Three Dimensional Displacement Measurement using Video Imaging

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Report No. WFW 96.089

Research Report
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Eindhoven, May 1996

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Abstract

Atherosclerosis is a major cause of death. Therefore, insight is needed in the genesis of atherosclerosis. It is generally assumed that atherogenesis is related to low and oscillating wall shear stresses at the endothelial cell’s surfaces. However, it is reasonable to assume that it is related to compressive strains. In this project, an experimental method is designed to measure the three-dimensional displacement field of a moving model of a curved artery under physiological conditions. The strains can be calculated from the displacements. The measurements are based upon optical registration of the movement of markers, using a mirror to be able to reconstruct three dimensional coordinates. The method to reconstruct the three-dimensional coordinates appears to be working correctly in a qualitative way, but should be validated before assigning quantitative value to it.

It appears compressive strains are present at the inner curve of the tube, and positive strains appear to be present at the outer curve. This relates to the location where atherosclerosis occurs, at the inside curve of an artery.
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Chapter 1

Introduction

Atherosclerosis is an arterial disease which may lead to narrowing (stenosis) and a decrease in elasticity of arteries. A relatively hard plaque develops at the inner surface of the artery. Atherosclerosis mainly sets in the large and medium-sized bent arteries and in bifurcations, and is a major cause of death. In the carotid bifurcation atherosclerosis may lead to cerebral vascular diseases. When atherosclerosis sets in the coronary arteries, fatalities in the heart function may occur.

The University of Limburg in Maastricht and the Eindhoven University of Technology cooperate in the Atherosclerosis project. The main goal of this project is early detection and better diagnostic treatment of atherosclerosis. To achieve this, better insight is needed in the genesis of atherosclerosis.

From a fluid-dynamical point of view, it is generally assumed that atherosclerosis is related to low and oscillating wall shear forces at the endothelial cell's surfaces. The endothelial cells align with the direction of the flow. However, from a solid-mechanical point of view, one may assume that at the locations where atherosclerotic lesions occur, the compressive strains are high. It is reasonable to assume that the strain distribution in the vessel wall may play an important role in atherogenesis. The means to test this assumption are designed in this project.

The goal here is to measure the dynamic strain distribution in a bent artery, such as the aorta. However, there is a restriction. The strain distribution cannot be measured directly, but has to be calculated from a displacement field. Here, a method to measure displacements in three dimensions is presented. This method is based on optical registration of the displacement of markers in two projections at the same time. The markers are attached to the outer surface of a model of a bent artery. The movement of the markers due to an instationary flow through the model of the bent artery represents the movement of the wall of the curved artery. In this case, the model of the artery resembles most to the aorta because of the size and the mechanical properties of the material from which the model is manufactured. This model will of course be a simplification of the real aorta, in order to gain some more insight into the strain distribution of artery walls in general and the aorta in particular.

In the following, the in-vivo behaviour of bent arteries and especially the aorta are looked at in some more detail. When a fully developed stationary velocity profile enters a bend, it will change due to different processes. The flow in a bend is determined by a balance of inertia-, pressure- and friction forces. Due to friction forces, a secondary flow field is induced. The fluid particles in the middle move to the outer surface of the bend and go back along the inner surface. The result is a flow field in which the maximum fluid velocity is nearer to the outer surface than to the inner surface. When an oscillating, instationary flow enters a bend, also a mean secondary flow develops, with the same orientation as in the case of the stationary flow. A splitting of the secondary flow field can occur, but the surface backflow always has the same orientation. The orientation of the secondary flow field is independent on the direction of the flow [van Dongen, 1995].

The aorta is the major conduit that distributes oxygenated blood into the various tissues and organs of the body. It acts as an auxiliary pump by maintaining blood pressure by virtue of its elastic retraction during filling of the right ventricle [Silver, 1994]. Because of a complex bloodflow in the aorta, and because of complex mechanical properties of the aorta wall, the behaviour of the aorta wall is expected to be very complex. Different branches are attached to the thoracic aorta, such as the carotid arteries and the subclavian arteries. Because of these branches, the behaviour of the aorta is even more complex. The wall of the aortic artery consists of different layers of tissue. The intima, the inner layer, consists of a basement membrane which carries the endothelial cells. The intima is very thin and plays an insignificant role in the mechanical properties of the vessel wall. The media consists of smooth muscle cells, collagen
fibres and elastic units. The auxiliary pumping ability of the aorta is believed to be associated with the elastic fibre network, while the collagen fibres prevent overdilatation and failure of the tissue. The thoracic aorta is more compliant than any other artery because the collagen component is relatively high. The outermost layer, the adventitia, does not contribute much to the mechanical properties of the aorta except in tethering the aorta to the surrounding tissue [Silver, 1994]. Because the aorta is embedded in the surrounding tissue the axial displacement is suppressed. However, in vivo there is a substantial radial displacement because of the dynamic pressure. The artery wall with its layers of tissue undergoes a dynamic strain, which at some places is a compressive strain.

