Determination of modal parameters based on sinus excitation

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Determination of Modal Parameters based on sinus excitation

Internship 4W409

Report DCT2003.89

Eindhoven, September 2003

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1. Introduction

Based on research on friction behavior of mechanical constructions, measurement techniques are currently investigated that describe the phenomena associated with friction. One of the phenomena is the friction-induced resonance [1]. This resonance is caused by spring-like behavior in the stick-phase combined with inertia effects of the excited mass. This behavior is essentially non-linear and therefore linear measurement techniques cannot be used. A measurement technique has been proposed that gives input/output relations based on one fixed frequency [2]. Another technique has been proposed that uses a swept sine technique [3].

The fixed frequency measurement technique is investigated further in this report. Based on a standard second-order dynamic model description, the modal parameters $\zeta$ and $\omega_n$ (respectively the damping ratio and undamped natural frequency) essentially describe the dynamic response of this model. Based on the measurement technique, the question is asked whether it is possible to estimate the modal parameters from the measurement technique used, thus resulting in a second-order description of the friction-induced resonance.

It is assumed that the reader of this report has read [1] and [2].
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2. Second-order model description

A model representation of a typical second-order rotational mechanical system is shown in figure(1). It consists of a mass with inertia effect J, a spring K and a damper B.

![Figure (1) Mechanical second-order model](image)

The sum of all forces acting on the body can be described as:

\[ J \ddot{\theta} + B \dot{\theta} + K \theta = F(t) \]

This can be written in standard notation as:

\[ \ddot{\theta} + 2\zeta \omega_n \dot{\theta} + \omega_n^2 \theta = u(t) \tag{2.1} \]

with

\[ 2\zeta \omega_n = \frac{B}{J}, \quad \omega_n^2 = \frac{K}{J} \quad \text{and} \quad u(t) = \frac{F(t)}{J} \]

from which follows that the undamped natural frequency \((\omega_n)\) and the damping ratio \((\zeta)\) can be described as:

\[ \zeta = \frac{B}{2\sqrt{JK}} \quad \omega_n = \sqrt{\frac{K}{J}} \tag{2.2} \]

in which the damping ratio depends on three modeling parameters and the undamped natural frequency only on two.
2.1. Laplace Transformation

When the linear equation (2.1) is transformed to the Laplace domain the following equation is obtained:

\[ Y(s)(s^2 + 2s\xi\omega_n + \omega_n^2) = U(s) \]

From which the transfer function \( H(s) = \frac{Y(s)}{U(s)} \) follows:

\[ H(s) = \frac{1}{s^2 + 2s\xi\omega_n + \omega_n^2} \] (2.3)

for \( \omega_n > 0 \) and \( 0 < \xi < 1 \) the poles of this system are:

\[ s_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1 - \xi^2} \]

If the poles are plotted in the complex plane, it can be seen that:

1. The poles lie on a circle with a radius
   \[ |s| = \omega_n\sqrt{\xi^2 + 1 - \xi^2} = \omega_n \]

2. The angle between the poles and the negative real axis is
   \[ \mu = \arctan\left(\frac{\sqrt{1 - \xi^2}}{\xi}\right) \]
2.2. Frequency response

The response of this system to a sinusoidal input \( u(t) = A_0 \sin \omega t \) can be determined by replacing \( s = j \omega \) in equation (2.3):

\[
H(j \omega) = \frac{1}{\omega_n^2 - \omega^2 + 2 j \xi \omega \omega_n}
\]

The complete amplitude response to this sinusoidal input reads [5]:

\[
|H(j \omega)| = \sqrt{\left( \text{Re}[H(j \omega)] \right)^2 + \left( \text{Im}[H(j \omega)] \right)^2} = \frac{1}{\sqrt{\left( \omega_n^2 - \omega^2 \right)^2 + (2 \xi \omega_n \omega)^2}} \tag{2.4}
\]

