) (run-total n sa (next-execs s execs)))}
    \end{itemize}
  \item have current-s-sa : \text{strict-equal (Some s) sa \rightarrow current s = current sa} unfolding strict-equal-def by auto
  \begin{itemize}
    \item assume equal-s-sa : \text{strict-equal (Some s) sa}
    \item assume realistic: \text{realistic-executions-ind execs}
    \item assume inv-sa: \text{precondition-ind sa execs}
  \end{itemize}
  \begin{itemize}
    \item from equal-s-sa have s-sa: s = sa unfolding strict-equal-def by auto
      \begin{itemize}
        \item let \text{?a} = next-action s execs
        \item let \text{?ns} = step (next-state s execs) ?a
        \item let \text{?na} = next-execs s execs
        \item let \text{?c} = control s (current s) (execs (current s))
      \end{itemize}
      \begin{itemize}
        \item have equal-ns-nsa : \text{strict-equal (Some ?ns) ?ns} unfolding strict-equal-def by auto
          \begin{itemize}
            \item from inv-sa equal-s-sa have inv-s: invariant s unfolding strict-equal-def precondition-ind-def by auto
          \end{itemize}
      \end{itemize}
  \end{itemize}
\end{itemize}

— Two things are proven inductive. First, the assumptions that the execution is realistic (statement realistic-na).
This proof uses lemma next-execution-is-realistic-partial. Secondly, the precondition: if the precondition holds for the current action, then it holds for the next action (statement invariant-na).
\begin{itemize}
  \item have realistic-na: \text{realistic-executions-ind ?na}
\end{itemize}
from next-execution-is-realistic-partial na-def True realistic thread-not-empty
  show realistic-AS-partial aseq ∧ set asseqs ⊆ AS-set by blast
qend

thus ?thesis unfolding realistic-executions-ind-def by auto
qend

have invariant-na: precondition-ind ?ns ?na
proof-
  from spec-of-invariant inv-sa next-state-invariant s-sa have inv-ns: invariant ?ns
  unfolding precondition-ind-def step-def
  by (cases next-action sa execs auto)
  have ∀ d. fst (control ?ns d (?na d)) → AS-precondition ?ns d
proof-
  { fix d
    { let ?a' = fst (control ?ns d (?na d))
      assume snd-action-not-none: ?a' ≠ None
      have AS-precondition ?ns d (the ?a')
      proof (cases d = current s)
        case True
        have ?thesis
        proof (cases ?a)
        case (Some a)
          — Assuming that the current domain executes some action a, and assuming that the action a’ after that is
          not None (statement snd-action-not-none), we prove that the precondition is inductive, i.e., it will hold for a’. Two
          cases arise: either action a is delayed (case waiting) or not (case executing).
          show ?thesis
          proof (cases ?a d = execs (current s) rule: case-split [case-names waiting executing])
            case executing — The kernel is executing two consecutive actions a and a’. We show that [a,a’] is a
            subsequence in some action in AS-set. The PO’s ensure that the precondition is inductive.
            from executing True Some control-spec [THEN spec, THEN spec, THEN spec, where x2 = s and x1 = d and
            x = execs d]
            have a-def: a = hd (hd (execs (current s)) ∧ ?na d = (tl (hd (execs (current s)))) ≠ (tl (execs
            (current s))))
            unfolding next-action-def next-execs-def Let-def
            by (auto)
            from a-def True snd-action-not-none control-spec [THEN spec, THEN spec, THEN spec, where x2 = ?ns
            and x1 = d and x = ?na d]
            second-elt-is-hd-tl [where x = hd (execs (current s)) and a = hd (tl (hd (execs (current s)))) and
            x' = tl (tl (hd (execs (current s))))]
            have na-def: the ?a' = (hd (execs (current s)))!1
            unfolding next-execs-def
            by (auto)
            from Some realistic [unfolded realistic-executions-ind-def, THEN spec, where x = d] True thread-not-empty
            obtain n aseq' where witness: n ≤ length aseq' ∧ aseq' ∈ AS-set ∧ hd(execs d') = lastn n aseq'
            unfolding realistic-AS-partial-def by (cases execs d, auto)
            from True executing length-lt-2-implies-tl-empty [where x = hd (execs (current s))]
            Some control-spec [THEN spec, THEN spec, THEN spec, where x2 = s and x1 = d and x = execs d]
            snd-action-not-none control-spec [THEN spec, THEN spec, THEN spec, where x2 = ?ns and x1 = d and
            x = ?na d]
            have in-action-sequence: length (hd (execs (current s))) ≥ 2
            unfolding next-action-def next-execs-def
            by auto
            from this witness consecutive-is-sub-seq [where a = a and b = the ?a' and n = n and y = aseq' and
            x = tl (tl (hd (execs (current s))))]
This holds, since the control mechanism will ensure that action a' is the start of a new action sequence in AS-set. If 

If \( a \neq \text{None} \) (statement snd-action-not-none), we prove that the precondition is inductive, i.e., it will hold for a'.

\[
\text{execs} \left( \text{current} s \right) \quad \text{execs} \left( \text{current} s \right)
\]

From True Some inv-sa[unfolded precondition-ind-def, THEN conjunct2, THEN spec, where \( x = \text{current} s \) s-sa

have 1: \( \exists \text{aseq}' \in \text{AS-set} . \text{is-sub-seq a} \left( \text{the} \ ?a' \right) \text{aseq}' \)

by(auto)

From True Some inv-sa[unfolded precondition-ind-def, THEN conjunct2, THEN spec, where \( x = \text{current} s \) s-sa

have 2: AS-precondition s \( \left( \text{current} s \right) a \)

unfolding strict-equal-def next-action-def B-def by auto

From executing True Some control-spec[THEN spec, THEN spec, THEN spec, where \( x 2 \neq s \) and \( x 1 = d \) and \( x = \text{execs} d \)]

have not-aborting: \( \neg\text{aborting} \left( \text{next-state s execs} \right) \left( \text{current} s \right) \left( \text{the} \ ?a \right) \)

unfolding next-action-def next-state-def next-execs-def

by auto

From executing True Some control-spec[THEN spec, THEN spec, THEN spec, where \( x 2 \neq s \) and \( x 1 = d \) and \( x = \text{execs} d \)]

have not-waiting: \( \neg\text{waiting} \left( \text{next-state s execs} \right) \left( \text{current} s \right) \left( \text{the} \ ?a \right) \)

unfolding next-action-def next-state-def next-execs-def

by auto

From True this

1 2 inv-s

sub-seq-in-prefixes[where \( X = \text{AS-set} \)] Some next-state-invariant

current-next-state[THEN spec, THEN spec, where \( x 1 = s \) and \( x = \text{execs} \)]

\( \text{AS-prec-after-step[THEN spec, THEN spec, THEN spec, where \( x 2 = \text{next-state s execs} \) and \( x 1 = a \) and} \)

next-state-precondition not-aborting not-waiting

show \( \text{thesis} \)

by auto

next case waiting — The kernel is delaying action a. Thus the action after a, which is a’, is equal to a.

From \( \text{tl-hd-x-not-tl.} x \left[ \text{where} \ x = \text{execs} d \right] \) True waiting control-spec[THEN spec, THEN spec, THEN spec, where \( x 2 = s \) and \( x 1 = d \) and \( x = \text{execs} d \)]

have a-def: \( \text{?na d} = \text{execs} \left( \text{current} s \right) \) \& next-state s execs = s \& waiting s d (the ?a)

unfolding next-action-def next-execs-def next-state-def

by(auto)

From Some waiting a-def True snd-action-not-none control-spec[THEN spec, THEN spec, THEN spec, where \( x 2 = s \) and \( x 1 = d \) and \( x = ?a' \)]

have na-def: \( \text{?a'} = \text{hd} \left( \text{hd} \left( \text{execs} \left( \text{current} s \right) \right) \right) \)

unfolding next-action-def next-execs-def

by(auto)

From spec-of-waiting a-def True

have no-step: \( \text{step s ?a = s unfolding step-def by (cases next-action s execs, auto)} \)

From no-step Some True a-def

inv-sa[unfolded precondition-ind-def, THEN conjunct2, THEN spec, where \( x = \text{current} s \)] s-sa

have 2: AS-precondition s \( \left( \text{current} s \right) \left( \text{the} \ ?a' \right) \)

unfolding next-action-def B-def

by(auto)

From a-def na-def this True Some no-step

show \( \text{thesis} \)

unfolding step-def

by(auto)

qed

next
case None

— Assuming that the current domain does not execute an action, and assuming that the action a’ after that is not None (statement snd-action-not-none), we prove that the precondition is inductive, i.e., it will hold for a’. This holds, since the control mechanism will ensure that action a’ is the start of a new action sequence in AS-set.
\[ \text{from None True snd-action-not-none control-spec[THEN spec,THEN spec,THEN spec,where } x2=\{na\} \text{ and } x1=d \text{ and } x=\{na\} d] \]
\[ \text{control-spec[THEN spec,THEN spec,THEN spec,where } x2=s \text{ and } x1=d \text{ and } x=\text{execs } d] \]
\[ \text{have na-def: the } a' = \text{hd (tl (execs (current s)))} \land ?na d = \text{tl (execs (current s))} \]
\[ \text{unfolding next-action-def next-exec-def} \]
\[ \text{by (auto)} \]
\[ \text{from True None snd-action-not-none control-spec[THEN spec,THEN spec,THEN spec,where } x2=\{na\} \text{ and } x1=d \text{ and } x=\{na\} d] \]
\[ \text{this}\]
\[ \text{have I: tl (execs (current s)) } \not\in [\] \land \text{hd (tl (execs (current s))) } \not\in [\] \]
\[ \text{by auto} \]
\[ \text{from this realistic[unfolded realistic-executions-ind-def,THEN spec,where } x=d] \text{ True thread-not-empty} \]
\[ \text{have hd (tl (execs (current s))) } \in \text{ AS-set} \]
\[ \text{by (cases execs } d,\text{auto)} \]
\[ \text{from True snd-action-not-none this} \]
\[ \text{inv-ns this na-def I} \]
\[ \text{AS-prec-first-action[THEN spec,THEN spec,THEN spec,where } x2=\{na\} \text{ and } x1=d \text{ and } x=\{na\} d] \]
\[ \text{show } \text{thesis by auto} \]
\[ \text{qed} \]
\[ \text{thus } \text{thesis} \]
\[ \text{using control-spec[THEN spec,THEN spec,THEN spec,where } x2=\{na\} \text{ and } x1=\text{current s and } x=\{na\} d] \]
\[ \text{thread-not-empty True snd-action-not-none} \]
\[ \text{by (auto simp add: Let-def)} \]
\[ \text{next case False} \]
\[ \text{from False have equal-na-a } ?na d = \text{execs } d \]
\[ \text{unfolding next-exec-def by auto} \]
\[ \text{from this False current-next-state next-action-after-step} \]
\[ \text{have ?a' = fst (control (next-state s execs ) d (next-exec s execs d))} \]
\[ \text{unfolding next-action-def by auto} \]
\[ \text{from inv-sa[unfolded precondition-ind-def,THEN conjunct2,THEN spec,where } x=d] \text{ s-sa equal-na-a this} \]
\[ \text{next-action-after-next-state[THEN spec,THEN spec,THEN spec,where } x=d \text{ and } x2=s \text{ and } x1=\text{execs} \]
\[ \text{snd-action-not-none False} \]
\[ \text{have AS-precondition s d (the } a' \text{)} \]
\[ \text{unfolding precondition-ind-def next-action-def B-def by (cases fst (control sa d (execs d)).auto)} \]
\[ \text{from equal-na-a False this next-state-precondition current-next-state} \]
\[ \text{AS-prec-dom-independent[THEN spec,THEN spec,THEN spec,THEN spec,where } x3=\text{next-state s execs and } x2=d \text{ and } x=\text{the } ?a \text{ and } x1=\text{the } a' \text{]} \]
\[ \text{show } \text{thesis} \]
\[ \text{unfolding step-def} \]
\[ \text{by (cases next-action s execs).auto)} \]
\[ \text{qed} \]
\[ \text{hence fst (control } ?ns d ( ?na d) \to \text{AS-precondition } ?ns d \text{ unfolding B-def by (cases fst (control } ?ns d ( ?na d),auto)} \]
\[ \text{thus } \text{thesis by auto} \]
\[ \text{qed} \]
\[ \text{from this inv-ns show } \text{thesis} \]
\[ \text{unfolding precondition-ind-def B-def Let-def by (auto)} \]
\[ \text{qed} \]
\[ \text{from equal-ns-nsa realistic-na invariant-na s-sa IH[where } sa=?ns] \]
have equal-ns-nt : strict-equal (run n (Some ns na) (run-total n (step (next-state sa execs) (next-action sa execs))) (next-exec sa execs)) (next-exec sa execs)) by(auto)
}
from this current-s-sa thread-not-empty not-interrupt prec show ?case by auto
qed
}
hence thm-inductive : \forall m s execs n. strict-equal m s \land realistic-executions-ind execs \land precondition-ind s execs → strict-equal (run n m execs) (run-total n s execs) by blast
have 1: strict-equal (Some s) s unfolding strict-equal-def by simp
have 2: realistic-executions-ind execs
proof-
{
fix d
have case execs d of [] ⇒ True | aseq # aseqs ⇒ realistic-AS-partial aseq \land set aseqs ⊆ AS-set
proof(cases execs d,simp)
case (Cons aseq aseqs)
from Cons realistic-exec[unfolded realistic-executions-def,THEN spec,where x=d]
have 0: length aseq ≤ length aseq \land aseq ∈ AS-set \land aseq = lastn (length aseq) aseq
unfolding lastn-def realistic-execution-def by auto
hence 1: realistic-AS-partial aseq unfolding realistic-AS-partial-def by auto
from Cons realistic-exec[unfolded realistic-executions-def,THEN spec,where x=d]
have 2: set aseqs ⊆ AS-set
unfolding realistic-execution-def by auto
from Cons 1 2 show ?thesis by auto
qed
}
thus ?thesis unfolding realistic-executions-ind-def by auto
qed
have 3: precondition-ind s execs
proof-
{
fix d
{
assume not-empty: fst (control s d (execs d)) ≠ None
from not-empty realistic-exec[unfolded realistic-executions-def,THEN spec,where x=d]
have current-aseq-is-realistic: hd (execs d) ∈ AS-set
using control-spec[THEN spec,THEN spec,THEN spec,where x=execs d and x1=d and x2=s]
unfolding realistic-execution-def by(cases execs d,auto)
from not-empty current-aseq-is-realistic invariant AS-prec-first-action[THEN spec,THEN spec,THEN spec,where x2=s and x1=d and x=hd (execs d)]
have AS-precondition s d (the (fst (control s d (execs d))))
using control-spec[THEN spec,THEN spec,THEN spec,where x=execs d and x1=d and x2=s]
by auto
}
hence fst (control s d (execs d)) → AS-precondition s d
unfolding B-def
by (cases fst (control s d (execs d)),auto)
}
from this invariant show ?thesis unfolding precondition-ind-def by auto
qed
from thm-inductive 1 2 3 show ?thesis by auto
qed

Theorem unwinding implies_isecure gives security for all realistic executions. For unrealistic executions, it holds vacuously and therefore does not tell us anything. In order to prove security for this refinement (i.e., for function run_total), we have to prove that purging yields realistic runs.
lemma realistic-purge:
shows ∀ execs d . realistic-executions execs → realistic-executions (purge execs d)
proof
{ 
  fix execs d 
  assume realistic-executions execs 
  hence realistic-executions (purge execs d) 
  using some
[where P=realistic-execution and x=execs d]
  unfolding realistic-executions-def purge-def by(simp) 
}
thus ?thesis by auto
qed

lemma remove-gateway-comm-subset:
shows set (remove-gateway-communications d exec) ⊆ set exec ∪ {[]} 
by(induct exec,auto)

lemma realistic-ipurge-l:
shows ∀ execs d . realistic-executions execs → realistic-executions (ipurge-l execs d)
proof
{ 
  fix execs d 
  assume 1 : realistic-executions execs 
  from empty-in-AS-set remove-gateway-comm-subset[where d=d and exec=execs d] 1 have realistic-executions (ipurge-l execs d) 
  unfolding realistic-executions-def ipurge-l-def by(auto) 
}
thus ?thesis by auto
qed

lemma realistic-ipurge-r:
shows ∀ execs d . realistic-executions execs → realistic-executions (ipurge-r execs d)
proof
{ 
  fix execs d 
  assume 1 : realistic-executions execs 
  from empty-in-AS-set remove-gateway-comm-subset[where d=d and exec=execs d] 1 have realistic-executions (ipurge-r execs d) 
  using some
[where P=λ x . realistic-execution x and x=execs d]
  unfolding realistic-executions-def ipurge-r-def by(auto) 
}
thus ?thesis by auto
qed

We now have sufficient lemma’s to prove security for run_total. The definition of security is similar to that in Section 3.2. It now assumes that the executions are realistic and concerns function run_total instead of function run.

definition NI-unrelated-total::bool
where NI-unrelated-total
≡ ∀ execs a n . realistic-executions execs → 
  (let s-f = run-total n s0 execs in 
    output-f s-f a = output-f (run-total n s0 (purge execs (current s-f))) a 
    ∧ current s-f = current (run-total n s0 (purge execs (current s-f)))))

definition NI-indirect-sources-total::bool
where NI-indirect-sources-total
≡ ∀ execs a n . realistic-executions execs →
(let s-f = run-total n s0 execs in
  output-f (run-total n s0 (ipurge-l execs (current s-f))) a =
  output-f (run-total n s0 (ipurge-r execs (current s-f))) a)

definition isecure-total :∶∶ bool
where
  isecure-total ≡ NI-unrelated-total ∧ NI-indirect-sources-total

theorem unwinding-implies-isecure-total:
shows isecure-total
proof−
  from assms unwinding-implies-isecure have secure-partial: NI-unrelated unfolding isecure-def by blast
  from assms unwinding-implies-isecure have isecure-l-partial: NI-indirect-sources unfolding isecure-def by blast

  have NI-unrelated-total: NI-unrelated-total
  proof−
    { fix execs a n
      assume realistic: realistic-executions execs
      from assms invariant-s0 realistic run-total-equals-run[where n=n and s=s0 and execs=execs]
      have 1: strict-equal (run n (Some s0) execs) (run-total n s0 execs) by auto

      have let s-f = run-total n s0 execs in output-f s-f a =
        output-f (run-total n s0 (ipurge-l execs (current s-f))) a ∧
        current s-f = current (run-total n s0 (purge execs (current s-f)))
      proof (cases run n (Some s0) execs)
        case None
          thus ?thesis using 1 unfolding NI-unrelated-total-def strict-equal-def by auto
        next
        case (Some s-f)
          from realistic-purge assms invariant-s0 realistic run-total-equals-run[where n=n and s=s0 and execs=purge execs (current s-f)]
          have 2: strict-equal (run n (Some s0) (purge execs (current s-f))) (run-total n s0 (purge execs (current s-f)))
            by auto
          show ?thesis proof (cases run n (Some s0) (purge execs (current s-f)))
            case None
              from 2 None show ?thesis using 2 unfolding NI-unrelated-total-def strict-equal-def by auto
            next
            case (Some s-f2)
              from run n (Some s0) execs = (Some s-f) Some 1 2 secure-partial[unfolded NI-unrelated-def,THEN spec,THEN spec,THEN spec,where n=n and x2=execs]
              show ?thesis
                unfolding strict-equal-def NI-unrelated-def
                by(simp add: Let-def B-def B2-def)
              qed
            qed
          } thus ?thesis unfolding NI-unrelated-total-def by auto
      qed
  have NI-indirect-sources-total: NI-indirect-sources-total
  proof−
    { fix execs a n
      assume realistic: realistic-executions execs
      from assms invariant-s0 realistic run-total-equals-run[where n=n and s=s0 and execs=execs]
      have 1: strict-equal (run n (Some s0) execs) (run-total n s0 execs) by auto

      have let s-f = run-total n s0 execs in output-f (run-total n s0 (ipurge-l execs (current s-f))) a = output-f
3.4 CISK (Controlled Interruptible Separation Kernel)

theory CISK
  imports ISK
begin

This section presents a generic model of a Controlled Interruptible Separation Kernel (CISK). It formulates security, i.e., intransitive noninterference. For a presentation of this model, see Section 2 of [31].