In conclusion, the complex flow and complex mechanical properties of the aorta wall in combination with the presence of sidebranches cause a very complex mechanical behaviour.
Chapter 2

Materials and Methods

An experimental method is presented to measure 3d deformation of a visco elastic model of the aorta. The model is placed inside a fluid-filled container, and markers are placed over the surface, in such a way that deformation at several axial positions can be determined. A mirror is used to be able to measure displacements of the markers in 3 dimensions. Flow and pressure are measured in phase with the displacements of the markers, so a correlation can be established between input and output.

2.1 Introduction

We would like to measure the strain distribution in the artery wall. However, as mentioned in the introduction, the strain distribution cannot be measured directly and has to be calculated from the displacement field. The displacement field can be measured. Therefore, markers are attached to the model. The model is placed inside a fluid-filled container. Optical registration of the markers is established using a videocamera and a digital framegrabber. Furthermore, the data are reconstructed to three-dimensional coordinates. A simulation is carried out for this reconstruction. The imageprocessing and the dataprocessing are described in separate chapters.

2.2 The Experimental Setup

2.2.1 The Aorta Model

The model of the aorta which was made in our lab is available. It is a transparent, bent tube with approximately the same dimensions as the natural aorta. The material used is EPDM rubber (ethylene-propylene-di-monomer). After cross-linking in an oven it changes into a polymer. It was made according to [Rutten, 1995]. Because EPDM rubber is transparent, it is possible to place markers all around the surface and still be able to see the markers through the tube. The placement of the markers is described in more detail in section 2.3.

The curvature radius of the aorta model is 62.5 [mm]. The radius of the flow area is 9 [mm] and the outer radius is 9.4 [mm]. The natural dimensions of the aorta come up to approximately an inner radius of 12.5 mm and a curvature of 50 [mm] [van Dongen, 1995].

2.2.2 The Fluid

The aorta model, hereafter called 'the tube', is placed into a rectangular fluid filled container. For a clear visibility of all markers, especially the ones at the bottom of the tube, the refraction index of the fluid should match the refraction index of the EPDM rubber. The refractive index of the rubber is 1.49 [-]. A fluid which can reach this refraction index is a solution of KSCN (Kaliumthiocyanate). It seems the EPDM rubber and the markers are resistant to the KSCN solution, so this should give no problem. However, the KSCN solution is very dangerous to one's health only when breathing it in or in the case of skin contact. Therefore it is decided to use water in the flow circulation and in the container. A disadvantage for the use of water is that its refractive index does not match the refractive index of the EPDM rubber, so the markers at the bottom of the tube are seen in a slightly different position. However, this refraction is the same for all moments in time.
2.2.3 The Camera

A camera is positioned right above the tube, so deformation can be visualized on a television screen. The depth of field of this camera has to be approximately 10 [cm]. The minimum height of the camera above the experimental setup thus is 1.3 [m]. The camera is attached to the ceiling. The ratio of width and height of the cameradisplay is 1 : 1.3. This influences the way in which the camera is positioned; the ratio of the area to be measured is 1 : 1.36. In order to obtain maximum contrast, halogen lights are used. The lights are placed just above the deforming tube, in such a way that reflections do not occur. The lamp power is 1300 [Watt], so a lot of heat is produced by the lights. Therefore, the lights are powered long before the measurements take place so there will be no thermal fluctuations during the measurements.

2.2.4 Visibility of the deforming Tube

A mirror is placed in the inner area confined by the tube (see figures 2.2, 2.1, 2.3). In this way two projections of the markers can be seen at the same time, so displacement of the markers in two directions can be measured.

Two conflicting factors can be identified concerning the placement of the mirror. In the first place, the area to be measured has to cover all markers in order to measure displacement of all markers. This means the area has to be as large as possible. Secondly, the resolution of the images has to be as high as possible. This means the area to be measured has to be as small as possible. It appears the resolution has the most effect on an accurate determination of the coordinates of a marker (see section 2.3.1). It is decided to place the mirror in the inner area of the tube because the area to be measured can be as small as possible in this way, thus increasing the resolution. A disadvantage for this position of the mirror is that not all markers are visible in one frame in two projections. This is because the mirror has a maximum size, confined by the tube. The area to be measured is 11 x 15 [cm]. The camera is positioned in such a way that this area fits into the cameradisplay. The direction in which the tube moves is parallel to the shortest side of the cameradisplay, so the resolution in this direction is the highest because the images are digitized in 512 pixels for both directions. The shortest side will then have the highest resolution.

Only four cross-sections can be seen in both views, but the accuracy of the measurements of these displacements is maximized because of a high resolution. Only for these four cross-sections full 3d reconstruction is possible (see figure 2.1).

![Figure 2.1: Distribution of the cross-sections over the surface of the tube](image)

Little ripples appear at the surface of the water because of the instationary flow in the tube. The tube will expand and shrink with the instationary flow, pushing the water away. The ripples would distort the optical projection in the camera, so a good determination of the position of the markers could not be
established. In order to avoid optical distortion, a thick glass plate is placed at the surface of the area to be measured. The water in the rectangular container can still expand, but only outside the area which is measured (see figure 2.2). The ripples now only occur outside the area which is visible for the camera.

