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On the theoretical relation between operating leverage, earnings variability, and systematic risk

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ON THE THEORETICAL RELATION BETWEEN OPERATING LEVERAGE, EARNINGS VARIABILITY, AND SYSTEMATIC RISK

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In order to determine the shareholder value of a business entity, we need to know its systematic risk as measured by $\beta$. This $\beta$ can be estimated from firm-specific characteristics, such as earnings variability and accounting beta. We show that the use of the accounting beta to determine the systematic risk of a business unit should be applied with care. This is because earnings variability is not directly related with the 'real' $\beta$.

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In the seventies and eighties, substantial attention has been paid to the components of the systematic risk of a firm, as measured by its $\beta$. The use of the $\beta$, and the capital asset pricing model (CAPM) from which it stems, is wide spread both in theory and practice. The well-known formulation is:

$$E(r_i) = r_{Ft} + \beta_i E(r_{Mt} - r_{Ft})$$  \hspace{1cm} (1)

with 

$r_{It}$ = return on asset $i$ in time $t$

$r_{Ft}$ = risk-free rate of return

$r_{Mt}$ = return on the market portfolio $M$

$\beta_{it}$ = risk measure

The risk measure $\beta$ is defined as

$$\beta_{it} = \frac{\text{cov}(r_{It}, r_{Mt})}{\sigma^2(r_{Mt})}$$  \hspace{1cm} (2)

Although the debate over the validity of the CAPM is heating up in the nineties (cf. Fama and French, 1992, 1996; Haugen and Baker, 1996), it is arguably the most popular model to explain asset prices. It is extensively covered in the leading textbooks on financial management and corporate finance (Brigham and Gapenski, 1995; Brealey and Myers, 1996; Ross, Westerfield and Jaffe, 1993).

The latest impetus for the popularity of CAPM is the growing emphasis on shareholder value as the goal of the firm. The shareholder value created by the firm as a whole is readily determined through its market value. In evaluating the performance of a new project or of the business units that make up the firm, this is not possible. Accounting income numbers (earnings and cash flow) are available, but they are perceived as incorrect measures of the economic value of a business entity. However, when we evaluate the book figures within a CAPM-framework, we can reach an indication of the shareholder value created.

In order to use the CAPM-equation as stated above, we need a value for $\beta$, the systematic risk of the business entity. There are several methods to estimate $\beta$. One of them is the use of accounting numbers in the mathematical formula for systematic risk. In this article, we will show that this can lead to incorrect conclusions about the 'true' $\beta$. We start with a short review of the literature on the determinants of
systematic risk. Then we will examine the relation between operating leverage, earnings variability and systematic risk. Finally, we will show that under certain conditions business entities with the same sales pattern but very different variabilities of earnings will have the same systematic risk.

**SYSTEMATIC RISK: ESTIMATION AND DETERMINANTS**

In the past, the problem of the estimation of systematic risk was treated under the heading of ‘divisional cost of capital’. With the increasing acceptance of the CAPM in the 1970s, it was realized that the business units or divisions of a firm need not face the same systematic risk as the firm as a whole\(^2\). As a consequence, a low-risk division was at a disadvantage in obtaining intrafirm capital because it projects were more likely to achieve a return below the firm-wide hurdle rate (cf. Fuller and Kerr, 1981). The problem of selecting the right \(\beta\) for a project, and thus the right hurdle rate, is still relevant today, but there has risen another need for estimating the cost of capital of a business unit: the ever-growing popularity of the shareholder value-concept. If a firm strives to create shareholder value, it should not judge the performance of its business units on accounting measures like return on equity or return on investment, but on the present value of the future cash flows to be generated by the unit. This requires a unit-specific cost of capital.

There are two basic ways to estimate the cost of capital of a business entity, be it a division, a business unit or an unlisted firm. First, one can look for a listed firm that has approximately the same operating characteristics as the business unit (Fuller and Kerr, 1981; Mohr, 1985). Although this is a relatively simple method, it is often not possible to find a shadow firm that is sufficiently similar, especially in smaller markets than the U.S. and U.K. The second way is to estimate the systematic risk from other, observable factors, or ‘fundamentals’, that make up this systematic risk.

