Comments on "A theoretical model of the pressure field arising from asymmetric intraglottal flows applied to a two-mass model of the vocal folds" [J. Acoust. Soc. Am. 130, 389-403 (2011)]

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I. INTRODUCTION

In a series of recent papers Erath et al.\textsuperscript{1–3} consider the interesting problem of the asymmetry of the flow in the downstream part of the glottis during the closing phase of the vocal fold oscillation cycle. The glottis is then a slit shaped converging-diverging channel with a neck (width $d_{\text{min}}$) at the upstream side of the glottis. In first approximation the flow is two-dimensional (see Fig. 1). The downstream part can be described as a diffuser of length $l$, terminated by a strong widening. The diffuser part is assumed to have straight walls at an angle $\gamma$ with the channel axis ($x$-direction), where $x = L$ for left and $x = R$ for right. At the beginning of the closing phase the opening angle $(\gamma_L + \gamma_R)$ is so small that the flow only separates from the walls at the diffuser exit $x = l_e$ where the channel height $a(x)$ increases abruptly. A free jet is formed down stream of the glottis. The radius of curvature of the walls at $x = l_e$ is $R_e$. The angle of divergence $(\gamma_L + \gamma_R)$ of the downstream part of the glottis increases as $d_{\text{min}}$ decreases during the closing phase. Above a critical value of the divergence angle $(\gamma_L + \gamma_R) \approx \pi/15$ flow separation occurs within the diffuser. The separation points move toward the neck of the glottis with increasing angle. Under steady flow conditions in such a configuration the jet will tend to attach to one of the walls of the diffuser (Fig. 1). This is the result of entrainment of the fluid surrounding the jet by momentum transfer from the jet. A small asymmetry results into a pressure difference that bends the jet toward the closest wall. In the case of the glottis the flow remains usually laminar in the diffuser so that this is driven by viscous (molecular) momentum transfer. When the jet attaches to one of the walls it remains there. We have assumed attachment to the right wall in Fig. 1 ($x = R$).

For a curved wall a pressure gradient necessary to curve the streamlines is established, resulting in a low pressure at the wall on which the jet is attached. Hence the jet exerts a transversal force on that wall. Once established this asymmetric flow can be described, in first order approximation, as a frictionless flow. Of course a viscous boundary layer will grow between the jet and the wall to meet the no-slip viscous boundary condition at the wall. The flow separation of the attached jet from the wall is determined by the momentum thickness $\theta$ of this boundary layer and adverse pressure gradient that occurs as the radius of curvature of the wall increases. The build-up and maintenance of this asymmetric flow is referred to as the Coanda effect.\textsuperscript{1–6} While Mittal et al.\textsuperscript{2} does not like this terminology, we will use it because it is the most commonly used by fluid dynamicists.

Asymmetry in the flow due to the Coanda effect was first observed by Teager and Teager\textsuperscript{7} downstream of the glottis in a static model of the upper airways (within the mouth). The intraglottal Coanda effect was observed in \textit{in vitro} static models of the glottis by, among others, Pelorson et al.\textsuperscript{8} and Scherer et al.\textsuperscript{9} It was argued by Hofmans et al.\textsuperscript{10} that the time scale needed to establish such an asymmetric flow was too long to allow asymmetry in glottal flow. It is one of the merits of Erath et al.\textsuperscript{1} that they have carried out experiments with an oscillating \textit{in vitro} model of vocal folds, demonstrating the existence of the Coanda effect during the closing phase of the oscillation...
cycle. Once this fact was established, Erath et al.\textsuperscript{2} made an attempt to estimate the impact of such an asymmetry on the motion of the vocal folds and the sound production. They have developed a semi-empirical model, which they call Boundary Layer Estimation of Asymmetric Pressure (BLEAP) aiming at calculating the transversal force on the

velocity decreases along the jet. As a consequence the pressure $p(x)$ is predicted to increase with increasing $x$ so that $p_i < p_e$. At sufficiently high Reynolds numbers $\text{Re} = [U_i d_{\text{min}}/\nu] \geq 10^2$ (with $\nu$ the kinematic viscosity of air) one expects a thin viscous boundary layer for which the equations of Prandtl are valid.\textsuperscript{11,12} This implies a vanishing normal pressure gradient $\partial p/\partial y = 0$ so that the pressure at the wall is equal to the pressure $p(x)$ in the jet. As the angles $|\gamma_x|$ are small we do not distinguish here between the direction normal to the wall and the $y$-direction.