First, a locale is defined that defines all generic functions and all proof obligations (see Section 2.3 of [31]).
locale Controllable-Interruptible-Separation-Kernel = — CISK

fixes kstep :: 'state-t ⇒ 'action-t ⇒ 'state-t — Executes one atomic kernel action
and output-f :: 'state-t ⇒ 'action-t ⇒ 'output-t — Returns the observable behavior
and s0 :: 'state-t — The initial state

and current :: 'state-t ⇒ 'dom-t — Returns the currently active domain
and cswitch :: 'time-t ⇒ 'state-t ⇒ 'state-t — Performs a context switch
and interrupt :: 'time-t ⇒ bool — Returns true iff an interrupt occurs in the given state at the given time
and kinvolved :: 'action-t ⇒ 'dom-t set — Returns the set of domains that are involved in the given action
and ifp :: 'dom-t ⇒ 'state-t ⇒ bool — The security policy.
and AS-set :: ('action-list t) set — Returns a set of valid action sequences, i.e., the attack surface
and invariant :: 'state-t ⇒ bool — Returns an inductive state-invariant
and AS-precondition :: 'state-t ⇒ 'dom-t ⇒ 'action-t ⇒ bool — Returns the preconditions under which the next action can be executed.

and aborting :: 'state-t ⇒ 'dom-t ⇒ 'action-t ⇒ bool — Returns true iff the action is aborted.
and waiting :: 'state-t ⇒ 'action-t ⇒ 'state-t ⇒ bool — Returns true iff execution of the given action is delayed.
and set-error-code :: 'state-t ⇒ 'action-t ⇒ 'state-t — Sets an error code when actions are aborted.

assumes vpeq-transitive: ∀ a b c u. (vpeq u a b ∧ vpeq u b c) ⟹ vpeq u a c
and vpeq-symmetric: ∀ a b u. vpeq u a b ⟹ vpeq u b a
and vpeq-reflective: ∀ a u. vpeq u a a
and ifp-reflective: ∀ a . ifp u u

and weakly-step-consistent: ∀ s t u a. vpeq u s t ∧ vpeq (current s) s t ∧ invariant s ∧ AS-precondition s (current s) a ∧ invariant t ∧ AS-precondition t (current t) a ∧ current s = current t — vpeq u (kstep s a) (kstep t a)
and locally-respects: ∀ a s u. ¬ifp (current s) u ∧ invariant s ∧ AS-precondition s (current s) a — vpeq u s (kstep s a)
and output-consistent: ∀ a s t. vpeq (current s) s t ∧ current s = current t — (output-f s a) = (output-f t a)
and step-atomicity: ∀ a s u. current (kstep s a) = current s
and cswitch-independent-of-state: ∀ n s t. current s = current t — current (cswitch n s) = current (cswitch n t)

and cswitch-consistency: ∀ a s t n . vpeq u s t — vpeq u (cswitch n s) (cswitch n t)
and empty-in-AS-set: [] ∈ AS-set
and invariant-s0 invariant s0
and invariant-after-cswitch: ∀ s n . invariant s — invariant (cswitch n s)
and preconditions-after-cswitch: ∀ s d n a. AS-precondition d s a — AS-precondition (cswitch n s) d a
and AS-prec-first-action: ∀ s d aseq . invariant s ∧ aseq ∈ AS-set ∧ aseq ≠ [] — AS-precondition s d (hd aseq)
and AS-prec-after-step: ∀ a s a t . (∃ aseq ∈ AS-set . is-sub-seq a a' aseq) ∧ invariant s ∧ AS-precondition s (current s) a ∧ ¬aborting s (current s) a ∧ ¬waiting s (current s a) — AS-precondition (kstep s a) (current s a a')
and AS-pred-dom-independent: ∀ s d a a' . current s ≠ d ∧ AS-precondition s d a — AS-precondition (kstep s a a') d a
and spec-of-invariant: ∀ a s a . invariant s — invariant (kstep s a)
and aborting-switch-independent: ∀ s n s . aborting (cswitch n s) = aborting s
and aborting-error-update: ∀ s d a a' . current s ≠ d ∧ aborting s d a — aborting (set-error-code s a') d a
and aborting-after-step: ∀ s a d . current s ≠ d — aborting (kstep s a) d = aborting s d

and invariant-after-cswitch: ∀ s n . invariant s — invariant (set-error-code s) a
and set-error-consistent: ∀ s n a . vpeq u s t — vpeq u (set-error-code s) a (set-error-code t a)
and set-error-locally-respects: ∀ s n a u . ¬ifp (current s) u — vpeq u s (set-error-code s) a
and current-set-error-code: ∀ s a . current (set-error-code s) a = current s

and precondition-after-set-error-code: ∀ s d a a' . AS-precondition s d a ∧ aborting s (current s) a' — AS-precondition (set-error-code s a') d a
and invariant-after-set-error-code: ∀ s a . invariant s — invariant (set-error-code s) a

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### 3.4.1 Execution semantics

Control is based on generic functions *aborting*, *waiting* and *set_error_code*. Function *aborting* decides whether a certain action is aborting, given its domain and the state. If so, then function *set_error_code* will be used to update the state, possibly communicating to other domains that an action has been aborted. Function *waiting* can delay the execution of an action. This behavior is implemented in function CISK\_control.

**Function CISK-control:**

\[
\text{CISK-control :: } \text{'state-t} \Rightarrow \text{'dom-t} \Rightarrow \text{'action-t execution} \Rightarrow (\text{'action-t option} \times \text{'action-t execution} \times \text{'state-t})
\]

**where**

1. \(\text{CISK-control s d []} = (\text{None},[],s)\) — The thread is empty
2. \(\text{CISK-control s d ([[]])} = (\text{None},[],s)\) — The current action sequence has been finished and the thread has no next action sequences to execute
3. \(\text{CISK-control s d ([[]#(#execs')] = (\text{None},as#execs',s)}\) — The current action sequence has been finished. Skip to the next sequence

**by** pat-completeness auto

**termination by** lexicographic-order

Function *run* defines the execution semantics. This function is presented in [31] by pseudo code (see Algorithm 1). Before defining the run function, we define accessor functions for the control mechanism. Functions next\_action, next\_execs and next\_state correspond to “control.a”, “control.x” and “control.s” in [31].

**Abstraction**

\[
\text{next-action:'state-t} \Rightarrow (\text{'dom-t} \Rightarrow \text{'action-t execution}) \Rightarrow \text{'action-t option}
\]

**where**

\[
\text{Kernel.next-action current CISK-control}
\]

**Abstraction**

\[
\text{next-exec:'state-t} \Rightarrow (\text{'dom-t} \Rightarrow \text{'action-t execution}) \Rightarrow (\text{'dom-t} \Rightarrow \text{'action-t execution})
\]

**where**

\[
\text{Kernel.next-exec current CISK-control}
\]

**Abstraction**

\[
\text{next-state:'state-t} \Rightarrow (\text{'dom-t} \Rightarrow \text{'action-t execution}) \Rightarrow \text{'state-t}
\]

**where**

\[
\text{Kernel.next-state current CISK-control}
\]

A thread is empty iff either it has no further action sequences to execute, or when the current action sequence is finished and there are no further action sequences to execute.

**Abstraction**

\[
\text{thread-empty:'action-t execution} \Rightarrow \text{bool}
\]

**where**

\[
\text{[] ∨ exec = [[]]}
\]

The following function defines the execution semantics of CISK, using function CISK\_control.

**Function run:**

\[
\text{run :: } \text{time-t} \Rightarrow \text{'state-t} \Rightarrow (\text{'dom-t} \Rightarrow \text{'action-t execution}) \Rightarrow \text{'state-t}
\]

**where**

\[
\text{run 0 s execs = s
}\]

1. \(\text{interrupt (Suc n) \Rightarrow run (Suc n) s execs = run n (cswitch (Suc n) s) execs}\)
2. \(\neg \text{interrupt (Suc n) \Rightarrow thread-empty(execs (current s)) \Rightarrow run (Suc n) s execs = run n s execs}\)
3. \(\neg \text{interrupt (Suc n) \Rightarrow \neg thread-empty(execs (current s)) \Rightarrow run (Suc n) s execs = \text{let control-a = next-action s execs; control-s = next-state s execs; control-x = next-exec s execs in case control-a of None \Rightarrow run n control-s control-x}
\]

**using** not0-implies-Suc by (metis prod-cases3, auto)

**termination by** lexicographic-order
3.4.2 Formulations of security

The definitions of security as presented in Section 2.2 of [31].

abbreviation kprecondition
  where kprecondition s a ≡ invariant s ∧ AS-precondition s (current s) a

definition realistic-execution
  where realistic-execution aseq ≡ set aseq ⊆ AS-set

definition realistic-executions :: ('dom-t ⇒ 'action-t execution) ⇒ bool
  where realistic-executions execs ≡ ∀ d. realistic-execution (execs d)

abbreviation involved where involved ≡ Kernel.involved

abbreviation step where step ≡ Kernel.step

abbreviation purge where purge ≡ Separation-Kernel.purge realistic-execution

abbreviation ipurge-l where ipurge-l ≡ Separation-Kernel.ipurge-l

abbreviation ipurge-r where ipurge-r ≡ Separation-Kernel.ipurge-r

definition NI-unrelated :: bool
  where NI-unrelated ≡ ∀ execs a n. realistic-executions execs/⇒
    (let s-f = run n s0 execs in
     output-f s-f a = output-f (run n s0 (purge execs (current s-f))) a)

definition NI-indirect-sources :: bool
  where NI-indirect-sources ≡ ∀ execs a n. realistic-executions execs/⇒
    (let s-f = run n s0 execs in
     output-f (run n s0 (ipurge-l execs (current s-f))) a =
     output-f (run n s0 (ipurge-r execs (current s-f))) a)

definition isecure :: bool
  where isecure ≡ NI-unrelated ∧ NI-indirect-sources

3.4.3 Proofs

The final theorem is unwinding_implies_isecure_CISK. This theorem shows that any interpretation of locale CISK is secure.

To prove this theorem, the refinement framework is used. CISK is a refinement of ISK, as the only difference is the control function. In ISK, this function is a generic function called control, in CISK it is interpreted in function CISK.control. It is proven that function CISK.control satisfies all the proof obligations concerning generic function control. In other words, CISK.control is proven to be an interpretation of control. Therefore, all theorems on run_total apply to the run function of CISK as well.

lemma next-action-consistent:
shows ∀ s t execs. vpeq (current s) s t ∧ (∀ d ∈ involved (next-action s execs) . vpeq d s t) ∧ current s = current t → next-action s execs = next-action t execs
proof-
{ fix s t execs
  assume vpeq: vpeq (current s) s t
  assume vpeq-involved: ∀ d ∈ involved (next-action s execs) . vpeq d s t
  assume current-s-t: current s = current t
  from aborting-consistent current-s-t vpeq
  have aborting t (current s) = aborting s (current s) by auto
  from current-s-t this waiting-consistent vpeq-involved
  have next-action s execs = next-action t execs
  unfolding Kernel.next-action-def
  by (cases s,(current s),execs (current s)) rule: CISK-control.cases,auto
} thus ⊢thesis by auto
qed
lemma next-execs-consistent:
shows ∀ s t execs . vpeq (current s) s t ∧ (∀ d ∈ involved (next-action s execs) . vpeq d s t) ∧ current s = current t → fst (snd (CISK-control s (current s) (execs (current s)))) = fst (snd (CISK-control t (current s) (execs (current s))))
proof-
{ fix s t execs
  assume vpeq: vpeq (current s) s t
  assume vpeq-involved: ∀ d ∈ involved (next-action s execs) . vpeq d s t
  assume current-s-t: current s = current t
  from aborting-consistent current-s-t vpeq
  have I: aborting t (current s) = aborting s (current s) by auto
  from I vpeq current-s-t vpeq-involved waiting-consistent[THEN spec,THEN spec,THEN spec,THEN spec,where x3=s and x2=t and x1=current s and x=the (next-action s execs)]
  have fst (snd (CISK-control s (current s) (execs (current s)))) = fst (snd (CISK-control t (current s) (execs (current s))))
    unfolding Kernel.next-action-def Kernel.involved-def
    by(cases (s,(current s),execs (current s)) rule: CISK-control.cases,auto split add: split-if-asm)
} thus ?thesis by auto
qed

lemma next-state-consistent:
shows ∀ s t u execs . vpeq (current s) s t ∧ vpeq u s t ∧ current s = current t → vpeq u (next-state s execs)
(next-state t execs)
proof-
{ fix s t u execs
  have vpeq u (next-state s execs) (next-state t execs)
    unfolding Kernel.next-state-def
    using aborting-consistent set-error-consistent
    by(cases (s,(current s),execs (current s)) rule: CISK-control.cases,auto)
} thus ?thesis by auto
qed

lemma current-next-state:
shows ∀ s execs . current (next-state s execs) = current s
proof-
{ fix s execs
  have current (next-state s execs) = current s
    unfolding Kernel.next-state-def
    using current-set-error-code
    by(cases (s,(current s),execs (current s)) rule: CISK-control.cases,auto)
} thus ?thesis by auto
qed

lemma locally-respects-next-state:
shows ∀ s u execs . ¬ifp (current s) u → vpeq u s (next-state s execs)
proof-
{
fix \( s \) \( u \) \( execs \)
assume \( \neg \text{ifp} \) (current \( s \)) \( u \)
hence \( \text{vpeq} \) \( u \) \( s \) \( \text{next-state} \) \( s \) \( \text{execs} \)

unfolding Kernel.next-state-def
using \( \text{vpeq-reflexive} \) \( \text{set-error-locally-respects} \)
by (cases \( (s, \text{current} \ s) \), execs (current \( s \))) \hspace{1em} \text{rule: CISK-control.cases,auto}

\}
thus \(?\text{thesis}\) by auto
qed

**lemma** CISK-control-spec:
shows \( \forall \) \( s \) \( d \) aseqs.

case CISK-control \( s \) \( d \) aseqs of
\( (a, \text{aseqs}', s') \Rightarrow \)

thread-empty aseqs \( \land \) \( (a, \text{aseqs}') = (\text{None}, \text{[]}) \lor \)

aseqs \( \neq \text{[]} \land \text{hd} \) aseqs \( \neq \text{[]} \land \neg \) aborting \( s' \) \( d \) \( \text{(the a)} \land \neg \) waiting \( s' \) \( d \) \( \text{(the a)} \land \neg \) \( a, \text{aseqs}' \) = (Some \( (\text{hd} \) \text{a}

\( \text{aseqs})), \text{tl} \) \( (\text{hd} \) \text{aseqs}) \# \text{tl} \) \text{aseqs} \) \lor 

aseqs \( \neq \text{[]} \land \text{hd} \) aseqs \( \neq \text{[]} \land \text{waiting} \) \( s' \) \( d \) \( \text{(the a)} \land \neg \) \( a, \text{aseqs}' \), \( s' \) = (Some \( (\text{hd} \) \text{a}

\( \text{aseqs}), \text{aseqs}, s) \lor \) \( a, \text{aseqs}' = (\text{None}, \text{tl} \) \text{aseqs})

proof~
\{
fix \( s \) \( d \) aseqs
have case CISK-control \( s \) \( d \) aseqs of
\( (a, \text{aseqs}', s') \Rightarrow \)

thread-empty aseqs \( \land \) \( (a, \text{aseqs}') = (\text{None}, \text{[]}) \lor \)

aseqs \( \neq \text{[]} \land \text{hd} \) aseqs \( \neq \text{[]} \land \neg \) aborting \( s' \) \( d \) \( \text{(the a)} \land \neg \) waiting \( s' \) \( d \) \( \text{(the a)} \land \neg \) \( a, \text{aseqs}' \) = (Some \( (\text{hd} \) \text{a}