This is quite a complicated result. The accompanying phase response reads:

\[
\angle H(s) = \arctan \left( \frac{\text{Im}[H(j \omega)]}{\text{Re}[H(j \omega)]} \right) = -\arctan \frac{2 \xi \omega_n \omega}{\omega_n^2 - \omega^2}
\]

When multiplying the numerator and denominator with a factor \( 1/\omega_n^2 \) the phase can be written as:

\[
\angle H(s) = -\arctan \left( \frac{2 \xi \left( \frac{\omega}{\omega_n} \right)}{1 - \left( \frac{\omega}{\omega_n} \right)^2} \right) \tag{2.5}
\]

or

\[
\angle H(s) = -\arctan \left( \frac{2 \xi \alpha}{1 - \alpha^2} \right), \quad \alpha = \left( \frac{\omega}{\omega_n} \right)
\]

Which is a relatively simple result.
One problem now arises when plotting the function (2.5) using a regular tangens function. Phase information is essentially $2\pi$ periodical while a tangens function is $\pi$ periodical (and thus also its inverse the arctan function). In MATLAB the phase can be correctly plotted by using the atan2 function.

Alternatively, the arctan function can be converted to a $2\pi$ periodical arccos function making use of the following property:

$$\arctan\frac{y}{x} = \text{sign}(y) \cdot \arccos\left( \frac{x}{\sqrt{x^2 + y^2}} \right)$$

To illustrate the obtained results (2.4) and (2.5) a plot of the functions can be made with a varying damping ratio to illustrate the change in shape.

Figure (3) Magnitude (top) and Phase (bottom) of the system $H(s)$. The Magnitude has been normalized with $\omega_n$, just like the phase, in this picture [5].
2.3. Time delay

A time delay is represented in the Laplace domain by

\[ L(s) = e^{-sT_d} \]

Where \( T_d \) is the delay time in seconds [5,6]. The frequency response (bode plot) of a time delay can be found by substituting

\[ s = j\omega \]

into the expression for the time delay.

\[ L(j\omega) = e^{-j\omega T_d} \]

Now, the magnitude is given by:

\[ |L(j\omega)| = |e^{j\omega T_d}| = \sqrt{\cos^2(\omega T_d) - j^2 \sin^2(\omega T_d)} \]

\[ = \sqrt{\cos^2(\phi) + \sin^2(\phi)}, \quad \text{where} \quad \phi = \omega T_d \]

\[ = 1 \]

And the phase is given by:

\[ \angle L(j\omega) = \angle(e^{-j\omega T_d}) = \angle(\cos(\omega T_d) - j\sin(\omega T_d)) \]

\[ = \arctan\left(\frac{-\sin(\omega T_d)}{\cos(\omega T_d)}\right) \]

\[ = -\arctan(\tan(\omega T_d)) \]

\[ = -\omega T_d \]

Thus a time delay does not affect the amplitude response but the phase has a lag \(-\omega T_d\).
2.4. In Practice

In theory the functions \((2.4+2.5)\) and \((2.6+2.7)\) describe the response of a second-order system accurately. However, in practice a considered friction system has a Magnitude and Phase (Bode) plot that has for example the following shape (for a complete description of this particular system the reader is referred to article [2]):

![Magnitude and Phase Plot](image)

Figure (4) Magnitude and Phase of a physical friction system. The first resonance (near 500 Hz) is the friction-induced resonance. FRF plot is obtained from feeding a physical system, modelled to the right, with white noise at an excitation signal \(V_{in}\) of 46 mV(red) and 1 mV(blue). Taken from [2].

It is clear that in this case the modal parameters \(\omega_n\) and \(\xi\) depend on the magnitude of the excitation signal. The phase does not start at 0° but at 180° and because of digital measurement delay time the phase information linearly descends.

Figure (4) however still resembles figure (3) and it is assumed that the amplitude and phase can be described accurately by \((2.4+2.5)\) and \((2.6+2.7)\).

