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Non-diagonal $\mathcal{H}_\infty$ weighting function design: Exploiting spatio-temporal deformations in precision motion control

Frank Boeren^{a,*}, Robbert van Herpen^a, Tom Oomen^a,
Marc van de Wal^b, Maarten Steinbuch^a

^a Eindhoven University of Technology, Department of Mechanical Engineering, Control Systems Technology Group, Eindhoven, The Netherlands

^b ASML, Research Mechatronics, Veldhoven, The Netherlands

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ABSTRACT

Model-based control design requires a careful specification of performance and robustness requirements. In typical norm-based control designs, performance and robustness requirements are specified in a scalar optimization criterion, even for complex multivariable systems. This paper aims to develop a novel approach for the formulation of this optimization criterion for multivariable motion systems that exhibit spatio-temporal deformations. To achieve this, characteristics of the underlying system are exploited to design multivariable weighting functions. In contrast to pre-existing approaches, which typically lead to diagonal weighting functions, the proposed approach enables the design of non-diagonal weighting functions. Extensive experimental results confirm that the proposed procedure can significantly improve the performance of an industrial motion system compared to earlier approaches.

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1. Introduction

The design of a high-performance controller for a complex multivariable system hinges on the specification of a suitable optimization criterion. In model-based control, a scalar criterion is typically adopted that should reflect the user-defined performance requirements, as in Zhou, Doyle, and Glover (1996), McFarlane and Glover (1990) and Skogestad and Postlethwaite (2005). These requirements are defined in the optimization criterion by means of weighting functions, see, e.g., Lanzon and Tsiotras (2005), Hu, Bohn, and Wu (2000), Graham and de Callafon (2007) and Lundström, Skogestad, and Wang (1991). Relevant examples of model-based control include $\mathcal{H}_2$ and $\mathcal{H}_\infty$ control.

The key advantage of $\mathcal{H}_\infty$ –optimization is that it is capable of delivering robust controllers by explicitly taking model uncertainty into account. In contrast, LQG and $\mathcal{H}_2$ designs have no guaranteed robustness margins, as is shown in Doyle (1978). Besides the ability of $\mathcal{H}_\infty$ –optimization to design robust controllers, it allows for the design of weighting functions using loop-shaping concepts, see, e.g., Doyle and Stein (1981) and McFarlane and Glover (1990). These techniques are particularly suitable for motion control, where controllers are traditionally being designed using manual loop-shaping, see, e.g., van de Wal, van Baars, Sperling, and Bosgra (2002).

Weighting function design for motion systems typically employs diagonal weighting functions to specify performance requirements, see, e.g., Steinbuch and Norg (1998), Schönhoff and Nordman (2002) and van de Wal et al. (2002). The underlying assumption for this approach is that the system is approximately diagonal. This assumption is in general not valid for multivariable motion systems that exhibit spatio-temporal deformations. For such systems, parasitic dynamics are typically relevant in more than one channel of the servo system. In fact, such dynamics have in general a specific directional effect, making the control problem inherently multivariable. In such cases, non-diagonal weighting selection in the performance channels of the $\mathcal{H}_\infty$ –optimization problem might be very effective to enhance the performance of multivariable motion systems.

Although $\mathcal{H}_\infty$ control is promising for the design of controllers for multivariable motion systems that exhibit spatio-temporal dynamics, at present there is no procedure to adequately specify the control goal in standard $\mathcal{H}_\infty$ control criteria. This paper aims to improve performance of such systems by exploiting non-diagonal weighting functions that address the directional effect of spatio-temporal dynamics. To achieve this, the designed weighting functions incorporate frequency-localized compensation of directionality in the system. The performance improvement obtained with non-diagonal weighting functions is illustrated by means of an experimental case study for an industrial high-performance motion system. This paper is an extension of a previously published conference paper (Boeren, van Herpen, Oomen, van de Wal,

* Corresponding author. Tel.: +31 402474517.

E-mail address: f.a.j.boeren@tue.nl (F. Boeren).

& Bosgra, 2013) and includes a complete derivation, analysis and experimental results.

This paper is organized as follows. In Section 2, loop-shaping for multivariable systems is revisited. In Section 3, a procedure is proposed to determine transformation matrices that provide frequency-localized compensation of the directionality in the system. In Section 4, multivariable weighting functions are proposed to specify performance requirements for motion systems. In Section 5, extensive experimental results for an industrial motion system are provided to evaluate the achievable performance enhancement of the proposed weighting function design. Finally, conclusions are provided in Section 6.

Notation: Let $P(s) \in \mathcal{R}^{n \times n}$ denote a square real-rational transfer function matrix, with s the Laplace operator. For ease of notation, $P(s)$ is also denoted P . Furthermore, let the singular values of a matrix $A \in \mathbb{C}^{n \times m}$ be denoted by $\sigma_i(A)$, with the maximum (resp. minimum) singular value denoted by $\bar{\sigma}(A)$ (resp. $\underline{\sigma}(A)$). The eigenvalues of a matrix $A \in \mathbb{C}^{n \times m}$ are denoted by $\lambda_i(A)$. A matrix $U \in \mathbb{C}^{n \times n}$ is unitary if $U^H U = U U^H = I$, where U^H is the conjugate transpose of U .

