Modal analysis of a thin cylindrical shell with top mass

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Modal analysis of a thin cylindrical shell with top mass

ing. L.J.A. den Boer

DCT 2007.086

Traineeship report

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Department Mechanical Engineering
Dynamics and Control Technology Group

Eindhoven, September, 2007
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Chapter 1

Introduction

Due to their high stiffness-to-mass ratio, thin cylindrical shells are used in a wide variety of applications. In practice, thin-walled structures are often subjected to a combination of static loading and dynamic loading. For example, aerospace structures are often dimensioned based on the loading conditions during launch. During launch, the propulsion forces will accelerate the structure resulting in inertia forces consisting of a summation of static loads and vibrational type of loads. The resistance of structures liable to buckling, to withstand such dynamic loading is often addressed as the dynamic stability of these structures. In the past, many studies already have been performed concerning the dynamic stability of thin-walled structures. Design strategies, taking rigourously the dynamics of such structures under dynamic loading into account, can be improved. The goal of the Ph.D. project of N.J. Mallon is a first step in deriving such design strategies. A part of this research concentrates on the dynamic stability problem of a base excited thin cylindrical shell with top mass [1].

This traineeship is part of the research project of N.J. Mallon. During this traineeship the linear vibration modes of a thin cylindrical shell with top mass will be investigated numerically (using FE analysis) and experimentally. In the future, the results of the present study may be used to verify the semi-analytical models developed by N.J. Mallon. Different types of vibrational mode shapes will be found, which are indicated in this report as main modes (figure 1.1(a)) and shell modes [5] (figure 1.1(b),(c)). The main modes are due to the elasticity of the cylindrical shell in combination with the inertia of the top mass. The shell modes are due to the elasticity of the cylindrical shell in combination with the inertia properties of the cylindrical shell itself. The shell modes have sinusoidal shapes in axial and circumferential direction. In the axial direction, the number of half sine waves are counted with the letter m. In the circumferential direction, the number of full sine waves are counted with the letter n. For example, in figure 1.1(b), a cylinder with m=1 and n=12 is shown and in figure 1.1(c), a cylinder with m=2 and n=12 is shown.

The outline for this report will be as follows. In chapter 2, the experimental setup will be discussed. In chapter 3, a numerical modal analysis (NMA) will be performed to determine which type of modes in which frequency band occur. In chapter 4, an experimental modal analysis (EMA) will be performed on the experimental setup and the results of the NMA and EMA will be qualitatively compared. In chapter 5, conclusions and recommendations will be given.
1. Introduction

Figure 1.1: Different modes shapes (top mass not shown)
Chapter 2
Experimental setup

This chapter describes the experimental test setup which is used for the experimental modal analysis of the thin cylindrical shell. The setup consists of the bottom part (figure 2.1 (1)), the cylindrical shell (figure 2.1 (2)) and the top mass (figure 2.1 (3)). The cylindrical shell is clamped between the bottom part and the top mass.

Figure 2.1: Experimental setup
2. experimental setup

2.1 Bottom part

The bottom part is placed on a table which can be rotated with a fixed angle of 9 degrees. The bottom part is placed on a rubber mat, to damp external vibrations.

2.2 Cylindrical shell

The cylindrical shell is cut out of a soft drink bottle made of PET. The shell thickness varies between 0.000225 and 0.000236 m, but is considered to have a constant value of 0.00023 m. The elastic modulus in circumferential and axial direction are measured with a tensile test. The cylindrical shell shows unisotropic material behaviour (i.e. the elastic axial modulus and the circumferential modulus differ significantly). The density is determined by weighting a number of pieces of the PET material. The Poisson ratio is estimated based on values from literature. The properties of the cylindrical shell are shown in table 2.1.

