Experimental Modal Analysis Of A Turbine Blade

M.L.J. Verhees

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Traineeship report

Coach(es): Dr. Ir. I. Lopez
Ing. W. Rooyakkers
Dr. Ir. R. Vijgen

Supervisor: Prof. Dr. H. Nijmeijer

Technische Universiteit Eindhoven
Department Mechanical Engineering
Dynamics and Control Group

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1 Introduction

This report is the result of an internship carried out at Sulzer Elbar B.V. Sulzer Elbar B.V. is a company that specializes in the repair and refurbishment of industrial gas turbines. Sulzer Elbar B.V. is situated in Lomm in the Netherlands. In this report the experimental modal analysis of a turbine blade is treated. The turbine blade under consideration is a second stage turbine blade from an ABB 13E2 gas turbine. In this general introduction first the working principles of gas turbines and the ABB 13E2 gas turbine are discussed. This is followed by a short discussion of the main cause of turbine blade failure and the role of modal analysis in preventing this failure. After this the current approach used to characterize the vibrational behavior of turbine blades is discussed. Finally the goals and structure of this report are discussed.

1.1 Gas Turbine Working Principle

A schematic drawing of a gas turbine is shown in figure 1.1. The working principle behind the gas turbine is as follows. Ambient air is compressed in the compressor. This compressed air is directed to the combustion chamber. In the combustion chamber the compressed air is mixed with vaporized fuel and burned under constant pressure. This burning process results in a hot gas with a high energy content. This hot gas is allowed to expand through the turbine where the energy in the gas is converted to a rotation of the turbine shaft. The turbine shaft powers both the compressor and a generator used to obtain electrical power from the gas turbine.

Figure 1.1: A schematic overview of a simple gas turbine.
Figure 1.2 shows a schematic drawing of the 165 MW ABB 13E2 gas turbine. Like the simple gas turbine schematically represented in figure 1.1 the ABB 13E2 gas turbine consists of a compressor, a combustion chamber and a turbine.

The compressor consists out of twenty one compressor stages. Each stage consists of a combination of a ring of rotor blades and a ring of stator vanes. The ring of rotor blades is attached to the turbine shaft while the ring of stator vanes is attached to the compressor casing. The rotor blades in the compressor are used to transport the air from one ring of stator vanes to the next ring of stator vanes. The stator vanes in the compressor are setup in such a way that their openings narrow from inlet to outlet so that at each stage the air is compressed. As the air is compressed the pressure and temperature of the air increase while its volume decreases. To accommodate this decrease in volume each consecutive ring of blades and vanes has a decreasing diameter.

In order to create a more symmetrical and even combustion process the ABB 13E2 gas turbine has an annular combustion chamber. In this combustion chamber fuel is vaporized into the air coming from the compressor. This mixture of vaporized fuel and compressed air is burned under constant pressure. This results in a hot gas with a high energy content. The burning process is continuous and only ignited once when starting the turbine.

The turbine consists of five turbine stages. Each of these stages contains a ring of stator vanes and a ring of rotor blades. The stator vanes in the turbine have two functions.
First they are used to direct the gas flow at the rotor blades in the direction of rotation. Second they are shaped in such a way that the gas is compressed when moving from one ring of rotor blades to the next ring of rotor blades. This increases the turbine efficiency. The purpose of the rotor blades is to convert the energy in the gas into mechanical energy available from the turbine shaft. The rotor blades are shaped in such a way that the amount of energy extracted from the gas flow is maximal.

1.2 Turbine Blade Failure

The main cause of turbine blade failure is high cycle fatigue. Fatigue failure is related to repeated cycling of the load on a structural member. The fatigue life of a structural member i.e. the number of load cycles it can survive is in general determined by the magnitude of the stress cycles. The exact relation between the magnitude of the stress and the fatigue life depends on the material properties of the structural member. In general higher stresses lead to a shorter fatigue life. For some materials fatigue only occurs if stresses exceed a certain minimum level for other materials there is no minimal stress level. If the stresses that are present on the turbine blade during operation and the material properties of the turbine blade are known then an estimation of the fatigue life of the turbine blade can be made.

Generally fatigue failure occurs as follows. After a number of load cycles a crack is initiated. This usually occurs at a point of relatively high stress concentration i.e. points with sharp geometrical discontinuities or points with relatively rough or soft surfaces. Once the crack is initiated it advances incrementally through the material with each stress cycle. In general this advance is very slow up to a certain point where it accelerates. The final failure occurs very rapidly. High cycle fatigue corresponds to failure after a relatively large number of load cycles. High cycle fatigue occurs at stress levels well below the yield strength of the material where deformation is elastic. The failure of a structural member is not caused by excessive loading but by the repeated cycling of the load.

In principle there are two ways in which the failure of turbine blades due to fatigue problems can be eliminated. These are a correct structural design and the prevention of cyclic loading. The correct design of a structural member can usually eliminate or dramatically reduce fatigue problems. For a turbine blade this is not always possible. The design of a blade is usually constrained by aerodynamic properties, weight, rotor length, etc. which can make the elimination of fatigue problems through design modifications very difficult if not impossible. The prevention of cyclic loading in gas turbines is virtually impossible in practice. The complex interior of the gas turbine makes the elimination of excitations difficult. There are numerous excitations that (can) occur within a gas turbine. These range from free vibrations like intermittent stalls, surges and liquid slugs to forced vibrations like unbalance, vane passing, rotor stator rubbing and cavitation. Self excited vibrations like blade flutter and shaft hysteresis generally do not occur in well designed turbines. The frequency of free vibrations can in general not be easily predicted as these mainly depend on the characteristics of a specific turbine. Forced and self excited vibrations in general depend on the running frequency.
of the turbine and are usually found between half to six times the running frequency of the turbine. Modal analysis can be a powerful tool to assist in the identification and elimination of fatigue problems. The most obvious use of modal analysis is in determining the natural frequencies of the turbine blades. Knowledge of these frequencies can be very useful in avoiding excessive excitations and thereby reducing the risk of fatigue failure. A less obvious application of modal analysis is in the validation of computer generated models of the turbine blades. These models can be very useful to investigate turbine and turbine blade properties under running conditions. Finite element models can be used to predict the influence of design changes on the stresses and strains acting on the turbine blade under running conditions.

1.3 Current Approach

The current approach used at Sulzer Elbar B.V. to identify turbine blade vibration characteristics is directed at the refurbishment of old turbine blades. In the current approach the natural frequencies of a turbine blade are not measured. Instead the turbine blade is excited using an unknown pulse signal. The frequency of the first dominant peak in the output spectrum of the turbine blade is measured. This measurement is performed both before and after the repair process. If the frequency of the dominant peak in the output spectrum has changed during the repair then either material is removed from the blade or added to the blade to adjust this frequency so that it coincides with the dominant frequency before repair.

The idea behind the current approach is as follows. Based on practical experience it is assumed that the first natural frequency of the turbine blade is the only natural frequency of interest. This frequency is assumed to be solely responsible for all vibration problems. The turbine blade that is to be refurbished has generally not failed. This means that the first natural frequency of the turbine blade must have been chosen correctly. If the location of the dominant resonance peaks in the output spectrum is identical both before and after repair then the repair process is assumed to not have changed this natural frequency. Consequently the characteristics of the turbine blade are assumed to be adequate to eliminate the chance of failure.

Although practice has shown that the current approach is adequate for the refurbishment of turbine blades it cannot be applied to evaluate new turbine blade designs or design modifications. The lack of reference material makes it impossible to evaluate the adequacy of the turbine blade design based on the output spectrum. Another problem is that the current approach does not provide detailed information on the vibration characteristics of a turbine blade. For proven turbine blade designs this may not be a problem as practice has already shown that the blade design is adequate to prevent vibration problems. For new or modified turbine blade designs the current approach will not work. In order for a new or modified design to be worthwhile the turbine blade design will have to be pushed to its limits. This can only be safely done if extensive knowledge about the vibrational behavior of the turbine blade is available. The current approach simply does not provide the needed information. In order to en-
sure that failure due to vibrational problems will not occur intimate knowledge of the stress and strain distributions across the turbine blade under operational conditions are needed. Currently only limited measurements under operational conditions are possible. An alternative to obtaining the required data from measurements is the use of finite element modeling. Finite element models can be used to accurately predict the stress and strain distributions across the turbine blade. An important condition for the successful application of finite element models is the validation of these models with experimental data. An approach often used to evaluate finite element models is the comparison of measured and calculated modal parameters.

1.4 Goals and Report Structure

This report addresses three main goals. The first of these goals is the introduction of experimental modal analysis at Sulzer Elbar B.V. This introduction includes both the investigation of the dynamical properties of a turbine blade as the investigation of the suitability of the current measurement setup in performing an experimental modal analysis. The second goal is the determination of the modal properties of the second stage ABB 13E2 turbine blade. Third a finite element model developed at the National Aerospace Laboratory NLR is to be validated.

This report is organized as follows. In chapter 2 the experiments needed to obtain the data used in performing the modal analysis are treated. This includes the setup of the experiments, the setup of the measurements and an evaluation of the quality of the measured data. Chapter 3 deals with the extraction of modal parameters from the measured data. This chapter is divided into three steps. First the number of modes is determined. Second an appropriate estimation method is chosen and finally different estimates of the modal parameters are compared in order to determine the best estimate. In chapter 4 the evaluation of the estimated modal parameters is performed. The estimated modal parameters themselves are examined, the modal parameter estimates are compared to theoretical values obtained from beam models and finally preliminary results obtained from a finite element model are discussed.
2 Experiments

In this chapter the experiments performed to obtain frequency response functions of the turbine blade are discussed. This discussion is divided into three parts. First the experimental setup is discussed. Second the measurements themselves are discussed and finally the quality of the measured frequency response functions is evaluated.

2.1 Experimental Setup

The experimental setup of the frequency response functions measurements consists of four parts. First the measurement method and measurement setup are discussed. This is followed by a short discussion on the properties of the measuring equipment. Finally the selection of the measurement points is treated.

2.1.1 Measurement Method

There are a number of methods available to measure the frequency response functions needed to perform a modal analysis. The most important differences between these methods are in the number of inputs and outputs and in the excitation method used. Common input output methods are the single input single output (SISO) and the multiple input multiple output (MIMO) methods. The two most common excitation methods are excitation using an impact hammer and excitation using an electrodynamical shaker. Each of the above methods has specific advantages and disadvantages that determine which measurement method should be used in a specific case. The advantages and disadvantages of each method are discussed in [1], [3], [12] and [7].

In order to measure the frequency response functions of the turbine blade a single input single output impact test with fixed boundary conditions is performed. The reasons behind the choice for this type of test are the following. With the exception of an impact hammer all the test equipment needed for an impact test is readily available. This makes the performance of an impact test relatively cheap when compared to alternative testing methods for which most of the needed equipment is not available. Secondly an impact test does not significantly differ from the current measurement approach as described in section 1.3 and therefore this should show if the current approach can be adapted to measure eigenfrequencies. Thirdly the extra sensors and data processing capability needed to implement an alternative testing method are not available. Alternatives to the above approach could be a SISO or a SIMO shaker test.
Shaker testing has as an advantage that there is more control over the input signal which in general should lead to better results especially at higher frequencies. The advantage of performing a SIMO test is that the response data is obtained simultaneously which in general leads to measurement data with a better consistency. The disadvantage of both approaches is that there is usually more test equipment needed and a test takes more time to setup.