The decomposition of systematic risk can be done from a firm perspective, from a macro-economic perspective, or from a combination. Research on firm-specific characteristics started with Beaver, Kettler, and Scholes (1970). This strand of
research is geared towards determining the influence of firm characteristics on the magnitude of $\beta$. Ross (1976) first formally modeled the macro-economic perspective in his Arbitrage Pricing Theory. In this model, it is tried to relate certain macro-economic variables (e.g. inflation, oil prices, and interest rates) to the returns on shares. It should be noted that this approach makes it less useful for estimating the systematic risk of a business unit, because it does not start from the firm itself, but from the way it reacts to macro-economic changes. Attempts have been made to combine the two approaches by incorporating firm-specific characteristics like financial leverage and sales growth into multifactor models. Whatever these models win in explanatory power – which is not very much in most cases – they lose in clarity, rendering them useless in estimating the systematic risk of an individual business entity (cf. Campbell and Mei, 1993).

By using only firm-specific characteristics (i.e. variables that are readily available, and do not require analyses like mathematically complex factor models), the estimation of the systematic risk of a business unit would be relatively simple. This requires knowledge of the determinants of $\beta$. There has been extensive research on this subject in the 70s and early 80s. Myers (1977) concludes from a literature review of empirical research that there are three, maybe four important determinants of systematic risk: cyclicality, earnings volatility, and financial leverage, and possibly growth. Cyclicality is defined by Myers as “the extent to which fluctuations in the firm’s earnings are correlated with fluctuations in earnings of firms generally” (1977: 53). Given this definition, it is not surprising that Myers considers the volatility of earnings to be important mainly because it is a good proxy for cyclicality. Myers’ intuitively appealing view on the relation between earnings variability and $\beta$ currently still prevails (cf. Ryan, 1997: 82). However, financial theory has long rejected this view.

Bowman (1979) examines the theoretical relation between $\beta$ and the determinants suggested in various papers. He concludes that financial leverage is theoretically related to systematic risk, but growth and earnings variability are not. Furthermore, there is a direct relation between the accounting beta and systematic risk.
The accounting beta is defined by Hill and Stone as “a generic term for the systematic sensitivity of some measure of accounting return to a broad-based index of that same return” (1980: 596). Bowman (1979) defines the accounting beta in accordance with Myers’ mentioned definition of cyclicality:

\[
B_{it} = \frac{\text{cov}(X_{it}, X_{Mt})}{\sigma^2(X_{Mt})}
\]

(3)

with \( B_{it} \) = accounting beta of firm \( i \)

\( X_{it} \) = accounting earnings of firm \( i \)

\( X_{Mt} \) = accounting earnings of the market portfolio \( M \)

It is obvious that relating the accounting beta as defined to the systematic risk requires some correction factor incorporating market values, because the CAPM is concerned with market returns, not book returns. It is not possible to derive a relation between book earnings and systematic risk incorporating book equity, because there is no theoretical basis for this4. Bowman (1979; 623) derives that

\[
\beta_{it} = \frac{S_{Mt}}{S_{it}} B_{it}
\]

(4)

with \( S_{Mt} \) = market value of the equity of the market portfolio

\( S_{it} \) = market value of the equity of firm \( i \)

The work of Bowman shows that there is a simple, linear relation between a certain form of the accounting beta and systematic risk. This would seem to make it attractive to use in estimating \( \beta \). Brealey and Myers claim that “firms with high accounting (..) betas should also have high stock betas” (1991; 199).