<table>
<thead>
<tr>
<th>Table 2.1: Cylindrical shell properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>diameter (neutral plane)</td>
</tr>
<tr>
<td>height</td>
</tr>
<tr>
<td>shell thickness</td>
</tr>
<tr>
<td>material</td>
</tr>
<tr>
<td>Poisson ratio</td>
</tr>
<tr>
<td>axial elastic modulus</td>
</tr>
<tr>
<td>circumferential elastic modulus</td>
</tr>
<tr>
<td>density</td>
</tr>
</tbody>
</table>

2.3 Top mass

The top mass consists of steel and aluminum parts and has a total mass of 4.0 kg. To investigate its inertia properties, the top mass is modeled in the 3D model program UNIGRAPHICS (figure 2.2). The four white parts are made of aluminum and the grey part is made of steel.

The rotational mass moments of inertia of the top mass are calculated by the program UNIGRAPHICS and are shown in table 2.2. The center of mass is measured from the bottom of the top mass. The mass moments of inertia are calculated with respect to the x-axis and y-axis trough the center of mass. Obviously, there also exists a mass moment of inertia with respect to the z-axis (I_{zz}), but this is not taken into account because torsional modes are not of interest. Only the mass moments of inertia (I_{xx} and I_{yy}) with respect to the x-axis and y-axis are considered. Due to the axi-symmetric form of the top mass, I_{xx} and I_{yy} are the same.
2. EXPERIMENTAL SETUP

Figure 2.2: Top mass modeled in Unigraphics

Table 2.2: Top mass properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average density of aluminum and steel parts</td>
<td>5404 kg/m³</td>
</tr>
<tr>
<td>Mass</td>
<td>4.0 kg</td>
</tr>
<tr>
<td>Center of mass</td>
<td>0.061 m</td>
</tr>
<tr>
<td>$I_{xx}$ and $I_{yy}$</td>
<td>$5.0 \cdot 10^{-3}$ kg m²</td>
</tr>
</tbody>
</table>
Chapter 3  
Numerical modal analysis  

This chapter describes the numerical modal analysis (NMA) of the test setup. The NMA is used to determine the different mode shapes and their corresponding eigenfrequencies. For all NMA analyses, the finite element method (FEM) software package MARC MENTAT [2] is used.

Figure 3.1: FEM model in Marc; 1: top mass; 2: cylindrical shell
3.1 Modeling

The cylindrical shell structure (figure 2.1) is modeled in MARC and the finite element model is shown in figure 3.1. The model consists of the bottom part, the cylindrical shell and the top mass. The geometry of the top mass is simplified but has the same mass and mass moments of inertia as the actual structure.

3.1.1 Bottom part

The bottom part is considered to be infinitely stiff. This is modeled as a fixed boundary condition (no translation and no rotation in all directions) for the lowest circle of nodes, as shown in figure 3.1 by arrows.

3.1.2 Cylindrical shell

The cylindrical shell is modeled in MARC using shell elements type 139 (four node thin shell element, based on Kirchhoff theory, six degrees of freedom per node [2]). The properties used for the model are shown in table 2.1. The cylindrical shell is shown in figure 3.1. For the model of the main modes a mesh of 7500 elements used and for the main modes a mesh of 35000 elements.

3.1.3 Top mass

The desired top mass properties are shown in table 2.2. The center of mass is the height (z) measured from the bottom of the top mass (figure 3.1). For the height of the solid cylinder, two times the distance of the center of mass (table 2.2) is chosen. The total top mass in MARC has dimensions as shown in table 3.1.

<table>
<thead>
<tr>
<th>Table 3.1: Marc top mass dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>height</td>
</tr>
<tr>
<td>radius</td>
</tr>
</tbody>
</table>

The top mass is modeled as 3D elements as shown in figure 3.1. The mass is calculated with the formula:

$$mass = \frac{I_{xx}}{\left(1/16 \cdot \text{diameter}^2 + 1/12 \cdot \text{height}^2\right)}$$  \hspace{1cm} (3.1)

and is 2.87 kg. This mass is used to calculate the density with the formula:

$$density = \frac{mass}{\left(\text{height} \cdot \text{diameter}^2 \cdot \pi/4\right)}$$  \hspace{1cm} (3.2)
and is 3852 kg/m³.