2.1.2 Measurement Setup

The measurement setup is shown in figure 2.1. Excluding the turbine blade it consists of four main parts: a turbine blade clamping device, an accelerometer, an impact hammer and a frequency analyser. The accelerometer, the impact hammer and the frequency analyser are discussed in section 2.1.3 and will not be discussed any further here. The turbine blade clamping device consists of three separate parts shown in figure 2.2. Essentially it consists out of a tool (A) that fits around the fir tree foot of the turbine blade. This tool has two holes at the bottom through which a pressure block (B) makes contact with the turbine blade. A hydraulic cylinder (C) is used to press the fir tree foot of the turbine blade onto the tool (A) using the pressure block (B). The turbine blade clamping tool is attached to a steel block with a mass of approximately 100kg. This steel block is attached to a granite block with a mass of approximately 1350kg.
The turbine blade clamping device as discussed above has two disadvantages. First
the clamping of the turbine blade through the tool (A) is statically over determined.
This causes difficulties in reproducing the clamping of turbine blade. Second the
pressure of the hydraulic cylinder (C) influences the measured natural frequencies but
it is difficult to maintain a constant pressure level and the pressure level cannot be
accurately controlled. This inability to accurately control the hydraulic pressure causes
inaccuracies in the measured frequency response functions.

2.1.3 Measuring Equipment

In this subsection the specifications of the measuring equipment are treated in more
detail. There are three pieces of measuring equipment used: an impact hammer,
an accelerometer and a dynamic signal analyser. These devices will all be treated
separately.

Impact Hammer

To apply the excitation to the turbine blade an Endevco impact hammer type 2302-50
is used. Some relevant physical properties of this impact hammer are presented in table
A.1 in appendix A. In order to measure the impact force imparted by the hammer a
piezoelectric force transducer is built into the hammer head. The hammer is powered
through an ISOTRON\textsuperscript{1} power supply. This means that a constant current is used
to power the hammer while the output signal is present as a voltage modulation on
the same power supply line. This has the advantage that the measurement signal and
power supply run trough the same cable but due to the modulation of the measurement
signal low frequency signals below approximately 0.1 Hz cannot be measured.

Accelerometer

The accelerometer that is used to measure the turbine blade accelerations is an
Endevco ISOTRON accelerometer type 2256A-100. The relevant properties of this ac-
celerometer are presented in table A.2 in appendix A. This type of accelerometer uses
an piezoelectric element to measure acceleration. The accelerometer has a built in sig-
nal conditioner to guarantee a suitable output signal. The accelerometer is attached
to the turbine blade using mounting wax.

Dynamic Signal Analyser

The measurement of the force and acceleration signals is performed using a HP 35670A
2-channel dynamic signal analyser. Some properties of this dynamic signal analyser
are shown in table A.3 in appendix A. The dynamic signal analyser samples the
voltage signals coming from the impact hammer and the accelerometer. The sensitivity
information of the sensors is used to convert the voltages to equivalent force and
acceleration values. The dynamic signal analyser also performs the transformations

\textsuperscript{1}An ISOTRON power supply is the Endevco equivalent of an ICP power supply.
and calculations necessary to convert the two measured time domain signals into a frequency response function.

2.1.4 Measurement Points

The two main points to take into consideration when selecting measurement points are: the number of measurement points and the location of each measurement point. The minimum number of measurement points used in a modal analysis should be chosen in such a way that all mode shapes can be uniquely described. Depending on the intended use of the resulting modal model more measurement points might be needed. If the only purpose of the modal analysis is to identify the different mode shapes than the above minimum is probably sufficient. If on the other hand the purpose of the modal model is to validate or update a finite element model than more measurement points are needed. The location of the measurement points on the turbine blade surface depends on the type of modes to be visualized. In order to correctly visualize both bending and torsional modes the measurement points have to be distributed both horizontally and vertically across the turbine blade surface.

The measurement points used in the modal analysis of the turbine blade are shown in figure 2.3. This figure shows 44 measurement points distributed across the convex side of the turbine blade. The convex side is used because the curvature of the blade makes it difficult to attach an accelerometer to its concave side. Across the length of the turbine blade eight measurement points are chosen. These measurement points are spaced at an equal distance between the top of the leading edge and the bottom of the leading edge of the turbine blade. Across the width of the turbine blade five measurement points are used. These measurement points are equally distributed over the distance between the leading edge and the trailing edge of the turbine blade. Because the turbine blade is wider at the top than at the bottom the distance between the horizontal measurement points differ at each vertical position.

2.2 Measurements

In this section the setup and execution of the frequency response function measurements is treated. In order to correctly setup the measurement the input range, the frequency range and resolution, triggering, windowing, averaging and hammer tip selection are treated. The measurement of frequency response functions is treated briefly. References to a more elaborate treatment of the measurement of frequency response functions are provided.

2.2.1 Input Range

The accuracy of the measured amplitude information is primarily determined by a correct setting of the input range of the frequency analyser. The conversion of analogue amplitude information to digital amplitude information is based on the input range of the frequency analyser and the number of bits of the integrated analogue to digital converter (ADC). In principle the ADC divides the input range into a discrete number
of steps and assigns the nearest discrete value to a measured analogue value. The number of input range divisions is fixed and determined by the number of bits of the ADC. The HP frequency analyser has an integrated 16-bit ADC which means that the input range is divided into $2^{16}$ steps. The width of these steps is determined by the width of the input range.

The analogue to digital conversion inherently causes so called quantization errors. These errors occur because the measured discrete amplitude value is generally not equal to the actual analogue amplitude value. If the input range is correctly set quantization errors are minimized. If the input range is too large then unnecessarily large quantization errors occur. Consequently the amplitude resolution of the measured signals is unnecessarily coarse. If the input range is too small then overloading occurs. Overloading occurs when the actual signal amplitudes are larger than the maximum signal amplitude allowed by the input range. In principle signals with an amplitude above the maximum value are truncated and recorded as the maximum value. More information on analogue to digital conversion and the quantization of signals can be
found in [1] and [6].

The input range of the measured signals is set using the auto up feature of the HP signal analyzer. This ensures that the input range is set to the minimum value available while avoiding overloads. The input range has to be determined for every combination of excitation and measurement points.

### 2.2.2 Frequency Range and Resolution

The frequency range and resolution are related to the sampling parameters through Shannon's sampling theorem and Rayleigh's criterion. In order to select the proper settings for both the frequency range and frequency resolution these criteria have to be taken into account.

According to Shannon's sampling theorem the maximum frequency that can be accurately described in a digitized signal is equal to half the sampling frequency \( f_s \). This maximum describable frequency is also known as the Nyquist frequency \( f_{Nyq} \).

In practice this means that the sampling frequency should be chosen in such a way that it is at least twice the maximum frequency present in the signal \( f_{max} \). The following relation between the sampling frequency \( f_s \), the Nyquist frequency \( f_{Nyq} \), the maximum frequency \( f_{max} \), and the sampling time interval \( \Delta t \) can be defined.

\[
f_s = \frac{1}{\Delta t} = 2f_{Nyq} \geq 2 \cdot f_{max}
\]  

(2.1)

If equation (2.1) is not obeyed aliasing will occur. Aliasing refers to the effect that signals with frequencies above the Nyquist frequency \( f_{Nyq} \) inaccurately appear as signals with lower frequencies. In practical conditions an analog low pass filter is used to ensure that \( f_s \geq 2f_{max} \). Physical limits in the design of these low pass filters require that the factor between the sampling frequency and the maximum frequency is larger than 2.

Rayleigh's criterion sets a limit on the minimum distinguishable difference between two harmonic components. This limit is also known as the frequency resolution \( \Delta f \) and is related to the length of the measurement time record \( T \) as follows.

\[
\Delta f = \frac{1}{T}
\]

(2.2)

In order to obtain accurate estimates of the modal parameters i.e. accurate fits of the resonance peaks the following rule of thumb is applied. The frequency resolution \( \Delta f \) should be chosen in such a way that in the frequency domain at least three points are available within the 3 dB bandwidth of each resonance peak. The reasoning behind this rule of thumb is as follows. In a complex plot of the frequency response function resonance peaks appear as circles. In order to define a circle a minimum of three points is needed. Consequently a minimum of three points is needed in order to be able to fit a function to a resonance peak. The 3 dB bandwidth is the point at which the power of the resonance peak is halved. In general a resonance peak will have a relatively large influence on the frequency response function within the 3 dB bandwidth. In order to obtain accurate estimates of the modal parameters sufficient measurement
points should lie within the 3 dB bandwidth of a resonance peak. In principle the above rule of thumb defines the absolute minimum number of data points needed to make accurate estimates of the modal parameters. In general a larger number of points “closer” to the resonance peak is preferable as this will lead to better estimates of the modal parameters.

Equation (2.1) and (2.2) define relations between the sampling parameters in the time and frequency domains. On the HP dynamic signal analyser these relations are not directly used. The selection of appropriate sampling parameters is performed as follows. First a measurement bandwidth is defined. Then a number of spectral lines is chosen. The combination of the measurement bandwidth and the number of spectral lines determines the frequency resolution \( \Delta f \). Based on the measurement bandwidth and the number of spectral lines the HP dynamic signal analyser automatically selects the appropriate sampling parameters. This selection is based on the relations provided by equations (2.1) and (2.2).

Here preliminary experiments are used to select appropriate values for the maximum frequency range of interest and the number of spectral lines. The maximum measurable frequency of interest is determined to be 1600 Hz. Above this frequency it becomes difficult to obtain accurate estimates of the frequency response function. The reproducibility and the reciprocity of the measurement degenerate and it is difficult to properly excite the turbine blade above this frequency. The number of spectral lines is set equal to the maximum value of 1600 lines. The measurement bandwidth now determines the frequency resolution. Measuring the entire frequency range of interest at once leads to a measurement bandwidth of 1600 Hz and frequency resolution of 1.0 Hz. Measuring the frequency range of interest in two steps leads to a measurement bandwidth of 800 Hz and a frequency resolution of 0.5 Hz.

Figures 2.4 and 2.5 show preliminary frequency response function measurements around the first and second resonance peaks respectively. In these figures the solid line represents a measurement with a frequency resolution of 0.5 Hz while the dashed lines indicate a measurement with a frequency resolution of 1.0 Hz. The horizontal lines in these figures indicate the 3 dB bandwidth of the respective resonance peaks. Figure 2.4 shows that a frequency resolution of 1.0 Hz is not sufficient to measure the first resonance peak. A frequency resolution of 0.5 Hz is only just sufficient. A lower frequency resolution would be preferable but this is leads to too many discontinuities in the measured spectrum. Figure 2.5 shows that both frequency resolutions are sufficient to measure the second resonance peak. A resolution of 0.5 Hz is preferable as this leads to more points within the 3 dB bandwidth. Based on figures 2.4 and 2.5 a spectral resolution of 1600 lines and a measurement bandwidth of 800 Hz is chosen. Consequently the frequency response functions are measured in two steps.

2.2.3 Triggering

In order to completely measure and synchronize the excitation signal and the resulting response of the structure the inputs of the signal analyzer are triggered. Three important properties of the trigger are the triggering level, the triggering slope and the pre-trigger delay. The triggering level is set to a percentage of the maximum input
signal and determines the signal amplitude at which the measurements are started. The triggering slope is used to start the measurements on a growing or decaying signal. Because the measurements are started on a slope of the signal a pre-trigger delay is needed to ensure that the entire signal is captured. Here the measurements are triggered on the excitation signal at a channel level of 5 % and on a positive slope. The pre-trigger delay is set to 20 ms.