Remark: In order to clearly illustrate the concepts in this paper, attention is restricted to square systems $P(s)$. Extensions to non-square systems are conceptually straightforward.

2. Problem definition

2.1. Loop-shaping

Closed-loop performance and robustness requirement can often be translated in a desired loop transfer function. The goal of loop-shaping is to attain this desired loop transfer function $L_{\text{des}} = PC$, with P being the system and C being the controller, that prescribes the desired gains of the system as a function of frequency. In particular, three requirements for L_{des} are commonly imposed, see, e.g., McFarlane and Glover (1990, Chap. 6) and Skogestad and Postlethwaite (2005, Chap. 9). These requirements are indicated in Fig. 1 by the black triangles and the cross-over region $f \in [f_1, f_2]$, given by $\underline{\sigma}(L_{\text{des}}(f_1)) = 1$ and $\bar{\sigma}(L_{\text{des}}(f_2)) = 1$.

- R1. Nominal stability: The maximum roll-off rate of $|\lambda_i(L_{\text{des}})|$, $\forall i$ in the cross-over region $f \in [f_1, f_2]$ is -40 db/decade.
- R2. A large open-loop gain needs to be attained for frequencies below f_1 , i.e., $\underline{\sigma}(L_{\text{des}}) \gg 1 \ \forall f \in [0, f_1]$.
- R3. A small open-loop gain needs to be attained for frequencies above f_3 , i.e., $\bar{\sigma}(L_{\text{des}}) \ll 1 \ \forall f \in [f_3, f_\infty]$.

Fig. 1. Singular values of the desired loop transfer function $L_{\text{des}}(s)$ for performance and robustness.

Requirement R1 ensures nominal closed-loop stability, while R2–R3 reflect classical closed-loop performance and robustness requirements. To illustrate the connection between R2–R3 and closed-loop requirements, consider the desired sensitivity function $S_{\text{des}} = (I + L_{\text{des}})^{-1}$ and desired complementary sensitivity function $T_{\text{des}} = L_{\text{des}}(I + L_{\text{des}})^{-1}$. As shown in McFarlane and Glover (1990), these expressions can be approximated in the relevant frequency ranges by

$$\begin{aligned} \bar{\sigma}(S_{\text{des}}) &\leq \frac{1}{\underline{\sigma}(L_{\text{des}})} \ll 1 \quad \text{where } \underline{\sigma}(L_{\text{des}}) \gg 1, \\ \bar{\sigma}(T_{\text{des}}) &\leq \bar{\sigma}(L_{\text{des}}) \ll 1 \quad \text{where } \bar{\sigma}(L_{\text{des}}) \ll 1. \end{aligned} \quad (1)$$

Expression (1) reveals that L_{des} implicitly determines the singular values of S_{des} and T_{des} . Complying with typical requirements, low-frequency disturbances are attenuated if $\bar{\sigma}(S_{\text{des}}) \ll 1$, while high-frequency robustness with respect to model uncertainty is obtained if $\bar{\sigma}(T_{\text{des}}) \ll 1$. This result shows that R2–R3 dictate closed-loop performance and robustness requirements.

In this paper, the $\mathcal{H}_\infty$ loop-shaping design procedure presented in McFarlane and Glover (1990) is used to attain L_{des} .

Goal 1. Given $\sigma_i(L_{\text{des}})$, the goal in loop-shaping is to design weighting functions $W_1(s), W_2(s)$ such that

$$\sigma_i(P_s) \approx \sigma_i(L_{\text{des}}) \quad i = 1, \dots, n.$$

where the shaped system P_s is given by

$$P_s(s) = W_2(s)P(s)W_1(s). \quad (2)$$

By loop-shaping $\sigma_i(P_s)$, Goal 1 ignores nominal closed-loop stability considerations as given in R1. In the $\mathcal{H}_\infty$ loop-shaping design procedure, closed-loop stability is ensured by subsequently using $\mathcal{H}_\infty$ -optimization based on the designed $P_s(s)$. A complete tutorial for the design of controllers using the $\mathcal{H}_\infty$ loop-shaping design procedure is provided in McFarlane and Glover (1990).

Remark 1. The open-loop weighting functions $W_1(s), W_2(s)$ in (2) can be directly translated into equivalent closed-loop weighting functions, as is used in common $\mathcal{H}_\infty$ -optimization algorithms including Skogestad and Postlethwaite (2005).