![Diagram](image)

**Figure 2.2:** *The experimental setup. Expanding of the water due to instationary flow is outside the area which is measured.*

### 2.2.5 The Referencebox

The goal is to reconstruct the real world coordinates of the objectmarkers from the two 2-dimensional projections. In order to do so, a reference is needed relative to which the objectcoordinates can be reconstructed. For total reconstruction, a minimum of 6 reference markers is needed. Generally, the **number of reference markers** is taken larger (12-20) in order to improve the accuracy by statistical averaging [Muitjens, 1995]. Here, it is chosen to use 6 reference markers. However, these are chosen larger than the objectmarkers so the centrepoints can be calculated more accurately. The reference markers are attached to a reference box and this is placed into the fluidfilled container with the tube in it. The reference markers surround the object markers as in figures 2.1 and 2.3. For more information on 3d-reconstruction, see section (4.5). The background material of the reference box is a clear white surface. Because the markers are black, maximum contrast is reached.

### 2.2.6 Design of the Flow Circuit

A stationary pump is used in the flow system to obtain a stationary flow and pressure in the tube. A dynamic flow is superimposed on this stationary flow, in order to simulate in-vivo blood flow. The dynamic flow has the same qualitative properties of in-vivo blood flow. In the experiments, only a qualitatively correct physiologically flow and pressure are established, i.e. the shape of the input flow is the same as the shape of the in-vivo blood flow at the entrance of the aorta. The dynamic pump is controlled by a computer.

A long, straight rigid tube is used to ensure a fully developed input flow. At the exit of the tube, a terminal impedance is mounted to ensure a good match between flow and pressure.
2.2.7 Measuring Devices

Both a flowmeter and a pressure catheter are used in the flowsystem. The measurements of pressure and flow are triggered in phase with the measurements of the displacements of the markers. Two computers are used in the measurements. One computer controls the dynamic part of the flow input, measures both pressure and flow, and triggers another computer. This other computer starts digitizing images from the camera when triggered. In this way, temporal correlation can be established between pressure, flow and displacements at any point in time. For further details on measuring of displacements, see chapter 3.

2.3 Markers

In order to measure deformation of the tube, markers are attached to the outer surface of the tube. The size, position and the way in which the markers are attached have to be determined.

2.3.1 Size

The images of the tube are digitized. This means that the image is transformed into a digital representation of reality. The image is described with pixels (picture elements). The edge of a round object such as a marker will then have a stairlike appearance. When digitizing a static object, the digitized image will not always have pixels at exactly the same positions in all images, but this changes due to statistic variations. The precision with which the position of a marker can be calculated depends on both the markersize as well as the resolution of the image processing system. These influences are described by formulas (2.1) [Peters, 1987].

\[
\sigma_x^2 = \frac{S_x^2 S_y}{22d} \quad \text{and} \quad \sigma_y^2 = \frac{S_x S_y^2}{22d}
\]  

(2.1)

In these formulas \(d\) represents the size of the marker in millimetres. The parameters \(S_x\) and \(S_y\) respectively represent the width and the height in millimetres of a pixel. These parameters are generally not equal to each other because the camera display is usually not square. In this case, the ratio of width and height is 1:1.3. The resolution is far more important than the markersize because the former influences the accuracy of the measurement to the cube, the latter is only linear. Considering the fact that the size of the screen is 512 pixels square and the accuracy with which the positions of the markers have to measured is approximately 0.1 [mm], a size of the markers and a size for the area to be measured can be determined. The area which has to be seen in one frame is approximately 100 [mm] square. When the size of the markers is 2 [mm], the real coordinate will be with 99 % certainty within 0.05 [mm] from the calculated coordinate. This will be accurate enough for the measurements.

2.3.2 Position of the Markers

The markers are distributed over the surface of the tube in such a way that cross-sections can be reconstructed at various axial positions (figure 2.1). To reconstruct an ellipse at a cross-section, a minimum of 4 markers is needed at each axial position.

To avoid the problem of markers overlapping with each other in one of the views, the markers at one axial position are distributed as in figure 2.3. In this way, the distance between each markerprojection in both views is equal. The angle \(\vartheta\) follows from \((\cos \vartheta - \sin \vartheta = 2 \sin \vartheta) \Rightarrow \vartheta = \arctan(1/3) = 18.43\)°.

2.3.3 Marker Processing

The markers have to be positioned over the surface of the EPDM tube rather accurately. It is difficult to do so because the tube is very soft and deforms when manipulating it. In order to place the markers accurately, the tube is placed over another, -rigid-, tube. This other tube is in this case a PVC tube, which was shaped into a bend with the same dimensions as the EPDM tube by heating it above its glass temperature with a hot air gun. Next, the positions of the markers are pre-marked at the surface of the PVC tube. This is done with the aid of a specially designed instrument (figure 2.4).