Erath et al.\textsuperscript{2} consider the normal pressure gradient induced by the rotation of the wall and come to the conclusion that it is negligible. They do consider the axial pressure gradient due to the centrifugal acceleration $\omega \Omega_x$, where $\Omega_x$ is the angular velocity of the wall. As they use Eq. (1) it is not clear to the author whether they actually do correct for this centrifugal effect. It seems, however, that neglecting this term would be consistent with the use of a quasi-steady approximation.

As a confirmation for the validity of their approach, Erath et al.\textsuperscript{2} show that the viscous boundary layer profile is correctly predicted from the measured velocity $U(x)$ by using a locally self-similar solution of Falkner and Skan.\textsuperscript{11,13,14} As they find an adverse pressure gradient $\partial p/\partial x > 0$ they worry about possible separation of this viscous boundary layer within the diffuser. Using the method of Thwaites,\textsuperscript{6,14,15} they come to the conclusion that this will not occur within the glottis.

The force $G_{z, \text{close}}$ per unit length of the vocal fold ($z$-direction) acting on the wall is calculated by the integration of $p(x)$,

$$ G_{z, \text{close}} = \int_0^x p(x)dx, $$

assumed to be valid when $|\gamma_x| + |\gamma_y| > \pi/15$ and for $d\theta_{\text{min}}/dt < 0$. This is the force per unit length of the vocal folds.

III. CRITICISMS

Using the same argument as for the viscous boundary layer one can demonstrate that the boundary layer approximation of Prandtl is also valid for a jet flow\textsuperscript{11,12,14} at a sufficiently high Reynolds number $\text{Re} = U_i d_{\text{min}}/\nu$. The typical
Reynolds number in glottal flow is \( \text{Re} = O(10^4) \). Actually when \( l_i/(a_{\text{min}}\text{Re}) \geq 1 \) the boundary layer cannot be distinguished from the jet and a self-similar wall-jet structure is approached if we assume a flow along a flat plate. For a wall-jet along a flat plate the boundary layer approximation of Prandtl implies, as for the boundary layer, that the normal gradient of the pressure vanishes, \( \partial p/\partial y = 0 \). This conclusion remains valid for a rotating flat plate. A pressure gradient \( \partial p/\partial y \) will be induced by the rotation of the wall and can be described as a result of the Coriolis acceleration experienced by an observer moving with the wall. The resulting wall pressure can be estimated by

\[
|\Delta p_{\text{Coriolis}}| \approx 2\rho \Omega_a U a_{\text{min}}. \quad (4)
\]

The neck width \( a_{\text{min}} \) is used here as an estimation for the wall-jet thickness because Erath et al.\(^2\) assume separation at the neck (\( x = 0 \)). The authors neglect this effect and we will accept this as it is consistent with the use of a quasi-steady approximation in Eq. (1).

Erath et al.\(^2\) assume that when \( |y_{\text{le}}| + |y_{\text{rt}}| \geq \pi/15 \) the flow separates at the neck \( x = 0 \) and a wall-jet is formed on one of the walls for \( x > 0 \). Assuming flow separation at the neck \( x = 0 \) and assuming that the pressure on the left wall is equal to \( p_{\text{r}} = p_{\text{le}}(x) \), by applying Bernoulli [Eq. (1)] in combination with Eq. (2), Erath et al.\(^2\) find a pressure \( p(x) < p_{\text{r}} \) in the wall-jet for \( 0 < x < l_c \). At the neck they find \( p_{\text{le}} > p_{\text{r}} \). Hence they predict a transversal pressure discontinuity across the shear layer separating the wall-jet from the dead-water region (left of the jet in Fig. 1). This is physically impossible because a shear layer (or streamline in a frictionless model) cannot sustain a pressure difference, \( \partial p/\partial y = 0 \). In the separated flow region the pressure gradient is negligible, because the velocity is low compared to that in the wall-jet. Hence the pressure \( p(x) \) should be the same for both walls \( p(x) = p_{\text{le}} \) if we assume that the wall is a flat plate.