2.2. Directionality in multivariable systems

For multivariable systems, the input and output directionality of $P(s)$ complicates the design of $W_1(s)$ and $W_2(s)$. This directionality determines the connection between the singular values and the individual entries of $P(s)$, as reflected in the singular value decomposition. The singular value decomposition at frequency ω_k is given by

$$P(j\omega_k) = U(j\omega_k)\Sigma(j\omega_k)V^H(j\omega_k), \quad (3)$$

with $\Sigma(j\omega_k) = \text{diag}(\sigma_1(j\omega_k), \sigma_2(j\omega_k), \dots, \sigma_n(j\omega_k)) \in \mathbb{R}^{n \times n}$, and unitary matrices $V(j\omega_k) \in \mathbb{C}^{n \times n}$ and $U(j\omega_k) \in \mathbb{C}^{n \times n}$.

In (3), Σ represents the singular values of the system, while V and U represent the corresponding input and output directionality. The key point is that this directionality is frequency dependent. Since performance and robustness requirements are specified by means of loop-shaping Σ , the directionality of P as defined in V and U should be accounted for in $W_1(s)$ and $W_2(s)$, as illustrated in Fig. 2. As a result, the singular values in Σ are accessible for loop-shaping.

In classical control design, directionality is often only addressed in the cross-over region, see, e.g., Maciejowski (1989, Section 4.3). Typically, this design methodology is focused on determining static transformation matrices $T_u, T_y \in \mathbb{R}^{n \times n}$ such that

$$P_{\text{diag}}(s) = T_y P(s) T_u, \quad (4)$$

Fig. 2. A need for directionality compensation in weighting function design for multivariable systems.

is diagonally dominant in $f \in [f_1, f_2]$. As a result, the singular values in $f \in [f_1, f_2]$ are approximately equal to the diagonal entries of $P_{\text{diag}}(s)$,

$$\sigma_i(P_{\text{diag}}) \approx |(P_{\text{diag}})_{ii}|,$$

where $(P_{\text{diag}})_{ii}$ is the (i, i) th entry of P_{diag} . Herein, V and U in (3) are compensated in $f \in [f_1, f_2]$ by $\tilde{T}_u = V$ and $\tilde{T}_y = U^H$. Then, the typically complex matrices $\tilde{T}_y, \tilde{T}_u$ are approximated by $T_u, T_y \in \mathbb{R}^{n \times n}$. For motion systems, this procedure corresponds to rigid-body decoupling, as is elaborated in Section 3.

Based on $P_{\text{diag}}(s)$ in (4), the requirements R2–R3 are translated into diagonal $W_1(s)$ and $W_2(s)$. As depicted in Fig. 3, W_1 defines integral action in $f \in [0, f_1]$, while W_2 specifies roll-off in $f \in [f_3, f_\infty]$. In addition, $W_1(s), W_2(s)$ scale $P(s)$ such that the cross-over region of $P_s(s)$ is in $[f_1, f_2]$. This diagonal weighting function design is successfully applied to motion systems in, e.g., van de Wal et al. (2002).

2.3. Extended requirements for multivariable motion systems with spatio-temporal dynamics

Lightly damped resonance phenomena associated with spatio-temporal dynamics hamper the performance of a mechanical system, see, e.g., Balas and Doyle (1994) and Smith, Chu, and Fanson (1994). Indeed, in Oomen et al. (2014, Section V) it is revealed that for an industrial motion system the dominant components in the servo error stem from such phenomena, as indicated by gray rectangles in Fig. 1.

A design for $W_1(s), W_2(s)$ is proposed that aims to specify performance requirements for lightly damped resonance phenomena in $[f_2, f_3]$, while retaining R2–R3. Since these dynamics have in general a specific directional effect, non-diagonal $W_1(s), W_2(s)$ are required to enable loop-shaping of the singular values in $[f_2, f_3]$. Non-diagonal $W_1(s), W_2(s)$ in $\mathcal{H}_\infty$ loop-shaping are considered in Papageorgiou and Glover (1997), Lanzon (2005), and Osinuga, Patra, and Lanzon (2012).

In this paper, properties of mechanical systems are exploited to address directionality in $W_1(s), W_2(s)$. In particular, frequency-localized compensation of the directionality is used to enable loop-shaping of the singular values of $P(s)$ in m distinct frequency ranges, as defined in the following sub-goal.

Sub-Goal 1. Determine matrices $T_{u,j}, T_{y,j} \in \mathbb{R}^{n \times n}$, for $j = 1, \dots, m$, such that $\forall i = 1, \dots, n$

$$\sigma_i(T_{u,j}P(s)T_{y,j}) \approx |(T_{u,j}P(s)T_{y,j})_{ii}| \quad \forall f \in [f_j, f_{j+1}],$$

i.e., account for the directionality of the system in the j th frequency range.

The static transformation matrices $T_{u,j}, T_{y,j}$ should be absorbed in $W_1(s), W_2(s)$ such that $P_s(s)$ satisfies Goal 1. In the next section, a procedure is proposed to determine $T_{u,j}, T_{y,j}$ that compensate for the directionality of the system in the j th frequency range.

