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Citation for published version (APA):

Oomen, T. A. E., Grassens, E., & Hendriks, F. B. J. W. M. (2015). Inferential motion control : identification and robust control for positioning an unmeasurable point of interest. *IEEE Transactions on Control Systems Technology*, 23(4), 1602-1610. <https://doi.org/10.1109/TCST.2014.2371830>

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DOI:
[10.1109/TCST.2014.2371830](https://doi.org/10.1109/TCST.2014.2371830)

Document status and date:
Published: 01/01/2015

Document Version:
Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
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Inferential Motion Control: Identification and Robust Control Framework for Positioning an Unmeasurable Point of Interest

Tom Oomen, *Member, IEEE*, Erik Grassens, and Ferdinand Hendriks

Abstract—Performance requirements in precision motion systems, including those used in integrated circuit manufacturing and printing systems, are ever increasing. For instance, internal deformations cannot be neglected anymore. As a result, measured signals at sensor locations cannot be used directly to evaluate performance at the point of interest. The aim of this brief is to develop an inferential motion control framework that explicitly distinguishes between performance variables and measured variables. In the proposed framework, a dynamic model is used to infer the performance variables from the measured variables. As the inferred performance variables depend on the model quality, an identification for robust inferential control approach is pursued that tightly captures the uncertainty. Experimental results on a prototype motion system reveal that ignoring internal deformations using traditional motion control design approaches can lead to disastrous performance at the point of interest. In addition, it is shown that the proposed inferential motion control framework leads to high performance at the unmeasurable point of interest.

Index Terms—Inferential control, mechatronics, motion control, robust control, system identification.

I. INTRODUCTION

INCREASING performance in precision motion systems necessitates distinguishing between performance and measured variables. Herein, measured variables refer to the sensor signals that are accessible by the feedback controller. The performance variables are the signals that are directly related to the system's performance but are not directly accessible to the controller. These variables are not identical in inferential motion control problems, as the following examples reveal.

A first example arises in the production of integrated circuits (ICs) (Fig. 1). Here, a silicon disc is exposed in a photolithographic process. This wafer, which is a silicon disc, must be repeatedly positioned in all six-motion degrees-of-freedom (DOFs) by the wafer stage using accurate sensors. Since the accuracy demands are tightening to the nanometer scale, the assumption that the system can be represented as a rigid body, e.g., as in [1], is no longer valid. Hence, the internal deformations in between the point-of-exposure (performance variable) and the sensor position (measured variable) have to be explicitly considered, as is also argued in [2].

Manuscript received July 11, 2014; accepted November 11, 2014. Date of publication December 9, 2014; date of current version June 12, 2015. Manuscript received in final form November 13, 2014. This work was supported in part by Philips Innovation Services, Eindhoven, The Netherlands, and in part by the Netherlands Organization for Scientific Research through the Ministry of Economic Affairs under Grant 13073. Recommended by Associate Editor L. Del Re.

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Color versions of one or more of the figures in this paper are available online at <http://ieeexplore.ieee.org>.

Digital Object Identifier 10.1109/TCST.2014.2371830

Fig. 1. Examples of inferential control. Left: inferential control in printer (side view). The medium positioning drive positions the paper for the printing process that takes place at the carriage. The paper is positioned using a motor and the motor position is measured using an optical encoder. Note that the scanner is not available in real time. Right: IC production. The lens system on top exposes a wafer that is being positioned using a sensor.

A second example is encountered in printer systems [3]. Here, the medium positioning drive positions the paper with respect to the carriage that contains the printheads. The measured variable is the motor position y , whereas the performance is clearly defined at the position where the carriage is located (z). Due to internal deformations in the medium positioning drive, high accuracy in terms of y does not necessarily imply good printing performance. Due to increasing performance requirements, this effect is of vital importance in next-generation printing systems.

Conceptually, the inferential control problem can be addressed through a model-based control [4], since a model-based design procedure enables the estimation of the unmeasured performance variables from the measured variables through the use of a model [2]. Model-based control design has been successfully applied to motion systems for the traditional case with measurable performance variables. For instance, robust motion control based on $\mathcal{H}_\infty$ -optimization is presented in [1] and [5]. As these techniques rely on the availability of an accurate model, joint identification and (robust) control design methodologies have been developed in [6] and [7], see also [8], [9] for a general overview. These identification and robust control methodologies are successfully applied to motion control in [2] and [10].

Although many identification and (robust) control design results have been obtained and successfully applied to traditional motion control problems, these results are not directly applicable to the inferential motion control problems that involve an explicit distinction between measured variables and performance variables. The reason is at least twofold.

First, model-based control design techniques have mainly focused on the traditional situation where measured variables are equal to performance variables and a single DOF controller structure is adopted, see for instance [1], [11], [12]. Such controller structures cannot be used in inferential motion control problems, since the required error signals cannot be computed directly from the measured and reference signals. Second,

Fig. 2. Photograph of the experimental flexible beam setup. ①—sensor s_1 . ②—sensor s_2 . ③—sensor s_3 . ④—actuator a_1 . ⑤—actuator a_2 . ⑥—actuator a_3 . ⑦—beam. ⑧—leaf spring.

in both control-relevant iterative identification schemes [13] and uncertainty structures for robust control, most importantly dual-Youla structures, it is assumed that performance variables are directly measurable for feedback.