When the dots are placed on the PVC tube, the EPDM tube is fitted over the PVC tube. Because the EPDM tube is transparent, the pre-positioned dots on the PVC tube are visible. The markers only have to be glued at the EPDM tube over the positions of the pre-marked dots. When the glue has cured, the EPDM tube is removed from the PVC tube. Soap is used to ease the removal.
The markers used in this experiment are polypropylene (PP) spheres with a density of \(0.9 \cdot 10^3 \text{[kg} \cdot \text{m}^{-3}]\). PP is the lightest plastic material available so the influence of the marker mass on the tube behaviour is minimised because the tube is in a container filled with water. In order to obtain maximum contrast, the markers are blackened with a waterproof pen. The markers are glued on the EPDM rubber with a waterbased glue. Because PP does not have any polar groups in the molecule, it is not affective to water. The markers thus are not attached optimally well to the tube. So, a non-waterbased glue is recommended instead of a waterbased glue.
Chapter 3

Image Processing

An image processing system is used to digitize camera images of the deforming tube and write
the digitized images to disk in real-time. The images are processed in a later stage because
of high computer times. The coordinates of the object markers and reference markers are
calculated and stored in different files for later processing in MATLAB.

3.1 Framegrabbing

3.1.1 Real Time Framegrabbing

The image processing system used here is TIM from DIFA measuring systems. This system consists
of a framegrabber, which is a piece of hardware, and software, with which the digitized images can be
processed. In the measurements, 25 images per second are produced by the camera. These are digitized
and written to disk in real-time. In an earlier version of TIM real time image-acquiring was not yet
possible. The images first had to be recorded on a videorecorder and in a later stage they had to be
digitized one at a time from the videotape. This resulted in loss of quality from recording and replaying
by the videorecorder. In the present version of TIM it is possible to digitize images in the computer
and write the data to disk or to memory in real-time. In this case, 25 images per second have to be
digitized. Each image contains 512 * 512 pixels (pixel elements), so this is 256 kB per image. The
resulting overall dataflow is 25 times 256 kB is 6,25 Mb per second. In order to attain this high speed
data acquisition, real-time framegrabbing is used in combination with Direct Memory Access (DMA).
Direct Memory Access means that data is transported over the databus from the framegrabber to the
RAM memory or disk without interference of the CPU (Central Processing Unit) registers. The DMA
controller is temporarily busmaster instead of the CPU ([Kopinga, 1993]). In this case, DMA takes place
in the burstmode, so a minimal overhead from the CPU is reached. Direct Memory Access in TIM can
be done either to harddisk or to RAM memory. When using DMA access to disk a lot of diskspace is
available (4 Gb in this case), but so called tiff-pictures are produced which need more work to process.
When using DMA access to RAM memory, a relative small amount of memory is available (32 Mb in this
case), but the processing can be done more simple. In this case, DMA access to disk is used because of
the great amount of data. The measurements last for 10 seconds, so 62,5 Mb of data is produced. The
RAM memory in the computer is 32 Mb, of which only 8 Mb can be addressed in Windows 3.1 for DMA
use. A digitized image looks like in figure (3.1.2).

3.1.2 Interlacing

The camera produces 25 images per second, so each image is produced in 0,04 sec. However, the way in
which the images are produced by the camera induces extra possibilities for the image processing. This
is explained here. The camera has a light sensitive chip. This chip is scanned one line at a time and the
electronic signal is immediately sent to an output device as an analog signal. In the first 0,02 seconds
all odd lines from the camerachip are scanned. These lines are sent to the output device while scanning.
During the next 0,02 seconds all even lines are scanned. These are also immediately sent to the output
device. The situation is that the camera produces an analog signal, of which the first 0,02 seconds of a
period represent the odd lines and the next 0,02 seconds represent the even lines. When the outputdevice
would be a television set, first the odd lines are projected on the screen and next the even lines. When the
even lines are projected, the odd lines still glow. For the human eye a fluently moving result is projected. In this case however, the output device is a digital framegrabber which grabs the images at 25 Hz. The grabbed image thus contains the odd and even lines superimposed to each other. The object was moving during the time that an image was grabbed, so an object has two slightly different positions superimposed to each other. In the image processing, advantage is taken from the interlacing. After digitizing, the odd and even lines are split into an 'odd' and an 'even image' and these are processed after each other because in fact they contain information which occurred after each other. When the odd and even lines are split into two different images, black lines arise between the actual image lines. The black lines between the lines in the odd and even images are filled with the line closest to that line. The result is a quasi 50 Hz digitizing.

![Digitized image before and after preprocessing](image)

**Figure 3.1:** A digitized image before and after preprocessing

### 3.2 Preprocessing

Each pixel in a grabbed image has a particular grey value. This value lies between 0 (black) and 255 (white), so for each grey value 8 bits are used \(2^8 = 256\). It is advantageous to establish a high contrast in the images in order to separate between objects and the background. This is reached by using high power halogen light (see section 2.2.3). In this case, markers in a digitized image have a grey value close to 70 and the background is more or less white and has a grey value higher than 180. After all images have been digitized and written to disk, they are preprocessed one at a time. Preprocessing means that the image is changed by various image processing routines. For instance, it is possible to change colours, or change shape and size of objects in an image or recognize patterns.