3. Frequency-localized directionality compensation

In this section, a procedure is proposed that is aimed to address Sub-Goal 1. To this purpose, $T_{u,j}, T_{y,j}$ for $j = 1, \dots, m$ are determined that compensate for the directionality in distinct frequency ranges by exploiting the characteristics of the underlying system. The

Fig. 3. Open-loop weighting functions $W_1(s)$ and $W_2(s)$ which reflect requirements R2–R3.

focus is on a class of motion systems which are described by the following definition.

Definition 1 (Gawronski, 2004). The dynamical response of a linear mechanical system $P(s) \in \mathbb{R}^{n \times n}$ with proportional damping can be written as a sum of N second-order subsystems

$$P(s) = \sum_{i=1}^n \frac{c_i^T b_i}{s^2} + \sum_{i=n+1}^N \frac{c_i^T b_i}{s^2 + 2\zeta_i \omega_i s + \omega_i^2}, \quad (5)$$

with n being the number of rigid-body modes, c_i^T being the i th column of the output matrix $C \in \mathbb{R}^{n \times N}$, b_i being the i th row of the input matrix $B \in \mathbb{R}^{N \times n}$, ζ_i being the dimensionless damping constant, ω_i being the natural frequency of the i th second-order subsystem.

In this paper, $P(s)$ is controlled in all n rigid-body degrees of freedom. As a result, the number of inputs and outputs of $P(s)$ is equal to the number of rigid-body modes.

For the sake of clarity, transformation matrices $T_{u,j}, T_{y,j}$ are determined for $f \in [0, f_2]$ and $f \in [f_2, f_3]$. Extension of the frequency-localized directionality compensation method to more frequency ranges is conceptually straightforward. First, transformation matrices $T_{u,1}, T_{y,1}$ are determined that result in compensation of the directionality in $f \in [0, f_2]$. Consider the following intermediate result.

Lemma 1. Let $P(s)$ be of the form of Definition 1. Then, there exist $T_{u,1}, T_{y,1} \in \mathbb{R}^{n \times n}$ such that

$$T_{y,1} s^2 P(s) T_{u,1},$$

is diagonal at $s=0$.

Proof. Since

$$s^2 P(s) = \sum_{i=1}^n c_i^T b_i + \sum_{i=n+1}^N \frac{c_i^T b_i s^2}{s^2 + 2\zeta_i \omega_i s + \omega_i^2},$$

it holds that for $|s| \rightarrow 0$

$$s^2 P(s) = \sum_{i=1}^n c_i^T b_i \in \mathbb{R}^{n \times n}. \quad (6)$$

The singular value decomposition of (6) is given by $U \Sigma V^T$ where $U, \Sigma, V \in \mathbb{R}^{n \times n}$. By defining $T_{y,1} = U^T$ and $T_{u,1} = V$ it follows that

$$T_{y,1} s^2 P(s) T_{u,1} = \Sigma \quad \text{for } s=0,$$

since U and V are unitary matrices. Hence, $T_{y,1} s^2 P(s) T_{u,1}$ is diagonal at $s=0$. $\square$

Next, Lemma 1 is used to propose rigid-body decoupling for motion systems. The concept of diagonal dominance is exploited, as defined in Maciejowski (1989, Section 2.10).

Proposition 1 (Rigid-body decoupling). Let $P(s)$ be of the form of Definition 1. Then,

$$P_{\text{diag}}(s) := T_{y,1} P(s) T_{u,1}, \quad (7)$$

with $T_{u,1}, T_{y,1} \in \mathbb{R}^{n \times n}$ from Proposition 1 being diagonally dominant for $|s| \rightarrow 0$.

Proof. Substitute (5) in (7) to obtain

$$T_{y,1}P(s)T_{u,1} = \frac{1}{s^2}\Sigma + \sum_{i=n+1}^N T_{y,1} \frac{c_i^T b_i}{s^2 + 2\zeta_i \omega_i s + \omega_i^2} T_{u,1}.$$

For $|s| \rightarrow 0$ it holds that

$$\frac{1}{s^2}\Sigma \gg \sum_{i=n+1}^N T_{y,1} \frac{c_i^T b_i}{s^2 + 2\zeta_i \omega_i s + \omega_i^2} T_{u,1},$$

i.e., $P_{\text{diag}}(s)$ is diagonally dominant for $|s| \rightarrow 0$. $\square$

The second-order rigid-body dynamical behavior at $s=0$ dominates the dynamical response of the system up to frequency f_2 . Therefore, for $i = 1, \dots, n$,

$$\sigma_i(T_{y,1}P(s)T_{u,1}) \approx |(T_{y,1}P(s)T_{u,1})_{ii}| \quad \forall f \in [0, f_2].$$