The main contribution of this brief is the development of a framework for identification and robust inferential control and its application to a prototype motion system, including the following steps.

- S1: Controller structures for inferential motion control are derived.
- S2: Weighting filters to appropriately define the inferential control goal in an $\mathcal{H}_\infty$ -optimal control framework are proposed.
- S3: An extension of control-relevant identification is used to obtain a model of the prototype motion system that includes the inferential response.
- S4: An extension of the dual-Youla model uncertainty structure is adopted to address the inferential control goal.
- S5: A robust controller is designed and experimentally implemented, revealing the necessity of the presented inferential control framework in next-generation motion systems.

The outline of this brief is as follows. In Section II, the prototype motion system is introduced and analyzed, followed by the problem formulation. Then, Steps S1–S5 are addressed sequentially in Sections III–VII. Finally, the conclusions are drawn in Section VIII.

Four aspects in the literature are relevant to the results presented here. First, performance limitations for inferential control are investigated in [14] and [15]. Second, global performance measured are investigated in [16]. Third, inferential process control is investigated in [17]–[19]. Fourth, this brief significantly extends earlier results in [20].

II. PROTOTYPE SETUP AND PROBLEM FORMULATION

A. Experimental Setup

The high precision prototype motion system in Fig. 2 has been developed for evaluating control strategies in next-generation motion systems that exhibit dominant flexible behavior [2], [21]. The movable steel beam has dimensions 500 mm $\times$ 20 mm $\times$ 2 mm. Four out of six-motion DOFs have been fixed by means of leaf springs. Thus, the system can move x and φ direction is shown in Fig. 3. The inputs consist of three current-driven voice-coil actuators, whereas the outputs are three contactless fiberoptic sensors with a resolution of $\sim 1 \mu\text{m}$, controlled using a PowerDAQ rapid prototyping with sampling frequency $f_s = 1$ [kHz].

Fig. 3. Schematic top view of flexible beam setup.

The following selection is made to show possible hazards in the inferential control situation and a solution. The goal is to control the translation of the center of the beam, i.e., at sensor location s_2 , hence

$$z_p = s_2$$

where s_2 is unavailable for the feedback controller. Regarding the measured variable, the center of the beam is determined by averaging the outer sensors s_1 and s_3

$$y_p = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} s_1 \\ s_3 \end{bmatrix} \quad (1)$$

which in fact corresponds to a sensor transformation based on static geometric relations, as shown in Fig. 3. Consequently, a discrepancy between the measured variable y_p and performance variable z_p may exist due to internal deformations of the beam. Only the outer actuators are used for control

$$a_1 = a_3 = u_p, \quad a_2 = 0. \quad (2)$$

Comparing (1) and (2) reveals that u_p and y_p are collocated. The resulting system is given by

$$\begin{bmatrix} z_p \\ y_p \end{bmatrix} = \begin{bmatrix} P_z \\ P_y \end{bmatrix} u_p = P u_p \quad (3)$$

where z_p denotes the performance variable, y_p is the measured variable, and u_p is the manipulated input. The resulting control problem above has closely resembles the examples in Section I and is investigated further in this brief.

B. Nonparametric Identification

To obtain insight in the prototype motion system, frequency response function estimates of $P_{z,o}$ and $P_{y,o}$ for a certain frequency grid $\omega_i \in \Omega^{\text{id}}$ are obtained using the approach in [2, App. A] and [22] (Fig. 4). The nonparametric models in Fig. 4 lead to the following observations.

- 1) At 4 and 10 Hz, two resonance phenomena are present. These correspond to the rigid-body modes of the flexible beam system. This conclusion is supported by the fact that $P_{z,o}$ and $P_{y,o}$ (3), have an identical frequency response function at these frequencies, hence the beam does not internally deform at these frequencies.
- 2) $P_{y,o}$ has a -2 slope up to 300 Hz where the antiresonances are directly followed by resonance phenomena. Typically, such behavior is obtained for mechanical systems with collocated actuator and sensor pairs [23]. Note that the delay is caused by the digital computer implementation and aliased resonance phenomena appear beyond 300 Hz [24].
- 3) The resonance phenomenon at 32 Hz is observed differently for $P_{z,o}$ than it is for $P_{y,o}$. As the slope of

Fig. 4. Frequency response function estimate for $\omega_i \in \Omega^{\text{id}}$. Black solid line: $P_{z,o}(\omega_i)$. Gray solid line: $P_{y,o}(\omega_i)$.

$P_{z,o}$ tends from -2 toward -4 , this behavior corresponds to noncollocated actuator and sensor pairs [23]. This is supported by the fact that a_1 and a_3 are used for excitation, whereas sensor s_2 is used as output here. It is hypothesized that the resonance phenomenon at 32 Hz corresponds to a deformation of the beam that can be visualised by the mode shape shown in Fig. 3, considering the following.