2.2.4 Windowing

In the computation of frequency response functions the discrete Fourier transform is used. This transform assumes that the measured signals are periodic with a period $T$ i.e. the period time is equal to the length of the measurement time record. In practice this is only the case if special precautions are taken. If the measured signals are not periodic leakage errors will occur. These errors lead to inadequate estimates of the frequency response function. To minimize the effect of leakage errors the measured signals are windowed. This means that before applying the Fourier transform the measured signals are multiplied with a so called window function. These window functions are defined in such a way that the resulting signal is periodic in the time domain. There are a large number of windowing functions available. Each of these windowing functions has certain advantages and disadvantages and should thereafter be applied with care. For impact testing two windowing functions are generally used. To the impact force signal a force window is applied and to the response signal an exponential window is applied. More information on windowing measured signals can be found in [1], [5] and [9].
Force Window

The force window (see figure 2.6) is applied to the impact force signal that is measured through the force sensor in the impact hammer. In general, this signal has an impulse-like form and a duration that is very small compared to the total measuring time. Consequently, leakage errors are generally not an issue with regard to the impact force signal. The main two reasons for applying the force window are as follows. Firstly, the force window is used to remove measurement noise that is present on the measured signal after the impact. Secondly, after the hammer impact, the force transducer in the hammer can give additional measurement signals due to noise, placing the hammer on a surface, swinging the hammer, etc. The force window is used to eliminate these additional forces from the measurement signal because they are not exerted on the structure and should therefore not be taken into account when calculating the frequency response function. As shown in figure 2.6, the force window has a value of one in a time band around the impulse signal and a value of zero elsewhere. This effectively cancels out all information that lies outside the width of the force window. It should be noted that the force window cannot be used to eliminate the effect of double hammer impacts because these impacts are exerted on the structure.

On the HP frequency analyser, the force window is set through the specification of the force width. Here the force width is set to 140 ms.

Exponential Window

The exponential window is shown in figure 2.7. The purpose of the exponential window is to make the measured response signal periodic thereby reducing the effect of leakage. The measured response signal is made periodic by bringing it to zero within the time record length. It is assumed that at the beginning of the time record, the measured response is zero. The application of an exponential window is of course only necessary if the response signal does not decay within the measurement time. Although the exponential window reduces the effect of leakage, it has the disadvantage...
that it introduces artificial damping in the measured signal. In the post processing phase of the modal analysis this artificial damping can be removed from the estimated modal damping. The exponential window should be applied with care because if too much artificial damping is added then it can become very difficult to estimate the actual damping level.

The exponential window is defined as follows:

$$w(t) = e^{-\frac{t}{\tau}} = e^{-\sigma_0 t}$$  \hspace{1cm} (2.3)

Where $\tau$ is known as the exponential time constant and $\sigma_0$ as the damping decay rate. In order to correct for the additional damping introduced by the exponential window both the unwindowed and windowed response signals are considered. Assuming that $n$ modes in the frequency range of interest are excited the unwindowed response signal $y(t)$ can be expressed as follows.

$$y(t) = \sum_{k=1}^{n} [R_k e^{-\sigma_k t} \sin(\omega_k t)]$$  \hspace{1cm} (2.4)

Where $\omega_k$ is the natural frequency, $\sigma_k$ is the damping decay rate and $R_k$ is a modal constant. The windowed response $y^*(t)$ is calculated from the unwindowed response through a multiplication with the exponential window function $w(t)$.

$$y^*(t) = w(t) y(t) = \sum_{k=1}^{n} [R_k e^{-\sigma_k t} \sin(\omega_k t)]$$  \hspace{1cm} (2.5)

The apparent modal damping $\sigma_k^*$ is defined as follows:

$$\sigma_k^* = \sigma_0 + \sigma_k$$  \hspace{1cm} (2.6)

The above equation simply states that the exponential window adds a constant amount of damping $\sigma_0$ to the actual modal damping $\sigma_k$. In order to correct modal damping estimates for the additional damping added through the exponential window the damping added by the window is subtracted from the measured damping.

2.2.5 Averaging

The accuracy of the measured frequency response functions is mainly determined by the measurement noise present in the system and by the skill of the person swinging the impact hammer. In order to improve the accuracy of the frequency response function measurements averaging is applied. The averaging process will reduce the effect of random errors but will have no effect on systematic errors like leakage. The averaging process used here is linear with no overlap. The linearity of the average implies that all measured frequency response functions are of equal importance when calculating the average. When averaging overlap can only be used if the measured signals are mutually
uncorrelated. As this is not the case for an impact test overlap cannot be used. More information on different averaging methods and their advantages and disadvantages can be found in [1].

On the HP frequency analyser a linear average with no overlap is specified. In order to keep the amount of measurements needed reasonable the average is calculated from five measurements. To ensure that inadequate measurements are not included into the average overload rejection and a manual average preview are specified.

2.2.6 Hammer Tip Selection

The selection of a hammer tip is a very important factor in an impact test as it is the only way to influence the shape of the input power spectrum. The input power spectrum is controlled by the length of the impact pulse. A long pulse in the time domain will result in a narrow frequency spectrum while a short pulse in the time domain results in a wide frequency spectrum [3]. The length of the impact pulse is determined by a combination of the stiffness of the hammer tip and the local stiffness of the structure. A soft hammer tip will produce a relatively long impact pulse and consequently a narrow frequency band is excited. Hard hammer tips produce a relatively short impact pulse and result in the excitation of a wide frequency range.

In order to properly excite the structure under test the input spectrum is required to have a sufficient amplitude and to be fairly even over the frequency range of concern. A hammer tip that is too soft will show a significant amount of roll-off of the input spectrum at frequencies of interest. This soft hammer tip will not produce enough excitation at higher frequencies. Consequently the structure will not respond to the excitation at higher frequencies and a low coherence at these frequencies can be expected. A hammer tip that is too hard will have an extremely flat input spectrum over the frequency range of interest. The use of this hammer tip will produce too much excitation at higher frequencies. Consequently eigenmodes above the frequency range of interest are excited. This causes a bad overall coherence especially near anti-resonances and at lower frequencies.

In order to select an appropriate hammer tip the following rule of thumb is used [3]. The structure is sufficiently excited if the roll-off of the input spectrum is around 20 dB over the frequency range of interest. For the Endevco impact hammer used here there are a rubber, a teflon and an aluminum hammer tip available. The input spectrum of the hammer on the turbine blade is shown in figure 2.8. In section 2.2.2 the maximum frequency range of interest is set to 1600 Hz. According to the above rule of thumb the teflon hammer tip is used in measuring the frequency response functions.

2.2.7 Frequency Response Function Estimation

The estimation of frequency response functions is an integral part in the experimental modal analysis approach followed here. It is also a process that is performed inside of the frequency analyser and is therefore "invisible" to the user. Here the basic ideas behind frequency response function estimation are presented. More information on
The basic idea behind frequency response function estimation is as follows. The input and output signals \( x(t) \) and \( y(t) \) are measured using a time sampling approach. To obtain frequency domain information from the sampled time signals \( x(t) \) and \( y(t) \) a fast fourier transform is applied. This leads to the transformed signals \( X(f) \) and \( Y(f) \). These frequency domain signals are used to calculate the auto power spectrum of the input and the cross power spectrum of the input and output.

\[
S_{xx}(f) = \frac{1}{T} X^*(f)X(f) \quad S_{xy}(f) = \frac{1}{T} X^*(f)Y(f)
\] (2.7)

Where * represents the complex conjugate and \( T \) the measurement time record length.

The so called \( H_1 \) frequency response function estimator can now be used to estimate the frequency response function.

\[
H_1(f) = \frac{S_{xy}(f)}{S_{xx}(f)}
\] (2.8)

In order to minimize the effects of measurement noise the estimated frequency response functions are averaged. The quality of the averaged frequency response function estimates can be evaluated using the coherence function which is defined as follows.

\[
\gamma_{xy}^2(f) = \frac{|S_{xy}(f)|^2}{S_{xx}(f)S_{yy}(f)}
\] (2.9)

The coherence function is a frequency dependent indicator that shows which part of the output \( y(t) \) is coming from the real input \( x(t) \) and which part is coming from the additional measurement noise \( n(t) \). If the coherence function has a value near one then a strong linear relation between input \( x(t) \) and output \( y(t) \) exists and the influence of
the measurement noise \( n(t) \) is negligible. If on the other hand the coherence function has a value near zero then no linear relation between the input \( x(t) \) and the output \( y(t) \) exists and the output spectrum is dominated by the measurement noise \( n(t) \). It has to be noted here that the coherence function \( \gamma_x^2(f) \) can only account for random errors. Bias errors do not influence the coherence function [1], [11].

### 2.3 Measurement Data Quality

The quality of the measured frequency response functions is important because this determines the quality of the estimated modal parameters. During the measurement process the quality of the measured frequency response functions is evaluated using the coherence function. As explained in section 2.2.7 the coherence function can only indicate the effects of random errors. The effects of bias errors do not show up in the coherence function. As a consequence additional checks are needed to ensure that the quality of the measured frequency response functions is sufficient to accurately estimate modal parameters.

#### 2.3.1 Driving Point Frequency Response Function

The first check of the quality of the measured frequency response function is made using the driving point frequency response function. The driving point frequency response function has specific properties that should be checked [3]. These properties are: all resonances should be separated by anti-resonances, phase differences larger than 180° cannot occur and the imaginary parts of the frequency response function should all have an identical sign.

Figure 2.9 shows the magnitude and phase of the driving point frequency response function. This figure shows that each resonance is followed by an anti-resonance. The figure also shows that phase differences larger than 180° do not occur. For frequencies
below approximately 25 Hz this is not true. The difference at low frequencies is caused by the inability of the sensor to measure at these low frequencies.

Figure 2.10 shows the real and imaginary part of the driving point frequency response function. The imaginary part of the driving point frequency response function has an identical sign as required.

2.3.2 Reproducibility and Reciprocity

The second check on the quality of the measured frequency response functions is made by evaluating the reproducibility and reciprocity of several measured frequency response functions.

The reproducibility of the measured frequency response functions is evaluated by measuring the same frequency response function multiple times. If a measured frequency response function is not reproducible this indicates that there is a problem somewhere in the experimental setup. The primary factors that influence the reproducibility of a measured frequency response function are: nonlinearities, insufficient clamping stiffness and inconsistencies in the measurement directions of excitation and response.

In theory Maxwell's reciprocity theorem $H_{xy}(f) = H_{yx}(f)$ should hold for all measured frequency response functions. In practice the reciprocity theorem will only be valid for relatively low frequencies. For higher frequencies the difference between $H_{xy}(f)$ and $H_{yx}(f)$ grows for increasing frequency. In impact tests the primary cause for deviations from the reciprocity condition is usually the attachment of the accelerometer to the structure. At higher frequencies the extra mass associated with the accelerometer has a relatively large influence on the local dynamics of the structure while at lower frequencies these effects are negligible.