In the following only the preprocessing routines of thresholding, dilatation, and erosion are described. When thresholding, all grey values below the threshold value will turn black (grey value 0) and all grey values above the threshold value will turn white (grey value 255). In this way it is possible to separate the objects from the background. After thresholding, a so called BITmap is produced, only one bit is needed for each grey value. This BITmap now has to be inverted for further preprocessing. When an image is thresholded and inverted, the markers are white and the background is black. At some places in the image little isolated groups of white pixels occur. Also discontinuities are present at the edges of the markers. These imperfections are due to contrast, intransparency of the tube, shadows etc. These redundant pixels can partly be removed by using erosion and dilatation. Erosion is a preprocessing routine which removes a layer of pixels around each white object. In this way, little objects disappear. The edges of the marker become smoother. A circumstance is that all objects shrink. This can be corrected by using dilatation. Dilatation is a preprocessing routine which dilatates an object, i.e. a layer of pixels grows around each white object. In this way, all objects have the original size and shape but the irregularities have vanished. After preprocessing, the image looks like in figure (3.1.2). Many more preprocessing routines are available in TIM but these are not discussed here.
3.3 Dataprocessing

After preprocessing, BITmaps are available in which white objects are present in a black background. The coordinates of the objects now have to be calculated. The coordinates of the reference markers are separated from the coordinates of the object markers and stored in different files.

3.3.1 Initialization

In order to separate the reference markers from the object markers, distinct has to be made between these. In the first image of a sequence of images, the user has to define which objects on the screen are reference markers and which objects are in fact object markers. Only in the first image of the sequence of images this has to be done. During initialization, the coordinates of the object markers are added to the first line in the object markers coordinate file and the coordinates of the reference markers are added to the first line of the reference markers coordinate file. In this way, the user is assured that the first line in a coordinate file contains all and only the right coordinates. The processing of all consequent images is automatically. The objects slightly change in position but the software is able to recognize them as being the same marker defined by the user in the first image. This is called a marker tracking routine, which is discussed in section 3.3.4.

3.3.2 Calculation of Coordinates

An object marker consists of approximately 80 pixels and a reference marker consists of approximately 320 pixels. The marker centrepoint represents the mean coordinate of all pixels belonging to that marker. In this way, the centrepoint is calculated as somewhere in between a number of pixels. Due to digitalization, the weighted centrepoint of all pixels differs from the real marker centrepoint. The difference has a random distribution, of which the properties can be determined (see section 2.3.1).

3.3.3 Coordinate files

For both the reference markers as well as for the object markers a separate coordinate file is opened. As mentioned, during initialisation the first lines of these files are already filled with the coordinates of the markers from the first image. The benefit of this method is that only and all object- and reference markers are taken into account. When processing all consequent images, the routine calculates the coordinates of an object, and then determines which coordinates from the preceding image fit best to these coordinates. It is at this place in the coordinate file the new coordinates are put. Each coordinate file contains a matrix in which each row contains the coordinates from one image. The coordinates are placed like x1 y1 x2 y2 x3 y3 etc. A column contains the coordinates of the same marker for each image.

3.3.4 Further Processing

In an early version of the processing routine, distinct between reference markers and object markers was made on the basis of the number of pixels. During initialisation, two intervals were defined by the initialisation routine. Each interval defined a number of pixels between which the markers were found. It soon appeared this algorithm did not answer to the processing conditions because of two reasons. The first reason is that the sequence in which the coordinates were added to the coordinate files sometimes changed. This was due to the way in which TIM scans an image: from top left, one line at a time, to bottom right. The sequence in which an object is encountered can change because the objects are moving. This was corrected later in a MATLAB routine, so this was not a grave problem. The second reason is that some objects in the image which were neither object- nor reference markers were seen as one of those because they accidently had a number of pixels which lied in one of the intervals. One of the consequences was that one or more of the last markers were discarded because all positions in the row were already taken by the non-markers.

The solution to this problem was found in distinguishing between object- and reference markers on the basis of coordinates of the marker rather than on the basis of the number of pixels. In this case, the objects do not move a lot between two images, so this is well possible. The maximum displacement during one period is 3 pixels. The displacement between two images thus approximately is 0.02 times 3 pixels is 0.06 pixels.
Chapter 4

Data Processing

4.1 Format of the Data

As mentioned in section 3.3.4, the TIM routine automatically processes the images and writes the coordinates of object markers and reference markers to two different files. The files are formatted in such a way that they can be processed in MATLAB. A file contains as many rows as there are images processed and two times as much columns as there were markers in an image, i.e. for each marker an X and a Y coordinate.

\begin{verbatim}
first image  x1 y1 x2 y2 x3 y3 . . . . xn yn
second image x1 y1 x2 y2 x3 y3 . . . . xn yn
third image  x1 y1 x2 y2 x3 y3 . . . . xn yn
\end{verbatim}

Two problems arise when processing these data: The first problem is that stereocorrespondence within one image has to be established between the two projections of a marker in one image. Each marker produces two projections, but the coordinates of these corresponding projections are not placed next to each other in the coordinate matrices. Corresponding coordinates in a row have to be placed next to each other. The second problem that arises is that time correspondence of one marker in consequent images has to be established. A marker changes slightly in position between consequent images. Therefore, difficulty arises when tracking a marker in time. This problem is already solved in the TIM routine (see section 3.3.4).