- The resonance phenomenon causes a 180° phase difference between $P_{z,o}$ and $P_{y,o}$, which corresponds to the deformation shown in Fig. 3.
- Based on a first principles model of a free-free beam [25], this is the first mode shape beyond the suspended rigid-body modes at 4 and 10 Hz.

C. Problem Formulation

The goal of the inferential servo problem is the minimization of $z_{\text{ref}} - z_p$, given the reference signal z_{ref} , by the design of an appropriate controller

$$u_p = C(z_{\text{ref}}, y_p). \quad (4)$$

To facilitate the exposition, the focus is on the optimal design of a feedback controller as defined next.

Definition 1: Given the true system P_o and interconnection defined by (3) and (4), determine

$$C^{\text{opt}} = \arg \min_C \mathcal{J}^9(P_o, C)$$

where $\mathcal{J}^9(P, C)$ is the control criterion, which is a function of the system P and controller C .

In $\mathcal{J}^9$, the superscript 9 will become clear in Section IV-A. Before proceeding to specifying $\mathcal{J}^9(P, C)$ in view of penalizing the inferential servo error $z_{\text{ref}} - z_p$, the general identification and robust control framework is presented.

D. General Approach: System Identification and Robust Control

The considered inferential motion control framework relies on models, since the performance variable z_p is not measurable during normal operation and must be predicted using

real-time measurements. Of course, at a certain point in the procedure some system knowledge is essential to obtain this model. The following assumption is imposed.

Assumption 1: During identification, both z_p and y_p are measured, whereas the controller only has access to y_p during normal operation of the system.

Assumption 1 is required to obtain the part of the model that corresponds to the performance variable z_p , since it cannot be measured in real time and will thus be inferred using the model. This assumption is nonrestrictive in many applications, including those in Fig. 1, and can also be relaxed by pursuing gray-box modeling.

The mathematical model of the true beam system in Fig. 2 is necessarily an approximation, since any physical system is too complex to be represented exactly by means of a model. In the inferential control case, the model quality is especially crucial, since the quality of the inferred performance variables hinges on the model quality. Model uncertainty is considered to guarantee that the model-based controller performs well when implemented on the true system. As in standard robust control designs [4], the model set $\mathcal{P}(\hat{P}, \Delta)$ is built up from a nominal model $\hat{P}$ and model uncertainty Δ , where it is assumed that

$$P_o \in \mathcal{P}(\hat{P}, \Delta). \quad (5)$$

Associated with the model set is a worst case control criterion

$$\mathcal{J}_{\text{WC}}^9(\mathcal{P}, C) = \sup_{P \in \mathcal{P}} \mathcal{J}^9(P, C). \quad (6)$$

Hence, using (5), the performance guarantee

$$\mathcal{J}^9(P_o, C) \leq \mathcal{J}_{\text{WC}}^9(\mathcal{P}, C)$$

is obtained. This leads to the following robust control design.

Definition 2: Given the model set $\mathcal{P}(\hat{P}, \Delta)$, determine

$$C^{\text{RP9}} = \arg \min_C \mathcal{J}_{\text{WC}}^9(\mathcal{P}, C). \quad (7)$$

The resulting robust controller (7) highly depends on the shape of the model set $\mathcal{P}(\hat{P}, \Delta)$. For instance, if the model set is large in some sense, an unnecessarily conservative control design results. Hence, the model set should be tight to achieve high performance. In this brief, this is done by considering the dual problem to the robust control synthesis problem in Definition 2, leading to the following robust-control-relevant model set identification problem.

Definition 3: Given a controller C^{exp} that is used during the identification experiment, determine

$$\begin{aligned} \mathcal{P}^{\text{RCR}} &= \arg \min_{\mathcal{P}} \mathcal{J}_{\text{WC}}^9(\mathcal{P}, C^{\text{exp}}) \\ \text{s.t. } P_o &\in \mathcal{P}. \end{aligned} \quad (8)$$

Several remarks are appropriate. First, the identification setting associated with (8) depends on the experimental conditions which are specified in the next sections. Second, in [6] and [7], related approaches have been presented that are restricted to the case where the set of measured variables is equal to the set of performance variables. Third, $\mathcal{P}^{\text{RCR}}$ in (8) depends on C^{exp} . For the considered application, the standard PID controller in Fig. 9 is used. The performance can be iteratively and monotonously improved by alternating between

Fig. 5. Controller structures. (a) Single DOF applied to inferential control. (b) Indirect two DOF inferential structure. (c) Inferential two DOF inferential structure. (d) General two DOFs controller structure used in this brief.

identifying a model set and synthesising a robust controller, as in [6] and [7]. Fourth, as already mentioned in Section I, the identification of a nominal model $\hat{P}$ and quantification of model uncertainty Δ will be dealt with separately. In Section VI, it will be shown that these independent steps jointly minimize the criterion in Definition 3.