However, it should be noted that there is a serious drawback to the accounting beta. To show this, we first take another look at the determinants of systematic risk. We will start with leverage and finally focus on the relation between earnings variability and systematic risk. As indicated before, empirical research sketches a strong relation, whereas theoretical research rejects this relation. Intuition, until now, took sides with the empirical relation. With presenting a stylized example we want to develop some intuition for the theoretical perspective.
SYSTEMATIC RISK AND LEVERAGE

What drives risk? It seems clear that the most important risk driver is the variability of sales. A firm with heavily fluctuating revenues is perceived as being more risky than one with a stable revenue development. The variability of sales can be seen as the connection between the overall market movements and the behavior of earnings; it is the mechanism through which the macro-economic influences do most of their work. This is even more so in a ceteris paribus-setting, with constant prices and constant costs. In this setting, there is only one other category of risk drivers: fixed charges.

When a firm faces fixed charges or fixed commitments, the variability of earnings as a result from the variability of sales is magnified. Therefore, these fixed charges are termed leverage. Financial leverage is a well-known firm characteristic; it is mostly measured as a ratio of debt and equity. Operating leverage is also frequently mentioned as a risk driver, but more as a qualitative indication of the cost structure than as a ratio. It results from the presence of fixed costs, and should ideally be measured with a ratio of fixed and variable costs. However, for the present analysis we shall use the elasticity formulation of leverage, i.e. the relative change in the dependent variable versus the relative change in the independent. This leads to the following definitions:

\[ dol = \frac{\Delta%Y}{\Delta%W} = \frac{\frac{Y_t}{W_t} - 1}{\frac{Y_{t-1}}{W_{t-1}} - 1} \]  
(5)

\[ dfl = \frac{\Delta%X}{\Delta%Y} = \frac{\frac{X_t}{Y_t} - 1}{\frac{X_{t-1}}{Y_{t-1}} - 1} \]  
(6)

with  
\[ dol \] = degree of operating leverage  
\[ dfl \] = degree of financial leverage  
\[ X_t \] = net earnings  
\[ Y_t \] = earnings before interest and taxes (ebit)  
\[ W_t \] = sales revenue
This formulation of the two types of leverage has enabled Mandelker and Rhee (1984) to derive the following relationship between financial leverage, operating leverage, and systematic risk (see appendix for the derivation):

\[
\beta = \text{dol} \cdot \text{dfl} \cdot \frac{\text{cov} \left( X_{t-1} \frac{W_t}{S_{t-1}}, r_{Mt} \right)}{\sigma^2(r_{Mt})}
\]

This is an exceptionally clear result: the systematic risk of a firm is the result of the presence of financial and operating leverage, and what Mandelker and Rhee (1984: 48) term 'intrinsic business risk'. They interpret the first term of the covariance as the product of the net profit margin and the turnover of the firm's common equity. It is clearer, however, to interpret it as the return on equity, \(X_{t-1}/S_{t-1}\), corrected for the change in sales. For a wholly unlevered firm, both \(\text{dol}\) and \(\text{dfl}\) are 1. In this case, any percentage change in sales is translated into an identical percentage change in the return on equity. If the firm is levered in any way, the intrinsic business risk is magnified, resulting in an increase in systematic risk.

Because the CAPM is a one-period model, care should be taken not to view the factor \(W_t/W_{t-1}\) as the growth in sales, and consequently conclude that the resulting 'growth in return' is a determinant of systematic risk. \(W_t\) is merely the only stochastic variable in the first term of the covariance, and as such the source of risk. If we want to introduce growth within this framework, we have to extend the analysis to a multi-period setting (see Bowman (1979) for such an analysis).