4.2 Stereocorrespondence

The position of a marker is defined by three coordinates. A stereoprojection of a marker provides 4 coordinates. So, each marker provides redundant information, which commonly is used to pair markers [Muijtjens, 1995]. However, this is not done here because of the small number of markers. In this case, stereocorrespondence is established by the user.

Stereocorrespondence in one image is interactively established in a MATLAB routine. An overview of all markers appears on the screen. The user has to define only in the first image which markers are stereocorrespondent to each other by clicking the mouse on the screen and the program automatically sorts out stereocorrespondence for all consequent images. On the screen, the corresponding projections are numbered the same.

While running this procedure, the matrix with coordinates is manipulated in such a way that for each set of stereocorrespondent marker projections the 4 columns with the x1 y1 x2 y2 coordinates of both projections are placed next to each other. These coordinates are still pixel coordinates in x and y direction for the above projection, and the x and y direction of the stereocorrespondent mirror projection of which the y movement in fact represents the z direction.

4.3 Temporal Correspondence

As mentioned in section 3.3.4, the early version of the TIM routine did not establish correspondence in time for the same object in consequent images. In order to establish this correspondence in time, a
MATLAB routine was developed which automatically sorted out marker coordinates in time on the basis of least distance from a marker found in the previous image. In a later version of the TIM routine a marker tracking routine is implemented, in a way that TIM recognizes a set of coordinates as belonging to a set of coordinates in the previous image on the basis of least distance. This is described in detail in section 3.3.4. Anyway, the result is that in the coordinatematrices a column contains the coordinates of one marker moving in time.

4.4 Ellipse Fitting

As already mentioned in chapter 2.3.2, cross-sections will be reconstructed at four axial positions in the tube. At each axial position, four markers are distributed over the surface of the tube. It is possible to fit an ellipse through four marker points. In order to do so, a plane is fitted through the markers at one cross-section. The coordinates of the four markers are projected into this plane. Next, the plane is rotated until it is perpendicular to the x-axis. In this way, only 2 coordinates define the position of each marker in the plane and now it is possible to fit an ellipse. After having fitted the ellipse, the plane is rotated back to the original position. The main axes of the ellipse are taken parallel and perpendicular to the main axes defined in the coordinatesystem in figure 4.5.2. It is possible that the axes of the ellipse have a slight rotation relative to this position, but when the rotation of the main axes of the ellipse has to be calculated as well, 5 coordinates at one cross-section should be needed. This was not possible in practice due to overlapping of markers in the two projections and besides, it is not expected that the main axes do not rotate much because of symmetry.

The equation for an ellipse is as follows:

\[
a(x_i - x_0)^2 + (y_i - y_0)^2 = R^2
\]

\[
a(x_i^2 - 2x_0x_i + x_0^2) + (y_i^2 - 2y_0y_i + y_0^2) = R^2; \text{ for all } i \in \{1, 2, 3, 4\}
\]

Actually, the above equations each represent four equations, one for all four markers at one cross-section. The x- and y coordinates in these equations are the two-dimensional projection coordinates in the plane fitted through the four three-dimensional coordinates.

\[
\begin{pmatrix}
    x_1^2 & -2x_0 & -2y_1 & 1 \\
    x_2^2 & -2x_0 & -2y_2 & 1 \\
    x_3^2 & -2x_0 & -2y_3 & 1 \\
    x_4^2 & -2x_0 & -2y_4 & 1
\end{pmatrix}
\begin{pmatrix}
a \\
anx_0 \\
y_0 \\
ax_0^2 + y_0^2 - r^2
\end{pmatrix}
\]

or \( \mathbf{A} \cdot \lambda = \mathbf{b} \)

The solution to this set of equations is given by:

\[
\lambda = \mathbf{A}^{-1} \cdot \mathbf{b}
\]

where

\[
\begin{cases}
a &= \lambda(1) \\
x_0 &= \lambda(2) \cdot \lambda^{-1}(1) \\
y_0 &= \lambda(3) \\
R^2 &= -\lambda(4) + \lambda^2(2) \cdot \lambda^{-1}(1) + \lambda^2(3)
\end{cases}
\]

4.5 Three Dimensional Reconstruction

As mentioned before, a matrix is now available with in each row the x1 y1 x2 y2 pixelcoordinates of the two projections of each object marker in one frame next to each other. Also a matrix is available with the pixel coordinates of the reference markers and a matrix with the real world coordinates of the reference markers. The coordinates of the referencemarkers are used to retrieve the real world coordinates of the objectmarkers.
4.5.1 Reconstruction Methods

The viewing projection of a 3D-point to the image plane is often described as a linear transformation, using a $3 \times 4$ matrix of 11 unknown elements and one element set equal to one [Faugeras, 1993], [Peters, 1987]. Per view, a reference marker provides 2 constraining equations, hence a minimum of 6 reference markers is needed. Also, specialized models are developed, such as the Maximum Likelihood (ML) method. This model does not use a reference object and the accuracy of the object reconstruction is improved [Muijtjens, 1995].