III. CONTROLLER STRUCTURES FOR INFERENCE MOTION CONTROL

The inferential control problem imposes additional requirements on the controller structure that are not encountered in traditional control applications. Suitable controller structures for inferential motion control are presented in this section, constituting Step S1 in Section I.

In the inferential servo control situation, the control goal is to let performance variable z_p track a certain reference z_{ref} , given measurements y_p . Note that in the typical noninferential control situation, where $y_p = z_p$, this is equivalent to the minimization of $z_{ref} - y_p$, in which case a suitable control structure is obtained using the error signal $z_{ref} - y_p$ as input to the controller, as a result the single DOF feedback controller in Fig. 5(a) is obtained. Here, r_2 should be selected as z_{ref} , whereas r_1 is an additional signal used in the further analysis.

However, the single DOF controller structure is inadequate in the inferential servo problem, considering the following.

- 1) In general $\dim z_p \neq \dim y_p$, e.g., both s_1 and s_2 can be used as measured signals, in which case an error signal cannot be computed based on a difference between z_{ref} and y_p .
- 2) Even in case $\dim z_p = \dim y_p$, then minimization of $z_{ref} - y_p$ does not imply that $z_{ref} - z_p$ is minimized. For example, the scaling of y_p in (1) is arbitrary, hence minimization of $z_{ref} - y_p$ is not sensible in general.
- 3) Joint tracking and disturbance attenuation may not be achievable. In particular, assume that $P_y C$ contains a factor $1/s$, i.e., an integrator. Then, application of a step in either r_1 or r_2 leads to $\lim_{t \rightarrow \infty} (r - y_p) = 0$. Next, it can be immediately verified that application of a step in r_1 leads to $\lim_{t \rightarrow \infty} z_p = 0$, whereas a step in r_2 leads to $\lim_{t \rightarrow \infty} z_p = (P_z(0)/P_y(0))r_2$.

The deficiencies of single DOF controller structures in view of the inferential control problem are further elaborated on and exemplified in [26, Sec. 3.3].

Extended controller structures that can deal with the inferential servo control problem are shown in Fig. 5(b) and (c). In Fig. 5(b), also referred as the indirect approach, controller C_2^a transforms the reference z_{ref} for z_p to a reference $C_2^a z_{ref}$ for y_p . Clearly, this can resolve a scaling difference in the z_p and y_p variables. In addition, it appropriately deals

with the situation $\dim z_p \neq \dim y_p$. Similarly, the situation in Fig. 5(c) corresponds to the case where C_2^b reconstructs the performance variable z_p from the measured variable y_p .

Although the controller structures in Fig. 5(b)–(d) can deal with inferential servo control and are intuitive to comprehend, a more detailed analysis of these controller structures reveals that these cannot be used directly in conjunction with standard optimization-based control algorithms [4]. Therefore, the unifying controller structure $C = [C_1 \ C_2]$ [see also Fig. 5(d)] is adopted in the framework in this brief, since it encompasses Fig. 5(a)–(c), and it enables the use of standard controller synthesis algorithms.

IV. $\mathcal{H}_\infty$ -OPTIMAL INFERENCE MOTION CONTROL FRAMEWORK

A. Toward a Nine-Block Setup

In this section, the general setup is presented and weighting filters are selected, thereby constituting Step S2 of this brief. The design of $\mathcal{H}_\infty$ -optimal robust motion controllers involves the specification of the control goal in terms of exogenous signals and associated weighting filters. As the inferential problem is a significant extension of traditional motion control problems, including [1] and [11], several modifications are required. Two requirements are imposed. The exogenous signals should be selected such that internal stability is guaranteed and various control requirements, e.g., loop-shaping requirements, can be appropriately specified. This extends preexisting design strategies, including [1], [4], [11], and [12], to the proposed controller structure in Section III and the explicit distinction between z_p and y_p .

A selection of these inputs and outputs is shown in Fig. 6. Regarding the general weighting filter selection, the exogenous signals can be given the following interpretation: 1) r_1 represents disturbances at the system input that have to be attenuated; 2) r_2 represents measurement noise that should not be tracked; and 3) r_3 represents the reference signal. In addition, e_z is the tracking error, y_p is the system output, and u_p is the system input. Here, y_p and u_p should also remain bounded in view of internal stability of the feedback loop. The closed-loop transfer function matrices are given by

$$\begin{aligned} \bar{z} &= \bar{M}(P, C) \bar{w} \\ \bar{M}(P, C) &= \begin{bmatrix} P_z \\ P_y \\ I \end{bmatrix} (I + C_2 P_y)^{-1} [C_1 \ C_2 \ I] - \begin{bmatrix} T_r & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ \bar{w} &= \begin{bmatrix} r_3 \\ r_2 \\ r_1 \end{bmatrix} \quad \bar{z} = \begin{bmatrix} e_z \\ y_p \\ u_p \end{bmatrix}. \end{aligned} \quad (9)$$

Fig. 6. Unweighted exogenous inputs and outputs.