The mass has a value of 2.87 kg, a difference of 1.13 kg compared to the desired 4.0 kg. This shortage of mass is compensated by adding an extra point mass of 1.13 kg to the center of mass of the top mass. The top mass is considered to be infinitely stiff so the elastic modulus is chosen very high compared to the elastic modulus of the shell, namely $2 \cdot 10^{13}$ N/m².

<table>
<thead>
<tr>
<th>Table 3.2: Extra Marc top mass properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass density</td>
</tr>
<tr>
<td>mass of the top mass</td>
</tr>
<tr>
<td>extra point mass</td>
</tr>
<tr>
<td>elastic modulus</td>
</tr>
</tbody>
</table>

The top mass is discretized using element type 7 (three dimensional arbitrarily distorted brick, eight nodes, six degrees of freedom per node [2]). The nodes along the bottom outer edge of the solid cylinder coincide with the nodes along the top edge of the cylindrical shell. In the rest of the cylinder the elements are automatically meshed by MARC. In the axial direction only one element height chosen is used.

3.2 NMA of main modes

The NMA analysis is done for an isotropic shell. Two values for the elastic modulus ($E$) are used: $3.5 \cdot 10^9$ N/m² and $5.5 \cdot 10^9$ N/m², because that are the boundary values of $E$ (see section 2.2). For the cylindrical shell a mesh size of 7500 elements is chosen. The main mode shapes are shown in figure 3.2 and the corresponding eigenfrequencies in table 3.3.

Main mode shapes 1 and 3 (figure 3.2 (a) and (c)) occur in pairs, because mode shapes of axisymmetric geometries occur in pairs if the mode shapes are not axisymmetric.

The influence of gravity on the setup is researched. This is done by adding a force, which represents the weight ($g = 9.81$ m/s²) of the top mass, to the point mass in negative z-direction. The mass is 4.0 kg (table 2.2) and this gives a constant prescribed gravity force of 39.2 N. Before the modal analysis, the static prestressed configuration is determined. The influence of the gravity on the eigenfrequency of the main modes is less than 0.1 %, so it has no significant influence.

<table>
<thead>
<tr>
<th>Table 3.3: Eigenfrequencies of the main modes without gravity</th>
</tr>
</thead>
<tbody>
<tr>
<td>mode</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>
3. Numerical Modal Analysis

(a) Mode 1  
(b) Mode 2  
(c) Mode 3

Figure 3.2: Main mode shapes

3.3 NMA of shell modes

The modes with a eigenfrequency higher than the main modes are all shell modes. The mass moment of inertia of the top mass has hardly any influence on the shell modes, so the top mass can be modeled as point mass. The nodes along the upper circle of the shell (figure 3.3) can only translate in the z-direction (one degree of freedom). Similar as in the previous section, the NMA analysis is done for an isotropic shell with the elastic moduli \(E\) of \(3.5 \cdot 10^9\) N/m\(^2\) and \(5.5 \cdot 10^9\) N/m\(^2\). For the cylindrical shell now a mesh size of 35000 elements is chosen. In figure 3.3, the NMA model in MARC is shown.

In figure 3.4, the shell mode with \(n=6\) and \(m=1\) is shown. Eigenfrequencies of the shell modes (with different \(n\) and \(m=1\)) are shown in table 3.4.