\( \text{aseqs})), \text{tl} \) \( (\text{hd} \) \text{aseqs}) \# \text{tl} \) \text{aseqs} \) \lor 

aseqs \( \neq \text{[]} \land \text{hd} \) aseqs \( \neq \text{[]} \land \text{waiting} \) \( s' \) \( d \) \( \text{(the a)} \land \neg \) \( a, \text{aseqs}' \), \( s' \) = (Some \( (\text{hd} \) \text{a}

\( \text{aseqs}), \text{aseqs}, s) \lor \) \( a, \text{aseqs}' = (\text{None}, \text{tl} \) \text{aseqs})
by (cases \( (s,d,\text{aseqs})\) \hspace{1em} \text{rule: CISK-control.cases,auto})
\}
thus \(?\text{thesis}\) by auto
qed

**lemma** next-action-after-cswitch:
shows \( \forall \) \( s \) \( n \) \( d \) aseqs . \( \text{fst} \) (CISK-control \( (\text{cswitch n s}) \) \( d \) aseqs) = \( \text{fst} \) (CISK-control \( s \) \( d \) aseqs)

proof~
\{
fix \( s \) \( n \) \( d \) aseqs
have \( \text{fst} \) (CISK-control \( (\text{cswitch n s}) \) \( d \) aseqs) = \( \text{fst} \) (CISK-control \( s \) \( d \) aseqs)
using aborting-switch-independent waiting-switch-independent
by (cases \( (s,d,\text{aseqs})\) \hspace{1em} \text{rule: CISK-control.cases,auto})
\}
thus \(?\text{thesis}\) by auto
qed

**lemma** next-action-after-next-state:
shows \( \forall \) \( s \) \( \text{execs d} \). current \( s \) \( \neq \) \( d \) \( \rightarrow \) \( \text{fst} \) (CISK-control \( (\text{next-state} s \) \( \text{execs} \) \( d \) \( \text{execs} \) \( d \)) = \( \text{None} \) \( \lor \) \( \text{fst} \) (CISK-control \( (\text{next-state} s \) \( \text{execs} \) \( d \) \( \text{execs} \) \( d \)) = \( \text{fst} \) (CISK-control \( s \) \( d \) \( \text{execs} \) \( d \))

proof~
\{
fix \( s \) \( \text{execs d aseqs} \)
assume \( \text{I: current} \ s \neq \) \( d \)
have \( \text{fst} \) (CISK-control \( (\text{next-state} s \) \( \text{execs} \) \( d \) \( \text{aseqs} \)) = \( \text{None} \) \( \lor \) \( \text{fst} \) (CISK-control \( (\text{next-state} s \) \( \text{execs} \) \( d \) \( \text{aseqs} \)) = \( \text{fst} \) (CISK-control \( s \) \( d \) \( \text{aseqs} \)
proof (cases \( (s,d,\text{aseqs})\) \hspace{1em} \text{rule: CISK-control.cases,simp,simp,simp})
case (4 sa da a as execs')
thus ?thesis
  unfolding Kernel.next-state-def
  using aborting-error-update waiting-error-update 1
  by (cases (sa, current sa, execs (current sa)) rule: CISK-control.cases, auto split: split-if-asm)
qed
}
thus ?thesis by auto
qed

lemma next-action-after-step:
shows ∀ s a d aseqs . current s ⊟ d → fst (CISK-control (step s a) d aseqs) = fst (CISK-control s d aseqs)
proof--
{ fix s a d aseqs
  assume 1: current s ⊟ d
  from this aborting-after-step
  have fst (CISK-control (step s a) d aseqs) = fst (CISK-control s d aseqs)
  unfolding Kernel.step-def
  by (cases (s, d, aseqs) rule: CISK-control.cases, simp, simp, simp, cases a, auto)
}
thus ?thesis by auto
qed

lemma next-state-precondition:
shows ∀ s d a execs . AS-precondition s d a → AS-precondition (next-state s execs) d a
proof--
{ fix s d a execs
  assume AS-precondition s d a
  hence AS-precondition (next-state s execs) d a
  unfolding Kernel.next-state-def
  using precondition-after-set-error-code
  by (cases (s, (current s), execs (current s)) rule: CISK-control.cases, auto)
}
thus ?thesis by auto
qed

lemma next-state-invariant:
shows ∀ s execs. invariant s → invariant (next-state s execs)
proof--
{ fix s execs
  assume invariant s
  hence invariant (next-state s execs)
  unfolding Kernel.next-state-def
  using invariant-after-set-error-code
  by (cases (s, (current s), execs (current s)) rule: CISK-control.cases, auto)
}
thus ?thesis by auto
qed

lemma next-action-from-execs:
shows ∀ s execs . next-action s execs → (λ a . a ∈ actions-in-execution (execs (current s)))
proof--
{ fix s execs

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\{
  \text{fix } a \\
  \text{assume } 1: \text{next-action } s \text{ execs} = \text{Some } a \\
  \text{from } \text{have } a \in \text{actions-in-execution } (\text{execs } (\text{current } s)) \\
  \text{unfolding Kernel.next-action-def actions-in-execution-def} \\
  \text{by } (\text{cases } (s, (\text{current } s), \text{execs } (\text{current } s)) \text{ rule: CISK-control.cases,auto split add: split-if-asm}) \\
\}

hence next-action s execs ↩ (\lambda a. a \in \text{actions-in-execution } (\text{execs } (\text{current } s)))

\text{unfolding B-def by } (\text{cases next-action } s \text{ execs,auto})

\}

\text{thus } \text{thesis unfolding B-def by } (\text{auto})

\text{qed}

\textbf{lemma next-execs-subset:}

\textbf{shows } \forall s \text{ execs } u. \text{actions-in-execution } (\text{next-execs } s \text{ execs } u) \subseteq \text{actions-in-execution } (\text{execs } u)

\textbf{proof}\texttt{–}

\{
  \text{fix } s \text{ execs } u \\
  \text{have } \text{actions-in-execution } (\text{next-execs } s \text{ execs } u) \subseteq \text{actions-in-execution } (\text{execs } u) \\
  \text{unfolding Kernel.next-execs-def actions-in-execution-def} \\
  \text{by } (\text{cases } (s, (\text{current } s), \text{execs } (\text{current } s)) \text{ rule: CISK-control.cases,auto split add: split-if-asm}) \\
\}

\text{thus } \text{thesis by } \text{auto}

\text{qed}

\textbf{theorem unwinding-implies-secure-CISK:}

\textbf{shows } \text{isecure}

\textbf{proof}\texttt{–}

\text{interpret int: Interruptible-Separation-Kernel kstep output-f s0 current cswitch interrupt kprecondition realistic-execution CISK-control kinvolved ifp vpeq AS-set invariant AS-precondition aborting waiting}

\textbf{proof } (\text{unfold-locales})

\text{show } \forall a b c u. \text{vpeq } u \text{ a b } \land \text{vpeq } u \text{ b c } \rightarrow \text{vpeq } u \text{ a c}

\text{using vpeq-transitive by } \text{blast}

\text{show } \forall a b u. \text{vpeq } u \text{ a b } \rightarrow \text{vpeq } u \text{ b a}

\text{using vpeq-symmetric by } \text{blast}

\text{show } \forall a u. \text{vpeq } u \text{ a a}

\text{using vpeq-reflexive by } \text{blast}

\text{show } \forall a s u. \text{ifp } u \text{ a}

\text{using ifp-reflexive by } \text{blast}

\text{show } \forall a s t u. \text{vpeq } s \text{ t u } \land \text{vpeq } (\text{current } s) \text{ s t } \land \text{kprecondition } s \text{ a } \land \text{kprecondition } t \text{ a } \land \text{current } s = \text{current } t \\
\rightarrow \text{vpeq } u \text{ (kstep } s \text{ a) (kstep } t \text{ a)}

\text{using weakly-step-consistent by } \text{blast}

\text{show } \forall a s t u. \text{~ifp } (\text{current } s) \text{ u } \land \text{kprecondition } s \text{ a } \rightarrow \text{vpeq } u \text{ s (kstep } s \text{ a)}

\text{using locally-respects by } \text{blast}

\text{show } \forall a s t. \text{vpeq } (\text{current } s) \text{ s t } \land \text{current } s = \text{current } t \\
\rightarrow (\text{output-f s a) = (output-f t a)}

\text{using output-consistent by } \text{blast}

\text{show } \forall s a. \text{current } (\text{kstep } s \text{ a) = current s}

\text{using step-atomicity by } \text{blast}

\text{show } \forall n s t. \text{current } s = \text{current } t \\
\rightarrow \text{current } (\text{cswitch } n \text{ s) = current } (\text{cswitch } n \text{ t)}

\text{using cswitch-independent-of-state by } \text{blast}

\text{show } \forall u s t n. \text{vpeq } u \text{ s t } \\
\rightarrow \text{vpeq } u \text{ (cswitch } n \text{ s) (cswitch } n \text{ t)}

\text{using cswitch-consistency by } \text{blast}

\text{show } \forall s t. \text{execs } \text{vpeq } (\text{current } s) \text{ s t } \land \text{(\forall d } \text{involved } (\text{next-action } s \text{ execs)} \cdot \text{vpeq } d \text{ s t)} \\
\land \text{current } s = \text{current } t \\
\rightarrow \text{next-action } s \text{ execs } = \text{next-action } t \text{ execs}

\text{using next-action-consistent by } \text{blast}
\[
\begin{align*}
\text{show } & \forall s \ t \ \text{execs}. \\
& \text{vpeq } (current s) \ s \ t \wedge (\forall d \in \text{involved } (next-action s \ \text{execs})). \ \text{vpeq } d \ s \ t \wedge current s = current t \rightarrow \\
& \ \text{fst } (\text{snd } (\text{CISK-control } s \ (current s)) \ (\text{execs } (current s)))) = \text{fst } (\text{snd } (\text{CISK-control } t \ (current s)) \ (\text{execs } (current s)))) \\
\text{using next-execs-consistent by blast} \\
\text{show } & \forall s \ t \ u \ \text{execs}. \ \text{vpeq } (current s) \ s \ t \wedge \text{vpeq } u \ s \ t \wedge current t = current u \rightarrow \text{vpeq } u \ (next-state s \ \text{execs}) \\
\text{(next-state } t \ \text{execs}) \\
\text{using next-state-consistent by auto} \\
\text{show } & \forall s \ \text{execs}. \ current(s) \ (\text{next-state } s \ \text{execs}) = current s \\
\text{using current-next-state by auto} \\
\text{show } & \forall s \ u \ \text{execs}. \ \sim \text{ifp } (current s) \ u \rightarrow \text{vpeq } u \ (next-state s \ \text{execs}) \\
\text{using locally-respects-next-state by auto} \\
\text{show } & [] \in \text{AS-set} \\
\text{using empty-in-AS-set by blast} \\
\text{show } & \forall s \ n. \ \text{invariant } s \rightarrow \text{invariant } (\text{cswitch } n \ s) \\
\text{using invariant-after-cswitch by blast} \\
\text{show } & \forall s \ d \ n \ a. \ \text{AS-precondition } s \ d \ a \rightarrow \text{AS-precondition } (\text{cswitch } n \ s) \ d \ a \\
\text{using precondition-after-cswitch by blast} \\
\text{show } & \text{invariant } s \text{0} \\
\text{using invariant-0 by blast} \\
\text{show } & \forall s \ a \ a'. \ (\exists a \text{seq}\in\text{AS-set}. \ \text{is-sub-seq } a \ a') \wedge \text{invariant } s \wedge \text{AS-precondition } s \ (current s) \ a \wedge \sim \text{aborting } s \ (current s) \ a \rightarrow \\
& \text{AS-precondition } (kstep s \ a) \ (current s) \ a' \\
\text{using AS-prec-after-step by blast} \\
\text{show } & \forall s \ d \ a a'. \ current s < d \wedge \text{AS-precondition } s \ d \ a \rightarrow \text{AS-precondition } (kstep s a') \ d \ a \\
\text{using AS-prec-dom-independent by blast} \\
\text{show } & \forall s \ a. \ \text{invariant } s \rightarrow \text{invariant } (\text{kstep } s \ a) \\
\text{using spec-of-invariant by blast} \\
\text{show } & \forall s \ a. \ \text{kprecondition } s \ a \equiv \text{kprecondition } s \ a \\
\text{by auto} \\
\text{show } & \forall s \ a. \ \text{realistic-execution } a \text{seq} = \text{set } a \text{seq} \subseteq \text{AS-set} \\
\text{unfolding realistic-execution-def by auto} \\
\text{show } & \forall s \ a. \ \forall d \in \text{involved } a. \ \text{kprecondition } s \ a \rightarrow \text{ifp } d \ (current s) \\
\text{using involved-ifp unfolding Kernel.involved-def by } (\text{auto split: option.splits}) \\
\text{show } & \forall s \ \text{execs}. \ \text{next-action } s \ \text{execs} \rightarrow (\lambda a. \ a \in \text{actions-in-execution } (\text{execs } (current s))) \\
\text{using next-action-from-execs by blast} \\
\text{show } & \forall s \ \text{execs } u. \ \text{actions-in-execution } (\text{next-execs } s \ \text{execs } u) \subseteq \text{actions-in-execution } (\text{execs } u) \\
\text{using next-execs-subset by blast} \\
\text{show } & \forall s \ d \ \text{aseqs}. \\
\text{case CISK-control } s \ d \ \text{aseqs} \ of \\
(a, a \text{seqs'}, \ s') \Rightarrow \\
\text{thread-empty aseqs } \wedge (a, a \text{seqs'}) = (\text{None, } []) \forall \\
\text{aseqs } \# [] \wedge \text{hd aseqs } \# [] \wedge \sim \text{aborting } s' \ d (\text{the } a) \wedge \sim \text{waiting } s' \ d (\text{the } a) \wedge (a, a \text{seqs'}) = (\text{Some } (\text{hd } (\text{hd aseqs})), \ \text{tl } (\text{hd aseqs}) \ # \ \text{tl aseqs}) \lor \\
\text{aseqs } \# [] \wedge \text{hd aseqs } \# [] \wedge \text{waiting } s' \ d (\text{the } a) \wedge (a, a \text{seqs}', s') = (\text{Some } (\text{hd } (\text{hd aseqs})), \ aseqs, s) \lor (a, a \text{seqs'}) = (\text{None}, \ tl aseqs) \\
\text{using CISK-control-spec by blast} \\
\text{show } & \forall s \ n \ d \ \text{aseqs}. \ \text{fst } (\text{CISK-control } (\text{cswitch } n \ s) \ d \ \text{aseqs}) = \text{fst } (\text{CISK-control } s \ d \ \text{aseqs}) \\
\text{using next-action-after-cswitch by auto} \\
\text{show } & \forall s \ \text{execs } d. \\
\text{current } s \# d \rightarrow \\
\text{fst } (\text{CISK-control } (\text{next-state } s \ \text{execs}) \ d (\text{execs } d)) = \text{None } \lor \text{fst } (\text{CISK-control } (\text{next-state } s \ \text{execs}) \ d (\text{execs } d)) = \text{fst } (\text{CISK-control } s \ d (\text{execs } d)) \\
\text{using next-action-after-next-state by auto} \\
\end{align*}
\]
4 Instantiation by a separation kernel with concrete actions

In the previous section, no concrete actions for the step function were given. The foremost point we want to make by this instantiation is to show that we can instantiate the CISK model of the previous section with an implementation that, for the step function, as actions, provides events and interprocess communication (IPC).

System call invocations that can be interrupted at certain interrupt points are split into several atomic steps. A communication interface of events and IPC is less “trivial” than it may seem at a first glance, for example the L4 microkernel API only provided IPC as communication primitive [16]. In particular, the concrete actions illustrate how an application of the CISK framework can be used to separate policy enforcement from other computations unrelated to policy enforcement.

Our separation kernel instantiation also has a notion of partitions. A partition is a logical unit that serves to encapsulate a group of CISK threads by, amongst others, enforcing a static per-partition access control policy to system resources. In the following instantiation, while the subjects of the step function are individual threads, the
information flow policy ifp is defined at the granularity of partitions, which is realistic for many separation kernel implementations.

Lastly, as a limited manipulation of an access control policy is often needed, we also provide an invariant for having a dynamic access control policy whose maximal closure is bounded by the static per-partition access control policy. That the dynamic access control policy is a subset of a static access control policy is expressed by the invariant sp_subset. A use case for this is when you have statically configured access to files by subjects, but whether a file can be read/written also depends on whether the file has been dynamically opened or not. The instantiation provides infrastructure for such an invariant on the relation of a dynamic policy to a static policy, and shows how the invariant is maintained, without modeling any API for an open/close operation.