In (9), $T_r \in \mathcal{RH}_\infty$ is a reference model, which ensures a sensible problem formulation. Due to the specific structure in $\overline{M}(P, C)$, the problem is referred to as a nine-block problem. The reasons for the specific structure are threefold.

First, the condition $\overline{M} \in \mathcal{RH}_\infty$ is equivalent to internal stability of the closed-loop system, which is a basic requirement for a satisfactory control system. Second, the specific formulation directly fits in the standard plant formulation, enabling the use of standard optimization algorithms. Let $\tilde{G}(P)$ be defined by

$$\begin{bmatrix} \tilde{z} \\ y \end{bmatrix} = \tilde{G}(P) \begin{bmatrix} \tilde{w} \\ u \end{bmatrix}$$

hence

$$\overline{M}(P, C) = \mathcal{F}_l(\tilde{G}(P), C).$$

Third, the setup in Fig. 6 enables a general weighting filter selection. In particular, the setup in Fig. 6 generally has to be extended with weighting filters to pose a sensible control problem. Let

$$W = \begin{bmatrix} W_e & 0 & 0 \\ 0 & W_y & 0 \\ 0 & 0 & W_u \end{bmatrix}, \quad V = \begin{bmatrix} V_3 & 0 & 0 \\ 0 & V_2 & 0 \\ 0 & 0 & V_1 \end{bmatrix}$$

be bistable weighting filters and define

$$z = W\tilde{z}, \quad \tilde{w} = Vw \quad (10)$$

$$M(P, C) = W\overline{M}(P, C)V \quad (11)$$

$$\mathcal{J}^0(P, C) = \|M(P, C)\|_\infty. \quad (12)$$

The control criterion in (12) is considered throughout. The selection of inferential weighting filters is presented in Section IV-B.

B. Weighting Filter Selection

In this section, weighting filter selection is considered, which is required prior to the control-relevant parametric identification procedure in Section V. The presented framework and associated control criterion enable a large freedom in the selection of the control goal. In this brief, the focus is on loop-shaping feedback control and the weighting filters are designed such that these achieve similar control properties as in the design of single DOF feedback controllers [1], [11]. Alternative signal-based approaches are pursued in [4].

The following steps lead to the design of the weighting filters. Throughout, the matrix $M(P, C)$ is partitioned according to the signals in (10)

$$M(P, C) = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}. \quad (13)$$

Fig. 7. Black solid line: shaped system $W_2 P_{y,o}(\omega_i) W_1$. Gray solid line: nonparametric frequency response estimate $P_{y,o}(\omega_i)$.

- 1) The weighting filters W_y , W_u , V_1 , V_2 are designed using standard loop-shaping techniques as in [1], [11], and [12]. In particular, W_1 and W_2 are used to specify the desired loop-shape $W_2 P W_1$ and are absorbed into the system to enable controller roll-off and integral action. Note that

$$W_y = W_2, \quad W_u = W_1^{-1}, \quad V_2 = W_2^{-1}, \quad V_1 = W_1.$$

As loop-shaping requires knowledge of the system, the nonparametric estimate in Fig. 4 is employed for designing the weighting functions (Fig. 7). This is in sharp contrast to preexisting approaches, including [12].

- 2) W_e allows for additional scaling. Due to the fact that z_p and y_p are defined in the same unit and have approximately the same scaling [see (1) and Fig. 4], no additional scaling of the M_{12} and M_{13} blocks is required.
- 3) A suitable reference model T_r is selected. To enforce similar feedback properties for the two DOF controller as for the four-block problem, which is used in the traditional situation, including [11] and [12], and the single DOF controller, a reference model T_r is chosen to have a low-pass characteristic with a bandwidth that corresponds to the desired closed-loop bandwidth in Step 1. In addition, a sufficiently high roll-off rate is selected to avoid a nonproper optimal C_1 .
- 4) Next, weighting filters $W_e = \beta_1$, $V_3 = \beta_2$ are defined, where $\beta_1, \beta_2 \in \mathbb{R}$. These constants are set to $\beta_1 = \beta_2 = 1.25$ to emphasize the importance of the block M_{11} in (13).

By the specific selection of weighting filters, all blocks are (indirectly) scaled. The lower-right block in (13) represents the standard feedback controller part, whereas the upper-left block directly affects the inferential servo part, which can be relatively weighted using β_1 and β_2 as in [27].

V. PARAMETRIC SYSTEM IDENTIFICATION

The inferential controller will (implicitly) incorporate a model to predict the performance variables from the measured variables using a model. Indeed, optimal controllers are typically observer-based [28], where the identified model is directly used. In this section, a new approach to identify a suitable model for (robust) inferential control is presented and applied to the prototype motion system, constituting Step S3.