Table 3.4: Results of the shell modes

<table>
<thead>
<tr>
<th>(n)</th>
<th>(f (E = 3.5 \cdot 10^9) N/m(^2))</th>
<th>(f (E = 5.5 \cdot 10^9) N/m(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1652 Hz</td>
<td>2071 Hz</td>
</tr>
<tr>
<td>4</td>
<td>1208 Hz</td>
<td>1514 Hz</td>
</tr>
<tr>
<td>5</td>
<td>933 Hz</td>
<td>1170 Hz</td>
</tr>
<tr>
<td>6</td>
<td>779 Hz</td>
<td>976 Hz</td>
</tr>
<tr>
<td>7</td>
<td>723 Hz</td>
<td>906 Hz</td>
</tr>
<tr>
<td>8</td>
<td>752 Hz</td>
<td>942 Hz</td>
</tr>
<tr>
<td>9</td>
<td>846 Hz</td>
<td>1061 Hz</td>
</tr>
<tr>
<td>10</td>
<td>987 Hz</td>
<td>1237 Hz</td>
</tr>
<tr>
<td>11</td>
<td>1162 Hz</td>
<td>1456 Hz</td>
</tr>
</tbody>
</table>
Similar as for the main modes, the axi asymmetrical modes occur in pairs.

Again similar as for the main modes, the influence of gravity on the eigenfrequencies of the shell modes is researched and appears to be less than 0.1 %, so has no significant influence.

### 3.4 Validation of mesh size

For a finite element approach yields that the more elements are used in the model, the more accurate the results are. But the disadvantage of many element is that the model becomes computationally expensive. That is why a good choice of the number of elements is important. An
elastic modulus of $5.5 \cdot 10^9$ N/m$^2$ is used. Three mesh sizes are considered to investigate convergence of results. The ratio between the number of elements in circumferential and height direction is chosen in such a way that the elements get a square form.

### Table 3.5: Eigenfrequencies for different mesh sizes

<table>
<thead>
<tr>
<th></th>
<th>number of elements:</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>150 $\cdot$ 50 = 7500</td>
<td>225 $\cdot$ 75 = 16875</td>
<td>300 $\cdot$ 100 = 30000</td>
<td></td>
</tr>
<tr>
<td>main mode</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>37 Hz</td>
<td>37 Hz</td>
<td>37 Hz</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>162 Hz</td>
<td>162 Hz</td>
<td>161 Hz</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>187 Hz</td>
<td>187 Hz</td>
<td>187 Hz</td>
<td></td>
</tr>
<tr>
<td>shell mode</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n=7, m=1</td>
<td>907 Hz</td>
<td>903 Hz</td>
<td>902 Hz</td>
<td></td>
</tr>
<tr>
<td>n=8, m=1</td>
<td>946 Hz</td>
<td>941 Hz</td>
<td>940 Hz</td>
<td></td>
</tr>
</tbody>
</table>

In table 3.5 using three different mesh sizes, three eigenfrequencies are given for each mode. Every mode is tested with a different number of elements. For the main modes the difference between the mesh of 7500 and 30000 elements is less than 1 % which is acceptable. Therefore, for these modes a mesh size of 7500 elements was used. Also for the shell modes the difference is less than 1 % in this case. A mesh size of 7500 elements in principle is acceptable but still a mesh of 35000 elements is chosen in this case.
Chapter 4
Experimental modal analysis

The aim of the experimental modal analysis is to experimentally find the modes and corresponding eigenfrequencies which characterize the dynamic behavior of the structure. These quantities subsequently can be used to verify the value of the numerical model by confronting the numerical modes and eigenfrequencies with the experimental modes and eigenfrequencies. This chapter describes the experimental test procedure and the modal parameter fit procedure to determine the different mode shapes and eigenfrequencies of the cylindrical shell with top mass. The tests are done in the DCT-lab of the Department of Mechanical Engineering at Eindhoven University of Technology, where the experimental setup is realized. The data acquisition and the estimation of the frequency response functions (FRF’s) is done with SIGLAB [3]. The experimental modal analysis (EMA) is done with the software ME’SCOPE [4].