Figures 2.11, 2.12 and 2.13 show the frequency response functions measured between the point 11 and the point 05, 43 and 81 respectively. The solid lines indicate the frequency response function measurements where the structure is excited at location 11 while the dashed lines indicate an excitation at the points 05, 43 and 81 (see figure 2.3) respectively. These figures show that in general the reproducibility and reciprocity of the measured frequency response functions is reasonably good. The differences between the measured frequency response functions are relatively small. Larger differences occur around resonances and anti-resonances and at lower frequencies. This is acceptable because at these points the estimates of the frequency response functions are not as reliable.
Figure 2.11: The frequency response functions $H_{11/05}(f)$ and $H_{05/11}(f)$ and the difference between both frequency response functions.

Figure 2.12: The frequency response functions $H_{11/43}(f)$ and $H_{43/11}(f)$ and the difference between both frequency response functions.

Figure 2.13: The frequency response functions $H_{11/81}(f)$ and $H_{81/11}(f)$ and the difference between both frequency response functions.
3 Modal Parameter Estimation

In this chapter the estimation of modal parameters from measured frequency response functions is treated. This estimation process is divided into three separate steps. First, the number of modes in the frequency band of interest is estimated. Then a modal parameter estimation method is chosen and the modal parameters and their associated mode shapes are calculated. These calculations result in a number of alternative estimates. From these estimates an appropriate modal parameter estimate is selected by using the MAC and COMAC criteria.

3.1 Determining The Number Of Modes

There are numerous methods that can be used to determine the number of modes present in measured frequency response functions. Here only two basic methods will be discussed.

The first method used to determine the number of modes in the frequency band of interest is to overlay all the measured frequency response functions and count the number of peaks. The idea behind this method is that each eigenmode will appear as a resonance peak in all measured frequency response functions. An exception to this rule are frequency response functions that are measured at nodal points of a specific mode. In these frequency response functions eigenmodes that have a nodal point at the excitation or measurement location will not appear. Figure 3.1 shows an overlayed plot of all measured frequency response functions. In this figure three dominant resonance peaks around approximately 250, 450 and 990 Hz can be identified. Figure 3.1 also shows a less prominent resonance peak around 1200 Hz. Around 850, 930 Hz possible resonance peaks are present but these are not distinct enough to identify them as resonance peaks.

The second method used to identify the presence of eigenmodes in the frequency band of interest is the use of a mode indicator function. There are numerous mode indicator functions available each with their own applications, advantages and disadvantages. More information on mode indicator functions can be found in [1], [12] and [8]. Here only the complex mode indicator function is discussed. The theory behind this mode indicator function is treated in appendix C. In this section only the results of applying this mode indicator function are discussed. The complex mode indicator functions is shown in figure 3.2. Based on the complex mode indicator function six eigenmodes are identified. Three dominant modes appear around approximately 250, 450 and 990 Hz.
Around approximately 1200 Hz a less dominant mode is shown. Around 850 and 930 Hz two very small peaks are shown. These peaks indicate mode shapes that are almost indistinguishable from the measured data. Based on the results obtained from overlaying the frequency response functions and the evaluation of the mode indicator function six eigenmodes are identified. Three of these modes are dominant modes as they appear clearly in both the measured frequency response functions and the mode indicator function. One mode is less dominant but still clearly appears as an eigenmode. Two modes are almost indistinguishable from the measured data but they are nevertheless taken into consideration. An explanation for their weak appearance cannot be formulated based on figures 3.1 and 3.2. The most likely reasons behind the weak appearance of both modes are: the modes are physically less relevant, the modes were insufficiently excited and/or the modes were incompletely measured.

### 3.2 Estimation Method

The selection of a modal parameter estimation method is an important step in the estimation of modal parameters. The modal parameter estimation problem is over determined i.e. there are many more data points than parameters available. Consequently the differences in the implementation of the different estimation algorithms result in differences in the modal parameter estimates. This makes the selection of an appropriate estimation method an important factor in obtaining accurate modal parameter estimates. The selection of an appropriate modal parameter estimation method is primarily based on the properties and the availability of the different estimation methods. The following three properties of a modal parameter estimation method can be used to distinguish between different methods. First is the method a single degree of freedom (SDOF) or a multiple degree of freedom method (MDOF), second is the method a local or a global method and third is the method a time or a frequency
The difference between SDOF and MDOF methods is in the assumptions made in the interaction between the different eigenmodes. SDOF methods assume that each eigenmode can be viewed as a single degree of freedom system and consequently there is no interaction between the different eigenmodes. A MDOF method is based on the assumption that each resonance peak in the measured frequency response functions can be viewed as the summed contribution of a number of modes in a particular frequency band. The usability of a SDOF or MDOF method is based on the amount of overlap that exists between different modes. If eigenmodes are well separated in frequency and the damping is relatively low then a SDOF method can be used. If eigenmodes are close together and/or a lot of damping is present then a MDOF method should be used. It should also be noted that in general more frequency values are taken into account in a MDOF method which generally leads to statistically better modal parameter estimates.

Local and global estimation methods differ in the number of measured frequency response functions that is taken into account when estimating modal parameters. A local estimation method considers each measured frequency response function separately leading to as many estimates of a particular resonance frequency and damping value as there are measured frequency response functions. Global methods use a formulation where all frequency response functions are considered simultaneously. This leads to one estimate of each resonance frequency and damping value. In general global methods deliver superior results when compared to local methods. On the other hand, the use of a global estimation method requires measurement data of reasonably high quality as global methods are sensitive to small variations in the data caused by e.g. mass loading effects and changes in temperature [3].

The difference between time domain and frequency domain methods is in the formulation of the underlying mathematical equations. Time domain methods are based on a model formulation in the time domain while frequency domain methods are based on a model formulation in the frequency domain. In principle there is no difference between time domain or frequency domain methods as the underlying mathematical equations are equivalent. In theory time domain and frequency domain methods should deliver identical estimation results. In practice time and frequency domain methods will deliver different results because the estimation problem is over determined.

MEScope provides estimation methods in all of the categories defined above [17]. Here a MDOF global frequency domain method known as the global polynomial method (see appendix D) is chosen. The reasons behind this choice are as follows. The separation between a number of eigenmodes is poor so a MDOF method should be used. The quality of the measurement data does not require the use of a local method so a global method which should lead to better results is used. MEScope provides two MDOF global estimation methods: the global complex exponential method and the global polynomial method. The global complex exponential method is a time domain method while the global polynomial method is a frequency domain method. According to [17] the global complex exponential method works best on wider frequency bands containing a relatively large number of modes while the global polynomial method works best on smaller frequency bands containing a relatively small number of modes.
The quality of the measurement data makes the correct application of the global complex exponential method difficult. The global polynomial method is chosen as this should lead to better results.

### 3.3 Modal Parameter Estimates

The estimation of modal parameters with the global polynomial method is performed using MEScope. During the estimation process all measured frequency response functions are taken into consideration. The global polynomial method delivers the best results if the measurement data is divided into relatively small frequency bands. Here the best results are obtained if three separate frequency bands are used. In each of these frequency bands separate modal parameter fits are performed. The first fit of modal parameters is performed on the first two resonance peaks shown in figure 3.1. In the second fit the modal parameters of the third, fourth and fifth resonance peaks are fitted. Finally the third fit is performed on the sixth resonance peak. During each of these fits the complex mode indicator function is used to weigh the measurement data around each resonance peak. To account for the effect of eigenmodes that lie outside the measurement bandwidth additional polynomial terms are used. In order to get the best possible estimates of the modal parameters each of the three fits mentioned above is performed using four, five and six additional polynomial terms. Due to limitations on the available computer memory the use of more than six additional polynomial terms is not possible. This leads to three estimates for different modal parameters and their associated mode shapes. The estimated modal parameters are shown in table 3.1.

In order to select the appropriate modal parameter estimate from the estimates presented in table 3.1 the different mode shapes are compared using MAC and COMAC matrices. More information on the calculation of MAC and COMAC matrices can be found in [2], [5] and [8].

The MAC matrix is a measure for the correlation between mode shapes. Values of one indicate a perfect correlation between mode shapes while values of zero indicate no correlation between mode shapes. The MAC matrices between the modal parameter estimates are shown in figures 3.3, 3.4 and 3.5. These figures indicate that the

<table>
<thead>
<tr>
<th>Mode Shape</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Freq. [Hz]</td>
<td>Damp. [%]</td>
<td>Freq. [Hz]</td>
</tr>
<tr>
<td>1</td>
<td>251.00</td>
<td>0.173790</td>
<td>251.13</td>
</tr>
<tr>
<td>2</td>
<td>453.97</td>
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<td>4</td>
<td>929.46</td>
<td>0.491210</td>
<td>930.13</td>
</tr>
<tr>
<td>5</td>
<td>987.65</td>
<td>0.058887</td>
<td>987.68</td>
</tr>
<tr>
<td>6</td>
<td>1202.70</td>
<td>0.374370</td>
<td>1202.80</td>
</tr>
</tbody>
</table>

Table 3.1: The estimated modal frequencies and damping using four, five and six additional polynomial terms.
Figure 3.3: MAC matrix for modal estimates using four and five additional polynomial terms.

Figure 3.4: MAC matrix for modal estimates using four and six additional polynomial terms.

Figure 3.5: MAC matrix for modal estimates using five and six additional polynomial terms.
correlation between the mode shapes estimated using four and five and four and six additional polynomial terms is not perfect i.e. the MAC has a value lower than one on its diagonal. The correlation between the mode shapes estimated using five and six additional polynomial terms is nearly perfect. This indicates that the inclusion of five additional polynomial terms should be sufficient to calculate all mode shapes and account for the effects of out of band modes.

The COMAC matrix gives a measure for the correlation between the degrees of freedom in two mode shape estimates. As with the MAC matrix values of one indicate a perfect correlation between coordinates while values of zero indicate that no correlation is present. The COMAC matrices calculated between mode shapes estimated using four five and six additional polynomial terms are shown in figures 3.6, 3.7 and 3.8. These COMAC matrices show that there is an almost perfect correlation between the contributions from the degrees of freedom when five and six additional polynomial terms are used. This indicates that again an estimate using five additional polynomial terms is sufficient to describe all mode shapes and account for the effects of out of band modes.

Both the MAC and COMAC matrices show that the modal parameter estimate using five additional polynomial terms sufficiently describes the modal parameters and their associated mode shapes. The estimated modal parameters are shown in table 3.1 and their associated mode shapes are presented in chapter 4.
Figure 3.6: COMAC matrix for modal estimates using four and five additional polynomial terms.

Figure 3.7: COMAC matrix for modal estimates using four and six additional polynomial terms.

Figure 3.8: COMAC matrix for modal estimates using five and six additional polynomial terms.
4 Evaluation

In this chapter the estimated modal parameters and their associated mode shapes are evaluated. This evaluation is performed in three steps. First the estimated modal parameters themselves are considered. Second the estimated modal parameters are compared to theoretical values. Finally the estimated modal parameters are compared to values obtained from a FEM analysis.

4.1 Estimated Modal Parameters

In this section the estimated modal parameters and their associated mode shapes are analysed. This analysis is performed in three steps. First the mode shapes are analysed, second the MAC values associated with the mode shapes are used to investigate the interaction and similarities between the different modes and finally synthesized frequency response functions are compared to measured frequency response functions.

4.1.1 Mode Shapes

The estimated modal parameters and their associate mode shapes are shown in table 4.1 and figures 4.1, 4.2, 4.3, 4.4, 4.5 and 4.6 respectively. Appendix E contains figures that show a more detailed quad view of the estimated mode shapes.