A. Control-Relevant Identification

As is motivated in Section II-C, a control-relevant nominal model is estimated by virtue of Definition 3. However,

control-relevant identification techniques, including the procedures in [7] and [13] are restricted to single DOF controller structures and to the situation where the set of performance variables equals the set of measured variables. Therefore, control-relevant identification techniques are extended appropriately, and the inferential-control-relevant identification criterion

$$\|M(P_o, C^{\text{exp}}) - M(\hat{P}, C^{\text{exp}})\|_{\infty} \quad (14)$$

is considered, which is to be minimized over $\hat{P}$. Here, $M(P_o, C^{\text{exp}})$ is the closed-loop transfer function matrix defined in (11), where M in fact has already been identified to obtain the results in Fig. 4. In particular, since the system operates in closed-loop with C^{exp} implemented, $M(P_o, C^{\text{exp}})$ is identified for a certain frequency grid $\omega_i \in \Omega^{\text{id}}$ using the approach in [2, App. A] and [22]. By appropriate manipulation of $M(P_o, C^{\text{exp}})$, frequency response function estimates of $P_{z,o}$ and $P_{y,o}$ have been obtained for $\omega_i \in \Omega^{\text{id}}$ (Fig. 4).

Note that details on the derivation of (14) are provided in [29]. In Section VI, it will be shown that the criterion in (14) indeed is useful to determine a robust-control-relevant model set $\mathcal{P}^{\text{RCR}}$ in view of Definition 3.

B. Control-Relevant Coprime Factor Identification

To appropriately extend the identified nominal model $\hat{P}$ of the beam system with a perturbation model that represents model uncertainty, the internal structure of the model $\hat{P}$ should be carefully specified. To anticipate on the model uncertainty structure in Section VI, the model is represented as a factorization

$$\hat{P} = \begin{bmatrix} \hat{N}_z \\ \hat{N} \end{bmatrix} \hat{D}^{-1} \quad (15)$$

where $\hat{N}_z \in \mathcal{RH}_{\infty}$ and $\hat{N}, \hat{D}$ constitute a coprime factorization of $\hat{P}_y$ over $\mathcal{RH}_{\infty}$. Substitution of (15) into (14) reveals that the control-relevant identification criterion in (14) is invariant under a change of internal structure of the model $\hat{P}$. Following the developments in [29, Prop. 8], a particular coprime factorization results in a connection to the control-relevant identification criterion in (14). In particular, this result states that (14) is equivalent to

$$\left\| W \left(\begin{bmatrix} N_{z,o} \\ N_o \\ D_o \end{bmatrix} - \begin{bmatrix} \hat{N}_z \\ \hat{N} \\ \hat{D} \end{bmatrix} \right) \right\|_{\infty}$$

where

$$\begin{bmatrix} N_z \\ N \\ D \end{bmatrix} = \begin{bmatrix} P_z \\ P_y \\ I \end{bmatrix} (\tilde{D}_e + \tilde{N}_{e,2} V_2^{-1} P_y)^{-1} \quad (16)$$

and $\{\tilde{N}_e, \tilde{D}_e\}$ is a left coprime factorization with coinner numerator of $[C_1 V_3 \ C_2 V_2 \ V_1]$, $\tilde{N}_e = [\tilde{N}_{e,3} \ \tilde{N}_{e,2} \ \tilde{N}_{e,1}]$.

C. Results

The criterion (16) can be minimized directly using the approach in [7]. The identified nonparametric factors $N_{z,o}, N_o,$

Fig. 8. Bode magnitude: frequency response functions P_z (top) and P_y (bottom). Dotted line: nonparametric estimate. Black solid line: nominal model $\hat{P}$. Shaded line: model set $\mathcal{P}$.

and D_o are computed from the identified frequency response function $\tilde{M}(P_o, C^{\text{exp}})$ for $\omega_i \in \Omega^{\text{id}}$. Given these identified factors, the criterion in (16) is minimized over $\hat{N}_z, \hat{N}$, and $\hat{D}$.

The resulting nominal model $\hat{P}$ in (15) is shown in Fig. 8. Control relevance can for instance be seen from the fact that the resonances within the controller bandwidth are accurately modeled, as well as the 180-Hz resonance that is known to endanger stability for typical control designs.

VI. MODEL UNCERTAINTY QUANTIFICATION

Inferential motion control depends on predictions of the performance variables through a model. This amplifies the need for addressing model uncertainty in a robust control design compared with traditional motion control [1], [6], [7].

In traditional motion control, the dual-Youla uncertainty structure has proved useful to minimize robust-control-relevant identification criteria of the form (8), where $z_p = y_p$. In this brief, the recent extension of dual-Youla toward inferential control, as first presented in [29], is exploited.

In particular, the model uncertainty structure

$$\left\{ P | P = \begin{bmatrix} \hat{N}_z + W_e^{-1} \Delta_z \\ \hat{N} + D_{c,22} \Delta_u \end{bmatrix} (\hat{D} - N_{c,2} \Delta_u)^{-1}, \left\| \begin{bmatrix} \Delta_z \\ \Delta_u \end{bmatrix} \right\|_{\infty} \leq \gamma \right\} \quad (17)$$

is adopted, where $\{N_{c,2}, D_{c,22}\}$ is any right coprime factorization of C^{exp} Section V-A. The key point is that (17) guarantees that $\mathcal{J}_{\text{WC}}^9(\mathcal{P}, C^{\text{exp}})$ in (8) remains bounded.