4.1 Theory

4.1.1 Transfer function measurement

To estimate the modes, first transfer functions need to be measured from different positions and several directions distributed over the setup. A transfer function is determined between two points on the setup. At one point a force is applied and at another point the response is measured. The applied force and the response are measured and converted via the computer program SIGLAB to a frequency response function (FRF). For a single reference measurement, the FRF’s must form one row or one column in the transfer matrix $H$ for the relation $x = H \cdot F$, where $x$ in this case is a column with velocities or accelerations and $F$ a column with the excitation forces. A multiple reference set of FRF’s corresponds to FRF’s from multiple rows or columns of the transfer matrix.
4. EXPERIMENTAL MODAL ANALYSIS

4.1.2 Mode indicator

Each resonance peak in an FRF measurement gives an indication of the presence of at least one mode. The first and most important step of the applied modal parameter estimation method is to determine how many modes have been excited (and are therefore represented by resonance peaks) in a certain frequency band of a set of FRF measurements [4].

Closely coupled modes and repeated roots structures can have two or more closely coupled modes that are very close in eigenfrequency with sufficient damping so that their resonance peaks sum together and appear as one resonance peak in the FRF’s. In our case where we have a geometrically symmetric structure, two shell modes can have the same eigenfrequency and damping but different mode shapes. This condition is called a repeated root. If a structure has closely coupled modes or repeated roots, and only one mode can be identified where there are really two or more, the resulting mode shape will look like the summation of two or more mode shapes when viewed in animation. The mode shape may also animate like a complex mode (a traveling wave instead of a standing wave) during animation [4].

Within ME'SCOPE there are three different mode indicators; the modal peaks function, the complex mode indicator function (CMIF) and the multivariate mode indicator function (MMIF). Any of the mode indicators can be used to count peaks from a set of single reference FRF’s. The CMIF and MMIF indicators provide more information from a multiple reference set of FRF’s. The complex mode indicator function (CMIF) performs a singular value decomposition of the FRF data, resulting in a set of multiple frequency domain curves. The number of mode indicator curves equals the number of references. Each peak in a curve is an indication of a resonance. The multivariate mode indicator function (MMIF) performs an energy minimization of either the real or imaginary part of the FRF data, resulting in a set of mode indicator curves. Like CMIF, the number of curves equals the number of references, and each peak in a curve is an indication of a resonance [4].

4.1.3 FRF fitting

Experimental modal parameters are estimated by applying curve fitting to a set of frequency response functions (FRF’s). The outcome of curve fitting is a set of modal parameters for each mode that is identified in the frequency band of the FRF measurements. Curve fitting is a process of matching a parametric form of an FRF to experimental data. The unknown parameters of the parametric form are the modal frequency, damping and residue for each mode [4].

4.1.4 Modal parameters

The modal parameters are the eigenfrequency, damping and the residue (representing the mode shape). The polynomial method is a multi-degree-of-freedom (MDOF) method that simultaneously estimates the modal parameters of multiple modes. The polynomial method is a frequency domain curve fitting method that utilizes the complex (real and imaginary) trace data in the frequency band of interest for curve fitting [4].
4.1.5 Mode shapes

After modal residues have been estimated for all modes of interest and at least one reference, modal parameters for each reference can be saved in a shape table as mode shapes. The mode shapes can be displayed in animation.

4.2 Influence of environment

It is important to know the influence of the environment on the test structure. Namely, if the supporting structure of the structure to be analyzed has modes in the frequency band of interest, this can influence the transfer functions which is undesirable.

For the supporting structure (bottom part in figure 2.1 (1)), the transfer functions of interest are the ones in the axial and radial direction. The excitation and measurement positions are shown in figure 4.1. The FRF with excitation and measurement in radial direction is shown in figure 4.2. The FRF with excitation and measurement in axial direction is shown in figure 4.3.