This contradiction is a consequence of the use of the equation of Bernoulli [Eq. (1)] in the jet. If the observed decrease of \( U(x) \) is due to viscous momentum transfer this would explain this paradox. The equation of Bernoulli is not valid. However part of the problem is due to the fact that the wall-jet is formed downstream of the neck rather than at the neck \( x = 0 \) as assumed by Erath et al.\(^2\). We now consider both aspects. The dimensionless parameter \( \text{Re}(a_{\text{min}}/l_c) = U a_{\text{min}}^2/(\nu l_c) \) allows to estimate the effect of viscosity on the wall-jet velocity \( U(x) \) for laminar flow conditions. When \( \text{Re}(a_{\text{min}}/l_c) \leq 10 \) the viscous boundary layers have reached the jet centerline. This will typically occur for normal phonation \( U_i = 20 \text{ m/s} \) and \( l_c \approx 3 \text{ mm} \) when \( a_{\text{min}} \leq 0.2 \text{ mm} \). This corresponds to the phase condition \( t/T_{\text{open}} > 0.80 \) following the data shown in Fig. 3 of Erath et al.\(^2\). For shorter times such as \( t/T_{\text{open}} = 0.7 \) or 0.60 the flow separates downstream of the neck. This can explain the strong decrease of \( U(x) \) just downstream of the neck, observed by Erath et al.\(^2\). Between the neck and the separation point \( x = x_s \) the equation of Bernoulli (1) can be used to estimate \( p(x) \); however, the same pressure applies to both walls. More information can be obtained by considering the study of Scherer et al.\(^9\).

The flow conditions shown by Erath et al.\(^2\) for \( t/T_{\text{open}} = 0.7 \) correspond to the experimental data and numerical simulation provided by Scherer et al.\(^9\). Scherer et al.\(^9\) consider steady flow conditions. In their \textit{in vitro} model \( |y_{\text{le}}| = |y_{\text{rt}}| = 5^\circ \). The jet separates from the left wall at about \( x = l_i/3 \). The difference in pressure shows that the left wall pressure is very close to the exit pressure \( p_c \) for \( x > l_i/3 \). The right wall pressure \( p(x) \) is lower than \( p_c \). This effect is most pronounced as the jet approaches the glottal exit where the wall curvature becomes strong, just before the jet separates from the right wall.

This confirms our assumption that a difference in pressure between the right wall and the glottal exit is mainly due to wall curvature or to the non-uniformity of the flow in the neighborhood of the separation point (\( x = x_s \)). For simplicity Erath et al.\(^2\) assume the fit constants \( n, c_0, \) and \( x_{\text{eff}} \) in Eq. (2) to be independent of \( a_{\text{min}} \). As a consequence the predicted force \( G_{\text{r,close}} \) is almost independent of \( a_{\text{min}} \) and displays a discontinuous behavior as \( a_{\text{min}} \rightarrow 0 \). Also the assumption that flow separation appears at the neck \( x = 0 \) when \( |y_{\text{r}}| + |y_{\text{l}}| > \pi/15 \) induces a discontinuous behavior of \( G_{\text{r,close}} \). Discontinuous time dependence of forces can induce chaotic behavior, making conclusions on the impact of asymmetry based on a two-mass lumped model rather hazardous.

**IV. A MODIFIED MODEL**

Using the basic assumptions of Erath et al.\(^2\): frictionless, quasi-steady incompressible flow, we now propose an alternative model. We assume attachment of the free jet to the right side of the diffuser and that the wall-jet separates from the right wall at the glottal exit \( x > l_i \). The jet width corresponds to the diffuser width \( a(x) = a_s \) at the position \( x = x_r \), where the flow separates from the left wall (see Fig. 1). The velocity \( U_B = \sqrt{2(p_c - p_{\text{r}})/\rho} \) in the jet is determined by applying Bernoulli from a position upstream of the glottis (where \( p = p_{\text{r}} \)) to the jet (where \( p(x) \approx p_c \)). Considering an integral y-momentum balance, for the volume enclosed by the control surface sketched in Fig. 1, we see that the jet leaving the glottis with an angle \( \gamma_{\text{r}} \) at \( x = x_r \) carries a y-momentum flux per unit length of the vocal folds,

\[
\Phi_y = a_s \rho U_B^2 \sin \gamma_{\text{r}}. \quad (5)
\]