### 4.1 Model of a separation kernel configuration

```plaintext
theory Step-configuration
  imports Main
begin

4.1.1 Type definitions

The separation kernel partitions are considered to be the “subjects” of the information flow policy ifp. A file provider is a partition that, via a file API (read/write), provides files to other partitions. The configuration statically defines which partitions can act as a file provider and also the access rights (right/write) of other partitions to the files provided by the file provider. Some separation kernels include a management for address spaces (tasks), that may be hierarchically structured. Such a task hierarchy is not part of this model.

typeddecl partition-id-t
typeddecl thread-id-t
typeddecl page-t — physical address of a memory page
typeddecl filep-t — name of file provider

datatype obj-id-t =
  PAGE page-t
  | FILEP filep-t

datatype mode-t =
  READ — The subject has right to read from the memory page, from the files served by a file provider.
  | WRITE — The subject has right to write to the memory page, from the files served by a file provider.
  | PROVIDE — The subject has right serve as the file provider. This mode is not used for memory pages or ports.

4.1.2 Configuration

The information flow policy is implicitly specified by the configuration. The configuration does not contain the communication rights between partitions (subjects). However, the rights can be derived from the configuration. For example, if two partitions p and p′ can access a file f, then p and p′ can communicate. See below.

consts
  configured-subj-obj :: partition-id-t ⇒ obj-id-t ⇒ mode-t ⇒ bool

  Each user thread belongs to a partition. The relation is fixed at system startup. The configuration specifies how many threads a partition can create, but this limit is not part of the model.

consts
  partition :: thread-id-t ⇒ partition-id-t
end
```
4.2 Formulation of a subject-subject communication policy and an information flow policy, and showing both can be derived from subject-object configuration data

theory Step-policies
  imports Step-configuration
begin

4.2.1 Specification

In order to use CISK, we need an information flow policy $ifp$ relation. We also express a static subject-subject $sp$-spec-subj-obj and subject-object $sp$-spec-subj-subj access control policy for the implementation of the model. The following locale summarizes all properties we need.

locale policy-axioms =
  fixes
  sp-spec-subj-obj :: 'a ⇒ obj-id-t ⇒ mode-t ⇒ bool
and
  sp-spec-subj-subj :: 'a ⇒ 'a ⇒ bool
and
  ifp :: 'a ⇒ 'a ⇒ bool

assumes
  sp-spec-file-provider: ∀ p1 p2 f m1 m2 .
  sp-spec-subj-obj p1 (FILEP f) m1 ∧
  sp-spec-subj-obj p2 (FILEP f) m2 → sp-spec-subj-subj p1 p2

and
  sp-spec-no-wronly-pages:
  ∀ p x . sp-spec-subj-obj p (PAGE x) WRITE → sp-spec-subj-obj p (PAGE x) READ

and
  ifp-reflexive:
  ∀ p . ifp p p

and
  ifp-compatible-with-sp-spec:
  ∀ a b . sp-spec-subj-subj a b → ifp a b ∧ ifp b a

and
  ifp-compatible-with-ipc:
  ∀ a b c x . (sp-spec-subj-subj a b ∧ sp-spec-subj-obj b (PAGE x) WRITE ∧ sp-spec-subj-obj c (PAGE x) READ) → ifp a c

begin end

4.2.2 Derivation

The configuration data only consists of a subject-object policy. We derive the subject-subject policy and the information flow policy from the configuration data and prove that properties we specified in Section 4.2.1 are satisfied.

locale abstract-policy-derivation =
  fixes
  configuration-subj-obj :: 'a ⇒ obj-id-t ⇒ mode-t ⇒ bool
begin

definition
  sp-spec-subj-obj a x m ≡ configuration-subj-obj a x m ∨ (∃ y . x = PAGE y ∧ m = READ ∧ configuration-subj-obj a x WRITE)

definition
  sp-spec-subj-subj a b ≡
  ∃ f m1 m2 . sp-spec-subj-obj a (FILEP f) m1 ∧ sp-spec-subj-obj b (FILEP f) m2

definition
  ifp a b ≡
  sp-spec-subj-subj a b ∨ sp-spec-subj-subj b a
  ∨ (∃ c y . sp-spec-subj-subj a c ∧ sp-spec-subj-obj c (PAGE y) WRITE)
Show that the policies specified in Section 4.2.1 can be derived from the configuration and their definitions.

lemma correct:
shows policy-axioms sp-spec-subj-obj sp-spec-subj-subj ifp
proof (unfold-locales)
show sp-spec-file-provider:
\( \forall \ p1 \ p2 \ f \ m1 \ m2 \ . \)
sp-spec-subj-obj p1 (FILEP f) m1 \( \land \)
sp-spec-subj-obj p2 (FILEP f) m2 \( \rightarrow \) sp-spec-subj-subj p1 p2
unfolding sp-spec-subj-subj-def by auto
show sp-spec-no-wronly-pages:
\( \forall \ p \ x \ . \ sp-spec-subj-obj p (PAGE x) WRITE \rightarrow sp-spec-subj-obj p (PAGE x) READ \)
unfolding sp-spec-subj-obj-def by auto
show ifp-reflexive:
\( \forall \ p \ . \ ifp p p \)
unfolding ifp-def by auto
show ifp-compatible-with-sp-spec:
\( \forall \ a \ b \ . \ sp-spec-subj-subj a b \rightarrow ifp a b \land ifp b a \)
unfolding ifp-def by auto
show ifp-compatible-with-ipc:
\( \forall \ a \ b \ c \ x \ . \ (sp-spec-subj-subj a b \land sp-spec-subj-obj b (PAGE x) WRITE \land sp-spec-subj-obj c (PAGE x) READ) \rightarrow ifp a c \)
unfolding ifp-def by auto
qed
end

type-synonym sp-subj-subj-t = partition-id-t \Rightarrow partition-id-t \Rightarrow bool
type-synonym sp-subj-obj-t = partition-id-t \Rightarrow obj-id-t \Rightarrow mode-t \Rightarrow bool

interpretation Policy-properties: policy-axioms Policy.sp-spec-subj-obj Policy.sp-spec-subj-subj Policy.ifp
using Policy.correct by auto

lemma example-how-to-use-properties-in-proofs:
shows \( \forall \ p \ . \ Policy.ifp p p \)
using Policy-properties.ifp-reflexive by auto
end

4.3 Separation kernel state and atomic step function

theory Step
imports Step-policies
begin

4.3.1 Interrupt points

To model concurrency, each system call is split into several atomic steps, while allowing interrupts between the steps. The state of a thread is represented by an “interrupt point” (which corresponds to the value of the program counter saved by the system when a thread is interrupted).

datatype ipc-direction-t = SEND | RECV
datatype ipc-stage-t = PREP | WAIT | BUF page-t
**4.3.2 System state**

**typedef** obj-t — value of an object

Each thread belongs to a partition. The relation is fixed (in this instantiation of a separation kernel).

**consts**

partition :: thread-id-t -> partition-id-t

The state contains the dynamic policy (the communication rights in the current state of the system, for example).

**record** thread-t =

- \( \text{ev-counter} \triangleq \text{nat} \) — event counter

**record** state-t =

- \( \text{sp-impl-subj-subj} \triangleq \text{sp-subj-subj-t} \) — current subject-subject policy
- \( \text{sp-impl-subj-obj} \triangleq \text{sp-subj-obj-t} \) — current subject-object policy
- \( \text{current} \triangleq \text{thread-id-t} \) — current thread
- \( \text{obj} :: \text{obj-id-t} \rightarrow \text{obj-t} \) — values of all objects
- \( \text{thread} :: \text{thread-id-t} \rightarrow \text{thread-t} \) — internal state of threads

Later (Section 4.4), the system invariant sp-subset will be used to ensure that the dynamic policies (sp_impl_) are a subset of the corresponding static policies (sp_spec_).

**4.3.3 Atomic step**

**Helper functions** Set new value for an object.

**definition** set-object-value :: obj-id-t => obj-t => state-t => state-t where

\[
\text{set-object-value} \text{ obj-id} \text{ val} \text{ state} = \text{ state}(\text{ obj} := \text{ fun-upd} (\text{ obj} \text{ state} \text{ obj-id} \text{ val})
\]

Return a representation of the opposite direction of IPC communication.

**definition** opposite-ipc-direction :: ipc-direction-t => ipc-direction-t where

\[
\text{opposite-ipc-direction} \text{ dir} = \text{ case dir of SEND \Rightarrow RECV | RECV \Rightarrow SEND}
\]

Add an access right from one partition to an object. In this model, not available from the API, but shows how dynamic changes of access rights could be implemented.

**definition** add-access-right :: partition-id-t => obj-id-t => mode-t => state-t => state-t where

\[
\text{add-access-right} \text{ part-id} \text{ obj-id} \text{ mode} \text{ state} = \text{ state}(\text{ sp-impl-subj-obj} := \lambda q q' q'' . (\text{ part-id} = q \land \text{ obj-id} = q' \land \text{ mode} = q''))
\]

Add a communication right from one partition to another. In this model, not available from the API.

**definition** add-comm-right :: partition-id-t => partition-id-t => state-t => state-t where

\[
\text{add-comm-right} \text{ p} \text{ p'} \text{ state} = \text{ state}(\text{ sp-impl-subj-subj} := \lambda q q' . (p = q \land p' = q') \lor \text{ sp-impl-subj-subj} \text{ s} q q')
\]
Model of IPC system call  We model IPC with the following simplifications:

1. The model contains the system calls for sending an IPC (SEND) and receiving an IPC (RECV), often implementations have a richer API (e.g. combining SEND and RECV in one invocation).
2. We model only a copying (“BUF”) mode, not a memory-mapping mode.
3. The model always copies one page per syscall.

\[
\text{definition ipc-precondition} :: \text{thread-id-t} \Rightarrow \text{ipc-direction-t} \Rightarrow \text{thread-id-t} \Rightarrow \text{page-t} \Rightarrow \text{state-t} \Rightarrow \text{bool}
\]

where

\[
\text{ipc-precondition} \quad \text{tid} \quad \text{dir} \quad \text{partner} \quad \text{page} \quad \text{s} \\
\equiv \text{let} \quad \text{sender} = \text{(case} \quad \text{dir} \quad \text{of} \quad \text{SEND} \quad \Rightarrow \text{tid} \quad \text{divides.alt0} \text{RECV} \quad \Rightarrow \text{partner} \text{in} \\
\text{let} \quad \text{receiver} = \text{(case} \quad \text{dir} \quad \text{of} \quad \text{SEND} \quad \Rightarrow \text{partner} \quad \text{divides.alt0} \text{RECV} \quad \Rightarrow \text{tid} \text{in} \\
\text{(sp-impl-subj-subj s (partition sender) (partition receiver)} \quad \text{local-access-mode}) \\
\wedge \text{sp-impl-subj-obj s (partition tid) (PAGE page) local-access-mode)}
\]

\[
\text{definition atomic-step-ipc} :: \text{thread-id-t} \Rightarrow \text{ipc-direction-t} \Rightarrow \text{ipc-stage-t} \Rightarrow \text{thread-id-t} \Rightarrow \text{page-t} \Rightarrow \text{state-t} \Rightarrow \text{state-t}
\]

where

\[
\text{atomic-step-ipc} \quad \text{tid} \quad \text{dir} \quad \text{stage} \quad \text{partner} \quad \text{page} \quad \text{s} \\
\equiv \text{case} \quad \text{stage} \quad \text{of} \\
\text{PREP} \Rightarrow \text{s} \quad / \text{divides.alt0} \text{WAIT} \Rightarrow \text{s} \quad / \text{divides.alt0} \text{BUF page'} \Rightarrow \text{(case} \quad \text{dir} \quad \text{of} \\
\text{SEND} \Rightarrow \text{(set-object-value (PAGE page') (obj s (PAGE page)) s)} \quad / \text{divides.alt0} \text{RECV} \Rightarrow \text{s})
\]

Model of event syscalls  \[
\text{definition ev-signal-precondition} :: \text{thread-id-t} \Rightarrow \text{thread-id-t} \Rightarrow \text{state-t} \Rightarrow \text{bool}
\]

where

\[
\text{ev-signal-precondition} \quad \text{tid} \quad \text{partner} \quad \text{s} \\
\equiv \text{(sp-impl-subj-subj s (partition tid) (partition partner))}
\]

\[
\text{definition atomic-step-ev-signal} :: \text{thread-id-t} \Rightarrow \text{thread-id-t} \Rightarrow \text{state-t} \Rightarrow \text{state-t}
\]

where

\[
\text{atomic-step-ev-signal} \quad \text{tid} \quad \text{partner} \quad \text{s} = \text{fun-upd} (\text{thread s partner} (\text{thread s tid}) (\text{ev-counter} := \text{Suc} (\text{ev-counter}) )) / \text{divides.alt0}
\]

\[
\text{definition atomic-step-ev-wait-one} :: \text{thread-id-t} \Rightarrow \text{state-t} \Rightarrow \text{state-t}
\]

where

\[
\text{atomic-step-ev-wait-one} \quad \text{tid} \quad \text{s} = \text{fun-upd} (\text{thread s tid}) (\text{ev-counter} := (\text{ev-counter} (\text{thread s tid}) - 1) / \text{divides.alt0}) / \text{divides.alt0}
\]

\[
\text{definition atomic-step-ev-wait-all} :: \text{thread-id-t} \Rightarrow \text{state-t} \Rightarrow \text{state-t}
\]

where

\[
\text{atomic-step-ev-wait-all} \quad \text{tid} \quad \text{s} = \text{fun-upd} (\text{thread s tid}) (\text{ev-counter} := 0 / \text{divides.alt0}) / \text{divides.alt0}
\]

Instantiation of CISK aborting and waiting  In this instantiation of CISK, the aborting function is used to indicate security policy enforcement. An IPC call aborts in its PREP stage if the precondition for the calling thread does not hold. An event signal call aborts in its EV-SIGNAL-PREP stage if the precondition for the calling thread does not hold.

\[
\text{definition aborting} :: \text{state-t} \Rightarrow \text{thread-id-t} \Rightarrow \text{int-point-t} \Rightarrow \text{bool}
\]

where

\[
\text{aborting} \quad \text{s} \quad \text{tid} \quad \text{a} \equiv \text{case} \quad \text{a} \quad \text{of} \text{SK-IPC dir PREP partner page} \Rightarrow \text{fun-upd} (\text{thread s tid}) (\text{ev-counter} := 0 / \text{divides.alt0}) / \text{divides.alt0}
\]
The *waiting* function is used to indicate synchronization. An IPC call waits in its WAIT stage while the precondition for the partner thread does not hold. An EV\_WAIT call waits until the event counter is not zero.

**Definition** waiting :: state-t ⇒ thread-id-t ⇒ int-point-t ⇒ bool

**Where**

- case a of SK-IPC dir WAIT partner page ⇒
  - ~ipc-precondition partner (opposite-ipc-direction dir) tid (SOME page\’. True) s
  - SK-EV-WAIT EV-PREP - ⇒ False
  - SK-EV-WAIT EV-WAIT - ⇒ ev-counter (thread s tid) = 0
  - SK-EV-WAIT EV-FINISH - ⇒ False
  - ⇒ False

**The atomic step function.** In the definition of atomic-step the arguments to an interrupt point are not taken from the thread state – the argument given to atomic-step could have an arbitrary value. So, seen in isolation, atomic-step allows more transitions than actually occur in the separation kernel. However, the CISK framework (1) restricts the atomic step function by the waiting and aborting functions as well (2) the set of realistic traces as attack sequences rAS-set (Section 4.8). An additional condition is that (3) the dynamic policy used in aborting is a subset of the static policy. This is ensured by the invariant sp-subset.

**Definition** atomic-step :: state-t ⇒ int-point-t ⇒ state-t

**Where**

- case ipt of
  - SK-IPC dir stage partner page ⇒
    - atomic-step-ipc (current s) dir stage partner page s
    - SK-EV-WAIT EV-PREP consume ⇒ s
    - SK-EV-WAIT EV-WAIT consume ⇒ s
    - SK-EV-WAIT EV-FINISH consume ⇒
      - case consume of
        - EV-CONSUME-ONE ⇒ atomic-step-ev-wait-one (current s) s
        - EV-CONSUME-ALL ⇒ atomic-step-ev-wait-all (current s) s
        - SK-EV-SIGNAL EV-SIGNAL-PREP partner ⇒ s
        - SK-EV-SIGNAL EV-SIGNAL-FINISH partner ⇒
          - atomic-step-ev-signal (current s) partner s
          - NONE ⇒ s

**End**

### 4.4 Preconditions and invariants for the atomic step

**Theory** Step-invariants

**Imports** Step

**Begin**

The dynamic/implementation policies have to be compatible with the static configuration.

**Definition** sp-subset s ≡

(∀ p1 p2 . sp-impl-subj-subj s p1 p2 → Policy.sp-spec-subj-subj p1 p2) ∧ (∀ p1 p2 m . sp-impl-subj-obj s p1 p2 m → Policy.sp-spec-subj-obj p1 p2 m)

The following predicate expresses the precondition for the atomic step. The precondition depends on the type of the atomic action.
definition atomic-step-precondition :: state-t ⇒ thread-id-t ⇒ int-point-t ⇒ bool where
atomic-step-precondition s tid ipt ≡
case ipt of
  SK-IPC dir WAIT partner page ⇒
    (* the thread managed it past PREP stage *)
  ipc-precondition tid dir partner page s
| SK-IPC dir (BUF page') partner page ⇒
    (* both the calling thread and its communication partner
      managed it past PREP and WAIT stages *)
  ipc-precondition tid dir partner page s
  ∧ ipc-precondition partner (opposite-ipc-direction dir) tid page' s
| - ⇒
    (* No precondition for other interrupt points. *)
  True

The invariant to be preserved by the atomic step function. The invariant is independent from the type
of the atomic action.

definition atomic-step-invariant :: state-t ⇒ bool where
atomic-step-invariant s ≡
sp-subset s

4.4.1 Atomic steps of SK_IPC preserve invariants

lemma set-object-value-invariant:
  shows atomic-step-invariant s = atomic-step-invariant (set-object-value ob va s)
proof −
  show ?thesis using assms
    unfolding atomic-step-invariant-def atomic-step-precondition-def ipc-precondition-def
    sp-subset-def set-object-value-def Let-def
    by (simp split add: int-point-t.splits ipc-stage-t.splits ipc-direction-t.splits)
qed

lemma set-thread-value-invariant:
  shows atomic-step-invariant s = atomic-step-invariant (s [thread := thrst])
proof −
  show ?thesis using assms
    unfolding atomic-step-invariant-def atomic-step-precondition-def ipc-precondition-def
    sp-subset-def set-object-value-def Let-def
    by (simp split add: int-point-t.splits ipc-stage-t.splits ipc-direction-t.splits)
qed

lemma atomic-ipc-preserves-invariants:
  fixes s :: state-t
  and tid :: thread-id-t
  assumes atomic-step-invariant s
  shows atomic-step-invariant (atomic-step-ipc tid dir stage partner page s)
proof −
  show ?thesis
  proof (cases stage)
  case PREP
    from this assms show ?thesis
      unfolding atomic-step-ipc-def atomic-step-invariant-def by auto
  next
  case WAIT
    from this assms show ?thesis
      unfolding atomic-step-ipc-def atomic-step-invariant-def by auto
next
case BUF
  show ?thesis
  using assms BUF set-object-value-invariant
  unfolding atomic-step-ipc-def
  by (simp split add: ipc-direction-t.splits)
qed
qed

lemma atomic-ev-wait-one-preserves-invariants:
  fixes s :: state-t
  and tid :: thread-id-t
  assumes atomic-step-invariant s
  shows atomic-step-invariant (atomic-step-ev-wait-one tid s)
proof –
  from assms show ?thesis
  unfolding atomic-step-ev-wait-one-def atomic-step-invariant-def sp-subset-def
  by auto
qed

lemma atomic-ev-wait-all-preserves-invariants:
  fixes s :: state-t
  and tid :: thread-id-t
  assumes atomic-step-invariant s
  shows atomic-step-invariant (atomic-step-ev-wait-all tid s)
proof –
  from assms show ?thesis
  unfolding atomic-step-ev-wait-all-def atomic-step-invariant-def sp-subset-def
  by auto
qed

lemma atomic-ev-signal-preserves-invariants:
  fixes s :: state-t
  and tid :: thread-id-t
  assumes atomic-step-invariant s
  shows atomic-step-invariant (atomic-step-ev-signal tid partner s)
proof –
  from assms show ?thesis
  unfolding atomic-step-ev-signal-def atomic-step-invariant-def sp-subset-def
  by auto
qed

4.4.2 Summary theorems on atomic step invariants
Now we are ready to show that an atomic step from the current interrupt point in any thread preserves invariants.

theorem atomic-step-preserves-invariants:
  fixes s :: state-t
  and tid :: thread-id-t
  assumes atomic-step-invariant s
  shows atomic-step-invariant (atomic-step s a)
proof (cases a)
  case SK-IPC
    then show ?thesis unfolding atomic-step-def
    using assms atomic-ipc-preserves-invariants
    by simp
  next case (SK-EV-WAIT ev-wait-stage consume)
then show \( \text{thesis} \)
proof (cases consume)
case EV-CONSUME-ALL
then show \( \text{thesis} \) unfolding atomic-step-def
using SK-EV-WAIT assms atomic-ev-wait-all-preserves-invariants
by (simp split: ev-wait-stage-t.splits)
next case EV-CONSUME-ONE
then show \( \text{thesis} \) unfolding atomic-step-def
using SK-EV-WAIT assms atomic-ev-wait-one-preserves-invariants
by (simp split: ev-wait-stage-t.splits)
qed
next case SK-EV-SIGNAL
then show \( \text{thesis} \) unfolding atomic-step-def
using assms atomic-ev-signal-preserves-invariants
by (simp add: ev-signal-stage-t.splits)
next case NONE
then show \( \text{thesis} \) unfolding atomic-step-def
by auto
qed

Finally, the invariants do not depend on the current thread. That is, the context switch preserves the invariants, and an atomic step that is not a context switch does not change the current thread.