In addition, for the top mass direct transfer functions are measured in the axial direction and radial direction. For this experiment the top mass is decoupled from the setup and is suspended in weak rubber bands. In this way the top mass is decoupled from the environment. The only influence is from the rubber bands and they are very low frequent (< 4Hz), which probably causes the peak from 0 to 10 Hz in figure 4.4. The FRF’s in axial and radial direction are approximately the same.
4.3 EMA of main modes

Transfer function measurement
The force is applied with a hammer (with force sensor PCB type SN 4995) and the response
in measured with an acceleration sensor (PCB type SN C104817) as shown in figure 4.2. The
measurement settings of Siglab are shown in table 4.1. The excitation positions are '1' (radial
and axial) and the measurement positions are '1', '2', '3' and '4' (radial and axial) as shown in
figure 4.6. Every measured FRF is five times averaged to minimize the influence of noise. Due
to the fact that the chosen excitation and measurement positions are in the same vertical plain,
the modes will not occur in pairs. Measuring from all these positions gives sixteen FRF's. A
representative example of a measured FRF is given in figure 4.7.

Table 4.1: Siglab settings

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>recordlength</td>
<td>4096 points</td>
</tr>
<tr>
<td>frequency range (FR)</td>
<td>1000 Hz</td>
</tr>
<tr>
<td>sample frequency</td>
<td>2.56 * FR = 2560 Hz</td>
</tr>
<tr>
<td>measure time</td>
<td>1.6 s</td>
</tr>
</tbody>
</table>
4. EXPERIMENTAL MODAL ANALYSIS

Mode indicator
Eight FRF’s of excitation reference ‘1 radial’ and eight FRF’s of excitation reference ‘2 axial’ are imported in ME’SCOPE. Two references are used, so the multiple reference function will be used. Based on the NMA results, closely coupled and repeated roots are not expected so for the mode indicator the modal peak function is used. This gives good results as shown in the lower diagram of figure 4.8.

Modal parameters
Because the three peaks are not closely coupled, the modal parameters of all the three peaks can be estimated in one step. So the frequency band is set from 0 till 400 Hz (dashed vertical lines in top diagram of figure 4.8) and ME’SCOPE fits all the FRF’s in that frequency band. The modal damping parameters are calculated for the three peaks. The frequencies ($f$) and damping are shown in table 4.2.

Mode shapes
The relative mode shape strengths are also calculated by ME’SCOPE as shown in table 4.2. Mode shape strength ranges from 0 (meaning the mode is not present) to 10 (meaning the mode is strongly represented at the reference). Mode shapes with high strengths will have large resonance peaks in the FRF’s, for a reference relative to the other modes. In general, if a mode shape has a high strength, its modal parameter estimates will be more accurate than the estimates of a mode.
4. Experimental Modal Analysis

Figure 4.4: FRF of the top mass

Figure 4.5: Above: acceleration sensor; below: hammer

Table 4.2: Main modes data

<table>
<thead>
<tr>
<th>Mode</th>
<th>f [Hz]</th>
<th>Strength ref: 1 radial</th>
<th>1 axial</th>
<th>damping [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>24</td>
<td>0.6</td>
<td>0.0</td>
<td>3.5</td>
</tr>
<tr>
<td>1</td>
<td>56</td>
<td>0.5</td>
<td>0.1</td>
<td>3.3</td>
</tr>
<tr>
<td>2</td>
<td>152</td>
<td>0.0</td>
<td>2.3</td>
<td>1.8</td>
</tr>
<tr>
<td>3</td>
<td>230</td>
<td>10.0</td>
<td>9.9</td>
<td>5.1</td>
</tr>
</tbody>
</table>
4. experimental modal analysis

shape with a lower strength.

A mode shape should be saved for one of the two references for each mode. Usually the reference
with the largest strength is the best choice so that is done for these modes. That means that for
mode 1 reference '1 radial', for mode 2 reference '1 axial' and for mode 3 reference '1 radial' is
chosen. For the chosen references for each mode the mode shapes are shown in figure 4.9.