This must be induced by a force from the walls on the flow. The reaction to this force is the net force of the flow on the walls in the y-direction. Hence we have: \( G_{\text{r,close}} = -\Phi_y \). This is actually the sum of the lateral forces (y-components) acting on the right and left walls of the glottis, respectively. As the deflection of the jet occurs around the separation point \( x = x_s \), which is the position where this force should be applied, in a dynamical model, on the wall on which the jet remains attached (right wall for the conditions shown in Fig. 1). Note that upstream from the separation point \( x < x_s \) the pressure \( p(x) \) can be estimated by using the equation of Bernoulli (1) in combination with the equation of continuity \( U(x)a(x) = U_B a_s \), where \( a(x) \) is the width of the glottal channel. We assume here a quasi-one-dimensional flow.
[uniform velocity \( U(x) \) within a cross section]. The lateral force on each wall (left and right) can be obtained by integration of the pressure \( p(x) \) along the wall. The y-component of these forces \((y < x_r)\) have equal magnitudes and opposite signs.

Obviously the main problem is to obtain an estimation for the position \( x_s \) of the separation point. A simplified method to obtain such an estimate is the method of Thwaites.\(^6,14–16\) The momentum thickness \( \theta \) of the viscous boundary layer is estimated by using the expression of Thwaites,

\[
\theta(x)^2 - \theta(0)^2 = \frac{0.45\nu}{U_B} \int_0^x \frac{a_s}{a(x)} \, dx.
\]

In principle we should integrate over a coordinate \( s \) along the wall. As we are using a Prandtl boundary layer approximation for the flow we can approximate this integration along \( s \) by an integration along \( x \). We also neglected here the effect of viscous boundary layer growth on the velocity \( U(x) \), which is only valid for \( 60/a(x) \ll 1 \). Using an iterative procedure one can correct for this effect. We will ignore this subtlety. Separation occurs when

\[
\left( \frac{\theta^2}{\nu} \right) (dU/dx) = -0.09.
\]

Assuming a diffuser with plane side wall, we have for small opening angles

\[
a(x) = a_{min} + x \sin(|\gamma_L| + |\gamma_R|),
\]

where we implicitly assumed a paraxial approximation (small angles corresponding to Prandtl’s boundary layer approximation). Combining Eqs. (6), (7), and (8) yields an implicit equation for \( x_s \), which should be solved numerically. An initial guess can be obtained by assuming a plane wall behavior \( \theta \approx \sqrt{\nu/9U_B} \) without pressure gradient, which combined with Eqs. (7) and (8) and yields an explicit expression for \( x_s \).

We finally consider the separation of the wall-jet at the glottal channel exit, using the method of Thwaites. Assuming a radius of curvature \( R_e \gg a_s \) at \( x = l_e \) we can estimate the adverse pressure gradient experienced by the wall boundary layer as

\[
(dp/dx)_e \approx \rho U_B^2 a_s / R_e^2.
\]

Using the plane wall approximation \( \theta_e \approx \sqrt{\nu l_e / 9U_B} \), we find from Eq. (7) that separation occurs at \( x = l_e \) when \( l_e a_s / R_e^2 \approx 0.8 \). This explains that for \( t/T_{\text{open}} = 0.9 \) the wall-jet does not separate from the wall at the wall exit in the experiment shown in Fig. 3 of Erath et al.\(^2\) For \( t/T_{\text{open}} = 0.8 \) we have \( l_e a_s / R_e^2 \approx 0.9 \) and the jet separates just downstream of \( x = l_e \). This example demonstrates the power of the theory of Thwaites. When the jet follows the wall further downstream than \( x = l_e \) an additional force should in principle be applied in the dynamical model at \( x = l_e \) to take into account the force induced on the wall by the bending of the jet at the exit of the glottis. As this only occurs for very thin jets, this can probably be neglected. While very crude, the proposed model does not display an unrealistic discontinuous behavior as the BLEAP model.

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