Figure 4.1 shows the first mode shape. This mode shape is the first bending mode of the turbine blade. It is associated with the resonance peak at 251 Hz. At 454 Hz the second mode shape is found. This mode shape corresponds to the first torsion mode of the turbine blade. The third mode shape at 849 Hz is shown in figure 4.3. This

<table>
<thead>
<tr>
<th>Shape</th>
<th>Freq. [Hz]</th>
<th>Damp. [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>251,13</td>
<td>0,217500</td>
</tr>
<tr>
<td>2</td>
<td>453,97</td>
<td>0,323950</td>
</tr>
<tr>
<td>3</td>
<td>849,02</td>
<td>0,500560</td>
</tr>
<tr>
<td>4</td>
<td>930,13</td>
<td>0,568920</td>
</tr>
<tr>
<td>5</td>
<td>987,68</td>
<td>0,65064</td>
</tr>
<tr>
<td>6</td>
<td>1202,80</td>
<td>0,426790</td>
</tr>
</tbody>
</table>

Table 4.1: The estimated modal frequencies and modal damping.
Figure 4.1: The first mode shape at 251 Hz.

Figure 4.2: The second mode shape at 454 Hz.

Figure 4.3: The third mode shape at 849 Hz.

Figure 4.4: The fourth mode shape at 930 Hz.

Figure 4.5: The fifth mode shape at 988 Hz.

Figure 4.6: The sixth mode shape at 1203 Hz.
mode shape appears to be the second bending mode of the turbine blade. The top left corner appears to have the wrong phase. This is probably due to measurement error. The fourth mode shape at 930 Hz is shown in figure 4.4. This mode shape appears to be a combination of a bending and torsion mode. The fifth mode shape is shown in figure 4.5 and is situated at 988 Hz. This mode shape appears to be related to some kind of flapping motion of the turbine blade. Figure 4.6 shows the sixth mode shape at 1203 Hz. This mode shape is similar to the second bending mode of the turbine blade. The similarity between mode shapes three and six indicates that one of these mode shapes is probably incompletely measured.

4.1.2 MAC Values

In order to obtain an indication of the quality of the fitted mode shapes the auto MAC is calculated. The calculation of the auto MAC is essentially identical to the calculation of the MAC with the exception that the auto MAC correlates the estimated mode shapes with themselves. A graphical representation of the auto MAC matrix is shown in figure 4.7. This figure shows that all the values on the diagonal of the MAC matrix are equal to one. This is expected as the mode shapes will of course correlate perfectly with themselves. All the off diagonal values of the MAC matrix are smaller than 0.9 which indicates that although there is a correlation between the estimated mode shapes all mode shapes are essentially different. Figure 4.7 shows that there is a correlation between mode shapes one and two and between mode shapes three and six. This correlation indicates that these mode shapes are somehow related. The correlation between the estimated mode shapes indicates that the measured degrees of freedom are not adequate to clearly distinguish the mode shapes from each other. Consequently either the number of measured degrees of
freedom is insufficient or the locations of the measured degrees of freedom are chosen incorrectly. Mode shapes one and two are shown in figures 4.1 and 4.2. These figures do not show a clear relationship between mode shapes one and two. Mode shapes three and six are shown in figures 4.3 and 4.6. These figures show that both mode shapes are very similar which explains the correlation between these mode shapes. The auto MAC shows that mode shapes four and five correlate with almost every other mode shape. This indicates that the information available in the measured data is not sufficient to clearly discriminate mode shapes four and five from the other mode shapes. For mode shape four the reason behind this becomes clear if figure 4.4 is compared to figure 2.3. The excitation point is located very close to nodal lines of the mode shape. This means that during the frequency response function measurements the fourth mode shape is not properly excited and can therefore not be accurately estimated. This conclusion is supported by the frequency response functions shown in figure 3.1. These frequency response functions do not show a clear resonance peak for mode number four. This reasoning does not hold for mode shape five as for this mode shape the frequency response functions shows a clear resonance peak. The most probable explanation for the correlation of the fifth mode shape with the other mode shapes is that this mode shape is incompletely measured. Due to the characteristics of the accelerometer and the geometry of the turbine blade all the measured degrees of freedom are taken perpendicular to the turbine blade surface. Motions parallel to the turbine blade surface are not measured and are therefore not taken into account when estimating the modal parameters and their associated mode shapes. If the motion of the fifth mode shape primarily takes place in a direction parallel to the turbine blade surface this motion is not measured which results in an incomplete estimate of the fifth mode shape.

4.1.3 Synthesized Frequency Response Functions

In order to evaluate the fitted modal parameters synthesized frequency response functions are compared to measured frequency response functions. The synthesized frequency response functions are calculated using MEScope. This calculation is based on the estimated modal parameters and the mode shapes associated to these modal parameters. Here frequency response functions are generated for the measurement points 05, 43 and 85 (see figure 2.3). The measured and synthesized frequency response functions are shown in figures 4.8, 4.9 and 4.10. Figures 4.8, 4.9 and 4.10 show that for all three considered points the synthesized frequency response functions approximate the measured frequency response functions around the resonance peaks. The quality of the fit decreases with an increase in the distance from a resonance peak. This indicates that the effect of out of band modes is not negligible and should be taken into consideration. During the estimation of modal parameters out of band modes are accounted for by the inclusion of additional polynomial terms. These additional terms are not included in the synthesized frequency response functions which explains the relatively poor fit outside the vicinity of resonance peaks.
Figure 4.8: The measured (solid) and synthesized (dashed) frequency response function for point 05.

Figure 4.9: The measured (solid) and synthesized (dashed) frequency response function for point 43.

Figure 4.10: The measured (solid) and synthesized (dashed) frequency response function for point 85.
Figures 4.8, 4.9 and 4.10 show that the amplitudes of the synthesized frequency response functions are larger than the amplitudes of the measured frequency response functions. This is caused by the use of the exponential window during the measurement process. Effectively the use of the exponential window increases the amount of measured damping. In the process of estimating the modal parameters this additional damping added by the exponential window is removed from the modal parameter estimates. The corrected modal parameters are then used to synthesize the frequency response functions. This leads to synthesized frequency response functions that have a lower damping level than the measured frequency response functions. This lower damping level causes higher resonance peaks in the synthesized frequency response functions.

Figures 4.8, 4.9 and 4.10 show that the first and second resonance peaks are represented relatively well. The third resonance peak is fitted reasonably well in the frequency response functions corresponding to points 43 and 85. The fit for this resonance peak is poorer for point 05. This is expected as the third resonance peak only appears weakly in the measured frequency response function. The fourth resonance peak is poorly fitted for points 43 and 85. Again this is expected as the fourth resonance peak appears very weakly in the measured frequency response functions. The fifth and sixth resonance peak are estimated relatively well. The differences between the measured and synthesized frequency response functions are larger for the fifth and sixth resonance peaks than the differences for the first two resonance peaks. This is not unusual as the error is expected to increase for higher frequencies.

4.2 Theoretical Mode Shapes

In this section the estimated modal parameters are compared to theoretical values obtained from Euler-Bernoulli beam models. This comparison is made for two reasons. First the Euler-Bernoulli beam models can possibly be used to predict the natural frequencies of a modified turbine blade. Second the Euler-Bernoulli beam models can possibly provide a coupling between the measured modal parameters and the finite element model. The comparison between estimated modal parameters and the Euler-Bernoulli beam model is made in two steps. First an Euler-Bernoulli cantilever beam model with a tip mass is compared to the estimated modal parameters. This should give an indication of the validity of the Euler-Bernoulli beam model and its ability to predict natural frequencies of the turbine blade. Second an Euler-Bernoulli cantilever beam model without a tip mass is compared to the beam model with tip mass. This comparison is made to evaluate the effect of removing the tip shroud on the natural frequencies of the turbine blade.

The derivation of the different beam models is presented in appendix F. For both beam models the natural frequency of the r-th mode can be written as:

\[ f_r = \frac{\omega_r}{2\pi} = (\beta_r L)^2 \sqrt{\frac{EI}{m_{\text{blade}} L^4}} \]  

(4.1)

In this equation \( \omega_r \) is the natural frequency of the r-th mode in \([\text{rad/s}]\), \( E \) is the modulus
of elasticity in \[ \frac{N}{m^2} \], \( I \) is the second area moment of inertia in \( m^4 \), \( m_{blade} \) the mass per unit length of the turbine blade \( \frac{kg}{m} \) and \( L \) the length of the turbine blade in \( m \).

In order to calculate the second area moment of inertia the cross section of the blade is assumed to be rectangular. The width \( b \) of the cross section is selected to be the chord length of the turbine blade, the height \( h \) of the turbine blade is taken as the maximal thickness of the turbine blade. The different physical parameters used in the Euler-Bernoulli beam model are presented in table 4.2.

The factor \( \frac{P}{L} \) is calculated from the characteristic equations (F.11) and (F.13) presented in appendix F. The Euler-Bernoulli beam model can only deliver estimates of the natural frequencies of the bending modes of the turbine blade. These estimates are somewhat crude and should not be expected to perfectly comply with the measured natural frequencies. At best the Euler-Bernoulli beam model provides an indication of the order of magnitude of the natural frequencies of the bending modes of the turbine blade. Differences between the measured and calculated natural frequencies have two main causes. First differences occur because the assumptions made in deriving the Euler-Bernoulli beam models do not comply with the physical reality of the turbine blade. Second differences occur because of the difficulty of representing the complex geometrical shape of the turbine blade in the second area moment of inertia \( I \).

The measured and calculated natural frequencies of the turbine blade are presented in table 4.3. The measured first natural frequency and the calculated first natural frequency are almost identical. The measured second natural frequency and the calculated second natural frequency differ substantially. This indicates that the Euler-Bernoulli does not deliver accurate predictions of the natural frequencies of the turbine blades. The good result obtained for the first natural frequency is more likely a coincidence than a physically relevant result. This becomes clear if equation (4.1) and the characteristic equation (F.13) are considered. Equation (4.1) shows that the natural frequencies are in principle determined by the characteristic equation through the factor \( (\beta, L)^2 \). The factor \( \sqrt{\frac{EI}{m_{blade} L^2}} \) can be seen as a constant scaling factor whose value depends on the material properties. In this scaling factor the modulus of elasticity \( E \), the blade mass per unit length \( m_{blade} \) and the blade length \( L \) can be determined with reasonable accuracy. The same is not true for the second moment of inertia \( I \). Due to

<table>
<thead>
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<th>Property</th>
<th>Value</th>
<th>Unit</th>
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<tbody>
<tr>
<td>( E )</td>
<td>200.6 \cdot 10^3</td>
<td>( \frac{N}{m^2} )</td>
</tr>
<tr>
<td>( b )</td>
<td>121.1 \cdot 10^{-3}</td>
<td>( m )</td>
</tr>
<tr>
<td>( h )</td>
<td>30.9 \cdot 10^{-3}</td>
<td>( m )</td>
</tr>
<tr>
<td>( L )</td>
<td>294.66 \cdot 10^{-3}</td>
<td>( m )</td>
</tr>
<tr>
<td>( I )</td>
<td>2.9774 \cdot 10^{-7}</td>
<td>( m^4 )</td>
</tr>
<tr>
<td>( m_{tip} )</td>
<td>677.77 \cdot 10^{-3}</td>
<td>( kg )</td>
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<td>( m_{blade} )</td>
<td>29,9236</td>
<td>( kg )</td>
</tr>
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</table>

Table 4.2: Physical properties of the Euler-Bernoulli beam.

