A stabilization technique for coupled convection-diffusion-reaction equations: multidimensional extension
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Published: 01/01/2016

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Download date: 02. Jan. 2019
Motivation

A variety of phenomena of scientific interest can be described by the Convection-Diffusion-Reaction equation:

$$\rho \frac{\partial u}{\partial t} - \frac{\partial}{\partial x} \left( \alpha \frac{\partial u}{\partial x} - \beta u \right) + \gamma u = f.$$ 

Moreover, interactions among different transported quantities can be modeled by using coupled CDR equations. Some of the branches of science where such systems arise are:

- **Bio-mechanics**: bone poro-elasticity, vaccine delivery.
- **Computer science**: Petri nets, optimization.
- **Ecology, epidemiology, neuroscience, physiology**: multiphase-flows, turbulence.
- **Economics, finance, stock market behavior**: Petri nets, optimization.
- **Fluid dynamics**: combustion, electro-analytical chemistry.
- **Mechanics of materials**: Continuum dislocation transport.

Exact solutions are available in extremely few cases making numerical approximation the most affordable strategy to deal with them.

Numerical example

Consider the following system of equations of interest in Petri nets systems simulation [2]:

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} u_x + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} u_y = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} .$$

After discretizing this system using the classical Bubnov-Galerkin FEM, numerical approximations plagued with spurious oscillations are obtained as can be observed in Figure 1.

Stabilization via perturbation

Several stabilization techniques have been developed to handle such transport equations by numerical means [1]. Recently, a new perturbation-based stabilization technique was proposed with dislocation transport as the main focus [3].

Yet, not extensive work has been done for systems of coupled equations. The reason of such immaturity is the lack of a maximum principle when going from a single transport equation towards systems of coupled equations [5].

The main goal of this communication is to present a stabilization technique for a system of multiple dimensional coupled CDR equations based on coefficient perturbations. This methodology extents the approach for a single equation [3] which in turn has been extended to a general 1D system of coupled equations to multiple dimensions [4]:

$$\rho_{pq} u_{pq,t} - \left( \delta_{ij} u_{ij,t} - \beta_{ij} u_{ij,t} \right) + \gamma_{pq} u_{pq} = f_{pq}.$$ 

These perturbations are optimally chosen in such a way that certain compatibility conditions analogous to a maximum principle are satisfied in each direction. Once the computed perturbations are injected in the classical Bubnov-Galerkin FEM, they render smooth and stable numerical approximations.

Numerical assessment

Figure 2 shows the results obtained when the stabilization technique has been applied to the previously shown system.

This time the numerical approximations are non-negative, smooth, and free of wiggles.

Conclusions

These results allow envisioning the use of the developed technique to simulate multi-dimensional dislocation transport in crystalline materials with an affordable computational effort. Also a thorough analysis of the stabilization technique is the further work to be carried out.

References