Here, another method is described to reconstruct the 3D-coordinates of the markers, the Line Reconstruction Method.

4.5.2 Line Reconstruction Method

Some assumptions are made here for the transformation from 2D-pixel coordinates to real world 3D-coordinates.

The first assumption is that the planes defined by the lower reference markers and by the back reference markers are respectively perpendicular and parallel to the optical axis of the camera system. The next assumption is that the decrease in the number of pixels per mm for an object is linearly with the distance from the camera. Also, the water surface and the glass plate do not influence the relative positions of the marker in the camera projection [Deurhof]. The coordinate system is defined as in figure 4.5.2. Both the entrance as well as the outlet of the bent tube are in the $xz$-plane, and the axis of the tube lies in the $xy$-plane.

From the above projection of a marker, a line can be matched through the coordinates in the top $(x_t, y_t, z_t)$ and in the bottom $(x_b, y_b, z_b)$ plane defined by the reference markers. This can also be done for the front $(x_f, y_f, z_f)$ and the rear $(x_r, y_r, z_r)$ plane. The coordinates are defined as pixel coordinates relative to the reference marker in $(0,0,0)$ defined in figure 4.5.2. Where these two lines intersect, the 3D coordinates of the marker lie. The equations for the two lines are as follows:

$$l_1 = \begin{pmatrix} x_f \\ y_f \\ z_f \end{pmatrix} + \lambda_1 \begin{pmatrix} x_r - x_f \\ y_r - y_f \\ z_r - z_f \end{pmatrix}$$  \hspace{1cm} $$l_2 = \begin{pmatrix} x_t \\ y_t \\ z_t \end{pmatrix} + \lambda_2 \begin{pmatrix} x_b - x_t \\ y_b - y_t \\ z_b - z_t \end{pmatrix}$$

Figure 4.1: The coordinate system for the real world coordinates is defined relative to one of the reference markers.
The above equations have to be equal to each other:

\[
\begin{pmatrix}
    x_f \\
    y_f \\
    z_f
\end{pmatrix}
+ \lambda_1 \begin{pmatrix}
    x_r - x_f \\
    y_r - y_f \\
    z_r - z_f
\end{pmatrix}
= \begin{pmatrix}
    x_t \\
    y_t \\
    z_t
\end{pmatrix}
+ \lambda_2 \begin{pmatrix}
    x_b - x_t \\
    y_b - y_t \\
    z_b - z_t
\end{pmatrix}
\]

This can be written as:

\[
\begin{pmatrix}
    x_r - x_f & x_t - x_b \\
    y_r - y_f & y_t - y_b \\
    z_r - z_f & z_t - z_b
\end{pmatrix}
\begin{pmatrix}
    \lambda_1 \\
    \lambda_2
\end{pmatrix}
= \begin{pmatrix}
    x_t - x_f \\
    y_t - y_f \\
    z_t - z_f
\end{pmatrix}
\quad \text{or} \quad \Delta \cdot \lambda = b
\]

This equation is overdetermined, i.e. there is no exact solution because both the parameters \( \lambda_1 \) and \( \lambda_2 \) have to agree with all three equations. The solution to this equation is given by the Minimum Least Square Solution:

\[
\lambda = (\Delta' \cdot \Delta)^{-1} \cdot \Delta' \cdot b
\]

Figure 4.2: The model to reconstruct the 3-dimensional coordinates from the two 2-dimensional projections.

4.6 Simulation

A simulation of the Line Reconstruction Method is implemented in MATLAB. For virtual object points distributed all over the volume where the object markers attached to the tube could possibly be, the projections in the boundary planes from the reference box are calculated using geometrical optics. Then a random signal is added. This random signal represents the disturbance introduced by the digitalization. When using the Line Reconstruction Method in the simulation, the difference between the calculated three dimensional coordinates and the coordinates of the virtual object points is in the order of the disturbance.
introduced by the digitalization. This means that the Line Reconstruction Method is a good method to reconstruct three dimensional coordinates from two two-dimensional projections when using a square reference box and the right conditions.

**Figure 4.3:** Results of the simulation which is carried out. At each virtual point in the object space a random signal is added 100 times. The mean difference between the real coordinate and the reconstructed coordinate is plotted in the left three plots for all virtual points. In the right three plots, the standard deviation of all 100 simulations at each virtual point is plotted.
Chapter 5

Results

The displacements of the markers appear to be in the order of maximally 3 pixels. This corresponds to a displacement of approximately 0.6 [mm]. The results are displayed in two different ways. The x, y and z positions are displayed separately as a function of time. A quantitative impression of the displacements can be achieved. Also, the displacements of the markers are displayed threedimensionally. In this way, a qualitative impression is given. Here, not all experimental results are presented. The software contains all measurements. Compressive strains are present at the inner curve and extensional strains at the outer curve, as expected.