To be able to compensate for the possible 180° shift and the time delay effect, \((2.5)\) can be adapted with one or more extra parameter(s) to compensate.
This results in the following set of phase descriptions:

2- parameter description: \[ \angle H(s) = -\arctan \left( \frac{\omega}{\omega_n} \right)^2 \] (2.5)

3- parameter description: \[ \angle H(s) = -\arctan \left( \frac{\omega}{\omega_n} \right)^2 - T_d \omega \] (2.8)

4- parameter description: \[ \angle H(s) = -\arctan \left( \frac{\omega}{\omega_n} \right)^2 - T_d \omega + \beta \] (2.9)

Where the second equation (2.8) essentially only compensates for the time delay effect while the third equation (2.9) compensates for both the phase shift of 180° and the time delay effect.

A small analysis has been performed to investigate the sensitivity of the four parameters of (2.9) to changes (a 100% increase of each parameter relative to an arbitrarily chosen original value). The result is presented in Figure (5):

![Figure (5) Sensitivity analysis of the four parameters to an increase of the four parameters, base values are:](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zeta</td>
<td>0.1 [c]</td>
</tr>
<tr>
<td>Omega_2</td>
<td>100 [Hz]</td>
</tr>
<tr>
<td>Td</td>
<td>0.0005 [s]</td>
</tr>
<tr>
<td>Beta</td>
<td>180 [°]</td>
</tr>
</tbody>
</table>
3. Swept Amplitude excitation

When a measurement is done with the swept amplitude method, the excitation frequency $\omega$ is fixed and the amplitude of the excitation signal is varied:

$$u(t) = A(t) \sin \omega_0 t$$

A restriction to use such an excitation signal on a nonlinear system is that the amplitude variation $A(t)$ itself is not a sinusoidal or sinus-like function with a frequency close to the operating frequency of $\omega_0$ and should be well below it. In other words:

$$\omega_{A(t)} \ll \omega_0$$

In [2] a swept amplitude signal is chosen with the following shape:

![Figure (6) Time-domain swept amplitude signal used, 139284 samples is 10.88 sec, $f_s=320$ Hz ($\omega_0=2\pi f_s$). For a description of the characteristics and advantages of this signal, see [2]](image)
The response of an observed system (in this case the same system as in figure 4) to this signal can be represented in a time-domain plot:

![Image of time-domain response](image)

Figure (1) Time-domain response of frictional system [2].

Which shows that the system has sometimes left the frictional stick phase and has rotated physically.

An amplitude domain plot (first harmonic only):

![Image of amplitude domain plot](image)

Figure (3) Top: x-axis motor current, y-axis observed gain (dB). Bottom: x-axis motor current, y-axis observed phase difference (degrees). Blue: upward motion (0-5.44 sec), red: downward motion (5.44-10.88 sec) [2].
3.1. Determination of modal parameters

The calculation of the modal parameters is now straightforward; making use of (2.5) (2.8) or (2.9) the complete phase description is given with a maximum of four parameters including the two modal parameters. Ideally, two parameters $\beta$ and $T_d$ can be estimated from the physical measurement system as $\beta$ describes the initial phase (e.g. 180°) and $T_d$ is the measurement time delay, resulting in a system with two parameters $\omega_n$ and $\xi$. However, for the rest of this chapter it is assumed that both $\beta$ and $T_d$ are unknown resulting in a four-parameter description of the phase.

From figure(10) it is clear that the modal parameters can vary with a varying input amplitude $A(t)$. This way, the phase information for one frequency $\omega_0$ can be evaluated with varying amplitude $A(t)$. The calculation of the four parameters $\omega_n$, $\xi$, $\beta$ and $T_d$ can then be done for one fixed amplitude $A_0$ when a maximum of four excitation frequencies $f_1$-$f_4$ are used resulting in a system of four unknown parameters and four known phases.

Standing assumption is that the modal parameters only depend on the input amplitude $A(t)$ and not on the input frequency $\omega_0$, thus $\omega_0=\omega_n(A)$ and $\xi=\xi(A)$. To prevent nearly singular systems and sensitivity to noise, the four frequencies $f_1$-$f_4$ should be chosen close to the undamped natural frequency $\omega_n$ and with enough space in between the four frequencies.