Second, transformation matrices $T_{u,2}, T_{y,2}$ are determined that provide local compensation of the directionality in $f \in [f_2, f_3]$. Consider the transformed system $P_{\text{flex}}(s)$, representing the resonance phenomena of $P(s)$, given by

$$P_{\text{flex}}(s) := P(s) - \sum_{i=1}^n \frac{c_i^T b_i}{s^2} = \sum_{i=n+1}^N \frac{c_i^T b_i}{s^2 + 2\zeta_i \omega_i s + \omega_i^2}.$$

Assumption 1. The lightly damped resonance phenomena in $[f_2, f_3]$ given by

$$\sum_{i=n+1}^{2n} \frac{c_i^T b_i}{s^2 + 2\zeta_i \omega_i s + \omega_i^2},$$

dominate the dynamical response of $P_{\text{flex}}(s)$ at $s=0$, i.e.,

$$\sum_{i=n+1}^{2n} \frac{c_i^T b_i}{\omega_i^2} \gg \sum_{i=2n}^N \frac{c_i^T b_i}{\omega_i^2},$$

where $\sum_{i=2n}^N c_i^T b_i / (s^2 + 2\zeta_i \omega_i s + \omega_i^2)$ correspond to the resonance phenomena in $[f_3, f_\infty]$.

Assumption 1 is graphically illustrated in Fig. 4 for a system with $n=2$. This assumption is in general non-restrictive for motion systems and is exploited in the following proposition.

Proposition 2. Given $P_{\text{flex}}(s)$, there exist $T_{u,2}, T_{y,2} \in \mathbb{R}^{n \times n}$ such that

$$P_{\text{flex,diag}} := T_{y,2}P_{\text{flex}}(s)T_{u,2},$$

where $P_{\text{flex,diag}}$ is diagonally dominant for $|s| \rightarrow 0$.

The proof of Proposition 2 follows along similar lines as the proof provided by Proposition 1. By virtue of Assumption 1, the dynamical response of $P_{\text{flex}}(s)$ is in $f \in [0, f_3]$ dominated by the first

Fig. 4. Illustration of Assumption 1: the directionality of the resonance phenomena in $f \in [f_2, f_3]$ (green) can be determined by means of Proposition 2 if these modes dominate the dynamical response of $P_{\text{flex}}(s)$ in $f \in [0, f_2]$ with respect to the contribution of resonance phenomena in $f \in [f_3, f_\infty]$ (red). (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

Fig. 5. The weighting function $W_{1,2}$ (left) reflects the specification of derivative action in $f \in [f_2, f_3]$ to provide damping. The multivariable weighting function W_1 (right) consists of $T_{u,2}W_{1,2}T_{y,2}$ and $W_{1,1}$, and specifies performance requirements for $f \in [0, f_3]$.

n lightly damped resonance phenomena. As a result, for $i = 1, \dots, n$,

$$\sigma_i(T_{y,2}P_{\text{flex}}(s)T_{u,2}) \approx |(T_{y,2}P_{\text{flex}}(s)T_{u,2})_{ii}|,$$

in the frequency range $f \in [0, f_3]$.

Even though $T_{u,2}, T_{y,2}$ diagonalize $P_{\text{flex}}(s)$ in $[0, f_3]$, the original system $P(s)$ is only diagonalized in the frequency range where the first n resonance phenomena dominate the dynamical response of $P(s)$. In Fig. 1, this corresponds to $[f_2, f_3]$. This implies that $T_{u,1}, T_{y,1}$ for rigid-body decoupling and $T_{u,2}, T_{y,2}$ for the flexible dynamics should be absorbed in W_1, W_2 to enable explicit loop-shaping of σ_i in respectively $[0, f_2]$ and $[f_2, f_3]$. The way to do this is to make use of frequency dependent weighting functions, as presented in the next section.

4. Multivariable weighting function design

In this section, multivariable weighting functions $W_1(s)$ and $W_2(s)$ are proposed for motion systems with spatio-temporal dynamics. These weighting functions reflect (i) classical requirements R2–R3 and (ii) performance requirements for the first resonance phenomenon in $[f_2, f_3]$.

In Preumont (2002) it is shown that for a SISO system, the inclusion of derivative action in the controller provides damping to lightly damped resonance phenomena. As a result, closed-loop performance is enhanced for systems with spatio-temporal dynamics. In terms of the desired loop transfer function L_{des} defined in Section 2.1, derivative action should be specified with respect to the singular value $\sigma_i(L_{\text{des}})$ associated with the resonance phenomenon in $[f_2, f_3]$. Frequency-localized directionality compensation as proposed in Section 3 is essential to accurately translate this performance requirement in $W_1(s)$ and $W_2(s)$.

The following design procedure for $W_1(s)$ and $W_2(s)$ is proposed, which implements the main contribution of this paper.