<table>
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<th></th>
<th>Measured [Hz]</th>
<th>Calculated [Hz]</th>
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<tr>
<td>Tip Mass</td>
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<td>252</td>
</tr>
<tr>
<td>No Tip Mass</td>
<td>1203</td>
<td>1619</td>
</tr>
</tbody>
</table>

Table 4.3: The measured and calculated natural frequencies.
the complex geometry of the turbine blade only a rough approximation of the value of $I$ can be made. That this value leads to a correct approximation of the first natural frequency does not indicate that the Euler-Bernoulli beam model is correct. It only indicates that the scaling factor is chosen "correctly". Another indication that the Euler-Bernoulli beam is only a very rough approximation of the natural frequencies of the turbine blade is the factor between the first and second natural frequencies. For the measured natural frequencies the factor between the first and second bending frequency is approximately 4.8 while for the Euler-Bernoulli beam model with a tip mass this factor is approximately 6.4. This indicates that the Euler-Bernoulli beam model is not an accurate approximation of the turbine blade bending mode behavior. Comparing the Euler-Bernoulli cantilever beam with tip mass with the Euler-Bernoulli cantilever beam without tip mass shows that removing the tip shroud will increase the natural frequency. The first natural frequency increases approximately 14.3 percent while the second natural frequency increases approximately 11.5 percent. This increase in natural frequency is expected as the removal of the tip shroud means that the mass of the turbine blade decreases. Because the natural frequency is proportional to the square root of the fraction between stiffness and mass a decrease of mass will lead to an increase in natural frequency.

4.3 Preliminary FEM Results

In this section the preliminary results from a finite element model are discussed. A complete comparison between the measured data and the data obtained from the finite element model cannot be made. The reason behind this is that the measured data is obtained from a turbine blade with a tip shroud while the finite element model is based on a turbine blade without a tip shroud. Consequently a direct comparison between measured and calculated data is not possible and some form of translation is needed. This section is divided into three parts. First the finite element model is discussed. Second the suitability of an Euler-Bernoulli beam model to provide the translation between measured and calculated data is discussed. Third the natural frequencies and mode shapes obtained from the finite element model are discussed.

The finite element model used here [10] is developed at the National Aerospace Laboratory NLR. The modeling and post-processing is performed using MSC Patran 2004. The finite element calculations are performed using MSC Marc 2003. The geometric model of the turbine blade is provided by Sulzer Elbar B.V. in the form of a Unigraphics Parasolid model. The finite element model generated from this Parasolid model has 56633 nodes and 33455 elements. Tetrahedral 10 node elements are used and the meshing of the Parasolid model is performed with the auto meshing features of MSC patran. The boundary conditions of the finite element model are based on the mechanical turbine blade clamp shown in figure 2.2. In the finite element model the clamp is simulated by fixing all contact points in all directions.

The Euler-Bernoulli beam model is not suited to provide the translation between measured and calculated modal parameters. In section 4.2 it is shown that the Euler-Bernoulli beam model cannot be used to predict the natural frequencies of the turbine
blade with a tip shroud. The preliminary frequency response function measurement in figure 4.11 shows that the first resonance peak of the turbine blade without tip shroud is measured at approximately 318 Hz. Comparing this value to the first natural frequency of the turbine blade with tip shroud leads to a measured increase in natural frequency of approximately 27 percent. The Euler-Bernoulli beam models presented in section 4.2 predict an increase of approximately 14.3 percent for the first natural frequency. The predicted increase and the measured increase differ significantly which indicates that the Euler-Bernoulli model is not adequate to translate the measured model parameters to the calculated finite element results.

Figure 4.11 shows the driving point frequency response function measured on the modified turbine blade i.e. the turbine blade without tip shroud. In this figure the estimates of the natural frequencies obtained from the finite element model are indicated by the vertical lines. Figures 4.12, 4.13 and 4.14 show the first, second and third eigenmode obtained from the finite element model. Figure 4.11 shows that the first natural frequency measured without tip shroud and the first natural frequency obtained from the finite element model correspond very well. A qualitative comparison of figures 4.1 and 4.12 shows that the mode shape measured with tip shroud and the mode shape calculated from the finite element model (without tip shroud) are very similar. This is expected as the removal of the tip shroud is expected to increase the natural frequency the eigenmodes while it is unlikely that the general shape of the eigenmodes is significantly altered. The similarity between the natural frequencies and mode shapes indicates that there is a high probability that the first mode shape calculated from the finite element model (without tip shroud) approximates the first mode shape of the turbine blade without tip shroud.

The second eigenmode calculated from the finite element model does not correspond to any of the measured resonance peaks shown in figure 4.11. Figure 4.13 shows that the
second eigenmode obtained from the finite element model primarily corresponds to a motion in a direction parallel to the turbine blade. In this direction no measurements are performed and therefore the eigenmode obtained from the finite element model does not correspond to any of the measured eigenmodes.

The third eigenmode obtained from the finite element model is shown in figure 4.14. This eigenmode corresponds to the first torsion mode of the turbine blade. Figure 4.11 shows that this eigenmode lies close to a resonance peak in the measured driving point frequency response function. Although this may seem a good reason to conclude that the measured resonance peak and the calculated mode shape belong together it is almost certain that this is not the case. The reason behind this is as follows. The removal of the turbine blade tip shroud will cause shifts in natural frequencies of the turbine blade but it is unlikely that the general shape of each of the eigenmodes...
will be significantly altered. The first torsion mode of the turbine blade with tip shroud corresponds to the second resonance peak in the measured frequency response functions. Consequently it is very likely that for the modified turbine blade the second resonance peak will also correspond to the first torsion mode. According to figure 4.11 the second resonance peak of the modified turbine blade lies at approximately 530 Hz which is about 40 percent of the frequency predicted by the finite element model. It has to be noted here that a definite conclusion on the mode shapes corresponding to each resonance peak can only be made after a complete experimental modal analysis is performed.

The mode shapes calculated using the finite element model do not correspond to the approximate natural frequencies obtained from the preliminary measured frequency response function. The first mode shape obtained from the finite element model might be correct but the second and third mode shapes are probably not correct. The three most likely sources of the deviations between the measured and calculated mode shapes are the following.

The first source of deviations is the modeling of the turbine blade geometry. The turbine blade has a complex internal geometry which makes the creation of an accurate solid model of the turbine blade difficult. The finite element model is based on the solid model of the turbine blade and consequently the accuracy of the finite element model is strongly influenced by the accuracy of the solid model.

The second source of deviations is the clamping of the turbine blade. In the finite element model the clamping of the turbine blade is simulated by setting the displacement of certain nodal points equal to zero. Implicitly this assumes that the clamping of the turbine blade is infinitely stiff. In practice the turbine blade is clamped using the tool described in section 2.1.2. The force of the clamping of the turbine blade depends on the pressure of the hydraulic cylinder. This clamp is not infinitely stiff and preliminary experiments show that the pressure of the hydraulic cylinder has a large influence on the measured frequency response functions.

The third source of deviations is the application of the accelerometer to measure the frequency response functions. The mass of the accelerometer has an influence on the local dynamics of the turbine blade. The mass loading by the accelerometer causes inconsistencies in the measured frequency response functions. These inconsistencies cause deviations in the estimates of the modal parameters and the mode shapes. The finite element model is based on an ideal situation without mass loading effects which causes differences between measured and calculated results.
5 Conclusion and Recommendations

The modal analysis performed in this report provides an introduction into the subject to Sulzer Elbar B.V. Through the performance of the modal analysis the current measurement setup available at Sulzer Elbar B.V. has been evaluated. The performed measurements show that the current measurement setup can be used to measure frequency response functions. The accuracy of the measured frequency response functions can be improved in a number of ways.

- The design of the turbine blade clamping device needs to be reconsidered. The construction of the turbine blade clamping device is statically over determined which makes a reproducible clamping of the turbine blade difficult. The pressure of the hydraulic cylinder cannot be accurately controlled which is problematic as this pressure has an influence on the measured natural frequencies.

- The application of a non-contact measurement approach makes measuring the response of the concave side of the turbine blade possible. As a non-contact method does not influence the local dynamics of the turbine blade this should also lead to a more accurate response measurement and an increase of the measurable frequency range. Consequently the accuracy of the measured frequency response functions will increase.

- The excitation of the turbine blade using an electrodynamic shaker gives more control over the input spectrum. This also makes the excitation of the turbine blade at relatively high frequencies above 1600 Hz possible. In general this leads to a more accurate frequency response function measurement.

- The minimum measurable frequency resolution is only just sufficient to measure the first resonance peak in the frequency response function. Measurements performed with a smaller frequency resolution lead to a more accurate frequency response function measurement which increases the accuracy of the estimated modal parameters.

The modal properties of the second stage ABB 13E2 turbine blade have been determined. An evaluation of the modal properties is performed. This evaluation shows that a number of eigenmodes cannot be clearly identified. In order to make the identification of these mode shapes possible the following improvements are needed.
• The excitation point should be chosen at a more suitable location that does not lie close to a nodal line of any of the modes of interest.

• The turbine blade is only excited and the response is only measured in directions parallel to the turbine blade surface. This leads to incomplete measurements of the mode shapes of the turbine blade. Exciting the turbine blade and measuring the response of the turbine blade in multiple independent directions should lead to improvements.

The validation of the finite element model is not completed. The reason behind this is twofold. First due to time constraints insufficient experimental data on the modified turbine blade was gathered. Second the data available from the developed finite element model is not sufficient to perform a detailed comparison of the calculated and "measured" mode shapes.
## A Measuring Equipment Characteristics

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Scale Range</td>
<td>445 $N$</td>
</tr>
<tr>
<td>Maximum Frequency Range</td>
<td>8 $kHz$</td>
</tr>
<tr>
<td>Resonance Frequency</td>
<td>50 $kHz$</td>
</tr>
<tr>
<td>Sensitivity</td>
<td>11.08 $mV/N$</td>
</tr>
<tr>
<td>Head Mass</td>
<td>100 $g$</td>
</tr>
</tbody>
</table>

Table A.1: Impact hammer specifications.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>±50 $g$</td>
</tr>
<tr>
<td>Sensitivity</td>
<td>102 $mV/N$</td>
</tr>
<tr>
<td>Resonance Frequency</td>
<td>20 $kHz$</td>
</tr>
<tr>
<td>Mass</td>
<td>4.3 $g$</td>
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</tbody>
</table>

Table A.2: Accelerometer specifications.

<table>
<thead>
<tr>
<th>Specifications</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency Range</td>
<td>51.2 $kHz$</td>
</tr>
<tr>
<td>Dynamic Range</td>
<td>90 $dB$</td>
</tr>
<tr>
<td>Accuracy</td>
<td>±0.15 $dB$</td>
</tr>
<tr>
<td>Chan. Match Amp.</td>
<td>±0.04 $dB$</td>
</tr>
<tr>
<td>Chan. Match Phase</td>
<td>±0.5 $°$</td>
</tr>
<tr>
<td>Maximum Resolution</td>
<td>1600 lines</td>
</tr>
</tbody>
</table>

Table A.3: Signal analyser specifications.
B Frequency Response Function Estimation

The estimation of frequency response functions is one of the most important steps in performing a modal analysis. The quality of the estimated modal parameters depends strongly on the quality of the measured frequency response functions. In this section estimators for the frequency response function are derived. The derivation of these estimators is divided in the following three steps. First a measurement model is adopted. Based on this measurement model expressions for the frequency response function as a function of the input and output signals are formulated. Finally the influence of measurement noise is investigated which results in three frequency response function estimators.