**OPERATING LEVERAGE AND SYSTEMATIC RISK**

The Mandelker-Rhee equation models the intuitive notion that the systematic risk of financially unlevered firms that face the same sales pattern differs only if their operations are dissimilar, i.e. if they do not have the same cost structure. From the equation, we also see that if these two firms have the same \(\text{dol}\), they have the same systematic risk. Is it possible for business entities with different cost structures to
have the same dol? In order to analyze this, we rewrite the elasticity-formulation of dol:

\[ \text{dol} = \frac{q(p-v)}{q(p-v)-F} \]

with \( q \) = unit sales

\( p \) = price per unit

\( v \) = variable costs per unit

\( F \) = fixed costs

This formula explicitly shows where operating leverage comes from, namely the presence of fixed costs.

\[ \frac{\text{dol}_1}{\text{dol}_2} = \frac{\frac{q(p-v_1)}{q(p-v_1)-F_1}}{\frac{q(p-v_2)}{q(p-v_2)-F_2}} = \frac{\frac{qm_1}{qm_1 - F_1}}{\frac{qm_2}{qm_2 - F_2}} = \frac{qm_1(qm_2 - F_2)}{qm_2(qm_1 - F_1)} = \frac{qm_1 F_2}{qm_2 F_1} \]

with \( m_i \) = contribution margin \((p-v_i)\) for firm \( i \)

When the ratio of fixed costs to contribution margin is the same, the two firms have an identical dol. This ratio is, of course, the well-known break-even point. This also means that two firms facing an identical sales pattern and having the same break-even point have the same systematic risk.

Although the previous conclusion is valid only under strict assumptions, it is not difficult to find situations in which it will be more or less appropriate to apply. Especially when we look at the project level, we can imagine a choice between a process with high fixed costs and low variable costs, and a process with low fixed costs and high variable costs; for example, the choice between a flexible manufacturing system or a traditional production process. Nevertheless, the analysis is theoretically valid for all firms.
Suppose we have two all-equity firms in a no-tax world, both having only one production process. Because they operate in the same market, manufacturing the same product, their sales pattern is identical. Firm 1 has fixed costs of $600 and variable costs of $2 per unit; firm 2 has fixed costs of $300 and variable costs of $6 per unit. With a price of $10 per unit the earnings are:

\[ Y_1 = 8q - 600 \]
\[ Y_2 = 4q - 300 \]

This leads to a break-even point for firm 1 and 2 of \( q = 75 \). Also, because both firms have the same break-even point, their \( \text{dol} \) is identical (see Figure 1).

**FIGURE 1**

Earnings Variability and Operating Leverage of Firm 1 and 2

The earnings of both firms follow the relation \( Y_1 = 2Y_2 \), meaning that the earnings line is twice as steep for firm 1. This results in \( \sigma^2(Y_1) = 4 \sigma^2(Y_2) \), so the variability of earnings is much larger for firm 1 than it is for firm 2. If we fill in these numbers in the accounting beta (\( B \)) and take into account that \( \text{cov}(ax, y) = a \text{cov}(x, y) \), it follows immediately that \( B_1 = 2B_2 \). The accounting beta of firm 1, the firm with the
larger earnings variability, is larger than firm 2. However, we established before that firms facing an identical sales pattern and having an identical *dol* have the same systematic risk. Where does it go wrong?

It would seem that leaving out the market value of equity in the accounting beta makes it a measure that should be interpreted with great care. This can be clarified by making some assumptions on the market value of the firms 1 and 2, in order to get an indication of their 'real' $\beta$'s. Because we know that $Y_1 = 2Y_2$, we can make a first estimate of their market values as being $S_1 = 2S_2$. The future earnings of firm 1 are twice that of firm 2. This would lead to the following $\beta$'s:

$$
\beta_2 = \frac{\text{cov}(\frac{Y_2}{S_2}, r_M)}{\sigma^2(r_M)} = \frac{\text{cov}(\frac{Y_2}{2S_2}, r_M)}{\sigma^2(r_M)} = \frac{\text{cov}(\frac{Y_2}{S_2}, r_M)}{\sigma^2(r_M)} = \beta_2
$$

Under the provisional assumption that the market value of firm 1 is twice that of firm 2, the systematic risk of both firms is identical. This means that, although firm 1 has an accounting beta that is twice that of firm 2, and an earnings variability that is four times as large, the systematic risk is the same. In fact, the earnings variability could differ by *any* factor, and still the two firms would have the same systematic risk.