Procedure 1. Given $\sigma_i(L_{\text{des}})$,
Classical loop-shaping

- Step 1: Determine $T_{y,1}$ and $T_{u,1}$ by applying Proposition 1 to locally compensate for the directionality in $[0, f_2]$.
- Step 2: Design $W_{1,1}$ and W_2 to reflect classical requirements R2–R3, see Fig. 3, with cross-over region $f \in [f_1, f_2]$, given by $\underline{\sigma}(P_s(f_1)) \geq 1$ and $\overline{\sigma}(P_s(f_2)) \leq 1$.

Extended loop-shaping

- Step 3: Determine $T_{y,2}$ and $T_{u,2}$ by applying Proposition 2 to locally compensate for the directionality in $[f_2, f_3]$.
- Step 4: Construct $W_{1,2}$ for $[f_2, f_3]$ as illustrated in Fig. 5.

Construction W_1 and W_2

- Step 5: Absorb $W_{1,1}$ and $T_{u,2}W_{1,2}T_{y,2}$ in W_1 , see Fig. 6.

Fig. 6. Multivariable weighting function design.

Fig. 7. Weighted feedback configuration.

Steps 1–2 have been presented in Section 2 for classical loop-shaping. Therefore, this section focuses on Steps 3–4 and Steps 5–6, which constitutes the novel aspect of the proposed weighting function design.

First, consider Steps 3–4 of Procedure 1. In this part of the procedure, damping is specified to the first lightly damped resonance phenomenon in $[f_2, f_3]$ to improve the closed-loop performance. Specifically, similar to the detailed approach provided in Cao and Hori (1997), derivative action is included in $W_1(s)$ for $f \in [f_2, f_3]$. Alternatively, (inverse) notch filtering can be included in $W_1(s)$ to specify damping. Local compensation of the directionality by means of $T_{y,2}$ and $T_{u,2}$ in Step 3 enables an extension of the approach in Cao and Hori (1997) to multivariable systems.

Second, Step 5 of Procedure 1 is considered. In Step 5, $W_1(s)$ is constructed from $W_{1,1}(s)$ and $T_{u,2}W_{1,2}(s)T_{y,2}$, as depicted in Fig. 6. The absorption of $T_{y,2}W_{1,2}(s)T_{u,2}$ in $W_1(s)$ enables weighting of the singular values corresponding to the first resonance phenomena in $[f_2, f_3]$, while leaving the remaining singular values unaffected in this frequency range. Furthermore, the performance requirements in $[0, f_1]$ by $W_{1,1}(s)$ and in $[f_2, f_3]$ by $T_{y,2}W_{1,2}(s)T_{u,2}$ do not interfere with each other due to the specific structure of $W_1(s)$, see Fig. 5.

The multivariable weighting functions constructed in Procedure 1 specify performance and robustness requirements in a scalar optimization criterion. As a final step, the optimal controller C_s^* is determined by minimizing this optimization criterion. The controller C_s^* results from

$$C_s^* = \arg \min_{C_s} J(\hat{P}_s, C_s), \quad (8)$$

where $\hat{P}_s = W_2 \hat{P} W_1$ is based on a parametric model $\hat{P}$ of the system P , and $J(\hat{P}_s, C_s)$ is a norm-based criterion given by $J(\hat{P}_s, C_s) = \left\| T(\hat{P}_s, C_s) \right\|_\infty$ with

$$T(\hat{P}_s, C_s) : \begin{bmatrix} r \\ d \end{bmatrix} \mapsto \begin{bmatrix} y \\ u \end{bmatrix} = \begin{bmatrix} \hat{P}_s \\ I \end{bmatrix} (I + C_s \hat{P}_s)^{-1} \begin{bmatrix} C_s \\ I \end{bmatrix},$$

as depicted in Fig. 7. The corresponding robust stability margin $\epsilon_{\max}(\hat{P}_s, C_s)$ is defined as

$$\epsilon_{\max}(\hat{P}_s, C_s) = \left\| \begin{bmatrix} \hat{P}_s \\ I \end{bmatrix} (I + C_s \hat{P}_s)^{-1} \begin{bmatrix} C_s \\ I \end{bmatrix} \right\|_\infty^{-1}. \quad (9)$$

If $\epsilon_{\max}(\hat{P}_s, C_s) \ll 1$, the performance requirements specified in terms of $\sigma_i(L_{\text{des}})$ are incompatible with robust stability requirements. In this case, L_{des} should be adjusted, see McFarlane and Glover (1990).

The synthesis of the $\mathcal{H}_\infty$ -controller C_s^* in (8) is extensively treated in McFarlane and Glover (1990). Finally, the controller

$$C = W_1 C_s^* W_2, \quad (10)$$

is implemented on P . The resulting controller (i) guarantees internal stability of the closed-loop system and (ii) is $\mathcal{H}_\infty$ -optimal with respect to the user-defined performance and robustness requirements, as specified in $W_1(s)$ and $W_2(s)$.