B.1 Measurement Model

In order to derive estimators for the frequency response function a measurement model has to be adopted. This measurement model conveys assumptions made in deriving frequency response function estimators. The measurement model used here is shown in figure B.1. In this measurement model the following assumptions are made. The measured system is assumed to be linear and time invariant. The system is assumed to be completely described by its (unknown) impulse response function \( h(t) \). Consequently the system is also completely described by its (unknown) frequency response function \( H(f) \). The signals \( u(t) \) and \( v(t) \) represent the actual input and output signals while the signals \( x(t) \) and \( y(t) \) represent the measured input and output signals. The measured input and output signals follow from the actual input and output signals.

\[
\begin{align*}
\text{u(t)} & \quad \text{h(t)} & \quad \text{v(t)} \\
\text{m(t)} & \quad \text{x(t)} & \quad \text{n(t)} & \quad \text{y(t)}
\end{align*}
\]

Figure B.1: The measurement model used in deriving FRF estimators.
by adding the measurement noise \( m(t) \) and \( n(t) \) respectively. The measurement noise \( m(t) \) and \( n(t) \) is assumed to be uncorrelated to each other and to the actual input and output signals. It is also assumed that the measurement noise does not pass through the measured system.

### B.2 Frequency Response Function

In order to find relations between the frequency response function and the input and output signals the system response in the absence of measurement noise is analyzed. In the absence of measurement noise the measured input and output signals \( x(t) \) and \( y(t) \) are identical to the actual input and output signals \( u(t) \) and \( v(t) \).

For linear time-invariant systems the the output \( v(t) \) of the system is related to its input \( u(t) \) by a convolution with the impulse response function \( h(t) \).

\[
v(t) = h(t) \ast u(t)
\]

\[
v(t) = \int_{-\infty}^{\infty} h(\sigma)u(t-\sigma)d\sigma
\]  

In this equation \( \ast \) denotes the convolution integral between two signals. Although equation (B.1) directly relates the input signal to the output signal through the impulse response function additional relations are formulated. Using equation (B.1) the cross correlation between the input \( u(t) \) and the output \( v(t) \) can be written in the following form.

\[
R_{uv}(\tau) = E[u(t)v(t+\tau)]
\]

\[
R_{uv}(\tau) = E\left[u(t)\int_{-\infty}^{\infty} h(\sigma)u(t+\tau-\sigma)d\sigma\right]
\]

\[
R_{uv}(\tau) = \int_{-\infty}^{\infty} h(\sigma)E[u(t)\int_{-\infty}^{\infty} u(t+\tau-\sigma)]d\sigma
\]

\[
R_{uv}(\tau) = h(\tau) \ast R_{uu}(\tau)
\]  

Where \( E \) denotes the expectation operator commonly used in statistics. The linearity of the expectation operator has been used to switch the order of integration and taking the expectation. In a similarly way the autocorrelation of the output \( v(t) \) can be written as.

\[
R_{vv}(\tau) = E[y(t)y(t+\tau)]
\]

\[
R_{vv}(\tau) = h(\tau) \ast R_{uu}(\tau)
\]  

Equations (B.1), (B.2) and (B.3) relate the impulse response function \( h(t) \) to the input \( u(t) \) and the output \( v(t) \). Applying a Fourier transform to these expressions leads to the following three equivalent expressions for the frequency response function \( H(f) \).

\[
H(f) = \frac{V(f)}{U(f)}
\]

\[
H(f) = \frac{S_{vv}(f)}{S_{uu}(f)}
\]

\[
H(f) = \frac{S_{vu}(f)}{S_{uv}(f)}
\]  

\[H(f) = \frac{S_{vu}(f)}{S_{vv}(f)}
\]

\[
H(f) = \frac{S_{uv}(f)}{S_{vv}(f)}
\]  

\[
43
\]
In the above equations $U(f)$ and $V(f)$ denote the Fourier transforms of the input $u(t)$ and the output $v(t)$ respectively. $S_{uu}(f)$ and $S_{uv}(f)$ denote the auto power spectrum of the input and output while $S_{uw}(f)$ denotes the cross power spectrum of the input and output.

**B.3 The Influence Of Measurement Noise**

In order to be able to use the equations (B.4) to estimate the frequency response function in a practical measurement situation the influence of measurement noise has to be taken into account. In the presence of measurement noise the signals $u(t)$ and $v(t)$ are not available. The measured signals $x(t)$ and $y(t)$ have to be used to estimate the frequency response function.

Replacing the actual input $u(t)$ by the measured input $x(t)$ and the actual output $v(t)$ by the measured output $y(t)$ in equations (B.4) leads to three estimators for the frequency response function. These estimators are known as the $H_0(f)$, $H_1(f)$ and $H_2(f)$ estimators and are defined as:

$$H_0(f) = \frac{Y(f)}{X(f)}$$  \hspace{1cm} (B.5)

$$H_1(f) = \frac{S_{xy}(f)}{S_{xx}(f)}$$  \hspace{1cm} (B.6)

$$H_2(f) = \frac{S_{xy}(f)}{S_{yy}(f)}$$  \hspace{1cm} (B.7)

Due to the presence of noise the $H_0(f)$, $H_1(f)$ and $H_2(f)$ estimators will not produce identical estimates of the frequency response function. Because of the presence of noise averaging of the frequency response function measurements is also necessary. The averaged frequency response function estimators are denoted as $\hat{H}_0(f)$, $\hat{H}_1(f)$ and $\hat{H}_2(f)$.

**B.3.1 The $\hat{H}_0(f)$ Estimator**

The $\hat{H}_0$ estimator is defined by the averages of the Fourier transforms of the input $x(t)$ and the output $y(t)$.

$$\hat{H}_0(f) = \frac{\hat{Y}(f)}{\hat{X}(f)}$$  \hspace{1cm} (B.8)

The $\hat{H}_0(f)$ estimator seems an appropriate frequency response function estimator but this is hardly the case. In practice the $\hat{H}_0(f)$ is rarely used because it is difficult to implement this estimator correctly. These practical implementation problems are caused by the direct use of the Fourier transforms of the input and output signals in the estimators. Time shifts in the measured signals $x(t)$ and $y(t)$ will cause phase shifts in the Fourier transforms $X(f)$ and $Y(f)$. If several realizations are averaged these phase shifts will cause the average to tend to zero. This makes a correct implementation of the $\hat{H}_0(f)$ estimator difficult and is also the reason why it is rarely used in practice.
B.3.2 The $\hat{H}_1(f)$ and $\hat{H}_2(f)$ Estimators

To analyze the effects of measurement noise on the $H_1(f)$ and $H_2(f)$ frequency response function estimators the auto- and cross power spectra of the signals $x(t)$ and $y(t)$ are considered. Using the measurement model shown in figure B.1 the auto- and cross-power spectra of the signals $x(t)$ and $y(t)$ can be written in the following form.

$$S_{xx}(f) = S_{uu}(f) + S_{mm}(f) \quad \text{(B.9)}$$

$$S_{yy}(f) = S_{vv}(f) + S_{nn}(f) \quad \text{(B.10)}$$

$$S_{xy}(f) = S_{uv}(f) \quad \text{(B.11)}$$

Where the fact that the measurement noise is uncorrelated is used to eliminate the cross spectral terms between the measurement noise and the input and output signals. Equations (B.9), (B.10) and (B.11) show that the auto-power spectra of the measured input and output are biased by the noise $m(t)$ and $n(t)$ respectively while the cross-power spectrum of $x(t)$ and $y(t)$ is unbiased by the measurement noise. Using equations (B.9), (B.10) and (B.11) the $H_1(f)$ and $H_2(f)$ estimators can be written in the following form.

$$H_1(f) = \frac{S_{uv}(f)}{S_{uv}(f)S_{mm}(f)} = H(f) \cdot \left(1 + \frac{S_{mm}(f)}{S_{uv}(f)}\right)^{-1} \quad \text{(B.12)}$$

$$H_2(f) = \frac{S_{vv}(f) + S_{nn}(f)}{S_{vv}(f)} = H(f) \cdot \left(1 + \frac{S_{nn}(f)}{S_{vv}(f)}\right) \quad \text{(B.13)}$$

In these equations the ratios $\frac{S_{mm}(f)}{S_{uv}(f)}$ and $\frac{S_{nn}(f)}{S_{vv}(f)}$ can be interpreted as signal to noise ratios in power terms on the input and output respectively. From equations (B.12) and (B.13) the following conclusions can be drawn.

- In the presence of noise the estimator $H_1(f)$ always underestimates the frequency response function while the estimator $H_2(f)$ always overestimates the frequency response function.
- The $H_1(f)$ estimator only produces biased results if measurement noise on the input is present. Consequently the $H_1(f)$ estimator will produce good estimates near anti-resonances where the system input has little influence on the system output.
- The $H_2(f)$ estimator only produces biased results if measurement noise on the output is present. This causes the $H_2(f)$ estimator to produce good estimates near resonance where the system input has a large influence on the system output.
Because auto-power spectra contain no phase information both estimators produce unbiased phase estimates.

To minimize the influence of noise, averages of the frequency response function estimators are used. These averages are denoted as $\bar{H}_1(f)$ and $\bar{H}_2(f)$.

The coherence function is a computed measurement that gives a measure of the linear dependence between two signals as a function of frequency. The coherence function $\gamma_{xy}^2$ is defined as:

$$\gamma_{xy}^2(f) = \frac{|S_{xy}(f)|^2}{S_{xx}(f)S_{yy}(f)}$$  \hspace{1cm} (B.14)

The coherence function $\gamma_{xy}^2$ is a dimensionless real valued function which has a value between zero and one. A coherence function value of zero indicates that there is no linear relationship between $x(t)$ and $y(t)$. If the value of the coherence function is equal to one then there exists a perfect linear relationship between $x(t)$ and $y(t)$.

Using equation (B.14) and equations (B.9), (B.10) and (B.11) the coherence function can be written in the following form:

$$\gamma_{xy}^2(f) = \frac{1}{\left(1 + \frac{S_{mm}(f)}{S_{xx}(f)}\right)\left(1 + \frac{S_{nn}(f)}{S_{yy}(f)}\right)}$$  \hspace{1cm} (B.15)

Equation (B.15) shows that the value of the coherence function is directly related to the signal to noise ratio of the input $x(t)$ and the output $y(t)$. Large signal to noise ratios will cause the coherence function to approach zero indicating that the measured output is primarily due to measurement noise and that there is no linear relationship between $x(t)$ and $y(t)$ exists. For relatively small signal to noise ratios the coherence function will tend to one indicating that the measured output signal is primarily caused by the measured input signal and that there is a linear relationship between $x(t)$ and $y(t)$ exists.
C Mode Indicator Functions

In MEscope three mode indicator functions are available. These are the modal peaks function, the complex mode indicator function and the multivariate mode indicator function. Here only the modal peaks function and the complex mode indicator function are treated.