Again, it should be noted that the example is simple, but it need not be very uncommon. What is even more important, however, is the notion that merely earnings variability is not a good indicator of the systematic risk of a business entity, and therefore should be handled with care. When we want to use the CAPM in estimating the cost of capital of a business unit, or an investment project, we cannot rely on the earnings variability as a good estimate of the systematic risk. This also means that the only definition of accounting beta that has a theoretical relation with $\beta$ should be handled with care.
CONCLUSION

In this paper we have tried to clarify the theoretical notion that earnings variability is not a direct source of systematic risk. We have done this through a simple example, using the concept of operating leverage. Next to drawing renewed attention to the elegant Mandelker-Rhee formula for the influence of financial and operating leverage on systematic risk, we showed that under certain conditions the systematic risk of a project is independent of the magnitude of its earnings variability. This result serves as a warning to the use of accounting numbers in estimating the systematic risk of a non-listed business entity.

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1 In the original shareholder value manifesto, Rappaport (1986: 20) lists the following important reasons for this: accounting policy changes, and the exclusion of risk, investment requirements, dividend policy, and the time value of money.

2 Although not everyone agrees on this point, see e.g. Reimann (1990).

3 The APT is theoretically empty. It is a mathematical model, and there is no necessary relation between an empirically derived factor and a ‘real’ macroeconomic variable.

4 Beaver, Kettler, and Scholes (1970) use a mix of accounting values and market values in their definition of accounting beta, in which they use earnings-price ratio’s in stead of absolute earnings. This cannot be used in estimating the \( \beta \) of business units. Hill and Stone (1980) perform extensive tests on the relation between market \( \beta \)’s and accounting betas, but use book returns (i.e. return on assets and return on equity). This can be questioned to a great extent.

5 This derivation is possible only under the conditions of the break-even analysis, where the main assumption is that units sold is the only variable; price, variable costs per unit, and fixed costs are assumed constant.

6 This result was also achieved by Lord (1995), but unfortunately he did not make it explicit.
REFERENCES


APPENDIX

Mandelker and Rhee (1984) derive their relation between financial leverage, operating leverage and systematic risk using the formula \( \text{cov} (ax + b, y) = a \text{cov} (x, y) \). In the notation used in this article, they find:

\[
\beta = \frac{\text{cov}(r_t, r_{Mt})}{\sigma^2(r_{Mt})}
\]

\[
= \frac{\text{cov}\left(\frac{X_t}{S_{t-1}} - 1, r_{Mt}\right)}{\sigma^2(r_{Mt})}
\]

\[
= \frac{\text{cov}\left(\frac{X_t}{S_{t-1}}, r_{Mt}\right)}{\sigma^2(r_{Mt})}
\]

\[
= \left(\frac{X_{t-1}}{S_{t-1}}\right) \frac{\text{cov}\left(\frac{X_t}{X_{t-1}} - 1, r_{Mt}\right)}{\sigma^2(r_{Mt})}
\]

The last step is taken by multiplying the first covariance term with \( X_{t-1}/X_{t-1} \), and subtracting a constant (-1). The resulting term is the relative change in net earnings, which is the numerator of \( dfl \). By rewriting the \( dfl \) and \( dol \)-formulas (equations (6) and (5) respectively) as

\[
\frac{X_t}{X_{t-1}} - 1 = dfl \cdot \left(\frac{Y_t}{Y_{t-1}} - 1\right)
\]

and

\[
\frac{Y_t}{Y_{t-1}} - 1 = dol \cdot \left(\frac{W_t}{W_{t-1}} - 1\right)
\]

and by successive substitution and rearranging we find

\[
\beta = \text{dol} \cdot dfl \cdot \frac{\text{cov}\left(\frac{X_{t-1}}{W_{t-1}}, \frac{W_t}{S_{t-1}}, r_{Mt}\right)}{\sigma^2(r_{Mt})}
\]

\[
= \text{dol} \cdot dfl \cdot \beta^0
\]