\textbf{Theorem} cswitch-preserves-invariants:
\begin{verbatim}
fixes s :: state-t
and new-current :: thread-id-t
assumes atomic-step-invariant s
shows atomic-step-invariant (s ( current := new-current ))
proof
  let \( ?s1 \) = s ( current := new-current )
  have sp-subset s = sp-subset \( ?s1 \)
  unfolding sp-subset-def by auto
  from assms this show \( \text{thesis} \)
  unfolding atomic-step-invariant-def by metis
qed
\end{verbatim}

\textbf{Theorem} atomic-step-does-not-change-current-thread:
\begin{verbatim}
shows current (atomic-step s ipt) = current s
proof --
  show \( \text{thesis} \)
  unfolding atomic-step-def
  and atomic-step-ipc-def
  and set-object-value-def Let-def
  and atomic-step-ev-wait-one-def atomic-step-ev-wait-all-def
  and atomic-step-ev-signal-def
  by (simp split add: int-point-t.splits ipc-stage-t.splits ipc-direction-t.splits
    ev-consume-t.splits ev-wait-stage-t.splits ev-signal-stage-t.splits)
qed
end

4.5 The view-partitioning equivalence relation

\textbf{Theory} Step-ypeq
\begin{verbatim}
imports Step Step-invariants
begin
  The view consists of
\end{verbatim}
1. View of object values.

2. View of subject-subject dynamic policy. The threads can discover the policy at runtime, e.g. by calling ipc() and observing success or failure.

3. View of subject-object dynamic policy. The threads can discover the policy at runtime, e.g. by calling open() and observing success or failure.

**Definition**

\[
\text{vpeq-obj} :: \text{partition-id-t} \Rightarrow \text{state-t} \Rightarrow \text{state-t} \Rightarrow \text{bool}
\]

where

\[
\text{vpeq-obj} \ x \ s \ t \equiv \forall \ \text{obj-id}. \ (\text{Policy} . \text{sp-spec-subj-obj} \ x \ \text{obj-id} \ \text{READ} \rightarrow (\text{obj} \ s) \ \text{obj-id} = (\text{obj} \ t) \ \text{obj-id})
\]

**Definition**

\[
\text{vpeq-subj-subj} :: \text{partition-id-t} \Rightarrow \text{state-t} \Rightarrow \text{state-t} \Rightarrow \text{state-t} \Rightarrow \text{bool}
\]

where

\[
\text{vpeq-subj-subj} \ x \ s \ t \equiv \forall \ v. \ ((\text{Policy} . \text{sp-spec-subj-subj} \ x \ v \rightarrow \text{sp-impl-subj-subj} \ s \ u \ v = \text{sp-impl-subj-subj} \ t \ u \ v) \\
\land \ (\text{Policy} . \text{sp-spec-subj-subj} \ v \ u \rightarrow \text{sp-impl-subj-subj} \ s \ v \ u = \text{sp-impl-subj-subj} \ t \ v \ u))
\]

**Definition**

\[
\text{vpeq-subj-obj} :: \text{partition-id-t} \Rightarrow \text{state-t} \Rightarrow \text{state-t} \Rightarrow \text{bool}
\]

where

\[
\text{vpeq-subj-obj} \ x \ s \ t \equiv \forall \ \text{ob m}. \ ((\text{Policy} . \text{sp-spec-subj-obj} \ x \ \text{ob m} \ \text{READ} \rightarrow \text{sp-impl-subj-obj} \ s \ \text{ob m} = \text{sp-impl-subj-obj} \ t \ \text{ob m}) \\
\land \ (\text{Policy} . \text{sp-spec-subj-obj} \ \text{ob m} \ \text{WRITE} \rightarrow \text{sp-impl-subj-obj} \ s \ \text{ob m} = \text{sp-impl-subj-obj} \ t \ \text{ob m}))
\]

**Definition**

\[
\text{vpeq-local} :: \text{partition-id-t} \Rightarrow \text{state-t} \Rightarrow \text{state-t} \Rightarrow \text{bool}
\]

where

\[
\text{vpeq-local} \ x \ s \ t \equiv \forall \ \text{tid}. \ (\text{partition} \ \text{tid}) = u \rightarrow (\text{thread} \ s \ \text{tid}) = (\text{thread} \ t \ \text{tid})
\]

**4.5.1 Elementary properties**

**Lemma** vpeq-rel:

shows vpeq-refl: vpeq u s s

and vpeq-sym [sym]: vpeq u s t \implies vpeq u t s

and vpeq-trans [trans]: [vpeq u s1 s2 ; vpeq u s2 s3] \implies vpeq u s1 s3

unfolding vpeq-def vpeq-obj-def vpeq-subj-subj-def vpeq-subj-obj-def vpeq-local-def

by auto

Auxiliary equivalence relation.

**Lemma** set-object-value-ign:

assumes eq-obs: " Policy.sp-spec-subj-obj u x \ READ

shows vpeq u s (set-object-value x y s)

proof –

from assms show ?thesis

unfolding vpeq-def vpeq-obj-def vpeq-subj-subj-def vpeq-subj-obj-def vpeq-local-def

by auto

qed

Context-switch and fetch operations are also consistent with vpeq and locally respect everything.

**Theorem** cswitch-consistency-and-respect:

fixes u :: partition-id-t

and s :: state-t

and new-current :: thread-id-t
assumes atomic-step-invariant \( s \)
shows \( \text{vpeq } u \; s \; (s \; (\text{current} := \text{new-current} )) \)
proof – 
show ?thesis
unfolding vpeq-def vpeq-obj-def vpeq-subj-subj-def vpeq-subj-obj-def vpeq-local-def
by auto
qed

end

4.6 Atomic step locally respects the information flow policy

theory Step-vpeq-locally-respects
imports Step Step-invariants Step-vpeq
begin

The notion of locally respects is common usage. We augment it by assuming that the atomic-step-invariant holds (see [31]).

4.6.1 Locally respects of atomic step functions

lemma ipc-respects-policy.
assumes nos \sim Policy.ifp (partition tid) \( u \)
and inv: atomic-step-invariant \( s \)
and prec: atomic-step-precondition \( s \; \text{tid} \) (SK-IPC dir stage partner \( \text{pag} \))
and ipt-case: ipt = SK-IPC dir stage partner page
shows vpeq \( u \; s \; (\text{atomic-step-ipc } \text{tid} \; \text{dir} \; \text{stage} \; \text{partner} \; \text{page} \; \text{s} ) \)
proof(cases stage)
case PREP
thus ?thesis
unfolding atomic-step-ipc-def
using vpeq-refl by simp
next
case WAIT
thus ?thesis
unfolding atomic-step-ipc-def
using vpeq-refl by simp
next case (BUF mypage)
show ?thesis
proof(cases dir)
case RECV
thus ?thesis
unfolding atomic-step-ipc-def
using vpeq-refl BUF by simp
next
case SEND
have Policy.sp-spec-subj-subj (partition tid) (partition partner)
and Policy.sp-spec-subj-obj (partition partner) (PAGE mypage) WRITE
using BUF SEND inv prec ipt-case
unfolding atomic-step-invariant-def sp-subset-def
unfolding atomic-step-precondition-def ipc-precondition-def opposite-ipc-direction-def
by auto
hence \sim Policy.sp-spec-subj-obj \( u \) (PAGE mypage) READ
using no Policy-properties.ifp-compatible-with-ipc
by auto
thus \(?thesis
using BUF SEND assms
unfolding atomic-step-ipc-def set-object-value-def
unfolding vpeq-def vpeq-obj-def vpeq-subj-obj-def vpeq-subj-subj-def vpeq-local-def
by auto
qed
qed

lemma ev-signal-respects-policy:
assumes no :~ Policy.ifp (partition tid) u
and inv: atomic-step-invariant s
and prec: atomic-step-precondition s tid (SK-EV-SIGNAL EV-SIGNAL-FINISH partner)
and ipt-case: ipt = SK-EV-SIGNAL EV-SIGNAL-FINISH partner
shows vpeq u s (atomic-step-ev-signal tid partner s)
proof
  from inv no have ~ sp-impl-subj-subj s (partition tid) u
  unfolding Policy.ifp-def atomic-step-invariant-def sp-subset-def
  by auto
  with prec have 1{ (partition partner) # u
  unfolding atomic-step-precondition-def ev-signal-precondition-def
  by (auto simp add: ev-signal-stage-t.splits)
  then have 2:vpeq-local u s (atomic-step-ev-signal tid partner s)
  unfolding vpeq-local-def atomic-step-ev-signal-def
  by simp
  have 3:vpeq-obj u s (atomic-step-ev-signal tid partner s)
  unfolding vpeq-obj-def atomic-step-ev-signal-def
  by simp
  have 4:vpeq-subj-subj u s (atomic-step-ev-signal tid partner s)
  unfolding vpeq-subj-subj-def atomic-step-ev-signal-def
  by simp
  have 5:vpeq-subj-obj u s (atomic-step-ev-signal tid partner s)
  unfolding vpeq-subj-obj-def atomic-step-ev-signal-def
  by simp
  with 2 3 4 5 show \(?thesis
  unfolding vpeq-def
  by simp
  qed

lemma ev-wait-all-respects-policy:
assumes no :~ Policy.ifp (partition tid) u
and inv: atomic-step-invariant s
and prec: atomic-step-precondition s tid ipt
and ipt-case: ipt = SK-EV-WAIT ev-wait-stage EV-CONSUME-ALL
shows vpeq u s (atomic-step-ev-wait-all tid s)
proof
  from assms have 1{ (partition tid) # u
  unfolding Policy.ifp-def
  by simp
  then have 2:vpeq-local u s (atomic-step-ev-wait-all tid s)
  unfolding vpeq-local-def atomic-step-ev-wait-all-def
  by simp
  have 3:vpeq-obj u s (atomic-step-ev-wait-all tid s)
  unfolding vpeq-obj-def atomic-step-ev-wait-all-def
  by simp
  have 4:vpeq-subj-subj u s (atomic-step-ev-wait-all tid s)
  unfolding vpeq-subj-subj-def atomic-step-ev-wait-all-def
  by simp
have 5: vpeq-subj-obj u s (atomic-step-ev-wait-all tid s)
unfolding vpeq-subj-obj-def atomic-step-ev-wait-all-def
by simp
with 2 3 4 5 show ?thesis
unfolding vpeq-def
by simp
qed

lemma ev-wait-one-respects-policy:
assumes no: ~ Policy.ifp (partition tid) u
and inv: atomic-step-invariant s
and prec: atomic-step-precondition s tid ipt
and ipt-case: ipt = SK-EV-WAIT ev-wait-stage EV-CONSUME-ONE
shows vpeq u s (atomic-step-ev-wait-one tid s)
proof –
from assms have 1: (partition tid) \not\in u
unfolding Policy.ifp-def
by simp
then have 2: vpeq-local u s (atomic-step-ev-wait-one tid s)
unfolding vpeq-local-def atomic-step-ev-wait-one-def
by simp
have 3: vpeq-obj u s (atomic-step-ev-wait-one tid s)
unfolding vpeq-obj-def atomic-step-ev-wait-one-def
by simp
have 4: vpeq-subj-subj u s (atomic-step-ev-wait-one tid s)
unfolding vpeq-subj-subj-def atomic-step-ev-wait-one-def
by simp
have 5: vpeq-subj-obj u s (atomic-step-ev-wait-one tid s)
unfolding vpeq-subj-obj-def atomic-step-ev-wait-one-def
by simp
with 2 3 4 5 show ?thesis
unfolding vpeq-def
by simp
qed

4.6.2 Summary theorems on view-partitioning locally respects

Atomic step locally respects the information flow policy (ifp). The policy ifp is not necessarily the same as sp_spec_subj_subj.

theorem atomic-step-respects-policy:
assumes no: ~ Policy.ifp (partition (current s)) u
and inv: atomic-step-invariant s
and prec: atomic-step-precondition s (current s) ipt
shows vpeq u s (atomic-step s ipt)
proof –
show ?thesis
using assms ipc-respects-policy vpeq-refl
  ev-signal-respects-policy ev-wait-one-respects-policy
  ev-wait-all-respects-policy
unfolding atomic-step-def
by (auto split add: int-point-t.splits ev-consume-t.splits ev-wait-stage-t.splits ev-signal-stage-t.splits)
qed

end
4.7 Weak step consistency

theory Step-vpeq-weakly-step-consistent
imports Step Step-invariants Step-vpeq
begin

The notion of weak step consistency is common usage. We augment it by assuming that the atomic-step-invariant holds (see [31]).

4.7.1 Weak step consistency of auxiliary functions

lemma ipc-precondition-weakly-step-consistent:
  assumes eq-tid: vpeq (partition tid) s1 s2
  and inv1: atomic-step-invariant s1
  and inv2: atomic-step-invariant s2
  shows ipc-precondition tid dir partner page s1 = ipc-precondition tid dir partner page s2
proof
  let ?sender = case dir of SEND ⇒ tid | RECV ⇒ partner
  let ?receiver = case dir of SEND ⇒ partner | RECV ⇒ tid
  let ?local-access-mode = case dir of SEND ⇒ READ | RECV ⇒ WRITE
  let ?A = sp-impl-subj-subj s1 (partition ?sender) (partition ?receiver)
       = sp-impl-subj-subj s2 (partition ?sender) (partition ?receiver)
  let ?B = sp-impl-subj-obj s1 (partition tid) (PAGE page) ?local-access-mode
       = sp-impl-subj-obj s2 (partition tid) (PAGE page) ?local-access-mode

  have A: ?A
    proof (cases Policy.sp-spec-subj-subj (partition ?sender) (partition ?receiver))
      case True
        thus ?A
          using eq-tid unfolding vpeq-def vpeq-subj-subj-def
          by (simp split add: ipc-direction-t.splits)
      next case False
        have sp-subset s1 and sp-subset s2
          using inv1 inv2 unfolding atomic-step-invariant-def sp-subset-def by auto
        hence ¬ sp-impl-subj-subj s1 (partition ?sender) (partition ?receiver)
            and ¬ sp-impl-subj-subj s2 (partition ?sender) (partition ?receiver)
          using False unfolding sp-subset-def by auto
        thus ?A by auto
      qed
    have B: ?B
      proof (cases Policy.sp-spec-subj-obj (partition tid) (PAGE page) ?local-access-mode)
        case True
          thus ?B
            using eq-tid unfolding vpeq-def vpeq-subj-obj-def
            by (simp split add: ipc-direction-t.splits)
        next case False
          have sp-subset s1 and sp-subset s2
            using inv1 inv2 unfolding atomic-step-invariant-def sp-subset-def by auto
          hence ¬ sp-impl-subj-obj s1 (partition tid) (PAGE page) ?local-access-mode
              and ¬ sp-impl-subj-obj s2 (partition tid) (PAGE page) ?local-access-mode
            using False unfolding sp-subset-def by auto
          thus ?B by auto
          qed
      show ?thesis using A B unfolding ipc-precondition-def by auto
      qed

lemma ev-signal-precondition-weakly-step-consistent:
  assumes eq-tid: vpeq (partition tid) s1 s2
  shows
and inv1: atomic-step-invariant s1
and inv2: atomic-step-invariant s2
shows ev-signal-precondition tid partner s1 = ev-signal-precondition tid partner s2

proof –

let ?A = sp-impl-subj-subj s1 (partition tid) (partition partner)
= sp-impl-subj-subj s2 (partition tid) (partition partner)

have A: ?A

proof (cases Policy.sp-spec-subj-subj (partition tid) (partition partner))
case True
thus ?A using eq-tid unfolding vpeq-def vpeq-subj-subj-def
by (simp split add: ipc-direction-t.split)
next case False
have sp-subset s1 and sp-subset s2
using inv1 inv2 unfolding atomic-step-invariant-def sp-subset-def by auto
hence ~ sp-impl-subj-subj s1 (partition tid) (partition partner)
and ~ sp-impl-subj-subj s2 (partition tid) (partition partner)
using False unfolding sp-subset-def by auto
thus ?A by auto
qed

show ?thesis using A unfolding ev-signal-precondition-def by auto
qed

lemma set-object-value-consistent:
assumes eq-obs: vpeq u s1 s2
shows vpeq u (set-object-value x y s1) (set-object-value x y s2)

proof –

let ?s1' = set-object-value x y s1 and ?s2' = set-object-value x y s2

have E1: vpeq-obj u ?s1' ?s2'

proof –
{ fix x'
 assume 1: Policy.sp-spec-subj-obj u x' READ
 have obj ?s1' x' = obj ?s2' x'
proof (cases x = x')
case True
thus obj ?s1' x' = obj ?s2' x' unfolding set-object-value-def by auto
next case False
hence 2: obj ?s1' x' = obj s1 x'
and 3: obj ?s2' x' = obj s2 x'

unfolding set-object-value-def by auto
have 4: obj s1 x' = obj s2 x'
using 1 eq-obs unfolding vpeq-def vpeq-obj-def by auto
from 2 3 4 show obj ?s1' x' = obj ?s2' x'
by simp
qed }

thus vpeq-obj u ?s1' ?s2' unfolding vpeq-obj-def by auto
qed

have E4: vpeq-subj-subj u ?s1' ?s2'

proof –

have sp-impl-subj-subj ?s1' = sp-impl-subj-subj s1
and sp-impl-subj-subj ?s2' = sp-impl-subj-subj s2

unfolding set-object-value-def by auto
thus vpeq-subj-subj u ?s1' ?s2'
using eq-obs unfolding vpeq-def vpeq-subj-subj-def by auto
qed

have E5: vpeq-subj-obj u ?s1' ?s2'

proof –
have \( sp-impl-subj-obj ?s1' = sp-impl-subj-obj s1 \)
and \( sp-impl-subj-obj ?s2' = sp-impl-subj-obj s2 \)
unfolding set-object-value-def by auto
thus vpeq-subj-obj u \( ?s1' ?s2' \)
  using eq-obs unfolding vpeq-def vpeq-subj-obj-def by auto
qed
from eq-obs have E6: vpeq-local u \( ?s1' ?s2' \)
  unfolding vpeq-def vpeq-local-def set-object-value-def
by simp
from E1 E4 E5 E6
show \(?\text{thesis}\) unfolding vpeq-def
by auto
qed