As an example, for the system belonging to figure(4) and with 45 mV as RMS value of the excitation noise, the frequencies could be chosen as $f_1$-$f_4=220$-$340$-$460$-$580$ (avoiding the “sensitive” 50 Hz and higher frequencies) when using a four-parameter description.

![Figure (9) Phase of the physical friction system from figure(4). Indicated are the proposed frequencies $f_1$-$f_4$ and the resulting approximate measurements for fixed amplitudes.](image-url)
How the modal parameters vary with the amplitude function $A(t)$ (the functions $\omega_n(A)$ and $\xi(A)$) can also be calculated given the same method which should result in (continuous) pictures like figure(10):

![Figure (10) Frequency and damping change with changing angular acceleration (thus also A(t)). The observed system is the same as in figure(4).][1]
4. Evaluation

4.1. Evaluation second order linear model

In order to test the estimation of modal parameters, a second order linear model was used to simulate the use of the 2-parameter description. The model used is SIMULINK-based, making use of the transfer function description (2.3). The model parameters as chosen in this experiment are $\zeta = 0.1$ and $\omega_n = 75$ Hz. The system was tested at 73 and 76 Hz making use of the following input signals:

![Graphs showing input and output signals](image)

Figure (11) Two input frequencies 73 Hz and 76 Hz excite the second order linear model (bottom). The resulting output is visible in the upper figures.

From the input and output signal the phase lag can be determined related to the amplitude of the input signal, making use of the MATLAB routine `demod`:

![Graph showing phase description](image)

Figure (12) Phase description of linear second order system. As expected, the phase description is independent of the input amplitude $A_i$ because the system considered is linear. At lower amplitudes (below 0.1) a difference in phase up and down movement is visible. This is investigated in more detail in chapter 4.3. Blue: 73 Hz and green: 76 Hz.
Now, for every frequency point the 2-parameter description (2.5) is fitted. The resulting two-dimensional system to be solved is however non-linear and use has been made of the MATLAB routine `fsolve` to solve this system numerically (substitution of one equation into the second would be another possibility to solve the system).

Figure (12) indicates that the -90° phase shift is exactly in between the two excitation frequency values whereas it should be closer to 76 Hz. As expected, this is also shown by the resulting modal parameter estimates:

![Figure (13) Estimation of the modal parameters of the linear second order system.](image)

The estimated value for omegan is 73.25 Hz, this is below the expected value of 75 Hz. The estimate for zeta is also just below the expected 0.1.

It is assumed that a measurement time delay is present in this model which accounts for the extra phase lag, resulting in lower estimates for both omegan and zeta. In fact, this measurement time delay is exactly 0.5*Ts which in this case is 0.0005 sec. At 75 Hz this typically accounts for a phase delay of approximately 7° or 0.23 rad, from (2.7).
To take this effect into account, the three-parameter description (2.8) is tested on the same second-order system with frequencies of 60, 70 and 80 Hz.

Figure (14) Phase description of linear second order system. Blue: 60 Hz, green: 70 Hz and red: 80 Hz.

Figure (15) Resulting estimates of Omegan and Zeta for the three-parameter description. Unsuccessful iteration results of the system of three equations have been deleted from the results.
The estimated $T_d$ is exactly as expected:

![Graph](image.png)

Figure (16) Estimate of $T_d$, this is a constant and also estimated as such.

In the two-parameter description, a system of two nonlinear equations had to be solved for each sampling point, thus 10.001 times. The MATLAB numerical estimation procedure `fsolve` succeeded in finding a least-square fit with high enough accuracy 10.000 times with 1 fail. Using the three-parameter description, the number of successes was 9985 with 16 failures in finding an optimal estimate. It is assumed that when using a four-parameter description the number of failures will raise again, however this is not investigated further.