Next, using the specific factorization of $\hat{P}$ that is identified in Section V in conjunction with a $\{W_u, W_y\}$ -normalized right coprime factorization of C^{exp} , which can directly be computed using the state-space formulas in [7], the important result

$$\mathcal{J}_{\text{WC}}^9(\mathcal{P}, C^{\text{exp}}) \leq \mathcal{J}(\hat{P}, C^{\text{exp}}) + \sup_{\Delta \in \Delta} \|\Delta\|_{\infty} \quad (18)$$

is obtained [29]. Clearly, the norm of Δ directly affects the control criterion, leading to a transparent connection between nominal model identification, model uncertainty quantification, and robust control. The result (18)

TABLE I
ROBUST-CONTROL-RELEVANT IDENTIFICATION AND ROBUST
CONTROLLER SYNTHESIS RESULTS

	C^{exp}	C^{NP4}	C^{NP9}	C^{RP4}	C^{RP9}
Minimized criterion	—	$\mathcal{J}^4(\hat{P}_y, C)$	$\mathcal{J}^9(\hat{P}, C)$	$\mathcal{J}_{\text{WC}}^4(\mathcal{P}^{\text{RCR}}, C)$	$\mathcal{J}_{\text{WC}}^9(\mathcal{P}^{\text{RCR}}, C)$
$\mathcal{J}^4(\hat{P}_y, C)$	123.01	3.12	—	4.09	—
$\mathcal{J}^9(\hat{P}, C)$	196.82	32.44	7.80	41.18	9.34
$\mathcal{J}_{\text{WC}}^4(\mathcal{P}^{\text{RCR}}, C)$	125.50	5.37	—	4.10	—
$\mathcal{J}_{\text{WC}}^9(\mathcal{P}^{\text{RCR}}, C)$	201.38	39.30	70.54	49.45	10.88

Fig. 9. Bode diagram of controller. Black solid line: optimal robust nine-block inferential controller C^{RP9} . Gray solid line: optimal robust four-block controller C^{RP4} . Black dashed line: initial controller C^{exp} .

extends the result in [7] to the inferential situation, and constitutes Step S4.

To evaluate the model quality of $\hat{P}$, the validation-based uncertainty modeling procedure as in [2] is applied to a large number of validation experiments. This leads to $\sup_{\Delta \in \Delta} \|\Delta\|_{\infty} \leq 5.6$.

The resulting model set (17) is formulated in an abstract coprime factor domain. To obtain insight in the open-loop model set $\mathcal{P}^{\text{RCR}}$, it is visualized in Fig. 8. It appears that the model indeed is most accurate around the desired crossover region at ~ 50 Hz and at the modeled resonance at ~ 180 Hz (Section V-C).

The performance of the model (set) can also be quantified in terms of the control criterion (6) (Table I). Clearly, (18) is satisfied, since $\mathcal{J}_{\text{WC}}^9(\mathcal{P}^{\text{RCR}}, C) = 201.38 \leq 196.82 + 5.6$.

VII. ROBUST CONTROL DESIGN

Next, controller synthesis is performed for different situations. First, both the inferential situation, i.e., involving $\mathcal{J}^9$, is considered, as well as the traditional noninferential case

$$\mathcal{J}^4(P_y, C) = \left\| \begin{bmatrix} W_y & 0 \\ 0 & W_u \end{bmatrix} \begin{bmatrix} P_y \\ I \end{bmatrix} (I + CP_y)^{-1} [C \ I] \begin{bmatrix} V_2 & 0 \\ 0 & V_1 \end{bmatrix} \right\|_{\infty}$$

where $\mathcal{J}_{\text{WC}}^4$ is defined analogous to (6). Second, both nominal controllers for $\hat{P}$ and robust controllers for $\mathcal{P}$ are computed using $\mathcal{H}_{\infty}$ -optimization and skewed- μ synthesis [2].

Fig. 10. Step responses using nominal model. Black solid line: optimal robust nine-block inferential controller C^{RP9} . Gray solid line: optimal robust four-block controller C^{RP4} . Black dashed line: initial controller C^{exp} .

Fig. 11. Experimental step responses. Black solid line: optimal robust nine-block inferential controller C^{RP9} . Gray solid line: optimal robust four-block controller C^{RP4} . Black dashed line: initial controller C^{exp} .

The resulting controllers are defined in Table I, including their computed performance. Bode diagrams of the controllers are shown in Fig. 9. Inspection reveals that the nominal controller achieves $\mathcal{J}^9(\hat{P}, C^{\text{NP9}}) = 7.80$, yet $\mathcal{J}_{\text{WC}}^9(\mathcal{P}^{\text{RCR}}, C^{\text{NP9}}) = 70.54$ is drastically higher. On the other hand, the robust controller achieves $\mathcal{J}^9(\hat{P}, C^{\text{RP9}}) = 9.34$, $\mathcal{J}_{\text{WC}}^9(\mathcal{P}^{\text{RCR}}, C^{\text{RP9}}) = 10.88$. As $\mathcal{J}^9(\hat{P}, C^{\text{RP9}})$ is close to $\mathcal{J}^9(\hat{P}, C^{\text{NP9}}) = 7.80$, $\mathcal{P}^{\text{RCR}}$ indeed is robust control relevant.