4.7.2 Weak step consistency of atomic step functions

lemma ipc-weakly-step-consistent:
assumes eq-obs: vpeq u s1 s2
  and eq-act: vpeq (partition tid) s1 s2
  and inv1: atomic-step-invariant s1
  and inv2: atomic-step-invariant s2
  and prec1: atomic-step-precondition s1 tid ipt
  and prec2: atomic-step-precondition s1 tid ipt
  and ipt-case: ipt = SK-IPC dir stage partner page
shows vpeq u
  (atomic-step-ipc tid dir stage partner page s1)
  (atomic-step-ipc tid dir stage partner page s2)
proof –
  have \( \forall \text{mypage} \cdot [[ \text{dir} = \text{SEND}; \text{stage} = \text{BUF \mypage} ]] \implies \text{?thesis} \)
proof –
  fix \( \text{mypage} \)
  assume dir-send: \( \text{dir} = \text{SEND} \)
  assume stage-buf: \( \text{stage} = \text{BUF \mypage} \)
  have Policy.sp-spec-subj-obj (partition tid) (PAGE page) READ
    using inv1 prec1 dir-send stage-buf ipt-case
  unfolding atomic-step-invariant-def sp-subset-def
  unfolding atomic-step-precondition-def ipc-precondition-def opposite-ipc-direction-def
  by auto
  hence obj s1 (PAGE page) = obj s2 (PAGE page)
  using eq-act unfolding vpeq-def vpeq-obj-def vpeq-local-def
  by auto
  thus vpeq u
    (atomic-step-ipc tid dir stage partner page s1)
    (atomic-step-ipc tid dir stage partner page s2)
  unfolding atomic-step-ipc-def
  by auto
  qed
  thus \text{?thesis}
  using eq-obs unfolding atomic-step-ipc-def
  by (cases stage, auto, cases dir, auto)
  qed

lemma ev-wait-one-weakly-step-consistent:
assumes eq-obs: vpeq u s1 s2
  and eq-act: vpeq (partition tid) s1 s2
  and inv1: atomic-step-invariant s1
  and inv2: atomic-step-invariant s2

and prec1: atomic-step-precondition s1 (current s1) ipt
and prec2: atomic-step-precondition s1 (current s1) ipt
shows vpeq u
    (atomic-step-ev-wait-one tid s1)
    (atomic-step-ev-wait-one tid s2)
using assms
unfolding vpeq-def vpeq-subj-subj-def vpeq-obj-def vpeq-subj-obj-def vpeq-local-def
atomic-step-ev-wait-one-def
by simp

lemma ev-wait-all-weakly-step-consistent:
assumes eq-obs: vpeq u s1 s2
and eq-act: vpeq (partition tid) s1 s2
and inv1: atomic-step-invariant s1
and inv2: atomic-step-invariant s2
and prec1: atomic-step-precondition s1 (current s1) ipt
and prec2: atomic-step-precondition s1 (current s1) ipt
shows vpeq u
    (atomic-step-ev-wait-all tid s1)
    (atomic-step-ev-wait-all tid s2)
using assms
unfolding vpeq-def vpeq-subj-subj-def vpeq-obj-def vpeq-subj-obj-def vpeq-local-def
atomic-step-ev-wait-all-def
by simp

lemma ev-signal-weakly-step-consistent:
assumes eq-obs: vpeq u s1 s2
and eq-act: vpeq (partition tid) s1 s2
and inv1: atomic-step-invariant s1
and inv2: atomic-step-invariant s2
and prec1: atomic-step-precondition s1 (current s1) ipt
and prec2: atomic-step-precondition s1 (current s1) ipt
shows vpeq u
    (atomic-step-ev-signal tid partner s1)
    (atomic-step-ev-signal tid partner s2)
using assms
unfolding vpeq-def vpeq-subj-subj-def vpeq-obj-def vpeq-subj-obj-def vpeq-local-def
atomic-step-ev-signal-def
by simp

The use of extend-f is to provide infrastructure to support use in dynamic policies, currently not used.
definition extend-f :: (partition-id-t ⇒ partition-id-t ⇒ bool) ⇒ (partition-id-t ⇒ partition-id-t ⇒ bool) ⇒
(partition-id-t ⇒ partition-id-t ⇒ bool) where
extend-f f g ≡ λ p1 p2 . f p1 p2 ∨ g p1 p2
definition extend-subj-subj :: (partition-id-t ⇒ partition-id-t ⇒ bool) ⇒ state-t ⇒ state-t where
extend-subj-subj f s ≡ s (sp-impl-subj-subj) := extend-f f (sp-impl-subj-subj)
definition extend-subj-subj-consistent:
fixes f :: partition-id-t ⇒ partition-id-t ⇒ bool
assumes vpeq u s1 s2
shows vpeq u (extend-subj-subj f s1) (extend-subj-subj f s2)
proof
  let ?g1 = sp-impl-subj-subj s1 and ?g2 = sp-impl-subj-subj s2
  have ∀ v . Policy.sp-spec-subj-subj u v →→ ?g1 u v = ?g2 u v
  and ∀ v . Policy.sp-spec-subj-subj v u →→ ?g1 v u = ?g2 v u
  using assms unfolding vpeq-def vpeq-subj-subj-def by auto
hence ∀ v. Policy.sp-spec-subj-subj u v ⎯→ extend-f f ?g1 u v = extend-f f ?g2 u v 
and ∀ v. Policy.sp-spec-subj-subj v u ⎯→ extend-f f ?g1 v u = extend-f f ?g2 v u

unfolding extend-f-def by auto

hence 1: vpeq-subj-subj u (extend-subj-subj f s1) (extend-subj-subj f s2)

unfolding vpeq-subj-subj-def extend-subj-subj-def
by auto

have 2: vpeq-obj u (extend-subj-subj f s1) (extend-subj-subj f s2)

using assms unfolding vpeq-def vpeq-obj-def extend-subj-subj-def by fastforce

have 3: vpeq-obj u (extend-subj-subj f s1) (extend-subj-subj f s2)

using assms unfolding vpeq-def vpeq-obj-def extend-subj-subj-def by fastforce

have 4: vpeq-local u (extend-subj-subj f s1) (extend-subj-subj f s2)

from 1 2 3 4 show ?thesis

using assms unfolding vpeq-def by fast

qed

4.7.3 Summary theorems on view-partitioning weak step consistency

The atomic step is weakly step consistent with view partitioning. Here, the “weakness” is that we assume that the two states are vp-equivalent not only w.r.t. the observer domain u, but also w.r.t. the caller domain Step.partition tid).

theorem atomic-step-weakly-step-consistent:
assumes eq-obs: vpeq u s1 s2 
and eq-act: vpeq (partition (current s1)) s1 s2 
and im1: atomic-step-invariant s1 
and im2: atomic-step-invariant s2 
and prec1: atomic-step-precondition s1 (current s1) ipt 
and prec2: atomic-step-precondition s2 (current s2) ipt 
and eq-curr: current s1 = current s2 
shows vpeq u (atomic-step s1 ipt) (atomic-step s2 ipt)

proof –

show ?thesis

using assms

ipc-weakly-step-consistent

ev-wait-all-weakly-step-consistent

ev-wait-one-weakly-step-consistent

ev-signal-weakly-step-consistent

vpeq-refl ev-signal-stage-t.exhaust

unfolding atomic-step-def

apply (cases ipt, auto)

apply (simp split add: ev-consume-t.splits ev-wait-stage-t.splits )

by (simp split add: ev-signal-stage-t.splits)

qed

end

4.8 Separation kernel model

theory Separation-kernel-model

imports . . ./step/Step 
 . . ./step/Step-invariants 
 . . ./step/Step-vpeq 
 . . ./step/Step-vpeq-locally-respects 
 . . ./step/Step-vpeq-weakly-step-consistent

CISK

begin

First (Section 4.8.1) we instantiate the CISK generic model. Functions that instantiate a generic
function of the CISK model are prefixed with an ‘r’, ‘r’ standing for “Rushby”; as CISK is derived originally from a model by Rushby [31]. For example, ‘rifp’ is the instantiation of the generic ‘ifp’.

Later (Section 4.8.5) all CISK proof obligations are discharged, e.g., weak step consistency, output consistency, etc. These will be used in Section 4.9.

4.8.1 Initial state of separation kernel model

We assume that the initial state of threads and memory is given. The initial state of threads is arbitrary, but the threads are not executing the system call. The purpose of the following definitions is to obtain the initial state without potentially dangerous axioms. The only axioms we admit without proof are formulated using the “consts” syntax and thus safe.

consts
  initial-current :: thread-id-t
  initial-obj :: obj-id-t ⇒ obj-t

definition s0 @ state-t where
  s0 ≡ ⟨⟨ sp-impl-subj-subj = Policy.sp-spec-subj-subj, sp-impl-subj-obj = Policy.sp-spec-subj-obj, current = initial-current, obj = initial-obj, thread = λ _ . (| ev-counter = 0 |) ⟩⟩

lemma initial-invariant:
  shows atomic-step-invariant s0
proof –
  have sp-subset s0
    unfolding sp-subset-def s0-def by auto
  thus ?thesis
    unfolding atomic-step-invariant-def by auto
qed

4.8.2 Types for instantiation of the generic model

To simplify formulations, we include the state invariant atomic-step-invariant in the state data type. The initial state s0 serves as witness that rstate-t is non-empty.

typedef rstate-t = { s . atomic-step-invariant s }
  using initial-invariant by auto

definition abs @ state-t ⇒ rstate-t (↑ - ) where abs = Abs-rstate-t
definition rep @ rstate-t ⇒ state-t (↓ - ) where rep = Rep-rstate-t

lemma rstate-invariant:
  shows atomic-step-invariant (↑s)
  unfolding rep-def by (metis Rep-rstate-t mem-Collect-eq)

lemma rstate-down-up [simp]:
  shows (↑ ↑s) = s
  unfolding rep-def abs-def using Rep-rstate-t-inverse by auto

lemma rstate-up-down [simp]:
  assumes atomic-step-invariant s
  shows (↓ ↑s) = s
  using assms Abs-rstate-t-inverse unfolding rep-def abs-def by auto

A CISK action is identified with an interrupt point.
type-synonym raction-t = int-point-t

definition rcurrent :: rstate-t ⇒ thread-id-t where
rcurrent s = current ↓ s

definition rstep :: rstate-t ⇒ raction-t ⇒ rstate-t where
rstep s a ≡ ↑ (atomic-step ↓ s) a

Each CISK domain is identified with a thread id.

type-synonym rdom-t = thread-id-t

The output function returns the contents of all memory accessible to the subject. The action argument of the output function is ignored.
datatype visible-obj-t = VALUE obj-t / EXCEPTION

type-synonym routput-t = page-t ⇒ visible-obj-t
definition routput-f :: rstate-t ⇒ raction-t ⇒ routput-t where
routput-f s a p ≡ if sp-impl-subj-obj ↓ s (partition rcurrent s) (PAGE p) READ then
VALUE (obj ↓ s (PAGE p))
else
EXCEPTION

The precondition for the generic model. Note that atomic-step-invariant is already part of the state.
definition rprecondition :: rstate-t ⇒ rdom-t ⇒ raction-t ⇒ bool where
rprecondition s d a ≡ atomic-step-precondition ↓ s d a

abbreviation rinvariant where
rinvariant s ≡ True — The invariant is already in the state type.

Translate view-partitioning and interaction-allowed relations.
definition rvpeq :: rdom-t ⇒ rstate-t ⇒ rstate-t ⇒ bool where
rvpeq u s1 s2 ≡ vpeq (partition u) ↓ s1 ↓ s2

definition rifp :: rdom-t ⇒ rdom-t ⇒ bool where
rifp u v = Policy.dfp (partition u) (partition v)

Context Switches
definition(rcswitch :: nat ⇒ rstate-t ⇒ rstate-t where
rcswitch n s ≡ ↑ ((↓ s \ current :: (SOME t. True)) \)

4.8.3 Possible action sequences

An SK-IPC consists of three atomic actions PREP, WAIT and BUF with the same parameters.
definition is-SK-IPC :: raction-t list ⇒ bool
where is-SK-IPC aseq ≡ \ dir partner page .
  aseq ≡ [SK-IPC dir PREP partner page, SK-IPC dir WAIT partner page, SK-IPC dir BUF (SOME page'. True) \] partner page]

An SK-EV-WAIT consists of three atomic actions, one for each of the stages EV-PREP, EV-WAIT and EV-FINISH with the same parameters.
definition is-SK-EV-WAIT :: raction-t list ⇒ bool
where is-SK-EV-WAIT aseq ≡ \ consume .
  aseq ≡ [SK-EV-WAIT EV-PREP consume, SK-EV-WAIT EV-WAIT consume, SK-EV-WAIT EV-FINISH consume]
An SK-EV-SIGNAL consists of two atomic actions, one for each of the stages EV-SIGNAL-PREP and EV-SIGNAL-FINISH with the same parameters.

**Definition**

\[ is-\text{SK-EV-SIGNAL} :: raction-t \xrightarrow{} \text{list} \Rightarrow \text{bool} \]

**Where**

\[ is-\text{SK-EV-SIGNAL} \ aseq \equiv \exists \ \text{partner} \ . \]
\[ aseq = [\text{SK-EV-SIGNAL} \ EV\text{-SIGNAL-PREP} \ \text{partner}, \]
\[ SK\text{-EV-SIGNAL} \ EV\text{-SIGNAL-FINISH} \ \text{partner}] \]

The complete attack surface consists of IPC calls, events, and noops.

**Definition**

\[ rAS\text{-}set :: raction-t \xrightarrow{} \text{list} \ \text{set} \]

**Where**

\[ rAS\text{-}set \equiv \{ \ aseq . \ is-\text{SK-IPC} \ aseq \lor is-\text{SK-EV-WAIT} \ aseq \lor is-\text{SK-EV-SIGNAL} \ aseq \} \cup \{[]\} \]

### 4.8.4 Control

When are actions aborting, and when are actions waiting. We do not currently use the set-error-code function yet.

**Abbreviation**

\[ raborting \]

**Where**

\[ raborting \ s \equiv \text{aborting} (\downarrow \ s) \]

**Abbreviation**

\[ rwaiting \]

**Where**

\[ rwaiting \ s \equiv \text{waiting} (\downarrow \ s) \]

**Definition**

\[ rset\text{-}error\text{-}code :: rstate-t \Rightarrow raction-t \Rightarrow rstate-t \]

**Where**

\[ rset\text{-}error\text{-}code \ s \ a \equiv s \]

Returns the set of threads that are involved in a certain action. For example, for an IPC call, the WAIT stage synchronizes with the partner. This partner is involved in that action.