A good initial estimate of the three parameters as a starting point for the iteration process is also required for a successful estimation.
4.2. Evaluation nonlinear Leuven friction model

In order to test the estimation of modal parameters on a frictional system, a "black-box" frictional model was used to simulate friction-like behavior. The working principles of this "Leuven friction model" are beyond the scope of this project. The input parameters of this particular model used are put in addendum 2.

To get a first feel of this model, supplying the model with white noise and an impulse response generated a frequency response function estimate:

From it, it is clear there is a lightly damped natural frequency near 50 Hz in the model. Two frequencies close to the 50 Hz are chosen to excite the system at variable amplitude inputs:

From it, it is clear there is a lightly damped natural frequency near 50 Hz in the model. Two frequencies close to the 50 Hz are chosen to excite the system at variable amplitude inputs:
The experiment is repeated with the three-parameter description in an attempt to further improve the results.
Because this model is considered "black box" it is unknown what the total time delay is and from the estimated \( T_d \) it even seems amplitude dependent. The starting value of \( 5 \times 10^{-4} \) sec however is in line with expectations, namely \( 0.5 \cdot T_s \).

Figure (21) Phase lag of the 3-parameter leuven frictional model experiment. 40 Hz (blue), 50 Hz (green), 60 Hz (red). Amplitude of input signal is 0.1, duration 10 sec, sampling time \( T_s=0.001 \) sec. 87 times of 1000 the algorithm failed to find a good fit for the three parameters.

Figure (22) Estimated parameter \( T_d \) of the 3-parameter leuven frictional model experiment.
In the last figure (23) the natural frequency is in line with the expected 50 Hz. Because of the chosen frequencies of 40, 50 and 60 Hz which are relatively far apart (figure (21)) the resulting damping ratio zeta is a bit noisy, it is more accurate when choosing frequencies closer together.

### 4.3. Observations

As the fitting process is an iterative (least square) process, a good starting guess for the parameters has to be given to the script. When choosing start guesses that are too far from the target values, the process might not converge at all. For convergence speeds and/or stability margins of the nonlinear equations (2.5, 2.8 and 2.9) an analysis can be performed, however this has not been done.

As mentioned in figure (12) at lower amplitude levels the back-and forth phase description differ. To investigate the source of this phenomenon, a SIMULINK model of a time-delay has been constructed and tested with an input signal of the same shape as figure (18). As a result, the same phase shape difference of the resulting phase description is visible as mentioned. Hypothetically, this difference might be caused by the way SIMULINK interprets discrete input signals, namely by interpolating the various input points to a continuous signal. Alternatively, the phase calculation algorithm itself might also cause it, for which the MATLAB command `demod` was used.

This effect can be minimized by choosing a higher sampling rate and a longer measurement time, however this also induces more measurement points and thus longer calculation time of corresponding modal parameters.

When choosing the test frequencies too close together, sometimes the resulting phase descriptions were too close together to estimate the modal parameters. The system was then too ill conditioned to find reliable results. The `fsolve` numerical solver does however tell when this situation happens.
5. Conclusion

A two-, three- and four-parameter description have been derived that describe the phase response near a resonance. When testing a nonlinear system with a fixed-frequency, swept-amplitude excitation signal this description can be used to determine the modal parameters $\omega_n(A)$ and $\zeta(A)$. The following conditions however are set:

1. Of the input signal $u(t) = A(t)\sin \omega_0 t$, the variation of the amplitude $A$ should be a lot smaller than the operating frequency, so $\omega_{A(t)} \ll \omega_0$. (e.g. choose $A(t)$ as a linear signal)
2. The phase response of the nonlinear system must resemble the phase response of a linear system in a white noise or impulse response experiment.
3. The excitation frequencies for the swept-amplitude excitation experiment have to be chosen near the resonance operation frequency. The space between these frequencies can be chosen arbitrarily (the resulting system of equations can however be near singular when these frequencies are chosen too close together and the results unreliable when too far apart).
Addendum 1 References


Addendum 2 Leuven friction model parameters

%m-script for the Leuven Integrated Friction Model
%doo LarS van Gerven februari 2003