5.1 Cross-section Reconstruction

As is mentioned in section (2.3), the cross-sections at four axial positions can be reconstructed. The points in the inner and outer curve of the bend can be interpolated from the ellipse which is fitted through the four markers at each cross-section. In figure 5.1, the reconstruction of the ellipses as well as the bend radius can be seen, as well as the position of the markers and the reconstruction of the inner and outer points.

Figure 5.1: The reconstruction of the ellipses at four cross-sections in order to obtain the interpolated points at the inner and outer curvature
5.2 Static Deformation

The flowmeter and the pressure catheter are calibrated first according to a standard procedure. The positions of the markers are then reconstructed for several stationary values of the pressure and the flow. The cross-sections of the tube grow when the pressure and flow rise. This can be seen in figures 5.2 and 5.3.

![Flow and pressure during static measurements](image)

**Figure 5.2:** *The cross-section rises non-linearly with the static pressure*

In the figure, the distance between the markers in the inner and outer curvature of the tube is called the long diameter. The distance between the marker on top and the marker at the bottom is called the short distance. The difference in diameter is caused by the flow, which pushes the tube to the outside.

![Static displacement of the four marker points at the first cross-section](image)

**Figure 5.3:** *The static displacement of the four marker points at the first cross-section*
5.3 Pressure and Flow

During the measurements, flow and pressure are measured in phase with the measurements of the positions of the markers (see section 2.2.7 for more details). The measured flow and pressure are shown in figure (5.4).

![Pressure and flow plots](image)

**Figure 5.4:** Pressure and flow during the measurements of positions. Notice the phasendifference

The flow and pressure, which can be adjusted by changing the stationary or instationary part of the input flow or the output impedance of the flowsystem, do not have physiologically correct values in this case. However, this is not important because the measurements can though be compared to a numerical model. The pressure is measured at the outlet of the tube, and the flow is measured at the input of the tube. The result is a difference in phase.

5.4 Dynamic Deformation

5.4.1 Two Dimensional Representation

The displacement of each marker can be visualized in the computer program. Here, this is done for only one marker. For all three directions a plot is presented for the displacements (figure 5.5 & figure 5.4.1).
Figure 5.5: The displacement in each direction for one marker for 8 periods in time. Each second 50 measurements were taken, so each period lasts for one second.

Also, for each displacement, the power spectrum is calculated. The power spectrum gives which frequencies are present in the measurements of the displacements. In figure 5.4.1, the powerspectrum for the above displacements can be seen.

Figure 5.6: Only one period of the x, y and z displacements from figure 5.5 at the left and the Power Spectrum of each of these displacements at the right.

In the z movement of this marker all frequency components except the frequency of 3 [Hz] seem to have vanished.
5.4.2 Cross-sections

In figure 5.7, the length of the main axes at the four cross-sections can be seen as a function of time for one period. As is to be seen, the mean length of the long axis decreases with each following cross-section, while the short axes approximately remains at a constant mean length. The flow-areas are calculated with \( A = d_l \times d_s \times \pi \) and presented in figure 5.8. It seems the mean flow area decreases with each following cross-section. This is possibly due to visco-elastic effects and inhomogenities of the tube. Also the stiffness of the tube may increase at each following cross-section.

![Figure 5.7: The relation between diameter, time and number of cross-section](image)

![Figure 5.8: Mean flow-area decreases due to inhomogenities in the tube](image)

5.4.3 Strains

The axial strains in the inner curvature and in the outer curvature are calculated. Also, the radial strains for the main axes of the reconstructed ellipses are calculated. As was the hypothesis, compression should
be present at the inner curvature and extension at the outer curvature. This can be seen in figures (5.9 and 5.10) at the next pages.

![Figure 5.9: The radial strain at the four cross-sections superimposed to each other](image)

![Figure 5.10: Axial strain at the inner and outer curvature](image)

### 5.4.4 Three-Dimensional Representation

The displacements of the markers can also be represented in three dimensions. This way of presenting does not always give a clear survey, but it can give some more insight into the movement of the markers (figure 5.11 & figure 5.12).
Figure 5.11: A three-dimensional reconstruction from the object markers, as seen from above, with the displacements 15 times enlarged.

Figure 5.12: A three-dimensional overview of the markers attached to the tube.
Chapter 6

Conclusion

The method to describe the three-dimensional reconstruction seems to work fine, but should be validated more because it is not clear what derivation from the real three-dimensional coordinates is introduced by this method. However, the results can be used to get a qualitative impression of the movement of the tube and the strains in the tube.

According to the measurements, compressive strains are present at the inner curve of the tube, and extensional strains at the outer curve.

The digitalization method using TIM works fine, and the error introduced by digitalization is very small when using high resolutions. Using a mirror is a good method to measure three-dimensional displacement of transparent objects such as the tube used in this experiment.

The diameter of each following cross-section seems to decrease in the direction of the flow. This can be due to various reasons, such as the reconstruction method or inhomogenities in the tube, where the stiffness may increase with each following cross-section.
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