**Definition**

\[ rkinvolved :: \text{int-point-t} \Rightarrow \text{rdom-t set} \]

**Where**

\[ rkinvolved \ a \equiv \]
\[ \text{case a of SK-IPC dir WAIT partner page} \Rightarrow \{ \text{partner} \} \]
\[ | \text{SK-EV-SIGNAL EV-SIGNAL-FINISH partner} \Rightarrow \{ \text{partner} \} \]
\[ | - \Rightarrow \{\} \]

**Abbreviation**

\[ rinvolved :: \text{int-point-t option} \Rightarrow \text{rdom-t set} \]

**Where**

\[ rinvolved \equiv \text{Kernel.involved} \ rkinvolved \]

### 4.8.5 Discharging the proof obligations

**Lemma**

\[ \text{inst-vpeq-rel} \]

**Shows**

\[ \text{rvpeq-refl} : \text{rvpeq u s s} \]

**And**

\[ \text{rvpeq-sym} : \text{rvpeq u s1 s2 } \Rightarrow \text{rvpeq u s2 s1} \]

**And**

\[ \text{rvpeq-trans} : [\text{rvpeq u s1 s2}; \text{rvpeq u s2 s3}] \Rightarrow \text{rvpeq u s1 s3} \]

**Unfolding**

\[ \text{rvpeq-def using vpeq-rel by metis} \]

**Lemma**

\[ \text{inst-ifp-refl} \]

**Shows**

\[ \forall \ u . \text{rifp u u} \]

**Unfolding**

\[ \text{rifp-def using Policy-properties.ifp-reflexive by fast} \]

**Lemma**

\[ \text{inst-step-atomicity [simp]} \]

**Shows**

\[ \forall \ s \ a . \text{rcurrent (rstep s a) = rcurrent s} \]

**Unfolding**

\[ \text{rstep-def using atomic-step-does-not-change-current-thread rstate-up-down rstate-invariant atomic-step-preserves-invariants by auto} \]

**Lemma**

\[ \text{inst-weakly-step-consistent} \]

**Assumes**

\[ \text{rvpeq u s t} \]
\begin{verbatim}
and rvpeq (rcurrent s) s t
and rcurrent s = rcurrent t
and rprecondition s (rcurrent s) a
and rprecondition t (rcurrent t) a
shows rvpeq u (rstep s a) (rstep t a)
using assms atomic-step-weakly-step-consistent rstate-invariant atomic-step-preserves-invariants
unfolding rcurrent-def rstep-def rvpeq-def rprecondition-def by auto

lemma inst-local-respect:
  assumes not-ifp : ¬rifp (rcurrent s) u
  and prec : rprecondition s (rcurrent s) a
  shows rvpeq u s (rstep s a)
using assms atomic-step-respects-policy rstate-invariant atomic-step-preserves-invariants
unfolding rifp-def rprecondition-def rvpeq-def rstep-def rcurrent-def by auto

lemma inst-output-consistency:
  assumes rvpeq : rvpeq (rcurrent s) s t
  and current-eq: rcurrent s = rcurrent t
  shows routput-f s a = routput-f t a
proof
  have ∀ a s t. rvpeq (rcurrent s) s t ∧ rcurrent s = rcurrent t → routput-f s a = routput-f t a
  proof
    { fix a :: action-t
      fix s t :: rstate-t
      fix p :: page-t
      assume 1 : rvpeq (rcurrent s) s t
      and 2 : rcurrent s = rcurrent t
      let ?part = partition (rcurrent s)
      have routput-f s a p = routput-f t a p
        proof (cases Policy.sp-spec-subj-obj ?part (PAGE p) READ
          rule: case-split [case-names Allowed Denied])
          case Allowed
          have 5: obj (↓s) (PAGE p) = obj (↓t) (PAGE p)
            using 1 Allowed unfolding rvpeq-def vpeq-def vpeq-obj-def by auto
          have 6: sp-impl-subj-obj (↓s) ?part (PAGE p) READ = sp-impl-subj-obj (↓t) ?part (PAGE p) READ
            using 1 2 Allowed unfolding rvpeq-def vpeq-def vpeq-subj-obj-def by auto
          show routput-f s a p = routput-f t a p
            unfolding routput-f-def using 2 5 6 by auto
          next case Denied
          hence sp-impl-subj-obj (↓s) ?part (PAGE p) READ = False
            and sp-impl-subj-obj (↓t) ?part (PAGE p) READ = False
            using rstate-invariant unfolding atomic-step-invariant-def sp-subset-def by auto
          thus routput-f s a p = routput-f t a p
            using 2 unfolding routput-f-def by simp
          qed
        } thus ∀ a s t. rvpeq (rcurrent s) s t ∧ rcurrent s = rcurrent t → routput-f s a = routput-f t a
          by auto
        qed
      thus ?thesis using assms by auto
    }
  }
\end{verbatim}
lemma inst-cswitch-independent-of-state:
assumes rcurrent s = rcurrent t
shows rcurrent (rcswitch n s) = rcurrent (rcswitch n t)
using rstate-invariant cswitch-preserves-invariants unfolding rcurrent-def rcswitch-def by simp

lemma inst-cswitch-consistency:
assumes rvpeq u s t
shows rvpeq u (rcswitch n s) (rcswitch n t)
proof-
  have 1: vpeq (partition u) (↓s) ↓(rcswitch n s)
  using rstate-invariant cswitch-consistency-and-respect cswitch-preserves-invariants
  unfolding rcswitch-def
  by auto
  have 2: vpeq (partition u) (↓t) ↓(rcswitch n t)
  using rstate-invariant cswitch-consistency-and-respect cswitch-preserves-invariants
  unfolding rcswitch-def
  by auto
  from 1 2 assms show ?thesis unfolding rvpeq-def using vpeq-rel by metis
qed

For the PREP stage (the first stage of the IPC action sequence) the precondition is True.

lemma prec-first-IPC-action:
assumes is-SK-IPC aseq
shows rprecondition s d (hd aseq)
using assms
unfolding is-SK-IPC-def rprecondition-def atomic-step-precondition-def
by auto

For the first stage of the EV-WAIT action sequence the precondition is True.

lemma prec-first-EV-WAIT-action:
assumes is-SK-EV-WAIT aseq
shows rprecondition s d (hd aseq)
using assms
unfolding is-SK-EV-WAIT-def rprecondition-def atomic-step-precondition-def
by auto

For the first stage of the EV-SIGNAL action sequence the precondition is True.

lemma prec-first-EV-SIGNAL-action:
assumes is-SK-EV-SIGNAL aseq
shows rprecondition s d (hd aseq)
using assms
unfolding is-SK-EV-SIGNAL-def rprecondition-def atomic-step-precondition-def
ev-signal-precondition-def
by auto

When not waiting or aborting, the precondition is “1-step inductive”, that is at all times the precondition holds initially (for the first step of an action sequence) and after doing one step.

lemma prec-after-IPC-step:
assumes pre: rprecondition s (rcurrent s) (aseq ! n)
  and n-bound: Suc n < length aseq
  and IPC: is-SK-IPC aseq
  and not-aborting: ~raborting s (rcurrent s) (aseq ! n)
  and not-waiting: ~rwaiting s (rcurrent s) (aseq ! n)

shows \( \text{rprecondition} \ (\text{rstep} \ s \ (\text{aseq} \ ! \ n)) \ (\text{rcurrent} \ s) \ (\text{aseq} ! \ Suc \ n) \)

\begin{verbatim}
proof
{
  fix dir partner page
  let ?page' = (SOME page'. True)
  assume IPC: aseq = [SK-IPC dir PREP partner page,SK-IPC dir WAIT partner page,SK-IPC dir (BUF ?page')
  partner page]
  { 
    assume 0: n=0
    from 0 IPC prec not-aborting
    have ?thesis
    by(auto)
  }
  moreover
  { 
    assume 1: n=1
    from 1 IPC prec not-waiting
    have ?thesis
    by(auto)
  }
  moreover
  from IPC
  have length aseq = 3
  by auto
  ultimately
  have ?thesis
  using n-bound
  by arith
  } 
thus ?thesis
using IPC
unfolding is-SK-IPC-def
by(auto)
qed
\end{verbatim}

When not waiting or aborting, the precondition is 1-step inductive.

\textbf{lemma} prec-after-EV-WAIT-step:
\begin{verbatim}
assumes prec: \text{rprecondition} \ s \ (\text{rcurrent} \ s) \ (\text{aseq} ! \ n)
  and n-bound: Suc n < length aseq
  and IPC: is-SK-EV-WAIT aseq
  and not-aborting: ~raborting \ s \ (\text{rcurrent} \ s) \ (\text{aseq} ! \ n)
  and not-waiting: ~rwaiting \ s \ (\text{rcurrent} \ s) \ (\text{aseq} ! \ n)
shows \text{rprecondition} \ (\text{rstep} \ s \ (\text{aseq} ! \ n)) \ (\text{rcurrent} \ s) \ (\text{aseq} ! \ Suc \ n)
\end{verbatim}

\begin{verbatim}
proof
{
  fix consume

  assume WAIT: aseq = [SK-EV-WAIT EV-PREP consume,
    SK-EV-WAIT EV-WAIT consume,
    SK-EV-WAIT EV-FINISH consume]
  { 
    assume 0: n=0
    from 0 WAIT prec not-aborting
    have ?thesis
  }
\end{verbatim}
When not waiting or aborting, the precondition is 1-step inductive.

**Lemma** prec-after-EV-SIGNAL-step:

**Assumes**
- \( \text{prec} \) \( \text{rprecondition} \) \( s \) ( \( \text{rcurrent} \) \( s \)) ( \( \text{aseq} ! n \))
- \( \text{n-bound} \): \( \text{Suc} \ n < \text{length} \ \text{aseq} \)
- \( \text{SIGNAL} \): \( \text{is-SK-EV-SIGNAL} \) \( \text{aseq} \)
- \( \text{not-aborting} \): \( \neg \text{raborting} \) \( s \) ( \( \text{rcurrent} \) \( s \)) ( \( \text{aseq} ! n \))
- \( \text{not-waiting} \): \( \neg \text{rwaiting} \) \( s \) ( \( \text{rcurrent} \) \( s \)) ( \( \text{aseq} ! n \))

**Shows** \( \text{rprecondition} \) \( (\text{rstep} \ s \ (\text{aseq} ! n)) \) ( \( \text{rcurrent} \) \( s \)) ( \( \text{aseq} \ \text{Suc} \ n \))

**Proof**

- **Fix** partner
- **Assume** \( \text{SIGNAL1} \): \( \text{aseq} = [\text{SK-EV-SIGNAL} \ \text{EV-SIGNAL-PREP} \ \text{partner}, \ \text{SK-EV-SIGNAL} \ \text{EV-SIGNAL-FINISH} \ \text{partner}] \)
  - **Assume** \( 0: n=0 \)
  - **From** \( 0 \ \text{SIGNAL1} \) prec not-aborting
  - **Have** \( ?\text{thesis} \)
  - **Unfolding** \( \text{rprecondition-def} \) \( \text{atomic-step-precondition-def} \) \( \text{ev-signal-precondition-def} \) \( \text{aborting-def} \) \( \text{rstep-def} \) \( \text{atomic-step-def} \)
    - **By** \( \text{auto} \)

- **Moreover**
  - **From** \( \text{SIGNAL1} \)
    - **Have** \( \text{length} \ \text{aseq} = 2 \)
      - **By** \( \text{auto} \)
    - **Ultimately**
      - **Have** \( ?\text{thesis} \)
        - **Using** \( \text{n-bound} \)
          - **By** \( \text{arith} \)

- **Thus** \( ?\text{thesis} \)
  - **Using** \( \text{assms} \)
  - **Unfolding** \( \text{is-SK-EV-SIGNAL-def} \)
by auto
qed

lemma on-set-object-value:
  shows sp-impl-subj-subj (set-object-value ob val s) = sp-impl-subj-subj s
  and sp-impl-subj-obj (set-object-value ob val s) = sp-impl-subj-obj s
unfolding set-object-value-def apply simp+ done

lemma prec-IPC-dom-independent:
  assumes current s ≠ d
  and atomic-step-invariant s
  and atomic-step-precondition s d a
  shows atomic-step-precondition (atomic-stepipc (current s) dir stage partner page s) d a
using assms on-set-object-value
unfolding atomic-step-precondition-def atomic-step-ipc-def ipc-precondition-def
  ev-signal-precondition-def set-object-value-def
  by (auto split add: int-point-t.splits ipc-stage-t.splits ipc-direction-t.splits
    ev-consume-t.splits ev-wait-stage-t.splits ev-signal-stage-t.splits)

lemma prec-ev-signal-dom-independent:
  assumes current s ≠ d
  and atomic-step-invariant s
  and atomic-step-precondition s d a
  shows atomic-step-precondition (atomic-step-ev-signal (current s) partner s) d a
using assms on-set-object-value
unfolding atomic-step-precondition-def atomic-step-ev-signal-def ipc-precondition-def
  ev-signal-precondition-def set-object-value-def
  by (auto split add: int-point-t.splits ipc-stage-t.splits ipc-direction-t.splits
    ev-consume-t.splits ev-wait-stage-t.splits ev-signal-stage-t.splits)

lemma prec-ev-wait-one-dom-independent:
  assumes current s ≠ d
  and atomic-step-invariant s
  and atomic-step-precondition s d a
  shows atomic-step-precondition (atomic-step-ev-wait-one (current s) s) d a
using assms on-set-object-value
unfolding atomic-step-precondition-def atomic-step-ev-wait-one-def ipc-precondition-def
  ev-signal-precondition-def set-object-value-def
  by (auto split add: int-point-t.splits ipc-stage-t.splits ipc-direction-t.splits
    ev-consume-t.splits ev-wait-stage-t.splits ev-signal-stage-t.splits)

lemma prec-ev-wait-all-dom-independent:
  assumes current s ≠ d
  and atomic-step-invariant s
  and atomic-step-precondition s d a
  shows atomic-step-precondition (atomic-step-ev-wait-all (current s) s) d a
using assms on-set-object-value
unfolding atomic-step-precondition-def atomic-step-ev-wait-all-def ipc-precondition-def
  ev-signal-precondition-def set-object-value-def
  by (auto split add: int-point-t.splits ipc-stage-t.splits ipc-direction-t.splits
    ev-consume-t.splits ev-wait-stage-t.splits ev-signal-stage-t.splits)

lemma prec-dom-independent:
  shows ∀ s d a a’. rercurrent s ≠ d ∧ rprecondition s d a → rprecondition (rstep s a’) d a
using atomic-step-preserves-invariants
rstate-invariant prec-IPC-dom-independent prec-ev-signal-dom-independent
prec-ev-wait-all-dom-independent prec-ev-wait-one-dom-independent
unfolding rcurrent-def rprecondition-def rstep-def atomic-step-def
by (auto split add: int-point-t splits ev-consume-t splits ev-wait-stage-t splits ev-signal-stage-t splits)

lemma ipc-precondition-after-cswitch [simp]:
shows ipc-precondition d dir partner page (↓ s) [current := new-current]
= ipc-precondition d dir partner page (↓ s)
using assms
unfolding ipc-precondition-def
by (auto split add: ipc-direction-t)

lemma precondition-after-cswitch:
shows ∀ s d n a. rprecondition s d a → rprecondition (rcswitch n s) d a
using cswitch-preserves-invariants rstate-invariant
unfolding rprecondition-def rcswitch-def atomic-step-precondition-def
ev-signal-precondition-def
by (auto split add: int-point-t ipc-stage-t splits ev-signal-stage-t splits)

lemma aborting-switch-independent:
shows ∀ n s. raborting (rcswitch n s s tid a) = raborting s tid a
proof−
{ fix n s
 { fix tid a
 have raborting (rcswitch n s) tid a = raborting s tid a
 using rstate-invariant cswitch-preserves-invariants ev-signal-precondition-weakly-step-consistent
cswitch-consistency-and-respect
 unfolding aborting-def rcswitch-def
 apply (auto split add: int-point-t splits ipc-stage-t splits
 ev-wait-stage-t splits ev-signal-stage-t splits)
 apply (metis (full-types))
 by blast
 }
hence raborting (rcswitch n s) = raborting s by auto
} thus ?thesis by auto
qed

lemma waiting-switch-independent:
shows ∀ n s. rwaiting (rcswitch n s) = rwaiting s
proof−
{ fix n s
 { fix tid a
 have rwaiting (rcswitch n s) tid a = rwaiting s tid a
 using rstate-invariant cswitch-preserves-invariants
 unfolding waiting-def rcswitch-def
 by (auto split add: int-point-t splits ipc-stage-t splits ev-wait-stage-t splits)
 }
hence rwaiting (rcswitch n s) = rwaiting s by auto
} thus ?thesis by auto
qed

lemma aborting-after-IPC-step:
assumes d1 ≠ d2
shows aborting (atomic-step-ipc d1 dir stage partner page s) d2 a = aborting s d2 a
 unfolding atomic-step-ipc-def aborting-def set-object-value-def ipc-precondition-def ev-signal-precondition-def
by(auto split add: int-point-t.splits ipc-stage-t.splits ipc-direction-t.splits ev-signal-stage-t.splits)

lemma waiting-after-IPC-step:
assumes \( d_1 \neq d_2 \)
shows waiting (atomic-step-ipc \( d_1 \) dir stage partner page \( s \)) \( d_2 \) a = waiting \( s \) \( d_2 \) a
unfolding atomic-step-ipc-def waiting-def set-object-value-def ipc-precondition-def
by(auto split add: int-point-t.splits ipc-stage-t.splits ipc-direction-t.splits ev-wait-stage-t.splits)

lemma raborting-consistent:
shows \( \forall s t u. \text{rvpeq } u s t \rightarrow \text{raborting } s u = \text{raborting } t u \)
proof –
{ fixed \( s t u \)
assume vpeq: rvpeq u s t
{ fixed \( a \) from vpeq ipc-precondition-weakly-step-consistent rstate-invariant
have \( \land \) tid dir partner page . ipc-precondition u dir partner page (\( s \)) = ipc-precondition u dir partner page (\( t \))
unfolding rvpeq-def
by auto
with vpeq rstate-invariant have raborting s u a = raborting t u a
apply (auto split add: int-point-t.splits ipc-stage-t.splits ev-signal-stage-t.splits)
by blast
} hence raborting s u a = raborting t u a by auto
}
thus \( ?\text{thesis by auto} \)
qed

lemma aborting-dom-independent:
assumes \( \text{rcurrent } s \neq d \)
shows raborting (rstep s a) d a’ = raborting s d a’
proof –
{ have \( \land \) tid dir partner page s . ipc-precondition tid dir partner page s = ipc-precondition tid dir partner page \( \text{(atomic-step s a)} \)
\land ev-signal-precondition tid partner s = ev-signal-precondition tid partner \( \text{(atomic-step s a)} \)
proof –
fix tid dir partner page s
let \( ?s = \text{atomic-step } s a \)
have \( (\forall p q . \text{sp-impl-subj-subj } p q = \text{sp-impl-subj-subj } ?s p q) \)
\land (\forall p x m . \text{sp-impl-subj-obj } p x m = \text{sp-impl-subj-obj } ?s p x m)
unfolding atomic-step-def atomic-step-ipc-def
atomic-step-ev-wait-all-def atomic-step-ev-wait-one-def
atomic-step-ev-signal-def set-object-value-def
by (auto split add: int-point-t.splits ipc-stage-t.splits ipc-direction-t.splits ev-wait-stage-t.splits ev-consume-t.splits ev-signal-stage-t.splits)
thus \( \text{ipc-precondition tid dir partner page } s = \text{ipc-precondition tid dir partner page } \text{(atomic-step s a)} \)
\( \text{ev-signal-precondition } tid \text{ partner } s = \text{ev-signal-precondition } tid \text{ partner } (\text{atomic-step } s \ a) \)

unfolding ipc-precondition-def ev-signal-precondition-def by simp

qed

moreover have \( \land \ b . (\{(\text{atomic-step } (\downarrow s) b)\}) = \text{atomic-step } (\downarrow s) b \)

using rstate-invariant atomic-step-preserves-invariants rstate-up-down by auto

ultimately show \( ?\text{thesis} \)

unfolding aborting-def rstep-def ev-signal-precondition-def

by (simp split add: int-point-t.splits ipc-stage-t.splits ev-wait-stage-t.splits ev-signal-stage-t.splits)

qed

lemma ipc-precondition-of-partner-consistent:

assumes vpeq: \( \forall d \in \text{rinvolved } (\text{SK-IPC } \text{dir } \text{WAIT partner page}) \cdot \text{rvpeq } d \ s \ t \)

shows ipc-precondition partner dir' u page' (\( \downarrow s \)) = ipc-precondition partner dir' u page' (\( \downarrow t \))

proof-

from assms ipc-precondition-weakly-step-consistent rstate-invariant

show \( ?\text{thesis} \)

unfolding rvpeq-def rkinvolved-def

by auto

qed

lemma ev-signal-precondition-of-partner-consistent:

assumes vpeq: \( \forall d \in \text{rinvolved } (\text{SK-EV-SIGNAL } \text{EV-SIGNAL-FINISH partner}) \cdot \text{rvpeq } d \ s \ t \)

shows ev-signal-precondition partner u (\( \downarrow s \)) = ev-signal-precondition partner u (\( \downarrow t \))

proof-

from assms ev-signal-precondition-weakly-step-consistent rstate-invariant

show \( ?\text{thesis} \)

unfolding rvpeq-def rkinvolved-def

by auto

qed

lemma waiting-consistent:

shows \( \forall s \ t \ u \ a . \text{rvpeq } (\text{rcurrent } s) \ s \ t \ \land (\forall d \in \text{rinvolved } a . \text{rvpeq } d \ s \ t) \)