%definitie van de hysterese eigenschappen
W_i = (1.5/2.15)*[0.10 0.15 0.25 0.40 0.55 0.70];
K_i = (1e5/180000)*[30000 30000 30000 30000 30000 30000];

%definitie van de 'sliding' parameters
n = 1;
delta = 2;
F_c = 1;
F_s = 1.5;
V_s = 0.001;

%definitie van de wrijvingscoëfficiënten
sigma1=20;
sigma2=0.4;
Addendum 3 Example script modal parameter estimation

%% Simuleren van leuven niet-lineair model voor bepalen van de geldigheid van multisinus
%% afschatting van modale parameters m.b.v. 2 parameter model
%% D.Hoekstra Juli 2003

clear; close all;

%% Volgend stuk script is onderdeel van de initialisatie van het Leuven frictie model

%% m-script for the Leuven Integrated Friction Model
%% door Lars van Gerven februari 2003

%% definitie van de hysterese eigenschappen
w_i = (1.5/2.15)*[0.10 0.15 0.25 0.40 0.55 0.70];
ki = (l65/180000)*[30000 30000 30000 30000 30000 30000];

%% definitie van de 'sliding' parameters
n = 1;
delta = 2;
F_c = 1;
F_s = 1.5;
v_s = 0.001;

%% definitie van de wrijvingscoefficienten
sigma1=20;
sigma2=0.4;

%% Tot dusver is het Leuven model bepaald. Nu wordt dit model als black box
%% beschouwd en aan het model wordt gemeten.

%% Opgegeven simulatie parameters
Ts=0.001; %sample tijd
fs=2*pi/Ts; %sample frequentie [rad/s]
fNyquist=0.5*fs; %Nyquist frequentie [rad/s]
T=5; %Meettijd
N=T/Ts; %N=aal ant meetpunten (is ter info)
Frequency=[48 52]; % Geef werkfrequenties op bij welke sinus frequenties het systeem ge-exciteerd moet worden (rad/s) [Hz]
A0=0.1; %maximale amplitude van ingangssignaal

%% Volgende loop rekent voor alle opgegeven frequenties (p stuks) de
response en fasedraaiing uit
for p=1:length(Frequency)
opfreq=Frequency(p);

%% Hier wordt het input signaal bepaald
t=[0:Ts:T]; %tijd

%% Eerste segment met klimmende amplitude
for i=1:floor(0.5*length(t))
    A(i)=A0*t(i)/t(floor(0.5*length(t)));
    f(i)=A(i)*sin(opfreq*t(i));
end

%% Tweede segment met dalende amplitude
m=[0.5*T-Ts:0];
for i=floor(0.5*length(t))+1:1:length(t)
    A(i)=A0*m(i-floor(0.5*length(t)))/t(floor(0.5*length(t)));
    f(i)=A(i)*sin(opfreq*t(i));
end
InSignal=[t' f'];
Simulation of modal parameters based on sinus excitation

```matlab
%% Simulation of mode shapes
sim('sim_eenheidsmassa', T);

%% Iddata object aanmaken
data=iddata(uecht, uecht, Ts, 'OutputName', 'Position', 'InputName', 'Input Force');

%% Plot data
figure;
tfplot(data);
subplot(2,1,1); grid on; xlabel('time [s]'); ylabel(' y [m]');
subplot(2,1,2); grid on; xlabel('time [s]'); ylabel(' x [input]');

%% Nu gaan we de fase berekening uitvoeren
data=[uecht yecht]; %input en output bij elkaar

% DC vrij maken
xOdc=(data(:,1)-mean(data(:,1)));

% Filteren van de input (bandpassfilter bij opfreq +- 10 rad/s). Is bij simulatie niet nodig
wp=[opfreq-10 opfreq+10]/fNyquist;
ws=[0.5*opfreq 1.5*opfreq]/fNyquist;
Rp=1;
Rs=60;
[n,wn]=buttord(wp,ws,Rp,Rs);
[b,a]=butter(n,wn);
xschoon=filtfilt(b,a,xOdc); %Is nodig als experiment uitgevoerd wordt
xschoon=xOdc;

% Filteren van de output (bandpassfilter bij opfreq +- 10 rad/s). Is bij simulatie niet nodig
wp=[opfreq-10 opfreq+10]/fNyquist;
ws=[0.5*opfreq 1.5*opfreq]/fNyquist;
Rp=1;
Rs=60;
[n,wn]=buttord(wp,ws,Rp,Rs);
[b,a]=butter(n,wn);
yschoon=filtfilt(b,a,yschoon);
yschoon=yschoon;

% Faseverschil tussen signaal xschoon en yschoon
faseverschil=(180/pi)*(demod(yschoon,opfreq,fs,'pm')-
demod(xschoon,opfreq,fs,'pm'));

% Filter aanmaken van het eindfilter (laagdoorlatfilter zodat hogere frequenties (verstoringen) niet zichtbaar zijn. Is bij simulatie niet nodig
wp=20/fNyquist;
ws=(opfreq)/fNyquist;
Rp=1;
Rs=80;
[d,c]=butter(n,wn);
%resultaat=filter(d,c,(faseverschil));
resultaat=faseverschil;
```
Voor het oog wat fijner...
for i=1:length(resultaat)
    if resultaat(i)>0
        resultaat(i)=resultaat(i)-360;
    else
        end
end
ampl=abs(hilbert(xschoon));
result(:,p)=resultaat;
end