4.4 EMA of shell modes

For the estimation of the shell modes, 40 equidistant measurement points are chosen on the
perimeter halfway the height of the cylinder, as shown in figure 4.10. The shell modes with m=1
have the lowest eigenfrequencies and are therefore the most of interest. It is evident that in this
way, modes with m=1 can very well and modes with m=2 cannot be identified. For the excitation
we use the hammer (PCB type SN 4995, figure 4.5) and for measuring the velocity response we use
the laser vibrometer (Ono Sokkie type LV 1500, figure 4.10 left in clamp). On the measurement
points small pieces of reflection paper are sticked to reflect the laser bundel. This is necessary
because the transparant shell does not reflect the laser bundel enough. The measurement settings
of SIGLAB are shown in table 4.3. From reference point 1 (figure 4.11), 40 FRF’s are measured and
from reference point 2 also 40 FRF’s are measured. A representative example of an measured
FRF is given in figure 4.13.
4. Experimental Modal Analysis

Figure 4.7: FRF from excitation point 1 radial to measurement point 2 radial

Table 4.3: Siglab settings

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>recordlength</td>
<td>2048 points</td>
</tr>
<tr>
<td>frequency range (FR)</td>
<td>2000 Hz</td>
</tr>
<tr>
<td>sample frequency</td>
<td>2.56 \cdot FR = 5120 Hz</td>
</tr>
<tr>
<td>measure time</td>
<td>0.4 s</td>
</tr>
</tbody>
</table>

Mode indicator

40 FRF’s of reference 1 and 40 FRF’s of reference 2 are imported in ME’SCOPE. Therefore the multiple reference function can be used. Based on the NMA results, closely coupled roots are expected and maybe even repeated roots (if modes with different \( n \) have the same eigenfrequency). For the mode indicator, the multivariate mode indicator function gives the best results (lower diagram of figure 4.4). Many peaks can be seen and the modes indeed seem to be closely coupled.

Modal parameters

Because the shell modes seem to be closely coupled, the choice is made to determine the modal parameters peak by peak. A typical obtained result is shown in the upper diagram of figure 4.12 where the frequency band (dashed vertical lines at 800 and 900 Hz) is set around a peak. The eigenfrequency \( (f) \) and damping are estimated for eight peaks and shown in table 4.4.
4. Experimental Modal Analysis

Figure 4.8: Above: fit of FRF from excitation point 1 radial to measurement point 2 radial
Below: mode indicator: modal peak function

Table 4.4: Eigenfrequencies and damping of the shell modes

<table>
<thead>
<tr>
<th>n</th>
<th>( f ) [Hz]</th>
<th>damping [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1794</td>
<td>0.92</td>
</tr>
<tr>
<td>4</td>
<td>1293</td>
<td>0.49</td>
</tr>
<tr>
<td>5</td>
<td>1030</td>
<td>0.70</td>
</tr>
<tr>
<td>7</td>
<td>886</td>
<td>0.99</td>
</tr>
<tr>
<td>8</td>
<td>997</td>
<td>1.57</td>
</tr>
<tr>
<td>9</td>
<td>1165</td>
<td>0.92</td>
</tr>
<tr>
<td>10</td>
<td>1496</td>
<td>0.65</td>
</tr>
<tr>
<td>11</td>
<td>1683</td>
<td>1.63</td>
</tr>
</tbody>
</table>

Mode shapes
The shell modes are close together so the different modes are difficult to distinguish. Therefore, are the mode shapes for reference 1 and 2 separately saved. During animation of the mode shapes visually is chosen the best mode shape (i.e. from which the number of \( n \) the best could be determined). The results are shown in figure 4.14.
4. Experimental modal analysis

Figure 4.9: Main modes
4. Experimental Modal Analysis

Figure 4.10: 40 measurement positions on the shell

Figure 4.11: Excitation points
Figure 4.12: Above: fit of FRF from excitation point 1 to measurement point 18
Below: mode indicator: multivariate mode indicator function

Figure 4.13: FRF from excitation point 1 to measurement point 18
4. Experimental Modal Analysis