\( \land \text{rvpeq } u \ s \ t \)

\( \rightarrow \text{rwaiting } s \ u \ a = \text{rwaiting } t \ u \ a \)

proof-

{
  fix s t u a

  assume vpeq: rvpeq (rcurrent s) s t

  assume vpeq-involved: \( \forall d \in \text{rinvolved } a . \text{rvpeq } d \ s \ t \)

  assume vpeq-u: rvpeq u s t

  have rwaiting s u a = rwaiting t u a proof (cases a)

  case SK-IPC

  thus rwaiting s u a = rwaiting t u a

  using ipc-precondition-of-partner-consistent vpeq-involved

  unfolding waiting-def by (auto split add: ipc-stage-t.splits)

  next case SK-EV-WAIT

  thus rwaiting s u a = rwaiting t u a

  using ev-signal-precondition-of-partner-consistent vpeq-involved vpeq-u

  unfolding waiting-def rinvolved-def ev-signal-precondition-def

  vpeq-def vpeq-def vpeq-local-def

  by (auto split add: ipc-stage-t.splits ev-wait-stage-t.splits ev-consume-t.splits)

  qed (simp add: waiting-def, simp add: waiting-def)

}
lemma ipc-precondition-ensures-ifp
assumes ipc-precondition (current s) dir partner page s
and atomic-step-invariant s
shows rifp partner (current s)
proof
  let ?sp = λ t1 t2. Policy.sp-spec-subj-subj (partition t1) (partition t2)
  have ?sp (current s) partner ∨ ?sp partner (current s)
  using assms unfolding ipc-precondition-def atomic-step-invariant-def sp-subset-def
  by (cases dir, auto)
  thus ?thesis
  unfolding rifp-def using Policy-properties.ifp-compatible-with-sp-spec by auto
qed

lemma ev-signal-precondition-ensures-ifp
assumes ev-signal-precondition (current s) partner s
and atomic-step-invariant s
shows rifp partner (current s)
proof
  let ?sp = λ t1 t2. Policy.sp-spec-subj-subj (partition t1) (partition t2)
  have ?sp (current s) partner ∨ ?sp partner (current s)
  using assms unfolding ev-signal-precondition-def atomic-step-invariant-def sp-subset-def
  by (auto)
  thus ?thesis
  unfolding rifp-def using Policy-properties.ifp-compatible-with-sp-spec by auto
qed

lemma involved-ifp
shows ∀ s a. ∀ d ∈ rkinvolved a . rprecondition s (rcurrent s) a → rifp d (rcurrent s)
proof
  { fix s a d
    assume d-involved: d ∈ rkinvolved a
    assume prec: rprecondition s (rcurrent s) a
    from d-involved prec have rifp d (rcurrent s)
    using ipc-precondition-ensures-ifp ev-signal-precondition-ensures-ifp rstate-invariant
    rkinvolved-def rprecondition-def atomic-step-precondition-def rcurrent-def Kernel.involved-def
    by (cases a simp auto split add: int-point-t.splits ipc-stage-t.splits ev-signal-stage-t.splits)
  }
  thus ?thesis by auto
qed

lemma spec-of-waiting-ev:
shows ∀ s a. rwaiting s (rcurrent s) (SK-EV-WAIT EV-FINISH EV-CONSUME-ALL) → rstep s a = s
unfolding waiting-def
by auto

lemma spec-of-waiting-ev-w:
shows ∀ s a. rwaiting s (rcurrent s) (SK-EV-WAIT EV-WAIT EV-CONSUME-ALL) → rstep s (SK-EV-WAIT EV-WAIT EV-CONSUME-ALL) = s
unfolding rstep-def atomic-step-def
by (auto split add: int-point-t.splits ipc-stage-t.splits ev-wait-stage-t.splits)

lemma spec-of-waiting:
shows ∀ s a. rwaiting s (rcurrent s) a → rstep s a = s
4.9 Link implementation to CISK: the specific separation kernel is an interpretation of the generic model.

theory Link-separation-kernel-model-to-CISK
imports Separation-kernel-model
begin

We show that the separation kernel instantiation satisfies the specification of CISK.

theorem CISK-proof-obligations-satisfied:
  shows
    Controllable-Interruptible-Separation-Kernel
    rstep
    routput-f
    (↑s0)
    rcurrent
    rcswitch
    rkinvolved
    rrf
    rvpeq
    rAS-set
    rinvariant
    rprecondition
    raborting
    rwaiting
    rset-error-code
  proof (unfold-locales)
  — show that rvpeq is equivalence relation
  show ∀ a b c u. (rvpeq u a b ∧ rvpeq u b c) → rvpeq u a c
  and ∀ a b u. rvpeq u a b → rvpeq u b a
  and ∀ a u. rvpeq u a a
  using inst-vpeq-rel by metis+
  — show output consistency
  show ∀ a s t. rvpeq (rcurrent s) s t ∧ rcurrent s = rcurrent t → routput-f s a = routput-f t a
  using inst-output-consistency by metis
  — show reflexivity of ifp
  show ∀ u. rrfp u u
  using inst-ifp-refl by metis
  — show step consistency
  show ∀ s t a. rvpeq u s t ∧ rvpeq (rcurrent s) s t ∧ rprecondition s (rcurrent s) a ∧ True ∧ rprecondition t (rcurrent t) a ∧ rcurrent s = rcurrent t → rvpeq u (rstep s a) (rstep t a)
  using inst-weakly-step-consistent by blast
  — show step atomicity
  show ∀ s a. rcurrent (rstep s a) = rcurrent s
  using inst-step-atomicity by metis
  show ∀ a s u. ¬ rrfp (rcurrent s) u ∧ True ∧ rprecondition s (rcurrent s) a → rvpeq u s (rstep s a)
  using inst-local-respect by blast
  — show caswitch is independent of state
  show ∀ n s t. rcurrent s = rcurrent t → rcurrent (rcaswitch n s) = rcurrent (rcaswitch n t)
  using inst-caswitch-independent-of-state by metis
  — show caswitch consistency
show \forall u s t n. rvpeq u s t \rightarrow rvpeq u (rcswitch n s) (rcswitch n t)
using inst-cswitch-consistency by metis
— Show the empt action sequence is in AS-set
show [] \in \text{rAS-set}
unfolding rAS-set-def
by auto
— The invariant for the initial state, already encoded in rstate-t
show True
by auto
— Step function of the invariant, already encoded in rstate-t
show \forall s n. True \rightarrow True
by auto
— The precondition does not change with a context switch
show \forall s d n a. rprecondition s d a \rightarrow rprecondition (rcswitch n s) d a
using precondition-after-cswitch by blast
— The precondition holds for the first action of each action sequence
show \forall s d aseq. True \land aseq \in \text{rAS-set} \land aseq \neq [] \rightarrow rprecondition s d (hd aseq)
using prec-first-IPC-action prec-first-EV-WAIT-action prec-first-EV-SIGNAL-action
unfolding rAS-set-def is-sub-seq-def
by auto
— The precondition holds for the next action in an action sequence, assuming the sequence is not aborted or delayed
show \forall s a a'. (\exists aseq \in \text{rAS-set}. is-sub-seq a a' aseq) \land True \land rprecondition s (rcurrent s) a \land \lnot raborting s (rcurrent s) a \land \lnot rwaiting s (rcurrent s) a \rightarrow
rprecondition (rstep s a') (rcurrent s) a'
unfolding rAS-set-def is-sub-seq-def
by auto
— Steps of other domains do not influence the precondition
show \forall s d a a'. current s \neq d \land rprecondition s d a \rightarrow rprecondition (rstep s a') d a
using prec-dom-independent by blast
— The invariant
show \forall s a. True \rightarrow True
by auto
— Aborting does not depend on a context switch
show \forall n s. raborting (rcswitch n s) = raborting s
using aborting-switch-independent by auto
— Aborting does not depend on actions of other domains
show \forall s d a. current s \neq d \rightarrow raborting (rstep s a) d = raborting s d
using aborting-dom-independent by auto
— Aborting is consistent
show \forall s t u. rvpeq u s t \rightarrow raborting s u = raborting t u
using raborting-consistent by auto
— Waiting does not depend on a context switch
show \forall n s. rwaiting (rcswitch n s) = rwaiting s
using waiting-switch-independent by auto
— Waiting is consistent
show \forall s t u a. rvpeq (rcurrent s) s t \land (\forall d \in \text{rinvolved a}. rvpeq d s t)
\land rvpeq u s t
\rightarrow rwaiting s u a = rwaiting t u a
unfolding Kernel.involved-def
using waiting-consistent by auto
— Domains that are involved in an action may influence the domain of the action
show \forall s a. \forall d \in \text{rinvolved a}. rprecondition s (rcurrent s) a \rightarrow rfp d (rcurrent s)
using involved-ifp by blast
— An action that is waiting does not change the state
show \forall s a. rwaiting s (rcurrent s) a \rightarrow rstep s a = s
using spec-of-waiting by blast
— Proof obligations for set-error-code. Right now, they are all trivial
show ∀s d a′ a. rcurrent s d a → raborting s d a → raborting (rset-error-code s a′) d a
unfolding rset-error-code-def
by auto
show ∀s t u a. rvpeq u s t → rvpeq u (rset-error-code s a) (rset-error-code t a)
unfolding rset-error-code-def
by auto
show ∀s u a. ¬rifp (rcurrent s) u → rvpeq u s (rset-error-code s a)
by (metis ∀a u. rvpeq u a a)
show ∀s a. rcurrent (rset-error-code s a) = rcurrent s
unfolding rset-error-code-def
by auto
show ∀s d a a′. rprecondition s d a ∧ raborting s (rcurrent s) a′ → rprecondition (rset-error-code s a′) d a
unfolding rset-error-code-def
by auto
show ∀s d a a′. rcurrent s d a → rwaiting s d a → rwaiting (rset-error-code s a′) d a
unfolding rset-error-code-def
by auto
qed

Now we can instantiate CISK with some initial state, interrupt function, etc.

interpretation Inst:
Controllable-Interruptible-Separation-Kernel
rstep — step function, without program stack
routput-f — output function
↑s0 — initial state
rcurrent — returns the currently active domain
rcswitch — switches the currently active domain
(op = 42) — interrupt function (yet unspecified)
rkinvolved — returns a set of threads involved in the give action
rifp — information flow policy
rvpeq — view partitioning
rAS-set — the set of valid action sequences
rinvariant — the state invariant
rprecondition — the precondition for doing an action
raborting — condition under which an action is aborted
rwaiting — condition under which an action is delayed
rset-error-code — updates the state. Has no meaning in the current model.
using CISK-proof-obligations-satisfied by auto

The main theorem: the instantiation implements the information flow policy ifp.

theorem risecure:
Inst.isecure
using Inst.unwinding-implies-secure-CISK
by blast
end

5 Related Work

We consider various definitions of intransitive (I) noninterference (NI). This overview is by no means intended to be complete. We first prune the field by focusing on INI with as granularity the domains: if the security policy states the act “v → u”, this means domain v is permitted to flow any information it has at its disposal to u. We do not consider language-based approaches to noninterference [26], which allow
finer granularity mechanisms (i.e., flowing just a subset of the available information). Secondly, several formal verification efforts have been conducted concerning properties similar and related to INI such as no-exfiltration and no-infiltration [9]. Heitmeyer et al. prove these properties for a separation kernel in a Common Criteria certification process [11] (which kernel and which EAL is not clear). Martin et al. proved separation properties over the MASK kernel [18] and Shapiro and Weber verified correctness of the EROS confinement mechanism [28]. Klein provides an excellent overview of OSs for which such properties have been verified [13]. Thirdly, INI definitions can be built upon either state-based automata, trace-based models, or process algebraic models [30]. We do not focus on the latter, as our approach is not based on process algebra.

Transitive NI was first introduced by Goguen and Meseguer in 1982 [7] and has been the topic of heavy research since. Goguen and Meseguer tried to extend their definition with an unless construct to allow such policies [8]. This construct, however, did not capture the notion of INI [17]. The first commonly accepted definition of INI is Rushby’s purging-based definition IP-secure [24]. IP-secure has been applied to, e.g., smartcards [27] and OS kernel extensions [7]. To the best of our knowledge, Rushby’s definition has not been applied in a certification context. Rushby’s definition has been subject to heavy scrutiny [22], [29] and a vast array of modifications have been proposed.

Roscoe and Goldsmith provide CSP-based definitions of NI for the transitive and the intransitive case, here dubbed as lazy and mixed independence. The latter one is more restrictive than Rushby’s IP-secure. Their critique on IP-secure, however, is not universally accepted [7]. Greve at al. provided the GWV framework developed in ACL2 [9]. Their definition is a non-inductive version of noninterference similar to Rushby’s step consistency. GWV has been used on various industrial systems. The exact relation between GWV and (IP)-secure, i.e., whether they are of equal strength, is still open. The second property, Declassification, means whether the definition allows assignments in the form of \( l := \text{declassify}(h) \) (where we use Sabelfeld’s notation for high and low variables). Information flows from \( h \) to \( l \), but only after it has been declassified. In general, NI is coarser than declassification. It allows where downgrading can occur, but now what may be downgraded [17]. Mantel provides a definition of transitive NI where exceptions can be added to allow de-classification by adding intransitive exceptions to the security policy [17].

To deal with concurrency, definitions of NI have been proposed for Non-Deterministic automata. Von Oheimb defined noninfluence for such systems. His definition can be regarded as a “non-deterministic version” of IP-secure. Engelhardt et al. defined nTA-secure, the non-deterministic version of TA-secure. Finally, some notions of INI consider models that are in a sense richer than similar counterparts. Leslie extends Rushby’s notion of IP-secure for a model in which the security policy is Dynamic. Eggert et al. defined i-secure, an extension of IP-secure. Their model extends Rushby’s model (Mealy machines) with Local security policies. Murray et al. extends Von Oheimb definition of noninfluence to apply to a model that does not assume a static mapping of actions to domains. This makes it applicable to OSs, as in such a setting such a mapping does not exist [20]. NI-OS has been applied to the seL4 separation kernel [20], [14].

Most definitions have an associated methodology. Various methodologies are based on unwinding [8]. This breaks down the proof of NI into smaller proof obligations (POs). These POs can be checked by some manual proof [24], [10], model checking [32] or dedicated algorithms [5]. The methodology of Murray et al. is a combination of unwinding, automated deduction and manual proofs. Some definitions are undecidable and have no suitable unwinding.

We are aiming to provide a methodology for INI based on a model that is richer in detail than Meal machines. This places our contribution next to other works that aim to extend IP-secure [15], [4] in Figure 2. Similar to those approaches, we take IP-secure as a starting point. We add kernel control mechanisms, interrupts and context switches. Ideally, we would simply prove IP-secure over CISK. We argue that this is impossible and that a rephrasing is necessary.

Our ultimate goal — certification of PikeOS — is very similar to the work done on seL4 [20][19]. There are two reasons why their approach is not directly applicable to PikeOS. First, seL4 has been developed from scratch. A Haskell specification serves as the medium for the implementation as well
as the system model for the kernel [6]. C code is derived from a high level specification. PikeOS, in contrast, is an established industrial OS. Secondly, interrupts are mostly disabled in seL4. Klein et al. side-step dealing with the verification complexity of interrupts by using a mostly atomic API [14]. In contrast, we aim to fully address interrupts.

With respect to attempts to formal operating system verifications, notable works are also the Verisoft I project [1] where also a weak form of a separation property, namely fairness of execution was addressed [3].

6 Conclusion

We have introduced a generic theory of intransitive non-interference for separation kernels with control as a series of locales and extensible record definitions in order to achieve a modular organization. Moreover, we have shown that it can be instantiated for a simplistic API consisting of IPC and events.

In the ongoing EURO-MILS project, we will extend this generic theory in order to make it sufficiently rich to be instantiated with a realistic functional model of PikeOS.

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