5. Experimental results

In this section, the multivariable weighting function design proposed in this paper is implemented on a prototype industrial motion system. The goal of this section is to experimentally validate

that the proposed multivariable weighting functions result in improved performance compared to classical weighting functions.

5.1. Experimental setup

The considered prototype industrial motion system is depicted in Fig. 8. The positioning stage of this prototype is explicitly designed to exhibit pronounced lightly damped resonance phenomena. Therefore, the positioning stage has dimensions $600 \times 600 \times 60$ mm, while it weighs 13.5 kg only, see van Herpen et al. (2014). For comparison, existing systems, as considered in Butler (2011), weigh approximately 40 kg.

The positioning stage in Fig. 8 is controlled in all six motion degrees-of-freedom (DOF) (i.e., three rotations and three translations), where gravity compensators enable contactless operation. The maximum displacement in the translational DOFs (x , y , and z) is equal to 0.5 mm, while the range in the rotational DOFs (R_x , R_y , and R_z) is given by 1 mrad, see Ronde, Schneiders, Kikken, van de Molengraft, and Steinbuch (2014). The positioning stage is actuated by six Lorentz motors. Furthermore, the measurement system consists of three linear incremental encoders with a resolution of 1 nm in the vertical plane (z , R_x , R_y), and three capacitive sensors in the horizontal plane, i.e., for the DOFs x , y and R_z . The control software is implemented in Matlab Simulink Real-Time.

The geometry of the experimental-setup results in significant interaction between the DOFs z , R_x and R_y , while the interaction terms are negligible between the vertical plane (z , R_x , R_y) and the horizontal plane (x , y , R_z). For this reason, a multivariable controller design is pursued for the vertical plane, while a multiloop SISO controller is used for the horizontal plane. The singular values of the measured frequency response function $P(\omega)$ corresponding to the vertical plane are depicted in Fig. 9.

5.2. Weighting function design

In this section, performance and robustness requirements for the experimental setup in Fig. 8 are expressed in weighting functions. It is emphasized that the weighting function design is solely based on the frequency response function $P(\omega)$. First, multivariable weighting

Fig. 8. Experimental setup.

functions W_1^{mimo} and W_2^{mimo} are designed according to Procedure 1. Second, classical weighting functions W_1^{class} , W_2^{class} are designed based on the guidelines provided in Section 2.2. The target crossover region $f=[f_1, f_2]$ is chosen as 28 Hz in both approaches for comparison.

From Fig. 9, it becomes clear that the first resonance of $P(\omega)$ appears at 129 Hz. Therefore, as depicted in Fig. 10, W_1^{mimo} is designed such that derivative action is specified in [28, 250] Hz. Due to local directionality compensation, only the singular value associated with this resonance is weighted in this frequency range. Hence, this approach is not affecting the remaining singular values.

The shaped system, as defined in (2), is given by

$$P_s^{\text{mimo}} = W_2^{\text{mimo}} P W_1^{\text{mimo}},$$

$$P_s^{\text{class}} = W_2^{\text{class}} P W_1^{\text{class}},$$

for multivariable and classical weighting functions, respectively. As depicted in Fig. 11, the target crossover region is for P_s^{class} and P_s^{mimo} equal to 28 Hz. Similar to the observations based on Fig. 10, the

Fig. 9. Singular values of $P(\omega)$.

Fig. 10. Singular values of weighting functions (a) W_1 and (b) W_2 : the classical design (red) as in Section 2.2 specifies only low- and high-frequency weighting, while for the multivariable design (dashed green) as in Section 4 derivative action is added with respect to the singular value associated with the first resonance of $P(\omega)$ at 129 Hz. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

shaped system P_s^{mimo} as shown in Fig. 11 contains explicit weighting of the first lightly damped resonance, while leaving the remaining singular values of the system unaffected.

Based on the designed weighting functions, the procedure in Section 4 is used to synthesize C_s^{mimo} , based on W_1^{mimo} , W_2^{mimo} , and C_s^{class} , based on W_1^{class} , W_2^{class} . To this purpose, the required parametric model $\hat{P}$ of the system P is determined by means of the approach in Oomen et al. (2014), see Fig. 15. The robust stability margin in (9) is for the proposed design (resp. classical design) given by $\epsilon_{\max}(\hat{P}_s, C_s^{\text{mimo}}) = 0.3195$ (resp. $\epsilon_{\max}(\hat{P}_s, C_s^{\text{class}}) = 0.3061$). Finally, C^{mimo} and C^{class} are determined according to (10).

5.3. Analysis

The design of W_1^{mimo} , W_2^{mimo} is motivated by the observation in Section 2.3 that lightly damped resonance phenomena hamper the performance of a system. To illustrate the performance enhancement that can be obtained by means of the proposed multivariable weighting function design, rejection of disturbances in [40, 200] Hz is considered.