Figure 4.14: Shell modes \((m=1)\)
4.5 Qualitative comparison between NMA and EMA

4.5.1 Qualitative comparison of the main modes

In Table 4.5, the eigenfrequencies of the three main modes of the NMA and EMA are shown. The main mode shapes of the NMA (figure 3.2) and EMA (figure 4.9) have the same shape and occur in the correct order of increasing eigenfrequency. EMA mode 0 and 1 look like each other. The NMA mode 1 corresponds probably with EMA mode 0, because the modes shapes look more like each other then NMA mode 1 and EMA mode 1.

<table>
<thead>
<tr>
<th>mode</th>
<th>NMA $f (E = 3.5 \text{ N/m}^2)$</th>
<th>NMA $f (E = 5.5 \text{ N/m}^2)$</th>
<th>EMA $f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-</td>
<td>-</td>
<td>24 Hz</td>
</tr>
<tr>
<td>1</td>
<td>30 Hz</td>
<td>37 Hz</td>
<td>56 Hz</td>
</tr>
<tr>
<td>2</td>
<td>129 Hz</td>
<td>162 Hz</td>
<td>152 Hz</td>
</tr>
<tr>
<td>3</td>
<td>149 Hz</td>
<td>187 Hz</td>
<td>230 Hz</td>
</tr>
</tbody>
</table>

4.5.2 Qualitative comparison of the shell modes

In figure 4.15, the eigenfrequencies of the NMA and the EMA are plotted for the shell modes. The modes shapes of the NMA (an example is shown in figure 3.4) and EMA (figure 4.14) correspond qualitative with each other. Both the NMA and EMA results show a typical parabolic form for different values of n. All the results of the EMA are for m=1, so we compare only for m=1 the NMA and EMA with each other. For the EMA, mode n=6 could not be found, probably because this mode is closely coupled to mode n=7.
Figure 4.15: Results of NMA and EMA compared
Chapter 5
Conclusions and recommendations

The linear vibration modes of a thin cylindrical shell with top mass are investigated. A numerical modal analysis (NMA) and an experimental modal analysis (EMA) are performed. The NMA analysis is done for an isotropic shell with the elastic moduli \( E \) of \( 3.5 \cdot 10^9 \) N/m\(^2\) and \( 5.5 \cdot 10^9 \) N/m\(^2\).

The influence of gravity on the test setup is researched. The influence of the gravity on the eigenfrequencies of the modes is less than 0.1 % so it has no significant influence.

5.1 Main modes

The conclusion is that the NMA and EMA results of the main modes correspond qualitatively. The modes have the same shape and occur in the correct order of increasing eigenfrequency.

The eigenfrequency of the EMA of mode 2 is in between the boundaries of the NMA, modes 1 and 3 are not. Mode 2 does not depend on the moment of inertia of the top mass whereas mode 1 and 3 do. An explanation of this can be that the mass moment of inertia of the top mass is not good implemented in the NMA model. It is recommended to make a more accurate estimation of the mass moment of inertia of the top mass.

Main mode 2 is an axial mode and should depend on the axial elastic modulus. But the eigenfrequency of the EMA of mode 2 is closer to the eigenfrequency of the NMA for the elastic modulus in circumferential direction, which is not expected. Therefore it is recommended to check if the measured elastic moduli of the shell are correct.

5.2 Shell modes

Similar as for the main modes the conclusion for the shell mode is that the NMA and EMA results of the shell modes correspond qualitatively. The modes have the same shape and occur in
The correct order of increasing eigenfrequency.

The eigenfrequencies of mode 3 till 7, determined by the EMA, are in between the frequencies (with different $E$) determined by the NMA. The modes 8 till 11 are not. The cylindrical shell shows anisotropic material behaviour (i.e. the axial elastic modulus and the circumferential elastic modulus differ significantly). It is recommended to include this anisotropic behaviour in the FE model.