Next, the controllers are implemented on both the model and the experimental setup. Only the results of C^{exp} , C^{RP4} , and C^{RP9} are shown. The implementation on the nominal model is shown in Fig. 10. Considering y_p , it is observed that C^{exp} has the slowest response. Clearly, the nominal four-block controller C^{RP4} significantly improves the response in terms of the measured variable y_p . However, when considering the performance variable z_p in Fig. 10, it is observed that C^{RP4} deteriorates the true system performance compared with the initial C^{exp} . The key point is that the response z_p is not measured during normal operation. Hence, the control designer typically does not realize from the measured data y_p in the control loop that C^{RP4} in fact is performing poorly. In contrast, C^{RP9} leads to a very satisfactory response in terms of z_p .

Furthermore, Fig. 10 reveals that C^{RP9} has a small oscillation in the y_p response at 0.07 s (right-hand side of Fig. 10). In the left of Fig. 10, the reason for this oscillation becomes clear: the trajectories of z_p and y_p are inherently related due to the use of only a single actuator u_p , see (2), and the

Fig. 12. Experimental step responses using optimal robust four-block controller C^{RP4} . The performance variable z_p (black solid line) is oscillating in counterphase with respect to the measured variable y_p (gray solid line).

small oscillation is incorporated to deliver a good response in z_p . Similar control inputs have been proposed in overhead cranes [30]. Concluding, the optimal inferential controller C^{RP9} is able to enhance the system performance, i.e., the response in terms of z_p .

Finally, it is noted that these results on the nominal models are directly confirmed by inspecting Bode plots of

$$\begin{bmatrix} z_p \\ y_p \end{bmatrix} = \begin{bmatrix} P_z \\ P_y \end{bmatrix} (I + C_2 P_y)^{-1} C_1 r_3$$

details of which are omitted due to space limitations.

Next, the controllers have been implemented on the experimental beam setup and the responses have been measured, see Fig. 11 for responses in the z_p and y_p variables. The simulation results reliably predict the experimental results, which confirm that the model is control relevant. Thus, C^{RP9} enhances the performance of the experimental beam system. In contrast, C^{RP4} deteriorates the system performance.

To further analyze the poor inferential performance corresponding to the controller C^{RP4} , observe from Fig. 11 that the highly oscillatory behavior in the z_p variable contains a dominant frequency component that coincides with the resonance phenomenon in Fig. 4 at ~ 32 Hz. When comparing the response of the z_p and y_p variables (Fig. 12), it is observed that the two variables are approximately in counterphase. This confirms that the deformation shown in Fig. 3 causes the oscillatory response. Again, it is emphasized that the measured signal y_p does not provide any indication that the performance variable z_p is highly oscillatory.

VIII. CONCLUSION

The results presented in this brief enable the next step in precision motion control by explicitly taking the distinction between performance variables and measured variables into account. This step is necessary as internal deformations cannot be neglected in view of the required accuracy in future motion systems. The presented framework connects system identification and robust control design, as a model-based approach is essential for the inferential motion control problem.

The experimental results confirm the necessity of the proposed framework. A common robust $\mathcal{H}_\infty$ -loop-shaping controller C^{RP4} is shown to significantly enhance the response of measured variables. However, this deteriorates the actual performance variables. This is a potentially dangerous

situation in high-precision systems, since the poor performance cannot be directly detected from the measured signals.

The proposed inferential motion framework effectively deals with the distinction between measured and performance variables. The key step is to use model knowledge to infer the performance variables from the measured variables.

This brief focuses on inferential feedback control, which is achieved through a reference model. This can be further enforced by a two-stage approach [31], structured controller optimisation, or designing a separate observer in the control structure of Fig. 5(c) as is done in [32]. In addition, this brief focusses on $\mathcal{H}_\infty$ -based control design, since model uncertainty is of key importance in motion control systems and the uncertainty in the model to predict the z_p variable has been explicitly addressed.

The results can be directly generalized. For instance, direct extensions along the lines of [33] and [34] are immediate, whereas implications toward Iterative Learning Control are recently investigated in [35]. Furthermore, the results in this brief might shed new light on ongoing challenges in motion systems (see [2], [36]–[38] for an overview). Current research focuses on the use of overactuation [21] to enhance inferential performance, addressing position dependent effects induced by position varying measurement of the mode shapes, and recently progress has been made on inferential control for thermal aspects in lithographic applications.

ACKNOWLEDGMENT

The authors would like to thank J. Bolder, O. Bosgra, R. van Herpen, M. Steinbuch, and M. van de Wal for their contribution.

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