The process sensitivity $\hat{S}\hat{P} = (I + \hat{P}C)^{-1}\hat{P}$ is a suitable measure for disturbance rejection, since $\hat{S}\hat{P}$ describes the transfer of input disturbances to the error signal. As depicted in Fig. 12, the singular value of $\hat{S}\hat{P}$ associated with the first resonance phenomenon is attenuated with 6 dB in [40, 200] Hz as a result of the proposed multivariable weighting function design. Note that due to local directionality compensation, W_1^{mimo} , W_2^{mimo} only affect the singular value of $\hat{S}\hat{P}$ associated with the resonance phenomenon, while keeping the remaining singular values unaltered with respect to the classical design.

Fig. 11. Singular values of shaped system P_s : classical weighting function design (red) and multivariable weighting function (dashed green). (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

Fig. 12. The singular values of the process sensitivity $\hat{S}\hat{P}$ for classical weighting (red) and multivariable weighting functions (dashed green) show that a reduction with 6 dB is obtained in [40, 200] Hz as a result of multivariable weighting functions. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

Table 1

Poles and dimensionless damping coefficient in the frequency interval [28, 250] Hz show that the proposed multivariable weighting significantly improves the damping of the resonance at 129 Hz in $\hat{P}(s)$ compared to the classical weighting.

	$\hat{P}(s)$	Classical	Multivariable
Pole	$-3.89 \pm 809i$	$-166 \pm 833i$	$-605 \pm 969i$
ζ	0.00437	0.195	0.529

Fig. 13. Measured time domain responses for C^{class} (left) and C^{mimo} (right) show a reduction of a factor two in the peak value of the error signal e due to C^{mimo} .

In $\mathcal{H}_\infty$ -optimization, the closed-loop poles in the frequency range [28, 250] Hz are explicitly assigned by the weighting functions and $\hat{P}$, see, e.g., [Cao and Hori \(1997\)](#). The closed-loop poles for W_1^{mimo} , W_2^{mimo} and W_1^{class} , W_2^{class} as presented in [Table 1](#) confirm that the proposed weighting functions significantly improve the damping of the performance-critical resonance phenomenon at 129 Hz when compared to a classical design.

5.4. Experimental results

In this section, the performance enhancement obtained by means of multivariable weighting functions is confirmed on the setup in [Fig. 8](#). The controllers C^{mimo} and C^{class} are implemented on the experimental setup and an input disturbance with frequency components in [40, 200] Hz is applied to the system. Time-domain measurements in [Fig. 13](#) and the cumulative power spectrum in [Fig. 14](#) of the measured error signals confirm that the performance is significantly improved if C^{mimo} , based on the proposed multivariable weighting functions in [Section 4](#), is implemented on the system. Indeed, [Table 2](#) confirms that a significant reduction is obtained in the standard deviation and peak value of the error signal in the DOFs z , R_x and R_y .

6. Conclusions

In this paper, a novel procedure is developed to specify performance requirements for multivariable motion systems with spatio-temporal dynamics in $\mathcal{H}_\infty$ control. The proposed weighting functions are non-diagonal to address the multivariable dynamical

Fig. 14. Cumulative power spectrum of the measured error signals: controller based on classical weighting functions C^{class} (red) and proposed C^{mimo} (dashed green) based on the proposed multivariable weighting functions. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

Table 2

The standard deviation σ and peak value p of the error signals e_z , e_{R_x} and e_{R_y} are significantly reduced by means of the proposed C^{mimo} compared to the classical C^{class} .

	σ_z	σ_{R_x}	σ_{R_y}	p_z	p_{R_x}	p_{R_y}
C^{class}	0.39	1.57	1.28	1.31	6.12	4.40
C^{mimo}	0.18	0.86	0.60	0.73	3.38	2.30

behavior of the system, thereby extending pre-existing literature on weighting function design. In particular, characteristics of the system are exploited to provide frequency-localized compensation of directionality. Extensive experimental verification on a prototype industrial motion system shows the attained performance improvement.

Motion systems as defined in [Definition 1](#) are a specific class of dyadic systems, see, e.g., [Owens \(1978\)](#), [Anthonius and Ramon \(2003\)](#) and [Boerlage \(2008\)](#). Extensions of the proposed frequency-localized directionality compensation to this general framework are beyond the scope of the present paper. Furthermore, the concept of frequency-localized directionality compensation can be directly used in manual loop-shaping of controllers for multivariable systems. In future research, systems are considered that contain a larger number of actuators and sensors than rigid-body DOFs. This introduces additional freedom to specify performance requirements for lightly damped resonances in $\mathcal{H}_\infty$ -control, see, e.g., [van Herpen et al. \(2014\)](#).

Fig. 15. Parametric model $\hat{P}$ of the system P : nonparametric $P(\omega)$ (black) and parametric model $\hat{P}$ (dashed red). The rigid-body dynamics and the first two resonance phenomena are accurately represented in the parametric model $\hat{P}$, while the higher-order flexible phenomena are not considered to be control-relevant. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

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