Einde van de loop die voor alle opgegeven frequenties (p stuks) de responsie en fasedraaiing uittrekt

Plaatje van de berekende fase t.o.v. amplitude van ingangssignaal
figure;
plot(amp1,result); xlabel('amplitude [-]'); ylabel('phase [degrees]');
grid on;

Resultaten voor de volgende stap:
% ampl Amplitude verloop van de sinus excitaties
% result Fase verdraaiing, i-de kolom is i-de excitatie frequentie
% Frequency Excitatie frequentie, i-de kolom is i-de frequentie [rad/s]

omega1=Frequency(1); omega2=Frequency(2); %omega3=Frequency(3); %rad/s
phase1=(pi/180)*result(:,1); phase2=(pi/180)*result(:,2);
%phase3=(pi/180)*result(:,3); %phase
X0=[10 0.1]; %beginschatting voor omega en zeta voor routine fsolve
options=optimset('ToIFun',1e-3,'Display','off');
FVAL=[];EXITFLAG=[0 0 0];
for q=1:length(phase1)
    [X,FVAL,EXITFLAGtemp]=fsolve('phase2b',X0,options,omega1,omega2,phase1,phase2,q);
    omegan(q)=abs(X(1));
    zeta(q)=abs(X(2));
    if EXITFLAGtemp<0
        EXITFLAG(1,1)=EXITFLAG(1,1)+1;
    elseif EXITFLAGtemp==0
        EXITFLAG(1,2)=EXITFLAG(1,2)+1;
    elseif EXITFLAGtemp>0
        EXITFLAG(1,3)=EXITFLAG(1,3)+1;
    else
        end
        X0=X; %maak slim gebruik van vorige oplossing als beginschatter
(versnelt proces aanzienlijk)
end

Fouten analyse:
['Number of times FSOLVE converged to a solution X:
' num2str(EXITFLAG(1,3))]
['Number of times maximum number of function evaluations was reached:
' num2str(EXITFLAG(1,2))]
['Number of times FSOLVE did not converge to a solution:
' num2str(EXITFLAG(1,1))]
figure;
plotyy(amp1,omegan/(2*pi),amp1,zeta)
The called function phase2b:

% 2 parameter description of phase information

function f = phase2b(X,omega1,omega2,phase1,phase2,q)
    f(1) = (1-(omega1./X(1)).^2)*tan(-phase1(q))-2.*X(2).*omega1./X(1);
    f(2) = (1-(omega2./X(1)).^2)*tan(-phase2(q))-2.*X(2).*omega2./X(1);