C.1 Modal Peaks Function

The modal peaks function is a very simple mode indicator function. It is usually defined as the sum of the magnitude of all measured frequency response functions. In certain situations alternative formulations based on the real or imaginary part of the measured frequency response functions can also be used.

\[ MPF(f) = \sum_{k=1}^{L} |H_k(f)| \]  

In the above equation \( k \) denotes the measurement number and \( L \) denotes the total number of measured frequency response functions. The idea behind the modal peaks function is as follows. In a measured frequency response function an eigenmode appears as a resonance peak at a certain frequency. A resonance peak belonging to a certain eigenmode appears in all measured frequency response functions with the exception of measurements made at nodal points of the eigenmode. Because of this property summing the magnitudes of all measured frequency response functions leads to a mode indicator function which shows peaks at the frequencies where resonances occur.

C.2 Complex Mode Indicator Function

The complex mode indicator function is based on the singular value decomposition of the frequency response function matrix \( H(f) \) at each spectral line. The frequency response function matrix is a column matrix that contains the measured frequency response functions and is defined as:

\[ H(f) = [H_1(f) \ H_2(f) \cdots H_{np}(f)] \]  

(C.2)
In this equation $H_1(f)$ to $H_{np}(f)$ denote the frequency response functions measured at each location. The singular value decomposition of this matrix is defined as:

$$H(f) = U(f)\Sigma(f)V^H(f)$$  \hspace{1cm} (C.3)

The $(L \times L)$ matrix $U(f)$ is known as the right singular matrix while the $(np \times np)$ matrix $V(f)$ is known as the left singular matrix. The $(L \times np)$ matrix $\Sigma$ is a diagonal matrix with the so called singular values on its diagonal. The number of nonzero singular values represents the order of the system. Using the singular value decomposition as defined above the complex mode indicator function is defined as:

$$CMIF(f) = \Sigma^T(f)\Sigma(f)$$ \hspace{1cm} (C.4)

Peaks in the complex mode indicator function indicate the presence of an eigenmode. The idea behind the complex mode indicator function is that because singular values are related to eigenvalues they can be used to indicate eigenmodes. A more elaborate treatment of the relation between the eigenmodes of a system and the singular values of the frequency response function matrix can be found in [12].
The global polynomial modal parameter estimation method is based on the assumption that the numerator and denominator of the frequency response function can be written in a polynomial form. If these polynomials can be fit to measured frequency response functions then the modal parameters can be calculated from the fitted polynomials. Usually the fitting of the polynomials is performed through a nonlinear least squares procedure. The frequency response function is written in the so called rational fraction form.

\[ H(j\omega) = \frac{\sum_{k=0}^{m} a_k j^k}{\sum_{k=0}^{n} b_k j^k} \quad (D.1) \]

An error \( e_i \) at a particular value of frequency \( \omega_i \) is defined as:

\[ e_i = H(\omega_i) - \bar{H}(\omega_i) \quad (D.2) \]

In this equation the term \( H(\omega_i) \) refers to the actual frequency response function while the term \( \bar{H}(\omega_i) \) refers to the measured frequency response function. If the denominator coefficient \( b_n \) is scaled to a value of 1 then the error \( e_i \) at a frequency \( \omega_i \) can be written as:

\[ e_i = \sum_{k=0}^{m} a_k (j\omega_i)^k - h_i \left[ \sum_{k=0}^{n-1} b_k (j\omega_i)^k + (j\omega_i)^n \right] \quad (D.3) \]

For \( L \) measured frequencies \( \omega_i \) the following squared error criterium can be defined:

\[ J = \sum_{i=1}^{L} e_i^2 e_i \]

\[ = E^T E \quad (D.4) \]

\[ = E^T E \quad (D.5) \]

\[ \text{(D.6)} \]

The least squares estimates of the polynomial coefficients \( a_k \) and \( b_k \) are obtained by minimizing the squared error criterium \( J \). In general the equations defined by the squared error criterium are ill-conditioned and therefore difficult to solve [16]. The solution of the squared error criterium is possible through the application of an appropriate optimization algorithm. One of a number of alternative approaches is to rewrite the above minimization problem using rational fractional polynomials [16] which leads to the so called rational fractional polynomial modal parameter estimation algorithm.
**E Estimated Mode Shapes**

In this appendix the estimated mode shapes are presented. The mode shapes are estimated using a global polynomial method with five additional polynomial terms. Using the driving point measurement the mode shapes are scaled to unit modal mass.

**Figure E.1:** The first mode shape.
Figure E.2: The second mode shape.

Figure E.3: The third mode shape.
Figure E.4: The fourth mode shape.
Figure E.5: The fifth mode shape.

Figure E.6: The sixth mode shape.
F Euler Bernoulli Beam

In this appendix the natural frequencies and mode shapes of the Euler-Bernoulli beam are calculated. More information on the calculation of natural frequencies and mode shapes of continuous systems can be found in [13] and [14]. An overview of the natural frequencies of different types of beams with different boundary conditions is presented in [4]. The derivation presented here is divided in two different steps. First the governing equations of the Euler-Bernoulli beam are derived. Second the natural frequencies and mode shapes of a cantilever beam and a cantilever beam with a tip mass are calculated.

F.1 Governing Equations

The Euler-Bernoulli beam considered here is shown in figure F.1. The Euler-Bernoulli beam has length $L$ and is not subjected to any external loads. Figure F.2 shows the differential beam element corresponding to the Euler-Bernoulli beam. The differential beam element is subjected to the transversal force $Q(x,t)$ and bending moment $M(x,t)$. Neglecting the effects of rotary inertia and shear deformation, applying a Newtonian approach and neglecting second order terms in $dx$ leads to the following equations:

$$
Q(x,t) + \frac{\partial Q(x,t)}{\partial x} \, dx
$$

$$
M(x,t) + \frac{\partial M(x,t)}{\partial x} \, dx
$$

Figure F.1: The Euler-Bernoulli beam. Figure F.2: The differential beam element.
force and moment balance for the differential beam element.

\[
\frac{\partial Q(x,t)}{\partial x} = m(x) \frac{\partial^2 y(x,t)}{\partial t^2} \quad \text{(F.1)}
\]

\[
\frac{\partial M(x,t)}{\partial x} dx + Q(x,t)dx = 0 \quad \text{(F.2)}
\]

Where the variable \( x \) is constrained by the length of the beam as \( 0 < x < L \) and \( m(x) \) is the mass of the beam per unit length. Substituting the second equation into the first equation leads to the following relation between the bending moment and the bending displacement.

\[
- \frac{\partial^2 M(x,t)}{\partial x^2} = m(x) \frac{\partial^2 y(x,t)}{\partial t^2} \quad \text{(F.3)}
\]

This equation relates the bending moment \( M(x,t) \) to the transversal displacement \( y(x,t) \). In order to obtain a relation that only depends on the transversal displacement the following relation originating from the mechanics of materials is used.

\[
M(x,t) = E I(x) \frac{\partial^2 y(x,t)}{\partial x^2} \quad \text{(F.4)}
\]

Where \( E \) is known as the modulus of elasticity and \( I \) is known as cross sectional or second area moment of inertia. The term \( EI(x) \) combining both \( E \) and \( I(x) \) is also known as the flexural rigidity. Equation (F.4) also implies a relation between the transversal force \( Q(x,t) \) and the bending displacement \( y(x,t) \) through equation (F.2). Substituting equation (F.4) into equation (F.3) leads to the following governing equation for the Euler-Bernoulli beam.

\[
- \frac{\partial^2}{\partial x^2} \left( EI(x) \frac{\partial^2 y(x,t)}{\partial x^2} \right) = m(x) \frac{\partial^2 y(x,t)}{\partial t^2} \quad \text{(F.5)}
\]

The above equation is a fourth order partial differential equation that describes the free bending vibration of an Euler-Bernoulli beam. The above governing equation is not sufficient to completely describe the problem. In order to obtain a complete problem description four boundary conditions are needed.

**F.2 Natural Frequencies**

In this section the natural frequencies of the cantilever beam with and without tip mass are determined. The natural frequencies are determined from a combination of the governing equation (F.5) and appropriate boundary conditions. Here only the results of solving the appropriate equations are presented. More information on solving the governing equation and the derivation of the boundary conditions is found in [13] and [14].

In order to be able to solve the governing equation (F.5) four boundary conditions are needed. At the clamped end at \( x = 0 \) the boundary conditions are found by requiring that the deflection and the slope of the deflection are equal to zero. These boundary conditions are:

\[
\frac{\partial y(x,t)}{\partial x} \bigg|_{x=0} = 0, \quad y(x,t) \bigg|_{x=0} = 0
\]

and

\[
\frac{\partial y(x,t)}{\partial x} \bigg|_{x=L} = 0, \quad y(x,t) \bigg|_{x=L} = 0
\]
conditions are known as geometrical boundary conditions and can be represented by
the following equations.

\[ y(x, t) = 0 \quad \frac{\partial y(x, t)}{\partial x} = 0 \quad x = 0 \quad (F.6) \]

At a free end at \( x = L \) the boundary conditions are found by requiring that the bending
moment and the transversal force are equal to zero. These boundary conditions are
known as natural boundary conditions and can be expressed as follows.

\[ M(x, t) = EI(x) \frac{\partial^2 y(x, t)}{\partial x^2} = 0 \quad x = L \quad (F.7) \]

\[ Q(x, t) = -\frac{\partial}{\partial x} \left( EI(x) \frac{\partial^2 y(x, t)}{\partial x^2} \right) = 0 \quad x = L \quad (F.8) \]

In the case of a tip mass \( M \) concentrated at \( x = L \) the boundary conditions are found
by requiring that the bending moment is equal to zero and that the transversal force
is equal to the acceleration force acting on the mass. The first of these boundary
conditions leads to boundary condition similar to equation \((F.7)\). The second of these
boundary conditions leads to the following equation.

\[ -Q(x, t) = \frac{\partial}{\partial x} \left( EI(x) \frac{\partial^2 y(x, t)}{\partial x^2} \right) = M \frac{\partial^2 y(x, t)}{\partial x^2} \quad x = L \quad (F.9) \]

Applying the boundary conditions \((F.6), (F.7), (F.8)\) to the governing equation
\((F.5)\) and assuming that the beam under consideration is uniform leads to the following
solutions for the cantilever Euler-Bernoulli beam. The natural frequency of the
cantilever Euler-Bernoulli beam is written as:

\[ f_r = \frac{(\beta_r L)^2}{2\pi} \sqrt{\frac{EI}{mL^4}} \quad (F.10) \]

The factor \( \beta_r L \) is determined from the following characteristic equation.

\[ \cos(\beta_r L) \cosh(\beta_r L) = -1 \quad (F.11) \]

The mode shapes of the cantilever beam are found by substituting the value of \( \beta_r L \)
into the following equation.

\[ Y_r(x) = A_r \left[ \sin(\beta_r x) - \sinh(\beta_r x) - \frac{\sin(\beta_r L) \sinh(\beta_r L)}{\cos(\beta_r L) + \cosh(\beta_r L)} \right] \quad (F.12) \]

The relations for the natural frequency and mode shapes of the cantilever beam with
tip mass are identical to those of the cantilever beam without tip mass. The only
difference lies in the characteristic equation. This difference occurs because for the
Euler-Bernoulli cantilever beam with tip mass the boundary condition \((F.8)\) is replaced
by boundary condition \((F.9)\). The characteristic equation for a cantilever beam with
a tip mass is defined as:

\[ 1 + \cos(\beta_r L) \cosh(\beta_r L) + \frac{M \beta_r L}{mL} \left[ \cos(\beta_r L) \sinh(\beta_r L) - \sin(\beta_r L) \cosh(\beta_r L) \right] = 0 \quad (F.13) \]
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